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Innovation and Patentability Requirement in a Globalized World

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Abstract

One important element of a patent regime is the non-obviousness requirement, which captures the minimum improvement to the best patented technology a new invention is required to make in order for it to be patentable. We call it the required inventive step. We explore the implications of a patenting regime based on required inventive step, by incorporating such intellectual property protection considerations in the quality-improvement model of technology, trade and growth developed by Eaton and Kortum (European Economic Review 2001). In considering whether to increase the inventive step, policy-makers trade off the benefits of a higher rate of innovation against the loss of consumer surplus as patent-holders enjoy their monopoly pricing power for a longer duration on average. We first establish that there exists an optimal binding inventive step that maximizes country welfare under certain reasonable conditions. We then proceed to formulate an open-economy model, in which countries interact through cross-border trade and firms patent internationally. We focus on the central case that domestic goods and foreign goods are highly substitutable with each other so as to contrast with earlier work of Grossman and Lai (American Economic Review 2004), who analyzed international IPR issues under an alternative regime based on patent length protection and product innovation. We find that there exists a stable Nash equilibrium of the inventive-step-setting game between national governments that features over-protection of intellectual property from a global perspective. Moreover, country governments set higher inventive steps in the open-economy relative to the closed-economy benchmark. We discuss how these results relate to Grossman and Lai (2004). Of note, globalization in the form of reduced trade frictions leads countries to raise their equilibrium inventive steps. Consequently, globalization leads to higher equilibrium research intensities in all countries.

Keywords: Intellectual property rights protection, Non-obviousness, Patentability, Required inventive step

JEL Classification: O31, O34, F13

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1 Introduction and Motivation

As globalization deepens, the protection of intellectual property has become increasingly important to developed countries. This is because the law of comparative advantage has steered developed countries towards specialization in R&D- and skill-intensive products, the profits from which are usually sensitive to intellectual property rights (IPR) protection. Not surprisingly, this has created political pressures on developed country governments to push for stronger IPR protection from less developed countries. A major step in this direction was the signing of the TRIPS agreement in 1994, which sought to provide a set of universal minimum standards for IPR protection and facilitate coordination among countries in this area. It is widely recognized that TRIPS required developing countries (the South) to substantially strengthen their IPR standards to catch up with those of the developed world (the North).

Many studies suggest that the North would gain, while the South lose, from a strengthening of IPR standards as a result of a TRIPS-like agreement. Nevertheless, would the world as a whole gain from better IPR protection in the South? Is there any theoretical basis for the view that international coordination in the protection of IPR can improve global welfare? Grossman and Lai (2004) shed light on this issue by modeling the international protection of IPR in a setting where innovation expands the available variety of differentiated products, and where newly-patented products are legally protected from imitation for a specified duration termed the patent length. A key result in their paper is that a country with both a larger domestic market for IPR-sensitive goods and a higher innovative capability would implement stronger IPR protection in equilibrium. Moreover, they identify the existence of a positive cross-border externality from raising protection, and hence conclude that there is global under-protection of IPR in Nash equilibrium when country governments set their (enforcement-adjusted) patent lengths non-cooperatively to maximize their own national welfare.

In practice however, patent protection is accorded in more ways than through the patent length. Whereas "patent length" captures the duration of monopoly power granted by law, we focus instead in this paper on an important alternative mode of IPR protection, which we term the "required inventive step": Many attempts at innovation often seek to build upon and improve existing products, rather than to engineer a radically new item. To serve its purpose of encouraging innovation, it is therefore also imperative for the IPR regime to protect patent-holders from incremental innovation that would compete away their profits too easily. To this end, patent laws often include clauses — called "non-obviousness" requirements in the US patent code – that disqualify an innovator from obtaining a new patent on the basis of minor or trivial changes to existing patents. The concept of "required inventive step" therefore refers to how much improvement the patent regime requires an innovator to make over the best patented technology before the new invention is eligible for a patent. In order to focus on the effects of patentability, we assume that any invention satisfying the inventive step requirement will not infringe on any existing patents. In other words, the "leading breadth" is sufficiently small that it is never greater than the required inventive step.[1] To the best of our knowledge, no work has yet been done to study the incentives for governments to increase required inventive steps in an open-economy setting, and the question remains to what extent the Grossman-Lai results would continue to hold when IPR protection takes this form. This paper presents a first attempt to fill this gap in the literature.

We develop a framework to analyze international IPR issues when protection is based on required inventive step, by incorporating such patenting considerations into Eaton and Kortum's (2001) benchmark model of technology, growth and trade. The underlying model of technology in this framework is the search-based model of innovation of Kortum (1997), in which research effort raises the cumulative stock of ideas, and innovation takes the form of new ideas that improve the quality of existing goods (rather than an expansion in the measure of varieties).[2] Here, the term quality is synonymous to the labor productivity of the production technique for the good, as is the common usage in the endogenous growth literature. In this setting, the natural mode of IPR protection by which to protect existing patent-holders would be to stipulate a minimum quality improvement (or inventive step) that any innovation would need to satisfy in order to be eligible for a new patent. We introduce these considerations into the model in the following way: If z is the quality (labor productivity) of the current best patented idea for a given variety, any innovation needs to be of quality at least Bz for it to be granted a new patent, where $B \geq 1$ is a country policy parameter. B therefore captures the required inventive step, along the quality dimension specifically, that is required under the country's IPR regime, since it is the minimum quality-improving step that an innovation must attain before it can be considered sufficiently non-obvious to be patentable.[3] In particular, if B is strictly greater than one, then the inventive step requirement has some real bite.

We show how to build this formulation of IPR protection based on inventive step into a fully-specified quality-improvement growth model, and use this to investigate how inventive step policy affects innovation and welfare in both closed and open economy settings. Of note, the stochastic treatment of the innovation process gives rise to a distribution of the best patented idea that is a function of the current cumulative stock of ideas and the inventive step policy parameter. The derivation of this patent frontier distribution is a key step that makes the entire model and the subsequent welfare analysis tractable. This patent

[1] In the formal patenting literature, a "non-obviousness" requirement captures how much improvement an invention has made relative to the prior art in order for it to be eligible for a new patent. On the other hand, "leading breadth" refers to how much a new innovation needs to better an existing patent before it can be considered not infringing on the latter's patent rights. (The innovation need not necessarily be seeking a patent, but the innovator would have to pay royalties to the existing patent-holder in order to market her innovation.) These two concepts are closely related but distinct. Since patenting is assumed to be costless in our model, any innovator whose invention satisfies the required inventive step will automatically patent her new idea.

[2] The model of Eaton and Kortum (2001) can be viewed as an extension of Kortum (1997) to a multi-country, open-economy setting.

[3] We assume that royalty payments are prohibitive, so that no innovator would consider infringing an existing patent. On the other hand, the cost of filing a patent is assumed to be negligible, while it is easy for competitors to imitate a production technique. This implies that any innovator would obtain a patent for her technology whenever and wherever it is patentable.

frontier is closely related to the concept of the technology frontier – the distribution of the most productive idea currently available – in the standard Eaton and Kortum framework without patenting. (We discuss this relationship at more length in Section 2.)

We first study the closed-economy version of our model, to obtain insights on the incentives for extending inventive step protection in this benchmark case. In the closed-economy setting, there is already a well-developed argument for the existence of an optimal patent length when innovation expands the available measure of varieties and IPR protection takes this alternative form (see for example, Nordhaus 1969, Tirole 1988, Grossman and Lai 2004): An increase in the patent length raises the incentives for and hence the rate of innovation, and this is traded off at the margin against the consumer surplus conceded to innovating firms.[4] We show that a similar existence result holds in our quality-based growth model, even though a more careful argument is now needed to establish that an increased inventive step will indeed raise the rate of innovation when innovation improves upon existing varieties rather than expanding the range of available products. This is because three distinct effects are now at play. First, for a given patent frontier, a higher inventive step B means that more resources need to be expended to obtain a new idea that surpasses the inventive step requirement, an effect which tends to lower the incentives for innovation. On the other hand, a higher B itself leads to a patent frontier distribution that reflects a lower average quality, since a larger margin of technologically-feasible ideas cannot be marketed; this actually increases the probability that a new idea can better the patent frontier by a given inventive step, and thus raises the incentives for innovation. In our model, it turns out that these first two effects offset each other exact, so that the probability that a new idea improves upon the patent frontier by a step of at least B is in fact independent of B. This leaves the third effect as the salient one, this being the direct role of a higher B in raising expected profits once a patent has been obtained, by allowing the patent-holder to charge a higher markup, while making it more difficult for competitors to leapfrog the patent.

In sum, the benefit of raising the inventive step is that it increases the overall incentives for innovation, so that consumers enjoy quality improvements at a faster rate. However, the cost of a higher B is that it confers patent-holders the ability to charge prices above marginal cost for a longer duration on average, until a better idea that surpasses the inventive step arrives. Our first key result shows that there exists a unique inventive step that resolves this tradeoff, and that this optimal B is binding ($B^* > 1$) under certain reasonable conditions. In other words, there is a scope for government intervention through the use of the inventive step as an instrument in IPR protection to improve country welfare.

Turning to the open-economy case, we now consider how cross-border interactions will affect each

[4] There have been several studies examining the optimal combination of patent breadth and patent length when the IPR regime features both forms of protection. For example, O'Donoghue, Scotchmer and Thisse (1998) study this in a quality-improvement model with some dynamic considerations, but theirs is not a fully specified dynamic growth model.

country's inventive step policies and equilibrium research intensities. We allow for international trade in goods, subject to the standard iceberg trade costs. Similarly, we allow for international patenting by firms, whereby firms always have incentives to obtain a patent in each potential destination country before they market there. We focus on the central case that domestic goods and foreign goods are differentiated but highly substitutable with each other so as to contrast with earlier work of Grossman and Lai (American Economic Review 2004), who analyzed international IPR issues under an alternative regime based on patent length protection and product innovation. We examine a world with national treatment in the granting of patents, so that countries do not discriminate across foreigners and award patents by the same rules to both their own nationals and foreigners alike. We focus our attention on the strategic interaction in the two-country case (for ease of tractability), when their governments set their respective inventive step policies non-cooperatively. We show that there exists a stable Nash equilibrium of this inventive step-setting game between national governments. Moreover, a formal analysis of the welfare properties of this equilibrium confirms that it features over-protection of IPR, in the sense that there is a scope for all countries to lower their inventive steps while achieving a Pareto improvement. This result is in sharp contrast with that of Grossman and Lai (2004), who find that there is under-protection of patent length in an expanding-varieties innovation model.

To understand the underlying intuition, it is important to grasp the nature of the cross-border externalities from a country's choice of inventive step. In this paper, the measure of varieties is fixed and innovation takes place along the quality dimension, and the natural concept of IPR protection is the inventive step. Firms from different countries compete directly with each other by selling goods that are highly substitutable with each other, and this direct competition gives rise to the negative cross-border externality. When Country 1 raises its inventive step, B_1, this raises incentive for Country 1's firms to invest in R&D and hence also the relative productivity position that Country 1 has in the differentiated goods sector. On the other hand, it discourages Country 2's firms from investing in R&D, as the profitability of research in country 2 now diminishes as the relative productivity position of Country 2's firms falls — as a result of the trade barriers and high substitutability between goods developed by Country-1 and Country-2. Overall, the technological progress enjoyed by consumers in Country 2 actually slows down, leading to lower welfare for their consumers. This is the source of the negative externality of Country 1's policy action on Country 2 welfare (Country 1 does not take this spillover into account when deciding, B_1). This underpins the over-protection result that we have derived regarding inventive step policy.

Grossman and Lai (2004) on the other hand, consider product innovation that expands the existing measure of varieties in the differentiated goods sector, in which the IPR protection instrument considered is the duration of the patent protection of each new variety. As a result, an increase in Country 1's patent length benefits both domestic and foreign firms by increasing the value of any global patent. This induces more product innovation from firms in all countries, even in the presence of trade barriers. The increased utility from the larger set of varieties that Country 2 can consume as well as increased profits for Country-2 firms selling in Country 1 generate a positive cross-border externality from raising Country 1's patent length, and hence implies a tendency towards the under-provision of patent length protection. Consequently, the conclusions with regards to whether patent protection is under- or over-provided by country governments can be sensitive to the underlying structure of innovation process, the patent instrument being considered, as well as the substitutability between domestic and foreign goods.

Our present model also facilitates an analysis of the effects of globalization (the reduction of distance barriers) on patent policies.[5] It turns out that, when domestic goods and foreign goods are differentiated but highly substitutable with each other, the equilibrium inventive step of a country is larger when it is open to trade and patenting by foreigners than when it is under autarky. In this sense, globalization increases the incentive that each government has to increase its inventive step. Evidently, the onset of globalization leads to a higher marginal benefit from raising inventive step protection relative to the closed-economy benchmark. Intuitively, globalization leads to increased competition for Country 1 firms, as they engage in direct competition with Country 2 firms. In this setting, a higher inventive step increases the profitability of home firms for two reasons. First, domestic patent-holding firms enjoy a longer average duration of monopoly profits, just like under autarky. Second, as the home inventive step increases, home firms' incentive to invest in R&D increases, while foreign firms' incentive to invest in R&D deceases. This in turn improves domestic firms' productivity relative to foreign firms, thus allowing domestic firms to compete more effectively in the product market, increasing the profitability of research in the home country. This latter effect is naturally absent in the closed-economy setting, hence explaining why inventive step protection confers a higher marginal benefit when countries can trade. Of note, this result also stands in contrast to Grossman and Lai (2004).

The concept of required inventive step that we apply in this paper draws upon an extensive literature on patenting in industrial organization. O'Donoghue (1998) models patent design in the context of cumulative innovation comprising a long sequence of product improvements. He finds that a patentability requirement can lead to an improvement in social welfare, namely that the benefit from a higher rate of innovation can exceed the loss from conferring more monopoly power to firms. This is consistent with our closed-economy result, although it is derived in a partial equilibrium setting.[6] Green and

[5] Grossman and Lai (2004) did not explicitly consider the role of distance barriers, so that all newly-innovated varieties are equally accessible by consumers in all markets.

[6] O'Donoghue (1998) also clarifies the distinction between a minimum patentable inventive step and a leading breadth requirement. The former governs the need for a new invention to be sufficiently novel before it can qualify for a patent. The latter governs whether a new invention can be considered as infringing upon an existing patent, and hence whether the inventor will need to pay royalty payments to the existing patent-holder.

Scotchmer (1995) are among the first to model patent breadth in a quality-improvement innovation setting. Similarly, Gallini (1992) proposed viewing the patent breadth as governing the cost of inventing around the patent.[7]

Several existing papers have built on Kortum's (1997) model of innovation to extend it to an open-economy setting. Eaton and Kortum (1999) provides an alternative way to incorporate patenting in a quality-based innovation model, where patenting reduces the hazard rate of imitation by potential competitors. On a similar note, Eaton and Kortum (2006) explores issues related to the cross-border diffusion of ideas, but in the absence of patenting constraints. Our contribution is instead to model the patenting regime explicitly through a inventive step requirement that specifies the minimum quality improvement needed for a new idea to be patentable and therefore to be marketed without being imitated.[8] It is also useful to clarify that while our model features quality-improving innovation, it is strictly speaking not a full model of cumulative innovation along the lines of the quality-ladder models of Grossman and Helpman (1991) and Aghion and Howitt (1992). In those papers, successful innovation confers a positive externality on future research effort since this effort is concentrated on ideas that would strictly improve on the technology frontier. In contrast, our framework follows Kortum (1997) in that the ongoing search process for future innovations can yield unsuccessful ideas that fall short of the patent frontier.

The paper proceeds as follows. Section 2 describes the innovation process based on Kortum (1997), and derives the patent frontier distribution. Section 3 sets up the full model of patenting, growth and trade. Section 4 analyzes the benchmark case of the closed-economy model. Section 5 tackles the Nash equilibrium properties in the open-economy setting, focusing on the two-country symmetric case to highlight the strategic interactions at play. Section 6 concludes. All detailed proofs are in the Appendix.

2 Innovation and the patent frontier

We first describe the innovation process, and show how this delivers a stochastic formulation of the patent frontier. This will be a key building block when we spell out the full model in Section 3. We derive the patent frontier in the closed-economy case, and then expand on this to develop useful results for the multi-country case. Not surprisingly, the patent frontier distribution will depend on (among other things) the inventive step B enacted by the country government.

[7] Note that our notion of required inventive step differs from that in Gilbert and Shapiro (1990) and Klemperer (1990), who study patent breadth under horizontal differentiation of products, rather than innovation along a quality dimension.

[8] In this light, the model of Kortum (1997), in which any research idea that advances the technological frontier is automatically patented, is essentially the special closed-economy case in which $B \equiv 1$.

2.1 The patent frontier in the closed economy

Consider a differentiated goods sector in which quality-based R&D takes place. There is a continuum of varieties in this sector, indexed by $j \in [0, 1]$. Since our focus is on innovation along the quality dimension, we hold the measure of varieties constant at 1. Labor is the sole factor of production, and all production technologies feature constant returns-to-scale. As a starting point, consider the closed-economy case. For now, suppose that the prevailing wage in the country, w, is pinned down by conditions in the economy that are exogenous from the perspective of this sector.

A defining feature of this differentiated goods sector is that productivity levels differ across varieties. Let $z_t(j)$ denote the productivity of the *best patented idea* for producing variety j at time t. Using this best patented idea, $1/z_t(j)$ is the labor input required to produce each unit of this variety, so that the corresponding unit production cost is: $c_t(j) = w/z_t(j)$. On the other hand, denote by $\tilde{z}_t(j)$ the productivity of the *best technologically-feasible idea* for producing this variety at time t. Note that $z_t(j) \leq \tilde{z}_t(j)$, as not all feasible technologies will be eligible for a patent. Given the stochastic features of the innovation process which we shall specify below, for each $j \in [0, 1]$, $z_t(j)$ and $\tilde{z}_t(j)$ will be realizations of random variables, whose cumulative distribution functions (cdfs) we denote by $F(z; t)$ and $\tilde{F}(z; t)$ respectively.[9] We refer to $F(z; t)$ as the patent frontier distribution, and to $\tilde{F}(z; t)$ as the technology frontier distribution.

