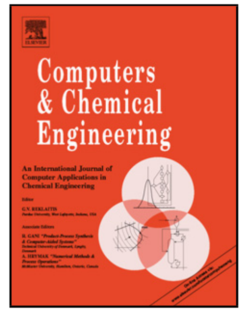
Accepted Manuscript



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PII:	S0098-1354(17)30376-9
DOI:	https://doi.org/10.1016/j.compchemeng.2017.10.021
Reference:	CACE 5927
To appear in:	*Computers and Chemical Engineering*
Received date:	28-7-2017
Revised date:	22-9-2017
Accepted date:	18-10-2017

Please cite this article as: Vázquez, Daniel., Torres, María J Fernández., Ruiz-Femenia, Rubén., Jiménez, Laureano., & Caballero, José A., MILP Method for Objective Reduction in Multi-objective Optimization.*Computers and Chemical Engineering* https://doi.org/10.1016/j.compchemeng.2017.10.021

MILP Method for Objective Reduction in Multi-objective Optimization

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Highlights

- MILP Objective reduction method based on the maintenance of the dominance structure
- Three models defined depending on the reduction grade the user wants to achieve
- Method to tackle high dimensional multi-objective optimization problems
- Method that quantifies the alterations in the dominance structure

Abstract

A procedure for reducing objectives in a multi-objective optimization problem given a set of Pareto solutions is presented. Three different models are detailed, which achieve three different degrees of objective reduction. These models are based on maintaining the dominance structure of the problem. To compare the performance of the proposed models, these are tested with pure mathematical cases and with actual data from previous works in the field of multi-objective optimization. The first model provides the reduced subset of objectives that do not alter the dominance structure of the problem at all. The second model determines the minimum subset of objectives that alters the dominance structure with an upper predefined limit for the error. The last model provides the subset of objectives with a previously defined cardinality, which achieves the minimum error. The possibility of different inputs introduces flexibility into the models, which accounts for the preferences of the decision-maker.

Keywords: Multi-objective Optimization; δ-MOSS; k-EMOSS; Objective Reduction; Dominance Structure; Pareto frontier.

Nomenclature

Sets

F, OBJ	$\left\{ \mathrm{i} \mid \text{ objectives } \right\}$
X	Feasible region for the original space of the decision variables
A	Subspace of X
SOL	$\left\{ s \mid \text{ solutions} \right\}$
$\mathrm{P}_{\mathrm{s,s'}}$	$\left\{ \mathrm{s,s'} \mid \text{subset of pairs of S where ord(s)} < \text{ord(s')} \right\}$

Data

$x_{s,i}$	Normalized value of the objective i in the solution s

Parameters

$\mathbf{x}, \mathbf{y}$	Solution in the original space of decision variables
$C_{1s,s',i}$, $C_{2s,s',i}$	Coefficients which have either the value 1 or 0, depending on the data
$Pen_{1s,s',i}$, $Pen_{2s,s',i}$	Measure the error commited between pairs when one objective is eliminated
$ObjNumber$	Number of objectives to maintain

Variables

Y,V,W	Boolean variables for y, v_1, v_2, w_1 and w_2
y_i	Binary variable that takes the value 1 when i exists and 0 when it does not
$v_{1s,s'}, v_{2s,s'}$	Binary variables that adopt the value 1 when the constraints of the set covering problem are broken
$w_{1s,s'}, w_{2s,s'}$	Continuous variables that act as binary
δ	Value of the approximation error

1. Introduction

One of the major challenges in the process systems engineering community consists of assessing a problem form a holistic point of view that considers not only the classical economic aspects, but also environmental, or social related aspects (e.g. safety, employment, region development, etc.). The problem becomes even more complicated if we take into account that for each one of these aspects we can draw an even larger set of impacts, sometimes contradictory among them. For example, if we follow the accepted Life Cycle Assessment (LCA) (Azapagic & Clift, 1999) approach for evaluating a process from an environmental point of view where we consider all the individual impacts, we must take into account a large set of indicators like global warming potential, acidification, ozone depletion etc.

There are different alternatives to deal with multi-criteria decision analysis, like analytical hierarchical process (AHP) (Saaty, 1990), analytic network process (ANP) (Saaty, 2001), case base reasoning (Aamodt & Plaza, 1994), data envelopment analysis (Banker et al., 1984), fuzzy set theory (Zadeh, 1965), etc. Recent reviews of all these techniques can be found for example in (Bandaru et al., 2017).

Among all the available techniques, multi-objective optimization (MOO) has been proven to be an excellent method to simultaneously optimize several objectives. Indeed, a search in Scopus for literature dealing with this technique surpasses the figure of 50000 publications. It has been employed in a large number of fields, for example in the area of process design (García et al., 2014), safety (Rubén Ruiz-Femenia et al., 2017) and other fields (Chang et al., 2015; R. Ruiz-Femenia et al., 2013; Salcedo et al., 2012; Sendín et al., 2009) .

The solution of a multi-objective optimization (MOO) problem is a set of Pareto optimal alternatives. Each point on this Pareto set has the property that, it is not possible to find any other feasible point that improves an objective without worsening in at least another one.

The major advantage of MOO is that it does not require any pre-optimization decision and provides the decision-maker with a set of “optimal solutions” (i.e. non feasible and non Pareto optimal solutions have been removed) to choose among them. For example, the decision-maker could identify regions in which an objective considerably increases with only marginally worsening the rest. The major drawbacks, however, are that the computational burden exponentially increases with the number of objectives and the analysis of the solution is difficult for more than two or three objectives.

In order to overcome the limitations of the exponential growth and difficultness in the results analysis when the number of objectives increases, some strategies can be followed. The most straightforward approach consists of omitting some of the objectives (e.g. selecting only two objectives) and therefore reducing the associated complexity. According to Guillén-Gosálbez (2011) this approach has the major drawback that the decision-maker must select a priori the objectives to include in the optimization based on his/her preferences/experience without any information of how good the selected objectives are. Another approach consists of using aggregated metrics in all or in a subset of objectives. For example, in environmental engineering a typical approach consists of assigning weights to each environmental impact, trying at the same time to cover all possible environmental impacts (ecotoxicity, land occupation, carcinogenic effects, climate change, ionizing radiation, ozone layer depletion, etc.). In this way, the problem is reduced to a bi-objective (economic and aggregated environmental) optimization problem. Those weights are usually defined by a panel of experts that should represent the opinions of the society or a group of stake holders. Different researchers have proposed different environmental metrics. Among those works, some examples are the aggregated indexes proposed by Mallick (Mallick et al., 1996), Elliot (Elliott et al., 1996) and Biwer (Biwer & Heinzle, 2004), among others (Goldberg, 2002). In this line, the aggregated metrics used on Life Cycle Assessment, which are able of provide a numerical score to the environmental impacts of a process, are likely the most successful approach, such as the Ecoindicator-95 (M. Goedkoop et al., 1995), the Ecoindicator – 99 (PRé-Consultants, 2000), ReCiPe (M. J. Goedkoop et al., 2009) and IMPACT 2002+ (Jolliet et al., 2003). In the field of safety analysis, inherent safety is defined by a set of indexes that finally are merged in a single final index which allows the user to compare the safety level of different processes or the same process under different operating conditions. Among those indexes, examples are the

I2SI (Khan & Amyotte, 2004) and the NUDIST (Ahmad et al., 2014), which are able to produce a single numerical value for a process regarding its safety.

In process engineering, bi-objective optimization problems typically optimize an economic objective versus an aggregated metric of another kind. The extreme situation of objective aggregation is when we reduce the problem to a single objective, for example by monetarizing the environmental impacts or the safety risks. While this has the advantage of simplifying the problem to a single-objective optimization (economic), it carries the disadvantage of not providing information about the monetarized objective, in these cases, the environmental or safety risks. Even when the aggregation of objectives is done at a lower scale, such as in the case of the aggregated environmental indexes, it can be observed that there may be a difference in the obtained results when considering the aggregated index versus considering the indexes as separated entities (Carreras et al., 2016).

In any case, both the objective aggregation and objective selection, could eventually modify the dominance structure of the optimization problem (Guillén-Gosálbez, 2011). For example, it would be possible to select an alternative that is the best in the subset of objectives selected but it is not the best if one of the objectives is removed. It would be also possible that two solutions show the same performance in the sub-set of objectives selected but one of them dominated the other in the original set of objectives. A dominance structure is a representation of the Pareto dominance relations between points. More insight in this matter is given in the Theoretical foundations section of this paper.

Due to this problems, it might be worth to reduce the dimension of the set of objectives, but not doing so in an arbitrary or aggregated manner. Dimensionality reduction is a problem that appears in areas like data mining or statistics and there are different approaches to deal with it (Agrell, 1997). It is possible to differentiate two approaches: feature extraction and feature selection. In feature extraction the objective is to extract a reduced set of arbitrary features, while in feature selection we try to find the smallest set of given features that best represent the data. In MOO the equivalent problem to feature selection is to find a sub-set of objectives that describes the original problem best (likely expressed as a combinations of the original ones). However, as pointed out by Purshouse and Flemming (2003), common dimensionality reduction techniques cannot be directly used because they cannot ensure that the Pareto-dominance structure is maintained in the space of reduced objectives.

One of the first contributions to objective reduction preserving the dominance structure was due to Gal and Leberling (1977). However, the approach is constrained to linear systems and cannot be applied to general problems.

Deb and Saxena (2005) presented an approach for objective reduction based on principal component analysis that was later extended to deal with non-linear dimensionality reduction techniques. The method selects the most important conflicting objectives and removes the redundant ones. To that end, those objectives less influential in the principal components are discarded. Since this approach considers the correlation among objectives as an indicator of the conflict among them, it cannot guarantee that the Pareto dominance is preserved in the reduced set of objectives and no quantitative measure is provided about how much the dominance changes with the removed objectives (Brockhoff & Zitzler, 2009).

Recently, Cheung and Liu (2016) formulated the problem of reducing the number of objectives as a linear combination of the original ones by maximizing the conflict between the reduced objectives. With this approach the dominance structure or the original MOO problem is preserved as much as possible.

In most MOO problems, however, there is a large redundancy among the objectives: some of them, and in some situations most of them, can be removed without changing the dominance structure of the problem. Brockhoff and Zitzler (2006a, 2006b, 2006c), in a seminal paper, formally stated the problem of calculating a minimum subset of objectives without losing information (δ-Minimum Objective Subset, δ-MOSS). They also stated the problem of computing the minimum objective subset of size k with minimum error (k-EMOSS problem). In this paper, we present Mixed Integer Linear Programming (MILP) models that efficiently allow solving those problems. An alternative MILP approach was presented by Guillén-Gosalbez (2011). However before going into the details of the model we need to introduce some basic definitions and concepts.

