Experimental and Numerical Analysis of PZT Bonded Laminated Composite Plate

Vikram Umakant Ukirde

Department of Mechanical Engineering
National Institute of Technology Rourkela

Experimental and Numerical Analysis of PZT Bonded Laminated Composite Plate

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by

Vikram Umakant Ukirde

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under the supervision of

Prof. Subrata Kumar Panda

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Department of Mechanical Engineering
National Institute of Technology Rourkela

Department of Mechanical Engineering
National Institute of Technology Rourkela

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Certificate of Examination

Roll Number: *214ME1280*
Name: *Vikram Umakant Ukirde*
Title of Thesis: *Experimental and Numerical Analysis of PZT Bonded Laminated Composite Plate*

I the below signed, after checking the thesis mentioned above and the official record book of the student, hereby state our approval of the thesis submitted in partial fulfillment of the requirements of the degree of *Master of Technology* in *Mechanical Engineering* at *National Institute of Technology Rourkela*. I am satisfied with the volume, quality, correctness, and originality of the work.

Subrata Kumar Panda

Department of Mechanical Engineering
National Institute of Technology Rourkela

Prof. Subrata Kumar Panda
Assistant Professor, Department of Mechanical Engineering

May 23, 2016

Supervisor's Certificate

This is to certify that the work presented in the Thesis entitled *Experimental and Numerical Analysis of PZT Bonded Laminated Composite Plate* submitted by *Vikram Umakant Ukirde*, Roll Number 214ME1280, is a record of original research carried out by him under my supervision and guidance in partial fulfillment of the requirements of the degree of *Master of Technology* in *Mechanical Engineering*. Neither this thesis nor any part of it has been submitted earlier for any degree or diploma to any institute or university in India or abroad.

Subrata Kumar Panda

हा शोधनिबंध मी माझ्या

प्रिय आई वडीलांना, पूज्य गुरुजनांना आणि परमेश्वराला

समर्पित करतो.

Declaration of Originality

I, *Vikram Umakant Ukirde*, Roll Number *214ME1280* hereby declare that this thesis entitled *Experimental and Numerical Analysis of PZT Bonded Laminated Composite Plate* presents my authentic work carried out as a Postgraduate student of NIT Rourkela and, to the best of my knowledge, contains no material previously published or written by another person, nor any material presented by me for the award of any degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the Thesis. Works of other authors cited in this Thesis have been duly acknowledged under the sections "Bibliography". I have also submitted my original research records to the scrutiny committee for evaluation of my thesis.

I am fully aware that in case of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present thesis.

May 23, 2016
NIT Rourkela

Vikram Umakant Ukirde

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May 23, 2016 *Vikram Umakant Ukirde*
NIT Rourkela Roll Number: 214ME1280

Abstract

In this work, the bending and vibration behaviour of the PZT bonded laminated composite plate is investigated. The structural responses computed using a simulation model with the help of ANSYS and compared with experimental results. In this analysis, the maximum central deflections and the natural frequencies of PZT bonded laminated composite plate for close and open circuit conditions have been computed numerically with the help of present simulation model and compared with the result of the published literature. Further, the efficacy of the simulation model has been checked for different geometrical parameters (thickness ratio and support conditions) and discussed in detail.

***Keywords*: *Vibration*; *Bending*; *FEM*; *FSDT*; *ANSYS APDL*.**

Contents

List of Figures

List of Tables

Nomenclature

$a, b, t \ and \ h$	Length, breadth, thickness of PZT plates and total thickness of laminated composite plate
$E_1, E_2 \ and \ E_3$	Young's modulus
$G_{12}, G_{23} \ and \ G_{13}$	Shear modulus
$\upsilon_{12}, \upsilon_{23} \ and \ \upsilon_{13}$	Poison's ratios
ρ	Density of the martial
u_0, v_0, w_0	Mid plane Displacement about x, y, z direction
$\theta_x, \theta_y, \theta_z$	Rotation perpendicular to $x, y \ and \ z$ axis
$\phi_0^{\ k}, \phi_1^k$	Electrical degree of freedom
$\{d\}$ and $\{d_0\}$	Global and mid-plane mechanical degree of freedoms
$[f_{mz}] \ and \ [f_{ez}]$	Mechanical and electrical functions of thickness coordinate
$\{\varepsilon_L\}$	Linear strain tensor
$[T_L]$	Thickness coordinates matrix of linear strain displacement relation
$[T_\phi]$	Thickness coordinates matrix of electric field potential
$\{E\}$	Electric field
$\{D^E\}$	Electric displacement
$\{\sigma\}$	Stress vectors
$\{\varepsilon\}$	Strain vectors
$[Q]$	Lamina constitutive relation
$[e]$	Piezoelectric stress matrix
$[\in]$	Dielectric constant matrix
$[M]$	Global mass matrix
$[K]$	Global stiffness matrix
$\{\psi\}$	Mode shapes of eigenvectors
ω_n	Natural frequency

Chapter 1

Introduction

1.1 Piezoelectric Concept

The concept of flexible structure creates a great revolution in today's worlds. It consists of laminated structure, sensors and actuators all are coordinated with each other by the help of some means of the controller. Flexible structures have low flexible rigidity and small material damping ratio. A small excitation may promote to destructive high amplitude vibration and long settling time. Vibration control of flexible structures is an important problem in several engineering applications, especially for the precise operation performances in aerospace systems, satellites, flexible manipulators, etc. Vibration can creat failure, fatigue damages or radiate redundant loud noise.

Piezoelectric (PZT) bonded structures have found its role in many engineering application like satellites, aircraft structures etc. There are different smart materials that are used as actuators and sensors like shape memory alloy, a magnetostrictive material, piezoelectric material, electrostrictive material etc. These materials can be bonded to the structures that can be used as actuators and sensors. Among the above stated smart materials, PZT has been very popular in use due to the insensitivity of temperature, easy to use and high strength. They also generate a large deformation for which they are used extensively nowadays. When a structure is experiencing some kind of vibration, there is a various method through which we can control the vibration:

1. Passive control involves some kind of structural modification or redesign, include the use of springs and dampers, which helps in reducing the vibration. But they reached to their extent in the field of development. They required extra mounting space and weight. And also it's not effective at low damping frequencies.
2. Active control involves the structure with sensors, actuators and some kind of electronic control system, which specifically functioned to reduce the vibration levels. New control design with sensor-actuator system have been proposed called smart material (PZT) because of their resilience property, light in weight, the high bandwidth of devices, fast expansion response very quickly, low power consumption, small space required and it can be operated at cryogenic temperature.

Smart materials have produced smaller and powerful actuators and sensors with high veracity in structures. These materials can modify their geometric mechanical (material) properties under the influence of any kind of electric (applied voltage), magnetic field. Piezoelectric (lead-Zirconium-Titanate) materials can be utilised sufficiently in the development of smart systems. By far most of the exploration in smart material has focussed on the control of structure produced using composite materials with installed or reinforced piezoelectric transducer due to their amazing mechanical electrical coupling attributes. A piezoelectric material reacts to mechanical power by producing an electric charge or voltage. This process is known as the direct piezoelectric impact, then again, when an electric field is connected to the material, mechanical push or strain is impelled, this phenomenon is known as the opposite piezoelectric impact. The immediate impact is utilised for sensing and the opposite impact for actuation.

1.1.1. Application of PZT bonded laminated composite plate

- By implementing their natural sensing and actuating abilities into structural components of vehicles smart materials have discovered tremendous use in car applications.
- They are extensively used in structural health monitoring (SHM). They sense the vibration produced in the structure during the earthquake and thereby counteract the vibration through actuation thus preventing the extent of damage to the structure.
- Another area of application of smart material is in the field of aerospace industry where a combination of sensors and actuators are used to detect any new crack or propagation of a crack in aircraft component.
- Energy harvesting
- Marine application

1.2 Introduction of Finite Element Method and ANSYS

With the advancement in technology, the design process is too close to precision, so the finite element method (FEM) is used widely and capable of drawing the complicated structure. Therefore, to solve the complex problems, various comparative techniques such as FEM, mesh-free method, finite difference method, etc. have been utilised in past to evaluate the desired responses by incorporating the real-life situations. Out of all comparative analysis, the FEM has been dominated the engineering computations since its invention and also expanded to a variety of engineering fields. FEM is widely used as the most reliable tool for designing of any structure because of the accuracy compare to other methods. It plays an significant role in divining the responses i.e. bending and vibration of various models. Nowadays, FEM is widely used by all industries which save their tremendous time of prototyping with reducing the damage and cost due to real test and enhances the innovation at a faster and more precise way.

ANSYS is being used in several engineering areas such as power generation, transportation, devices, and household appliances as well as to analyse the vehicle simulation and in aerospace industries. In the present work, the bending and vibration analysis of PZT bonded homogeneous isotropic and laminated composite plates is done by taking SOLID5 for PZT and SOLID45 for homogeneous isotropic and laminated composite plate from the ANSYS library. SOLID5 is an eight noded six degrees of freedom (DOF) at each node and having the mechanical, electrical and mechanical capability and SOLID45 is an eight noded three DOF at each node at having a large strain, the large deflection.

1.3 Motivation of the Present Work

The laminated composite plates are of prominent consideration to the designers because of efficient, lightweight structures, due to theirs infinite beneficial properties. The increased complex analysis of the composite structures is mainly due to its excellent use in the field of aerospace/aeronautical engineering. These structural elements are subjected to multiple types of mixed loading in their service life which leads diminish the natural frequency of the material and highest central deflection. As non-destructive testing (NDT) methods like mechanical impedance, ultrasonic inspection, etc. have been employed to precisely evaluate the effect of the laminated composite plate. Moreover, most of these techniques are mainly time-consuming, labour intensive, and cost ineffective when massive structures are included. Hence, to examine the structural responses, the numerical approaches can be implemented especially when the geometry, the material, and the loading types are complicated in essence. Therefore, a general simulation model is developed which can compute the true bending and vibration responses of PZT bonded homogeneous isotropic and laminated composite plates.

1.4 Organization of the Thesis

The summary and motivation of the present work are discussed in this chapter. This Chapter 1 is divided into four different sections, the first section, a basic introduction to problem and theories used in past. Consequently, a concise introduction of FEM and FEA tool, ANSYS is used in the second section and the motivation of present work is explained in the third section. The remaining part of the thesis is organised in the following fashion. In Chapter 2, some significant augmentations to the bending and the vibration behaviour of PZT bonded homogeneous isotropic and laminated composite plate are highlighted. Based on the literature survey the objective and scope of the work is discussed in next section. In Chapter 3, the general mathematical formulation for the bending and vibration of PZT bonded homogeneous isotropic and laminated composite plate is explained. Finite element (FE) formulation for coupled field analysis is examined. First-order shear deformation theory (FSDT) is used for the present analysis of homogeneous isotropic and laminated composite plate. Chapter 4 shows the bending and vibration behavior of PZT bonded homogeneous isotropic and laminated composite plate. Convergence and validation study is carried out for bending and vibration analysis of homogeneous isotropic and laminated composite plate. In addition to that, an experimental study is also carried out for vibration analysis of homogeneous isotropic and laminated composite plate for closed and open circuit condition. Some bending and vibration problems are solved for different geometrical parameters and discussed in Numerical illustration section. Chapter 5 reviews the whole work and it contains the concluding remarks based on the present study and the future scope of the work is discussed.

Chapter 2

Literature Review

2.1 Literature Survey

Many researchers have studied the bending and vibration responses of isotropic, orthotropic, anisotropic and laminated composite plates using various plate theories. Few of the important work on bending and vibration of PZT bonded plate carried out by various researchers are discussed in detail.

