BUCKLING ANALYSIS OF TWISTED CANTILEVER FGM PLATES WITH AND WITHOUT CUT-OUTS

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Master of Technology

Submitted by

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CERTIFICATE

This is to certify that the thesis entitled "**BUCKLING ANALYSIS OF TWISTED CANTILEVER FGM PLATES WITH AND WITHOUT CUT-OUTS**", submitted by **METKU VIVEKANAND SAGAR** bearing **Roll no. 214CE2061** in partial fulfilment of the requirements for the award of **Master of Technology** in the Department of Civil Engineering, National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university/Institute for the award of any Degree or Diploma.

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DECLARATION OF ORIGINALITY

I, **Metku Vivekanand Sagar**, Roll Number **214CE2061** hereby declare that this thesis **Buckling Analysis of Twisted Cantilever FGM Plates with and without Cut-outs** presents my original work carried out as a post graduate student of NIT Rourkela and, to the best of my knowledge, contains no material previously published or written by another person, nor any material presented by me for the award of any degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the dissertation. Works of other authors cited in this dissertation have been duly acknowledged under the sections "*Reference*" or "*Bibliography*". I have also submitted my original research records to the scrutiny committee for evaluation of my dissertation.

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ABSTRACT

The use and applications of composites are expanding these days. Due to light weight, high specific strength and stiffness, composite materials are being widely used as a part of wind turbine blades and ship building. For high temperature applications, Functionally Graded Materials (FGM) are preferred over laminated composites because of its good performance in the thermal field. Chopper blades, turbine cutting blades, marine propellers, compressor blades, fan shape blades, and mostly gas turbines use pre-twisted cantilever plates. Often they are subjected to thermal environments, and thus FGMs are a decent option to metal plates. Composite structures with cut-outs are usually employed in engineering structures. In structural components cut-outs are provided sometimes to lighten the structure and for proper ventilation. Cut-outs in aircraft components (for example, fuselage, ribs and wing spar) are required for inspection, access, fuel lines and electric lines or to minimize the general weight of the aircraft.

Study of buckling of cantilever twisted functionally graded material plates with and without holes and with varying applied in-plane loads is dealt in the present work. The analysis is carried out by using ANSYS. An element having six degrees of freedom per node SHELL-281 is used. The FGM plate is assumed to be a laminated section containing a number of layers with a steady variation of the material property through the thickness, where each layer is taken as isotropic. Material properties in each layer are determined using power law. Results obtained from convergence studies, carried out by using 12 number of layers and 12 by 12 mesh, are found to be quite accurate. Buckling behavior of cantilever twisted FGM plates with and without cut-outs and for different non-uniform applied in-plane loads are studied for the effect of various parameters like material gradient index, aspect ratio, side to thickness ratio, diameter of cut-outs and twist angle.

KEYWORDS: Functionally Graded Materials, Twisted Plates, Buckling

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ABBREVATIONS

The principal symbols used in this thesis are presented for reference. Every symbol is used for different meanings depending on the context and defined in the text as they occur.

a, b	Length and width of twisted panel
a/ b	Aspect ratio
h	Thickness of Plate
b/ h	Width to thickness ratio
Φ	Angle of twist
n	Gradient index
E	Modulus of elasticity
G	Shear Modulus
ϑ	Poisson's ratio
K	Shear correction factor
kx, ky, kxy	Bending strains
Mx, My, Mxy	Moment resultants of the twisted panel
[N]	Shape function matrix
Nx, Ny, Nxy	In-plane stress resultants of the twisted panel
${N_x}^0$, ${N_y}^0$	External loading in the X and Y directions respectively
Aij, Bij, Dij and Sij	Extensional, bending-stretching coupling, bending and transverse shear stiffness
dx, dy	Element length in x and y-direction
dV	Volume of the element
Qx , Qy	Shearing forces
Rx, Ry, Rxy	Radii of curvature of shell in x and y directions and Radius of twist
Γ	Shear strains
σx σy τxy	Stresses at a point

τxy, τxz, τyz	Shear stresses in xy, xz and yz planes respectively
εx, εy, γxy	Strains at a point
θx, θy	Rotations of the mid surface normal about x- and y-axes respectively
N_{cr}	Critical Buckling load
$\overline{N}$	Non-dimensional buckling load
ρ_j	Density at j^{th} layer

CHAPTER 1

INTRODUCTION

Introduction

In all engineering applications, unadulterated metals are of little use. To deal with this issue, blending (in liquid state) of one metal with various metals or non-metals is used. This mix of materials is termed alloying that gives a property that is not the same as the guardian materials. There is limit to which a material can be dissolved in a solution of another material in view of thermodynamic equilibrium limit. Powdered metallurgy (PM) is another technique for creating combinations in powdered structure. In spite of the brilliant attributes of powdered metallurgy, there exist a few restrictions, which include: complex shapes can't be created utilizing PM; the parts are permeable and have poor quality. Another technique for delivering materials with blend of properties is by combining materials in solid state, which is alluded as composite material.

Composite materials are formed by combining two or more materials to achieve some superior material with distinct chemical and physical properties which aren’t similar to their individual parent materials. Composite materials when compared to their parent materials are considered stronger, tougher and lighter in weight. Under extreme working conditions, laminated composite materials will fail through a procedure called delamination (partition of strands from framework) which happens during high temperature applications where materials with different coefficient of expansion are used. To tackle this issue, in mid-1980’s scientists in Japan, came up with a novel material called Functionally Graded Material (FGM).

Functionally Graded Material (FGM), are a new class of engineered materials with varying properties over changing dimensions. The FGM idea started in the year 1984 in Japan as part of a space research program. This system visualized the manufacturing of a temperature safe material having a thickness less than 10mm and able to resist a temperature of 2000 Kelvin and a temperature gradient of 1000 Kelvin [Koizumi and Niino, 1995]. FGM, takes out the sharp interfaces existing in composite material from where delamination gets started and replaces this sharp interface with smooth interface which produces continuous transition in properties from one surface to next thereby eliminating the stress concentration. One remarkable aspect for FGM is the capacity to tailor a material for particular application. FGMs have distinctive applications particularly for aviation, automobiles, engineering and hardware.

Twisted Cantilever panel finds its application as turbine blades, compressor blades, fan blades, marine propellers and many more. Cut-outs are often found in composite structures. They are provided in structural components for ventilation and to lighten the structure. The twisted plates have become key structural units which are being researched a lot presently. In view of the utilization of twisted FGM panels in aviation, turbomachinery, and aeronautical businesses, it is important to have knowledge of buckling characteristics of pre-twisted panels with and without cut outs.

CHAPTER 2

LITERATURE REVIEW

2.1 Literature Review

2.1.1 FGM Twisted Plates without Cut-outs

The literature on the behavior of functionally graded materials is very rich due to their versatility of behavior and its wide range of applications.

Abrate (2006) presented the problems of functionally graded plates regarding free vibration, static deflections and buckling, where material properties were assumed to vary along the thickness direction. It was demonstrated that, every other parameter remaining same, the natural frequencies of homogeneous isotropic plates and of functionally graded plates were found to be proportional.

Birman and Byrd (2007) displayed a survey on functionally graded materials with priority on the works published since 2000. Diverse areas significant to various forms of uses and theory of FGM were reflected in the paper which included heat transfer issues, dynamic analysis, stress, homogenization of particulate FGM, applications, design and manufacturing, testing and fractures. The critical areas where further research was required for an effective execution of FGM was outlined in the conclusion.

Cherradi *et al*. (1994) displayed the advancements and a short theoretical description of FGMs, their applications, handling techniques, etc. along with their analysis, in Europe specifically. They showed that the functionally graded material can be tailored to meet for specific requirements. The related properties were not just mechanical (hardness, wear resistance, static and dynamic strength), but also chemical (oxidation resistance and corrosion), thermal (heat conductivity and isolation), and so forth.

Javaheri and Eslami (2002) derived stability and equilibrium equations for rectangular functionally graded (FG) plates. They assumed that the material properties varied with power law along thickness direction. The buckling behavior of functionally graded plates were studied using the stability and equilibrium equations for the case where all edges are simply supported and was subjected to in-plane loading conditions. By considering zero as the index of power law, the derived equations were identical to the equations of laminated composite plates.

Jha et al. (2012) conducted a basic survey on the reported studies in the range of thermo-elastic and vibration analyses of functionally graded (FG) plates since 1992. This survey deals with the stress, deformation, stability and vibration problems of FG plates. An effort was made to incorporate all the essential contributions in the present area of interest.

Kang and Leissa (2005) developed an exact solution for the buckling analysis of rectangular plates acted by in- plane linearly varying normal stresses where two opposite edges are simply supported and the other two may be simply supported, free or clamped.

Mahadavian (2009) considered buckling of simply supported rectangular functionally graded (FG) plates subjected to non-uniform in-plane compression loads and derived equilibrium and stability equations for these cases. The paper presented the effects of power law index on buckling coefficient.