The paper is structured in the following manner. The next section introduces the theoretical foundations previous to our work. After that, the next section states the problem we want to solve. The mathematical models, with their associated examples, are defined next. The real cases are solved following to that, and their results are presented before ending with the conclusions of the paper.

2. Theoretical foundations

To solve the δ-MOSS and/or the k-EMOSS problem we must know what happens if a new objective is added or removed from a problem, when an objective could be removed, and how to measure the conflict among objectives. The definitions that follow are mainly based on the works by Brockhoff and Zitzler (2006a, 2006b, 2006c, 2009).

As commented above, when we have more than three objectives it is difficult to visualize the dominance structure of the problem. In those cases it is useful to use a parallel coordinates plot. Fig. 1a is an example of a Pareto chart in 2D for a bi-objective problem. Notice that the data represented in the two axis has been normalized from 0 to 1 and therefore are dimensionless. A Pareto curve consists of an infinite number of points. To generate a parallel coordinate plot we must select a subset of points (solutions) over the Pareto frontier (i.e. solutions 1, 2 and 3) highlighted in Fig. 1a. The representation of the values of this subset of solutions for each objective forms the parallel coordinate plot. (Fig. 1b).

< Fig.1 >

Fig. 1: a) Example of Pareto chart b) Parallel coordinate plot of the same data.

Let *X* to be a decision space defined over a set *F* of k objective functions: $f_i : X \to \mathbb{R} \quad i \in F$ that must be minimized. A solution $\mathbf{x} \in X$ is said to ***weakly dominate*** another solution $\mathbf{y} \in X$ if and only if **x** is not worse than **y** in all the objectives. Here we use the notation $\mathbf{x} \preceq_{F'} \mathbf{y}$ to denote that the weak Pareto dominance relation is used with relation to a particular subset of objectives, as shown in eq.(1).

$$
\begin{aligned}
& F' \subseteq F := \left\{ f_1, f_2, f_3 \ldots f_k \right\} \;\; k = |F| \\
& \mathbf{x} \preceq_{F'} \mathbf{y} \Leftrightarrow \forall f_i \in F' \colon f_i(\mathbf{x}) \leq f_i(\mathbf{y})
\end{aligned}
\tag{1}
$$

In this paper we use the weak Pareto dominance relation as the structure according to the optimization is to be carried out.

As we cannot compare the complete space *X*, we must select a finite number of solutions $A \subseteq X$, therefore the weak dominance is referred only to that subset of solutions, as shown in eq.(2):

$$\mathbf{x} \preceq_{F'} \mathbf{y} = \left\{ (\mathbf{x}, \mathbf{y}) \in A \times A \,/\, \forall f_i \in F' \colon f_i(\mathbf{x}) \leq f_i(\mathbf{y}) \right\} \quad A \subseteq X \tag{2}$$

We say that x, y are ***comparable*** if either $\mathbf{x} \preceq_{F'} \mathbf{y}$ or $\mathbf{y} \preceq_{F'} \mathbf{x}$, and **x**, **y** are ***incomparable*** if neither $\mathbf{x} \preceq_{F'} \mathbf{y}$ nor $\mathbf{y} \preceq_{F'} \mathbf{x}$. The Pareto optimal set is formed by all solutions **x** that either weakly dominate or are incomparable to any other solution **y**.

Two objective sets *F₁*, *F₂* are say to be ***conflicting*** if the weak Pareto dominance relations differ $\left(\preceq_{F_1} \neq \preceq_{F_2} \right)$ and ***non-conflicting*** otherwise $\left(\preceq_{F_1} = \preceq_{F_2} \right)$.

A set $F_1 \subseteq F$ of objectives is say to be **redundant** if and only if exists a subset $F_2 \subseteq F_1$ that is not-conflicting with F_1.

Brockhoff and Zitzler (2009) proved that a solution **x** dominates another solution **y** with respect to a set of objectives *F* if and only if **x** weakly dominated **y** for every subset $F' \subseteq F$, as shown in eq.(3).

$$\mathbf{x} \preceq_{F} \mathbf{y} \Leftrightarrow f_i(\mathbf{x}) \leq f_i(\mathbf{y}) \;\; \forall f_i \in F \equiv \mathbf{x} \preceq_{\{f_i\}} \mathbf{y} \tag{3}$$

Let us start by showing what happens to the dominance structure when we add a new objective function. Consider the two solutions A and B in Fig. 2. If we only have objective function 1 (f_1), solution B weakly dominates solution A. If we add the objective function 2 (f_1), solution B still dominates solution A, because solution B has lower values than A in both objectives f_1 and f_2. However, if we add the third objective function (f_3), the two solutions become incomparable because solution B weakly dominates A with respect to objectives 1 and 2 $\left(B \preceq_{\{f_1, f_2\}} A \right)$ but solution A weakly dominates B with respect to objective 3. $\left(B \preceq_{\{f_3\}} A \right)$.

< Fig.2 >

Fig. 2: Dominance structure for 2 hypothetical solutions, A and B, in 3 objectives; i_1, i_2, i_3. Both solutions are Pareto Optimal in the original set (a) and in the reduced set 2 (c), while B completely dominates A in the reduced set 1 (b)

To study whether an objective can be omitted or not we use the concept of redundancy as necessary and sufficient condition. Consider again Fig. 2, if we remove objective 1 the dominance structure does not change – the objective set $\left\{ f_2, f_3 \right\}$ induces the same dominance relation as $\left\{ f_1, f_2, f_3 \right\}$. Therefore the set $\left\{ f_1, f_2, f_3 \right\}$ is redundant. A similar situation appears if instead of objective 1 we remove objective 2.

The requirement of strictly maintaining the dominance structure of the problem is sometimes too strict for practical applications, especially if the minimum number of non-redundant objectives is still too large. In those cases we will be able to introduce some error to get a more substantial decrease of the objective set. However, in that case a metric to measure the changes of the dominance relations is necessary to control the degeneration in the dominance structure of the problem. To that end we use the additive ***δ-conflicting objective sets*** (eq(4)).

$$\mathbf{x} \preceq_{F'}^{\delta} \mathbf{y} \Leftrightarrow \forall f_i \in F': f_i(\mathbf{x}) - \delta \leq f_i(\mathbf{y}) \quad (4)$$

Therefore, if we have two sets F_1 and F_2, we say that F_1 is ***δ-nonconflicting*** with F_2 if and only if both $\preceq_{F_1} \subseteq \preceq_{F_2}^{\delta}$ and $\preceq_{F_2} \subseteq \preceq_{F_1}^{\delta}$ hold true; otherwise F_1 and F_2 are said to be ***δ-conflicting***.

Extending the concept of redundancy then we can define a set $F_1 \subseteq F$ is ***δ-redundant*** if and only if there exists an objective subset $F_2 \subset F_1$ that is δ -nonconflicting with F_1.

To illustrate the concept of δ-redundancy consider again Fig. 2. If we remove objective 3, the remaining objectives do not induce the same dominance relation than the original set. However, if we admit a 0.4-conflict (the delta error shown in the figure) the remaining subset $\left\{f_1, f_2\right\}$ is 0.4-redundant with the original set $\left\{f_1, f_2, f_3\right\}$.

The question that naturally arises is, what is the minimum nonredundant (or δ-redundant) set of objectives? In this context the following definitions are of interest:

An objective set $F_2 \subseteq F_1$ is denoted as ***minimal*** (δ-minimal) if and only if, it is both not redundant (δ-redundant) and nonconflicting (δ-nonconflicting) with F_1. It is denoted as ***minimum*** (δ-minimum) if and only if, it is the smallest minimal objective set. For example, in relation to Fig. 2, the subsets formed by $\left\{f_1, f_3\right\}$ and $\left\{f_2, f_3\right\}$ are both minimal and at the same time minimum. However, the subset $\left\{f_1, f_2\right\}$ is δ-minimal (δ=0.4) but it is not minimum. The subsets $\left\{f_1\right\}$ and $\left\{f_2\right\}$ are both δ-minimal and δ-minimum (δ=0.4).

In the rest of the paper we present a MILP formulation for solving the following problems.

1. Given a set of $A \subseteq X$ of *s* solutions together with the objective values $f_i(\mathbf{x})\, i \in F,\ \mathbf{x} \in A$, the problem is to find a subset (or all of them) $F_1 \subseteq F$ that are minimum with relation to *F*.
2. Given a set of $A \subseteq X$ of *s* solutions and a $\delta_{MAX} \in \mathbb{R}$ together with the objective values $f_i(\mathbf{x})\, i \in F,\ \mathbf{x} \in A$, the problem is to find a subset (or all of them) $F_1 \subseteq F$ that are δ-minimum with relation to F (δ-MOSS problem).
3. Given a set of $A \subseteq X$ of *s* solutions together with the objective values $f_i(\mathbf{x})\ \ i \in F,\ \mathbf{x} \in A$, and the cardinality of a subset $F_1 \subseteq F$, the problem is to find a subset the elements (objective

functions) with fixed cardinality that is δ-minimum with relation to *F* with minimum δ value (k-EMOSS problem).

3. Problem Statement

Given a set of objectives (or objective functions) and a number of Pareto solutions, the aim of this paper is to present a method to solve the three previously mentioned problems, namely: minimum subset of objectives, δ-minimum subset of objectives, and to find a subset of objectives with fixed cardinality that achieves minimum δ. From now on, these three will be designed as: minimum subset problem, δ-MOSS problem and k-EMOSS problem.

We refer to the set of objectives as OBJ, and to the set of solutions as SOL. To indicate any element of OBJ or SOL we use the indexes "i" and "s" respectively. The index "i" will represent a particular objective in the set OBJ, meanwhile the index "s" corresponds to a concrete Pareto solution. For our methodology, we need to compare the values of pairs of solutions for a given objective. For example, if we were to compare the values of solutions 3 and 5 for objective "i", this needs to be established in a general way. In other words, we need to provide two indexes to refer to the solution set, SOL, one set of the first solution chosen (here s=3) and another set for the second solution to be compared with (here s'=5).

OBJ	[i \ i is an objective]
SOL	[s \ s is a Pareto solution.] This is an ordered set
$P_{s,s'}$	$\left\{ s, s' \in SOL \middle| \, (s < s') \right\}$

P is another set based on s and s'. The inequality in the definition of the set *P* makes reference to the position in the ordered set s. This will be useful for referring to the pairs of solutions that we need to compare without incurring in a needless repetition. For example, if we want to compare the solutions s_3 and s_5, the set $P_{s,s'}$ will not allow the simultaneous election of both pairs s- s' (s_3-s_5) and s'-s (s_5-s_3) , but only the first one. This will allow us to avoid considering twice the relation between the first and second solutions, for example, by limiting the possibility to (s_1,s_2) and not including (s_2,s_1).