Innovation: The innovation process that generates these technology and patent frontiers is based on Kortum (1997) and Eaton and Kortum (2001). Innovation takes place along the quality dimension. The actual innovation event itself is the outcome of a stochastic search process for ideas that potentially improve upon the productivity of the current technologies. These ideas are generated by the research sector in this economy. Specifically, each worker in the research sector obtains ideas at a Poisson arrival rate α. These ideas represent productivity levels, q, drawn from a stationary search distribution, $H(q)$. The specific variety j to which each idea applies is determined by a random draw from the uniform distribution on $[0, 1]$, so that the varieties are symmetric from the perspective of the innovation process.[10] Let $R_t \leq L_t$ be the number of workers employed in the research sector at time t, with L_t being the total workforce of the country. (The remaining $L_t - R_t$ workers are employed in production.) The cumulative stock of ideas in the knowledge pool of the economy at time t is thus: $T(t) = \alpha \int_0^t R_s ds$, where we initialize time at $t = 0$, with $T(0) = 0$.

To complete the description of the innovation process, we follow Kortum (1997) in specifying the stationary search distribution to be the standard Pareto distribution: $H(q) = 1 - q^{-\theta}$, $q \geq 1$.[11] As is

[9] $F(z; t)$ and $\tilde{F}(z; t)$ do not depend on j, since the varieties are *ex ante* symmetric.

[10] We rule out the possibility of directed search or innovation targeted towards specific goods, which would significantly complicate the considerations involved.

[11] That this search distribution is fixed across time periods means that researchers may sample ideas that are below

well-known, the Pareto search distribution gives rise to a technology frontier distribution across varieties $j \in [0,1]$ which is Fréchet with cdf: $\tilde{F}(z;t) = \exp\left\{-T(t)z^{-\theta}\right\}$.[12] This distribution is characterized by two key parameters: The scale parameter, $T(t)$, governs the mean of the distribution, so that an economy with a larger stock of ideas will on average feature a more productive technology frontier. On the other hand, a larger shape parameter, θ, corresponds to a $H(q)$ distribution with a lower variance, meaning that researchers draw from a search distribution that places less weight on high productivity draws.[13] For this reason, θ is also referred to as the inverse spread parameter, as it leads to a technology frontier distribution with a thinner right-tail.

Patenting: We extend this treatment of the innovation process by incorporating IPR considerations in the form of a inventive step requirement. As imitation costs of domestically sold goods are zero, innovators with new ideas need to apply for a patent before the idea can be marketed without being imitated. To improve the incentives for innovation, the patent authority or government protects existing patent-holders by stipulating that a new idea can only be patented if it improves upon the productivity of the current best patented idea by a minimum inventive step of $B \geq 1$. Formally, if the current best patented idea for variety j is of productivity z, a new patent will only be awarded if an idea of productivity at least Bz is drawn.

We refer to B as the required inventive step of the country's IPR regime. This inventive step requirement offers protection to patent-holders in that one cannot obtain a new patent with a quality improvement smaller than the required inventive step, as only ideas that exceed Bz in quality are awarded a new patent. Thus, at any one time, for each variety j, there is (generically) one patent-holder who commands the best available patent and the market for that variety. (We assume that there are no costs in applying for and enforcing a patent, so that all innovators will patent their technology whenever and wherever it qualifies for a patent.)

The stochastic arrival of ideas and the inventive step requirement jointly give rise to a patent frontier distribution, $F(z;t)$, across varieties $j \in [0,1]$. Our first key result derives a parametric form for this

the current technological frontier, so that innovation is not truly cumulative in the sense of the quality-ladder models of Grossman and Helpman (1991) and Aghion and Howitt (1992). Nevertheless, Kortum (1997) shows that parameterizing the search distribution as Pareto gives rise to a steady state that features a constant rate of patenting in spite of a constant positive growth rate in the research workforce, matching the US growth experience in the past half-century.

[12] See Eaton and Kortum (2001). This is derived by considering the probability that exactly k ideas with quality $\leq z$ are drawn when the stock of ideas is $T(t)$, and summing over all $k \geq 0$:

$$\tilde{F}(z;t) = \sum_{k=0}^{\infty} \frac{\exp\{-T(t)\}(T(t))^k}{k!} H(z)^k = \exp\{-T(t)\} \sum_{k=0}^{\infty} \frac{(T(t)(1-z^{-\theta}))^k}{k!} = \exp\left\{-T(t)z^{-\theta}\right\}$$

The above holds for $z \geq 1$, which is the support of the Pareto search distribution. Kortum (1997) shows that this cdf remains asymptotically valid for all $z > 0$, as the probability mass for $0 < z < 1$ becomes infinitesimally small as the stock of ideas grows large.

[13] The mean of $\log Z$ is $\frac{1}{\theta}(\gamma + \log T(t))$, which is increasing in $T(t)$; γ is the Euler-Mascheroni constant. Also, the standard deviation of $\log Z$ is $\pi/(\theta\sqrt{(6)})$, which is decreasing in θ.

patent frontier distribution:

Lemma 1: Suppose that $T'(t)$ is bounded. Then, as the stock of ideas grows arbitrarily large ($T(t) \to \infty$), the patent frontier at time t across varieties $j \in [0,1]$ has a Fréchet distribution with cdf $F(z;t) = \exp\left\{-T(t)\,(Bz)^{-\theta}\right\}$, where B is the inventive step established by the country.

Proof. See the Appendix. Heuristically, the proof sets up a differential equation with respect to t for $F(z;t)$ that captures the evolution of the patent frontier over small time periods. Solving this differential equation yields the above functional form for $F(z;t)$. (This is similar to the solution strategy in Evenson and Kislev (1976), Kortum (1997), and Alvarez et al. (2008).) ■

The patent frontier therefore belongs to the family of Fréchet distributions with the same shape parameter as the technology frontier, but with a (weakly) smaller scale parameter, $T(t)B^{-\theta}$, since $B \geq 1$. Intuitively, the patent frontier features ideas that are on average less productive than those on the technology frontier, as some ideas which are technologically feasible are not patented because they fail to surpass the inventive step requirement. Of note, a larger inventive step (a higher B) tends to restrict more ideas from being patented, and hence results in a patent frontier distribution with a lower mean productivity.

2.2 The patent frontier in the open-economy

The above discussion demonstrates how one can introduce inventive step considerations into the quality-improvement growth model of Kortum (1997), while retaining much of the convenience of the stochastic formulation of innovation in that model. We now extend this discussion to a setting with $N \geq 1$ countries. When there is international trade, the differentiated goods sector in each country now has available a larger set of varieties of goods than that available in a closed economy. In particular, firms in each of the N countries are producers of their own unit measure of varieties $j_i \in [0,1]$, $i = 1, 2, ...N$, and sell them to all countries in the world. In doing so, they compete with not only varieties produced by firms in their own countries but also those produced by foreign firms. Instead of having varieties of measure one available to them, consumers in each country now are faced with a wider choice of varieties with a measure of N. The aggregate utility derived from the sets of varieties originated from different countries are differentiated technologically and utility-wise. In the central case we consider, they are highly substitutable with each other.

Innovation: Production ideas emerge from the research sectors in all the N countries. The idea-generating process within each country is similar: Each worker in the research sector obtains production ideas with a Poisson arrival rate of α_i, where i denotes the country. When an idea arrives, the specific variety j to which it applies is a random draw from a standard uniform distribution, while the quality of the idea is drawn from the stationary Pareto search distribution, $H(q) = 1 - q^{-\theta}, q \geq 1$. Letting R_{it} denote the number of workers in country i employed in its research sector at time t, the current cumulative stock of ideas generated by country i is then: $T_i(t) = \alpha_i \int_{s=0}^{t} R_s ds$. This innovation process yields a technological frontier within each country, defined as the most productive idea obtained by that country's research sector. As we have seen, this technological frontier across varieties j is described by the Fréchet cdf, $\exp\{T_i(t)z^{-\theta}\}$, where the relevant scale parameter is the cumulative idea stock attributable to country i. Our task moving forward is to understand how patenting affects this innovation process.

Patenting: The open-economy model features a global market for ideas. Varieties originated from different countries are technologically and utility-wise differentiated from each other. They are therefore treated as distinct varieties by all patent authorities. A producer seeking to market a differentiated variety in a country can obtain a patent from that country's patent authority (regardless of the country of origin of the producer). IPR protection takes the form of an inventive step in each country: If the existing best patented idea in country n for a particular variety is of productivity z, a new patent will only be awarded if the quality of the new idea is at least $B_n z$, where $B_n \geq 1$ is the required inventive step established by country n's patent board.

We adopt a setting in which international patenting is governed by the principles of non-discrimination and national treatment. Each country therefore accords all ideas the same treatment under its patenting regime, regardless of their country of origin. In particular, the same procedure for determining whether an idea qualifies for a patent – namely, whether that idea exceeds the minimum inventive step of B_n – is applied to innovations from all countries. Apart from the patenting requirement, we assume that there are no other impediments to the global flow of ideas. In particular, there are no costs to obtaining a patent in any country. This implies that producers will seek out and obtain a patent for their products in every market where it is patentable. The imitation cost of a good that is sold locally is zero, but the imitation cost of a good that is sold only in foreign countries is prohibitive, even if the technology has been patented there.[14] Goods from different countries are differentiated and treated as technologically distinct by patent authorities.

The following lemma states the distribution of the patent frontier in a country n fro varieties originated from each source country i:

Lemma 2: Suppose that $T_i'(t)$ is bounded for all countries i. Then, as the stock of ideas from each country grows arbitrarily large ($T_i(t) \to \infty$), under national treatment, the patent frontier in country n at time t across varieties originated from country i $j_i \in [0, 1]$ has a Fréchet distribution with cdf:

[14]Specifically, the cost of imitating a technology of a good that is sold only in a foreign country (even if it is patented there) and marketing it at home is higher than the expected profit from obtaining a new patent in the home country.

$\exp\left\{-T_i(t)\left(B_n z\right)^{-\theta}\right\}$, where B_n is the inventive step established by country n.

Proof. This follows directly from Lemma 1. Since there are no barriers to international patenting of ideas in the open-economy, the result follows by replacing $T(t)$ by $T_i(t)$ and B by B_n in the closed-economy patent frontier distribution from Lemma 1. ■

This is a close parallel of the expression for the patent frontier in the closed-economy case, being from the same family of Fréchet distributions with shape parameter θ. Note that the scale parameter is now $T_i(t)B_n^{-\theta}$, since the relevant stock of ideas is that which originates from country i only. Naturally, the mean of this best patented idea distribution is increasing in the source country stock of ideas, and decreasing in the inventive step in the recipient country.

3 Setting up the full model of innovation, patenting and trade

Having laid out the underlying innovation and patenting process, we proceed now to set up the full model. Here, we build on the generality that the Eaton-Kortum framework affords us to develop the model for the N-country case (where $N \geq 1$). The case of $N = 1$ corresponds to the closed-economy setting, which will be analyzed in Section 4. We also focus on the two-country case in more detail in Section 5, in order to derive more explicit results regarding inventive steps in the Nash equilibrium, although much of the intuition here is applicable to the general N-country case.

3.1 The market for goods

Utility: There are two sectors in the world economy: (i) a homogeneous goods sector, and (ii) a differentiated goods sector featuring N sets of continuum of varieties $j_i \in [0,1]$ for $i = 1, 2, ...N$ where i corresponds to country i. The preferences of the representative consumer in each country $n \in \{1, \ldots, N\}$ at time t are given by following Cobb-Douglas utility function:

$$U_{nt} = \left(\frac{y_{nt}}{\eta}\right)^{\eta} \left(\frac{\left[\sum_{i=1}^{N}\left(\exp\{\int_0^1 \ln x_{nit}(j)dj\}\right)^{\beta}\right]^{\frac{1}{\beta}}}{1-\eta}\right)^{1-\eta} \tag{1}$$

Here, i indexes the country of origin of the differentiated variety, while n denotes the country of consumption. The elasticity of substitution that prevails between the consumption of varieties that originate from different countries is: $\phi = 1/(1-\beta)$, where $0 < \beta < 1$. For each country, utility of varieties is aggregated, with elasticity of substitution equal to 1. The aggregates of differentiated goods from different countries are imperfect substitutes of each other. Here, y_{nt} denotes consumption of the homogenous good, and $x_{nit}(j)$ denotes consumption of variety j of differentiated goods from country i. A constant share of income $\eta \in (0, 1)$ is devoted to purchases of the homogenous good. In equilibrium, the expenditure on varieties originated from country i is distributed equally among these varieties. The differentiated goods sector consists of varieties from all N countries. The differentiated goods originated from each country is as described in Section 2, and features innovation and patenting over time. On the other hand, there is no technological progress in the homogenous goods sector; for this reason, we shall also refer to this sector as the "traditional" sector, in the sense that no R&D effort is channeled towards it.

Production and market structure: The traditional sector has a simple constant returns-to-scale structure. In country n, production of this good uses a_n units of labor. Trade in this good encounters no cross-border frictions, so its price is equalized across all countries and can be normalized to one. In what follows, we shall focus on equilibria in which a positive quantity of this numeraire good is produced in each country, so that the wage in country n is given by $w_n = 1/a_n$, being pinned down by technological conditions in this traditional sector.[15] In practice, this means that the labor force L_n in each country, as well as the consumer demand for homogenous goods (captured by η), must all be sufficiently large, to support such an equilibrium. (We shall make these parameter conditions more explicit when we explore the two-country model in Section 5.)

Production in the differentiated goods sector likewise takes place under constant returns-to-scale. In the multi-country world, however, countries will in general differ in their productivity levels. These underlying productivity differences generate a Ricardian basis for trade between countries, as varieties can potentially be sourced from each of the N countries. However, the flow of goods across borders is impeded by transport costs of the iceberg form: $d_{ni} \geq 1$ units of a goods must be shipped out from country i in order for one unit to arrive in country n.[16] We make the standard assumptions that: (i) $d_{ii} = 1$ for all i, so that there are no shipping costs within each country; and (ii) $d_{ni} \leq d_{nk}d_{ki}$ for all $k \neq i, n$, so that transhipment through a third-country port is not pursued.

Let $z_{nit}^{(1)}(j)$ denote the productivity of the best idea originating from country i which has successfully obtained a patent in country n at time t for variety j_i. The marginal cost of production for this patent-holder is: $c_{nit} = w_i d_{ni}/z_{nit}(j)$, after taking the labor and shipping costs into account. To determine the market outcome in country n, note that only producers who hold a country-n patent will participate in that market in equilibrium. For each variety j_i from country i, all producers from i who hold a country-n patent for that variety engage in head-to-head Bertrand competition. The country i producer who possesses the patented idea with the lowest marginal cost will thus command the entire country-n market for variety j_i, and limit-prices at the level of the patent-holder with the second-lowest marginal

[15] It will turn out that when all countries have the same labor force growth rate, the steady state exhibits a constant wage in each country that does not vary over time, so that we need not attach a time subscript to w_n. The assumption of a constant marginal product of labor is thus consistent with the long-run equilibrium of the model.

[16] We follow the convention in EK, where the second subscript denotes the exporting country, while the first subscript refers to the importing country.

cost. This rule ensures that all other country i patent-holders with less cost-efficient ideas cannot earn positive profits, and therefore leave the market to the lowest-cost patent-holder as the sole supplier.

We seek to evaluate the distribution of market prices for differentiated varieties, in order to later derive an expression for welfare. Given the limit-pricing rule, it follows that we need to specify the stochastic distribution that jointly describes both the best and second-best patented ideas that originate from country i in country n. We have already seen in Lemma 2 that the distribution of this best patented idea from i in n, denoted by the random variable $Z_{nit}^{(1)}(j)$, is given by the Fréchet cdf: $Pr(Z_{nit}^{(1)}(j) \leq z) = BP_{ni}(z;t) = \exp\left\{-T_i(t)\left(B_n z\right)^{-\theta}\right\}$, with B_n being the inventive step required by country n. Moving forward, following Bernard et al. (2003), we specify the joint distribution of the best and second-best patented ideas from country i in country n to be the multivariate analogue of the Fréchet distribution in Lemma 2:

$$Pr(Z_{nit}^{(1)} \leq z_1, Z_{nit}^{(2)} \leq z_2) = \left[1 + T_i(t)\left((B_n^2 z_2)^{-\theta} - (B_n z_1)^{-\theta}\right)\right] \exp\left\{-T(B_n^2 z_2)^{-\theta}\right\},\ 0 \leq B_n z_2 \leq z_1 \quad (2)$$

Note that we require $B_n z_2 \leq z_1$, since the best patented idea needs to satisfy the inventive step requirement vis-à-vis the second-best patented idea. Also, as $B_n z_2 \to z_1$, (2) reduces to: $\Pr\left(Z_{nit}^{(1)} \leq z_1\right) = \exp\left\{-T_i(t)(B_n z_1)^{-\theta}\right\}$, which is precisely the best patented idea distribution from Lemma 2.

Welfare: The joint productivity distribution for the best and second-best patented ideas from each country given by (2) implies that the price of each variety j_i originated from country i in country n, $p_{nit}(j)$, can be viewed as a realization from a price distribution. Define $P_{nit} = \exp\{\int_0^1 \ln p_{nit}(j)dj\}$. This is the ideal price index of country i differentiated goods in country n. Then, solving the utility-maximization problem based on (1), one can show that the demand $x_{nit}(j)$ for this variety by a country-n individual who earns wage w_n is:

$$x_{nit}(j) = \frac{(1-\eta)w_n P_{nit}^{1-\phi}}{\sum_{k=1}^{N} P_{nkt}^{1-\phi}} \frac{1}{p_{nit}(j)} \quad (3)$$

In particular, if $\phi > 1$, then when P_{nit} increase, the demand level for each variety from country i in country n will fall (holding $p_{nit}(j)$ constant).

Let's now specialize to the two-country case. Define d as the iceberg trade cost between the two countries. Based on the expression for the ideal price index in the closed-economy case derived in the Appendix (setting $N = 1$), we have: $P_{nit} = T_i^{-\frac{1}{\theta}} w_i d_{ni} B_n^2 \gamma$, where γ is a constant that does not depend on B_n. In particular:

$$\begin{aligned} P_{11t} &= T_{1t}^{-\frac{1}{\theta}} w_1 B_1^2 \gamma \\ P_{12t} &= T_{2t}^{-\frac{1}{\theta}} (w_2 d) B_1^2 \gamma \\ P_{21t} &= T_{1t}^{-\frac{1}{\theta}} (w_1 d) B_2^2 \gamma \\ P_{22t} &= T_{2t}^{-\frac{1}{\theta}} w_2 B_2^2 \gamma \end{aligned} \quad (4)$$

Based on (1), welfare (the real wage) is now given by:

$$U_n = \frac{w_n}{\left(\sum_{i=1}^{N} P_{ni}^{1-\phi}\right)^{\frac{1-\eta}{1-\phi}}} \tag{5}$$

The task of maximizing welfare thus boils down to the task of maximizing $\left(\sum_{i=1}^{N} P_{ni}^{1-\phi}\right)^{-\frac{1}{1-\phi}}$, ie minimizing the price index. Using the expressions for the price indices, this reduces in the two-country case to:

$$\max_{B_1} \; B_1^{-2}\left((T_1^{-\frac{1}{\theta}}w_1)^{1-\phi} + (T_2^{-\frac{1}{\theta}}w_2 d)^{1-\phi}\right)^{-\frac{1}{1-\phi}} \tag{6}$$

and

$$\max_{B_2} \; B_2^{-2}\left((T_1^{-\frac{1}{\theta}}w_1 d)^{1-\phi} + (T_2^{-\frac{1}{\theta}}w_2)^{1-\phi}\right)^{-\frac{1}{1-\phi}} \tag{7}$$

The above welfare expressions highlight the key tradeoff involved in inventive step policy. On the one hand, as we shall soon see below, a larger inventive step (a higher B_n) will raise research incentives in all countries, and will hence increase the stock of ideas (each country's T_i) in the aggregate. However, this decreases consumer surplus, as a larger inventive step protects incumbent patent-holders for a longer duration on average, by preventing a larger margin of lower-priced ideas from securing a patent. This loss is captured by the B_n^{-2} term ($n = 1, 2$) in (6) and (7), which reflects the effect of a higher B_n on welfare holding the stock of ideas constant. The optimal patent policy will thus be the choice of B_n that equates the marginal benefit of a larger idea stock against the marginal cost of the consumer surplus lost.