Reddy (2000) performed the analysis of functionally graded plates based on third-order shear deformation theory. The plate was considered to be isotropic with modulus of elasticity and material being distributed based on power-law distribution along thickness direction. Numerical outcome of the non-linear first-order theory and linear third-order theory were displayed to demonstrate the impact of the material distribution on the stresses and deflections.

Singha et al. (2011) studied the nonlinear behavior of functionally graded material (FGM) plates under transverse distributed load using a high precision plate bending finite element. By using three-dimensional equilibrium equations and constitutive equations, the transverse normal stresses and transverse shear stress components were obtained. The nonlinear representing conditions were attained following a standard finite element methodology and determined through Newton–Raphson cycle system to anticipate the lateral pressure load versus central displacement relationship.

Woodward and Kashtalyan (2012) presented a review on the bending of an isotropic functionally graded plate under localized transverse load through a combination of computational and analytical means.. The plate considered was simply supported, with exponentially varying Young’s and shear moduli along the thickness and the Poisson's ratio was assumed constant. Similar analysis of displacement and stress fields in homogeneous plates and functionally graded plates subjected to uniformly distributed and patch loadings was carried out.

2.1.2 FGM Twisted Plates with Cut-outs

Ghannadpour et al. (2006) performed finite element analysis of rectangular symmetric cross ply laminates to predict the effect of cut-outs on its buckling behavior. The unloaded edges of the plate were modeled in simply supported and clamped boundary condition and the plate was loaded to study the effect of boundary conditions. Several behavioral characteristics and key findings like the effects of plate aspect ratio, cut-out size, boundary conditions, and shape were discussed. Some of the imperative discoveries of this study was that the plates with cut-out can buckle at loads, higher than the buckling loads for corresponding plates without a cut-out.

Lee et al. (1989) presented the characteristics of mode shape and buckling strength of orthotropic plates with central circular holes subjected to both uniaxial and biaxial compression for various boundary conditions. Materials with varying degrees of orthotropy were inspected. With the help of finite element method, the buckling behavior of orthotropic square plate, either with or without a central circular hole, was analyzed with the help of FEM.

Rockey et al. , Azizian et al., Shanmugam et al., Sabir et al. concluded that with an increase in the cut-out dimension, the elastic buckling stress increased up to a certain limit, but due to the reduction in the overall plate cross section, decreased its ultimate load carrying capacity. With the help of finite element method, the buckling of square plates with perforations subjected to uniaxial or biaxial compression was analyzed.

Shanmugam et al. (1999) utilized the finite element method to develop a formula to define the ultimate load carrying capacity of axially compressed square plates with centrally located cut-outs (square and circular). They determined that the ultimate load capacity of square perforated plate was affected considerably by the hole size and its slenderness ratio. They also concluded that plates with circular holes generally have a higher ultimate load carrying capacity compared to plates with square holes.

Shimizu et al. (1991) investigated buckling of isotropic plates with a hole under tensile loads using finite element analysis. Stress distributions and buckling behaviors of such plates were studied. Variations of buckling coefficients and buckling modes with aspect ratios were obtained. The effects of the hole shapes on the buckling strength were also discussed.

2.2 Objective of Present Study

FGMs are being increasingly used nowadays in engineering structures. FGMs allow a gradual variation of material property from one layer to another and hence stress discontinuities are eliminated when compared to laminated composite materials. Composite materials have greater advantages than the materials they are composed of. The plates are subjected to in-plane forces like aerodynamic or hydrodynamic forces. There are many studies on the buckling analysis of FGM plates, but buckling of FGM Twisted plates with or without holes subjected to loading such as in the engine blades of jet engines and certain types of ships, particularly naval ships have not been studied much.

A review of literature shows there is much work done on buckling of flat FGM plates. But there is no work found on the buckling of twisted FGM plates subjected to non-uniform in-plane loads. Hence, the present work is to determine the buckling behavior of twisted FGM plates, with and without cut-outs, subjected to in-plane varying loads.

CHAPTER 3

THEORY AND FORMULATION

3.1 Governing Differential Equation

Figure 3.1 depicts a Twisted Functionally Graded Material plate.

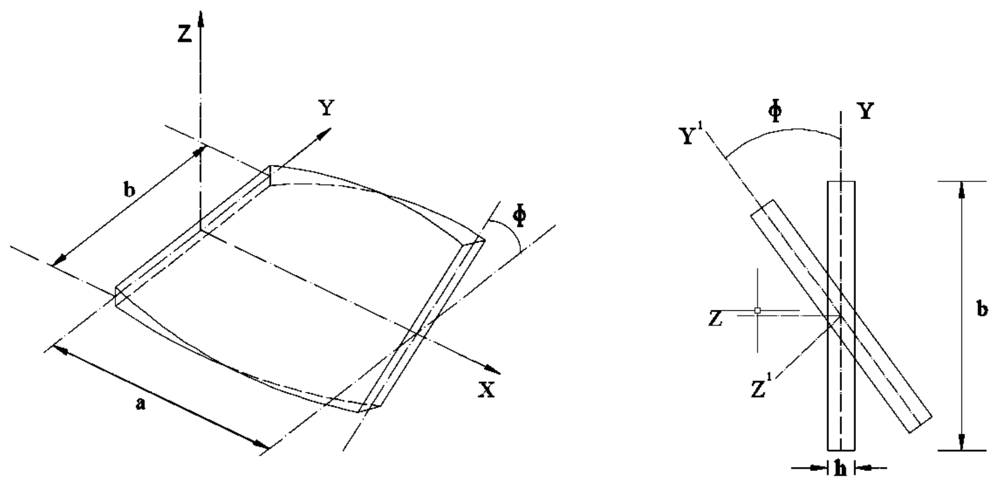


Figure 3.1: *Twisted Functionally Graded Material plate.*

In figure 3.1, ‘a’, ‘b’ and ‘h’ signifies the dimensions of the plate i.e. (length, width and thickness) respectively and ‘Φ’ is the angle of twist.

Figure 3.2 demonstrates a differential element of the twisted panel. Internal axial forces are depicted as N_x ,N_y and N_{xy} and shear forces as Q_x and Q_y and $M_x, \ M_y \ and \ M_{xy}$ as the moment resultants.

For a pretwisted doubly curved shell panel (Chandrasekhara [5], Sahu and Datta [17]) gave the differential equations of equilibrium as:

$$\frac{Q_x}{R_x} + \frac{\partial N_x}{\partial x} + \frac{Q_y}{R_{xy}} + \frac{\partial N_{xy}}{\partial y} - \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial x} = R_1\frac{\partial^2 u}{\partial t^2} + R_2\frac{\partial^2 \theta_x}{\partial t^2}$$

$$\frac{Q_x}{R_y} + \frac{Q_y}{R_{xy}} + \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial x} = R_1 \frac{\partial^2 v}{\partial t^2} + R_2 \frac{\partial^2 \theta_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} - \frac{N_x}{R_x} + {N_x}^0 \frac{\partial^2 w}{\partial x^2} + \frac{\partial Q_y}{\partial y} - \frac{N_y}{R_y} - 2\frac{N_{xy}}{R_{xy}} + {N_y}^0 \frac{\partial^2 w}{\partial y^2} = R_1 \frac{\partial^2 w}{\partial t^2} \qquad (3.1)$$

$$R_3 \frac{\partial^2 \theta_x}{\partial t^2} + P_2 \frac{\partial^2 u}{\partial t^2} = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = R_2 \frac{\partial^2 v}{\partial t^2} + R_3 \frac{\partial^2 \theta_y}{\partial t^2}$$

Where,

External in-plane forces are expressed as '${N_x}^0$' and '${N_y}^0$' along X and Y axes.

'R_x ' and 'R_y' are curvature radii in the X and Y axis.

R_{xy} as the radius of twist.