The static and dynamic responses of the cantilever laminated composite structure embedded with piezoelectric layer are investigated by Crawley and luist [1]. The dynamic analysis of composite structure having piezoelectric layers was studied by Ray et al. [2]. They obtained the solution for stresses and mathematical deflections by using the field equations under the application of the electric potential. Lee [3] developed a new piezoelectric laminate theory to investigate the effect of distributed sensing to control the torsional, bending, shrinking, stretching and shearing of the flexible laminate composite plate under electromecanical and mechanoelectrical loading. Wang and Rogers [4] analyse the classical laminated plate theory to find out pure extension and pure bending of piezoelectric patch bonded and/or embedded in laminated structure under thermal loading condition. Huang and Wu [5] developed mathematical modeling for studying the response of fully coupled hybrid multilayered composite plate and piezoelectric plate by using first order shear deformation theory to find out the coefficient of displacement and electric field. Wu and Syu [6] presents asymptotic formulations to evaluate structural behavior of FG piezoelectric shell in the electro-elastic coupled field and also studied the effect of gradient index of properties of the material on the variables mechanical and electric fields. By further modifying the Stroh formalism an analytical solution is obtained for the frequency response of cylindrical bending of piezoelectric laminated structure to the plane strain vibrations of piezoelectric materials obtained by Vel *et al.* [7]. Also, coefficients of the endless series solution found out from the continuity condition at the interface and the different boundary condition at the edge of the laminate and they satisfied according to consideration of Fourier series sense. For different boundary conditions, the results are shown. The bending responses of the laminated plate embedded with piezoelectric actuators and sensors subjected to electrical and mechanical loading are obtained by Vel and Batra [8] using state space method in the framework of first order shear deformation theory (FSDT). Mallik and Ray [9] investigated the performance of PFRC materials for smart composite plates as the distributor actuator and studied the static behaviour of the composite plate made of PFRC material. Mitchell and Reddy [10] analysed mechanical displacement using equivalent single layer theory whereas layerwise theory is used to find out potential function of piezoelectric bonded laminated composite panel. Mitchell and Reddy [11] presents the refined theory for simply supported piezoelectric bonded laminated composite plate to show equation of motions based on electromechanical coupling and linear piezoelectricity. Liao and Yu [12] constructed coupled electromechanical field of Reissner-Mindlin model designed for Piezoelectric bonded laminated plate by means of any electric potential in the thickness direction by using the variational asymptotic method. Dumir *et al.* [13] developed a mathematical model based on the third order theory and zigzag theory to obtain the vibration and buckling responses of laminated piezoelectric plates considering the nonlinearity in von-Karman sense. kumari *et al.* [14] presents new improved third order theory to find out bending and natural frequency response of simply supported hybrid horizontal plate integrated with piezoelectric under thermal loading condition. Moita *et al.* [15] developed a mathematical model of higher-order theory for static and vibration analysis of magnetostrictive elastic plates to find out the mechanical deformation, electric and magnetic potentials. Lage *et al.* [16] presented a three-dimensional analytical solution of finite element formulation by applying Reissner mixed principle of variational for adaptive plate structure which is somewhat mixed layer-wise in nature. Reddy [17] developed a mathematical formulation of finite element model and Navier solutions by implementing classical laminate and shear deformation theories of the plate for the investigation of simply supported PZT bonded rectangular laminated composite plate subjected to both electrical and mechanical loading. Saviz and Mohammafpourfard [18] revealed a mathematical model for dynamic analysis of simply supports PZT bonded layered cylindrical shell subjected to pinch/ring load by applying Galerkin’s finite element method to find out the natural frequency response. Torres and Mendonca [19] developed a mathematical model for simply supported piezoelectric laminated composite plate based on the equivalent single layer theory to find out the mechanical deflection and electrical potential under different loading and stacking sequence. Vnucec [20] performed the mathematical model by using classical laminate plate theory to find out the engineering properties of the profound plate and stress-strain distribution for various angles of laminations of the composite structure under combined loading condition. Liang [21] developed a mathematical model to obtain an analytical solution to analyze steady frequency response of a simply supported laminated composite structure with piezoelectric patches as sensors and actuators, either bonded to or embedded in it to its surface by using Fourier series method. The transient deformations of plate or beam bonded by piezoelectric patches are simulated by using the FE method. Sahoo *et al.* [20] developed mathematical modeling by using HSDT and Green-Lagrangian nonlinearity to present nonlinear flexural behaviour of laminated Carbon/Epoxy structure. Vel and Batra [23] developed mathematical modeling of piezoelectric bonded laminated homogeneous panel and are analyzed by using the Eshelby-Stroh formalism to find out quasistatic deformation by considering Fourier series sense. Kant and Shiyekar [24] obtained an analytical solution by using higher order normal and shear deformation theory for the flexural of cylindrical piezoelectric plates to find out displacement and electrical potential. Shiyekar and Kant [25] presented an analytical solution for integrated piezoelectric fiber-reinforced composite actuators in cross-ply composite laminates by using higher order normal and shear deformation theory subjected to electromechanical loading under bi-directional bending. Kerur and Ghosh [26] developed a mathematical model of the laminated composite plate for studying the finite element formulation of coupled electromechanical field for controlling the non-linear transient response by using Von Karman and first order shear deformation theory. Sladek *et al.* [27] investigated static and dynamic analysis of piezoelectric bonded laminated plates using Reissner-Mindlin theory under electrical potential or pure mechanical load on the upper surface of laminated plate. Saravanos *et al.* [28] investigated dynamic and quasi-static analysis of piezoelectric bonded smart laminated composite structure using layerwise theory. Godoy and Trindade [29] used the equivalent single layer theory and third-order shear deformation theory designed for the study of piezoelectric patches embedded in the laminated composite panel and patches connected to resonant shunt circuit which is in an active-passive sense by considering the mechanical and electrical degree of freedom. Qing et al. [30] investigated static and dynamic analysis of clamped aluminum plates with piezoelectric patches using modified mixed variational principle. Dash and Singh [31] developed mathematical modeling using higher order shear deformation theory (HSDT) for piezoelectric bonded laminated composite plate to find out zero transverse shear strain at the top surface of panel considering Von-Karman logic. Rogacheva [32] developed mathematical modeling for piezoelectric laminated bars which are symmetrically arranged about the middle plane to calculate stresses, vibration, deflection and electrical quantities. Zhang *et al.* [33] investigated free vibration analysis of multilayered piezoelectric laminated composite plate by using differential quadrature techniques to resolve three-dimensional piezoelasticity equations under different boundary conditions at the plate edge. Bendigeri *et al.* [34] presents the use of piezoelectric material properties to study the static and dynamic responses of smart composite structures using FEM. Heylinger [35] developed an exact solution for the three-dimensional static response of piezoelectric embedded laminated cylindrical shells under simply supported condition. Cinefra *et al.* [36] investigated the free vibration responses of the plate embedded with piezoelectric patches using virtual displacement principle and finite element method (FEM). Yakub *et al.* [37] presents vibration control using equivalent single layer third-order shear deformation theory to get natural frequency of piezoelectric sensors and actuator bonded to plate under electromechanical coupling. Malgaca [38] solved the vibration control problem by using finite element programs in the single analysis step. In addition to that, for different lay-ups and piezoelectric actuators, the closed loop time responses and natural frequency responses are calculated. Rahman and Alam [39] presented vibration control of smart beams by using coupled layerwise (zig-zag) theory for cantilever piezoelectric bonded laminated composite beam to obtain undamped natural frequency.

Based on the above study it is observed that there is no study found on the bending and vibration behaviour of PZT bonded homogeneous isotropic and laminated composite plate using simulation model to the best of author’s knowledge. It is also important to mention that an experiment is also carried out to obtain the vibration responses of closed and open circuit conditions. The convergence and comparison studies are provided for the bending and vibration responses. Based on the convergence and validation study, some new problems are also solved which are not provided here.

2.2 Objective and Scope of the Present Thesis

The main object of the work is to develop a simulation model for PZT bonded homogeneous isotropic and the laminated composite plate to investigate the bending and vibration analysis. In addition to that, a simulation model is developed in commercial FE package (ANSYS 15.0) based on ANSYS parametric design language (APDL). The study is further continued to analyse the structural responses of PZT bonded homogeneous isotropic and laminated composite plates. Experimental studies carried out to investigate the free vibration responses of PZT bonded homogeneous isotropic and laminated composite plates for closed and open circuit conditions and their results were validated with ANSYS model. Further to find out the effect of different thickness ratios and support conditions on the bending and vibration of PZT bonded laminated. The description of the the present study is discussed below:

- As a first step, the bending responses of PZT bonded homogeneous isotropic and laminated composite plate considering various geometrical parameters have been studied using the proposed simulation model with the help of Graphical User Interface (GUI) in ANSYS 15.0 environment.
- The models are extended to study the dynamic responses, i.e., the free vibration responses of PZT bonded laminated composite plates through the developed simulation model.
- The vibration responses of PZT bonded laminated plates for closed and open circuit conditions have also been extended for the experimental validation and their responses are compared with those of developed model in ANSYS.
- Finally, the parametric study of laminated and delaminated composite plates has been carried out using ANSYS APDL.
- Study the effect of various thickness ratios and support condition on PZT bonded laminated composite plate for bending and vibration behavior.
- The bending and vibration response with and without PZT bonded laminated composite plates are shown finally.

Few parametric studies of laminated composite plates with and without PZT are solved using the simulation model. In order to check the efficacy of the presently developed simulation models, the convergence behaviour with minimum mesh size has been estimated and compared for the PZT bonded homogeneous isotropic and laminated composite plates. In addition to that, the present bending and vibration responses of PZT bonded homogeneous isotropic and laminated composite plates are also compared with that literature and experimental results shows the accuracy of the developed models.

Chapter 3

Mathematical Modeling and Solution Techniques

3.1 Introduction

The analysis of smart composite structures have always been a complicated task due to the coupling effect of different types of loading either two or more (electrical, mechanical, thermal and external stimuli) need to be modelled as a single unit by considering the individual effects of the parent and the functional materials. Thispresent chapter provides the step of generalised mathematical formulation for the bending and the vibration behaviour of the PZT bonded homogenous isotropic and laminated composite plates i.e. piezoelectric actuators and sensors. In order to address the issue, a generalised linear FE model has been developed in ANSYS environment.

3.2 Assumptions

The present general mathematical formulation is based on the following fundamental assumptions:

- A perfect bonding exists between fibres and matrix of the laminated composite plates and there is no slippage occurs at the interface of the plate.
- The reference plane is considered as the middle plane of the plate.
- The loads are applied either in parallel or perpendicular to the fibre direction.
- The piezoelectric layers and elastic substrate of the laminated composite plate are bonded perfectly.

3.3 Field Variables

As discussed earlier, the simulation model is developed in ANSYS environment using ANSYS APDL code. For the present work solid element, two fields variable are taken i.e. mechanical displacement (u, v, w) and electric potential (ϕ^k) in the function of thickness (z). The SOLID5 element is taken for a PZT having eight nodes with up to six

DOF at each node and the SOLID45 element is taken for homogeneous isotropic and a laminated composite plate having eight nodes having three DOF at each node. The present simulation model is based on the FSDT kinematics for the solid element at the mid-plane.

The displacement field is considered as:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y) + z\theta_x(x,y) \\ v(x,y,z,t) &= v_0(x,y) + z\theta_y(x,y) \\ w(x,y,z,t) &= w_0(x,y) + z\theta_z(x,y) \end{aligned} \qquad (3.1)$$

The electric potential can be defined as:

$$\phi^k(x,y,z) = \phi_0^k(x,y) + z\phi_1^k(x,y) \qquad (3.2)$$

where, ($u_{0,}$ $v_{0,}$ w_0) denote the displacements at mid-plane along the (x, y, z) coordinates axes respectively and θ_x, θ_y and θ_z are the shear rotations normal to midplane about the x, y and z-axes respectively.

For any k^{th} piezoelectric layers or patch, the electrical DOF are $\left(\phi_0^k \quad \phi_1^k\right)$ and they are shown are as follows.

$$\phi_0^k = \frac{\phi_+^k + \phi_-^k}{2}, \phi_1^k = \frac{\phi_-^k - \phi_+^k}{h_p} \qquad (3.3)$$

where, the electrical potentials ϕ_+ for the upper surfaces and ϕ_- for the bottommost surfaces of piezoelectric layers or patch i.e. k^{th}.

The matrix form of the Equations (3.1) and (3.2) are shown as follows:

$$\{d\} = [f_{mz}]\{d_0\} \qquad (3.4)$$

$$\{\phi^k\} = [f_{ez}]\{\phi_{e0}^k\} \qquad (3.5)$$

where, $\{d\} = \{u \; v \; w\}^T$ is global and $\{d_0\} = \left[u_0, v_0, w_0, \theta_x, \theta_y, \theta_z\right]^T$ is the midplane nodal DOFs and $\{\phi^k\}$ is global and $\{\phi_{e0}^k\} = \{\phi_0^k \quad \phi_1^k\}^T$ nodal electrical DOFs in a piezoelectric layers or patch. $\left[f_{mz}\right]$ is the mechanical function and $\left[f_{ez}\right]$ is the electrical function of thickness coordinate.

