Control of Cart-Inverted Pendulum System Using Pole Placement

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by

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CERTIFICATE

This is to certify that the thesis entitled **Control of Cart-Inverted Pendulum System Using Pole Placement**, submitted by **Sudipto Ghosh**, to the National Institute of Technology, Rourkela, is a record of bonafide research work under my supervision and I consider it worthy of consideration for award of the degree of Master of Technology in Electrical Engineering with specialization in Control and Automation from the institute.

The embodiment of this thesis is not submitted in any other university and/or institute for the award of any degree or diploma to the best of our knowledge and belief.

Dr. Sandip Ghosh

Supervisor

ACKNOWLEDGEMENTS

This thesis is one of my proud possessions and it wont be in its final form without the help of others.

Firstly, I express my sincere gratitude to my supervisor, Dr. Sandip Ghosh. Without his valuable guidance and suggestions this wouldnt have been possible. I also thank him for his motivation, making himself available at the time of need, bearing me with my silly doubts without losing patience. I also express my earnest thanks to Prof. Jitendriya Kumar Satapathy, Head of the Department of Electrical Engineering, NIT Rourkela and Prof. Bidyadhar Subudhi, Coordinator, Control and Automation, for providing all the possible facilities towards this work. Also, thanks also to other faculty members in the department for the course work which established my foundation for research.

I want to thank friends from here, for bearing me two years and giving such love. Thanks friends for making my stay memorable.

My warmest thanks go to my family for their support, love, encouragement and patience.

Sudipto Ghosh

Rourkela

DECLARATION

I certify that

a. The work contained in this thesis is original and has been done by me under the general supervision of my supervisors.

b. The work has not been submitted to any other Institute for any degree or diploma.

c. I have followed the guidelines provided by the Institute in writing the thesis.

d. I have conformed to the norms and guidelines given in the Ethical Code of Conduct of the Institute.

e. Whenever I have used materials (data, theoretical analysis, figures, and text) from other sources, I have given due credit to them in the text of the thesis and giving their details in the references.

f. Whenever I have quoted written materials from other sources, I have put them under quotation marks and given due credit to the sources by citing them and giving required details in the references.

SANDIP GHOSH

Abstract

The Cart Inverted Pendulum has many real life applications like missile launching system, balancing systems like human walking, aircraft landing pad in sea etc. Moreover this is a highly unstable and non-linear system and so designing a controller to bring the system to a stable condition is a challenging task. This thesis includes system and hardware description of Inverted Pendulum System, dynamics of the system, state space model. In this thesis, pole placement methods like two-loop PID and PID+PI have been implemented for Inverted Pendulum System and this control strategies gives stable responses. With the recent development of LMIs tool, regional pole placement can achieve the goal as well. A regional pole placement controller is also synthesized, where desired specifications are transformed into LMI regions. In present case, a conical sector in the left half plane is taken and the method is implemented. Lastly, a reduced order controller is also designed and its bode magnitude plot is compared with that of the full order controller. The reduced order simplification method has an almost identical frequency response, showing that it can be utilised as well for stabilizing the CIPS.

Key words: Two-loop PID, PID-PI, LMI, Pole Placement

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List of Symbols and Acronyms

List of Symbols

$\in$	:	Belongs to
$<$ ($\leq$)	:	Less than (Less than equal to)
$>$ ($\geq$)	:	Greater than (Greater than equal to)
$\neq$	:	Not equal to
$\forall$	:	For all
$\rightarrow$	:	Tends to
I	:	An identity matrix with appropriate dimension
X^T	:	Transpose of matrix X
X^{-1}	:	Inverse of X
$\lambda(X)$	:	Eigenvalue of X
$\det(X)$	:	Determinant of X
$\mathrm{diag}(x_1, \ldots, x_n)$	:	A diagonal matrix with diagonal elements as x_1,x_2,$\ldots$,x_n
$X > 0$	:	Positive definite matrix X
$X \geq 0$	:	Positive semidefinite matrix X
$X < 0$	:	Negative definite matrix X
$X \leq 0$	:	Negative semidefinite matrix X

List of Acronyms

PID	:	Proportional Integral Derivative
CIPS	:	Cart Inverted Pendulum System
DOF	:	Degrees of Freedom
RPP	:	Regional Pole Placement
LMI	:	Linear Matrix Inequality
LTI	:	Linear Time Invariant
PI	:	Performance Index
IFAC	:	International Federation of Automatic Control

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CHAPTER 1

INTRODUCTION

1.1 OVERVIEW

The Cart-Inverted Pendulum System is a pendulum system that has its center of mass above its pivot point. The pivot point is mounted on a cart that can move horizontally. This horizontal movement keeps the bob in an upright position as shown in the image below.

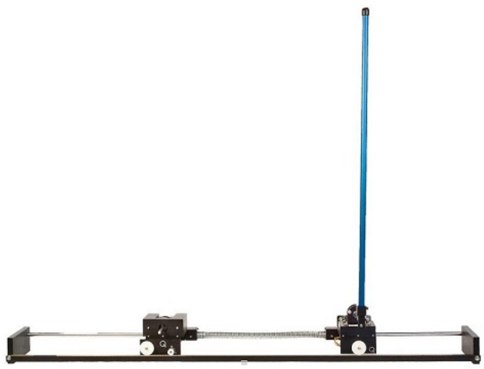

Figure 1.1: Cart-Inverted Pendulum System

The cart and the pendulum both has one DOF each. The cart is attached to a dc motor which makes the cart to move horizontally to and fro. The horizontal motion keeps the bob in an upright stable position. This arrangement is analogous to a ruler balanced upright by the palm of a person. The CIPS is a non-linear highly unstable system. The friction of the cart wheels with the rails and the gravitational pull of the pendulum bob accounts for the non linearity. Being inverted, the pendulum has an inherent tendency to fall and attain its position of minimum potential energy, which renders the system unstable. Yet the system finds wide range of applications in the field of robotics and space explorations, such as humanoid biped robot and rocket launching. A standstill human being is a natural example of an inverted pendulum. He has to constantly make adjustments in order to stay upright. Mobile inverted pendulum (Segway) used for personal transportaion employs the same principle. Seismometers also happen to make use of this system. Thus we see that the CIPS has a quite significant position in the field of Control Systems.

Hence it becomes necessary stabilizing a system that has such wide range of applications. An external force has to be applied to keep the bob stable. Controllers are designed to stabilize the system. In this thesis, we have taken the following strategies to fulfil our objective.

a) Design of a PID-PI controller

b) Design of a two-loop PID controller and

c) Using the technique of Regional Pole Placement to achieve the same.

These methods are enumerated in subsequent chapters.

1.2 LITERATURE REVIEW

Since the 1960's, the Inverted pendulum has been a benchmark problem in control systems. The inverted pendulum first found its application in the year 1844 in Great Britain in seismometers to detect earthquakes. Later on, the same principle of inverted pendulum has been applied to a number of real-time systems as in [1] and thus it has gained its popularity. The detailed method starting from formulating the dynamical equations of the CIPS to finding its state-space representation is stated in here.

The interface of the CIPS with MATLAB environment is done and the steps for real-time built are followed as in [5].

In [7], work has been done towards both swinging and stabilisation of inverted pendulum. For many control system applications PID control is the basic controller which provides optimal performance of the system. Several methods of tuning a PID controller are available as in [3] . In [8] PID tuning is done by pole placement technique. In [4] there is a study which shows a pole placement technique with and without state estimation. This also gives satisfactory response from the CIPS.