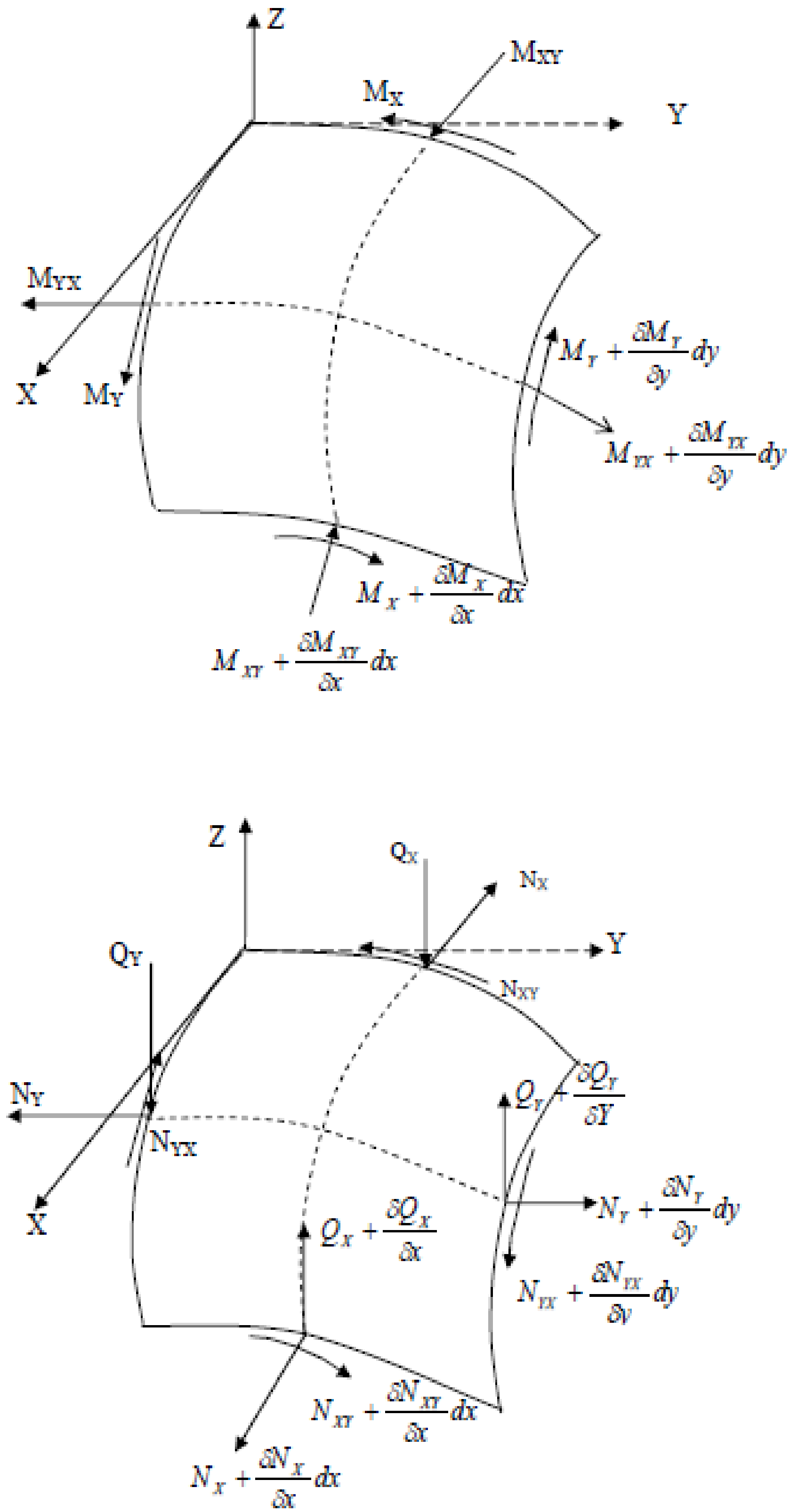


Figure 3.2: An Element of a shell panel.

$$(R_1, R_2, R_3) = \sum_{j=1}^{n} \int_{z_{j-1}}^{z_j} (\rho)_j (1, z, z^2)\, \partial z \tag{3.2}$$

Where 'n' is the number of layers considered in a plate and 'ρ_j' is the density at j^{th} layer.

3.2 Constitutive Relations

The Functionally Graded Material plate considered in this study consists of ceramic on one side and metal on the other. The variation in the material and its properties is shown by a parameter 'n' known as the material property index which varies along the thickness. The plate is considered to be fully ceramic for n = 0, and as fully metal for n = α. According to (Reddy [14]), material property index 'n' and the position of the layer in plate signifies its material properties which vary according to the power law.

$$P_z = (P_t - P_b)v_f + P_b \tag{3.3}$$

$$V_f = \left(\frac{2Z + h}{2h}\right)^{\mathrm{n}} \tag{3.4}$$

Where 'P' denotes the relevant material property, 'P_t' and 'P_b' alludes to the material property at the top and bottom layers of the plate and 'Z' indicates the distance from the mid surface of the plate to the point under consideration. Volume fraction index and material property index are interpreted as'V_f' and 'n' correspondingly. Here, in this study, Poisson's ratio 'ϑ' and material density 'ρ' is considered constant while Young's modulus 'E' is considered to vary according to power law.

In the present study, it is assumed that the material property varies along its thickness. The ANSYS model is separated into different layers with a specific end goal to model the gradual change in material properties of the FGM. Every layer is assumed to be isotropic. Material properties are ascertained at the mid-point of each of these layers from the mid surface utilizing power law, (Reddy [14]). Despite the fact that the layered structure does not mirror the gradual change in material property, by utilizing an adequate number of layers the variation can be approximated.

The linear fundamental relations are

$$\begin{bmatrix} S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & 0 & 0 & 0 \\ 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}$$

Where,

$$\frac{E}{(1-\nu^2)} = S_{11} \tag{3.5}$$

$$\frac{\nu E}{(1-\nu^2)} = S_{12} \tag{3.6}$$

$$\frac{E}{2(1+\nu)} = S_{44} = S_{55} = S_{66} \tag{3.7}$$

The fundamental relations related to FGM plate are given by:

$\{F\} = [D]\{\epsilon\}$

Where

$\{N_x\ N_y\ N_{xy}\ M_x\ M_y\ \ M_{xy}\ Q_x\ Q_y\} = \{F\}$

$\{\epsilon_x\ \epsilon_y\ \gamma_{xy}\ K_x\ K_y\ K_{xy}\ \varphi_x\ \varphi_y\}^T = \{\epsilon\}$

$$D = \begin{bmatrix} G_{11} & G_{12} & G_{16} & H_{11} & H_{12} & H_{16} & 0 & 0 \\ G_{21} & G_{22} & G_{26} & H_{21} & H_{22} & H_{26} & 0 & 0 \\ G_{16} & G_{22} & G_{66} & H_{16} & H_{22} & H_{66} & 0 & 0 \\ H_{11} & H_{12} & H_{16} & I_{11} & I_{12} & I_{16} & 0 & 0 \\ H_{21} & H_{22} & H_{26} & I_{21} & I_{22} & I_{26} & 0 & 0 \\ H_{16} & H_{22} & H_{66} & I_{16} & I_{22} & I_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & T_{44} & T_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & T_{45} & T_{55} \end{bmatrix} \tag{3.8}$$

Stiffness coefficients are defined as:

$\sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [Q_{ij}]_k \,(1, z\,, z^2)dz = (G_{ij}, H_{ij}, I_{ij}\,)$, For (i, j = 1, 2, 6)

$$\kappa \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [Q_{ij}]_k \, dz = T_{ij}$$

The forces and moment resultants are obtained as follows

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_Y \\ M_{xY} \\ Q_x \\ Q_Y \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_x \\ \tau_{xy} \\ \sigma_x z \\ \sigma_{yz} \\ \tau_{xy} z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{3.9}$$

Where σ_x, σ_y are the normal stresses along X and Y direction and τ_{xy},τ_{xz} and τ_{yz} are shear stresses in xy, xz and yz planes respectively.

3.3 Finite Element Formulation

ANSYS 15.0, a finite element software is used for the present study. The element used is SHELL 281. This element has eight nodes. Each node has six degrees of freedom: they are three translations in x-, y- and z- directions and three rotations about x-, y- and z- axes. The first-order shear deformation theory is the basis of the displacement formulation in ANSYS. The plate is presumed to be made up of a number of layers. Each layer is considered to be homogeneous and isotropic. The values of Young's modulus of elasticity is varied for each layer and is calculated by the power law (Reddy [14]) using a MATLAB program.

3.4 Strain Displacement Relations

Green-Lagrange's strain displacement relations are given below. The linear strain is used to derive the elastic stiffness matrix and the nonlinear strain component is used to derive the geometric stiffness matrix which is used in buckling formulation. The linear strain displacement relations for a shell element are:

$$zk_x+\frac{w}{R_x}+\frac{\partial u}{\partial x}=\xi_{xl},\ \frac{\partial v}{\partial y}+\frac{w}{R_y}+zk_y=\xi_{yl}$$

$$zk_{xy}+\frac{2w}{R_{xy}}+\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}=\gamma_{xyl} \tag{3.10}$$

$$\frac{\partial w}{\partial x}+\theta_x-\frac{u}{R_x}-\frac{v}{R_{xy}}=\gamma_{xzl}$$

$$\frac{\partial w}{\partial y}+\theta_y-\frac{u}{R_y}-\frac{u}{R_{xy}}=\gamma_{yzl}$$

Where the bending strains are

$$\frac{\partial \theta_x}{\partial x}=k_x,\ \frac{\partial \theta_y}{\partial y}=k_y$$

$$\frac{\partial \theta_x}{\partial y}+\frac{\partial \theta_y}{\partial x}+\frac{1}{2}\left(\frac{1}{R_y}-\frac{1}{R_x}\right)\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)=k_{xy} \tag{3.11}$$

The linear strains are related to the displacements by the following relation

$$\{\varepsilon\} = [B]\{d_e\} \tag{3.12}$$

Where,

$$\{d_e\} = \{u_1 v_1 w_1 \theta_{x1} \theta_{y1} \dots\dots\dots\dots\dots\dots. u_8 v_8 w_8 \theta_{x8} \theta_{y8}\} \tag{3.13}$$

$$[B] = [[B_1], [B_2]. . \dots\dots\dots\dots\dots\dots. [B_8]] \tag{3.14}$$

The strain displacement matrix [B] is

$$[B_i]=\begin{bmatrix} N_{i,x} & 0 & \frac{N_i}{R_x} & 0 & 0 \\ 0 & N_{i,y} & \frac{N_i}{R_y} & 0 & 0 \\ N_{i,y} & N_{i,x} & 2\frac{N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} \\ 0 & 0 & N_{i,x} & N_i & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i \end{bmatrix} \tag{3.15}$$

3.4 Buckling Analysis

For buckling analysis, the governing equation is expressed as the following eigenvalue problem

$$([K] + \lambda[S])\{U\} = \{0\} \tag{3.16}$$

where matrix [K] denotes the stiffness matrix and matrix [S], the geometric stiffness matrix due to the in-plane stresses.