4. Mathematical Model

The procedure of eliminating redundant objectives starts with a list of data provided in the form of $x_{s,i}$, that stands as the normalized value (from 0 -1 range) of the objective *i* in the Pareto solution *s*. This normalization has been undertaken as shown in eq.(5) :

$$x_{s,i} = \frac{x_{s,i}^{original} - \min\limits_{s \in SOL}(x_{s,i})}{\max\limits_{s \in SOL}(x_{s,i}) - \min\limits_{s \in SOL}(x_{s,i})} \qquad \forall s \in SOL, \forall i \in OBJ \tag{5}$$

Even though it is out of the scope of this paper, it is worth mentioning that normalization is critical when removing or comparing objectives and deserves some comments. Without further information, the previous normalization is a good alternative to deal with objectives with different units and different physical meaning. However, if we do have information about typical max and min values of certain objectives, if not all of them, then it would be more realistic to normalize over those values. In this way, we could remove a priory some objectives that have negligible variations, or avoid overestimating the importance of some objectives. Alternatively, if we do not have information about max and min values, and we are sure that all objectives are important, we can normalize only over the minimum value of each objective. For example, if the economic objective improves a 30% in relation to its worse value and an environmental objective improves a 20%, this normalization gives a relative weight of 3/2 to the economic versus the environmental objective instead of the same weight to both objectives.

As stated before, we want to solve three problems. We present below a model for each one of them, from Model 1 to Model 3.

4.1 Model 1: Minimum subset of objectives problem

The aim of this first model is to select the minimum number of objectives that preserve the dominance structure of the original problem: Let y_i be a binary variable that takes the value 1 when that objective is selected to be part of the minimum set of objectives and 0 otherwise. For our model, we also need to calculate $C_{1s,s',i}$ and $C_{2s,s',i}$ which are two parameters defined by eq(6):

$$\begin{aligned}
&C_{1s,s',i} = 1 && \forall (P_{s,s'}, i \in OBJ) \mid (x_{s',i} - x_{s,i}) > 0 \\
&C_{1s,s',i} = 0 && \forall (P_{s,s'}, i \in OBJ) \mid (x_{s',i} - x_{s,i}) \le 0 \\
\\
&C_{2s,s',i} = 1 && \forall (P_{s,s'}, i \in OBJ) \mid (x_{s',i} - x_{s,i}) \le 0 \\
&C_{2s,s',i} = 0 && \forall (P_{s,s'}, i \in OBJ) \mid (x_{s',i} - x_{s,i}) > 0
\end{aligned} \tag{6}$$

For the sake of clarity, let's apply eq(6) to the data contained in Fig. 2. We must compare all the solutions in pairs (s with s'). Since Fig. 2 only contains two solutions, A and B, only one pair is possible.

There are three objectives, for the first objective, i_1, solution A has a higher value than solution B, i.e., solution B dominates solution A, $x_{B,1} < x_{A,1}$. So, according to equation 1, $C_{1,A,B,i1}$ must be 0 for objective i_1 and $C_{2,A,B,i1}$ must be 1. Similarly for the remaining two objectives $C_{1,A,B,i2}$ must be 0 for solution i_2 and $C_{2,A,B,i2}$ must be 1; $C_{1,A,B,i3}$ must be 1 for solution i_3 and $C_{2,A,B,i3}$ must be 0.

$$\begin{aligned} \min \sum_{i \in OBJ} y_i & \\ s.t. \quad \sum_{i \in OBJ} C_{1,s,s',i}\; y_i \geq 1 & \quad \forall (s,s') \in P_{s,s'} \\ \sum_{i \in OBJ} C_{2,s,s',i}\; y_i \geq 1 & \quad \forall (s,s') \in P_{s,s'} \\ y_i \in \{0,1\} & \end{aligned} \qquad \text{(Model 1)}$$

In Model 1, we minimize the number of objectives selected. The first constraint ensures that among all the objectives in which the solution s dominates solution s', at least one of those is selected. The second constraint selects at least an objective in which solution s' dominates solution s. These two constraints warrant that the dominance structure of the problem is maintained.

It is worth pointing out that the Model 1 corresponds to a «*Set Covering Problem*». The set covering problem is NP-Hard. Brockhoff and Zitzler (2009) proved that the minimum subset of objectives problem is NP-Hard as well. It is now clear that there exists a one-to-one relationship between these two problems.

This model is further explained in the first example.

4.1.1 Example 1: Finding a minimum subset of objectives

In this first case, it will be shown how our first model rejects redundant objectives with zero approximation error (δ=0).

The data used for this example is shown in Table 1.

As stated before, the first step is to normalize each solution for each objective from 0 to 1, since their measures could refer to various scales. In this problem we normalize based on the presented data, but using the bounds of each objective would provide a more accurate normalization. The normalized data are shown in Table 2:

The data in Table 2 is shown as a parallel coordinate plot in Fig. 3. This Figure shows that $x_{3,1}<x_{2,1}<x_{1,1}$; $x_{2,2}<x_{1,2}<x_{3,2}$; $x_{1,3}<x_{3,3}<x_{2,3}$; and $x_{3,4}<x_{2,4}<x_{1,4}$. It is clear that both, the first and fourth objective, behave in

the same manner, and thus one can be pruned. Model 1 will achieve the goal of removing either objective 1 or 4. The following step in the model is to calculate parameters $C_{1,s,s',i}$ and $C_{2,s,s',i}$, from eq(6). These results are shown in Table 3.

< Fig.3 >

Fig. 3: Dominance structure of the first example case

Once the values of $C_{1,s,s',i}$ and $C_{2,s,s',i}$ are calculated, the constraints shown in Model 1 can be written, and from those, we can analyze what exactly each constraint does. From Fig. 3, it can be observed that objective i_2 cannot be removed, otherwise solution s_3 would completely dominate solution s_2. Likewise, if objective i_3 were removed, solution s_2 would completely dominate solution s_1.

Now, for this simple case, it is possible to compare what is obvious from Fig. 3 with the constraints obtained with Model 1. This model applied to example 1 results as:

$$\min z = y_1 + y_2 + y_3 + y_4$$

$$\begin{array}{l}
s.t. \\
\text{First block of constraints (from } \mathrm{C}_{1,s,s',i}) \\
\qquad y_3 \geq 1 \\
\qquad y_2 + y_3 \geq 1 \\
\qquad y_2 \geq 1 \\
- \\
\text{Second block of constraints (from } \mathrm{C}_{2,s,s',i}) \\
\qquad y_1 + y_2 + y_4 \geq 1 \\
\qquad y_1 + y_4 \geq 1 \\
\qquad y_1 + y_3 + y_4 \geq 1
\end{array}$$

From the first block of constraints it is clear that objectives i_2 and i_3 are necessary. This is due to the fact that their associated binary variables must have a value of 1 in order to fulfill the constraints. Thus, $y_2 = y_3 = 1$. Keeping this result in mind, from the second block of constraints we can be sure that the first and last equations are already fulfilled, no matter which value y_1 or y_4 take, but for the second equation, at least one of them must take the value 1 in order to fulfill the constraint. Thus, either i_4 or i_1 is redundant.

We know that it will only pick one of the two because the objective function demands a minimum in the amount of objectives. This will drag the amount objective to the least possible that fulfills the constraints.

Note that this first model does not make any distinction between the redundant objectives i_1 and i_4 although they provide different solutions. The reason for this is because the model only checks the dominance order of the solutions, i.e., s_3 dominates s_2 which dominates s_1 in both objectives.

4.2 Model 2: δ-MOSS problem

The objective of the second model is to obtain a subset of objectives that will be δ-minimum with relation to the initial set. In order to know how much the dominance structure would change after removing some objectives, we need to introduce a parameter which allows us to measure how much the dominance structure is being altered. To that end, we introduce two penalty factors, Pen_1 and Pen_2 which are defined as shown in eq(7).

$$
\begin{array}{lll}
Pen_{1,s,s',i} = x_{s',i} - x_{s,i} & \forall (P_{s,s'}, i) \mid (x_{s',i} \geq x_{s,i}) \\
Pen_{1,s,s',i} = 0 & \forall (P_{s,s'}, i) \mid (x_{s',i} < x_{s,i}) \\
\\
Pen_{2,s,s',i} = x_{s,i} - x_{s',i} & \forall (P_{s,s'}, i) \mid (x_{s',i} < x_{s,i}) \\
Pen_{2,s,s',i} = 0 & \forall (P_{s,s'}, i) \mid (x_{s',i} \geq x_{s,i})
\end{array}
\tag{7}
$$

The resulting value of $Pen_{1s,s',i}$ and $Pen_{2s,s',i}$ will act as the value of the δ error previously defined.

We also need to add new binary variables ($v_{1,s,s'}$ and $v_{2,s,s'}$) to the constraints already presented in Model 1 as shown in eq(8). The reason behind this is that for some cases where an objective is eliminated, the sum of $C_{1,s,s',i}y_i$ can be zero, and then the constraint will not be met, so we need to add this variable which allows the constraint to be violated. In other words, the binaries introduced in eq(8), allow to change the dominance structure as compared to Model 1 (if needed) and also is able to track the change introduced:

$$
\begin{array}{l}
\sum_{i \in OBJ} C_{1,s,s',i} \cdot y_i + v_{1s,s'} \geq 1 \quad \forall (s, s') \in P_{s,s'} \\
\\
\sum_{i \in OBJ} C_{2s,s',i} \cdot y_i + v_{2s,s'} \geq 1 \quad \forall (s, s') \in P_{s,s'}
\end{array}
\tag{8}
$$

It is possible to fix some of those binaries beforehand by knowing that we are only working on the subset P of solutions. In other words, those binaries associated with pairs outside the subset P are set to zero as shown in eq(9).

$$\left.\begin{aligned} v_{1s,s'} &= 0 \\ v_{2s,s'} &= 0 \end{aligned}\right\} \forall (s,s') \notin P_{s,s'} \tag{9}$$

In order to calculate the approximation error we need to use new binary variables that we name as $w_{1,s,s',i}$ and $w_{2,s,s',i}$. These binary variables allow us to calculate the approximation error as shown in eq(10). This means that δ will be affected or not by the value of $Pen_{1,s,s',i}$ or $Pen_{2,s,s',i}$ depending on the value of $w_{1,s,s',i}$ (or $w_{2,s,s',i}$).