In [2] pole placement in LMI regions are discussed and required conditions for stability are given. One such condition has been applied in this thesis for our regional pole placement study. [6] shows two types of regions which constitute the largest class of S-plane regions till date. The results, when this criterion is utilised in both open and closed regions, are used in designing methodology for control systems.

1.3 MOTIVATION

Stabilizing an unstable system is the prime motivation of a control systems engineer. A system finds its application in real life only after it has attained stability under all conditions. The CIPS we are studying in this thesis has numerous applications, provided it is stable. In nature this system is not only non-linear, but highly unstable as well. So to bring the system to stability is a complex and also a much needed task. Keeping this objective in mind, in this thesis we have designed a number of controllers, each of which makes the system stable. Thus attaining stability of the CIPS is the prime motivation of this study.

1.4 OBJECTIVES

With the above motivation in mind, we have the following list of objectives:

a) Modelling the CIPS and finding the transfer function from state-space representation.

b) Linearizing the system by removing the cart friction and gravitational force related non-linearities.

c) Developing controllers to stabilize the system.

1.5 ORGANIZATION OF THE THESIS

The thesis consists of five chapters. A brief summary of each is provided below for quick reference.

Chapter1: It gives an introduction regarding the problem at hand and how it has been dealt with. The previous works in this same field have been mentioned here also.

Chapter2: A brief description of the CIPS is provided here. It also describes the system dynamics using Newtons's Laws of Motion. The non-linear equations have been linearized and the final transfer function model is determined, using the values of the system constants.

Chapter3: This chapter deals with the design methodology of a PID-PI controller. The simulation results and the experimental output are provided here.

Chapter4: In this chapter, two methods of pole-placement approach have been discussed. These include the two-loop PID controller design method and the Regional Pole Placement method. The results are provided alongwith.

Chapter5: This chapter concludes the study. A brief comparison of the three controller outcomes is also added. Any future work possible in this field of control systems is mentioned here.

CHAPTER 2

Modelling of Cart-Inverted Pendulum System

2.1 Introduction

This chapter deals with the dynamics of the inverted pendulum system. The fundamental Newton's Laws of Motion are applied here to obtain the ordinary differential equations which govern the dynamics of the system. Subsequent assumptions have been made to linearize the system about its point of equilibrium. Lastly we reach the final transfer function model of the CIPS.

2.2 Modelling of Inverted Pendulum

The inverted pendulum system is shown below with its various parameters. The cart has only one degree of freedom along the x-axis and the pendulum also has one degree of freedom in the xy-plane.
Here, x represents the position of the cart from the x-axis in meters, while θ represents the angular position of the pendulum from the vertical.
The parameters represent:
M- Cart mass
m- Pendulum mass

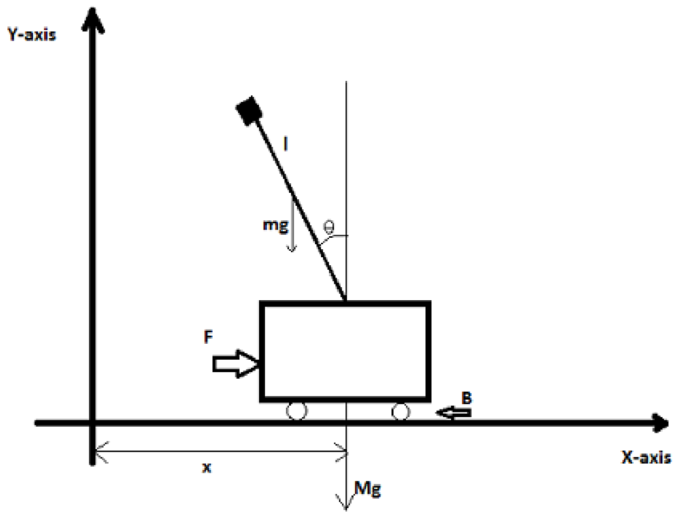

Figure 2.1: Parametric representation of Inverted Pendulum System

J- Moment of inertia

L- Pendulum length

b- Cart friction co-efficient

g Gravitational force

The free body diagram of the cart is shown below:

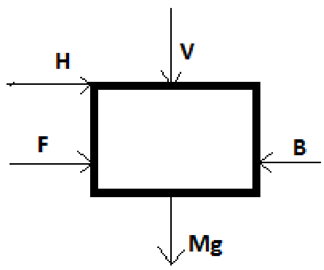

Figure 2.2: FBD of cart

Taking the forces acting on the cart in the horizontal direction we can write,

$M\ddot{x} = F + H - B$, where $\ddot{x}$ represents the accelaration along the x-axis.

Now the equation of the bob is written as:

$H = m\frac{d^2}{dt^2}(x + l\sin\theta) = m(\ddot{x} + \ddot{\theta}\, l\cos\theta - \dot{\theta}^2 l\sin\theta)$

The free body diagram of the pendulum is shown below.

The forces in the vertical plane are now considered. We can write,

$P + mg = m\frac{d^2}{dt^2}(l\cos\theta) = m(l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta - g)$

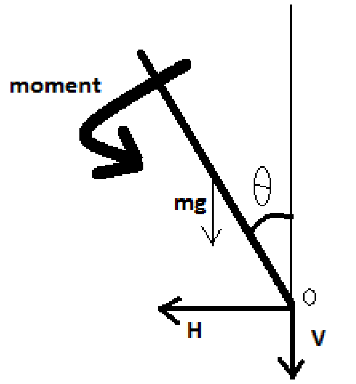

Figure 2.3: FBD of pendulum

The moment due to the reaction forces H and V are resolved into X and Y directions. After summing all the moments across the centre O, we will get:

$ml\,\ddot{x}\cos\theta - (ml^2 + J)\,\ddot{\theta} = -mgl\sin\theta$

By substituting the second equation in the first, we get

$\ddot{\theta} = \frac{ml}{\sigma}[(F - Bx)\cos\theta - m(\dot{\theta})^2 l\cos\theta\sin\theta + (M+m)g\sin\theta]$

By solving for $\ddot{x}$, we get

$\ddot{x} = \frac{1}{\sigma}[(J + ml^2)(F - B\,\dot{x} - ml(\dot{\theta})^2\sin\theta) + ml^2 g\sin\theta + ml^2 g\sin\theta\cos\theta]$

where $\sigma = ml^2(M + m(\cos\theta)^2) + J(M+m)$

This gives the cart-pendulum system dynamics.

2.3 Linearization of the model

the non-linear nodel thus obtained is now linearized using the following asumptions about the equilibrium point $\theta = 0$ as,

$\sin\theta = \theta$ and

$\cos\theta = 1$

The linearized model dynamics are as follows:

$\ddot{\theta} = \frac{ml[(F - B\dot{x}) + (M+m)g\theta]}{\sigma'}$ and

$\ddot{x} = \frac{[(ml^2+J)(F - B\dot{x}) + ml^2 g\theta]}{\sigma'}$ where,

$\sigma' = Mml^2 + J(m+M)$

The state space for the Inverted Pendulum system is thus given by:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \theta \\ \ddot{\theta} \end{bmatrix}^{\cdot} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(J+ml^2)b}{\sigma'} & \frac{m^2l^2g}{\sigma'} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{\sigma'} & \frac{mgl(m+M)}{\sigma^1} & 0 \end{bmatrix} \begin{bmatrix} x \\ x^{\cdot} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{(J+ml^2)}{\sigma'} \\ 0 \\ \frac{ml}{\sigma'} \end{bmatrix} F$$

$$y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}^T \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

The following parameter values are incorporated in the state space system:

$M = 2.4kg$

$m = 0.23kg$

$J = 0.099kg - m^2$

$l = 0.4m$

$B = 0.05Ns/m$

$g = 9.81m/s^2$

We finally obtain the transfer functions of the CIPS.