3.5 ANSYS Methodology

ANSYS, Inc. is American Computer-aided engineering software, performs engineering analysis across a range of disciplines including structural analysis, computational fluid dynamics, finite element analysis, implicit and explicit methods, and heat transfer.

In present work, modelling is done using ANSYS 15.0, to perform buckling analysis and to obtain the mode shapes. The details regarding the modelling are presented in the following sub-sections. First, terms relevant to the topic are explained. Then, the modelling procedure is presented.

TERMINOLOGIES

Shell 281: **Shell 281** is an element, having eight nodes with six degrees of freedom at each node with translations along the x, y, and z axes, and rotations about the x, y, and z-axes and is used as an element type for thin to moderately thick shell structures. SHELL281 might be utilized for layered applications for modelling sandwich construction or composite shells. The precision in displaying composite shells is governed by the first-order shear-deformation theory. The figure shown below illustrates the co-ordinate system, nodes and geometry of a shell element[1].

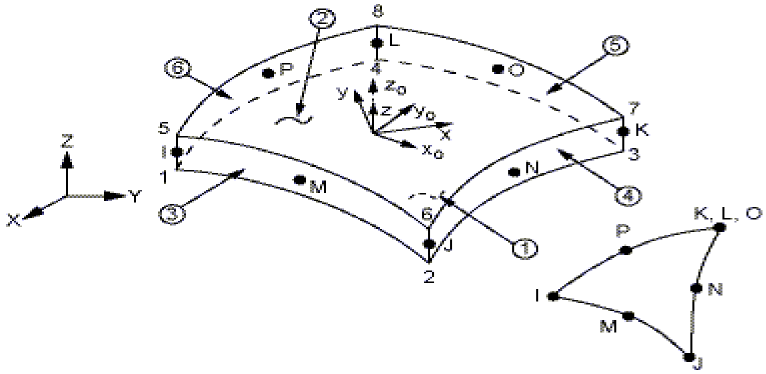


Figure 3.3: *A SHELL 281 Element.* ***[1]***

PROCEDURE

The following flow chart depicts an outline of the steps to be followed in ANSYS.

PRE-PROCESSOR

- Element type, material and geometric properties are defined.
- Defining of key points/lines/areas/volumes.
- Modelling the required structural element to the requirement.
- Meshing the structural element, containing lines/areas/volumes.

PROCESSOR

- In this stage, loads, boundary conditions and the method adopted are assigned to the problem and is solved to obtain the required result.

POST-PROCESSOR

- This stage helps to see and interpret the results. The frequencies, buckling loads, mode shapes, nodal displacements, element forces and moments may be obtained as per the problem specifications.

Initially, a laminated composite plate was solved under uniform compressive load in Ansys 15.0 in order to validate the methodology and to compare with previous results for buckling. Once the results matched closely with the referred ones, buckling analysis of FGM plate using Ansys 15.0 is performed using different types of in-plane loads.

CHAPTER 4

RESULTS AND DISCUSSIONS

CHAPTER 4

Results and Discussions

4.1 Introduction

The FGM plate considered here comprises of ceramic on top and metal at the base. In FGM plates, by varying the material gradient index (n), the material properties can be varied continuously along the thickness. Material properties generally considered are Young's modulus (E), Poisson's (υ) ratio and density (ρ). Material properties rely on gradient index (n). By utilizing the power law (Reddy[2000]) we can calculate the material properties.

$$P_z = (P_t - P_b)v_f + P_b \tag{4.1}$$

$$V_f = \left(\frac{2Z + h}{2h}\right)^{n} \tag{4.2}$$

Where 'P' denotes the relevant material property, 'P_t' and 'P_b' alludes to the material property at the top and bottom layers of the plate and 'Z' indicates the distance from the mid surface of the plate to the point under consideration. Volume fraction index and material property index are interpreted as'V_f' and 'n' correspondingly. Here, in this study, Poisson's ratio 'ϑ' and material density 'ρ' is considered constant while Young's modulus 'E' is considered to vary according to power law.

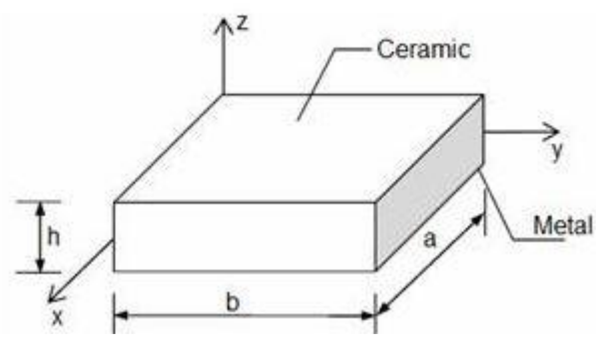


Figure 4.1: *Typical FGM square plate* ***[4]***

By utilizing the MATLAB software, the Young's modulus of every layer was calculated. Poisson's ratio and density were kept as constants. At this point, a model of FGM plate was created by utilizing ANSYS. For this, results were contrasted with the results displayed by Reddy et al. [14] to pick a mesh size and number of layers to model the FGM plate precisely. Then the twisted FGM plate is modeled and then results shall be studied for plate with varying in plane loads.

Figure 4.2 depicts the variation of volume fraction (V_f) along the plate thickness.

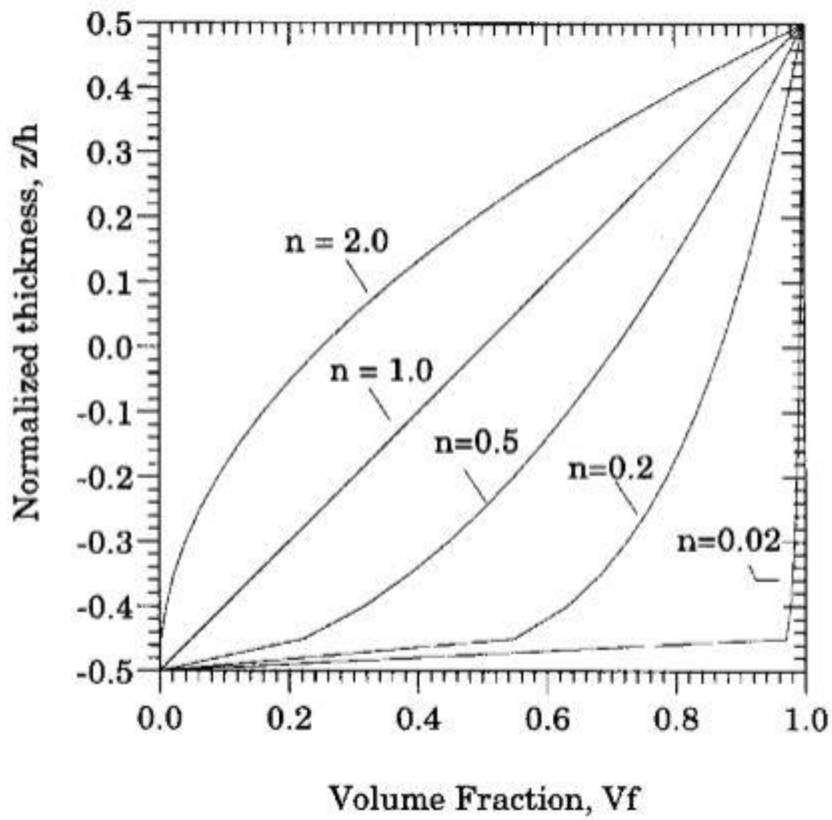


Figure 4.2: *Variation of Volume fraction (V_f) through plate thickness* ***[14]***

When n = 0, the plate is completely ceramic and when n = infinity, the plate is completely metal.

In this section, the results regarding to the buckling analysis of cantilever twisted functionally graded material plates subjected to in-plane loads are presented. By using ANSYS-finite element software, the evaluation is carried out making use of SHELL281 element. The SHELL281 element has eight nodes and each and every node has six degrees of freedom. The functionally graded material plate section is modelled in the form of laminated composite section consisting of number of layers by considering each layer as isotropic and approximating the uniform variation of the material property along the thickness. The material properties of each layer is determined using power law. Convergence studies are carried out to fix up the number of layers and mesh size as well, and results are compared with the prior studies.