We are interested in the maximum approximation error δ (infinite norm), calculated as in eq.(10):

$$\delta = \max_{\substack{s,s' \in P \\ i \in OBJ}} (Pen_{1s,s',i}\, w_{1s,s',i}, Pen_{2s,s',i}\, w_{2s,s',i}) \tag{10}$$

Eq.(10) can then be reformulated as two inequalities as shown in eq.(11):

$$\begin{aligned} \delta &\geq Pen_{1s,s',i}\, w_{1s,s',i} \quad \forall P_{s,s'}, \forall i \in OBJ \\ \delta &\geq Pen_{2s,s',i}\, w_{2s,s',i} \quad \forall P_{s,s'}, \forall i \in OBJ \end{aligned} \tag{11}$$

Since the objective is to maintain the subset as a δ-minimum subset, we have to impose a limit to the δ error. This is shown in eq.(12), where δ_{MAX} is a parameter chosen by the user for the maximum permitted value of δ.

$$\delta \leq \delta_{MAX} \tag{12}$$

The objective function is, like in the previous minimum subset problem, to minimize the amount of objectives, thus eq(13).

$$z = \sum_{i \in OBJ} y_i \tag{13}$$

In order to force the correct relations between v and w variables, we must introduce the following logical relationships: If the objective "i" is not chosen and there exists a binary variable $v_{1,s,s'}$ or $v_{2,s,s'}$, then the variable correspondent to that binary, $w_{1,s,s',i}$ or $w_{2,s,s',i}$ must exist so that the error δ is accounted for in that specific pair of solutions. In other words, when an objective is not chosen, and the

Pareto structure is being compromised, the w binary variable is activated so that it will provide an approximation error.

The easiest way of writing this relations is using logic propositions in terms of Boolean variables (presented here in capital letters) and then transform these relations to algebraic equations in terms of binary variables.

This statement can easily be written with Boolean logic propositions as shown in eq(14), which means: if an objective is not chosen ($\neg \mathrm{Y}_i$) and $\mathrm{V}_{1,s,s'}$ or $\mathrm{V}_{2,s,s'}$ are true, then the variable $\mathrm{W}_{1,s,s',i}$ (or $\mathrm{W}_{2,s,s',i}$) must be true. These Boolean implications work as well in the opposite direction.

$$\begin{array}{ll} \neg \mathrm{Y}_i \wedge \mathrm{V}_{1s,s'} \Leftrightarrow \mathrm{W}_{1s,s',i} & \forall P_{s,s'}, \forall i \in OBJ \\ \neg \mathrm{Y}_i \wedge \mathrm{V}_{2s,s'} \Leftrightarrow \mathrm{W}_{2s,s',i} & \forall P_{s,s'}, \forall i \in OBJ \end{array} \tag{14}$$

We can now convert these Boolean expressions methodically as described by Raman and Grossmann (Raman & Grossmann, 1991) into the algebraic expressions eq(15), which allows us to add those logical relations as algebraic constraints to the model.

$$\begin{array}{rl} y_i + (1 - v_{1s,s'}) + w_{1s,s',i} \geq 1 & \forall P_{s,s'}, \forall i \in OBJ \\ y_i + (1 - v_{2s,s'}) + w_{2s,s',i} \geq 1 & \forall P_{s,s'}, \forall i \in OBJ \\ (1 - y_i) + (1 - w_{1s,s',i}) \geq 1 & \forall P_{s,s'}, \forall i \in OBJ \\ (1 - y_i) + (1 - w_{2s,s',i}) \geq 1 & \forall P_{s,s'}, \forall i \in OBJ \\ v_{1s,s'} + 1 - w_{1s,s',i} \geq 1 & \forall P_{s,s'}, \forall i \in OBJ \\ v_{2s,s'} + 1 - w_{2s,s',i} \geq 1 & \forall P_{s,s'}, \forall i \in OBJ \end{array} \tag{15}$$

The expressions in eq.(15) also allow to define the variables w as continuous, with bounds between 0 and 1. Integer values are forced by the binaries y and v.

Thus, the complete δ-MOOS model is formed by the minimization of eq.(13), subject to the constraints of eq(8),(11), (12) and (15).

$$
\begin{aligned}
&\min z = \sum_{i \in OBJ} y_i \\
&s.t. \\
&\left.\begin{aligned}
&\sum_{i \in OBJ} C_{1,s,s',i} y_i + v_{1s,s'} \geq 1 \\
&\sum_{i \in OBJ} C_{2s,s',i} y_i + v_{2s,s'} \geq 1
\end{aligned}\right\} \forall (s,s') \in P_{s,s'} \\
&\delta \leq \delta_{MAX} \\
&\left.\begin{aligned}
&\delta \geq Pen_{1s,s',i}\, w_{1s,s',i} \\
&\delta \geq Pen_{2s,s',i}\, w_{2s,s',i} \\
&y_i + (1 - v_{1s,s'}) + w_{1s,s',i} \geq 1 \\
&y_i + (1 - v_{2s,s'}) + w_{2s,s',i} \geq 1 \\
&(1 - y_i) + (1 - w_{1s,s',i}) \geq 1 \\
&(1 - y_i) + (1 - w_{2s,s',i}) \geq 1 \\
&v_{1s,s'} + 1 - w_{1s,s',i} \geq 1 \\
&v_{2s,s'} + 1 - w_{2s,s',i} \geq 1
\end{aligned}\right\} \forall (s,s') \in P_{s,s'};\ \forall i \in OBJ \\
&y_i \in \{0,1\}
\end{aligned}
\qquad \text{(Model 2)}
$$

While this model provides a subset of objectives that is δ-minimum in relation with the original set, it does not provide any of the degenerate solutions, in the case that they exist. To obtain all the degenerate solutions we can either ask the solver to provide the rest of solutions or, if the solver has not that capacity, then we can introduce a canonical cut (Balas & Jeroslow, 1972) and solve the problem again. The cut procedure is shown in eq.(16)

$$\sum_{i \in B} y_i - \sum_{i \in N} y_i \leq |B| - 1 \qquad (16)$$

Where B_i is the sub-set of objectives that have already been selected ($y_i = 1$), *N* the subset of objectives that were not selected ($y_i = 0$) and $|B|$ is the cardinality of set *B*.

While this is useful and provides a solution fast enough, it can be more interesting to fix the number of objectives to be maintained and obtain the minimum δ-error that could be achieved with that cardinality: the k-EMOSS problem.

4.3 Model 3: k-EMOSS problem

This model can be obtained from a small modification in the δ-MOSS problem.

In this third model, we fix the number of objectives (ObjNumber) we want to maintain. The main idea of this model is to minimize the maximum approximation error (δ) when reducing a set of objectives to a subset of a determined size. We add then constraint (17).

$$\sum_{i \in OBJ} y_i = ObjNumber \tag{17}$$

As we have stated before, the intention of this third model is to minimize the maximum approximation error between pairs of Pareto solutions that, when eliminating objectives, one solution of one of the pairs stops being Pareto optimal. Thus we exchange the previous objective function for (18).

$$z = \delta \tag{18}$$

We also want to remove (12). While it would work with it, it is better to not restrain the objective function for the calculations.

The model is thus as shown in Model 3.

$$\begin{aligned}
&\min z = \delta \\
&s.t. \\
&\quad \sum_{i \in OBJ} y_i = ObjNumber \\
&\left.\begin{aligned}
&\sum_{i \in OBJ} C_{1,s,s',i} \cdot y_i + v_{1s,s'} \geq 1 \\
&\sum_{i \in OBJ} C_{2s,s',i} \cdot y_i + v_{2s,s'} \geq 1
\end{aligned}\right\} \forall (s,s') \in P_{s,s'} \\
&\left.\begin{aligned}
&\delta \geq Pen_{1s,s',i} \cdot W_{1s,s',i} \\
&\delta \geq Pen_{2s,s',i} \cdot W_{2s,s',i} \\
&y_i + (1 - v_{1s,s'}) + W_{1s,s',i} \geq 1 \\
&y_i + (1 - v_{2s,s'}) + W_{2s,s',i} \geq 1 \\
&(1 - y_i) + (1 - W_{1s,s',i}) \geq 1 \\
&(1 - y_i) + (1 - W_{2s,s',i}) \geq 1 \\
&v_{1s,s'} + 1 - W_{1s,s',i} \geq 1 \\
&v_{2s,s'} + 1 - W_{2s,s',i} \geq 1
\end{aligned}\right\} \forall (s,s') \in P_{s,s'},\ \forall i \in OBJ \\
&\quad y_i \in \{0,1\}
\end{aligned} \tag{Model 3}$$

It is worth noting that it is not necessary to calculate the minimum subset of objectives (Model 1) previously to running the k-EMOSS problem (Model 3). However, it may be useful to firstly take care of the redundant objectives and start the pruning from the results of the first model. We must be careful when doing this, since we are pruning possible combinations that may be better. For example, in a scenario with 4 objectives, maybe i_3 is redundant with i_1, thus we prune i_1 (for example). When now we check the δ-error for two objectives, if i_3 is present, this may result in a greater error than if we had pruned i_3 and maintained i_1. It is only advisable to use for the third model the remaining objectives from the first one when the number of objectives is substantially large and by doing this we will eliminate an important part of them. If not, it is recommended to run the third model directly in order to see the full spectrum of solutions.

This model could also be simply used to calculate δ for a fixed and known set of objectives that it is wished to maintain, but then, instead of dealing with an optimization problem, we simply end up with an algebraic problem by previously fixing equal to one the binary variables of those objectives that we want to keep.

This third model is further illustrated with Example 2.

4.3.1 Example 2: k-EMOSS problem

This example intends to clarify the details of the third model. The data chosen to illustrate how the model works are already normalized and shown in Table 4.

Its dominance structure is illustrated in Fig. 4. This particular data has been chosen because we cannot prune any objective to strictly maintain the dominance structure of the pairings. Since this statement is not so straightforward, the first model is applied to assess. The parameters obtained for the first model are shown in Table 5.

< Fig.4 >

Fig. 4: Dominance structure of the second example case

It becomes obvious from the data in Table 5 that a larger number of solutions under consideration will increase the total amount of pairs to be studied, which as a consequence will also raise the number of binary variables present in the third model. First model's constraints result in a considerable number of equations to being expanded here, but from a quick inspection to Table 5 and parameter $C_{1,s,s',i}$, we can withdraw the following conclusions: the pair s_1-s_2 indicates that we will need to maintain i_3; the pair s_1-s_3

indicates to retain i_4; for the pair s_1-s_4, i_1 and for the pair s_4-s_5, i_2. Thus, in order to maintain the dominance structure completely we cannot remove any of the objectives, i.e., the first model reveals that it is necessary to keep the four objectives in order to achieve δ=0.

Applying now the k-MOOS model, we obtain that if the intention is to diminish the number of objectives to 3, we would be incurring in an error of 66.7% if we remove i_2, and a 100% error if we remove any of the others. This is illustrated with the penalty parameters, $P_{1,s,s',i}$ and $P_{2,s,s',i}$, shown in Table 6.