$\frac{X(s)}{F(s)} = \frac{5.841}{s^2}$

$\frac{\theta(s)}{F(s)} = \frac{3.96}{s^2-6.807}$

From the transfer functions we can conclude that the system is unstable. We now proceed with the controller design which will stabilize the system.

2.4 Experimental Setup

The CIPS consists of the following components:

1) PC with PCI-1711 card

2) Feedback SCSI Cable Adaptor

3) Digital Pendulum Controller

4) DC Motor (Actuator)

5) Cart and Pendant Pendulum with weight

6) Optical encoders with HCTL2016 ICs

7) Track of 1m length with limit switches.

8) Adjustable feet with belt tension adjustment.

9) Software: MATLAB, SIMULINK, Real-Time Workshop, ADVANTECH PCI-1711 Device driver, Feedback Pendulum Software.
10) Connection cables and wires.

The two main components of the CIPS include the cart and the pendulum. The cart moves on the track and the pendulum is fixed to it. The Inverted Pendulum has its centre of gravity above its reference point which leads to instability. The DC servo motor drives the cart and the force with which it drives the cart is proportional to the control voltage applied. The carts moment on the track is limited by using two limit switches which will shut down the power to the cart when activated. The cut way diagram shows the location of sensors and switches on the track. There are digital encoders mounted on the track which outputs a signal that is a combination of two signals 90 degrees apart one representing shaft position and the other direction of rotation. Two such digital encoders are used, one for the cart and the other for the pendulum.
The implementation of controller in real time is done by designing a specific controller in MATLAB Simulink and then applying it on the real time model.
The cutaway diagram showing the sensors and the whole mechanical setup are shown in the figures.
The control algorithm for the CIPS is shown below.

2.5 Real-Time Workshop

The steps for real-time built are as follows:
1. At first analyzing block diagram and after that this is compiled to intermediate hierarchical representation in the form of model.rtw.
2. Now model.rtw is read by TCL and converted into C code.
3. A make file is now constructed by TCL and that placed into built directory. 4. For compiling the source code the make file is now read by system make utility and an executable file model.exe is generated.
Now system can easily understand the file as because the file is in binary format.