3.2 Research incentives and the country idea stocks

We turn now to the task of understanding this steady-state relationship between the inventive step policies of the N countries and the country idea stocks. This is a key step towards characterizing the benefit of raising IPR protection through a higher inventive step, and hence towards evaluating the optimal inventive step. This requires us to consider the incentives faced by the research sector, specifically how innovation activities are affected when the inventive step is raised.

Markups and profits: Since innovation effort is motivated by the expectation of a future flow of profit returns, we compute the expected profits of differentiated varieties for firms in each country. Recall that varieties originated from different countries are technologically differentiated and are treated as different goods by the patent authorities. Therefore, only innovators from i who comes up with an idea for variety $j_i \in [0, 1]$ will be considered for a patent in country n for variety j_i. Let $b_{nit}(m)$ be the probability that a given idea originating from country i undercuts the current best patented idea in n from i by a factor of at least m, where $m > 1$. Then:

$$b_{nit}(m) = \int_1^{\infty} \exp\left\{-T_i(t)B_n^{-\theta}\left(\frac{m}{q}\right)^{\theta}\right\}\theta q^{-\theta-1}dq \simeq \frac{1}{B_n^{-\theta}m^{\theta}T_i(t)} \tag{8}$$

where we have used the fact that the probability of the best patented idea in country n from country i at time t being $\leq c$ is equal to: $1 - \exp\left\{-T_i(t)B_n^{-\theta}c^\theta\right\}$.[17] The approximation taken in (8) is valid when $T_i(t) \to \infty$.[18]

Observe in particular that the probability of a new idea from country i being eligible for a patent in country n is: $b_{nit}(B_n) = \frac{1}{T_i(t)}$. Not surprisingly, the larger the effective stock of ideas available in country i (a higher $T_i(t)$), the more difficult it is for a new idea to secure a patent in country n. More interestingly, this probability turns out to be independent of the inventive step, B_n. There are in fact two countervailing effects of an increase in B_n. First, given the current best patented idea from i in n, a larger inventive step tends to reduce the probability that a new idea from i would qualify for a patent in n. On the other hand, however, this would also lead to a patent frontier distribution of technologies from i in n with a lower mean productivity level, since a larger margin of ideas which are technologically feasible are not patented when B_n is high; this tends to raise the probability of an idea from i of successfully qualifying for a patent in n. In our model, it turns out these two effects offset each other exactly, so that B_n does not affect the probability of a new idea from i obtaining a country-n patent, as asserted in the introduction. As discussed in the Introduction, the inventive step set by a country thus affects the expected value of an idea only through its impact on the profits accruing to the innovator conditional on successfully obtaining a patent in that country. On this last count, we know that a higher B_n tends to raise flow profits, both because the successfully-patented idea would command a higher markup on average, and because it makes it harder for competitor ideas to displace the patent-holder. A larger inventive step therefore raises the overall incentive to innovate in our model.

The markup that a new patented idea from country i can command in country n is stochastically determined, and is given by:

$$\Pr\left[M \leq m \mid M \geq B_n\right] = \frac{b_{nit}\left(B_n\right) - b_{nit}\left(m\right)}{b_{nit}\left(B_n\right)} = 1 - \left(\frac{m}{B_n}\right)^{-\theta}, \quad m \geq B_n \tag{9}$$

This markup thus inherits a Pareto distribution; naturally, its support is defined only for $m \geq B_n$, since any newly-patented idea must by definition secure a markup of at least B_n vis-à-vis the second-best patented idea. Of note, conditional on the idea surpassing the inventive step, the distribution of the markup that the new idea commands does not depend on its country of origin.

We are now in a position to compute the profits that accrue to successful innovators.

Let X_{nit} be the total expenditure by country-n residents on differentiated goods originated from country i. Let Π_{nit} denote the profits earned by country i producers from sales in the country n market.

[17] Let $G_{ni}(c)$ denote the cost distribution for the best patented idea in country n from country i. Then $G_{ni}(c) = 1 - \exp\left\{-T_{it}B_n^{-\theta}c^\theta\right\}$.

[18] There is a discrepancy in that the stationary search distribution is defined for ideas $q \geq 1$, whereas the patent frontier distribution is defined for all $z \geq 0$. Kortum (1997) shows that the probability mass for ideas with productivity between 0 and 1 can be ignored as $T_i(t) \to \infty$.

Using the markup distribution (9), we have:

$$\Pi_{nit} = X_{nit} \int_{B_n}^{\infty} \left(1 - m^{-1}\right) \theta m^{-\theta-1} B_n^{\theta}\, dm = \frac{X_{nit}}{1+\theta} \left[1 + \theta \left(\frac{B_n - 1}{B_n}\right)\right] \tag{10}$$

Profits are therefore equal to the fraction $\frac{1}{1+\theta}\left[1+\theta\left(\frac{B_n-1}{B_n}\right)\right] \in (0,1)$ of consumer expenditures on the differentiated goods. Note the natural properties that a more protective patent regime (a higher B_n) and a larger variance for the distribution of ideas (a smaller θ) both lead to a larger profit share.[19]

Using (3), (4) and (10), we can deduce that:

$$\frac{X_{11t}}{X_{12t}} = \frac{\Pi_{11t}}{\Pi_{12t}} = \left(\frac{P_{11t}}{P_{12t}}\right)^{1-\phi} = \left(\frac{T_1}{T_2}\right)^{\frac{\phi-1}{\theta}} \left(\frac{w_2 d}{w_1}\right)^{\phi-1} \tag{11}$$

$$\frac{X_{21t}}{X_{22t}} = \frac{\Pi_{21t}}{\Pi_{22t}} = \left(\frac{P_{21t}}{P_{22t}}\right)^{1-\phi} = \left(\frac{T_1}{T_2}\right)^{\frac{\phi-1}{\theta}} \left(\frac{w_2}{w_1 d}\right)^{\phi-1} \tag{12}$$

These expenditures and profits in the differentiated goods sector are further linked as a matter of national income accounting. Specifically, consumption expenditure on all goods (including the numeraire good) from all countries must be equal to the sum total of labor income in the production sector and income earned from profits. In the two-country case, this means that:

$$\frac{X_{11t} + X_{12t}}{1-\eta} = (1 - r_1)\, w_1 L_{1t} + \Pi_{11t} + \Pi_{21t}$$

$$\frac{X_{21t} + X_{22t}}{1-\eta} = (1 - r_2)\, w_2 L_{2t} + \Pi_{12t} + \Pi_{22t}$$

Note that $r_{it} = R_{it}/L_{it}$ is the share of the labor force engaged in the research sector in country i at time t, so that $1 - r_{it}$ is the relevant share of workers who earn the labor wage w_i.

Define $h(B) = \frac{1}{1-\eta}\frac{B(1+\theta)}{B(1+\theta)-\theta}$. Then, using (10), we can re-write the above equations as:

$$h(B_1)(\Pi_{11t} + \Pi_{12t}) = (1 - r_1)\, w_1 L_{1t} + \Pi_{11t} + \Pi_{21t}$$

$$h(B_2)(\Pi_{21t} + \Pi_{22t}) = (1 - r_2)\, w_2 L_{2t} + \Pi_{12t} + \Pi_{22t} \tag{13}$$

Research incentives: To close the model fully, we need to specify how profits motivate innovation activities. Let V_i denote the expected value of an idea that originates in country i. In equilibrium, the expected value of such an idea multiplied by the arrival rate of ideas per worker needs to be equal to the domestic wage: $\alpha_i V_{it} = w_i$, so that workers are indifferent between engaging in production or research activities. In turn, the expected value of an idea originated from country i, V_{it}, is the expected discounted flow of future profits in each of the N potential consumer markets:

$$V_{it} = P_{it} \int_t^{\infty} e^{-\rho(s-t)} \sum_{n=1}^{N} b_{nit}(B_n) \times \frac{\Pi_{nis}}{P_{is}} \frac{b_{nis}(1)}{b_{nit}(B_n)}\, ds = \int_t^{\infty} e^{-\rho(s-t)} \frac{P_{it}}{P_{is}} \sum_{n=1}^{N} \Pi_{nis} \frac{B_n^{\theta}}{T_i(s)}\, ds \tag{14}$$

[19] In the limiting case where $B_n = 1$, (10) implies a profit share of $1/(1+\theta)$, consistent with Eaton and Kortum (2001)'s model without patenting.

The term $\frac{\Pi_{ns}}{P_{is}}$ captures the real profits earned in country n at time s; these are pre-multiplied by P_{it} to express these profits in terms of time-t prices. Note also that $\frac{b_{nis}(1)}{b_{nit}(B_n)}$ is the probability that the innovation continues to have the market to itself at time $s > t$, conditional on its having secured the market at time t. (The numerator is $b_{nis}(1)$ since the incumbent patent-holder does not need to satisfy the required inventive step constraint vis-à-vis new entering ideas.) For simplicity, all countries share the same discount rate parameter ρ, where ρ satisfies: $\rho > \frac{g_L}{\theta}(1-\eta)$, to guarantee the existence of a steady state where the country idea stocks are finite.

We evaluate V_{it} for a steady state in which: (i) the labor force grows in all countries at a constant rate, g_L (so that $\dot{L}_i/L_i = g_L$ for all i), and (ii) the share of labor in the research sector is constant in each country (so that $r_{it} = r_i$). Since $T_i(t) = \alpha_i \int_0^t R_{is} ds$, the time path of the stock of ideas is therefore governed by: $\dot{T}_i(t) = \alpha_i r_i L_{it}$. This implies that T/L converges to a constant value, $\frac{\alpha_i r_i}{g_L}$, in the steady state. The stock of ideas therefore grows at the same rate as the labor force: $\dot{T}/T = \dot{L}/L = g_L$. Now, observe that the country price index derived in the Appendix is $\left(\sum_{i=1}^N \left(T_i^{-\frac{1}{\theta}} w_i d_{ni} B_n^2 \exp\left\{\frac{1-\gamma}{\theta}\right\}\right)^{1-\phi}\right)^{\frac{1-\eta}{1-\phi}}$. This implies that: $P_{it}/P_{is} = (T_i(t)/T_i(s))^{-\left(\frac{1-\eta}{\theta}\right)}$ since idea stocks in all countries are growing at the same rate g_L in steady state. Therefore, $P_{it}/P_{is} = \exp\left\{\left(\frac{1-\eta}{\theta}\right) g_L(s-t)\right\}$. Also, a quick inspection of (10) and (13) implies that profits earned in each country market, Π_{nit}, will grow in tandem with L_i and T_i at rate g_L. In particular, this means that $\Pi_{nit}/T_i(t)$ is constant over time.

Using these properties, the expected value of an idea from country i from (14) can be expressed as:

$$V_{it} = \int_t^\infty e^{-\left[\rho - \frac{g_L}{\theta}(1-\eta)\right](s-t)} \sum_{n=1}^N \Pi_{nt} \frac{B_n^\theta}{T_i(t)} \, ds = \frac{1}{\left[\rho - \frac{g_L}{\theta}(1-\eta)\right]} \sum_{n=1}^N \Pi_{nt} \frac{B_n^\theta}{T_i(t)}$$

Substituting this into the equilibrium condition $\alpha_i V_{it} = w_i$, we have, in the two country case:

$$B_1^\theta \Pi_{11} + B_2^\theta \Pi_{21} = \frac{w_1 T_1}{\alpha_1}\left(\rho - \frac{g_L}{\theta}(1-\eta)\right) \tag{15}$$

$$B_1^\theta \Pi_{12} + B_2^\theta \Pi_{22} = \frac{w_2 T_2}{\alpha_2}\left(\rho - \frac{g_L}{\theta}(1-\eta)\right) \tag{16}$$

This delivers a set of equations in (16) that capture how profits, Π_{nit}, and the stock of ideas, T_{it}, must be related in the steady state in which each country features a constant share of the workforce, $r_i \in (0,1)$, engaged in the research sector.

Moreover, we can now substitute the steady state expression for $r_i = \frac{g_L}{\alpha_i}\frac{T_{it}}{L_{it}}$ back into the accounting equations (13) that equate national expenditure to income. This yields, in the two-country case:

$$h(B_1)(\Pi_{11t} + \Pi_{12t}) = w_1 L_{1t} - \frac{w_1 T_{1t} g_L}{\alpha_1} + \Pi_{11t} + \Pi_{21t} \tag{17}$$

$$h(B_2)(\Pi_{21t} + \Pi_{22t}) = w_2 L_{2t} - \frac{w_2 T_{2t} g_L}{\alpha_2} + \Pi_{12t} + \Pi_{22t} \tag{18}$$

3.3 Equilibrium

The equilibrium in the two-country model is now fully-determined by the equations (11)-(12) and (15)-(18), which represent a system of 6 equations in the 6 unknowns, Π_{11}, Π_{12}, Π_{21}, Π_{22}, T_1, and T_2 . The profit levels, expenditure levels, and stock of ideas in the differentiated goods industry in each country can be solved for as a function of the underlying conditions in the two countries, including the labor force situation (L_{it}, w_i, g_L), the deep parameters of the model related to the innovation process (α_i, θ), preferences (η, ρ), distance barriers (d), as well as the country patent policies chosen by each country government (the B_n's).

Several remarks are worth highlighting. First, the share of labor effort in the economy devoted towards innovation activities is an endogenous outcome of this equilibrium, being equal to $r_i = \frac{g_L}{\alpha_i}\frac{T_{it}}{L_{it}}$. We shall refer to r_i as the research intensity of this country. Second, using a labor market-clearing condition, one can show that the wages in each country, w_i, are in fact constant in this steady state; the argument for this is analogous to that in Eaton and Kortum (2001). Finally, the equilibrium of the model does feature balanced trade: Countries that are net exporters of differentiated goods are in turn net importers of the traditional good (and vice versa), so that the overall trade balance is zero for each country.

4 The closed-economy benchmark

We first study the closed-economy case of the model, which provides an important benchmark. This allows us in particular to establish a baseline result on the existence of a non-trivial optimal inventive step that improves country welfare. We will build on this result when we turn to the two-country model with strategic interaction in Section 5.

4.1 Equilibrium

It is fairly straightforward to solve explicitly for the equilibrium in the closed-economy case. When $N = 1$, (10) and (13) are a system of two equations in precisely two unknowns, X_t and Π_t, which are respectively the consumption expenditures and profits for the differentiated goods sector in the sole country. (We drop the country subscript for convenience.) Solving these two equations simultaneously yields:

$$X_t = \left[\frac{B(1+\theta)}{\frac{\eta}{1-\eta}B(1+\theta)+\theta}\right](1-r_t)w_tL_t \tag{19}$$

$$\Pi_t = \left[\frac{B(1+\theta)-\theta}{\frac{\eta}{1-\eta}B(1+\theta)+\theta}\right](1-r_t)w_tL_t \tag{20}$$

Based on (19) and (20), one can show that for a given country income level, w_tL_t, and for a given research intensity, r_t, both consumption and profits in this differentiated goods sector will increase if: (i) the country's inventive step, B, is raised; or (ii) the stationary search distribution of ideas has a larger spread ($1/\theta$ is higher). Naturally, both X_t and Π_t fall as the share of income spent on the numeraire good, η, rises.

From the closed-economy analogue of (16), the steady-state stock of ideas for this country is therefore:

$$T(t) = \frac{\alpha L_t}{g_L} \cdot \frac{B^\theta \left[B(1+\theta) - \theta\right]}{B^\theta \left[B(1+\theta) - \theta\right] + \left[\frac{\rho}{g_L} - \frac{1-\eta}{\theta}\right] \left[\frac{\eta}{1-\eta} B(1+\theta) + \theta\right]} \tag{21}$$

where we have used the expression for Π_t from (20). Observe that the larger the labor force, L_t, the greater the stock of ideas the economy is able to generate in equilibrium, by virtue of being able to devote more workers to the research sector. It is moreover straightforward to differentiate (21) and verify that $T(t)$ is in fact increasing in B. Thus, a more stringent inventive step requirement indeed expands the stock of research ideas accessible to the economy.

Since $T/L = \frac{\alpha r}{g_L}$ in steady state, the share of the labor force employed in the research sector is:

$$r = \frac{B^\theta \left[B(1+\theta) - \theta\right]}{B^\theta \left[B(1+\theta) - \theta\right] + \left(\frac{\rho}{g_L} - \frac{1-\eta}{\theta}\right) \left[\frac{\eta}{1-\eta} B(1+\theta) + \theta\right]} \tag{22}$$

Note that $r \in (0,1)$, so long as $\rho - \left(\frac{g_L}{\theta}\right)(1-\eta) > 0$. Importantly, the equilibrium value of r also increases with the inventive step. Intuitively, as B increases, if we hold r (and hence also T) constant, this will increase the value of a patent V_t. This is evident by re-writing (15) in the closed-economy case:

$$V_t = \frac{1}{\left[\rho - \frac{g_L}{\theta}(1-\eta)\right]} \Pi_t \frac{B^\theta}{T_t}$$

With a higher B, the per-period profits Π_t increase as we saw from (20); this happens because the more stringent inventive step requirement raises the expected markup which existing patent-holders can charge. Moreover, the B^θ term also increases, which reflects the fact that the higher B enables the incumbent patent-holder to retain the market for a longer expected duration. (Recall from (8) that $b_s(1) = \frac{B^\theta}{T_s}$ is equal to the probability that an idea secures a patent at date t and continues to hold the market at date $s > t$.) An increase in B therefore tends to raise the value of a patent through these two channels. However, we know that V_t must stay constant in steady-state, as dictated by the labor market equilibrium condition, $\alpha V_t = w$. Thus, workers move out of production into the R&D sector in response to the higher inventive step, so that r and T both rise endogenously; in turn, the expanded stock of ideas lowers the expected duration over which the patent-holder can command the market, which provides the countervailing force that keeps V_t constant in equilibrium.