From the constraints and logical relations of the third model, it is forced that the pseudo-binary variable $W_{1s4,s5,i2}$ adopts the value 1 in order to fulfill the model, which provides a maximum approximation error (and unique error in this case) of 66.7%. Up to now we have seen both models separately. In example 3, a more comprehensive problem is shown where both models are tested.

4.3.2 Example 3: Finding a minimum subset problem, δ-MOOS problem, k-EMOSS problem

The second example case was already too big to show the whole explicit model equations. The number of variables grows very fast with the number of solutions because of the increasing number of pairings. For the last example, we have considered a bigger example with again 5 solutions but 9 objectives. The data is shown in Table 7.

The dominance structure of the third example case is shown in Fig. *5*.

< Fig.5 >

Fig. 5: Dominance structure of the third example case

Due to the amount of data contained in this example, it is not straightforward to distinguish which objectives are redundant, hence the utility of the methodologies described in this work. Using the first model it is possible to conclude that 5 out of the original 9 objectives are redundant and that the remaining 4 can completely maintain the dominance structure. These 4 objectives are, according to Model 1, i_1, i_2, i_4 and i_8.

By using the third model, we can obtain the error that will be introduced when we intentionally want to eliminate one or two extra objectives from the minimum 4 obtained with model 1. These results are shown in Table 8. For this small example, all 9 objectives were taken into consideration as original data for the third model instead of using just those corresponding to the 4 objectives selected by Model 1. The results obtained with the third model when the number of pruned objectives are between one and five provide, as expected, a δ error of zero. Thus it is proved that those five pruned objectives were redundant. Again, it is worth noting that when applying the third model, the pruned objectives resultant from the first model (i3, i5, i6 and i7) are still under consideration as original data. This is apparent in many of the rows shown in Table 8. We cannot say that if we lower the number of objectives the best results will still be achieved starting the third model with the non-pruned ones. As explained previously, it is only recommended to prune initial data with Model 1 if the amount of data is too extensive and many objectives are found redundant, so to ease the calculation for the third model.

As stated before, the third model only gives a solution for each number of remaining objectives (rows #1 and #5). In order to obtain the rest of the solutions (rows #2, 3, 4, 6, 7 and 8) for that number of pruned objectives we introduced a binary cut (canonical cut) in each iteration.

Knowing the results from Model 3, we can check if Model 2 produces the same results. These are shown in Table 9. The results are consistent with those of Model 3. It is important to remark that it is impossible to know the exact δ-error from Model 2, since it will only fulfill the constraints and not calculate it. Thus, we cannot be sure that the results from Model 2 for a set δ-error provide the minimum δ for the amount of objectives resultant.

5. Case Studies

So far pure mathematical examples have been presented to demonstrate the use of the two Models described above. The intention now, is to use these Models for two real problems with physical meaning that have already been solved in literature. In what follows, we will compare our results to those already published~~.~~

5.1 First case study

In this case study, we use the data shown in Table 10, obtained from an article with real physical meaning (Guillén-Gosálbez, 2011). In that article, the objective is to optimize a petrochemical Supply Chain taking into account not only the economic objective, but the environmental one as well.

This data was employed by the mentioned authors but using another algorithm for objective reduction. The aim in this optimization is to maximize the economic objective while minimizing the rest, which are environmental ones.

The dominance structure of the data contained in Table 10 is shown in Fig. 6. It is noticeable that the first objective, the economic one (NPV), goes against all the others, since the intention is to maximize this objective while minimizing the rest, those associated to the environmental impact. However, even though the environmental impact objectives seem to operate in a very similar way, it is difficult to pinpoint just by examining them, which of them are redundant.

< Fig.6 >

Fig. 6: Dominance Structure for the data used in Case Study 1 (Table 10)

We apply to this data the first model described in this work but we add a binary cut strategy with the intention of obtaining all the options of the objectives that have to remain under consideration in order to completely maintain the dominance structure. The results are shown in Table 11. Thus, with any of the two combinations shown in the Table, the dominance structure would be maintained. The model is very fast, with a CPU time << 1 s.

Similarly to the previous example, we will study now in how much error do we incur from pruning more objectives. Again, a binary cut strategy will be applied to obtain the full range of solutions. These results are shown in Table 12.

While our results differ in scale from the ones shown by (Guillén-Gosálbez, 2011), the choice of the eliminated objectives remains the same. The change in scale can be explained by the normalization method used.

5.2 Second case study

For the second case, a real example that will test the limits of our model is chosen. A sub-space of 200 Pareto solutions out of the original 7776 solutions with 12 objectives from (Carreras et al., 2016) is used as original data to test our models. In the original problem, the objective was to optimize the construction of a building (cubicle) in regard to the economic and environmental objectives.

A plot of 200 solutions would not be illustrative. Therefore, a sample of sixteen random solutions (out of the 200 chosen) have been normalized and plot in Fig. 7.

< Fig.7 >

Fig. 7: Normalized data for 16 of the 200 Pareto solutions

Given the high number of pairs of solutions present in the second case study, the CPU time escalates as compared to the previous problems shown.

For this reason, our Model 3 was modified. The binary variables $v_{1,s,s'}$ and $v_{2,s,s'}$ were changed to continuous variables bounded between 0 and 1. The logical constraints employed guarantee that when they are active, their value tops at a maximum of 1, acting as a binary, but there is nothing that forces them to adopt a value of zero when required. For this reason, it was necessary to introduce a change in the objective function as shown in eq(19), where λ is a penalty parameter multiplied by a factor F that is also described in eq(19). This new term ensures that when the intention is to minimize the objective function, if those variables are not necessary, their value drop to 0. The value of λ must be carefully chosen since the lower it is, the larger the time required for reaching the solution, but if it is given a high value, the model loses precision. Let's see an example of the latest assertion: suppose two solutions, A and B. Solution A exhibits a value of δ=0.2, meanwhile solution B displays the value δ = 0.15. In this case clearly B would be the preferred solution. But in the case where λ is given the value of 0.1 and if F for solution A is 1 and 2 for solution B, we would end up with an objective function that would have the value of z=0.3 for A and z = 0.35 for B leading us to choose A before B. Therefore, for our results, a value of λ = 0.001 was considered, which would in the worst cases move in a rank of error of 0.1 – 0.2 % between the minimum maximum error and z.

$$\begin{aligned} z &= \delta + \lambda \cdot F \\ F &= \sum_{P} (V_{1s,s'} + V_{2s,s'}) \end{aligned} \tag{19}$$

It is important to note that if eq(19) is used, the results must be seriously verified. If many of $V_{1,s,s'}$ and $V_{2,s,s'}$ take a value of 1, this method is not effective since it is possible to incur in a significant error. Thus, for those cases, the previous model with the binaries and eq(18) for the objective function must be used. The w can be put as binaries in this case in order to make sure that no different values than zero or one are being selected for them. The reduction in calculation time is still notable.

The study was repeated a second time with a different group of 200 random solutions out of the 7776 possible. The results for the first sample of 200 random Pareto solutions can be seen in Table 13. The models were run considering two scenarios. A first one where the aggregated objective EI – 99 was considered as one objective more, despite being an accumulation of the previous environmental objectives, and a second one where that objective was removed and the models were run with the other 11 objectives. The results of the second sample of 200 random solutions are presented in Table 14. Comparing the results obtained for the first model in Tables 12 and 13, it can be concluded that the second sample of 200 solutions gave the same results than with the first random sample, with only a small difference in the CPU time. Now, the results are not so similar when we compare the results in both Tables for the third model: while both sample's results agreed on the maintained objectives, they disagree in the size of the minimum maximum error. This can be attributed to the fact that an infinite norm is heavily influenced by the data, and by the effect of normalization. Thus, between those two different samples, the maximum difference when maintaining the resultant objectives was quite different Despite that difference in the size of the error, since both samples agreed on the objectives to maintain, it can be considered that those 200 points of data represent in a good measure the 7776 original Pareto points. Also, the minimum number of resulting objectives are in agreement with those of (Carreras et al., 2016) and provides all of the zero error subsets of objectives calculated by them, plus more that our model finds.

In order to show the difference in the difficulty of calculation between the two models, and thus the difference in the calculation time, we can look at Table 15. In the same way as in the examples before, we ran the second model with the parameters of the third model in order to check the results since those are the same that were shown in the previous tables, we only check the difference in calculation time. The statistics are shown also in Table 15.

All the results of this paper were obtained with a PC using Windows 7 Professional 64-bits as OS, with an Intel® Core ™ i7-4790 CPU of 3.60 GHz and a RAM of 8 Gb. The optimization was performed in GAMS using Cplex as solver for the MILP problems.

6. Conclusions

Three different MILP models to reduce objectives in the context of multi-objective optimization are presented. Those three models answer the following problems;

1. Minimum subset of objectives with relation to the original set of objectives. Thus, it eliminates only the redundant objectives that do not break at all the dominance structure of the problem.
2. δ-minimum subset of objectives with relation to the original set of objectives. Given a maximum δ-error, it gives the minimum amount of objectives that does not break the dominance structure with more error than that limit.
3. Minimum δ-error for a subset of objectives of a determined cardinality. Given a number of objectives (cardinality), it gives which must be selected in order to minimize the δ-error of the resultant subset.

From the results it is clear that the third model needs much more computational time than the previous two. This is due to the fact that it has to fix the summation of objectives to a determined number, which is much more restrictive than giving a maximum to the δ-error, like in the second model. For this, it must be studied in each particular case what could be the best methodology to follow. For small problems (Number of solutions < 100, Number of objectives < 12), while it is true that the third model is slower, it provides the result in a small window of time. For bigger problems it could be better to perform a parametric study with the second model varying the δ_{MAX} parameter, which provides a much faster result, as seen in Table 15.

This may raise the question of why bother to resolve the k-EMOSS problem. The reason is that it tends to be more useful to fix a number of objectives and know which are needed to choose in order to minimize the δ-error. The third model provides the user with that answer, which is in the end the objective of the philosophy of reducing objectives, to minimize the error. Besides, as mentioned previously, the second model does not allow the user to know the error incurred when eliminating the objectives. It only allows to bind it to a certain upper valor.

While it is not in any way the objective of the paper, it is worth noting again that the normalization of the solutions is a critical step in the methodology. The way the approximation error δ is calculated produces that if the range of one objective is much larger than the others will incur in the method always trying to not eliminate that objective. In this paper, the chosen normalization is done by bounding the values of the solutions provided for each objective with the highest and the lowest values in the space of solutions for that objective. It is therefore highly recommended that if the upper and lower real bounds for any objective are known, despite they not being in the space of solution, to use those values instead of the maximum and the minimum in the space of solutions under study.

Acknowledgments

The authors acknowledge financial support from the Spanish “Ministerio de Economía, Industria y Competitividad” (CTQ2016-77968-C3-2-P, AEI/FEDER, UE).