3.4 Strain-Displacement Relations

The strain-displacement relation for the PZT bonded plate structure is as follows:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{yz}\\ \varepsilon_{xz}\\ \varepsilon_{xy}\end{Bmatrix}=\begin{Bmatrix}\left(\dfrac{\partial u}{\partial x}\right)\\ \left(\dfrac{\partial v}{\partial y}\right)\\ \left(\dfrac{\partial w}{\partial z}\right)\\ \left(\dfrac{\partial v}{\partial z}+\dfrac{\partial w}{\partial y}\right)\\ \left(\dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x}\right)\\ \left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right)\end{Bmatrix} \tag{3.6}$$

where, $\{\varepsilon\}$ is the strain tensors this is shown as follows:

$$\{\varepsilon\}=[T]\{\bar{\varepsilon}\}=\begin{Bmatrix}\varepsilon_{xx}{}^{0}\\ \varepsilon_{yy}{}^{0}\\ \varepsilon_{zz}{}^{0}\\ \varepsilon_{yz}{}^{0}\\ \varepsilon_{xz}{}^{0}\\ \varepsilon_{xy}{}^{0}\end{Bmatrix}+z\begin{Bmatrix}k_{xx}{}^{1}\\ k_{yy}{}^{1}\\ 0\\ k_{yz}{}^{1}\\ k_{xz}{}^{1}\\ k_{xy}{}^{1}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_{x}\\ \varepsilon_{y}\\ \varepsilon_{z}\\ \varepsilon_{yz}\\ \varepsilon_{xz}\\ \varepsilon_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_{x}{}^{0}+zk_{x}{}^{1}\\ \varepsilon_{y}{}^{0}+zk_{y}{}^{1}\\ \varepsilon_{z}{}^{0}\\ \varepsilon_{yz}{}^{0}+zk_{yz}{}^{1}\\ \varepsilon_{xz}{}^{0}+zk_{xz}{}^{1}\\ \varepsilon_{xy}{}^{0}+zk_{xy}{}^{1}\end{Bmatrix} \tag{3.7}$$

where, [*T*] is the thickness coordinate matrix of strain displacement relation and $\{\bar{\varepsilon}\}$ is strain matrix at mid-plane is shown below:

$$\{\bar{\varepsilon}\}=\left\{\varepsilon_{x}{}^{0}\varepsilon_{y}{}^{0}\varepsilon_{z}{}^{0}\varepsilon_{yz}{}^{0}\varepsilon_{xz}{}^{0}\varepsilon_{xy}{}^{0}k_{x}{}^{1}k_{y}{}^{1}k_{yz}{}^{1}k_{xz}{}^{1}k_{xy}{}^{1}\right\}^{T} \tag{3.8}$$

The electric field '*E*' can be articulated as the negative of electric potential (ϕ):

$$\{E\} = \{E_x \; E_y \; E_z\}^T = \left\{ -\partial\phi/\partial x \; -\partial\phi/\partial y \; -\partial\phi/\partial z \right\}^T \tag{3.9}$$

By substituting Equation (3.2) in the Equation (3.9), the electric field components along x, y and z- axes can be written as follows:

$$\{E\} = \{E_x \; E_y \; E_z\}^T = [T_\phi]\{E^0\} \tag{3.10}$$

$[T_\phi]$ is the thickness coordinate matrix of electric field potential.

3.5 Constitutive Relations

The plate structure is consisting of the numeral elastic substrate bonded with piezoelectric actuator and sensor patches/layers. Each lamina is considered as piezoelectrically and elastically orthotropic. The constitutive equation having fiber direction (Θ) with respect to material axes is as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}^k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} & 0 \\ 0 & 0 & 0 & Q_{54} & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix}^k \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}_L^k \tag{3.11}$$

The constitutive relation for piezoelectric layers having to couple between electric field and elastic lamina can be written as follows:

$$\{D^E\} = [e]^T \{\varepsilon\} - [\in]\{E\} \tag{3.12}$$

$$\{\sigma\} = [Q]\{\varepsilon\} - [e]\{E\} \tag{3.13}$$

where, $\{E\}$ is the electric field, $\{D^E\}$ is the electric displacement, $\{\sigma\}$ is the stress vectors and $\{\varepsilon\}$ is the strain vectors while $[Q]$ is the constitutive matrix, $[e]$ is the piezoelectric stress matrix and $[\in]$ is the dielectric constant matrix. Equations (3.12) and (3.13) shows the sensor and actuator equations i.e. direct piezoelectric effect and converse piezoelectric effect respectively.

The stress coefficient matrix for PZT is as follows:

$$\left[e^k\right]_{PZT}=\begin{bmatrix}0 & 0 & 0 & e_{14} & e_{15} & 0\\ 0 & 0 & 0 & e_{24} & e_{25} & 0\\ e_{31} & e_{32} & e_{33} & 0 & 0 & e_{36}\end{bmatrix} \tag{3.14}$$

Whereas, dielectric constant matrix is as follows:

$$\left[\in^k\right]=\begin{bmatrix}\in_{11} & \in_{12} & 0\\ \in_{12} & \in_{22} & 0\\ 0 & 0 & \in_{33}\end{bmatrix}^k \tag{3.15}$$

The resultants of force and moment in terms of material stiffness can be written as follows:

$$\begin{Bmatrix}\{N\}\\ \{M\}\\ \{Q_s\}\end{Bmatrix}=\begin{bmatrix}A_{ij} & B_{ij} & 0\\ B_{ij} & D_{ij} & 0\\ 0 & 0 & A_{sij}\end{bmatrix}\begin{Bmatrix}\{\varepsilon\}\\ \{k\}\\ \{\varepsilon_{ts}\}\end{Bmatrix}-\begin{Bmatrix}\{N^E\}\\ \{M^E\}\\ \{Q^E{}_s\}\end{Bmatrix} \tag{3.16}$$

3.6 Energy Equation

This present mathematical formulation deals by means of the structural displacements due to externally applied electro- mechanical loading. The overall potential energy of the system is given by as follows:

$$T_p=\frac{1}{2}\left[\sum_{i=1}^{n}\int_v\left\{\varepsilon^k\right\}^T\left\{\sigma^k\right\}dv-\int_v\left\{E_i\right\}^T\left\{D_s\right\}dv\right] \tag{3.17}$$

Whereas, the subscript '*i*' represents actuator and sensor i.e. '*a*' and '*s*' respectively.

3.6.1 Variational principle for the bending analysis

The principle of minimum potential energy (PMPE) affirms that the equilibrium form of a system can be defined if an appropriate force state satisfies the geometric constraints and the change in potential energy (δT_p) vanishes for arbitrarily force variations.

i.e. $$\delta T_p=\delta\left(U-W_E\right)=0 \tag{3.18}$$

3.6.2 Variational principle for vibration analysis

To obtain the governing equation of system under dynamic load by using Hamilton's principle (HP).This is also known as the dynamic version of principle of minimum potential energy (PMPE) and can be revealed in terms of Lagrangian '*L*' as:

$$I = \int_{t1}^{t2} L dt = \int_{t1}^{t2} \left(T_P - T_{KE} \right) dt \tag{3.19}$$

where, kinetic energy (T_{KE}) is given by

$$T_{KE} = \frac{1}{2} \sum_{k=1}^{n} \int_{v} \left\{ \dot{d} \right\}^{T} \rho^k \left\{ \dot{d} \right\} dv \tag{3.20}$$

ρ^k is the mass density of the k^{th} layer. From the minimum potential energy and Hamilton's principle the total Lagrangian formulation for deformed configuration of structure is written as follows:

$$\delta \int_{t1}^{t2} \left(T_P - T_{KE} \right) dt = 0 \tag{3.21}$$

3.7 Finite Element Formulation

To determine the approximate mathematical solution of displacement fields in terms of desired field variables we employed FEM steps. The first order shear deformation theory used to widen a finite element formulation by incorporating the geometrical linearity for the bending and vibration analysis of piezoelectric bonded homogeneous isotropic and the laminated composite structure.

The mid-plane linear strain vectors in terms of their nodal displacement vector from Equation (3.9) written as follows:

$$\left\{ \overline{\varepsilon_L} \right\} = \left[B_L \right] \left\{ d_o \right\} \tag{3.22}$$

where, $[B_L]$ is the linear strain-displacement matrix in the product form of differential operator and shape function.

Lagrange equation for the conservative system can be expressed as:

$$\frac{d}{dt}\left(\frac{\partial T_{KE}}{\partial\left\{\ddot{d}\right\}}\right)+\frac{\partial T_P}{\partial\{d\}}=0 \tag{3.23}$$

By substituting the Equations (3.18) and (3.21) in the above Eqn. (3.24) it deduces to a general linear finite element equation which can be written as follows:

$$[M]\left\{\ddot{d}\right\}+\left[\overline{K}\right]\{\mathrm{d}\}=\{F_{mech}\}+\{F_{elect}\} \tag{3.24}$$

$$\left[\overline{K}\right]=[K_d]-[K_{di}]\left[K_{\phi\phi}\right]^{-1}[K_{id}] \tag{3.25}$$

3.8 Governing Equation

The governing equilibrium equations for the bending and vibration analysis can be obtained by dropping the appropriate terms from Equation (3.24) which are shown individually as follows.

3.8.1 Bending analysis

To obtain governing equilibrium equation for the bending analysis of the piezoelectric bonded homogeneous isotropic and the laminated composite plate from, dropping inertia matrix in Equation (3.24) and by using total Lagrangian virtual work principle can be shown as follows:

$$[K]\{\mathrm{d}\}=\{F_{mech}\}+\{F_{elect}\} \tag{3.26}$$

where, [K] is the total resultant stiffness function of displacement {d}.

3.8.2 Vibration analysis

To obtain governing equilibrium equation for free vibration analysis of piezoelectric bonded laminated composite plate from Equation (3.24) we have to drop electro-mechanical load vector and by substituting $\{d\}=ae^{i\omega t}\{\psi\}$ can be written as:

$$\left([K]-\omega_n^2[M]\right)\{\psi\}=0 \tag{3.28}$$

where, ω_n is the natural frequency (eigenvalue) and $\{\psi\}$ is the mode shapes (eigenvector) of the freely vibrated Piezoelectric bonded laminated composite panel in the above eigenvalue equation. The fundamental natural frequency is the smallest value of the eigenvalue.

3.9 Solution Technique

3.9.1 Simulation study

An FE model of the homogeneous isotropic and laminated composite plate bonded with the PZT has been developed using ANSYS APDL. The SOLID5 [40] (Figure 3.1) element suitable for coupled field analysis has been for the PZT-4 layers, SOLID45 [40] (Figure 3.2) for the homogeneous isotropic plate and laminated composite plate for the Epoxy/carbon and carbon fibre reinforced polymer (CFRP).

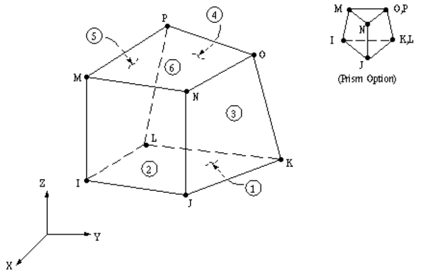


Figure 3.1: SOLID5-3D Coupled-Field element

SOLID5 has a three-dimensional (3D) magnetic, thermal, electric, piezoelectric, and field capability with limited coupling between the fields. The element has eight nodes with up to DOF at each node.

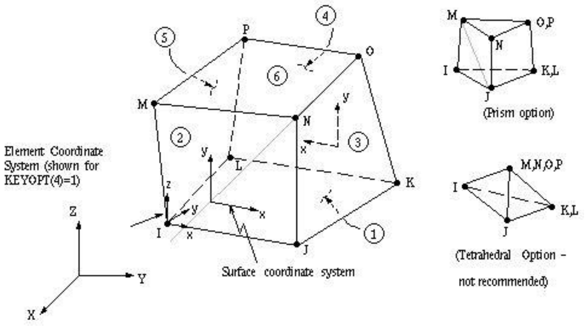


Figure 3.2: SOLID45 3-D structural solid

SOLID45 is used for the 3-D modelling of solid structures. The element is defined by eight nodes having three DOF at each node translations in the nodal x, y, and z directions. The element has plasticity,creep, swelling, stress stiffening, largedeflection, and large strain capabilities. A reduced integration option with hourglass control is available.