4.2 Convergence Study

For this study, a square FGM plate with aspect ratio as unity(a/b=1), b/h=100, (side to thickness ratio) comprising of aluminum oxide in aluminum (Al/Al2O3) matrix (where a, b, and h are the width, length, and thickness) was used. The properties of constituents are E_c =380GPa, $\vartheta = 0.3$ for aluminum oxide and E_m= 70GPa, $\vartheta = 0.3$ for Aluminum.

Table 4.1 shows the convergence study of simply supported flat FGM plate, whose material gradient index n = 0 for determining, the mesh division to be considered in the study. The results demonstrate better convergence for 12×12 mesh division and the same is utilized for the further study. The non-dimensional buckling load is expressed as

$$\bar{N} = N_{cr} \frac{a^2}{E_m h^3}$$

Table 4.1: *Convergence of Non- dimensional buckling load of flat simply supported FGM plate with varying mesh size (n= 0, a/b =1, b/h=100)*

Mesh Size	**Buckling Load N_{cr} (kN)**	**Non-Dimensional Buckling Load ($\bar{N}$)**
4 x 4	696.59	19.90
8 x 8	686.60	19.62
10 x 10	686.53	19.61
12 x 12	686.52	19.61
Reddy et al. [14]		19.57

Table 4.2 shows, convergence study of simply supported flat FGM plate, with material gradient index n =1 for determining, the number of layers to be considered in the study. The results illustrate better convergence for 12 number of layers and the same is used in the further study.

Table 4.2: Convergence of non-dimensional buckling load of flat simply supported FGM plate with varying number of layers (n=1, a/b=1, b/h=100).

No. of Layers	**Buckling Load N_{cr} (kN)**	**Non-Dimensional Buckling Load ($\bar{N}$)**
4	352.87	10.06
8	347.74	9.93
10	346.76	9.91
12	346.41	9.89
Saha et al. [16]		9.77

4.3 Convergence Study on Cantilever Twisted FGM Plate

Table 4.3 shows the convergence study of cantilever twisted FGM plate, whose material gradient index n = 0 for determining the mesh division to be considered in the study. The results demonstrate better convergence for 12×12 mesh division and the same is utilized for the further study. The non-dimensional buckling load is expressed as

$$\bar{N} = N_{cr} \frac{a^2}{E_m h^3}$$

Table 4.3: *Convergence results of non-dimensional buckling load of cantilever twisted FGM plate with varying mesh size (n = 0, a/b =1, b/h =100, Φ = 15°).*

Mesh Size	**Buckling Load N_{cr} (kN)**		**Non-Dimensional Buckling Load ($\bar{N}$)**	
	1st Buckling Mode	**2nd Buckling Mode**	**1st Buckling Mode**	**2nd Buckling Mode**
4 x 4	40.350	357.48	1.1529	10.2137
8 x 8	40.246	355.90	1.1499	10.1686
12 x 12	40.234	355.74	1.1495	10.1640
16 x 16	40.232	355.69	1.1495	10.1626

Table 4.4 shows the convergence study of cantilever twisted FGM plate, whose material gradient index n =1 for determining the number of layers to be considered in the study. The results

demonstrate better convergence for 12 number of layers and the same is utilized for the further study.

Table 4.4: *Convergence results of non- dimensional buckling load of cantilever twisted FGM plate with varying number of layers (n =1, a/b =1, b/h =100, Φ =15°.*

Number of Layers	**Buckling Load N_{cr} (kN)**		**Non-Dimensional Buckling Load ($\bar{N}$)**	
	1st Buckling Mode	**2nd Buckling Mode**	**1st Buckling Mode**	**2nd Buckling Mode**
4	20.998	184.79	0.5999	5.2797
8	20.652	181.75	0.5901	5.1928
12	20.568	181.17	0.5882	5.1763
16	20.563	180.96	0.5875	5.1703

4.4 Comparison with Previous Studies

4.4.1 Buckling Analysis of Laminated Composite Plate with In-Plane Loading

Buckling of laminated composite plates with simply supported condition subjected to in-plane loads was first executed for validating the ANSYS formulation for the case of in-plane loading. The results from previous studies conducted by Zhong and Gu [22] were compared with the present ones. The loading is given by the expression for compressive force

$$N_x = N_0(1 - \eta \frac{y}{b})$$

Here η characterizes the linear variation of the in-plane load. The present discussion is limited to the case for $0 \leq \eta \leq 2$. η = 0 and η = 2 corresponds to the case of uniform compression and pure in-plane bending respectively. The non-dimensional critical load (K) is given by:

$$K = \frac{N_o b^2}{E_T h^3}$$

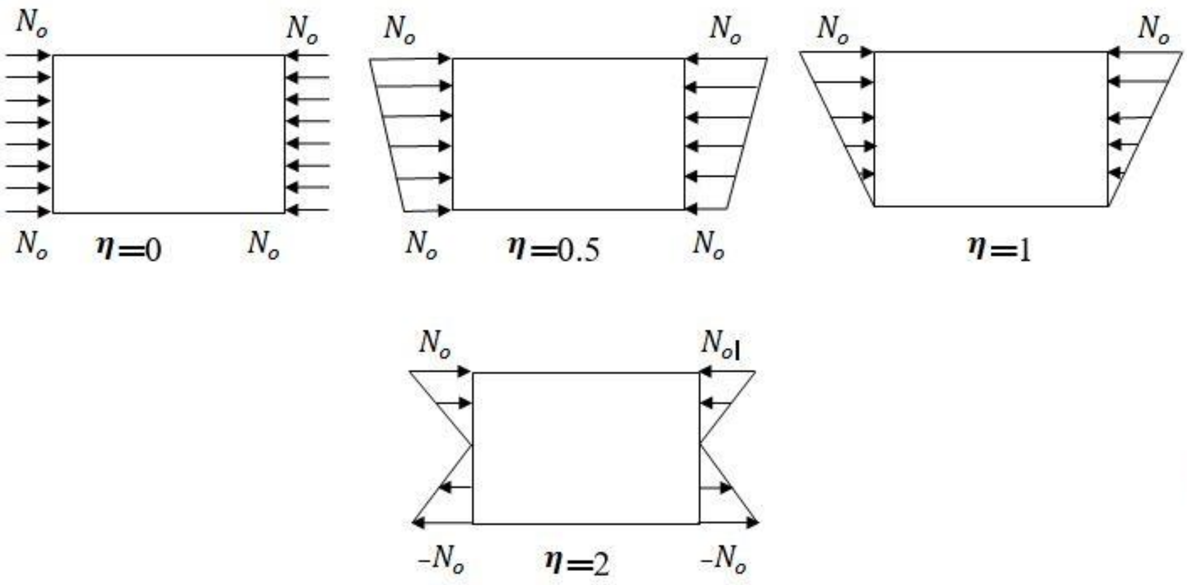


Figure 4.3: Different load parameters

The comparative study is done through making use of symmetric cross-ply laminated square plate $[0^0/90^0/0^0]$ subjected to various linearly varying loads as shown in figure 4.3.

Given properties of the laminated composite plate are [22]

$$\frac{E_L}{E_T} = 40, \frac{G_{LT}}{E_T} = \frac{G_{LZ}}{E_T} = 0.6, \frac{G_{ZT}}{E_T} = 0.5, \ \frac{h}{b} = 0.1, \ \vartheta = 0.25$$

Table 4.5: *Comparative study of variation of non-dimensional buckling load for symmetric cross-ply square plate with varying in plane loads (η=0, 0.5, 1).*

Linear Variation of In-Plane Load (η)	**Zhong and Gu[22]**	**Present Study**
$\eta = 0$(uniform compression)	22.317	21.759
$\eta = 0.5$	29.432	28.781
$\eta = 1$	40.999	40.380

The above table indicates that the results are quite similar with that of Zhong and Gu [22].

4.4.2 Convergence Study on Rectangular Plate, Cross-ply Laminate $[0/90]_{2s}$ with Eccentric Circular Cut-outs.

The impact of size of eccentric circular cut-out is considered in this section. Rectangular plates with cross- ply laminate $[0/90]_{2s}$ is considered for this study. The thickness of each and every layer of this eight layer laminate is 0.15mm. The material properties of the lamina is mentioned in the Table 4.6. Rectangular plates with width (a =120mm), side to thickness ratio (b/h =100), aspect ratio (a/b = 1) was taken.

The comparison of results for present formulation was done with simply supported cross-ply laminate $[0/90]_{2s}$ for different diameter/length (d/b) ratio of circular cut-out subjected to uniform compression as shown in Table 4.7.

Table 4.6: Material properties of the lamina [7].