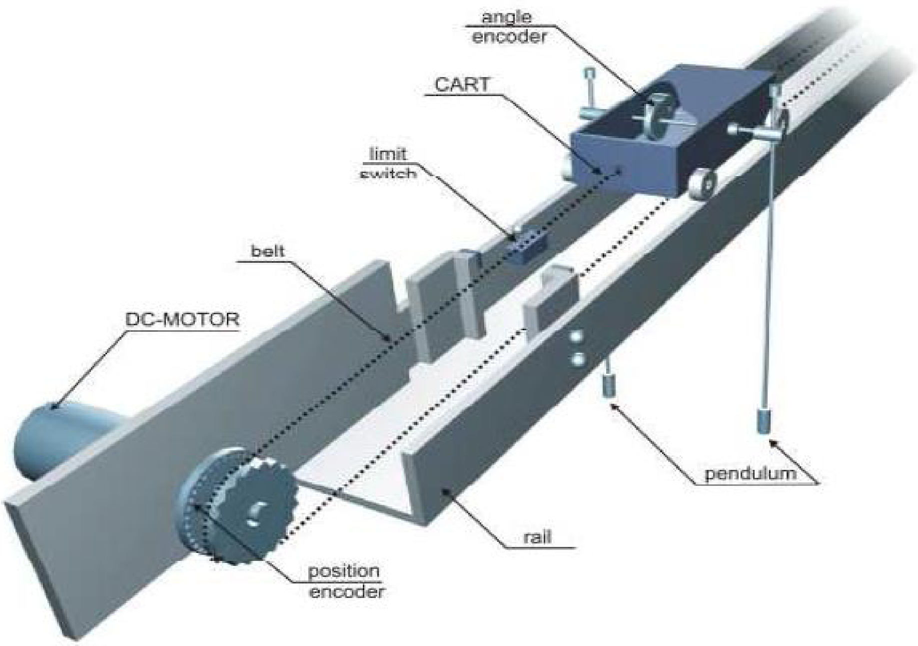

Figure 2.4: Cutaway Diagram showing the sensors

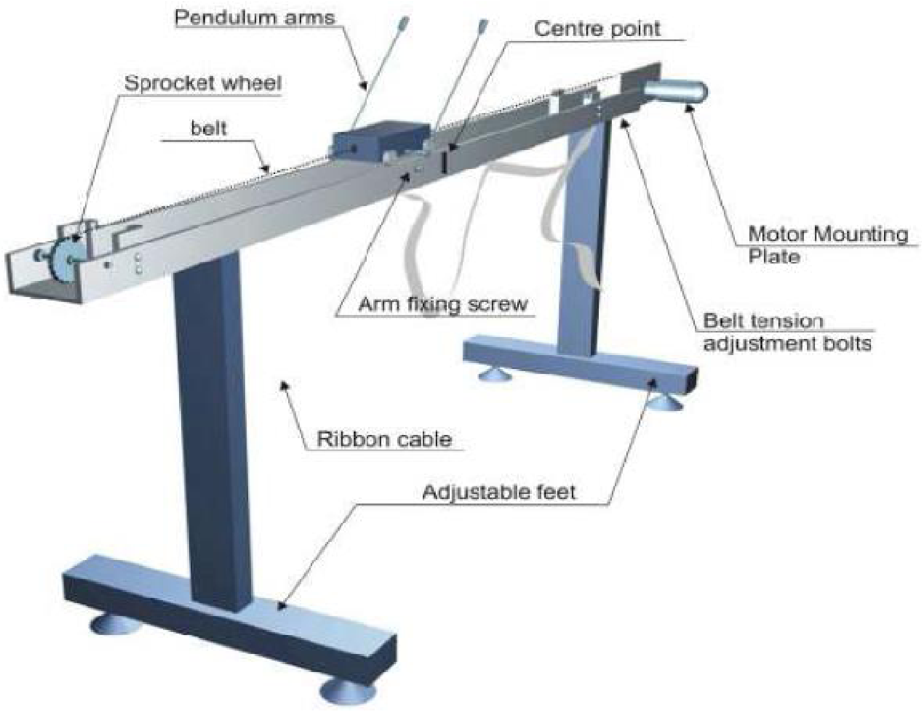

Figure 2.5: Complete experimental setup of CIPS

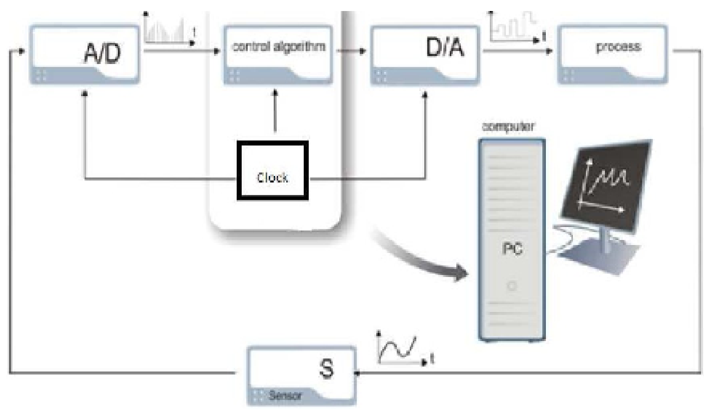

Figure 2.6: Control Algorithm for CIPS

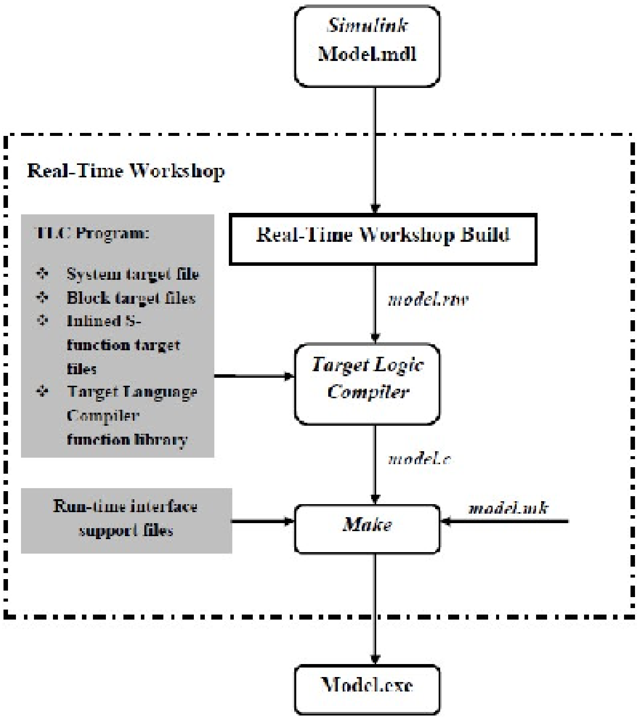

Figure 2.7: Working Scheme in Real-time

CHAPTER 3

Control of Inverted Pendulum

In this chapter, we have designed a conventional method of stabilizing the inverted pendulum system, ie using a PID-PI controller. The reason behind using two controllers is that we need to control not only the angular position of the pendulum but also the linear displacement of the cart. The method is enumerated below.

3.1 PID-PI method

The PID-PI method block diagram is shown below.

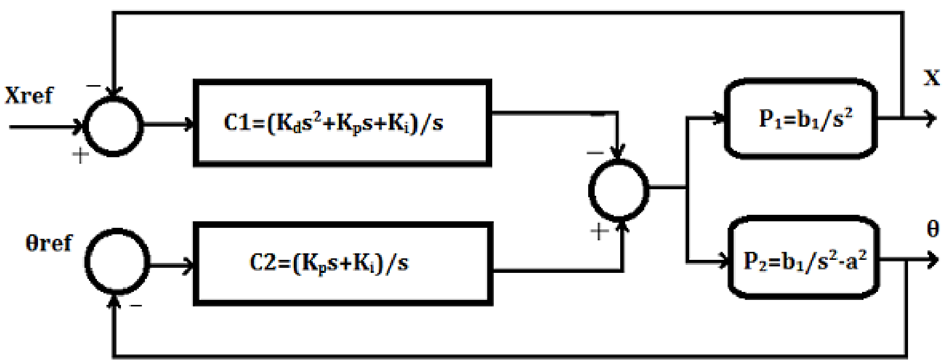

Figure 3.1: PID-PI Block Diagram

$b_1 = 5.841$

Here, $b_2 = 3.96$

$a^2 = 6.807$

In the block diagram shown, C_1 and C_2 represent the controllers that have to be designed. One controller is dedicated for controlling the cart position and the other for controlling the pendulum swing. P_1 and P_2 represent the plant transfer functions. It is to be noted that the angle reference is given as zero, since we wnat the inverted pendulum system to be stable about the equilibrium point $\theta = 0$. Now after simplifying the block diagram, we get the characteristic equation as:

$$1 - P_1C_1 + P_2C_2 = 0$$

The derivation of the above equation is enumerated below:

We have, from the block diagram,

$[(\theta_{ref} - \theta)C_2 - (x_{ref} - x)C_1]P_1 = x$ and

$[(\theta_{ref} - \theta)C_2 - (x_{ref} - x)C_1]P_2 = \theta$

Now, to find the transfer function for a MIMO system, we choose one of the reference inputs to be zero, say $\theta_{ref} = 0$. Now substituting this value in the second equation we have the value of θ as

$\theta = \frac{(x - x_{ref})P_2C_1}{1+P_2C_2}$

Now, we again substitute this value of θ in the first equation and we get

$\frac{(x_{ref} - x)P_1P_2C_1C_2}{1+P_2C_2} - P_1C_1(x_{ref} - x) = x$

From this, after manipulation, we find the transfer function of the cart displacement x to the reference input x_{ref} as

$\frac{x}{x_{ref}} = \frac{-P_1C_1}{1-P_1C_1+P_2C_2}$

Proceeding similarly and taking $x_{ref} = 0$, we find the transfer function $\frac{\theta}{\theta_{ref}}$.

Thus we have the characteristic polynomial to be

$$1 - P_1C_1 + P_2C_2 = 0$$

Substituitng the values of the transfer functions we get,

$1 - \frac{b_1(K_{d1}s^2+K_{p1}s+K_{i1})}{s^3} + \frac{b_2(K_{d2}s^2+K_{p2}s+K_{i2})}{s(s^2-a^2)} = 0$

Now, from the deired pole location, we get the characteristic equation as,

$s^5 + 26s^4 + 216.6s^3 + 872.7s^2 + 1723.3s + 1342.2 = 0$

After substituting values and comparing we get PID Gains as follows,

$K_{p1} = 43.4$
$K_{i1} = 33.5$ and $K_{p2} = 121.7$
$K_{d1} = -4.52$ $K_{i2} = 316.4$

3.1.1 Simulation and Experimental Results

The simulation results are shown below:

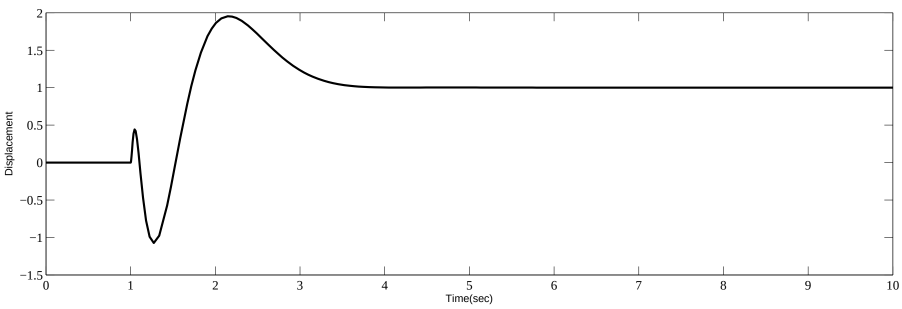

Figure 3.2: Simulation Result for Displacemnt

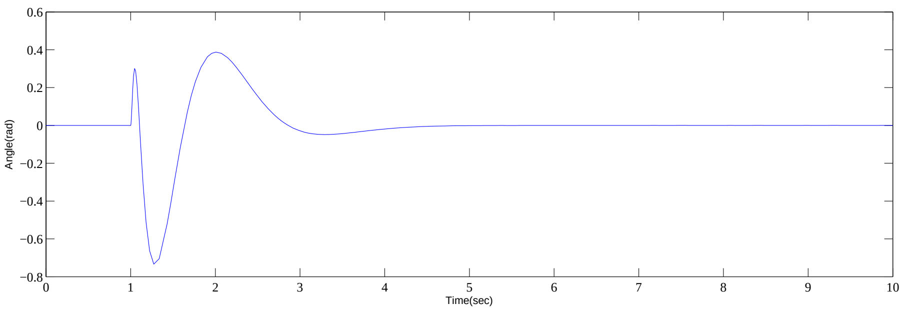

Figure 3.3: Simulation Result for Angle

The experimental results for PID-PI Controller is shown below. The responses obtained are quite satisfactory.