The present work is carried out using the element SOLID5 for PZT layer and SOLID45 for the plate (aluminium and composite) from the ANSYS APDL library for bending and vibration analysis.

- The basis of the FEM is the representation of a body or a structure by an assemblage of subdivisions called finite elements.
- The FEM translates partial differential equation problems into a set of linear algebraic equations.

a) Steps involved in FE simulation:

❖ Preference – static, electric.

❖ Preprocessor-

✓ Element type- SOLID5 for the PZT layer and SOLID45 for the aluminum plate and laminated composite plate has been taken.

✓ Material property- different mechanical and electrical properties are entered according to literature.

✓ Sections- layers added for a composite plate this is not for the homogeneous plate.

✓ Modeling- generation of the model as per required dimensions.

✓ Meshing- meshing has been discretized in the finite mesh and from the convergence is calculated and this refined mesh size is used for further analysis.

❖ Solution –

✓ Analysis type-New analysis- we can choose Static or Modal analysis for bending and vibration analysis respectively as required.

✓ Define Loads- Applying Structural and electrical loads and we can constrain the model according to loading and boundary conditions.

❖ Preprocessor – coupling conditions – again have to come back to coupling conditions in preprocessor and we have to give electric voltage condition layer-wise to the PZT patches/plates.

❖ General post process –

✓ For bending –

a. Plot result - deformed + undeformed as required

b. Contour plot – nodal solution (to obtain the deflection)

✓ For vibration –

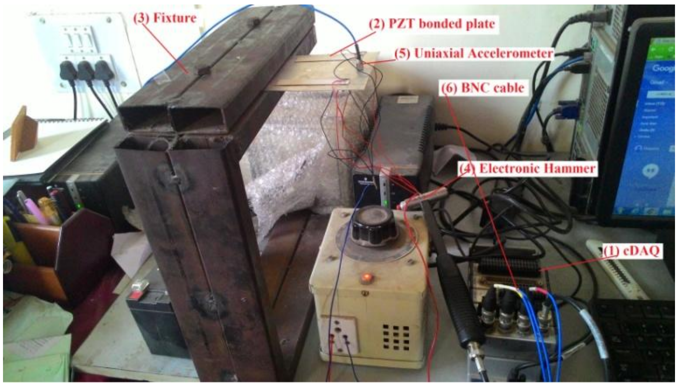
a. Result summary

b. Read result – first set

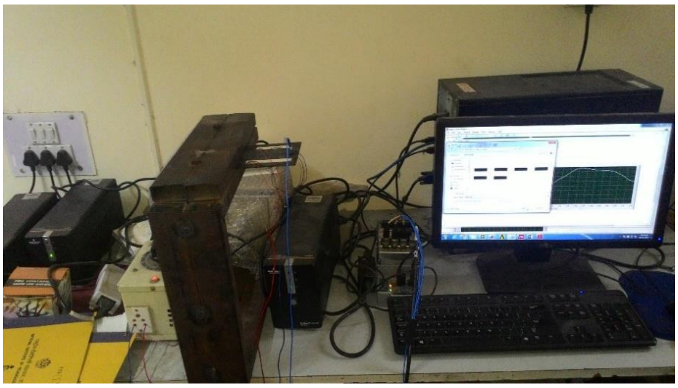
c. Plot result – nodal solution (to obtain the deformed shape of 1st mode natural frequency)

3.9.2 Experimental Study

In the present work, the natural frequency of the PZT bonded homogeneous isotropic laminated composite plate is obtained for closed circuit and open circuit condition using cDAQ (1) i.e. compact data acquisition. The experimental setup is shown in Figure 3.3 (a) and Figure 3.3 (b) for homogeneous isotropic (2) and laminated composite plate (2*), respectively with fixtures (3). The PZT bonded plate (2) is fixed in such a way that cantilever support condition obtained which is subjected to an initial excitation with the help of hammer (4) (SN 33452, National Instruments) and mechanical response is sensed by an uniaxial accelerometer (5) which is mounted at the edge. Uniaxial accelerometer transforms the mechanical signal into an electrical signal and is fed to the cDAQ-9178 (National Instruments) through the BNC cable (6). The corresponding electric signal in the form of acceleration in the time domain is obtained. For the sensor configuration condition, the accelerometer is placed at a different position and the voltage output is obtained and the circuit diagram is presented in Figure 3.4. On the output window through the graphical programming language LABVIEW 14.0 the frequency responses are extracted by transformed signals in the frequency domain, using the inbuilt Fast Fourier transform (FFT) function in the and the first mode of frequency response obtained.



(a) PZT bonded homogeneous isotropic plate



(b) PZT bonded laminated composite plate (CFRP)

Figure 3.3 (a) and (b): Experimental set-up

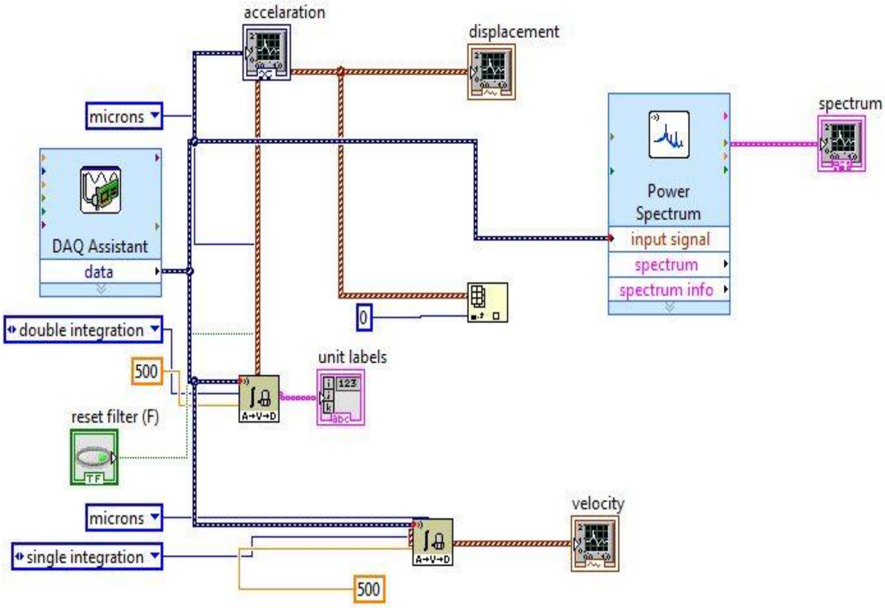


Figure 3.4: Circuit diagram in LABVIEW

3.10 Boundary Conditions

In order to restrict the rigid body motion a number of different support constraints such as (clamped (C), simply supported (S) and free (F)) have been imposed on the edges of the plate in the present work. The details of the restricted DOF in different support conditions are presented here in Table 3.1.

For V_{top} and $V_{bottom} = 0$ for sensor configuration

And $V_{top} = v$ volt and $V_{bottom} = 0$ for actuator configuration

Table 3.1 Details of support conditions

Type	Locations	Restricted degrees of freedom
Clamped (C)	*x = 0, a* and	$u_0 = v_0 = w_0 = \theta_x = \theta_y = \theta_z$
Simply Supported (S)	*x = 0, a*	$v_0 = w_0 = \theta_y = \theta_z$
	y = 0, b	$u_0 = w_0 = \theta_x = \theta_z$

3.11 Summary

A general simulation model has been developed in ANSYS environment in this present chapter to analyse the bending and vibration responses of the piezoelectric patches under electro-mechanical loading. In this present work, the homogeneous isotropic and laminated composite plate bonded/embedded with piezoelectric layers/patches (i.e. PZT) as distributed sensors and actuators considered to achieve the central deflection and frequency. The effect of different geometrical parameters on bending and vibration behaviour of the piezoelectric bonded homogeneous isotropic and composite plate is discussed in detail in the following chapter.

Chapter 4

Bending and Vibration Analysis

4.1 Overview

In this chapter, the bending and vibration responses of PZT bonded homogeneous isotropic and laminated composite plates have been investigated with the help of simulation model as discussed in chapter 3. It is well known that these are under a combined type of loading which affects the bending strength and the frequency of the plate. In addition to that to predict the actual bending and parameter are necessary. Section 4.3 presents the convergence behaviour of the simulation models for the analysis of bending and vibration analysis of homogeneous isotropic and the laminated composite plates and then the responses were compared with the results obtained from previously published literature and succeeding simulation and experimental studies. In section 4.6 the efficacy of the presently developed model has been through a parametric study. The effects of various thickness ratio and the support conditions on the structural responses of the PZT bonded homogeneous isotropic and laminated composite plates are studied.

4.2 Results and Discussion

A simulation model is developed in ANSYS APDL to study the bending and vibration responses of the homogeneous isotropic and the laminated composite plate. The convergence study is carried out for PZT bonded homogeneous isotropic and laminated composite plate. Subsequently, in order to examine the validation and accuracy of the presently developed model, numerous examples are solved for the validation purposes and compared with that available published literature. After the comprehensive testing of the presently developed simulation model, a comprehensive parametric study of the homogeneous isotropic and the laminated composite plate are performed. The effect of different combinations of parameters such as the thickness ratio (*t/h*) and the support condition on homogeneous isotropic and the laminated composite plate responses are discussed. The bending and vibration responses of the PZT bonded homogeneous isotropic and the laminated composite plate is computed for convergence andcomparison purpose by considering various material configurations as tabulated in Table 4.1.

Table 4.1: Material properties of PZT patches/layers and laminated composite plate

Material Properties	Aluminium [19]	PZT-4^{1} [19]	Graphite/ Epoxy [0°/90°/0°][19]]	PZT-4^{2} [19]	CFRP [0°/90°] [22]	PZT-5H [41]
Dimensions (cm)	(10×10×8)	(10×10×2)	(10×10×8)	(10×10×2)	(10×10×1)	(8×1.2×0.5)
E_1 (GPa)	70	94.95	132.38	81.30	6.695	60.60
E_2 (GPa)	-	94.95	10.76	81.30	6.314	60.60
G_{12} (GPa)	26	35.90	3.61	25.60	2.7	23.47
G_{23} (GPa)	-	25.40	5.65	30.60	1.35	22.98
υ_{12}	0.33	0.32	0.24	0.33	0.17	0.289
υ_{23}	-	0.38	0.49	0.43	-	0.512
ρ (kg/m^3)	2700	7800	1600	7800	1388	7500
$e_{31} = e_{32}$ $\left(C/m^2\right)$	-	-2.10	-	-	-	-6.6228
e_{33}(C/m^2)	-	9.5	-	-	-	23.2403
$e_{15} = e_{24}$ $\left(C/m^2\right)$	-	9.20	-	-	-	17.0345
$d_{31} = d_{32}$ (C/N)	-	-	-	-122×10^{-12}	-	-
$d_{33}(C/N)$	-	-	-	-285×10^{-12}	-	-
$\chi_{11}(F/m)$	-	4.07×10^{-9}	-	-	-	-
$\chi_{22}(F/m$)	-	4.07×10^{-9}	-	-	-	-

4.3 Bending Analysis

In order to check the convergence and the validation of proposed FE models, the bending response of the simply supported (SSSS) PZT bonded homogeneous isotropic and laminated composite plates are computed for different mesh size refinement. The convergence is performed by employing the simulation model developed in ANSYS 15.0 environment. In addition to that, the validation of the present simulation model has been checked by comparing the responses with those of available published literature.