4.2 Optimal patent policy

We now analyze the standpoint of a government that seeks to maximize consumer welfare through its choice of patent policy instrument. The closed-economy analogue for welfare can be obtained from the expression for the real wage in (5) by setting $N = 1$ and $w_n = w$. Simplifying, we have:

$$\frac{w}{P} = \gamma' w^{\eta} \left(T^{\frac{1}{\theta}} B^{-2}\right)^{1-\eta} \tag{23}$$

where we have suppressed the time argument for notational simplicity. (In steady state, maximizing the present discounted flow of real wages over time is equivalent to maximizing the per period real wage.) Since γ' and w are constants, the optimal inventive step needs to equate the marginal benefit of an increase in B to the economy (as captured by an increase in T) with the marginal cost from paying higher prices for goods that are under patent protection (as captured by a decrease in the term B^{-2}).

Substituting in the expression for the steady-state stock of ideas from (21), and ignoring various constants that do not affect the maximization outcome, one can show that the optimal inventive step policy is given by:

$$\begin{aligned} B^* \quad &= \quad \arg\max_{B} \; B^{-2\theta} \left[1 + \left(\frac{\rho}{g_L} - \frac{1-\eta}{\theta} \right) \frac{\frac{\eta}{1-\eta} B\,(1+\theta) + \theta}{B^{\theta} [B(1+\theta) - \theta]} \right]^{-1} \\ & \quad\quad \text{subject to } B \geq 1 \end{aligned} \tag{24}$$

Solving this maximization problem delivers the first key result of this paper, which addresses the scope for inventive step protection to improve outcomes in the closed-economy setting:

Proposition 1 [***Optimal Patent Policy in the Closed Economy***] *If* $\frac{\rho}{g_L} - \frac{1-\eta}{\theta} > 2$, *the optimal policy calls for a binding inventive step,* $B^* > 1$. *This optimal inventive step,* B^*, *is moreover unique. Otherwise, for* $0 < \frac{\rho}{g_L} - \frac{1-\eta}{\theta} \leq 2$, *the optimal policy is* $B^* = 1$.

The above proposition is in line with a set of results from the endogenous growth literature on how there can be under-investment in innovation activities in the laissez-faire equilibrium in such models (Aghion and Howitt 1992, Grossman and Helpman 1991). In our model, it is useful to inspect the condition $\frac{\rho}{g_L} - \frac{1-\eta}{\theta} > 2$, to obtain some intuition for understanding when such under-investment is more likely. Research activities are more likely to be inefficiently low from a social planner's perspective when: (i) ρ is large, so that consumers weigh the short-term consumer surplus loss heavily, leading to a low incentive to undertake R&D; (ii) g_L is small, so that the stock of ideas does not advance sufficiently fast in steady state ($\frac{\dot{T}}{T}$ is small); (iii) $1 - \eta$ is small, namely when expenditure on differentiated varieties and hence the profit motive for innovation in this sector are both small; and (iv) $1/\theta$ is small, so that the right-tail of the stationary search distribution for ideas is thinner, hence lowering the likelihood of obtaining a very high productivity innovation.

It should moreover be stressed that this condition is readily satisfied by standard parameter values. For example, consider $\rho = 0.05$ and $g_L = 0.02$, which imply an annual discount factor of about 0.95 and a 2% labor force growth rate respectively. Eaton and Kortum (2002) offer a range of estimates for

θ from fitting their model of trade to the OECD data, of which $\theta = 8.28$ is a commonly-used central value. Taken together, we have: $\frac{\rho}{g_L} - \frac{1-\eta}{\theta} > \frac{\rho}{g_L} - \frac{1}{\theta} = 2.38 > 2$. If we further identify the differentiated goods sector with manufacturing industries, then $1 - \eta$ is the share of income spent by consumers on manufactured goods, which in Eaton and Kortum (2002) is calibrated to equal 0.13. The relatively small value of this parameter makes it even easier to satisfy the desired condition for an interior solution to the welfare maximization problem. In short, there is good reason to believe that a binding inventive step will indeed be the optimal policy for a wide range of relevant parameterizations.

5 Nash equilibrium analysis for the two-country case

We turn next to a detailed analysis of the open-economy model. This will allow us to expand the discussion into issues relating to the cross-border externalities from national inventive step policies, and the effect of globalization on the domestic research intensity and the overall incentive to extend inventive step protection.

5.1 Solving for the equilibrium

While specifying the equilibrium is conceptually straightforward, this being described in Section 3.3, solving explicitly for closed-form expressions is algebraically complicated for the general N-country case. For the sake of tractability, we will therefore focus most of our attention on the two-country case. This will allow us to derive some sharp results concerning the policy game between the two countries in terms of how strongly each chooses to protect inventive step. It will moreover facilitate a more direct comparison with the benchmark results in Grossman and Lai (2004) regarding optimal patent length policy in their two-country setting.

Suppose that there are only two countries, $i = 1, 2$, in the global economy. From (11) and (12), define:

$$K_1 = \left(\frac{T_1}{T_2}\right)^{\frac{1-\phi}{\theta}} \left(\frac{w_2 d}{w_1}\right)^{1-\phi}$$

$$K_2 = \left(\frac{T_1}{T_2}\right)^{\frac{\phi-1}{\theta}} \left(\frac{w_2}{w_1 d}\right)^{\phi-1}$$

Then, we have that $\Pi_{12} = K_1\Pi_{11}$, and $\Pi_{21} = K_2\Pi_{22}$. (Moreover, observe that $K_1K_2 = d^{2(1-\phi)}$.)

Substitute $\Pi_{12} = K_1\Pi_{11}$, and $\Pi_{21} = K_2\Pi_{22}$ into (15) and (16). This gives a system of two equations in two unknowns, Π_{11} and Π_{22}. The solution to this system is:

$$\Pi_{11} = \frac{\rho - \frac{g_L}{\theta}(1-\eta)}{1 - d^{2(1-\phi)}} \frac{1}{B_1^\theta} \left(\frac{T_1 w_1}{\alpha_1} - K_2 \frac{T_2 w_2}{\alpha_2}\right) \quad (25)$$

$$\Pi_{22} = \frac{\rho - \frac{g_L}{\theta}(1-\eta)}{1 - d^{2(1-\phi)}} \frac{1}{B_2^\theta} \left(\frac{T_2 w_2}{\alpha_2} - K_1 \frac{T_1 w_1}{\alpha_1}\right) \quad (26)$$

First, substitute $\Pi_{12} = K_1 \Pi_{11}$, and $\Pi_{21} = K_2 \Pi_{22}$ into (17) and (18). Then, substitute the expressions for Π_{11} and Π_{22} into (17) and (18). Finally, using the fact that $r_i = \frac{g_L}{\alpha_i} \frac{T_i}{L_i}$, we get the following two equations in the two unknowns T_1 and T_2.

$$[h(B_1)(1+K_1)-1] \frac{1}{B_1^\theta} \left[1 - K_2 \frac{T_2}{T_1} \frac{w_2/\alpha_2}{w_1/\alpha_1}\right] - K_2 \frac{1}{B_2^\theta} \frac{T_2}{T_1} \frac{w_2/\alpha_2}{w_1/\alpha_1} + d^{2(1-\phi)} \frac{1}{B_2^\theta} = \left(\frac{\alpha_1 L_1}{T_1} - g_L\right) \frac{1-d^{2(1-\phi)}}{\rho - \frac{g_L}{\theta}(1-\eta)} \tag{27}$$

$$[h(B_2)(1+K_2)-1] \frac{1}{B_2^\theta} \left[1 - K_1 \frac{T_1}{T_2} \frac{w_1/\alpha_1}{w_2/\alpha_2}\right] - K_1 \frac{1}{B_1^\theta} \frac{T_1}{T_2} \frac{w_1/\alpha_1}{w_2/\alpha_2} + d^{2(1-\phi)} \frac{1}{B_1^\theta} = \left(\frac{\alpha_2 L_2}{T_2} - g_L\right) \frac{1-d^{2(1-\phi)}}{\rho - \frac{g_L}{\theta}(1-\eta)} \tag{28}$$

Note the symmetry in the two equations. Now, observe from the definition of K_1 that K_1 is decreasing in T_1/T_2 when $\phi > 1$. On the other hand, $K_2 \frac{T_2}{T_1}$ is increasing in T_1/T_2 when $\phi > 1 + \theta$. Also, K_2 is increasing in T_1/T_2 when $\phi > 1$, while $K_1 \frac{T_1}{T_2}$ is decreasing in T_1/T_2 when $\phi > 1 + \theta$.

In order to determine the sign of $\hat{T}_1$ and $\hat{T}_2$, we need to totally differentiate the system of equations in (27) and (28) with respect to B_1. To simplify the notation, let us define the following:

$$\begin{aligned}
A_{11} &\equiv [h(B_1)(1+K_1)-1] \frac{1}{B_1^\theta} \left[1 - K_2 \frac{T_2}{T_1} \frac{w_2/\alpha_2}{w_1/\alpha_1}\right] \\
A_{12} &\equiv -K_2 \frac{1}{B_2^\theta} \frac{T_2}{T_1} \frac{w_2/\alpha_2}{w_1/\alpha_1} \\
A_{13} &\equiv d^{2(1-\phi)} \frac{1}{B_2^\theta} \\
A_{21} &\equiv [h(B_2)(1+K_2)-1] \frac{1}{B_2^\theta} \left[1 - K_1 \frac{T_1}{T_2} \frac{w_1/\alpha_1}{w_2/\alpha_2}\right] \\
A_{22} &\equiv -K_1 \frac{1}{B_1^\theta} \frac{T_1}{T_2} \frac{w_1/\alpha_1}{w_2/\alpha_2} \\
A_{23} &\equiv d^{2(1-\phi)} \frac{1}{B_1^\theta}
\end{aligned}$$

Note that A_{11}, A_{12}, and A_{13} are respectively the first, second and third summands on the left-hand side of (27), while A_{21}, A_{22}, and A_{23} are the corresponding summands in (28). We focus our attention on equilibria in which differentiated products firms earn positive profits from their sales in both countries. From (25) and (26), we require that $1 - K_2 \frac{T_2}{T_1} \frac{w_2/\alpha_2}{w_1/\alpha_1} > 0$ and $1 - K_1 \frac{T_1}{T_2} \frac{w_1/\alpha_1}{w_2/\alpha_2} > 0$ in such equilibria. Moreover, note that $K_1, K_2 \geq 0$, and that $h(B_1), h(B_2) > 1$ for all $B > 1$. It follows that $A_{11}, A_{21} \geq 0$, $A_{12}, A_{22} \leq 0$, and $A_{13}, A_{23} \geq 0$.

Define $\hat{T}_i \equiv \frac{1}{T_i} \frac{dT_i}{dB_1}$. To facilitate the differentiation with respect to B_1, observe that:

$$\begin{aligned}
\frac{dK_1}{dB_1} &= K_1\frac{1-\phi}{\theta}(\hat{T}_1-\hat{T}_2)\\
\frac{dK_2}{dB_1} &= K_2\frac{\phi-1}{\theta}(\hat{T}_1-\hat{T}_2)\\
\frac{d\left(K_1\frac{T_1}{T_2}\right)}{dB_1} &= K_1\frac{T_1}{T_2}\frac{1-\phi+\theta}{\theta}(\hat{T}_1-\hat{T}_2)\\
\frac{d\left(K_2\frac{T_2}{T_1}\right)}{dB_1} &= K_2\frac{T_2}{T_1}\frac{\phi-\theta-1}{\theta}(\hat{T}_1-\hat{T}_2)
\end{aligned}$$

Using these expressions, we can now totally differentiate (27) and (28) with respect to B_1. This yields

$$\begin{aligned}
&\left[A_{11}\left(\frac{h(B_1)K_1}{h(B_1)(1+K_1)-1}\frac{1-\phi}{\theta}+\frac{K_2\frac{T_2}{T_1}\frac{w_2/\alpha_2}{w_1/\alpha_1}}{1-K_2\frac{T_2}{T_1}\frac{w_2/\alpha_2}{w_1/\alpha_1}}\frac{1-\phi+\theta}{\theta}\right)-A_{12}\frac{1-\phi+\theta}{\theta}\right](\hat{T}_1-\hat{T}_2)\\
&+\frac{\alpha_1L_1}{T_1}\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}\hat{T}_1\\
=\;& -A_{11}\left(\frac{h'(B_1)(1+K_1)}{h(B_1)(1+K_1)-1}-\frac{\theta}{B_1}\right) \qquad (29)
\end{aligned}$$

and

$$\begin{aligned}
&\left[A_{21}\left(\frac{h(B_2)K_2}{h(B_2)(1+K_2)-1}\frac{\phi-1}{\theta}+\frac{K_1\frac{T_1}{T_2}\frac{w_1/\alpha_1}{w_2/\alpha_2}}{1-K_1\frac{T_1}{T_2}\frac{w_1/\alpha_1}{w_2/\alpha_2}}\frac{\phi-\theta-1}{\theta}\right)-A_{22}\frac{\phi-\theta-1}{\theta}\right](\hat{T}_1-\hat{T}_2)\\
&+\frac{\alpha_2L_2}{T_2}\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}\hat{T}_2\\
=\;& (A_{22}+A_{23})\frac{\theta}{B_1} \qquad (30)
\end{aligned}$$

For notational ease, let us define:

$$\begin{aligned}
D_1 &\equiv A_{11}\left(\frac{h(B_1)K_1}{h(B_1)(1+K_1)-1}\frac{\phi-1}{\theta}+\frac{K_2\frac{T_2}{T_1}\frac{w_2/\alpha_2}{w_1/\alpha_1}}{1-K_2\frac{T_2}{T_1}\frac{w_2/\alpha_2}{w_1/\alpha_1}}\frac{\phi-\theta-1}{\theta}\right)-A_{12}\frac{\phi-\theta-1}{\theta},\\
D_2 &\equiv A_{21}\left(\frac{h(B_2)K_2}{h(B_2)(1+K_2)-1}\frac{\phi-1}{\theta}+\frac{K_1\frac{T_1}{T_2}\frac{w_1/\alpha_1}{w_2/\alpha_2}}{1-K_1\frac{T_1}{T_2}\frac{w_1/\alpha_1}{w_2/\alpha_2}}\frac{\phi-\theta-1}{\theta}\right)-A_{22}\frac{\phi-\theta-1}{\theta}.
\end{aligned}$$

Thus, D_1 is the negative of the coefficient of $\hat{T}_1-\hat{T}_2$ in (29), and D_2 is the coefficient of $\hat{T}_1-\hat{T}_2$ in (30). Let us consider a case where the differentiated varieties from the two countries are sufficiently strong substitutes. In particular, assume that: $\phi>1+\theta$. This would imply that $D_1, D_2\geq 0$ (bearing in mind that $A_{11}, A_{21}\geq 0$ and $A_{12}, A_{22}\leq 0$).

A slight rearrangement of (29) and (30) yields:

$$\left[\frac{\alpha_1 L_1}{T_1}\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}-D_1\right]\hat{T}_1+D_1\hat{T}_2 = -A_{11}\left(\frac{h'(B_1)(1+K_1)}{h(B_1)(1+K_1)-1}-\frac{\theta}{B_1}\right) \quad (31)$$

$$D_2\hat{T}_1+\left[\frac{\alpha_2 L_2}{T_2}\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}-D_2\right]\hat{T}_2 = (A_{22}+A_{23})\frac{\theta}{B_1} \quad (32)$$

which is a system of linear simultaneous equations in $\hat{T}_1$ and $\hat{T}_2$. Using Cramer's Rule, it follows that: $\hat{T}_1=\frac{de-bf}{ad-bc}$ and $\hat{T}_2=\frac{af-ce}{ad-bc}$, where:

$$de-bf = -A_{11}\left(\frac{h'(B_1)(1+K_1)}{h(B_1)(1+K_1)-1}-\frac{\theta}{B_1}\right)\left[\frac{\alpha_2 L_2}{T_2}\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}-D_2\right]-D_1(A_{22}+A_{23})\frac{\theta}{B_1}$$

$$af-ce = (A_{22}+A_{23})\frac{\theta}{B_1}\left[\frac{\alpha_1 L_1}{T_1}\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}-D_1\right]+A_{11}\left(\frac{h'(B_1)(1+K_1)}{h(B_1)(1+K_1)-1}-\frac{\theta}{B_1}\right)D_2$$

$$ad-bc = \left[\frac{\alpha_1 L_1}{T_1}\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}-D_1\right]\left[\frac{\alpha_2 L_2}{T_2}\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}-D_2\right]-D_1 D_2$$

For tractability, from now on, we shall assume symmetry, i.e. $w_1=w_2=w$, $\alpha_1=\alpha_2=\alpha$, $L_1=L_2=L$, $B_1=B_2=B$, $d>1$. Therefore, $T_1=T_2=T$. It will be seen that the results we derive in the rest of the paper do seem to be robust to generalization to the asymmetric cases.

In the appendix, it is shown that a sufficient condition for $\hat{T}_2<0$ is

$$\frac{\theta B^{\theta}}{h(B)}\left[\frac{h(B)-1}{B^{\theta}}+\frac{g_L}{\rho-\frac{g_L}{\theta}(1-\eta)}\right]>\frac{2d^{1-\phi}}{1-d^{2(1-\phi)}}\left[2(\phi-1)-\theta\left(1+d^{1-\phi}\right)\right] \quad (33)$$

For any given B, θ, η, ρ, and g_L, it can be easily shown that this holds when ϕ is sufficiently large, or d is sufficiently large. The fact that $\widehat{T}_1+\widehat{T}_2>0$ and $\widehat{T}_1>\widehat{T}_2$ means that $\widehat{T}_2<0$ implies that $\widehat{T}_1>0$. Therefore, the following lemma establishes how the country stock of ideas responds to inventive step policy:

Lemma 3: Consider equilibria of the two-country model in which both countries produce a positive amount of output in both the homogeneous and differentiated goods sectors. Then, $\frac{\partial T_i}{\partial B_i}>0$ and $\frac{\partial T_j}{\partial B_i}<0$, where $i\in\{1,2\}, i\neq j$ as long as ϕ is sufficiently large or d is sufficiently large.

Proof: See the Appendix for the derivation of (33) and an analysis of its property.