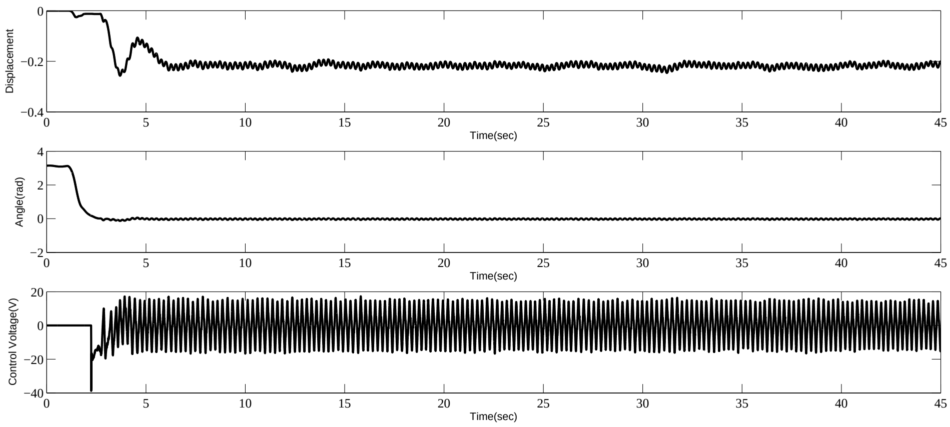

Figure 3.4: Experimental Result for PID-PI

3.2 Summary

This chapter describes very basic of PID-PI controller to solve the stability issue associated with Inverted Pendulum. Here pole-placement technique is used for PID+PI controller design and designed controller gives satisfactory response both in Simulation and Realtime.

CHAPTER 4

Pole Placement Design for Inverted Pendulum

4.1 Conventional Pole Placement: Two-Loop PID

The PID controllers are hugely popular owing to their simplicity in working. These controllers are also easy to implement with the help of electronic components. In this chapter, a control scheme involving two PID loops have been implemented for stabilizing the CIPS. The PID controller structure is shown below:

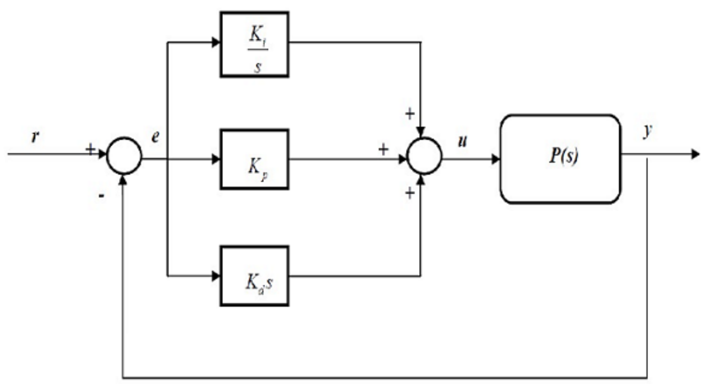

Figure 4.1: PID Controller Structure

The control signal u fed to the plant depends on the actuating error signal e . The reference input r is also called the set-point weighting in process control literature. The mathematical representation of the control action is:

$u = r - y$

$u = K_p e + K_i \int edt + K_d \frac{de}{dt}$

It is seen that with the increase in the value of proportional gain, the value of error becomes greatly reduced but the response becomes highly oscillatory, though there exists a constant steady state error. There comes the application of the Integral term. The integral term ensures that the steady state error is zero. But, a large value of the integral gain makes the response sluggish leading to unsatisfactory performance. The role of the derivative gain is to damp the oscillatory behaviour of the process output. Use of high value of the derivative gain may lead to instability. Thus we see, that there is a trade-off while choosing the values of the proportional, integral and derivative constants. In order to achieve satisfactory performance, we use one or the other PID tuning methods available, out of which Ziegler-Nichols tuning is the most popular one. In this section this tuning method has been made use of, the details of which is provided in [3].

4.2 Controller Design

The block diagram of the control strategy is shown below:

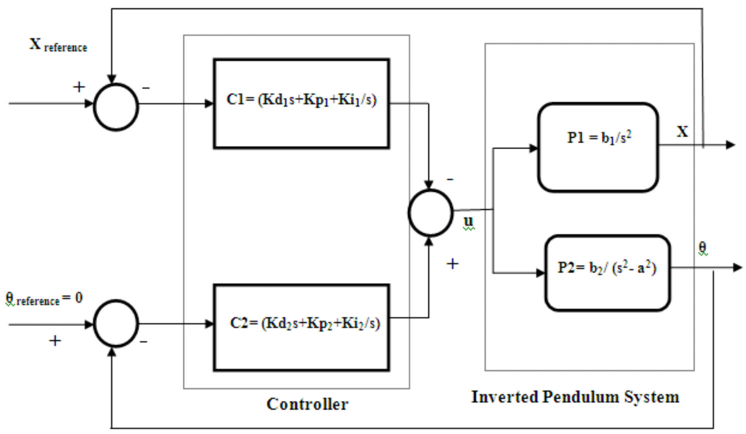

Figure 4.2: Two-Loop PID Controller Block Diagram

Here, $b_1 = 5.841$
$b_2 = 3.96$
$a^2 = 6.807$

In the block diagram shown, C_1 and C_2 represent the two PID controllers to be designed. One controller is dedicated for controlling the cart position and the other for controlling the pendulum swing. P_1 and P_2 represent the plant transfer functions. It is to be noted that the angle reference is given as zero, since we wnat the inverted pendulum system to be stable about the equilibrium point $\theta = 0$. Now after simplifying the block diagram by following the manipulation as shown before, we get the characteristic equation to be:

$$1 - P_1C_1 + P_2C_2 = 0$$

Substituitng the values of the transfer functions we get,
$1 - \frac{b_1(K_{d1}s^2+K_{p1}s+K_{i1})}{s^3} + \frac{b_2(K_{d2}s^2+K_{p2}s+K_{i2})}{s(s^2-a^2)} = 0$
Now, from the deired pole location, we get the characteristic equation as,
$s^5 + 26s^4 + 216.6s^3 + 872.7s^2 + 1723.3s + 1342.2 = 0$
We compare the two characteristic equations and obtain the PID parameters. But here we find that there are six unknowns and five equations. So we choose $K_{d2} = 10$. Thus we get the following PID gain values.

$K_{p1} = 43.4$ $\quad$ $K_{p2} = 119.7$
$K_{i1} = 34.5$ and $K_{i2} = 245.7$
$K_{d1} = 2.34$ $\quad$ $K_{d2} = 10$

4.2.1 Simulation and Experimental Results

The MATLAB simulation results are shown below for both the displacement and angle outputs. The results are satisfactory.

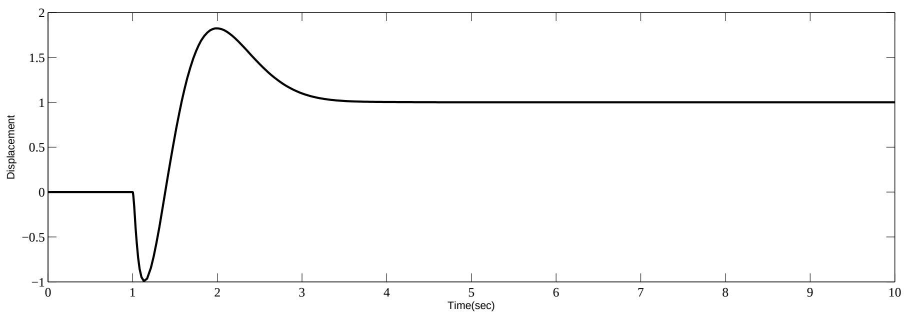

Figure 4.3: Simulation Result for Displacemnt

The Experimental result is shown below.