4.3.1 Convergence study of PZT bonded homogeneous isotropic plate

As a first step, the convergence behaviour of the presently developed models is obtained and presented in Table 4.2. For the computation purpose, SSSS homogeneous isotropic plate integrated with PZT^1 layers is subjected to uniformly distributed load (UDL). The non-dimensional central deflections are obtained for the thickness ratio (*t/h*=0.10) and aspect ratio (*a/h*=7) by taking the materia properties as aluminum and PZT-4^1 of Table 4.1. The non-dimensional of the central deflection is normalised using the formula $\overline{w}=\left(w/h\right)\times10^9$, where w, is the maximum central deflection and *h* is the total thickness of PZT bonded plate. It can be seen from the obtained results that they are converging well with the mesh refinement and a (15×15) mesh is sufficient enough to compute the bending response of the plate. Hence, based on the convergence study a (15×15) mesh has been used in the further analysis to obtain the bending responses of the homogeneous isotropic plate.

Table 4.2: Convergence behaviour of SSSS PZT bonded homogeneous isotropic plate subjected to UDL (*t/h*=10 and *a/h*=7)

Mesh size	Non-dimensional central deflection
10×10	2
13×13	1.98
15×15	1.97

4.3.2 Comparison study of PZT bonded homogeneous isotropic plate

In this example, the non-dimensional central deflection ($\overline{w}$) of SSSS PZT bonded homogeneous isotropic plate subjected to UDL is computed using the proposed model and the results are compared with the previously published literature [19] and presented in Table 4.3. The material properties considered as aluminum and PZT-4[1] of Table 4.1. The non-dimensional central deflections are obtained for the thickness ratios (*t/h*=0.1, 0.2, 0.3, 0.4) and aspect ratio (*a/h*=7 and 10) and normalized to the formula $\overline{w} = \left(w/h\right) \times 10^9$. It is clear from the table that the present numerical results are showing good agreement with those of the references.

Table 4.3: Comparison study of SSSS homogeneous isotropic plate subjected to UDL

t/h	*a/h*=7		*a/h*=10	
	Present	Torres and Mendonca [19]	Present	Torres and Mendonca [19]
0.1	1.97	1.4337	5.93	5.6295
0.2	1.66	1.3087	5.23	5.1106
0.4	1.19	1.178	4.91	4.562

4.3.3 Convergence study of PZT bonded laminated composite plate

The convergence behavior of the present developed model is obtained for PZT bonded laminated composite plate (Graphite/Epoxy) and presented in Table 4.4. The non-dimensional bending deflections are computed for different layered square SSSS PZT bonded laminated composite plates subjected to UDL. The deflections are obtained for three different thickness ratios (*t/h* = 0.4) and aspect ratio (*a/h*=10) using the same material properties as Graphite/Epoxy and PZT[2] of Table 4.1. The non-dimensional central deflection is normalised by using the formula $\overline{w} = \left(wE_2h^3/a^4q_0\right) \times 10^3$, where w, is the maximum central deflection and *h* is the total thickness of PZT bonded laminated plate. It is clearly observed that the present results are converging well and an (8×8) mesh is sufficient enough to obtain the bending responses and the same is being used for the further computation purposes.

Table 4.4: Convergence behaviour of SSSS PZT bonded laminated Graphite/Epoxy plate subjected to UDL (*t/h*=0.4 and *a/h*=10)

Mesh size	Non-dimensional central Deflection
6×6	0.26
8×8	0.259
10×10	0.259
12×12	0.259
14×14	0.259
15×15	0.258

4.3.4 Comparison study of PZT bonded laminated composite plate

The non-dimensional central deflection ($\overline{w}$) of SSSS PZT bonded composite plates subjected to UDL is computed using the proposed model and the results are compared with the available published literature [19] and presented in Table 4.5. The material are considered as Graphite/Epoxy and PZT^2 of Table 4.1. It is evident that the present numerical results are showing good agreement with those of the published literature. The non-dimensional central deflections are obtained for the thickness ratios (*t/h* = 0.4 and 0.6) and aspect ratio (*a/h* = 10) and normalized to formula $\overline{w} = \left(wE_2h^3 / a^4q_0\right)\times 10^3$. It is clear from the results that the present numerical results are showing good agreement with published literature.

Table 4.5: Comparison study of SSSS PZT bonded laminated Graphite/Epoxy plate subjected to UDL (*a/h*=10)

t/h	Present	Torres and Mendonca [19]
0.2	6.248	6.96
0.4	5.668	5.035
0.6	4.935	4.328

4.4 Vibration Analysis

Now, in order to show the efficiency of the proposed simulation model, the present model is extended to study the free vibration response of the PZT bonded homogeneous isotropic and laminated composite plate. The convergence behaviour of the presently developed simulation model has been obtained for homogeneous isotropic and laminated composite plate. For the validation purpose, the frequencies are obtained for closed and open circuit condition using simulation model and experimental study and the results are compared.

4.4.1 Convergence study of PZT bonded homogeneous isotropic plate

The convergence behaviour of the presently developed model has been obtained and the responses are shown in Figure 4.1. The responses are obtained for clamped (CCCC) PZT bonded homogeneous isotropic plate by considering the material properties as aluminum and PZT-5H of Table 4.1. From the Figure, it can be clearly noted that the model is well converging with the mesh refinement and (8×8) mesh size is sufficient enough to compute the free vibration response of the PZT bonded homogeneous isotropic plate for further analysis.

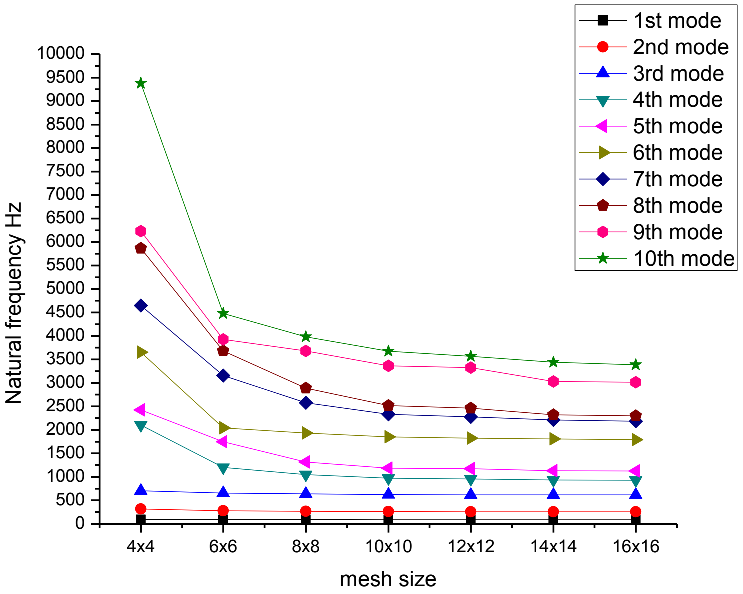


Figure 4.1: Convergence study of CCCC PZT bonded homogeneous isotropic plate

4.4.2 Convergence study of PZT bonded laminated composite

The convergence behaviour of the presently developed model has been obtained and the results are presented in Table 4.6. The frequencies are obtained for CCCC PZT bonded laminated composite plate (CFRP) by taking the material Properties as CFRP and PZT-5H of Table 4.1. The laminate plate has two layers [0°/90°] of Carbon/Epoxy and two layers of PZT-5H. From the table, it can be clearly noted that for the model is converging well with the mesh refinement and (14×14) mesh size is sufficient enough to compute the free vibration response of the PZT bonded laminated composite plate for further analysis.

Table 4.6: Convergence study for CCCC PZT bonded laminated composite plate (CFRP plate)

Mesh size	Natural frequency
10×10	39.292
12×12	38.166
14×14	36.771
16×16	36.746

4.4.3 Comparison study of PZT bonded laminated composite plate (Graphite/Epoxy)

In addition to that, the non-dimensional natural frequency (ϖ) of square SSSS PZT bonded laminated composite plate have been obtained and the results are compared with the previously published literature [19] and presented in Table 4.7. For the computation purpose different thickness ratio (*t/h* = 0.2, 0.4 and 0.6) and aspect ratio (*a/h* = 10) are considered with the material properties as Graphite/Epoxy and PZT-4^2 of Table 4.1. It is clear from the table that the results are in good agreement with those of the published literature. The non-dimensional natural frequency is normalized using the formula $\varpi = \left(\omega a^2 / h\sqrt{\rho}\right)\times 10^3$ where, ω is the natural frequency of PZT bonded laminated composite plate and $\rho = 1\, kg/m^2$ is the density considered for all the substrate layers.

Table 4.7: Comparison study of SSSS PZT bonded laminated composite plate
(Graphite/Epoxy plate) (*a/h* = 10)

t/h	Present	Torres and Mendonca [19]
0.2	205.49	203.73
0.4	225.43	225.62
0.6	237.75	242.44

4.5 Vibration Study – An Experiment

Now, in order to build more confidence in the presently developed model, vibration responses of homogeneous isotropic and laminated composite plates are examined experimentally and compared with those of the developed simulation model. The responses of specimens from CFRP plate with different lamination schemes are obtained experimentally and compared with the model.

4.5.1 Power generation by sensor configuration

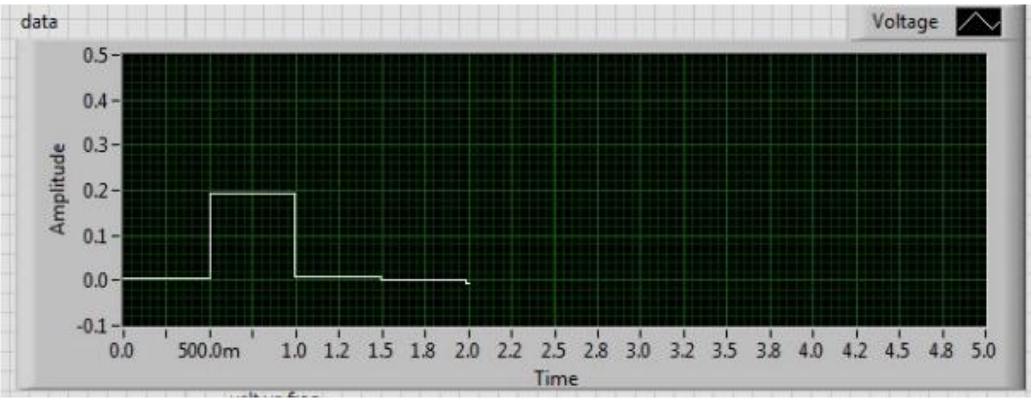
The experimental study for sensor configuration has been already discussed in Chapter 3. In this study, PZT patches are bonded on the upper and bottom surface of the plate to get the required voltage from an external force. The are obtained for the homogeneous isotropic and laminated composite plate are discussed in detail and presented below:

a) PZT bonded homogeneous isotropic plate

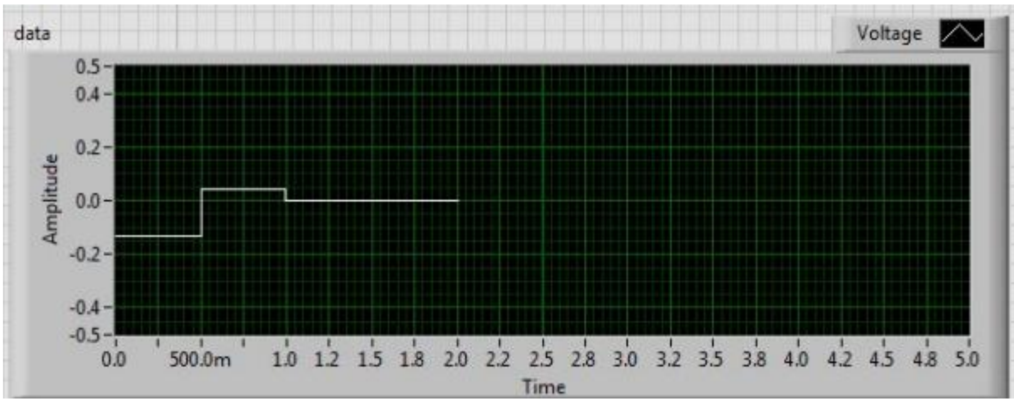
The voltage response of one edge clamped and other edges free (CFFF) PZT bonded homogeneous isotropic plate is shown in Table 4.8. The voltage responses of the homogeneous isotropic plate are decreased as the position of accelerometer changes from free edge (which is opposite to clamped edge) to clamped edge. The responses obtained through LABVIEW are presented in Figure 4.2 (a) and (b) for position 0-2 and 2-4 cm, respectively.