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Figure Captions

Fig.1: *a) Example of Pareto chart b) Parallel coordinate plot of the same data*

Fig.2: *Dominance structure for 2 hypothetical solutions, A and B, in 3 objectives;* i_1, i_2, i_3. *Both solutions are Pareto Optimal in the original set (a) and in the reduced set 2 (c), while B completely dominates A in the reduced set 1 (b)*

Fig.3: Dominance structure of the first example case

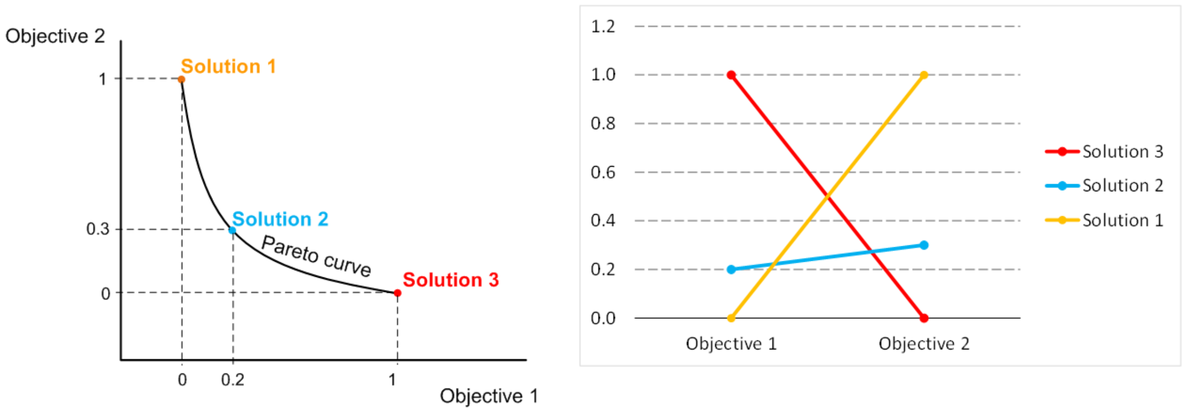
Fig.4: Dominance structure of the second example case

Fig.5: Dominance structure of the third example case

Fig.6: Dominance Structure for the data used in Case Study 1 (Table 9)

Fig.7: Normalized data for 16 of the 200 Pareto solutions

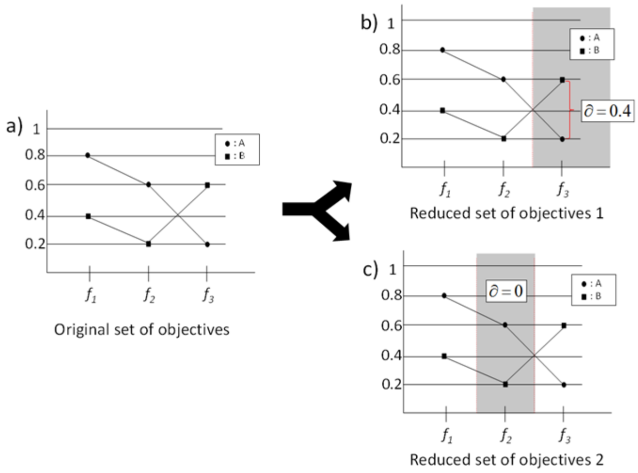
Figr-1


Objective 2
Solution 1
1
0.3
Solution 2
Pareto curve
Solution 3
0
0
0.2
1
Objective 1
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Objective 1
Objective 2
Solution 3
Solution 2
Solution 1

a)

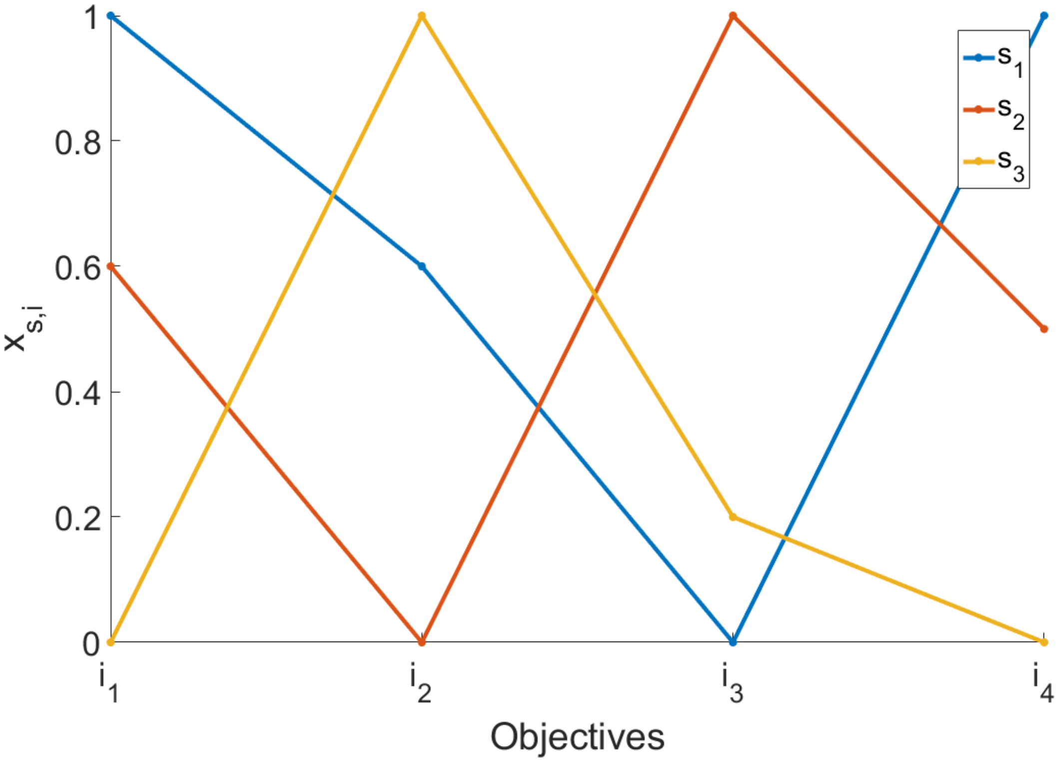
b)

Figr-2

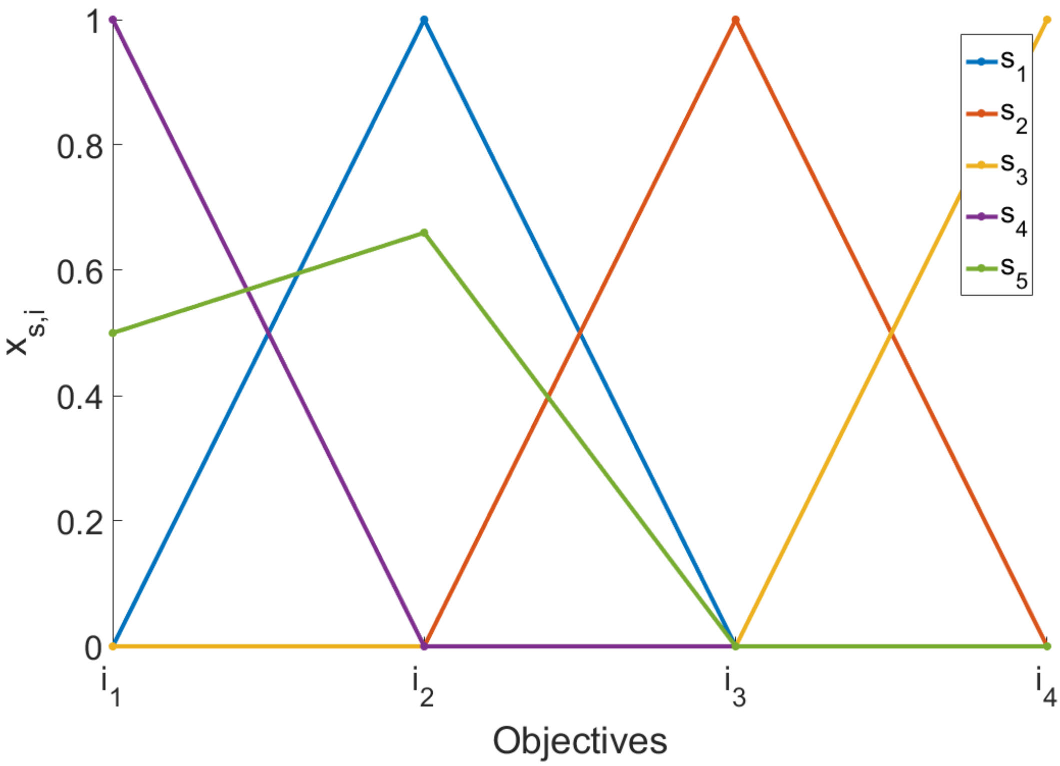


a)
1
0.8
0.6
0.4
0.2
• : A
▪ : B
f_1
f_2
f_3
Original set of objectives
b)
1
0.8
0.6
0.4
0.2
• : A
▪ : B
$\partial = 0.4$
f_1
f_2
f_3
Reduced set of objectives 1
c)
1
0.8
0.6
0.4
0.2
$\partial = 0$
• : A
▪ : B
f_1
f_2
f_3
Reduced set of objectives 2