Mechanical Properties	Values
E_1	130.0 GPa
E_2	10.0 GPa
E_3	10.0 GPa
$G_{12} = G_{13}$	5.0 GPa
$\vartheta_{12} = \vartheta_{13}$	0.35
$\vartheta_{23} = \vartheta_{32}$	0.49

Table 4.7: *Comparative study of variation of non- dimensional buckling load of simply supported square cross-ply laminate $[0/90]_{2s}$ with varying circular cutout diameter/length (d/b).*

Diameter/Length (d/b)	Non-Dimensional Buckling Load($\bar{N}$) Ghannadpour et al. [7]	Non-Dimensional Buckling Load ($\bar{N}$)
0.0	13.79	13.79
0.025	13.71	13.78
0.05	13.51	13.34
0.1	12.82	12.62
0.8	4.43	5.43

From the observations given in Table 4.7, it is concluded that the same procedure can be followed for FGM plates with circular cut-outs.

4.5 Results and Discussions

Initially, buckling analysis is carried out for a twisted cantilever FG material plate with circular cut-outs of varying diameter to determine the effects caused by it. The material plate considered in the study is (*Al/Al$_2$O$_3$*) Aluminum/Aluminum oxide with material properties as *Al* - (E_m=70GPa, υ = 0.3), *Al/Al$_2$O$_3$* - (E_c =380GPa, υ = 0.3).

Non-dimensional buckling load is given by

$$\bar{N} = N_{cr} \frac{a^2}{E_m h^3}$$

Table 4.8: *Variation of Non-dimensional buckling load of Twisted FGM plate with circular cut-out, with varying cut-out (circular) diameter (b/h=100, Φ =10°, n=1).*

Diameter to Side ratio (d/b)	Buckling Load N_{cr} (kN)		Non-Dimensional Buckling Load ($\bar{N}$)	
	1st Buckling Mode	2nd Buckling Mode	1st Buckling Mode	2nd Buckling Mode
0	20.445	182.41	0.584	5.212
0.2	19.161	172.26	0.547	4.922
0.5	13.535	123.53	0.387	3.529
0.8	6.255	56.794	0.179	1.623

It is seen from Table 4.8 that as the diameter of the hole increases, the buckling load decreases as the stiffness of the plate decreases.

The following tables shows the variation of non-dimensional buckling load of twisted FGM plates with and without cut-outs subjected to different parameters like linearly varying load , aspect ratio(a/b), b/h ratio, material gradient index and varying twist angle.

In Table 4.9, different varying in-plane loads are applied to the twisted plate. It is seen that for η = 2, the non-dimensional buckling load is the highest for both cases.

Table 4.9: *Variation of non-dimensional buckling load of twisted FGM plate, with & without cut-out (circular) for varying loading conditions (d = 0.25m, a/b =1, b/h=100, n=1, Φ =10°).*

Linear Variation of the In-Plane Load (η)	Non-dimensional Buckling Load ($\bar{N}$)			
	Without Cut-out		With Cut-out	
	1st Buckling Mode	2nd Buckling Mode	1st Buckling Mode	2nd Buckling Mode
0	0.584	5.212	0.387	3.529
0.5	0.778	6.810	0.514	4.653
1	1.153	9.127	0.763	6.479
2	8.402	40.131	5.725	22.882

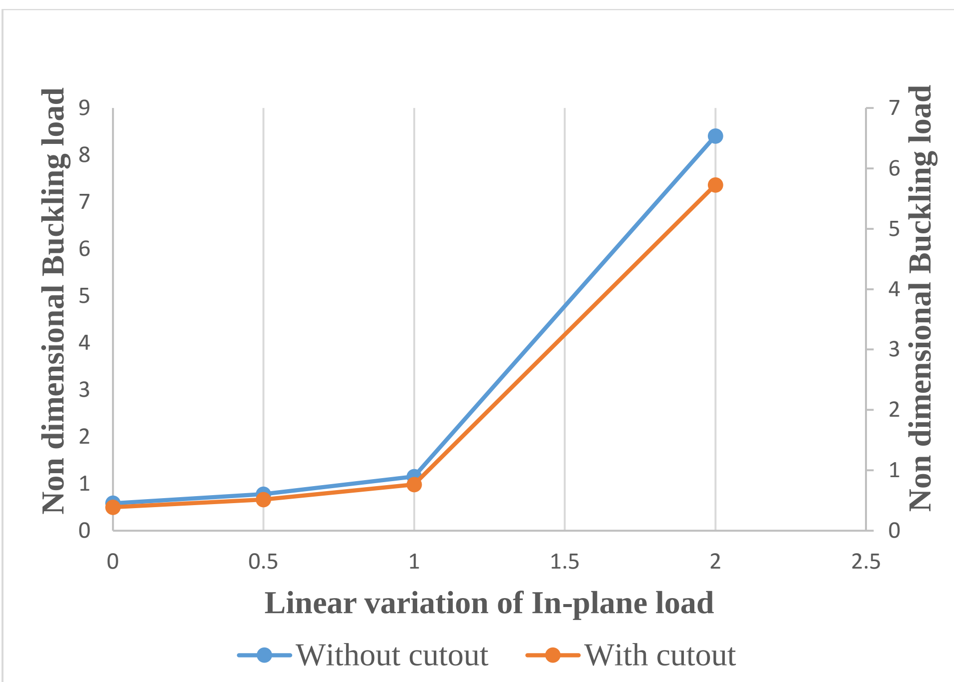
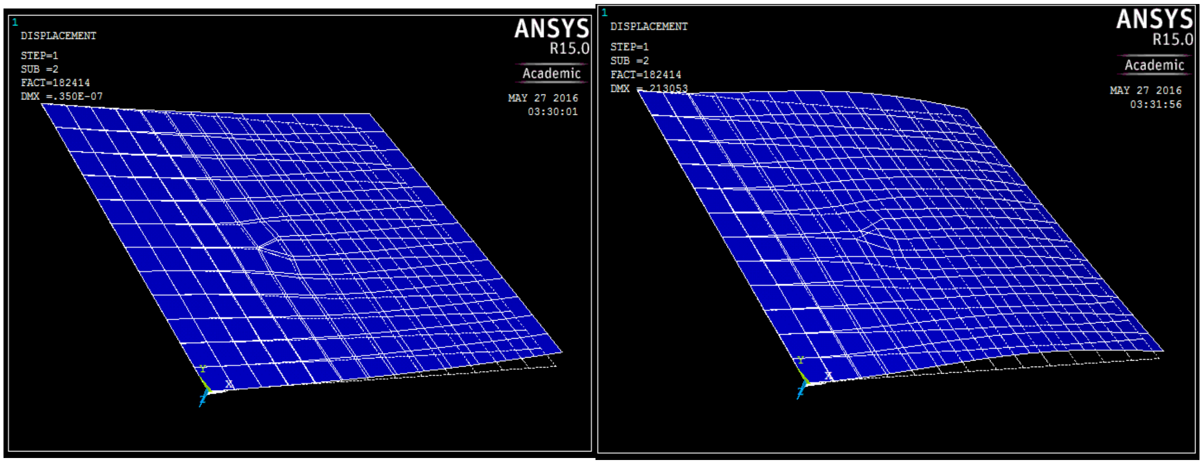


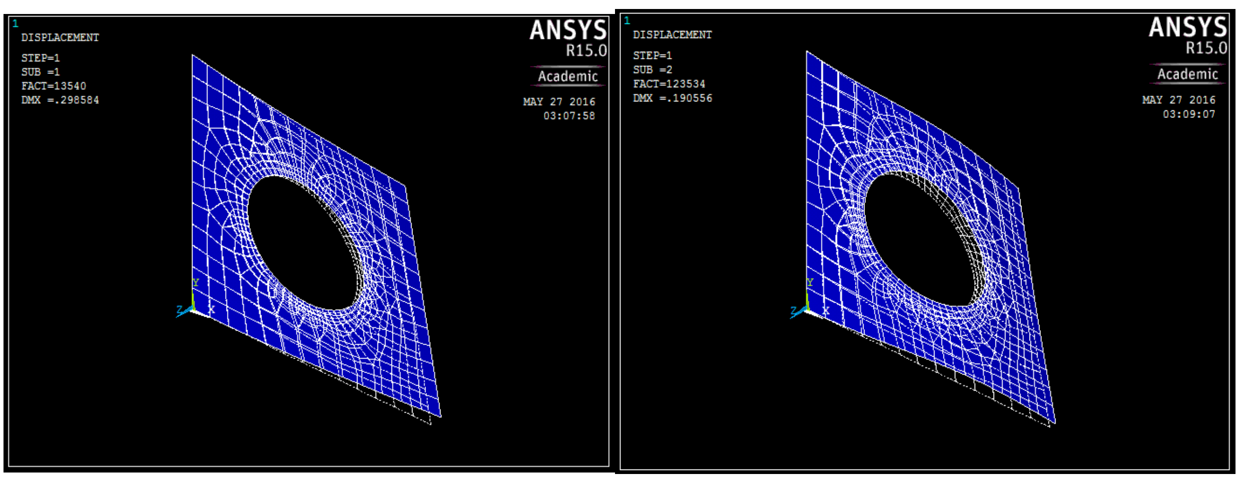
Figure 4.4: *Variation of non-dimensional buckling load with varying In-plane loads.*

Figure 4.5 and Figure 4.6 respectively, shows the 1st and 2nd buckling modes of twisted cantilever FGM plates with and without cut-outs.