Table 4.8: Voltage response of CFFF PZT bonded homogeneous isotropic plate

Position (cm)	Voltage (volt)
0.-2	1.023293
2-4	1.017419
4-6	1.002305
6-8	1.002305
8-10	1.001728



(a) 0-2 cm



(b) 2-4 cm

Figure 4.2 (a) and (b): Voltage response of CFFF PZT bonded homogeneous isotropic plate

b) PZT bonded laminated composite plate (CFRP plate)

The voltage response of CFFF PZT bonded laminated composite plate is presented in Table 4.9. The responses are decreasing as the position of accelerometer changes from free edge (which is opposite to clamped edge) to clamped edge. The voltage responses of CFFF PZT bonded square laminated composite plate obtained using LABVIEW are presented in Figure 4.3 (a) and (b) for position 0-2 and 2-4 cm, respectively.

Table 4.9: Voltage response of PZT bonded laminated composite plate

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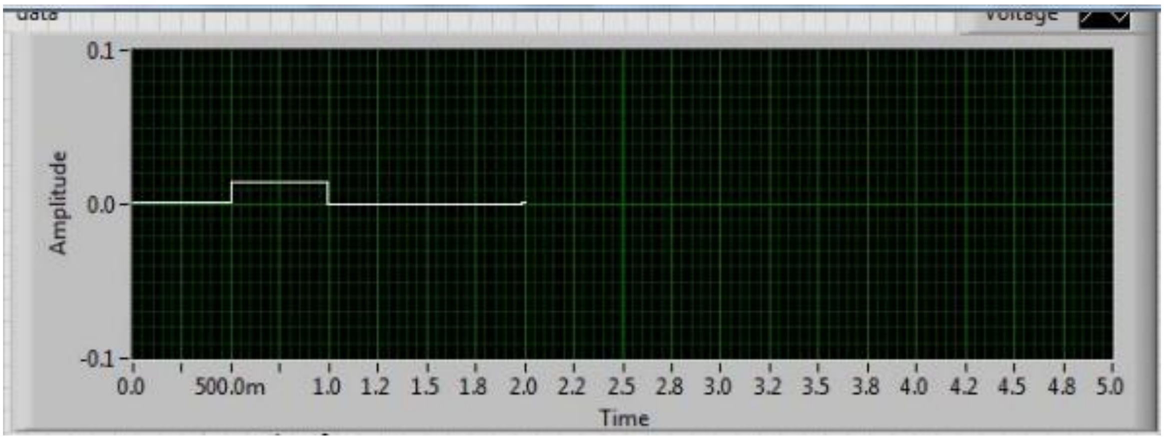
(a) 0-2 cm



(b) 2-4 cm

Figure 4.3 (a) and (b): Voltage response of CFFF PZT bonded laminated composite plate

4.5.2 Vibration response of closed and open circuit condition

In this study, the vibration responses of CFFF PZT bonded homogeneous isotropic and laminated composite plate for closed and open circuit condition is computed. The frequency responses of plates are recorded experimentally by using PXIe-1071 (National Instruments) at NIT Rourkela and which is further compared with simulation model.

a) Homogeneous isotropic plate bonded by PZT patches

The frequency response of CFFF PZT bonded homogeneous isotropic plate for closed and open circuit condition is presented in Table 4.10. From the experimental study, it is observed that the frequency responses in open circuit condition are higher than that of closed circuit condition. This is because of the stiffness in open circuit condition is higher than that of the closed circuit condition. Also, these experimental results are compared with the simulation model developed using ANSYS APDL and it is observed that the difference is small. The responses are obtained using a mesh size of (8×8) using a simulation model. The first two mode shapes of frequency responses of closed and open circuit condition obtained through simulation model and experimental study are presented in Figures 4.4 - 4.7.

Table 4.10: Natural frequency for CFFF PZT bonded homogeneous isotropic plate for closed and open circuit condition

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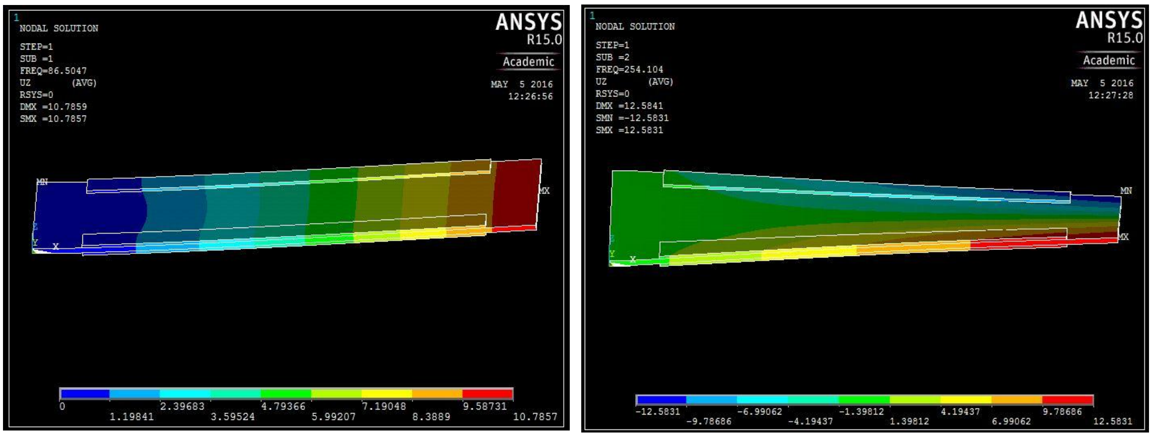
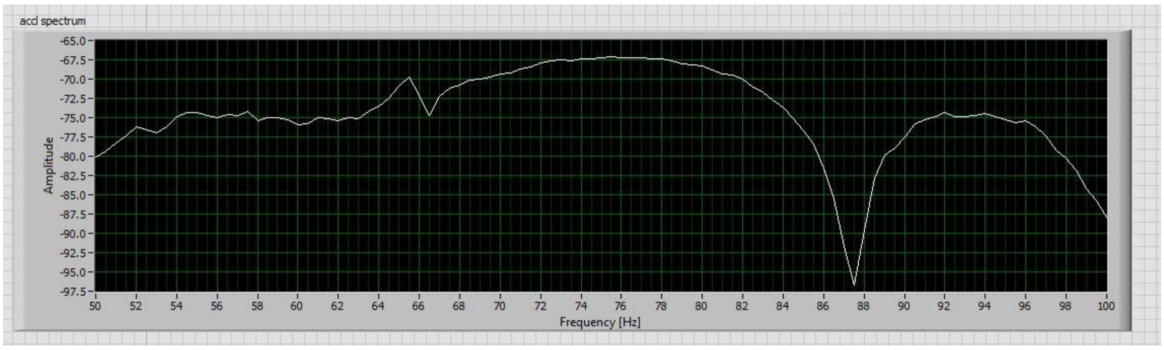
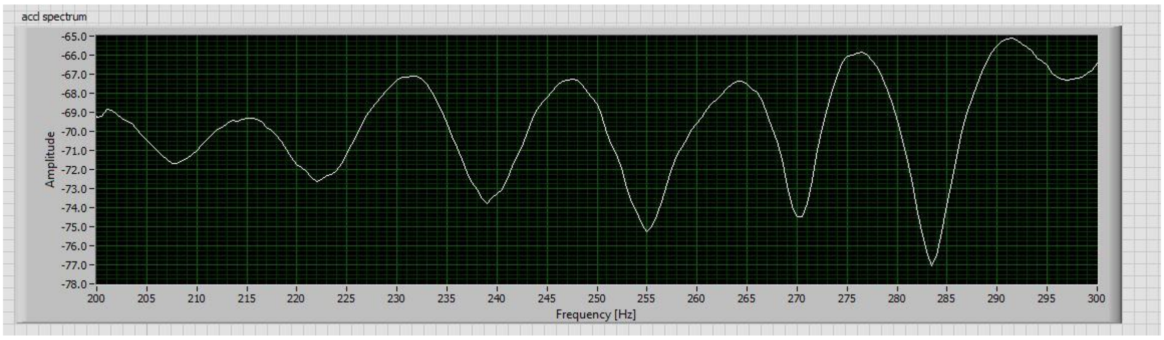


(a) 1st mode (b) 2nd mode

Figure 4.4 (a) and (b): Mode shape of CFFF homogeneous isotropic plate for closed circuit condition in ANSYS environment

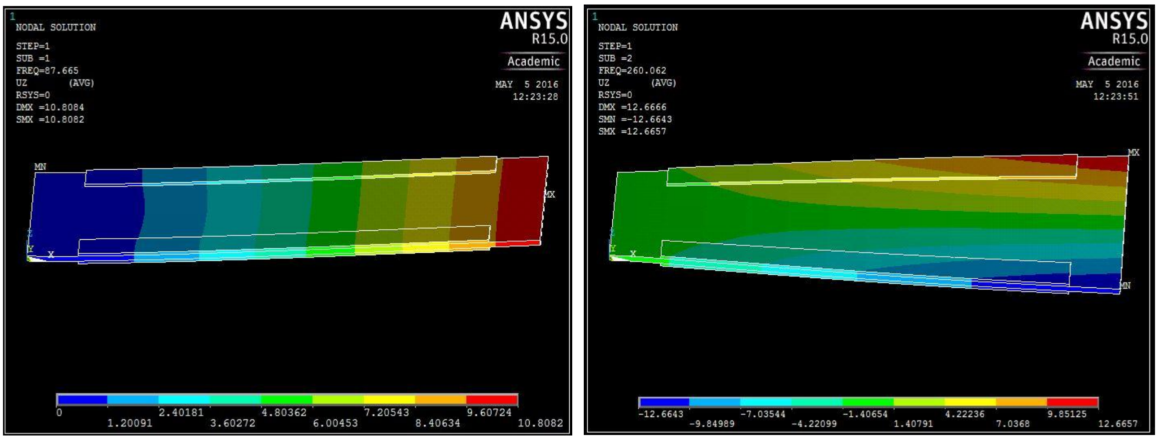


(a) 1st mode



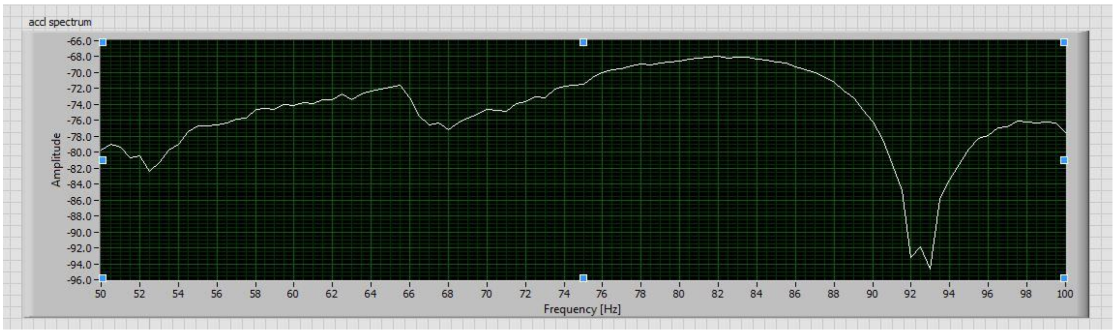
(b) 2nd mode

Figure 4.5 (a) and (b): Mode shapes of CFFF homogeneous isotropic plate for closed circuit condition by an experiment

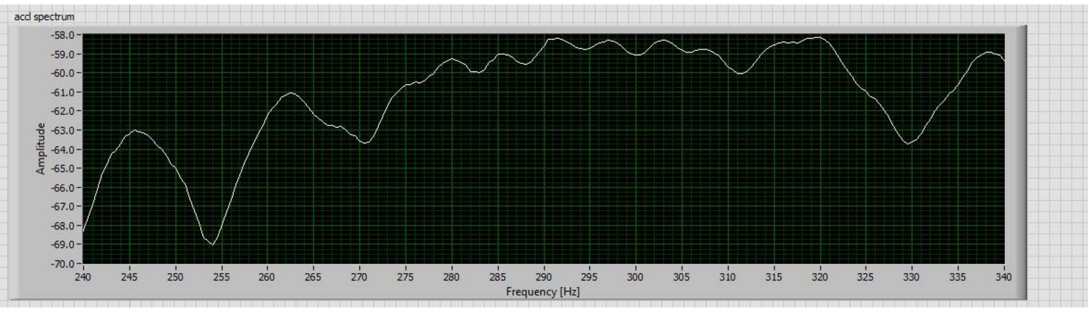


(a) 1st mode (b) 2nd mode

Figure 4.6 (a) and (b): Mode shape of CFFF homogeneous isotropic plate for an open circuit condition in ANSYS environment



(a) 1st mode



(b) 2nd mode

Figure 4.7 (a) and (b): Mode shape of CFFF homogeneous isotropic plate for an open circuit condition by an experiment

b) Laminated composite plate bonded by PZT patches

The frequency response of CFFF PZT bonded laminated composite plate (CFRP) are obtained for closed and open circuit condition and presented in Table 4.13. From the experimental study, it is noted that the responses in open circuit condition are higher than that of closed circuit condition. The responses are follows the same trend as in the case of homogeneous plate and the results are also compared with the results obtained through simulation model. The responses are using a mesh size of (14×14) using a simulation model. The first two mode shapes of closed and open circuit condition obtained in ANSYS environment and experimentally are presented in Figures 4.8-4.11.