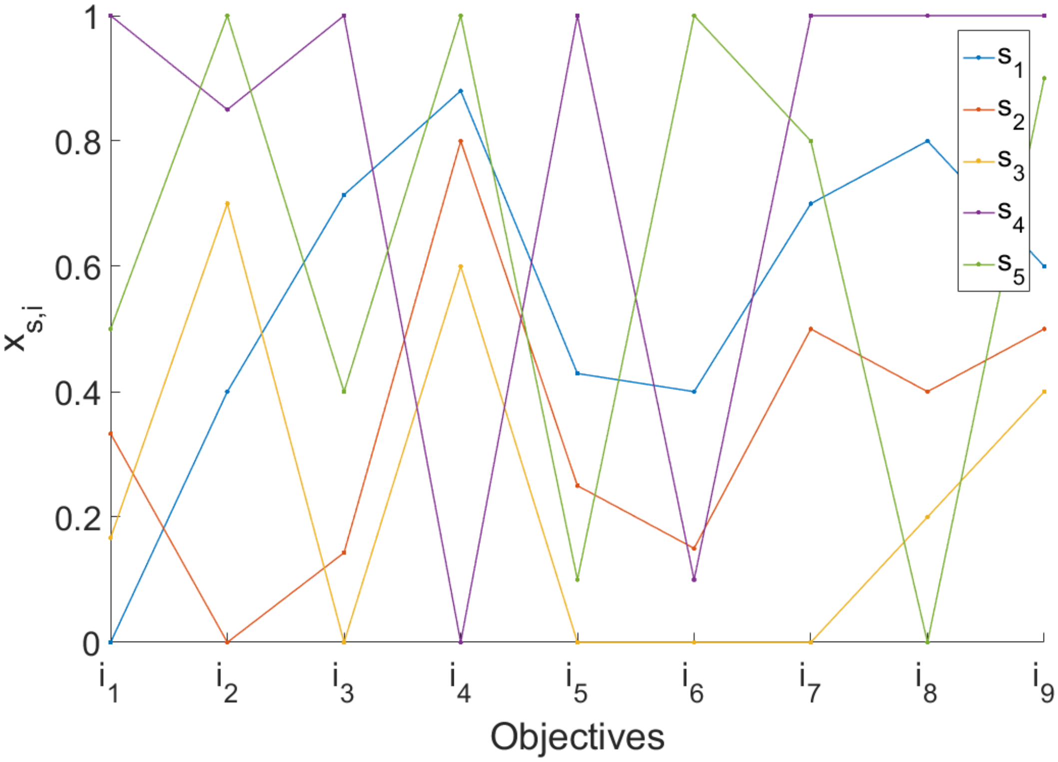
Figr-3



Figr-4

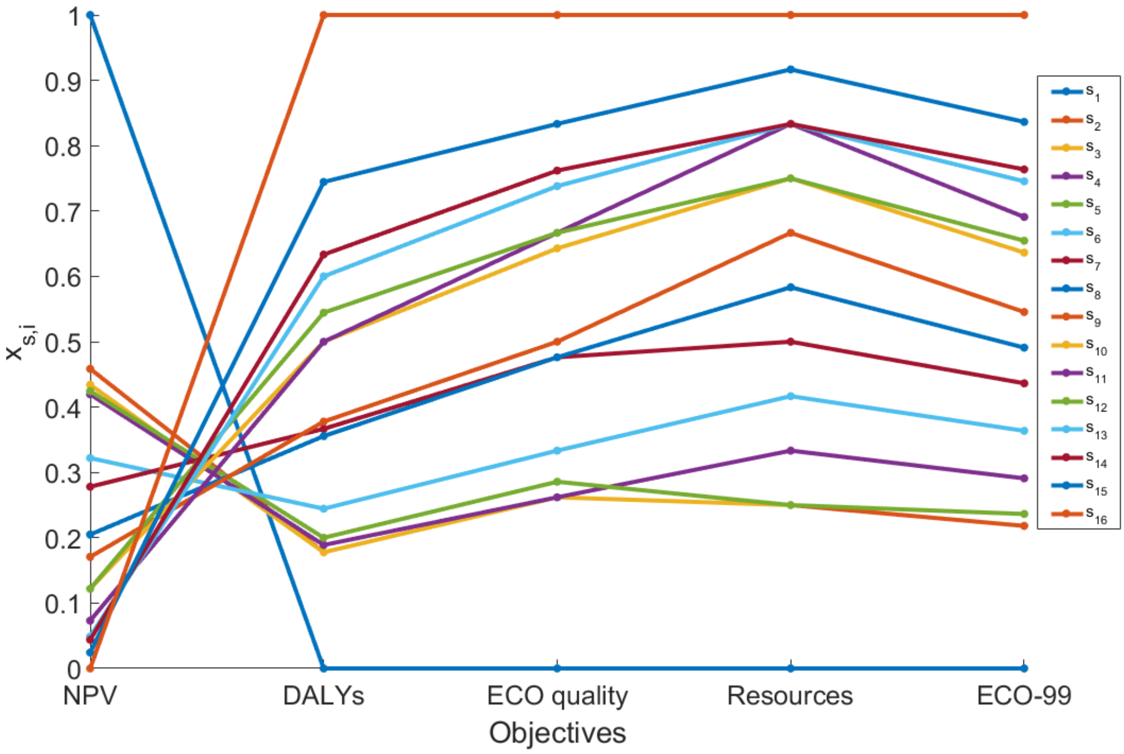


Figr-5



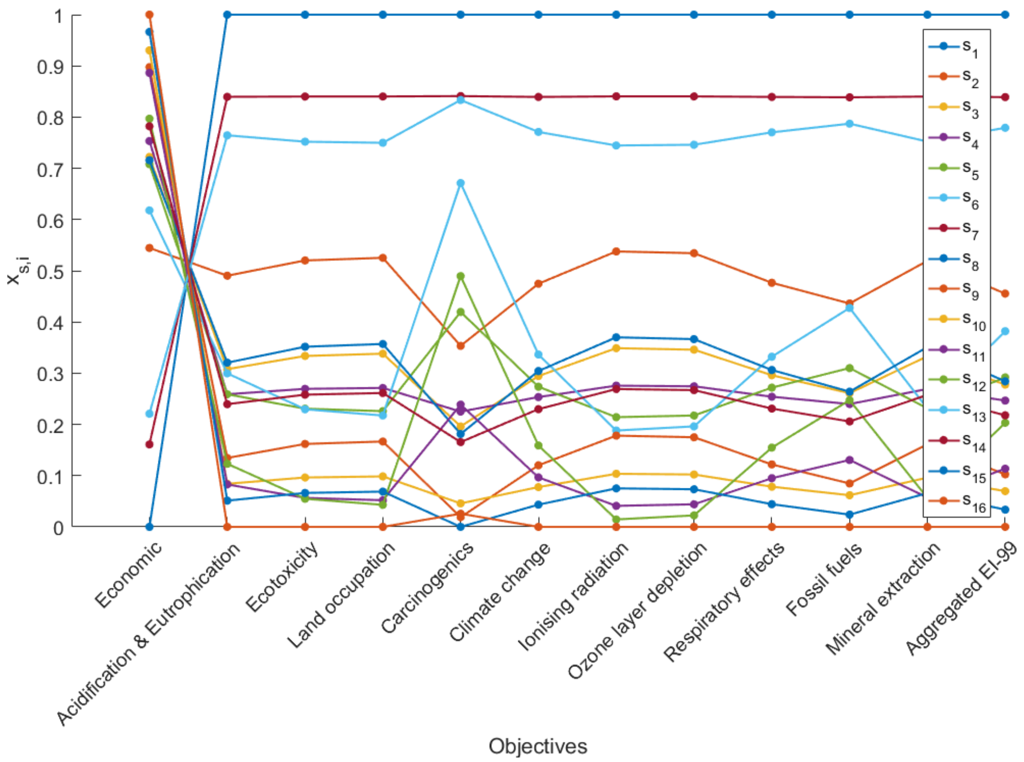
1
0.8
0.6
0.4
0.2
0
$x_{s,i}$
i_1
i_2
i_3
i_4
i_5
i_6
i_7
i_8
i_9
Objectives
s_1
s_2
s_3
s_4
s_5

Figr-6



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$x_{s,i}$
NPV
DALYs
ECO quality
Resources
ECO-99
Objectives
s_1
s_2
s_3
s_4
s_5
s_6
s_7
s_8
s_9
s_{10}
s_{11}
s_{12}
s_{13}
s_{14}
s_{15}
s_{16}

Figr-7



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$x_{s,i}$
s_1
s_2
s_3
s_4
s_5
s_6
s_7
s_8
s_9
s_{10}
s_{11}
s_{12}
s_{13}
s_{14}
s_{15}
s_{16}
Economic
Acidification & Eutrophication
Ecotoxicity
Land occupation
Carcinogenics
Climate change
Ionising radiation
Ozone layer depletion
Respiratory effects
Fossil fuels
Mineral extraction
Aggregated EI-99
Objectives

Table 1: Data for the first example with 4 objectives and 3 solutions

$x_{s,i}^{original}$	i_1	i_2	i_3	i_4
s_1	100	600	30.0	27.8
s_2	60.0	0.00	80.0	20.5
s_3	0.00	1000	40.0	13.2

Table 2: Normalized data for the first example shown in Table 1

$x_{s,i}$	i_1	i_2	i_3	i_4
s_1	1.0	0.6	0.0	1.0
s_2	0.6	0.0	1.0	0.5
s_3	0.0	1.0	0.2	0.0

Table 3: Parameters $C_{1,s,s',i}$ and $C_{2,s,s',i}$ of the first example.

$C_{1,s,s',i}$	i_1	i_2	i_3	i_4
s_1, s_2	0	0	1	0
s_1, s_3	0	1	1	0
s_2, s_3	0	1	0	0
$C_{2,s,s',i}$	i_1	i_2	i_3	i_4
s_1, s_2	1	1	0	1
s_1, s_3	1	0	0	1
s_2, s_3	1	0	1	1

Table 4: Normalized data for example 2 with 4 objectives and 5 solutions

$x_{s,i}$	i_1	i_2	i_3	i_4
s_1	0.00	1.00	0.00	0.00
s_2	0.00	0.00	1.00	0.00
s_3	0.00	0.00	0.00	1.00
s_4	1.00	0.00	0.00	0.00
s_5	0.50	0.66	0.00	0.00

Table 5: Parameters $C_{1,s,s',i}$ and $C_{2,s,s',i}$ of the second example

$C_{1,s,s',i}$	**i_1**	**i_2**	**i_3**	**i_4**
s_1, s_2	0	0	1	0
s_1, s_3	0	0	0	1
s_1, s_4	1	0	0	0
s_1, s_5	1	0	0	0
s_2, s_3	0	0	0	1
s_2, s_4	1	0	0	0
s_2, s_5	1	1	0	0
s_3, s_4	1	0	0	0
s_3, s_5	1	1	0	0
s_4, s_5	0	1	0	0
$C_{2,s,s',i}$	**i_1**	**i_2**	**i_3**	**i_4**
s_1, s_2	1	1	0	1
s_1, s_3	1	1	1	0
s_1, s_4	0	1	1	1
s_1, s_5	0	1	1	1
s_2, s_3	1	1	1	0
s_2, s_4	0	1	1	1
s_2, s_5	0	0	1	1
s_3, s_4	0	1	1	1
s_3, s_5	0	0	1	1
s_4, s_5	1	0	1	1

Table 6: Penalty parameters $P_{1,s,s',I}$ and $P_{2,s,s',i}$ for the second example

$P_{1s,s',i}$	i_1	i_2	i_3	i_4
s_1, s_2	0	0	1	0
s_1, s_3	0	0	0	1
s_1, s_4	1	0	0	0
s_1, s_5	0.5	0	0	0
s_2, s_3	0	0	0	1
s_2, s_4	1	0	0	0
s_2, s_5	0.5	0.67	0	0
s_3, s_4	1	0	0	0
s_3, s_5	0.5	0.67	0	0
s_4, s_5	0	0.67	0	0
$P_{2s,s',i}$	i_1	i_2	i_3	i_4
s_1, s_2	0	1	0	0
s_1, s_3	0	1	0	0
s_1, s_4	0	1	0	0
s_1, s_5	0	0.33	0	0
s_2, s_3	0	0	1	0
s_2, s_4	0	0	1	0
s_2, s_5	0	0	1	0
s_3, s_4	0	0	0	1
s_3, s_5	0	0	0	1
s_4, s_5	0.5	0	0	0

Table 7: Normalized data of example 3 with 9 objectives and 5 solutions

$x_{s,i}$	i_1	i_2	i_3	i_4	i_5
s_1	0.000	0.400	0.714	0.880	0.429
s_2	0.333	0.000	0.143	0.800	0.250
s_3	0.167	0.700	0.000	0.600	0.000
s_4	1.000	0.850	1.000	0.000	1.000
s_5	0.500	1.000	0.400	1.000	0.100

	i_6	i_7	i_8	i_9
s_1	0.400	0.700	0.800	0.600
s_2	0.150	0.500	0.400	0.500
s_3	0.000	0.000	0.200	0.400
s_4	0.100	1.000	1.000	1.000
s_5	1.000	0.800	0.000	0.900

Table 8: δ results for different number of objectives of the third example when a higher value of the maximum number of objectives (according to the first model) are pruned.