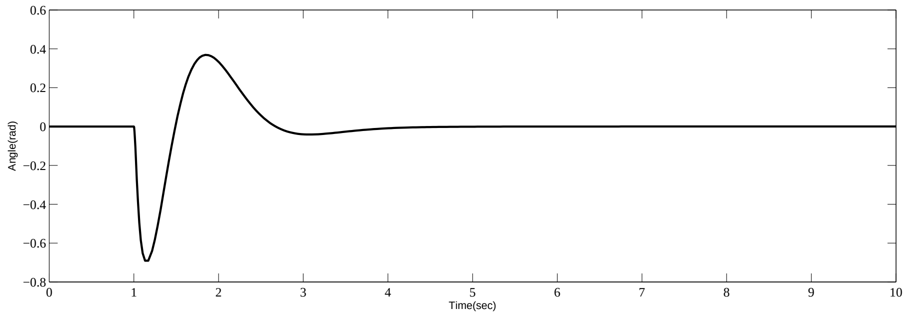

Figure 4.4: Simulation Result for Angle

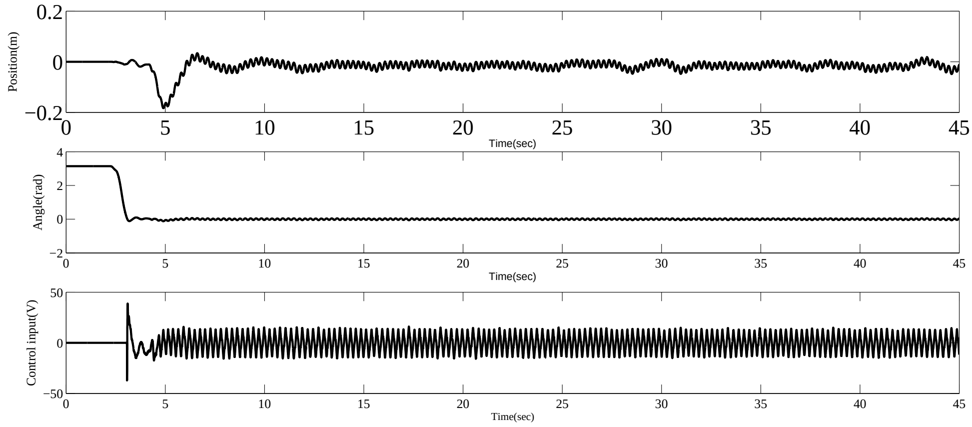

Figure 4.5: Experimental Result for Two-Loop PID Controller

4.2.2 Summary

This chapter describes very basic of PID controller to solve the stability issue associated with Inverted Pendulum. Here pole-placement Technique is used for PID controller design and designed controller gives satisfactory response both in simulation and real-time experiment

4.3 Control of CIPS using Robust Pole Placement

This section discusses the method of regional pole placement to stabilize the CIPS. M.Chilali et al. [2] has studied the method of designing an output feedback controller using LMI. The concept of choosing a region in S-plane based on time domain specifications is also provided. Regional pole placement refers to the process of choosing specific region in the complex plane where the poles should ge placed so that stability of the system can be achieved. This is in contrary to the traditional pole placement techniques where poles are placed at specific points only.Complex regions include a circular disc, a conical sector, etc.

4.3.1 LMI Region

This subsection enumerates how a region in the complex plane is represented by an LMI. A dynamical system described as the state equation $\dot{x} = Ax$ is said to be stable if there exists a positive definite matrix $P \succ 0$ such that $A^T P + PA \prec 0$

This is considered to be the first LMI and is otherwise known as Lyapunov inequation. P matrix is called the Lyapunov function matrix. As another example, we can write the quadratic inequality $A^T P + PA + PBR^{-1}B^T P + Q < 0$ in the form of an LMI as follows:
$\begin{pmatrix} -A^T P - PA - Q & PB \\ B^T P & R \end{pmatrix} < 0$

In fact, a general representation of a LMI is given as $F_0 + \sum F_i x_i < 0$, where F'is are real, symmetric matrices and x'i's are unknown, real scalar decision variables. Finding out whether such x exists or not which satisfies the LMI is called the matrix feasibility. In context to this paper, the LMI is used to describe a region of the complex plane, D which is as follows:

$$L + zM + z^* M^T < 0$$

,where L and M are symmetric matrices and z is a complex variable.

For an example, the above LMI with L = 0 and M = 1 gives D as the entire left half complex plane, as shown below:

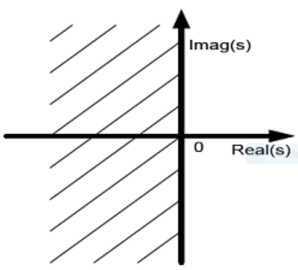

Figure 4.6: Left Half S-plane represented by LMI

When an unit circle centred at the origin is to be represented as the region D, then the values of L and M are,

$$L = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

and $M = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

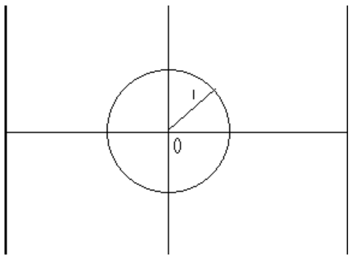

Figure 4.7: Unit Circle Centred at Origin represented by LMI

When a conical sector is to be represented as the region D, then the values of L and M are, $L = 0$ and $M = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Here the apex of the sector lies at the origin as shown below.

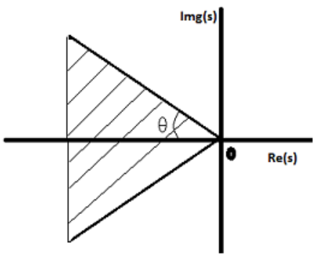

Figure 4.8: A Conical Sector represented by LMI

4.3.2 Pole Placement by Output Feedback Controller

This subsection deals with the design of an output feedback controller that will place the poles in the specified LMI region, represented by D. Thus even after the nominal system is perturbed with an uncertainty Δ, the system remains quadraticaly stable, which, in other words, means the poles are robustly assigned. The controller dynamics is as follows. The block diagram showing the plant, controller and the uncertainty block is shown below:

$\dot{x}_c = A_c x_c + B_c y$. (1)