Table 4.11: Frequency responses for CFFF PZT bonded laminated composite plate for closed and open circuit condition

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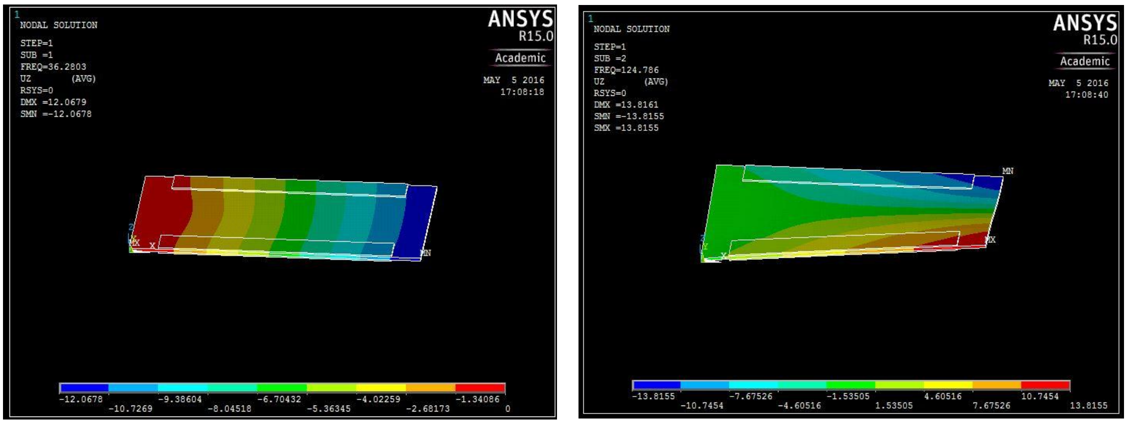
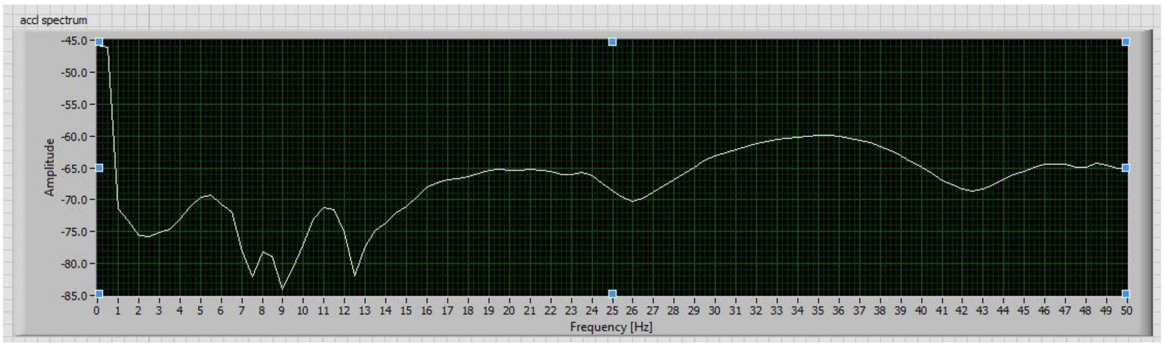


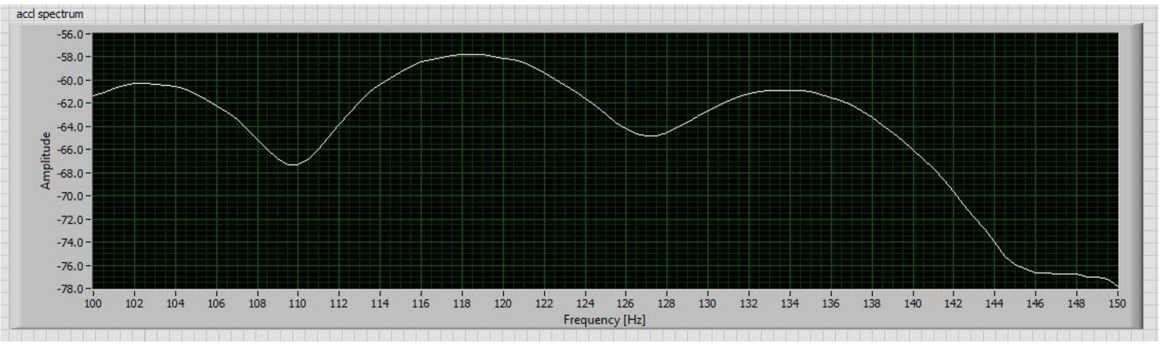
(a) 1st mode

(b) 2nd mode

Figure 4.8 (a) and (b): Mode shape of CFFF laminated composite plate for closed circuit condition in ANSYS environment

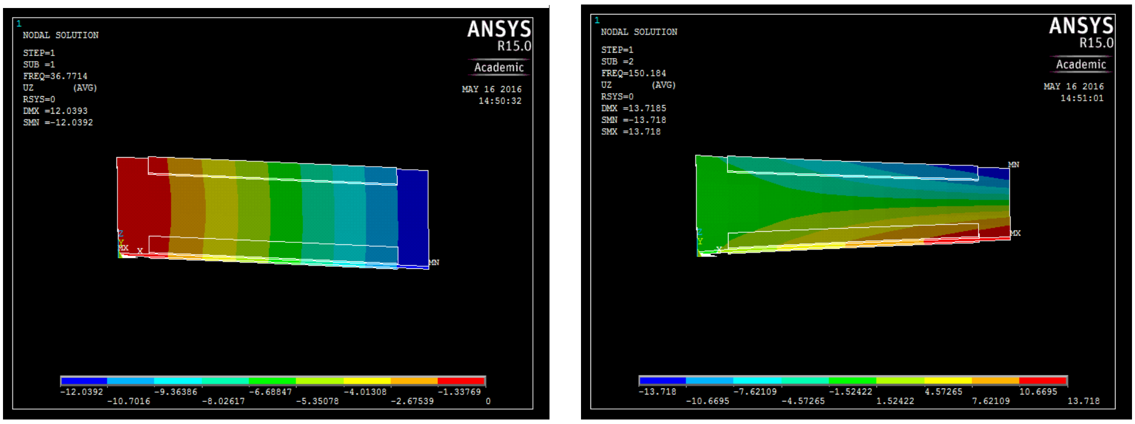


(a) 1st mode



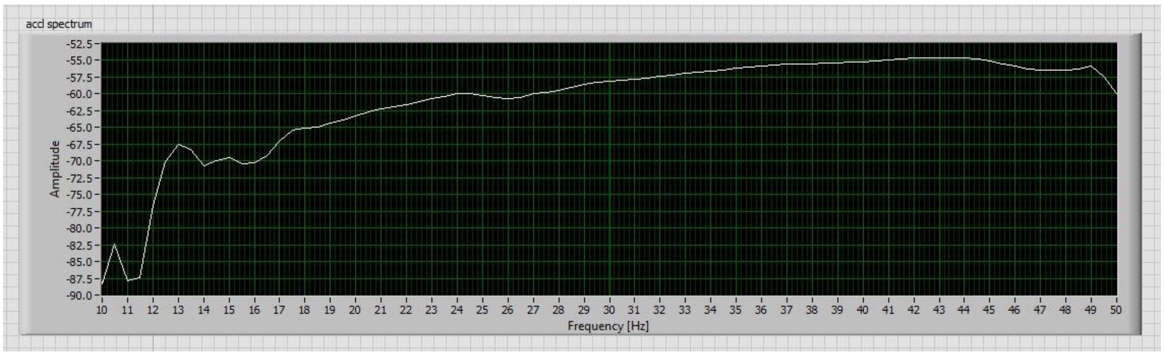
(b) 2nd mode

Figure 4.9 (a) and (b): Mode shape of CFFF laminated composite plate for closed circuit condition by an experiment

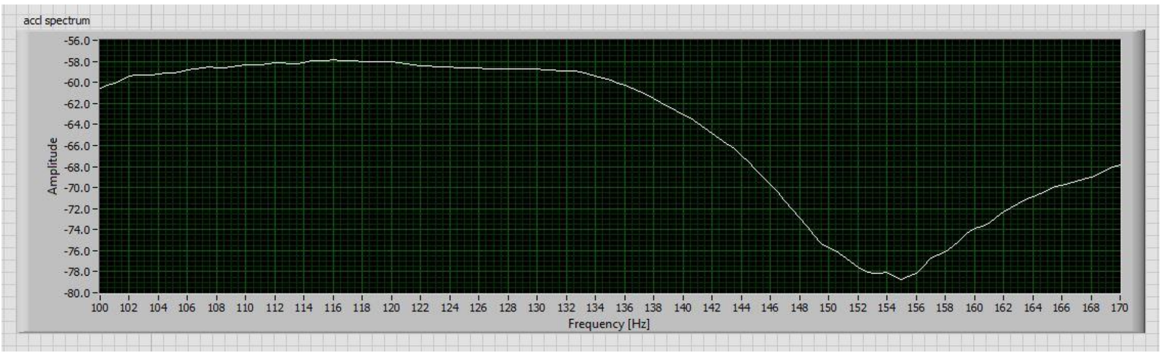


(a) 1st mode (b) 2nd mode

Figure 4.10 (a) and (b): Mode shape of CFFF laminated composite plate for an open circuit condition in ANSYS environment



(a) 1st mode



(b) 2nd mode

Figure 4.11 (a) and (b): Mode shape of CFFF laminated composite plate for an open circuit condition by an experiment

4.6 Numerical Illustration

The present simulation model is extended to study the bending vibration responses of the homogeneous isotropic composite plate by varying the geometrical parameters.

4.6.1 Bending analysis

Central deflection of homogeneous isotropic and laminated composite plate with and without PZT

In this study, the bending behavior of isotropic plate and laminated composite plate are investigated with without PZT plate. The responses are obtained for two opposite edges clamped and two opposite edges free (CFCF) support condition as shown in Figure 4.12. It is important to mention that as *t/h*=0 that mean there is no PZT plates are bonded to plate. The central deflection of the homogeneous plate and the composite plate is inversely proportional to the *t*/*h*. This because the stiffness increases with an electrical potential and mechanical loading.

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(a) Homogeneous isotropic plate

This data is intentionally made blank as the data is used under journal paper preparation

(b) Laminated composite plate

Figure 4.12 (a) and (b): Deflection of homogeneous isotropic and laminated composite plate with and without PZT

Effect of thickness ratio and boundary condition on PZT bonded homogeneous isotropic plate

In this study, the bending behavior of PZT homogeneous isotropic plate is obtained for different support conditions and in Figure 4.13. For all support conditions, the central deflection of homogeneous isotropic plate with an increase in *t/h.* The clamped support condition shows the less deflection in comparison to the other support condition and the reason behind this is structure stiffer. It is also important to mention that the structure becomes stiffer as the plate is loaded which results in generation of electrical voltage in PZT plate.