1st mode 2nd mode

Figure 4.5: *Buckling modes of cantilever twisted FGM plate with an angle of twist as 10° subjected to uniform compression.*



1st mode 2nd mode

Figure 4.6: *Buckling modes of cantilever twisted FGM plate with circular cut-out for an angle of twist of 10° subjected to uniform compression.*

Table 4.10 presents the variation of aspect ratio on critical buckling load and on non-dimensional buckling load for 10° twisted plate with and without cut-outs subjected to uniform compression($\eta = 0$). Figure 4.7 exhibits this variation graphically.

Table 4.10: *Variation of non-dimensional buckling load of twisted FGM plate, with & without cut-out (circular) for varying aspect ratio (a/b) (d = 0.1m, b/h=100,n=1, Φ =10°).*

Aspect Ratio (a/b)	Buckling Load N_{cr} (kN)		Non-Dimensional Buckling Load ($\bar{N}$)	
	Without Cut-out	With Cut-out	Without Cut-out	With Cut-out
0.5	82.511	72.389	1.179	1.034
1	20.445	19.161	0.584	0.548
2	5.005	4.854	0.286	0.277
3	2.200	2.156	0.189	0.185

From the results, it can be deduced that the non-dimensional buckling load decreases considerably with increasing (a/b) i.e. aspect ratio for the twisted FGM plate considered. This behavior is due to the fact that with the increase in aspect ratio, the length of the plate increases along the direction of in plane compressive load resulting in lower stiffness of plate and consequently, decreasing the critical buckling capacity of the plate.

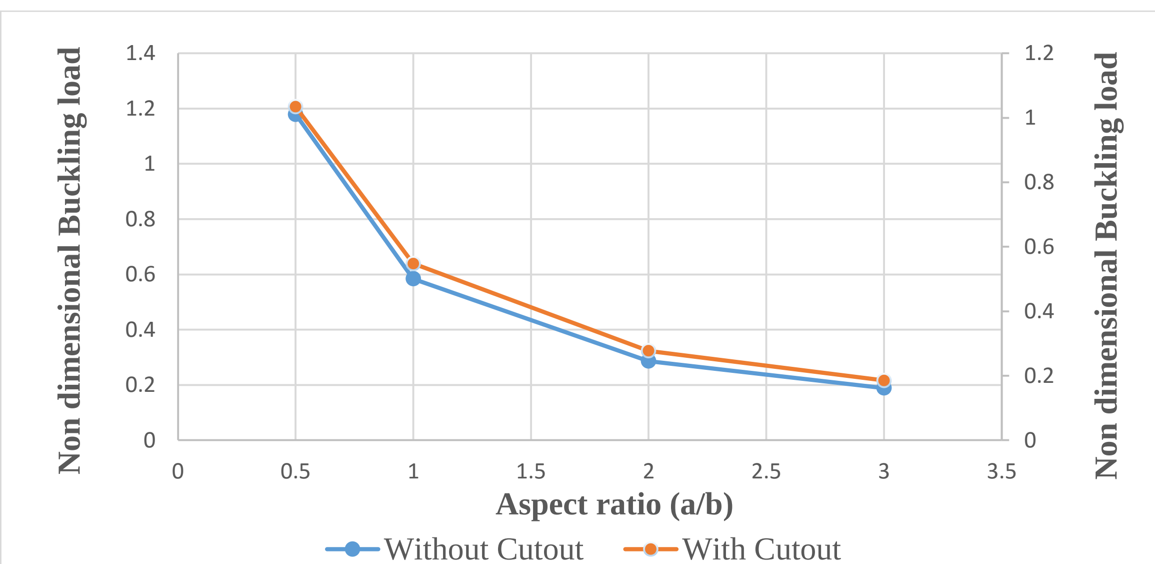


Figure 4.7: Variation of non-dimensional buckling load with varying aspect ratio (a/b).

Table 4.11 presents the variation of thickness on critical buckling load and on non-dimensional buckling load for 10° twisted plate with and without cut-outs subjected to uniform compression (η = 0). Figure 4.8 exhibits this variation graphically.

Table 4.11: *Variation of Non-dimensional buckling load of Twisted FGM plate, with & without cut-out (circular) for varying side to thickness condition (d = 0.25m, a/b =1,n=1, Φ =10°).*

Side to Thickness Ratio (b/h)	**Buckling Load N_{cr} (kN)**		**Non-Dimensional Buckling Load ($\bar{N}$)**	
	Without Cut-out	**With Cut-out**	**Without Cut-out**	**With Cut-out**
10	20142	13088	0.5755	0.3739
20	2537	1663.7	0.5799	0.3803
30	757.741	498.482	0.5810	0.3822
40	318.110	209.601	0.5817	0.3833
50	163.041	107.451	0.5820	0.3838
60	94.421	62.329	0.5826	0.3846
70	59.490	39.300	0.5830	0.3851
80	39.880	26.364	0.5834	0.3857
90	28.095	18.586	0.5838	0.3862
100	20.445	13.535	0.5841	0.3867

Table 4.11 concludes that, for a 10° twisted cantilever plate studied, with increased side to thickness ratio, the critical buckling load shows a decreasing trend while the non-dimensional buckling load shows an increasing trend and the graphical representation of this variation is shown in Figure 4.8. Increase in side to thickness ratio (b/h), increases non-dimensional buckling load for a twisted FGM plate with and without cut-out's. Here, (b/h) i.e. side to thickness ratio for the plate is increased by decreasing the thickness of the plate, that decreases the stiffness of plate which in turn, decreases the critical buckling load of the twisted plate, but, when non-dimensional buckling load's expression is concerned the term 'h' resides in the denominator which increases its value with decreasing thickness of the plate.

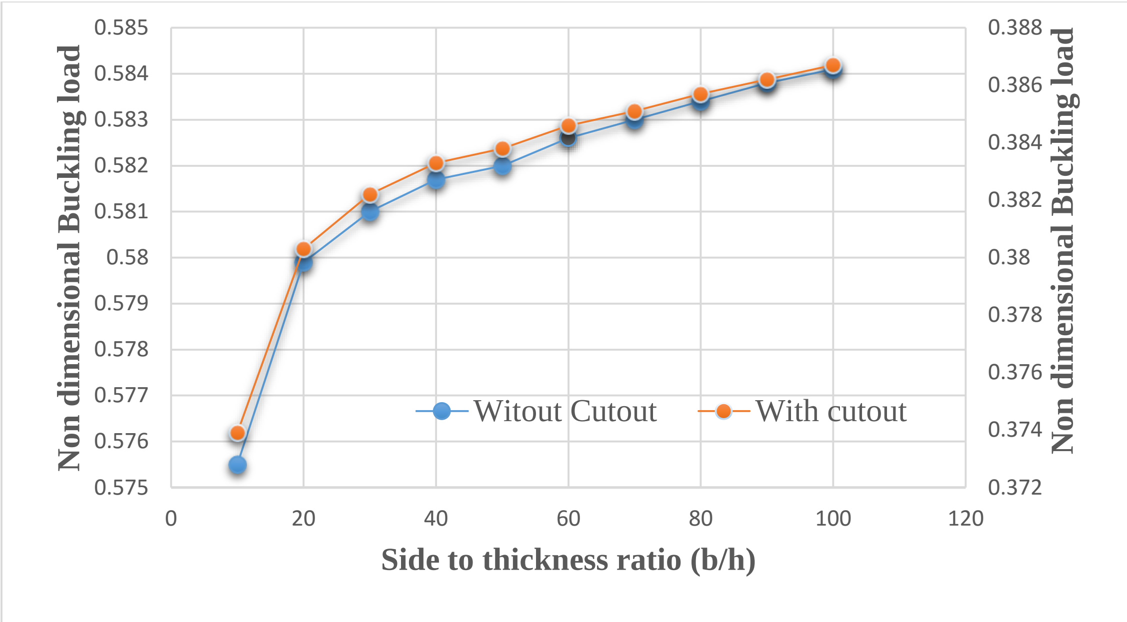


Figure 4.8: *Variation of non-dimensional buckling load with varying side to thickness ratio (b/h).*

In the next study, the angle of twist is varied for uniform compression (η = 0) applied on the twisted plate. The results are shown in Table 4.12.