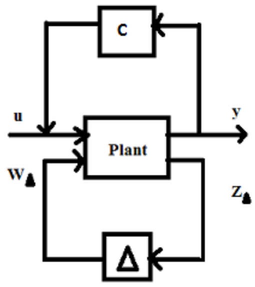

Figure 4.9: Block Diagram showing Plant, Controller and Uncertainty

$u = C_c x_c + D_c y$. (2)
where A_c, B_c, C_c, D_c are the controller parameters to be determined. Note here, in case of an output feedback controller, u acts as the output and y acts the input signal.
The uncertain state space model of the system is given by:
$\dot{x} = Ax + B_\Delta w_\Delta + B_u u$ .. (3)
$z_\Delta = C_\Delta x + D_{\Delta_\Delta} w_\Delta + D_{\Delta_u} u$ (4)
$y = C_y x + D_{y_\Delta} w_\Delta$ (5)
$w_\Delta = \Delta z_\Delta$. (6)
From the above two systems, a combined equivalent closed loop system is computed having parametersA_e, B_e, C_e, D_e. The computational procedure is enumerated below:
The state equation of the system is written as:
$\dot{x} = Ax + B_\Delta w_\Delta + B_u C_c x_k + B_u D_c C_y x + B_u D_c D_{y_\Delta} w_\Delta$. (7)
The above equation is obtained by substituting equation (5) in equation (2) and then putting the modified equation (2) in equation (3). Similarly we put equation (5) in equation (1) and obtain the following controller state equation:
$\dot{x_c} = A_c x_c + B_c C_y x + B_c D_{y_\Delta} w_\Delta$ (8)
Now the output equation (4) is written as (by substituting equation (5) in (2) and then substituting modified equation (2) in (4)),
$z_\Delta = C_\Delta x + D_{\Delta_\Delta} w_\Delta + D_{\Delta_u} C_c x_c + D_{\Delta_u} D_c C_y x + D_{\Delta_u} D_c D_{y_\Delta} w_\Delta$ (9)
Combining equations (7), (8) and (9) we can write the following state space representation:
$\begin{bmatrix} \dot{x} \\ \dot{x_c} \end{bmatrix} = \begin{pmatrix} A + B_u D_c C_y & B_u C_c \\ B_c C_y & A_c \end{pmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} + \begin{bmatrix} B_\Delta + B_u D_c D_{y_\Delta} \\ B_c D_{y_\Delta} \end{bmatrix} w_\Delta$ .... the combined state equation and

$z_\Delta = [C_\Delta + D_{\Delta_u} D_c C_y \;\; D_{\Delta_u} C_c] \begin{bmatrix} x \\ x_c \end{bmatrix} + [D_{\Delta_\Delta} + D_{\Delta_u} D_c D_{y_\Delta}] \, w_\Delta$ .... the output equation.

Thus we find the values of the parameters A_e, B_e, C_e and D_e as follows:

$A_e = \begin{pmatrix} A + B_u D_c C_y & B_u C_c \\ B_c C_y & A_c \end{pmatrix}$

$B_e = \begin{bmatrix} B_\Delta + B_u D_c D_{y\Delta} \\ B_c D_{y\Delta} \end{bmatrix}$

$C_e = [C_\Delta + D_{\Delta_u} D_c C_y \;\; D_{\Delta_u} C_c]$

$D_e = [D_{\Delta_\Delta} + D_{\Delta_u} D_c D_{y\Delta}]$

The equivalent block diagram thus obtained is shown below:

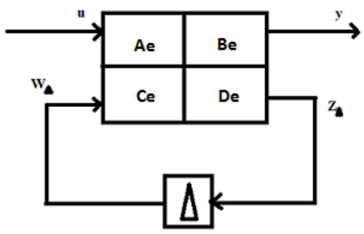

Figure 4.10: Equivalent Block Diagram

For robust D-stability of the system, there should exist a positive definite matrix X such that,

$$\begin{pmatrix} M_D(A_e, X) & {M_1}^T \otimes (X B_e) & {M_2}^T \otimes {C_e}^T \\ M_1 \otimes ({B_e}^T X) & -\Upsilon I & I \otimes {D_e}^T \\ M_2 \otimes C_e & I \otimes D_e & -\Upsilon I \end{pmatrix} \prec 0 \text{ ,where } M = {M_1}^T M_2$$

4.3.3 Controller Design

The region to put the closed loop poles is chosen as a conic sector, represented by an LMI, with its tip at $s = -2$. The angle the edge of the section makes with the negative real axis is taken as $\cos^{-1}(0.99)$ The poles of the closed loop function obtained is shown in the figure.

Transfer function of the controller for cart position obtained is

$G_1(s) = \frac{.0075s^6 - 9390s^5 - 4.2*10^5 s^4 - 6120s^3 - 3.613*10^7 s^2 - 7.28*10^7 s - 3.5*10^7}{s^6 + 180s^5 + 10780s^4 + 2.7*10^5 s^3 + 3.2*10^6 s^2 + 1.6*10^7 s + 2.4*10^7}$

Transfer function of the controller for the output pendulum angle obtained is

$G_2(s) = \frac{.081s^6 + 2.4*10^7 s^5 + 1.13*10^6 s^4 + 1.82*10^7 s^3 + 124s^2 + 3.5*10^8 s + 3.5*10^8}{s^6 + 180s^5 + 10780s^4 + 2.7*10^5 s^3 + 3.2*10^6 s^2 + 1.6*10^7 s + 2.4*10^7}$

4.3.4 Results and Discussions

The simulation and experimental results are shown below.

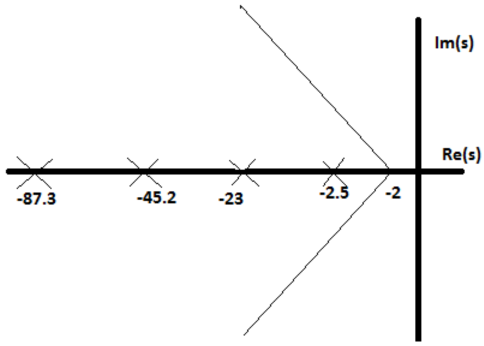

Figure 4.11: Conical Sector chosen for Controller

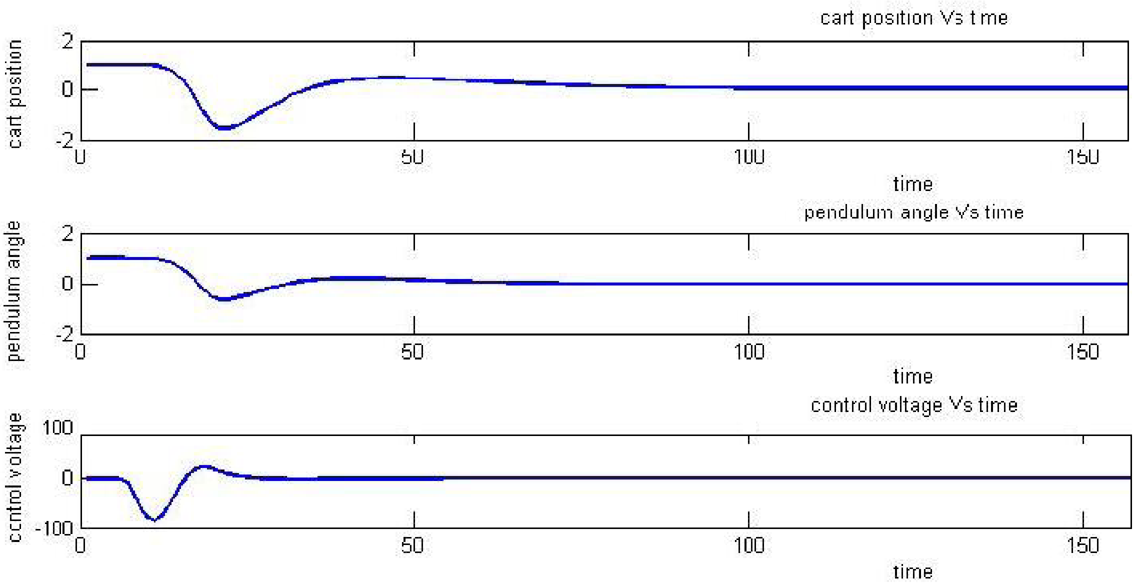

Figure 4.12: Simulation Result for Regional Pole Placement Method

The chapter contains a brief discussion on Regional Pole placement technique. Various LMI regions along with their characteristic equations have been described. The controller design is being illustrated. Based on time domain specifications, we have located the region of S-plane where the closed loop poles should be located. The pole locations are shown in figure 4.11. The controllers are designed and the simulation and experimental results are provided at the end. We can infer that the stability of the CIPS has been achieved.