Thus, an increase in the inventive step raises the domestic stock of ideas, so that *ceteris paribus* this patent instrument does indeed have its presumed intended effect of promoting innovations. On the other hand, the model features a negative cross-border effect, in that a higher inventive step lowers the stock of ideas in the foreign country. Intuitively, an increase in B_1 promotes domestic innovation and raises the average productivity of domestic firms in the differentiated goods sector. This erodes the profitability of R&D for foreign firms in this sector, leading to a shift of labor out of the foreign differentiated goods sector into the traditional good sector there. Therefore, the research intensity and hence the stock of ideas falls in the foreign country.

To address the issue of welfare, refer to equation (6). To facilitate exposition, define $\Psi_i \equiv \left((T_i^{-\frac{1}{\theta}} w_i)^{1-\phi} + (T_j^{-\frac{1}{\theta}} w_j d)^{1-\phi}\right)^{-\frac{1}{1-\phi}}$ where $i \neq j$. Differentiating (6) with respect to B_1, we get:

$$\theta \frac{d \ln B_1^{-2} \Psi_1}{dB_1} = \frac{-2\theta}{B_1} + \frac{(T_1^{-\frac{1}{\theta}} w_1)^{1-\phi} \hat{T}_1 + (T_2^{-\frac{1}{\theta}} w_2 d)^{1-\phi} \hat{T}_2}{(T_1^{-\frac{1}{\theta}} w_1)^{1-\phi} + (T_2^{-\frac{1}{\theta}} w_2 d)^{1-\phi}}$$

Treating the inventive step as a policy instrument, what then is the nature of the tradeoff facing each country in deciding on its optimal patent policy? Without loss of generality, let us illustrate this for country 1 based on (6). A more active patenting regime corresponding to a higher B_1 should in principle raise the aggregate stock of ideas available, hence providing a boost to consumption in country 1. This benefit is reflected in the Ψ_1 term in (6) for country 1 welfare, although it is not *a priori* obvious from the algebraic expression how this will vary with B_1. Since an increase in B_1 would raise T_1, but lower T_2, so the net effect on Ψ_1 needs to be formally worked out from the above equation. Assuming symmetry as before, we get

$$\theta \frac{d \ln B_1^{-2} \Psi_1}{dB_1} = \frac{-2\theta}{B_1} + \frac{d^{1-\phi} \hat{T}_2 + \hat{T}_1}{d^{1-\phi} + 1} \tag{34}$$

It is clear that $sgn\left(\frac{d\Psi_1}{dB_1}\right) = sgn\left(d^{1-\phi}\hat{T}_2 + \hat{T}_1\right)$. Since $\widehat{T}_1 + \widehat{T}_2 > 0$ and $\widehat{T}_1 > \widehat{T}_2$, it is clear that $d^{1-\phi}\hat{T}_2 + \hat{T}_1 > 0$. Therefore, we have the following:

Lemma 4: Consider equilibria of the two-country model in which both countries produce a positive amount of output in both the homogeneous and differentiated goods sectors. We have: $\frac{\partial \Psi_i}{\partial B_i} > 0$, where $i \in \{1, 2\}$.

Thus, it is indeed the case that an increase in B_1 would raise Ψ_1 and thus confer a net benefit to country 1's consumers. In particular, the increase in the domestic stock of ideas T_1 outweighs the negative effect on T_2, intuitively because the effect of T_2 on Ψ_1 is more muted due to the distance barrier, d.

On the cost side however, ideas that improve on the current best patented idea by a factor smaller than B_1 are not eligible for a patent and hence cannot enter the market, so that consumers have to pay a higher price for these particular varieties even though a more productive technology has become feasible. The higher is B_1, the larger the consumer surplus loss due to the protection that the inventive step affords to existing patent holders. This cost of patent protection is captured by the B_1^{-2} term in (6). From the perspective of country 1, the optimal patent policy choice thus entails trading off the benefit of more innovation against this short-term consumer surplus loss.[20]

Incidentally, it is straightforward to verify as a consistency check that as $d \to \infty$, (5) simplify to the expression for welfare in the closed-economy case, namely (23). In other words, the cross-border spillover of ideas vanishes if goods cannot be traded due to a prohibitive transport cost, and so welfare would simply be given by the closed-economy expression with the domestic country stock of ideas.

5.2 Nash equilibrium analysis

Having derived the country welfare expressions, we are now ready to tackle the question of how inventive step policies are determined. Following Grossman and Lai (2004), we model this policy question as a one-shot non-cooperative game, with B_1 and B_2 chosen simultaneously by the respective country governments.

We assume that each country government seeks to maximize their respective country's welfare (they are national welfare-maximizers), but that they do not internalize the spillover effects which their policy choice has on the foreign country. More formally, given the current choice of inventive step protection being exercised by Country 2, the Country 1 government would do best by choosing: $B_1^* = \arg\max_{B_1} B_1^{-2}\Psi_1$. This delivers Country 1's best-response function (BRF), which expresses B_1^* as a function of B_2. Likewise, given B_1, Country 2's BRF is given by solving $B_2^* = \arg\max_{B_2} \; B_2^{-2}\Psi_2$. The Nash Equilibrium outcome is then determined as a point of intersection of the two best-response functions.

At first glance, this view of how inventive steps are set may seem at odds with the observation that *de jure* patent laws are rarely changed in practice. Nevertheless, we take B_1 and B_2 in our model to reflect the actual *de facto* implementation of these formal laws, namely how stringently patent laws are applied. The degree of this enforcement clearly varies more substantially across countries and across time, and is plausibly more responsive to the policy choices made by a country's trading partners.

To obtain the BRF for Country 1 (BRF_1), we calculate $\frac{\partial \ln(B_1^{-2}\Psi_1)}{\partial B_1}$ given B_2, and set this partial derivative to zero. This computation lead to the following equation for BRF_1:

$$\frac{-2\theta}{B_1} + \frac{(T_1^{-\frac{1}{\theta}} w_1)^{1-\phi}\hat{T}_1 + (T_2^{-\frac{1}{\theta}} w_2 d)^{1-\phi}\hat{T}_2}{(T_1^{-\frac{1}{\theta}} w_1)^{1-\phi} + (T_2^{-\frac{1}{\theta}} w_2 d)^{1-\phi}} = 0 \tag{35}$$

In particular, the marginal benefit term is a weighted average of $\hat{T}_1$ and $\hat{T}_2$, namely a weighted average of the percentage change in the idea stocks in each country. Likewise, solving for $\frac{\partial \ln(B_2^{-2}\Psi_2)}{\partial B_2} = 0$ yields the BRF for Country 2 (BRF_2):

$$\frac{-2\theta}{B_2} + \frac{(T_2^{-\frac{1}{\theta}} w_2)^{1-\phi}\hat{T}_2' + (T_1^{-\frac{1}{\theta}} w_1 d)^{1-\phi}\hat{T}_1'}{(T_2^{-\frac{1}{\theta}} w_2)^{1-\phi} + (T_1^{-\frac{1}{\theta}} w_1 d)^{1-\phi}} = 0 \tag{36}$$

[20] It should also be clear that the problem of maximizing the present discounted flow of real wages will boil down to choosing B_1 to maximize the per-period flow welfare given by (6).

where $\hat{T}_i' \equiv \frac{1}{T_i}\frac{dT_i}{dB_2}$.

Focusing first on BRF_1, we show in the Appendix that, under symmetry, for a given level of B_2, the expression in the curly brackets on the left-hand side of (35) is in fact a monotonically decreasing function of B_1, so that the value of B_1 that maximizes Country 1 welfare is in fact unique. Given the underlying symmetry between (35) and (36), it follows that for each given level of B_1, there is a unique B_2^* that maximizes welfare in Country 2. In short, the BRFs for both countries are well-defined functions, and the Nash Equilibrium inventive steps are determined by the intersection point of the two BRFs in B_1-B_2 space. We have found through extensive numerical simulations that B_1 and B_2 are typically strategic complements, in the sense that both BRFs tend to be upward-sloping functions when $B_1, B_2 \geq 1$. Moreover, for a wide range of standard parameterizations, we have found that there is a unique, stable Nash equilibrium to the inventive step-setting game. (We have found no examples to the contrary, and it is our conjecture that this can be proven formally.)

A central question in this line of research concerns the efficiency of the patent policies that countries adopt in the strategic equilibrium. A key result in Grossman and Lai (2004) is that the Nash equilibrium in their patent policy game features under-protection of patent lengths, in the sense that a Pareto improvement can be achieved if both countries could jointly coordinate to raise their patent length protection. This results rests on the existence of a net positive cross-border externality as a country extends its patent length: This raises the incentives to innovate in the foreign country, but this benefit is not fully internalized in the setting of the domestic patent length. What then is the nature of the cross-border externality in our context, where the nature of innovation and the instrument of patent protection is markedly different? To tackle this question, we need to understand the nature of the cross-border externality present in our model. Observe from (7) that Country 1's choice of inventive step B_1 affects the welfare of Country 2 through the term Ψ_2, namely through its effect on the stock of ideas that Country 2 effectively consumes. In fact, it can be shown that

$$\theta\frac{d\ln B_2^{-2}\Psi_2}{dB_1} = \frac{(T_1^{-\frac{1}{\theta}}w_1 d)^{1-\phi}\hat{T}_1 + (T_2^{-\frac{1}{\theta}}w_2)^{1-\phi}\hat{T}_2}{(T_1^{-\frac{1}{\theta}}w_1 d)^{1-\phi} + (T_2^{-\frac{1}{\theta}}w_2)^{1-\phi}} = \frac{d^{1-\phi}\hat{T}_1 + \hat{T}_2}{d^{1-\phi}+1} \qquad \text{under symmetry}$$

Therefore, there is negative (positive) externality when $d^{1-\phi}\hat{T}_1 + \hat{T}_2$ is less (greater) than zero. In the appendix, it is shown that a sufficient condition for $d^{1-\phi}\hat{T}_1 + \hat{T}_2 < 0$ is

$$\frac{1}{1-d^{1-\phi}}\left[2(\phi-1) - \theta\left(1+d^{1-\phi}\right)\right] > \frac{\theta B^\theta}{h(B)}\left[\frac{h(B)-1}{B^\theta} + \frac{g_L}{\rho - \frac{g_L}{\theta}(1-\eta)}\right] > \frac{2d^{1-\phi}}{1-d^{2(1-\phi)}}\left[2(\phi-1) - \theta\left(1+d^{1-\phi}\right)\right] \tag{37}$$

In fact, it can be shown that the expression on the left hand side increases with ϕ, while the one on the right hand side has a hump-shaped relationship with ϕ as well as decreases with d. The expression on the right hand side tends to zero as ϕ gets large, whereas the expression on the left hand side increases without bound with ϕ. This is shown in Figure 1. It can be seen that for any given B, θ, η, ρ, g_L, the above set of inequalities (37) holds when d or ϕ is sufficiently large.[21]

[FIGURE 1 ABOUT HERE]

Therefore, we have:

Proposition 2 ***[Negative externality of inventive step protection]*** *Consider equilibria of the two-country model in which both countries produce a positive amount of output in both the homogeneous and differentiated goods sectors. We have:* $\frac{\partial U_j}{\partial B_i} < 0$*, where* $i \in \{1, 2\}$ *and* $j \neq i$ *as long as* ϕ *or* d *is sufficiently large .*

Thus, an increase in Country 1's inventive step tends to lower Ψ_2 when ϕ is sufficiently large and hence lower welfare in Country 2. In other words, we in fact have a negative cross-border spillover of Country 1's inventive step policy on Country 2 , and vice versa when foreign and domestic goods are highly substitutable with each other. In the Appendix, we show that it is possible to find conditions under which positive externality occurs, as Figure 1 indicates.[22]

Now we turn to the highly policy-relevant issue of whether there is global over- or under-protection of IPR protection in Nash equilibrium. To answer this question, we show below that under symmetry the Nash equilibrium inventive step is greater than the globally optimal one for any given d as long as there is negative cross-border externality of raising the inventive step requirement.

First, define

$$\hat{T}_i' \equiv \frac{1}{T_i}\frac{dT_i}{dB_2}$$

Due to symmetry, to calculate the globally optimal B, we only need to focus on maximizing $B_1^{-2}\Psi_1$ by choosing B, bearing in mind that (1) both B_1 and B_2 change as B changes, (2) $dB_1 = dB_2$, and (3) each of B_1 and B_2 affects T_1 as well as T_2. Therefore, to obtain the globally optimal B under symmetry, we need to solve

$$\frac{dB_1^{-2}\Psi_1}{dB_1} + \frac{dB_1^{-2}\Psi_1}{dB_2} = 0$$

and then set $B_1 = B_2 = B$. Note that $\frac{dB_1^{-2}\Psi_1}{dB_2}$ and $\frac{dB_2^{-2}\Psi_2}{dB_1}$ are mirror images of each other. Therefore, the above equation is equivalent to

$$\frac{B_1}{\theta}\left[\frac{-2\theta}{B_1} + \frac{(T_1^{-\frac{1}{\theta}}w_1)^{1-\phi}\hat{T}_1 + (T_2^{-\frac{1}{\theta}}w_2 d)^{1-\phi}\hat{T}_2}{(T_1^{-\frac{1}{\theta}}w_1)^{1-\phi} + (T_2^{-\frac{1}{\theta}}w_2 d)^{1-\phi}}\right] + \frac{B_2}{\theta}\left[\frac{(T_2^{-\frac{1}{\theta}}w_2 d)^{1-\phi}\hat{T}_2' + (T_1^{-\frac{1}{\theta}}w_1)^{1-\phi}\hat{T}_1'}{(T_2^{-\frac{1}{\theta}}w_2 d)^{1-\phi} + (T_1^{-\frac{1}{\theta}}w_1)^{1-\phi}}\right] = 0$$

[21] To be more exact, there is negative externality when d is sufficiently large so that case (a) shown in Figure 1 occurs, and ϕ is sufficiently large so that the externality falls into the negative range indicated in case (a).

[22] It is shown in the Appendix that a positive cross-border externality occurs when d and ϕ are intermediate in values.

Symmetry of the two countries implies that $\hat{T}_1 = \hat{T}'_2$ and $\hat{T}_2 = \hat{T}'_1$. So, we have

$$\frac{B}{\theta}\left[\frac{-2\theta}{B} + \frac{(T^{-\frac{1}{\theta}}w)^{1-\phi}\left(1+d^{1-\phi}\right)\hat{T}_1 + (T^{-\frac{1}{\theta}}wd)^{1-\phi}\left(1+d^{1-\phi}\right)\hat{T}_2}{(T^{-\frac{1}{\theta}}w)^{1-\phi}\left(1+d^{1-\phi}\right)}\right] = 0$$

$$\Leftrightarrow \quad \frac{-2\theta}{B} + \hat{T}_1 + \hat{T}_2 = 0$$

On the other hand, the condition that determines the Nash equilibrium B under symmetry is obtained by setting the expression in (34) to zero, which becomes

$$\frac{-2\theta}{B} + \frac{\hat{T}_1 + d^{1-\phi}\hat{T}_2}{1+d^{1-\phi}} = 0$$

Now, it can be easily shown that

$$\frac{\hat{T}_1 + d^{1-\phi}\hat{T}_2}{1+d^{1-\phi}} > \hat{T}_1 + \hat{T}_2 \quad \text{for any given } d > 1$$

$\Longleftrightarrow$

$$\hat{T}_2 + d^{1-\phi}\hat{T}_1 < 0 \qquad \text{for any given } d$$

which is true iff there is negative cross-border externality, as shown in (**??**).

Therefore, Nash $B >$ globally optimal B for any given d iff there is negative cross-border externality from raising B. ■

Therefore, we have

Proposition 3 ***[Global over-provision of inventive step requirement in equilibrium]*** *Under symmetry, when ϕ or d is sufficiently large, and suppose that both countries set strictly binding inventive steps in Nash Equilibrium ($B_1^*, B_2^* > 1$ where $B_1^* = B_2^* = B^*$). There is over-provision of inventive step requirement in Nash Equilibrium, in that there exist B_1^{**} and B_2^{**} (where $B_1^{**} = B_2^{**} = B^{**}$) with $1 < B^{**} < B^*$, such that welfare in both countries will increase if their inventive step policies are lowered to B^{**}.*

We illustrate this finding in Figure 2, which highlights the contrast between our result and Grossman and Lai (2004). In Figure 2a, we sketch the BRFs in our present model for the typical case in which the BRFs are upward-sloping (inventive steps are strategic complements). Given the negative nature of the effect of a higher B_1 on Country 2's welfare, the iso-welfare curves along BRF_2 correspond to higher levels of welfare as we move in a leftward direction (the direction of decreasing B_1). Similarly, the iso-welfare curves along BRF_1 reflect increased levels of welfare as we move downward (in the direction of decreasing B_2). The shaded area to the southwest of the Nash equilibrium thus corresponds to a region of stronger inventive step protection in both countries that nevertheless leads to a Pareto improvement in welfare. In short, there is over-provision of inventive step protection in Nash equilibrium. This stands in contrast with the situation in Grossman and Lai (2004), which is shown in Figure 2b. There, patent length policies are in fact strategic substitutes in the central case they consider, hence the downward-sloping best-response functions.[23] Here, as the patent length policy in Country 1, Ω_1, increases, welfare in Country 2 also increases, reflecting the positive cross-border externality. The direction of increasing welfare for the Country 2 iso-welfare curves is thus to the right (the direction of higher Ω_1). An analogous argument applies for Country 1's iso-welfare curves. Taken together, there is a region to the northeast of the Nash equilibrium that illustrates combinations of patent length policies that would yield Pareto superior outcomes, so that the Nash equilibrium features under-protection of patent lengths instead.

[FIGURE 2 ABOUT HERE]

The contrast between Proposition 3 and the conclusions in Grossman and Lai (2004) have an intuitive explanation that hinges on the differences in the nature of innovation in the two papers. In this paper, the measure of varieties from each country is fixed and innovation takes place along the quality dimension, and the natural concept of IPR protection is the inventive step. Firms from different countries compete directly with each other by selling goods that are highly substitutable with each other, and this direct competition gives rise to the negative cross-border externality. When Country 1 raises its inventive step, B_1, this raises T_1 and hence also the relative productivity position that Country 1 has in the differentiated goods sector. As domestic and foreign goods are highly substitutable, and Country-2 firms competes at an disadvantage in Country-1 market (due to trade barriers), the profitability of R&D for firms in Country 2 falls, leading to a decrease in T_2. Overall, consumers in Country 1 gain because Ψ_1 increases overall; the increase in T_1 outweighs the effect of the fall in T_2, as consumers can only access Country 2 technologies after paying an iceberg transport cost d. Conversely, for Country 2, the effect of T_2 falling dominates the increase in T_1 for much the same reason, and Ψ_2 falls, generating a negative externality of Country 1's policy action on Country 2 welfare (Country 1 does not take this spillover into account when deciding B_1). This underpins the over-protection result that we have derive inventive step policy.