Run #	Number of pruned objectives	Remaining objectives	Minimum maximum error δ
1	6	i_2, i4, i8	33.30%
2	6	i2,i4,i5	33.30%

3	6	i2,i3,i4	40.00%
4	6	i2,i6,i8	60.00%
5	7	i4,i5	70.00%
6	7	i4,i8	70.00%
7	7	i3,i4	70.00%
8	7	i6,i8	70.00%

Table 9: Model 2 results for example 3

Run #	δ_{MAX}	Remaining objectives
1	33%	i_1, i_2, i_4, i_8
2	34%	i_2,i_4,i_8
3	71%	i_3,i_4

Table 10: Data of the First Case Study originally used by Guillén-Gosálbez, 2011. The data contains 5 objectives, the first one is tied to the economical measure (NPV) while the other 4 are related to environmental measures. The example contains 16 solutions

SOL	NPV (G$)	Human health (x10^3 DALYs)	Ecosystem quality (x10^8 PDF m^2 year)	Resources depletion (x10^{10} MJ)	Eco-indicator 99 (x10^8 points)
s1	1.04	7.00	4.24	1.57	5.88
s2	1.15	7.16	4.35	1.60	6.00
s3	1.15	7.16	4.35	1.60	6.01
s4	1.16	7.17	4.35	1.61	6.04
s5	1.16	7.18	4.36	1.60	6.01
s6	1.18	7.22	4.38	1.62	6.08
s7	1.19	7.33	4.44	1.63	6.12
s8	1.20	7.32	4.44	1.64	6.15
s9	1.21	7.34	4.45	1.65	6.18
s10	1.22	7.45	4.51	1.66	6.23
s11	1.23	7.45	4.52	1.67	6.26
s12	1.22	7.49	4.52	1.66	6.24
s13	1.23	7.54	4.55	1.67	6.29
s14	1.23	7.57	4.56	1.67	6.30
s15	1.24	7.67	4.59	1.68	6.34
s16	1.24	7.90	4.66	1.69	6.43

Table 11: Results of the first model applied to case study 1 which shows the redundant number of objectives (2 in this example)

Remaining objectives	Minimum maximum error δ
NPV, DALYs, ECO-99	0%
NPV, Ecosystem quality, ECO-99	0%

Table 12: Results of the third model applied to case study 1 which shows the error produced by pruning more objectives than 2.

Remaining objectives	Minimum maximum error δ	CPU time (s)
NPV, ECO-99	2.38%	0.22
NPV, DALYs	8.30%	0.24
NPV, ECO quality	8.30%	0.25
NPV, Resources	13.33%	0.15
Rest of combinations	100%	-

Table 13: Results of the first 200 solutions sample of the Second case study. Two different scenarios were considered: including or not the aggregated objective EI-99.

With aggregated EI-99		
MODEL 1 : Non-redundant objectives: 3	**Error δ**	**CPU time(s)**
Economic, Acidification & Eutrophication, Carcinogenics	0.00%	0.078
Economic, Fossil fuels, Aggregated EI-99	0.00%	0.093
Economic, Carcinogenics, Mineral extraction	0.00%	0.078
Economic, Carcinogenics, Respiratory effects	0.00%	0.093
Economic, Carcinogenics, Ozone layer depletion	0.00%	0.093
Economic, Carcinogenics, Ionizing radiation	0.00%	0.078
Economics, Carcinogenics, Climate change	0.00%	0.078
Economics, Land occupation, Carcinogenics	0.00%	0.093
Economics, Ecotoxicity, Carcinogenics	0.00%	0.093
Economics, Acidification & Eutrophication, Carcinogenics	0.00%	0.171
Economic, Ecotoxicty, Fossil fuels	0.00%	0.093
Economic, Land occupation, Fossil fuels	0.00%	0.078
Economic, Ionising radiation, Fossil fuels	0.00%	0.093
Economic, Respiratory effects, Fossil fuels	0.00%	0.107
Economic, Fossil fuels, Mineral extraction	0.00%	0.093
Economic, Climate change, Fossil fuels	0.00%	0.093
Economic, Carcinogenics, Aggregated EI-99	0.00%	0.078
Economic, Ozone layer depletion, Fossil fuels	0.00%	0.109
MODEL 3 : Remaining Objectives: 2	**Error δ**	**CPU time (s)**
Economic, Aggregated EI-99	24.19%	4,032
Without aggregated EI-99		
MODEL 1 : Non-redundant objectives: 3	**Error δ**	**CPU time(s)**
Economic, Acidification & Eutrophication, Carcinogenics	0.00%	0.094
Economic, Fossil fuels, Mineral extraction	0.00%	0.093
Economic, Carcinogenics, Respiratory effects	0.00%	0.093
Economic, Carcinogenics, Ozone layer depletion	0.00%	0.093
Economic, Carcinogenics, Ionising radiation	0.00%	0.171
Economic, Carcinogenics, Climate change	0.00%	0.078
Economic, Land occupation, Carcinogenics	0.00%	0.171
Economic, Ecotoxicity, Carcinogenics	0.00%	0.093
Economic, Acidification & Eutrophication, Fossil fuels	0.00%	0.093

Economic, Ecotoxicity, Fossil fuels	0.00%	0.093
Economic, Land occupation, Fossil fuels	0.00%	0.078
Economic, Ionising radiation, Fossil fuels	0.00%	0.094
Economic, Respiratory effects, Fossil fuels	0.00%	0.078
Economic, Climate change, Fossil fuels	0.00%	0.093
Economic, Carcinogenics, Mineral extraction	0.00%	0.078
Economic, Ozone layer depletion, Fossil fuels	0.00%	0.093
MODEL 3 : Remaining Objectives: 2	**Error δ**	**CPU time (s)**
Economic, Fossil fuels	60.76%	5,098

Table 14: Results of the second 200 solutions sample of the Second case study. As in Table 13, two different scenarios were considered: including or not the aggregated objective EI-99.

With aggregated EI-99		
MODEL 1 : Non-redundant objectives: 3	**Error δ**	**CPU time(s)**
Economic, Acidification & Eutrophication, Carcinogenics	0.00%	0.088
Economic, Fossil fuels, Aggregated EI-99	0.00%	0.094
Economic, Carcinogenics, Mineral extraction	0.00%	0.074
Economic, Carcinogenics, Respiratory effects	0.00%	0.066
Economic, Carcinogenics, Ozone layer depletion	0.00%	0.088
Economic, Carcinogenics, Ionizing radiation	0.00%	0.098
Economics, Carcinogenics, Climate change	0.00%	0.079
Economics, Land occupation, Carcinogenics	0.00%	0.095
Economics, Ecotoxicity, Carcinogenics	0.00%	0.090
Economics, Acidification & Eutrophication, Carcinogenics	0.00%	0.177
Economic, Ecotoxicty, Fossil fuels	0.00%	0.096
Economic, Land occupation, Fossil fuels	0.00%	0.079
Economic, Ionising radiation, Fossil fuels	0.00%	0.083
Economic, Respiratory effects, Fossil fuels	0.00%	0.113
Economic, Fossil fuels, Mineral extraction	0.00%	0.083
Economic, Climate change, Fossil fuels	0.00%	0.073
Economic, Carcinogenics, Aggregated EI-99	0.00%	0.088
Economic, Ozone layer depletion, Fossil fuels	0.00%	0.099
MODEL 3 : Remaining Objectives: 2	**Error δ**	**CPU time (s)**
Economic, Aggregated EI-99	53.87%	4,192
Without aggregated EI-99		
MODEL 1 : Non-redundant objectives: 3	**Error δ**	**CPU time(s)**
Economic, Acidification & Eutrophication, Carcinogenics	0.00%	0.094
Economic, Fossil fuels, Mineral extraction	0.00%	0.093
Economic, Carcinogenics, Respiratory effects	0.00%	0.093
Economic, Carcinogenics, Ozone layer depletion	0.00%	0.093
Economic, Carcinogenics, Ionising radiation	0.00%	0.171
Economic, Carcinogenics, Climate change	0.00%	0.078
Economic, Land occupation, Carcinogenics	0.00%	0.171
Economic, Ecotoxicity, Carcinogenics	0.00%	0.093

Economic, Acidification & Eutrophication, Fossil fuels	0.00%	0.093
Economic, Ecotoxicity, Fossil fuels	0.00%	0.093
Economic, Land occupation, Fossil fuels	0.00%	0.078
Economic, Ionising radiation, Fossil fuels	0.00%	0.094
Economic, Respiratory effects, Fossil fuels	0.00%	0.078
Economic, Climate change, Fossil fuels	0.00%	0.093
Economic, Carcinogenics, Mineral extraction	0.00%	0.078
Economic, Ozone layer depletion, Fossil fuels	0.00%	0.093
MODEL 3 : Remaining Objectives: 2	**Error δ**	**CPU time (s)**
Economic, Fossil fuels	55.90%	5,718

Table 15: GAMS' Model Statistics of the performed optimizations

Model 1: Minimum subset of objectives problem			
Blocks of equations :	4	Single equations :	39,801
Blocks of variables :	2	Single variables :	13
Non-zero elements :	238,813	Discrete variables :	12
Time (s) :	0.078		
Model 2: δ-MOOS problem			
Blocks of equations :	11	Single equations :	1,950,201
Blocks of variables :	7	Single variables :	517,414
Non-zero elements :	4,338,213	Discrete variables :	39,812
Time (s) :	63.320		
Model 3: k-EMOSS problem			
Blocks of equations :	12	Single equations :	1,950,202
Blocks of variables :	7	Single variables :	517,414
Non-zero elements :	4,378,014	Discrete variables :	477,612
Time (s) :	4032		