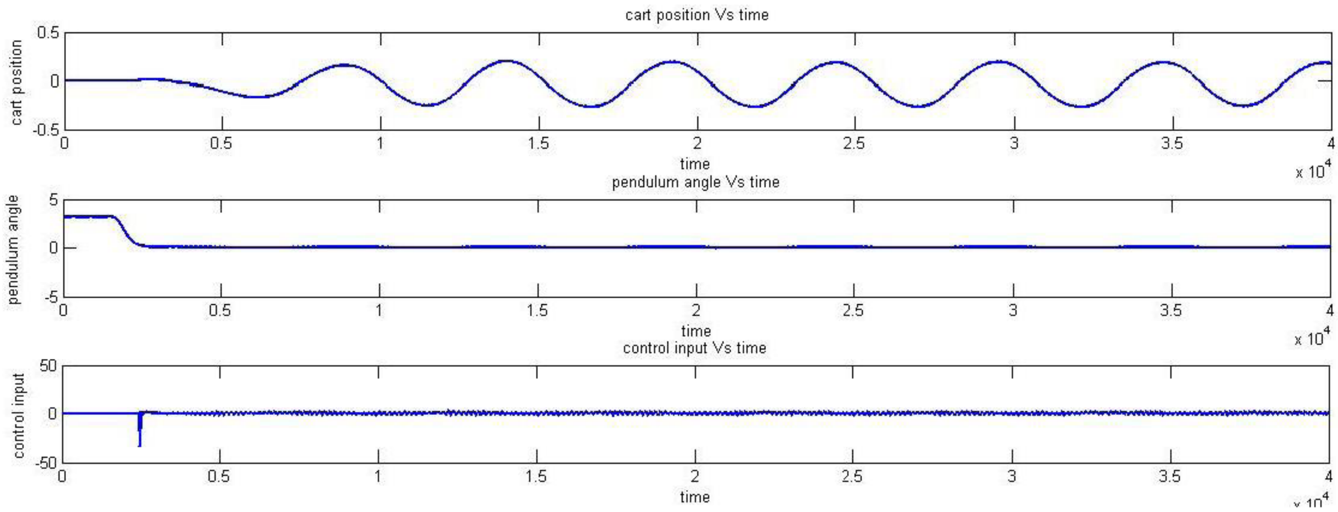

Figure 4.13: Experimental Result for Regional Pole Placement Method

4.3.5 Pole Placement Using Reduced Order System

Now we are reducing the order of the transfer functions of the cart position and that of the pendulum swing angle. The reduced order transfer functions are as given below:

$$R_1(s) = \frac{-s^4 - 8993s^3 + 2.9*10^4 s^2 - 7.7*10^5 s - 4.2*10^5}{s^4 + 124.7s^3 + 4395s^2 + 5.1*10^4 s + 2.83*10^5}$$

$$R_2(s) = \frac{881s^4 + 2.327*10^7 s^3 - 1.2*10^8 s^2 + 2.69*10^8 s + 2.95*10^7}{s^4 + 170s^3 + 9825s^2 + 2.14*10^5 s + 2.02*10^6}$$

The motivation behind using a reduced order system is that we have to handle a simpler, easily factorisable transfer function. But we have to make certain that by using a reduced order transfer function, the system responses do not change drastically. In our study, we have made a comparison of the Bode magnitude plots of the full order and the reduced order transfer functions. The comparison is shown below:

Here, the red plot refers to the full order transfer function, while the blue one refers to the reduced order plot.

We can infer that the reduced order transfer functions can be used safely instead of the higher order ones with a minimum percentage error.

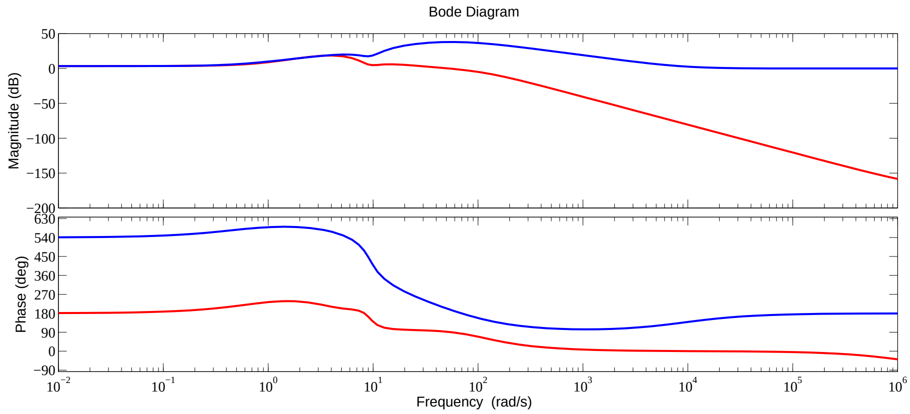

Figure 4.14: Comparison of Bode Plot for Cart Transfer Functions

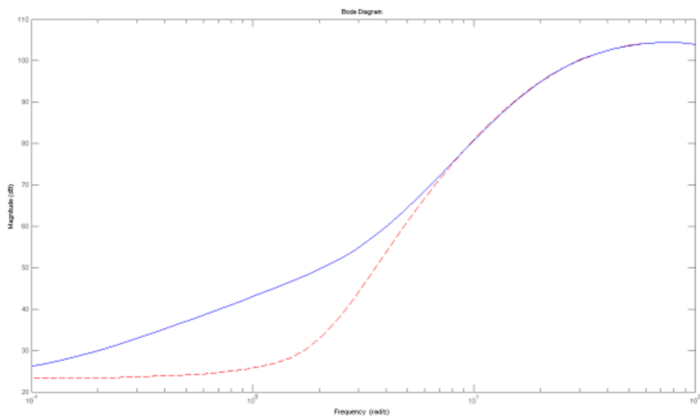

Figure 4.15: Comparison of Bode Plot for Pendulum Angle Transfer Functions

CHAPTER 5

Conclusion and Future Work

5.1 Conclusion

The thesis addresses the inherent instability related with the CIPS by designing various control methodologies, for example, Twoloop PID Controller, PID-PI Controller and Regional pole placement technique. These controllers have satisfactorily stabilized the CIPS. Due to the presence of non-linear friction term in the system dynamics there has been a difficulty in obtaining the idealistic behaviour for the cart position.

Regional pole placement technique is used and here the pole locations are chosen automatically within a specified region in the s-plane. For a Two- Loop PID controller, two loops have been used and the controller is designed based on pole placement. A Two-Loop PID controller provides a good overall performance.

A comparison between the PID-PI and Two-Loop PID is provided here based on pendulum swing angle and displacement of cart and certain inferences can be drawn.

From the comparative responses, we see that the Overshoot and Undershoot is almost similar for the two types of controllers. In simulation study, performance of Two-loop PID and PID-PI almost same. In case of Two-loop PID controller overshoot is minimum as compared to PID-PI controller but control voltage requirement is higher. In case of PID-PI controller overshoot is slightly more than Two-loop PID controller but settling time, control voltage is lesser than Two-Loop PID.

Thus we come to the conclusion that overall response of PID+PI is satisfactory than its Two-Loop counterpart.

5.2 Future Work

The stability of the Cart-Inverted Pendulum System can be achieved by designing Artificial Neural Network or Fuzzy Logic or Integral Sliding Mode controllers also. All these methods will also give satisfactory results.

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