Table 4.12: *Variation of Non-dimensional buckling load of twisted FGM plate, with & without cut-out (circular) for varying angle of twist (d = 0.25m, a/b =1, b/h=100,n=1).*

Angle of Twist	**Buckling Load N_{cr} (kN)**		**Non-Dimensional Buckling Load ($\bar{N}$)**	
	Without Cut-out	**With Cut-out**	**Without Cut-out**	**With Cut-out**
0°	20645	13659	0.5899	0.3903
10°	20435	13535	0.5839	0.3867
15°	20122	13353	0.5749	0.3815
20°	19647	13065	0.5613	0.3733
30°	18249	12179	0.5214	0.3480

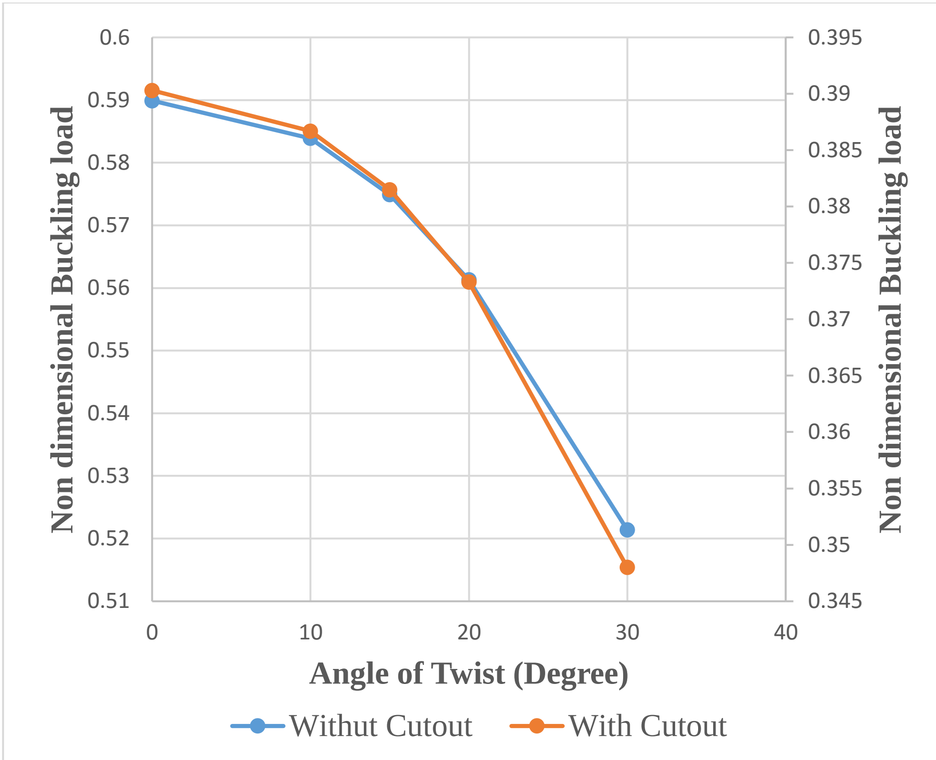


Figure 4.9: *Variation of non-dimensional buckling load with varying angle of Twist.*

It is seen from Table 4.12 and figure 4.9 that as the angle of twist increases, the buckling load decreases showing a decreased stiffness with increasing angle of twist.

Table 4.13: *Variation of Non-dimensional buckling load of Twisted FGM plate, with & without cut-out (circular) for varying material gradient index (d = 0.25m, a/b =1, b/h=100, Φ =10°).*

Material gradient index (n)	**Buckling load N_{cr} (kN)**		**Non-dimensional buckling load ($\bar{N}$)**	
	Without cut-out	**With cut-out**	**Without cut-out**	**With cut-out**
0	40.892	27.054	1.168	0.773
1	20.445	13.535	0.584	0.387
2	15.952	10.559	0.456	0.301
5	13.420	8.876	0.383	0.254
10	12.306	8.136	0.352	0.232
100	10.495	6.939	0.300	0.198

The variation of non-dimensional buckling load for Al/Al_2O_3 FGM with increasing material index for a 10° twisted FG plate with and without cut-outs is considered next. The results are displayed in Table 4.13. From the results, it can be concluded that with increase in gradient index, the non-dimensional buckling load decreases. This is considering that, when material property index is zero, the plate is speculated as completely ceramic which could be very stiff, consequently, increasing the value of critical buckling load. As the gradient index increases, the critical buckling load continues to diminish due to the rise in metal material content in the plate bringing about reduced stiffness. At the point when the material property index reaches infinity, the plate is speculated as totally metallic which is less stiff when compared to ceramic thereby, reducing the critical buckling load. The graphical representation of this variation is shown in Figure 4.10.

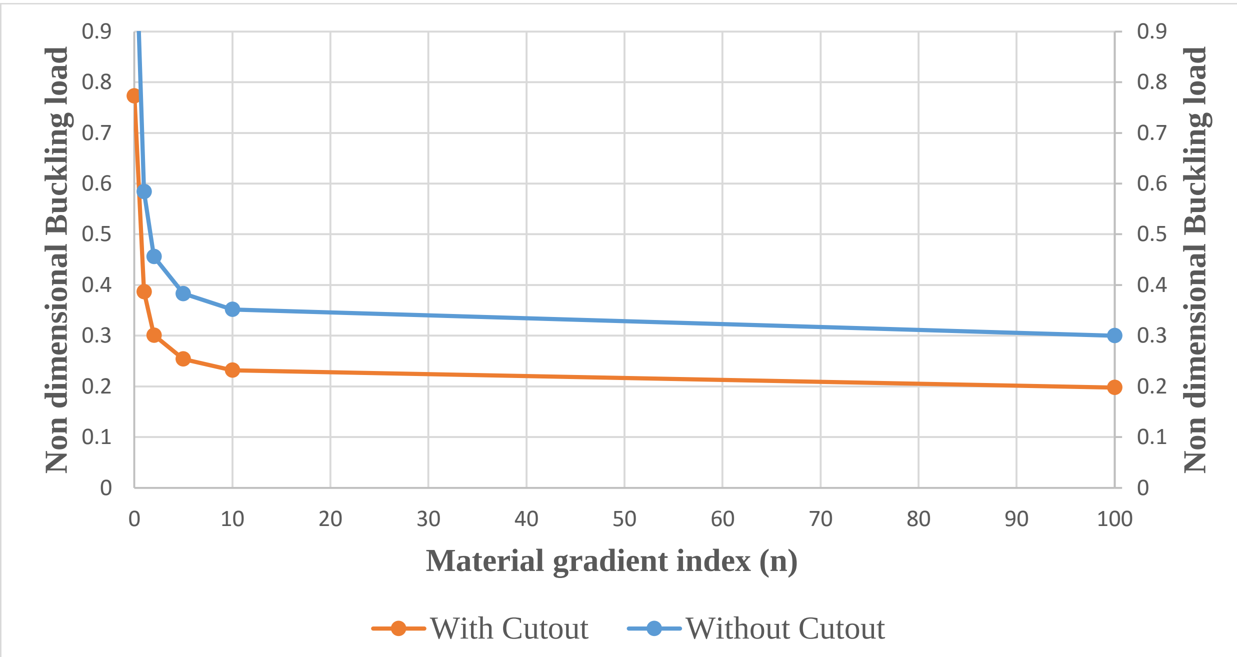


Figure 4.10: *Variation of non-dimensional buckling load with varying Material gradient index (n).*

CHAPTER 5

CONCLUSIONS

5.1 Conclusions

Important conclusions following the present work are discussed:

1. With increase in diameter to side ratio of cut-outs, non-dimensional buckling load of twisted FG plate decreases due to the fact that an increase in the diameter of the cut-out reduces the effective plate area, which in turn reduces the stiffening capacity of plate thereby decreasing its critical buckling load.

2. The non-dimensional buckling load decreases considerably with increasing (a/b) i.e. aspect ratio for the twisted FGM plate considered. This behaviour is due to the fact that with the increase in aspect ratio, the length of the plate increases along the direction of in plane compressive loads resulting in lower stiffness of plate and consequently decreasing the critical buckling capacity of the plate.

3. Increase in side to thickness ratio (b/h) increases non-dimensional buckling load for a twisted FGM plate with and without cut-outs. Here (b/h) i.e. side to thickness ratio for the plate is increased by decreasing the thickness of the plate, that decreases the stiffness of plate which in turn, decreases the critical buckling load of the twisted plate, but when considering the non-dimensional buckling load, the term ‘h’ resides in the denominator which increases its value with decreasing thickness of the plate.

4. With increase in gradient index, the non-dimensional buckling load decreases. This is considering that, when material property index is zero, the plate is speculated as completely ceramic which could be very stiff, consequently, increasing the value of critical buckling load. As the gradient index increases, the critical buckling load continues to diminish due to the rise in metal material content in the plate bringing about reduced stiffness. At the point when the material property index reaches infinity, the plate is speculated as totally metallic which is less stiff when compared to ceramic thereby, reducing the critical buckling load.

5.2 Scope of Future Work

FGMs have many applications in engineering and aerospace structures. The plates are subjected to in-plane forces often. These are due to aerodynamic or hydrodynamic forces acting on it. Twisted Cantilever panel finds its application as turbine blades, compressor blades, fan blades, marine propellers and many more. Cut-outs are often found in composite structures. They are provided in structural components for ventilation and to lighten the structure.

In the present study, the twisted plate has been studied for a central circular cut-out only. Eccentric cut-outs and cut-outs of different shapes can also be studied.

Effect of biaxial non-uniform loading may also be considered as a future study.

CHAPTER 6

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REFERENCES

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