Grossman and Lai (2004) on the other hand, consider product innovation that expands the existing measure of varieties in the differentiated goods sector, in which the IPR protection instrument considered is the duration of the patent protection of each new variety. As a result, an increase in Ω_1 (Country 1's patent length) benefits both domestic and foreign firms by increasing the value of any global patent. This induces more product innovation from firms in all countries, even in the presence of trade barriers. The increased utility from the larger set of varieties that Country 2 can consume as well as increased profits for Country-2 firms selling in Country 1 generate a positive cross-border externality from raising

[23] It is stated in Grossman and Lai (2004) that the patent lengths can be strategic complements too for some values of β in their model (where the elasticity of substitution between skilled and unskilled labor in R&D is $1/(1-\beta)$. But those cases are considered to be rarer.

Ω_1, and hence implies a tendency towards the under-provision of patent length protection.

In sum, the verdict with regards to whether there is over- or under-protection of IPR in Nash equilibrium of the patent-setting game hinges on the sign of the cross-border externality that the patent instrument generates. In the model which we have presented above, the sign of this spillover is negative, in contrast with Grossman and Lai (2004), because goods from different countries are highly substitutable with each other and an increase in home inventive step disproportionately favors the competitiveness of home firms versus foreign firms because of the existence of trade barriers. Consequently, the conclusions with regards to whether patent protection is under- or over-provided by country governments can be sensitive to the underlying structure of innovation process and the patent instrument being considered.

We turn next to the key question of how globalization affects research intensity in each country. The can be two approaches to answering this question. First, we can assume that countries respond to globalization by changing their patent protection policies in response to it. Within the context of our model, this boils down to a question of how d affects the equilibrium inventive steps, which would in turn affect research intensities. The second approach will be based on the assumption that countries do not adjust their required inventive steps as globalization deepens, perhaps because history and institutional inertia create hurdles for governments to change their patent laws. Under such circumstances, what would be the effect of globalization on research intensities of countries? To analyze this case, we have to assume that B_1 and B_2 are exogenous and determine the effects of a decrease in d on T_1 and T_2. We shall start with the second approach, as it is very simple to analyze.

(27) and (16) imply that under symmetry

$$\frac{h(B)-1}{B^{\theta}} = \frac{\frac{\alpha L}{T} - g_L}{\rho - \frac{g_L}{\theta}(1-\eta)} \qquad \text{for any given } d \tag{38}$$

This implies that globalization (decrease in d) does not affect research intensities in both countries if $B_1 = B_2 = B$ are exogenous given. Therefore, we have

Proposition 4 ***[Effect of globalization when inventive steps are exogenous]*** *Given that countries set the same exogenous inventive steps, the research intensities of all countries remain unchanged as distance barriers are lowered.*

What is the intuition for this result? As Eaton and Kortum (2001) point out, there are two effects on the incentive to allocate resource to R&D as d decreases: 1. an increase in profitability from R&D as the increased access to the foreign market increases profit opportunities; 2. a decrease in profitability from R&D as there is increased competition from foreign firms in the home market. Each of these effects increases with the required inventive step of the market in question. When $B_1 = B_2$, the two effects are equal.

Note that equation (38) holds for $d = \infty$, which is the closed economy. Therefore, the globally optimal B for any d is the same as the optimal B in a closed economy. Since we have proved previously that when ϕ or d is sufficiently large the Nash $B >$ globally optimal B for any given d, it must be true that Nash B $(\infty > d > 1) >$ closed economy optimal $B =$ Nash B $(d = \infty)$. Consequently, the equilibrium inventive steps chosen in the open-economy setting will be strictly higher than that chosen in autarky. We record this finding in:

Proposition 5 ***[Effect of globalization when inventive steps are endogenous]*** *The equilibrium inventive step of a country is larger when it is open to trade and patenting by foreigners than when it is under autarky when ϕ or d is sufficiently large.*

This result merits some discussion. Observe first that the $\frac{-2\theta}{B_1}$ term on the left-hand side of (35) comes from the log-derivative of the B_1^{-2} term in the real wage, and hence represents the marginal cost to Country 1 of raising its inventive step. On the other hand, the remaining summand on the left-hand side of (35) correspond to the marginal benefit of raising B_1, namely from the increase in the stock of ideas, Ψ_1, accessible to Country 1's consumers. These marginal cost and marginal benefit functions are illustrated in Figure 1 for a given value of B_2; both curves are downward-sloping, but the marginal benefit function descends more steeply and cuts the marginal cost curve from above.[24]

[FIGURE 3 ABOUT HERE]

Evidently, the onset of globalization leads to a higher marginal benefit from raising inventive step protection relative to the closed-economy benchmark. Intuitively, when domestic goods and foreign goods are highly substitutable with each other, globalization leads to increased competition for Country 1 firms, as they engage in direct competition with Country 2 firms . In this setting, a higher inventive step encourages domestic innovation at the expense of that in the foreign country (T_1 increases, while T_2 decreases), improving domestic firms' productivity relative to foreign firms, and thus allowing domestic firms to compete more effectively in the product market. This effect is naturally absent in the closed-economy setting, hence explaining why inventive step protection confers a higher marginal benefit when countries can trade. (Refer to Figure 1 again)

Of note, this result also stands in contrast to Grossman and Lai (2004). In their Proposition 1, they find that globalization actually leads to a decreased propensity to extend patent length protection, the intuition being that patent policy in an open-economy setting is less effective at stimulating innovation as firms earn part of their profits in overseas markets where the patent protection extended in their

[24] To see this, note that after factoring out $\frac{\theta}{B_1}$, the marginal cost component is a constant (-2). On the other hand, the marginal benefit component remains a decreasing function of B_1; this follows from the proof that the expression in the curly brackets on the left-hand side of (35) is monotonically decreasing in B_1.

home countries does not apply. Therefore, the marginal benefit of extending patent length is lower in an open economy than in autarky. Moreover, the marginal cost of extending patent length is higher in an open economy than in autarky, as a fraction of goods consumed is imported, and an increase in patent length benefits foreign firms instead of home firms. It appears therefore that inventive step protection in quality-improvement innovation in the presence of direct competition between domestic and foreign goods that are highly substitutable in our open-economy model in fact raises the marginal benefit of patent protection without changing the marginal cost, leading to the converse result that the incentives to protect inventive steps are higher in the open-economy.

6 Conclusion

In the patent protection literature, it is widely recognized that patent length protection leads to a tradeoff between the dynamic gains from induced innovation and static losses from the increased duration of monopoly power of the patent-holders. Grossman and Lai (2004) conducted an analysis of a multi-country patent-length-setting game, based on a model of on-going product innovation. One of their major findings is that there are positive cross-border externalities from extending patent length protection. This implies that there is global *under-protection* of IPR, and that there is room for international coordination in patent policy to improve global welfare by *increasing* the strength of global patent protection. This is because an increase in patent length protection for new products benefits both domestic and foreign firms even with the presence of trade barriers. As firms do not invest in R&D so as to improve their productivity, there is no sense in which an increase in patent length hurt the competitiveness of foreign firms. Thus, there are positive cross-border externalities from extending patent length.

In the present paper, we find that if one considers required inventive step (or non-obviousness) as the instrument of patent protection in quality-improvement innovation (which we can call process innovation), there can be negative cross-border externalities from increaseing the strength of patent protection if domestic and foreign goods are sufficiently substitutable with each other. This means that there is global *over-protection* of IPR and international coordination should aim at *decreasing* global IPR protection. This is because an increase in inventive step requirement at home disproportionately increases the profitability of quality-improvement R&D at home vis-a-vis that of foreign firms, due the existence of trade barriers. High substitutability between home and foreign goods exacerbate this disadvantage of foreign firms, lowering the profitability their R&D activities and slowing their technological progress. Meanwhile, the existence of trade barriers limits the gains of foreign consumers from productivity improvement in the home country. This is the cause of the negative cross-border externalities.

Consequently, the conclusions with regard to whether patent protection is under- or over-provided by country governments can be sensitive to the underlying structure of innovation process, the patent instrument being considered, as well as the substitutability between domestic and foreign goods.

Interestingly, the existence of negative externalities in our model also leads to the conclusion that globalization leads to an increase in the Nash equilibrium inventive steps and higher equilibrium research intensities in all countries.

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8 Appendix: Details of Proofs

Proof of Lemma 1: [Patent Frontier in the Closed Economy]

Proof. The objective is to obtain an expression for $F(z;t)$, the time-t distribution of the patent frontier. Consider a short window of time between t and $t+\epsilon$ (with ϵ small) during which the patent office is open and ideas can be patented. Let $Z_{\Delta t}$ be a random variable denoting the most productive idea that arrives between time t and $t+\epsilon$. During this short time period, the stock of ideas increases by $\Delta T(t) = T(t+\epsilon) - T(t)$. Thus:

$$Pr(Z_{\Delta t} < z) = \sum_{k=0}^{\infty} \frac{\exp\{-\Delta T(t)\}(\Delta T(t))^k}{k!} H(z)^k = \exp\{-\Delta T(t)\} \sum_{k=0}^{\infty} \frac{(\Delta T(t)(1-z^{-\theta}))^k}{k!} = \exp\left\{-\Delta T(t) z^{-\theta}\right\} \tag{39}$$

(To obtain this last expression, we sum over the probability that exactly k ideas $\leq z$ arrive between time t and $t+\epsilon$, these being drawn from the incremental stock of ideas $\Delta T(t)$, where k ranges from 0 to ∞.)

We write down an expression for $F(z;t+\epsilon)$ in terms of $F(z;t)$. The probability that the patent frontier is $\leq z$ at time $t+\epsilon$ is the union of two events: (i) all ideas that arrive between t and $t+\epsilon$ do not surpass the required inventive step, and the patent frontier at time t is itself already $\leq z$, and (ii) the best idea that arrives between t and $t+\epsilon$ surpasses the required inventive step, and this new best idea is itself $\leq z$. Note that only the best idea that arrives during this short time period of length ϵ will be considered for a patent.

Let PF_t be the random variable denoting the quality of the idea at the patent frontier at time t. We have:

$$F(z;t+\epsilon) = \int_{\zeta=0}^{z} Pr(Z_{\Delta t} < B\zeta | PF_t = \zeta)\, dPr(PF_t \leq \zeta) + \int_{\zeta=0}^{z/B} Pr(B\zeta \leq Z_{\Delta t} < z | PF_t = \zeta)\, dPr(PF_t \leq \zeta)$$

The first summand captures the probability of event (i): Conditioning on the current value of the patent frontier ($PF_t = \zeta$), we evaluate the probability that the best idea that arrives does not exceed $B\zeta$. The integral sign aggregates this probability over the possible values of $\zeta \in [0,z]$, these being consistent with $PF_{t+\epsilon}$ being $\leq z$. The second summand captures event (ii): Conditioning once again on the value of the time-t patent frontier, we work out the probability that the best idea that arrives surpasses the required inventive step but nevertheless remains $\leq z$. (The upper limit of the integral is z/B, to ensure that there are ideas which surpasses the required inventive step, but nevertheless remain $\leq z$.)

Substituting in the cdf for $Z_{\Delta t}$ from (39), and simplifying, we get:

$$\begin{aligned} F(z;t+\epsilon) &= \int_{\zeta=0}^{z} \exp\{-\Delta T(t)(B\zeta)^{-\theta}\}\, dPr(PF_t \leq \zeta) \\ &\quad + \int_{\zeta=0}^{z/B} \left(\exp\{-\Delta T(t) z^{-\theta}\} - \exp\{-\Delta T(t)(B\zeta)^{-\theta}\}\right)\, dPr(PF_t \leq \zeta) \\ &= \int_{\zeta=0}^{z/B} \exp\{-\Delta T(t) z^{-\theta}\}\, dF(\zeta;t) + \int_{\zeta=z/B}^{z} \exp\{-\Delta T(t)(B\zeta)^{-\theta}\}\, dF(\zeta;t) \\ &= \exp\{-\Delta T(t) z^{-\theta}\} F(z/B;t) + \int_{\zeta=z/B}^{z} \exp\{-\Delta T(t)(B\zeta)^{-\theta}\}\, dF(\zeta;t) \\ &= \exp\{-\Delta T(t)(Bz)^{-\theta}\} F(z;t) - \int_{\zeta=z/B}^{z} \theta \Delta T(t) B^{-\theta} \zeta^{-\theta-1} \exp\{-\Delta T(t)(B\zeta)^{-\theta}\} F(\zeta;t)\, d\zeta \end{aligned}$$

where the last step follows from integrating by parts.

Now, substitute the above expression for $F(z;t+\epsilon)$ into $\frac{\partial F(z;t)}{\partial t} = \lim_{\epsilon \to 0} \frac{1}{\epsilon}\left[F(z;t+\epsilon) - F(z;t)\right]$, and apply L'Hospital's Rule. One obtains:

$$\frac{\partial\, F(z;t)}{\partial t} = -T'(t)(Bz)^{-\theta} F(z;t) - \int_{\zeta=z/B}^{z} T'(t)\theta B^{-\theta} \zeta^{-\theta-1} F(\zeta;t)\, d\zeta$$

$$\Longrightarrow \quad \log F(z;t) = \int_{s=0}^{t}\left[-T'(s)(Bz)^{-\theta} - \int_{\zeta=z/B}^{z} T'(s)\theta B^{-\theta}\zeta^{-\theta-1}\frac{F(\zeta;s)}{F(z;s)}\, d\zeta\right] ds$$

$$= -T(t)(Bz)^{-\theta} - \int_{s=0}^{t} T'(s)\int_{\zeta=z/B}^{z}\theta B^{-\theta}\zeta^{-\theta-1}\frac{F(\zeta;s)}{F(z;s)}\, dz\ ds$$

$$\Longrightarrow \quad F(z;t) = \exp\left\{-T(t)(Bz)^{-\theta}\right\}\times\exp\left\{\int_{s=0}^{t} -T'(s)\int_{\zeta=z/B}^{z}\theta B^{-\theta}\zeta^{-\theta-1}\frac{F(\zeta;s)}{F(z;s)}\, d\zeta\ ds\right\} \tag{40}$$

Note that we adopt the normalization, $T(0) = 0$ (the stock of ideas is zero in the initial time period).

Since the integrand in (40) is non-positive, it follows that $F(z;t) \leq \exp\left\{-T(t)(Bz)^{-\theta}\right\}$. Together with the assumption that $T'(t)$ is bounded with $|T'(t)| \leq k$, we have:

$$\begin{aligned}
\int_{\zeta=z/B}^{z} T'(t)\theta B^{-\theta}\zeta^{-\theta-1}F(\zeta;t)\, d\zeta &\leq \int_{\zeta=z/B}^{z} T'(t)\theta B^{-\theta}\zeta^{-\theta-1}\exp\left\{-T(t)(B\zeta)^{-\theta}\right\}\, d\zeta \\
&\leq k\left[\frac{\exp\left\{-T(t)(B\zeta)^{-\theta}\right\}}{T(t)}\right]_{\zeta=z/B}^{z} \\
&= \frac{k}{T(t)}\left(\exp\left\{-T(t)(Bz)^{-\theta}\right\} - \exp\left\{-T(t)(z)^{-\theta}\right\}\right) \\
&\to 0 \quad \text{as } T(t)\to+\infty
\end{aligned}$$

Hence, as the stock of ideas grows large, the second term in the product on the right-hand side of (40) tends to 1. The patent frontier distribution thus tends asymptotically to the Fréchet cdf: $F(z;t) = \exp\left\{-T(t)(Bz)^{-\theta}\right\}$. This distribution applies to each variety $j \in [0,1]$, since varieties are *ex ante* identical. ∎

Proof of Lemma 2: [Best Patented Idea from each Country]

Proof. We present a short proof below. An alternative proof derived from first principles in a manner similar to Lemma 1 above is available on request.

Recall that B_n is the inventive step in country n. Country n awards country-specific patents, namely where it has a separate patenting system for ideas originating from each country. Let $CSP_{ni}(t)$ denote the country-specific patent frontier for ideas from country i in country n at time t. Thus, an idea from country i receives this country-specific patent so long as it is more productive than the previous best idea holding a country-i specific patent by an inventive step of B_n.

Moreover, applying Lemma 1, as $T_i(t) \to +\infty$ for all countries i, we have:

$$\Pr\left(CSP_{ni}(t) \leq z\right) = \exp\{-T_i(t)(B_n z)^{-\theta}\}. \tag{41}$$

∎

Derivation of the Price Index and Welfare Expressions:

Proof Recall that in the open-economy model, the price index for the differentiated goods sector in country n is given by: $\left(\sum_{i=1}^{N} P_{ni}^{1-\phi}\right)^{\frac{1}{1-\phi}}$, where P_{ni} is the ideal price index for varieties originating from country i. We thus need to compute P_{ni}, the derivation of which follows closely that in the Mathematical Appendix to Bernard et al. (2003).

Consider a new idea originating from country i. (Without loss of generality, suppose this new idea is for some variety $j \in [0,1]$.) This new idea needs to surpasses the minimum inventive step requirement of B_n in order to qualify for a patent in country n. Specifically, this means that the new idea needs to improve on the existing best idea from country i for variety j which holds a patent in country n by an inventive step of at least B_n. Note in particular that if this new idea satisfies this inventive step requirement, it would automatically secure the market for that country-i variety in country n: If the current best patented idea from country i in country n for that variety is z, then the current market price for that variety would be $w_i d_{ni}/z$. A new idea that satisfies the minimum inventive step would have a market price of at most $w_i d_{ni}/B_n z$, which is $\leq w_i d_{ni}/z$. Hence, a new idea that is patentable would automatically be competitive and secure the country-n market for that variety.