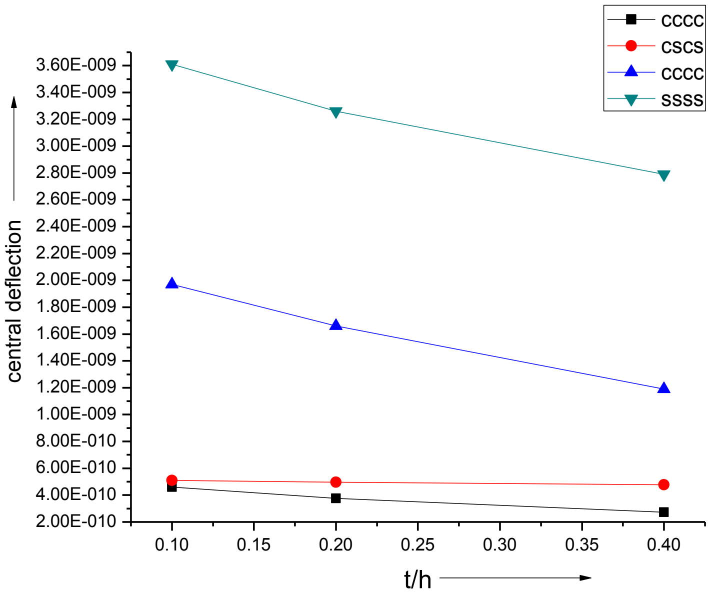

Figure 4.13: Effect of *t/h* ratio and support conditions on the central deflection of PZT bonded homogeneous isotropic plate

Effect of thickness ratio and boundary condition on PZT bonded laminated composite plate

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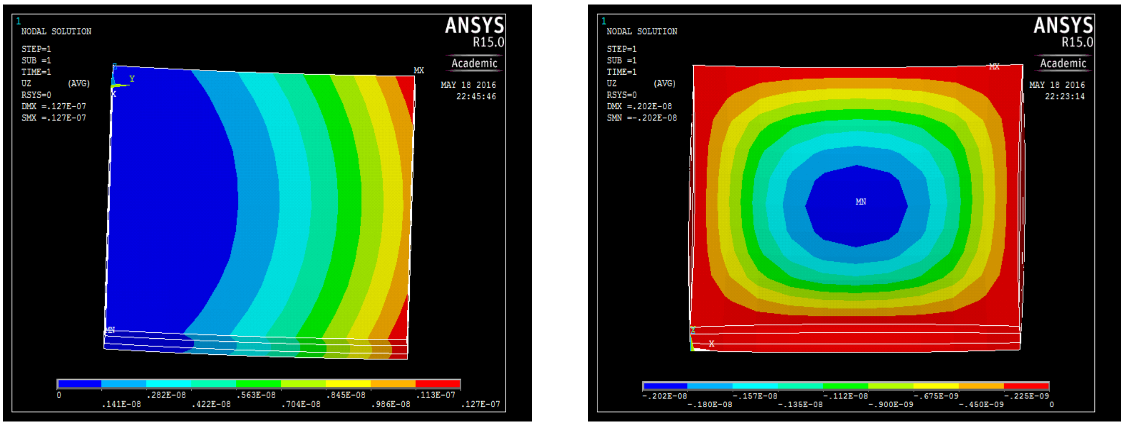
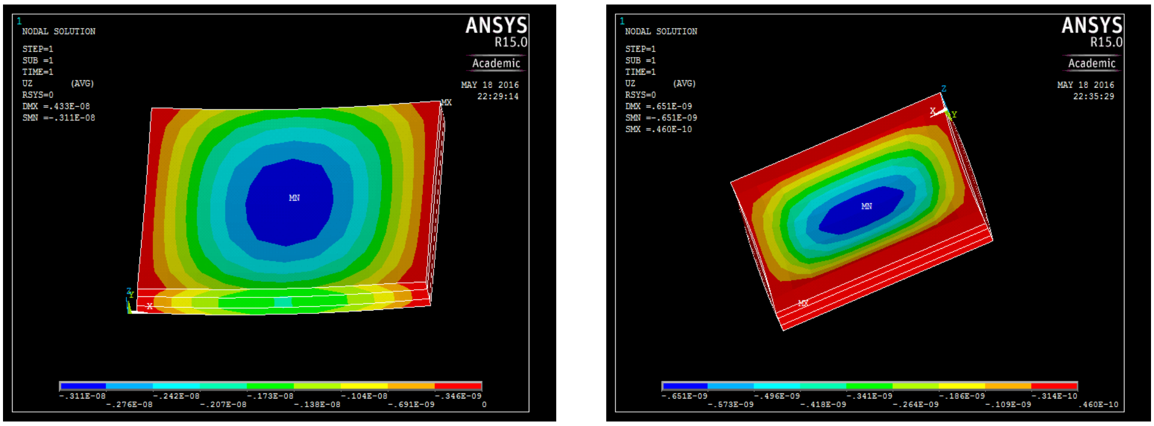
Figure 4.14: Effect of *t/h* ratio and support conditions on the central deflection of PZT bonded laminated composite plate



(a) CFFF (b) SSSS



(c) CFCF (d) CSCS

Figure 4.15 (a), (b), (c) and (d): Variation of central deflection for PZT bonded laminated composite plate

4.6.2 Vibration analysis

Natural frequency of homogenous isotropic and laminated composite plate with and without PZT

In this study, the vibration responses of homogeneous isotropic and laminated composite plate are investigated with and without PZT plate for CFCF support condition and presented in Figure 4.16. As earlier discussed in the present case also, the *t/h*=0 that mean there is no PZT plate is bonded to plate. It is important to mention that the frequency response of homogeneous and the composite plate are directly proportional to the *t*/*h* and the reason behind this is the stiffness increases with electrical potential inside the PZT.

This data is intentionally made blank as the data is used under journal paper preparation

(a) Homogeneous isotropic plate

This data is intentionally made blank as the data is used under journal paper preparation

(b) Laminated composite plate

Figure 4.16 (a) and (b): Frequency responses of homogeneous isotropic and laminated composite plate with and without PZT

Effect of thickness ratio and boundary condition on PZT bonded homogeneous isotropic plate

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This data is intentionally made blank as the data is used under journal paper preparation

Figure 4.17: Effect of *t*/*h* ratio and boundary conditions on frequency response of PZT bonded homogeneous isotropic plate

Effect of thickness ratio and boundary condition on PZT bonded laminated composite plate for vibration analysis

In the similar manner, the vibration behaviour of PZT bonded laminated composite plate is obtained for different support conditions and presented in Figure 4.18. For all the support conditions, the vibration response of laminated composite plate increasing with an increase in *t/h.* The clamped support condition shows the high frequency among all other support condition and the reason behind this is already discussed earlier. The frequency response obtained in ANSYS APDL environment for different support conditions such as CCCC, CSCS, SSSS and SFSF are presented in Figure 4.19 (a), (b), (c) and (d), respectively.

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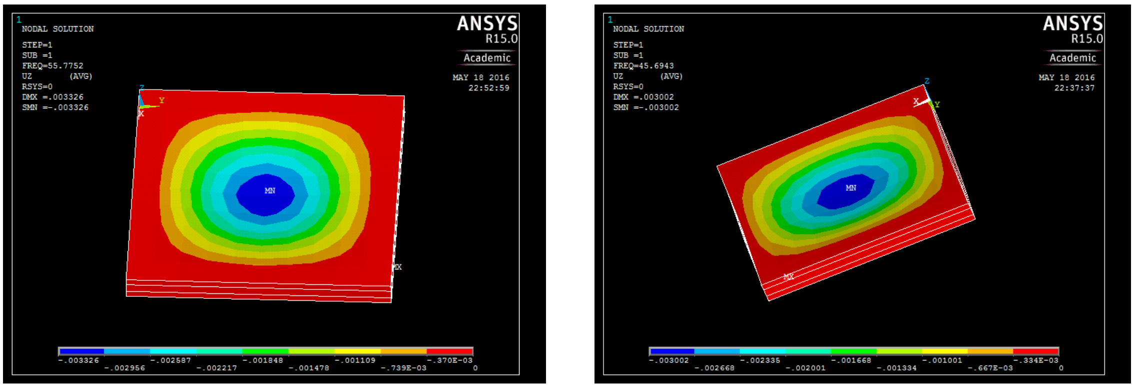
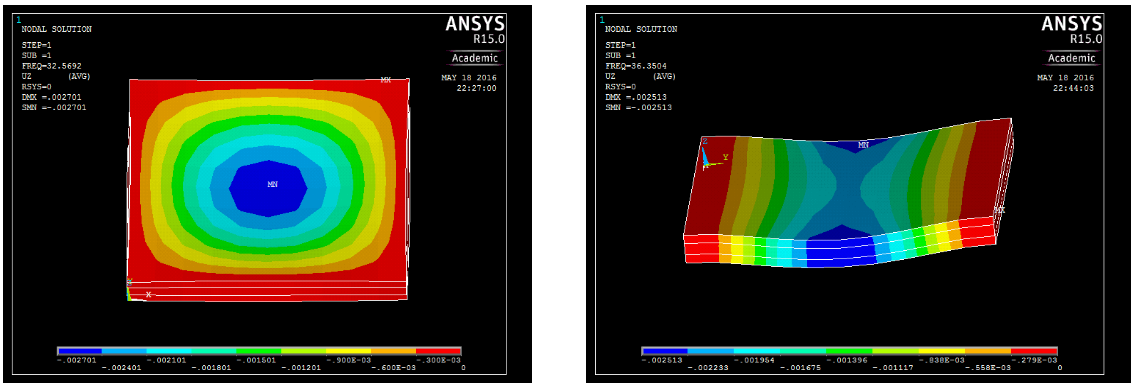
Figure 4.18: Effect of *t/h* ratio and boundary conditions on frequency response of PZT bonded laminated composite plate



(a) CCCC (b) CSCS



(c) SSSS (d) CFCF

Figure 4.19 (a), (b), (c) and (d): Variation of natural frequency for PZT bonded laminated composite plate

Chapter 5

Closure

5.1 Concluding Remarks

In this present work, the bending and vibration behaviour of the PZT bonded homogeneous isotropic and laminated composite plate are examined through experimentally and simulation model developed in ANSYS APDL environment. The bending and vibration responses have been computed and validated with those of the available published literature. In addition to that, the present results obtained vibration analysis using simulation model are also validated with an experimental results. It is observed that as the PZT thickness ratio (*t/h*) increases the maximum central deflection decreasing. It is also observed from the responses that the support conditions affect the responses significantly. A parametric study has been carried out for the bending and vibration behaviour of the PZT bonded homogeneous isotropic and laminated composite plates. The most specific conclusions as a result of the present investigation are stated below:

- Convergence study of the present developed simulation model is performed by refining the mesh density for bending and vibration problems. The comparison study for different cases indicates the necessity and requirement of the present mathematical model for an accurate prediction of the structural behaviour.

- The homogeneous isotropic plate and Graphite/Epoxy composites plates have been examined by taking the different thickness ratio and support conditions. Effects on the homogeneous and composite plate with and without PZT plates on the bending and vibration response are also studied in detailed.

- The natural frequency increases and central deflection decreases when the thickness of PZT to substrate increases. For open circuit condition, the natural frequency is more than that of the closed circuit condition.

5.2 Significant Contribution of the Thesis

The contribution of the present research work is as follows:

- The bending responses of PZT bonded homogeneous isotropic and laminated composite plate is investigated by using FE simulation model developed in ANSYS environment.
- The vibration response of PZT bonded homogeneous isotropic and laminated composite plate is investigated for closed and open circuit conditions experimentally as well as using FE simulation model developed in ANSYS environment.
- The results obtained shows that the good efficiency of the experimental model and developed simulation model.
- The effect of different thickness ratio and support condition on the bending and vibration responses are investigated.
- The deflection and frequency responses are obtained for homogeneous isotropic and laminated composite plate with and without PZT.

5.3 Future Scope of the Work

- The present study can be extended further for the controlled vibration suppression in smart composite shell structure.
- The linear frequency response obtained in the present study can be further used for the computation of nonlinear responses.
- The study can be extended for different geometry & material properties delaminating, cracked composite laminate, buckling analysis also for the hygrothermal environment.
- The present study can be extended to investigate the nonlinear forced/damped vibration and thermomechanical postbuckling behaviour of PZT bonded laminated composite structures by taking temperature dependent material properties based on thenonlinear mathematical model.

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Dissemination

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1 V.U. Ukirde, V.K. Singh, P.V. Katariya, K. Mehar and S.K. Panda, “Bending and Vibration analysis of PZT bonded plates – A Numerical and Experimental Study”