Let $C_{ni}^{(1)}$ and $C_{ni}^{(2)}$ be the unit production costs corresponding to the best and second-best patented ideas respectively originating from country i in country n. These costs are random variables across varieties j from country i's differentiated goods sector. (We suppress the time subscripts for ease of notation.) Using the joint distribution for the productivity of these two best ideas from (2), we have:

$$\begin{aligned} G_{ni}^c(c_1,c_2) &= \Pr(C_{ni}^{(1)} \geq c_1, C_{ni}^{(2)} \geq c_2) \\ &= \Pr\left(Z_{ni}^{(1)} \leq \frac{w_i d_{ni}}{c_1}, Z_{ni}^{(2)} \leq \frac{w_i d_{ni}}{c_2}\right) \\ &= \left\{1 + T_i[(B_n^2 w_i d_{ni})^{-\theta} c_2^\theta - (B_n w_i d_{ni})^{-\theta} c_1^\theta]\right\} \exp\{-T_i(B_n^2 w_i d_{ni})^{-\theta} c_2^\theta\}, \quad 0 \leq c_1 \leq \frac{c_2}{B_n} \end{aligned}$$

From this, the joint distribution of the lowest and second lowest costs corresponding to ideas from country i that have been patented in country n, which we denote by $G_{ni}(c_1,c_2)$, is:

$$\begin{aligned} G_{ni}(c_1,c_2) &= \Pr(C_{ni}^{(1)} \leq c_1, C_{ni}^{(2)} \leq c_2) \\ &= 1 - \Pr(C_{ni}^{(1)} \geq c_1, C_{ni}^{(2)} \geq c_1) - \Pr(C_{ni}^{(1)} \geq 0, C_{ni}^{(2)} \geq c_2) + \Pr(C_{ni}^{(1)} \geq c_1, C_{ni}^{(2)} \geq c_2) \\ &= 1 - (1 + T_i(w_i d_{ni})^{-\theta}(B_n^{-2\theta} - B_n^{-\theta})c_1^\theta) \exp\left\{-T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} c_1^\theta\right\} \ldots \\ &\quad -T_i(w_i d_{ni})^{-\theta} B_n^{-\theta} c_1^\theta \exp\left\{-T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} c_2^\theta\right\} \end{aligned}$$

Since the firm with the best patented idea sets a price equal to the marginal cost of the second-best patented idea (limit-pricing), the prevailing market price for country i varieties in country n will be governed by the distribution of $C_{ni}^{(2)}$:

$$\begin{aligned} \Pr\left(C_{ni}^{(2)} \leq c_2\right) &= \Pr\left(C_{ni}^{(1)} \leq c_2, C_{ni}^{(2)} \leq c_2\right) \\ &= 1 - \exp\left\{-T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} c_2^\theta\right\} - T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} c_2^\theta \exp\left\{-T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} c_2^\theta\right\} \end{aligned}$$

The probability distribution function (pdf) of the market price $p_{ni}(j)$ for variety j from country i in country n is thus given by:

$$\begin{aligned} g_2(p_{ni}) &= \frac{\partial}{\partial p_{ni}} \left[1 - \exp\left\{-T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} p_{ni}^\theta\right\} - T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} p_{ni}^\theta \exp\left\{-T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} p_{ni}^\theta\right\}\right] \\ &= \theta p_{ni}^{2\theta-1} T_i^2 (w_i d_{ni})^{-2\theta} B_n^{-4\theta} \exp\left\{-T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} p_{ni}^\theta\right\} \end{aligned}$$

The ideal price index for differentiated varieties from country i in country n is therefore:

$$\begin{aligned} \exp\left\{\int_0^1 \ln p_{ni}(j)\ dj\right\} &= \exp\left\{\int_0^\infty (\ln p) g_2(p)\ dp\right\} \\ &= \exp\left\{\int_0^\infty (\ln p)\theta p_{ni}^{2\theta-1} T_i^2 (w_i d_{ni})^{-2\theta} B_n^{-4\theta} \exp\left\{-T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} p_{ni}^\theta\right\}\ dp\right\} \\ &= \exp\left\{\int_0^\infty \left[\frac{1}{\theta}\ln x + \ln\left(T_i^{-\frac{1}{\theta}} w_i d_{ni} B_n^2\right)\right] \cdot x \exp\{-x\}\ dx\right\} \end{aligned}$$

where we have performed the change of variables, $x = T_i(w_i d_{ni})^{-\theta} B_n^{-2\theta} p^\theta$, to get to this last step.

It is straightforward first to show through integration by parts that: $\int_0^\infty \ln\left(T_i^{-\frac{1}{\theta}} w_i d_{ni} B_n^2\right) x \exp\{-x\}\ dx =$

$\ln\left(T_i^{-\frac{1}{\theta}}w_i d_{ni} B_n^2\right)$. As for $\int_0^\infty \frac{1}{\theta}(\ln x)x\exp\{-x\}\ dx$, perform the change of variables, $y = -\ln x$:

$$\begin{aligned}
\int_0^\infty \frac{1}{\theta}(\ln x)x\exp\{-x\}\ dx &= \frac{1}{\theta}\int_{-\infty}^{\infty} -\exp\{-y\}\exp\{-\exp\{-y\}\}\,y\exp\{-y\}\,dy \\
&= -\frac{1}{\theta}\left[\exp\{-\exp\{-y\}\}\,y\exp\{-y\}\right]_{-\infty}^{\infty} \\
&\quad + \frac{1}{\theta}\int_{-\infty}^{\infty}\exp\{-\exp\{-y\}\}\left[\exp\{-y\} - y\exp\{-y\}\right]dy \\
&= \frac{1}{\theta}\int_{-\infty}^{\infty}\exp\{-\exp\{-y\}\}\left[\exp\{-y\} - y\exp\{-y\}\right]dy \\
&= \frac{1}{\theta}\left[\exp\{-\exp\{-y\}\}\right]_{-\infty}^{\infty} - \frac{1}{\theta}\int_{-\infty}^{\infty} y\exp\{-y\}\exp\{-\exp\{-y\}\}\,dy \\
&= \frac{1}{\theta} - \frac{1}{\theta}\int_{-\infty}^{\infty} y\exp\{-y\}\exp\{-\exp\{-y\}\}\,dy
\end{aligned}$$

But $\int_{-\infty}^{\infty} y\exp\{-y\}\exp\{-\exp\{-y\}\}\,dy$ is the mean of the Gumbel distribution (with pdf: $\exp\{-\exp\{-y\}\}\,dy$). This is precisely equal to γ, the Euler-Mascheroni constant. So, $\int_0^\infty \frac{1}{\theta}(\ln x)x\exp\{-x\}\ dx = \frac{1-\gamma}{\theta}$. The ideal price index, $P_{ni} = \exp\left\{\int_0^1 \ln p(j)_{ni}\ dj\right\}$ is thus equal to $T_i^{-\frac{1}{\theta}}w_i d_{ni} B_n^2 \exp\left\{\frac{1-\gamma}{\theta}\right\}$.

The differentiated goods sector price index for country n is thus: $\left(\sum_{i=1}^N\left(T_i^{-\frac{1}{\theta}}w_i d_{ni} B_n^2\exp\left\{\frac{1-\gamma}{\theta}\right\}\right)^{1-\phi}\right)^{\frac{1}{1-\phi}}$, which is equal to: $B_n^2\left(\sum_{i=1}^N\left(T_i^{-\frac{1}{\theta}}w_i d_{ni}\right)^{1-\phi}\right)^{\frac{1}{1-\phi}}$ up to a multiplicative constant. In particular, for the two-country case which we consider in Section 5 of the paper, this price index is equal to: $B_1^2\left(\left(T_1^{-\frac{1}{\theta}}w_1\right)^{1-\phi} + \left(T_2^{-\frac{1}{\theta}}w_2 d\right)^{1-\phi}\right)^{\frac{1}{1-\phi}}$ for country 1; and equal to: $B_2^2\left(\left(T_2^{-\frac{1}{\theta}}w_2\right)^{1-\phi} + \left(T_1^{-\frac{1}{\theta}}w_1 d\right)^{1-\phi}\right)^{\frac{1}{1-\phi}}$ for country 2. The corresponding welfare expressions (the real wage) for country 1 and 2 respectively are: $w_1 B_1^{-2(1-\eta)}\left(\left(T_1^{-\frac{1}{\theta}}w_1\right)^{1-\phi} + \left(T_2^{-\frac{1}{\theta}}w_2 d\right)^{1-\phi}\right)^{-\frac{1-\eta}{1-\phi}}$ and $w_2 B_2^{-2(1-\eta)}\left(\left(T_2^{-\frac{1}{\theta}}w_2\right)^{1-\phi} + \left(T_1^{-\frac{1}{\theta}}w_1 d\right)^{1-\phi}\right)^{-\frac{1-\eta}{1-\phi}}$.

Proof of Proposition 1: [Optimal inventive step in the Closed Economy]

Proof. Log-differentiating (24), one obtains the following first-order condition:

$$-2 + \frac{\left(\rho - \frac{g_L}{\theta}(1-\eta)\right)\frac{\left(B(1+\theta)^2-\theta^2\right)\left(\frac{\eta}{1-\eta}B+1\right)+\frac{\eta}{1-\eta}B}{B^\theta[B(1+\theta)-\theta]^2}}{g_L + \left(\rho - \frac{g_L}{\theta}(1-\eta)\right)\frac{\left(\frac{\eta}{1-\eta}\right)B(1+\theta)+\theta}{B^\theta[B(1+\theta)-\theta]}} = 0$$

Note that the denominator of the fraction on the left-hand side is positive, since: (i) $(1+\theta)B - \theta > 0$, as $B \geq 1 > \theta/(1+\theta)$; and (ii) $\frac{\rho}{g_L} - \frac{1-\eta}{\theta} > 0$. Upon further simplification, the first-order condition is equivalent to:

$$-2B^\theta\left[B(1+\theta)-\theta\right]^2 + \left(\frac{\rho}{g_L} - \frac{1-\eta}{\theta}\right)\left[-\frac{\eta}{1-\eta}B^2(1+\theta)^2 + \frac{\eta}{1-\eta}B(1+\theta)^2 + B(1-\theta^2) + \theta^2\right] = 0$$

Let Γ denote the expression on the left-hand side of the above equation. We have: Differentiating this with respect to B yields:

$$\begin{aligned}
\frac{d\Gamma}{dB} &= -2B^{\theta-1}\left(B(1+\theta)-\theta\right)\left[\theta\left(B(1+\theta)-\theta\right) + 2B(1+\theta)\right] \\
&\quad + \left(\frac{\rho}{g_L} - \frac{1-\eta}{\theta}\right)\left[\frac{\eta}{1-\eta}(1-2B)(1+\theta)^2 + (1-\theta^2)\right] \\
&< 0
\end{aligned}$$

for all $B \geq 1$ and $\theta > 1$.

Observe that: (i) $\Gamma < 0$ as $B \to \infty$; and (ii) $\Gamma = -2 + \left(\frac{\rho}{g_L} - \frac{1-\eta}{\theta}\right)$ when $B = 1$. Since $\frac{d\Gamma}{dB} < 0$ for all $B > 1$, the intermediate value theorem implies that a necessary and sufficient condition for a solution to the first-order condition to exist is: $\frac{\rho}{g_L} - \frac{1-\eta}{\theta} > 2$. When this holds, the graph of Γ as a function of B starts out at a positive value at $B = 1$ and declines monotonically, eventually taking on a negative value for B sufficiently big. There thus exists a unique $B^* > 1$ that satisfies the first-order condition. Moreover, $sign(\Gamma) > 0$ for $1 \geq B < B^*$ and $sign(\Gamma) < 0$ for $B > B^*$, which means that the slope of the original objective function is positive for $1 \geq B < B^*$ and negative for $B > B^*$. In other words, the turning point at B^* is indeed a maxima.

On the other hand, when $0 < \frac{\rho}{g_L} - \frac{1-\eta}{\theta} < 2$, the Γ function always takes on a negative value, which means that original objective function is decreasing for all $B \geq 1$. The maximum is therefore achieved at a corner solution, $B^* = 1$. ■

Proof that there in negative cross-border externality when ϕ is sufficiently large and positive externality when ϕ and d are intermediate in value (or, externality would never be positive when d is large)

Proof We can be easily show that under symmetry

$$
\begin{aligned}
K_1 &= d^{1-\phi} \\
K_2 &= d^{1-\phi} \\
A_{11} &= \frac{\left[h(B)\left(1+d^{1-\phi}\right)-1\right]\left(1-d^{1-\phi}\right)}{B^\theta} > 0 \\
A_{12} &= -\frac{d^{1-\phi}}{B^\theta} < 0 \\
A_{13} &= \frac{d^{2(1-\phi)}}{B^\theta} > 0 \\
A_{21} &= A_{11} \\
A_{22} &= A_{12} \\
A_{23} &= A_{13} \\
D_1 &= \frac{d^{1-\phi}h(B)}{\theta B^\theta}\left[2(\phi-1)-\theta\left(1+d^{1-\phi}\right)\right] > 0 \text{ given } \phi > \theta+1 \\
D_2 &= D_1 = D
\end{aligned}
$$

Referring to (27) and (28), it can be easily shown that under symmetry

$$
\begin{aligned}
\frac{h(B)-1}{B^\theta} &= \frac{\frac{\alpha L}{T} - g_L}{\rho - \frac{g_L}{\theta}(1-\eta)} \qquad \text{for all } d \\
\Rightarrow \quad \frac{\alpha L}{T\left[\rho - \frac{g_L}{\theta}(1-\eta)\right]} &= \frac{h(B)-1}{B^\theta} + \frac{g_L}{\rho - \frac{g_L}{\theta}(1-\eta)} \qquad \text{for all } d
\end{aligned}
$$

This equation will be useful below.

Recall that there is negative (positive) externality when $d^{1-\phi}\hat{T}_1 + \hat{T}_2$ is less (greater) than zero.

1. Negative externality

A sufficient condition for negative externality (i.e. $d^{1-\phi}\hat{T}_1 + \hat{T}_2 < 0$) is $ad - bc > 0$ and $(af - ce) + (de - bf)\, d^{1-\phi} < 0$

A sufficient condition for $ad - bc > 0$, $af - ce < 0$ and $de - bf > 0$ is

$$
\frac{\alpha L}{T}\left[\frac{1-d^{2(1-\phi)}}{\rho - \frac{g_L}{\theta}(1-\eta)}\right] - D > D
$$

which is equivalent to

$$
\begin{aligned}
&\left[\frac{h(B)-1}{B^\theta} + \frac{g_L}{\rho - \frac{g_L}{\theta}(1-\eta)}\right]\left(1-d^{2(1-\phi)}\right) \\
&> \frac{2d^{1-\phi}h(B)}{\theta B^\theta}\left[2(\phi-1)-\theta\left(1+d^{1-\phi}\right)\right] \qquad (42)
\end{aligned}
$$

Given that $\frac{\alpha L}{T}\left[\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}\right]-D>D$, a sufficient condition for $\frac{|af-ce|}{d^{1-\phi}}>de-bf$ (so as to get negative externality) is

$$\frac{D}{d^{1-\phi}}>\frac{\alpha L}{T}\left[\frac{1-d^{2(1-\phi)}}{\rho-\frac{g_L}{\theta}(1-\eta)}\right]-D$$

which is equivalent to

$$\begin{aligned}&\frac{\left(1+d^{1-\phi}\right)h(B)}{\theta B^{\theta}}\left[2(\phi-1)-\theta\left(1+d^{1-\phi}\right)\right]\\ >\ &\left[\frac{h(B)-1}{B^{\theta}}+\frac{g_L}{\rho-\frac{g_L}{\theta}(1-\eta)}\right]\left(1-d^{2(1-\phi)}\right)\end{aligned} \tag{43}$$

Combining (42) and (43), we have

$$\begin{aligned}&\frac{\left(1+d^{1-\phi}\right)h(B)}{\theta B^{\theta}}\left[2(\phi-1)-\theta\left(1+d^{1-\phi}\right)\right]\\ >\ &\left[\frac{h(B)-1}{B^{\theta}}+\frac{g_L}{\rho-\frac{g_L}{\theta}(1-\eta)}\right]\left(1-d^{2(1-\phi)}\right)\\ >\ &\frac{2d^{1-\phi}h(B)}{\theta B^{\theta}}\left[2(\phi-1)-\theta\left(1+d^{1-\phi}\right)\right]\end{aligned} \tag{44}$$

which is equivalent to (37):

$$\frac{1}{1-d^{1-\phi}}\left[2(\phi-1)-\theta\left(1+d^{1-\phi}\right)\right]>\frac{\theta B^{\theta}}{h(B)}\left[\frac{h(B)-1}{B^{\theta}}+\frac{g_L}{\rho-\frac{g_L}{\theta}(1-\eta)}\right]>\frac{2d^{1-\phi}}{1-d^{2(1-\phi)}}\left[2(\phi-1)-\theta\left(1+d^{1-\phi}\right)\right]$$

The above condition (44) together with $\phi>\theta+1$ would guarantee that

1. $ad-bc>0$
2. $de-bf>0$
3. $af-ce<0$
4. $\frac{|af-ce|}{d^{1-\phi}}>de-bf$
5. and therefore, an increase in B_1 leads to a decrease in U_2 — there is negative cross-border externality from increasing the inventive step requirement at home.

The effect of ϕ on the LHS of (37) is always positive for all ϕ, since the effects of $\phi-1$ and $\phi-\theta-1$ are going to dominate that of $\frac{1+d^{1-\phi}}{1-d^{1-\phi}}$. Proof:

$$\begin{aligned}&\frac{\partial}{\partial\phi}\left[\frac{2(\phi-1)-\theta\left(1+d^{1-\phi}\right)}{1-d^{1-\phi}}\right]\\ =\ &\frac{\left(1-d^{1-\phi}\right)^2+1-d^{2(1-\phi)}-2d^{2(1-\phi)}(\phi-\theta-1)\ln d}{\left(1-d^{1-\phi}\right)^2}\end{aligned} \tag{45}$$

Note that the above expression is positive for all $1<\phi\leq\theta+1$. Now, taking derivative of the numerator,

$$\begin{aligned}&\frac{\partial}{\partial\phi}\left[\left(1-d^{1-\phi}\right)^2+1-d^{2(1-\phi)}-2d^{1-\phi}(\phi-\theta-1)\ln d\right]\\ =\ &\ln d\left[2d^{2(1-\phi)}+4d^{2(1-\phi)}(\phi-\theta-1)(\ln d)-2d^{2(1-\phi)}+2\left(1-d^{1-\phi}\right)d^{1-\phi}\right]\\ =\ &\ln d\left[4d^{2(1-\phi)}(\phi-\theta-1)(\ln d)+2\left(1-d^{1-\phi}\right)d^{1-\phi}\right]>0\quad\text{for }\phi\geq\theta+1\end{aligned}$$

We conclude that

$$\frac{\partial}{\partial\phi}\left[\frac{2(\phi-1)-\theta\left(1+d^{1-\phi}\right)}{1-d^{1-\phi}}\right]>0\quad\text{for all }\phi>1$$

Moreover, the value of $\frac{2(\phi-1)-\theta\left(1+d^{1-\phi}\right)}{1-d^{1-\phi}}$ ranges from 0 to infinity as ϕ increases from small to large. Hence, for a given d, we can always find a ϕ^* such that when $\phi > \phi^*$ condition (44) is satisfied. The graph of this expression as a function of ϕ is shown in Figure 1.

2. Positive externality

A sufficient condition for positive externality (i.e. $d^{1-\phi}\hat{T}_1 + \hat{T}_2 > 0$) is $ad - bc < 0$ and $(af - ce) + (de - bf)\, d^{1-\phi} < 0$.

The sufficient conditions for $ad - bc < 0$ and $af - ce < 0$ are

$$\frac{\alpha L}{T}\left[\frac{1-d^{2(1-\phi)}}{\rho - \frac{g_L}{\theta}(1-\eta)}\right] - D < D$$

and

$$\frac{\alpha L}{T}\left[\frac{1-d^{2(1-\phi)}}{\rho - \frac{g_L}{\theta}(1-\eta)}\right] - D > 0$$

These two conditions together is equivalent to

$$0 < \frac{\frac{\alpha L}{T}\left[\frac{1-d^{2(1-\phi)}}{\rho - \frac{g_L}{\theta}(1-\eta)}\right] - D}{D} < 1 \tag{46}$$

As $\hat{T}_1 > \hat{T}_2$, we know that $\hat{T}_2 > 0 \Rightarrow \hat{T}_1 > 0 \Rightarrow dU_2/dB_1 > 0$. But (46) is already sufficient for $ad - bc < 0$ and $af - ce < 0$, which implies that $\hat{T}_2 > 0$. Therefore, (46) is a sufficient condition for $dU_2/dB_1 > 0$.

(46) implies that

$$D < \frac{\alpha L}{T}\left[\frac{1-d^{2(1-\phi)}}{\rho - \frac{g_L}{\theta}(1-\eta)}\right] < 2D$$

which is equivalent to

$$\begin{aligned} & \frac{d^{1-\phi}h(B)}{\theta B^\theta}\left[2(\phi-1) - \theta\left(1+d^{1-\phi}\right)\right] \\ < \; & \left[\frac{h(B)-1}{B^\theta} + \frac{g_L}{\rho - \frac{g_L}{\theta}(1-\eta)}\right]\left(1-d^{2(1-\phi)}\right) \\ < \; & \frac{2d^{1-\phi}h(B)}{\theta B^\theta}\left[2(\phi-1) - \theta\left(1+d^{1-\phi}\right)\right] \end{aligned}$$

which is equivalent to (**??**):

$$\frac{d^{1-\phi}}{1-d^{2(1-\phi)}}\left[2(\phi-1) - \theta\left(1+d^{1-\phi}\right)\right] < \frac{\theta B^\theta}{h(B)}\left[\frac{h(B)-1}{B^\theta} + \frac{g_L}{\rho - \frac{g_L}{\theta}(1-\eta)}\right] < \frac{2d^{1-\phi}}{1-d^{2(1-\phi)}}\left[2(\phi-1) - \theta\left(1+d^{1-\phi}\right)\right]$$

The right hand side is exactly two times the left hand side. It can be shown that both left hand side and right hand side have a hump-shaped relationship with ϕ, and the expressions tend to zero as ϕ tends to infinity. Moreover, for any given ϕ, both expressions decrease with d, and increase without bound as d decreases towards one. The expressions are graphed as functions of ϕ in Figure 1. Note that the right hand side expression is the same as the right hand side expression in the last set of inequalities. The left hand side expression is exactly one half times the right hand side expression. Therefore, it also has a hump-shaped relationship with ϕ and also decreases with d. It tends to infinity as d tends to one. Therefore, from case (b) of Figure 1, we see that for any given ϕ, it is possible for there to be positive externality when d is sufficient large but not too large. Therefore, we have: $\frac{\partial U_j}{\partial B_i} > 0$ where $i \in \{1, 2\}$ and $j \neq i$, i.e. there is positive cross-border externality of strengthening inventive step requirement by a country, when ϕ and d are both intermediate in value.

Best Response Functions under Symmetry

In this notes, we try to derive the best response function under symmetry, and then show that the second order condition is satisfied.

$$
\begin{aligned}
& \widehat{T}_1 (ad - bc) \\
= & \frac{-\left[h(B_1)(1+d^{1-\phi})-1\right](1-d^{1-\phi})}{B_1^\theta}\left[\frac{h'(B_1)(1+d^{1-\phi})}{h(B_1)(1+d^{1-\phi})-1}-\frac{\theta}{B_1}\right] \\
& \times\left\{\left[\frac{h(B_2)-1}{B_2^\theta}+\frac{g_L}{\rho-\frac{g_L}{\theta}(1-\eta)}\right]\left[1-d^{2(1-\phi)}\right]-\left[\frac{d^{1-\phi}h(B_2)}{\theta B_2^\theta}\left(\phi-\theta d^{1-\phi}-1\right)+(\phi-\theta-1)\frac{d^{1-\phi}}{\theta B_1^\theta}\right]\right\} \\
& +\left[\frac{d^{1-\phi}h(B_1)}{\theta B_1^\theta}\left(\phi-1-\theta d^{1-\phi}\right)+(\phi-\theta-1)\frac{d^{1-\phi}}{\theta B_2^\theta}\right]\left[\frac{d^{1-\phi}\left(1-d^{1-\phi}\right)}{B_1^\theta}\right]\frac{\theta}{B_1}
\end{aligned}
$$

$$
\begin{aligned}
& \widehat{T}_2 (ad - bc) \\
= & -\left[\frac{d^{1-\phi}\left(1-d^{1-\phi}\right)}{B_1^\theta}\right]\frac{\theta}{B_1} \\
& \times\left\{\left[\frac{h(B_1)-1}{B_1^\theta}+\frac{g_L}{\rho-\frac{g_L}{\theta}(1-\eta)}\right]\left[1-d^{2(1-\phi)}\right]-\left[\frac{d^{1-\phi}h(B_1)}{\theta B_1^\theta}\left(\phi-\theta d^{1-\phi}-1\right)+(\phi-\theta-1)\frac{d^{1-\phi}}{\theta B_2^\theta}\right]\right\} \\
& +\frac{\left[h(B_1)(1+d^{1-\phi})-1\right](1-d^{1-\phi})}{B_1^\theta} \\
& \times\left[\frac{h'(B_1)(1+d^{1-\phi})}{h(B_1)(1+d^{1-\phi})-1}-\frac{\theta}{B_1}\right]\left[\frac{d^{1-\phi}h(B_2)}{\theta B_2^\theta}\left(\phi-1-\theta d^{1-\phi}\right)+(\phi-\theta-1)\frac{d^{1-\phi}}{\theta B_1^\theta}\right]
\end{aligned}
$$

The best response function of country 1 is given by (35), which can be re-written as:

$$
-2\theta+\left\{\frac{\left[\left(\frac{T_1}{T_2}\right)^{-\frac{1}{\theta}}\times\frac{w_1}{w_2 d}\right]^{1-\phi}B_1\hat{T}_1+B_1\hat{T}_2}{\left[\left(\frac{T_1}{T_2}\right)^{-\frac{1}{\theta}}\times\frac{w_1}{w_2 d}\right]^{1-\phi}+1}\right\}=0
$$

Since an increase in B_1 leads to an increase in $\frac{T_1}{T_2}$, the effect of B_1 on $\left[\left(\frac{T_1}{T_2}\right)^{-\frac{1}{\theta}}\times\frac{w_1}{w_2 d}\right]^{1-\phi}$ is negative. Since $\hat{T}_1 > \hat{T}_2$, the effect of $\left[\left(\frac{T_1}{T_2}\right)^{-\frac{1}{\theta}}\times\frac{w_1}{w_2 d}\right]^{1-\phi}$ on the second term on the LHS is positive. Therefore an increase in B_1 leads to a decrease of the second term on the LHS so far as its effect on $\left[\left(\frac{T_1}{T_2}\right)^{-\frac{1}{\theta}}\times\frac{w_1}{w_2 d}\right]^{1-\phi}$ is concerned. Under symmetry, $\left[\left(\frac{T_1}{T_2}\right)^{-\frac{1}{\theta}}\times\frac{w_1}{w_2 d}\right]^{1-\phi} = d^{\phi-1} > 1$. From the first two equations above, we believe it is true that for any given B_2, an increse in B_1 leads to a decrease in $\frac{d^{\phi-1}B_1\hat{T}_1+B_1\hat{T}_2}{d^{\phi-1}+1}$. Therefore, we believe that an increase in B_1 leads to a decrease in the second term on the LHS, which means that the second order condition is satisfied.

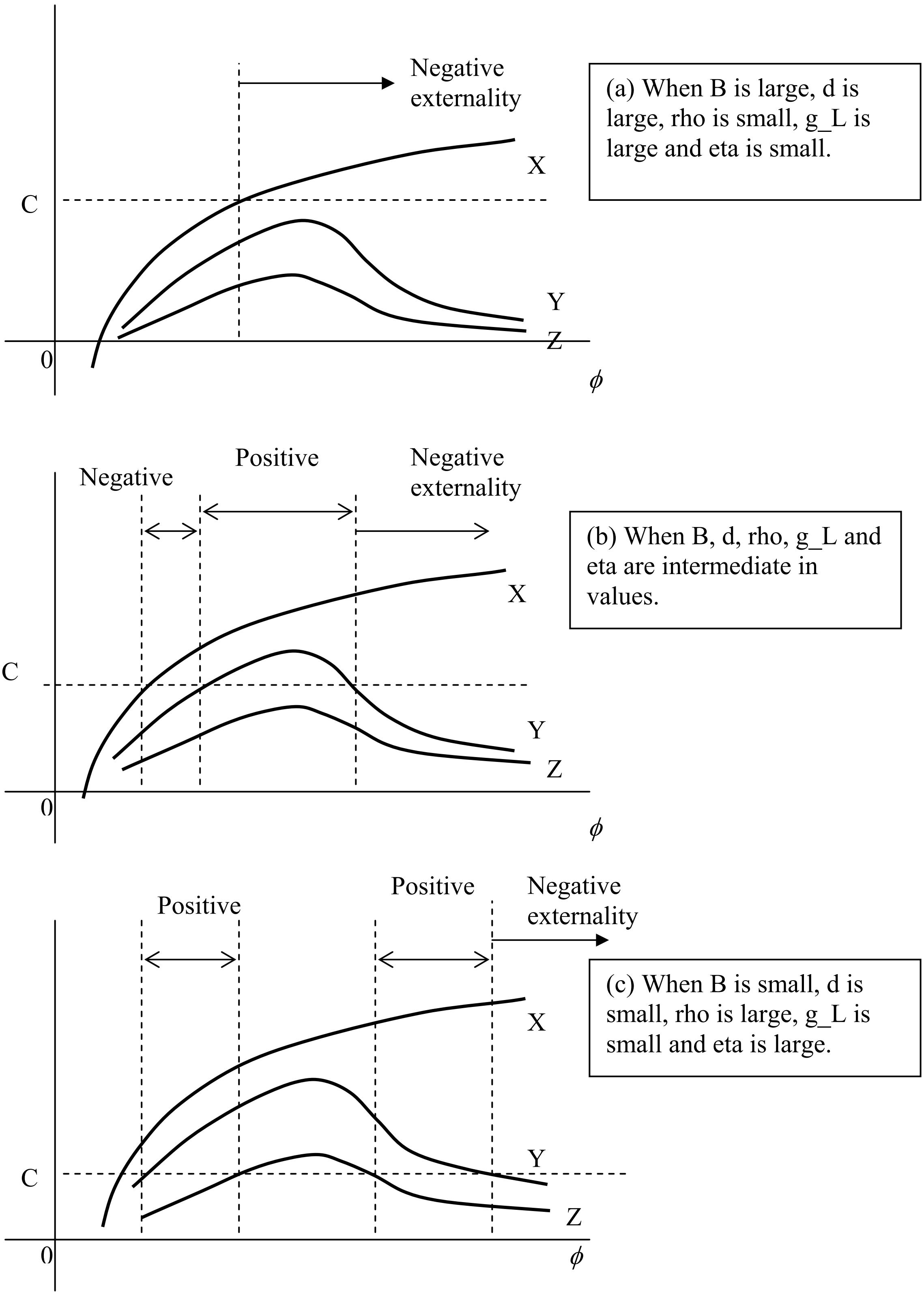

Figure 1. Curve X = LHS of equation (37); curve Y = RHS of equation (37); curve Z = LHS of equation (38); C is the value of the middle expression in both (37) and (38).

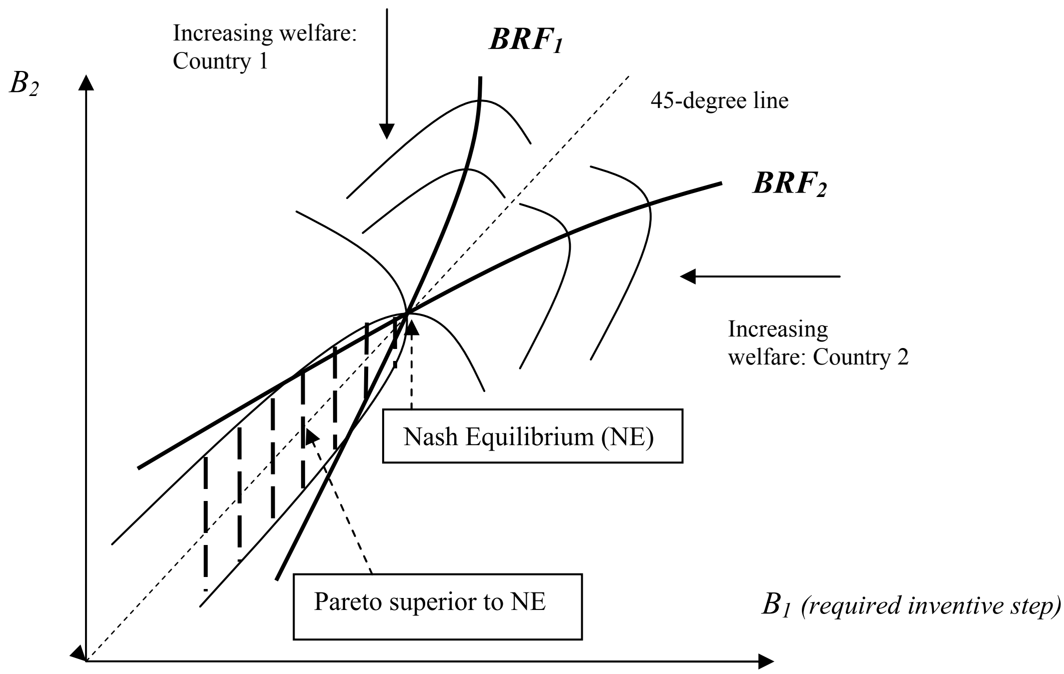

Figure 2A
Nash Equilibrium under a Required Inventive Step IP Regime for Chor and Lai (2009)

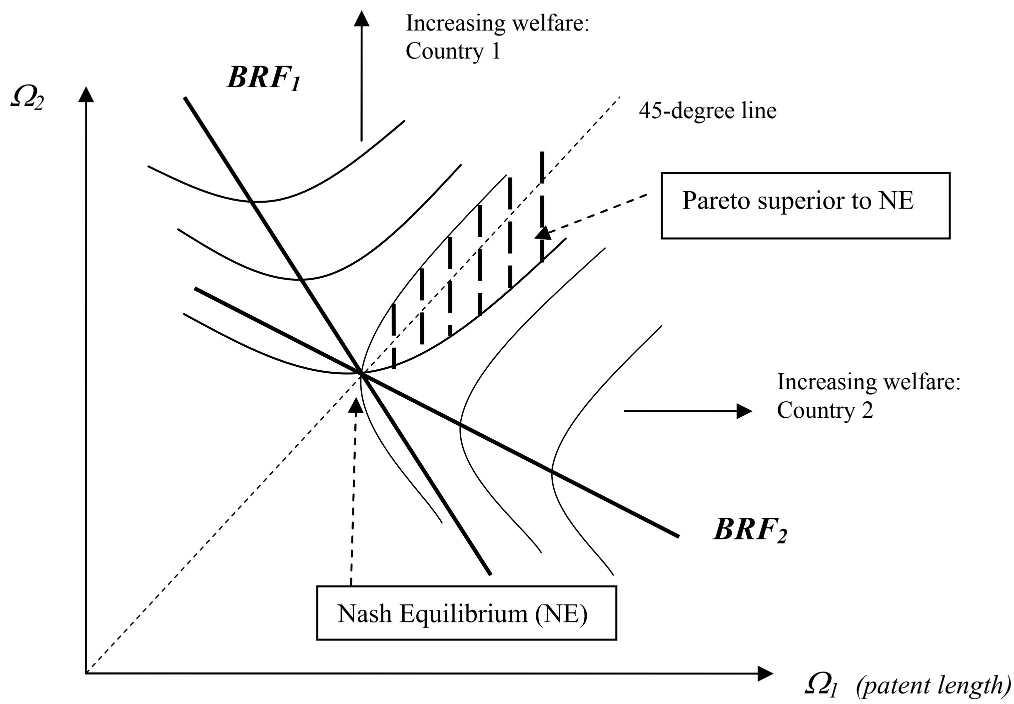

Figure 2B
Nash Equilibrium under a Patent Length IP Regime for Grossman and Lai (2004)

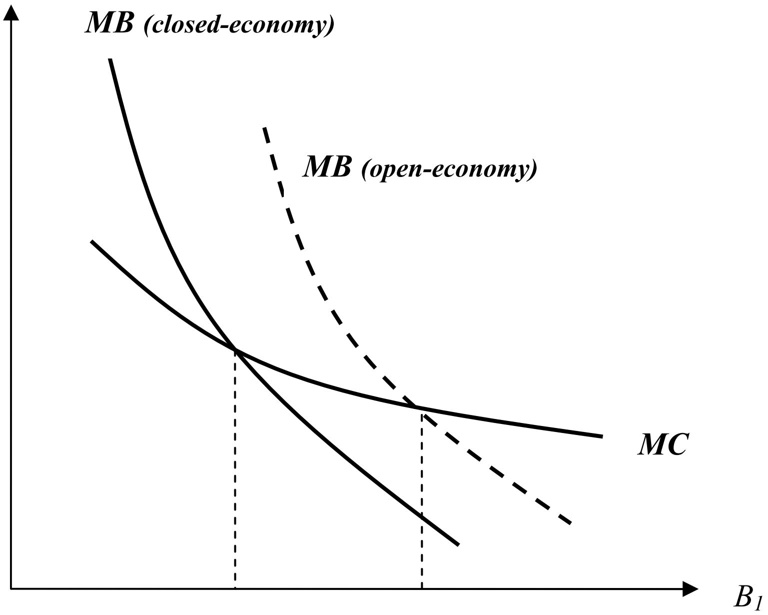

Figure 3
The marginal benefit and marginal cost of increased required inventive step