Classes of Ordinary Differential Equations Obtained for the Probability Functions of Frĕchet Distribution

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***Abstract*— In this paper, the differential calculus was used to obtain some classes of ordinary differential equations (ODE) for the probability density function, quantile function, survival function, inverse survival function, hazard function and reversed hazard function of the Frĕchet distribution. The stated necessary conditions required for the existence of the ODEs are consistent with the various parameters that defined the distribution. Solutions of these ODEs by using numerous available methods are a new ways of understanding the nature of the probability functions that characterize the distribution. The method can be extended to other probability distributions and can serve an alternative to approximation.**

***Index Terms*— Quantile function, Frĕchet distribution, reversed hazard function, calculus, differentiation, probability density function.**

I. Introduction

FRÊCHET distribution is one of the mostly applied distributions in extreme value theory. Detailed information about the distribution can be obtained from the early works of [1] and [2] and book written by [3]. Different methods of estimation of the parameters of the distribution were discussed extensively by [4]. Some of the applications are as follows: Zaharim et al. [5] used the distribution to model wind speed data, Harlow [6] worked on the usefulness of the distribution in modeling engineering problems, Nadarajah and Kotz [7] discussed extensively on the application of the Fréchet random variables to sociological models. Vovoras and Tsokos [8] used the distribution to model and analyze precipitation data. Details on the application of the distribution in modeling extremal data can be found in [9].

Many authors and researchers have proposed modifications or developed generalizations of the distribution. Some of them are: beta Fréchet distribution [10] [11], Kumaraswamy Fréchet distribution [12], transmuted Fréchet distribution [13], transmuted exponentiated Fréchet distribution [14], gamma extended Fréchet distribution [15], Marshall–Olkin Fréchet distribution [16], transmuted Marshall–Olkin Fréchet distribution [17], Weibull Fréchet distribution [18], six-parameter Fréchet distribution [19], beta exponential Fréchet distribution [20]

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The aim of this research is to develop ordinary differential equations (ODE) for the probability density function (PDF), Quantile function (QF), survival function (SF), inverse survival function (ISF), hazard function (HF) and reversed hazard function (RHF) of Fréchet distribution by the use of differential calculus. Calculus is a very key tool in the determination of mode of a given probability distribution and in estimation of parameters of probability distributions, amongst other uses. The research is an extension of the ODE to other probability functions other than the PDF. Similar works done where the PDF of probability distributions was expressed as ODE whose solution is the PDF are available. They include: Laplace distribution [21], beta distribution [22], raised cosine distribution [23], Lomax distribution [24], beta prime distribution or inverted beta distribution [25].

II. Probability Density Function

The probability density function of the Frêchet distribution is given as;

$$f(x)=\beta\alpha^{\beta}x^{-(\beta+1)}\,\mathrm{e}^{-\left(\frac{\alpha}{x}\right)^{\beta}} \quad (1)$$

To obtain the first order ordinary differential equation for the probability density function of the Frêchet distribution, differentiate equation (1), to obtain;

$$f'(x)=\left\{-\frac{(\beta+1)x^{-(\beta+2)}}{x^{-(\beta+1)}}+\frac{\frac{\alpha\beta}{x^2}\left(\frac{\alpha}{x}\right)^{\beta-1}\mathrm{e}^{-\left(\frac{\alpha}{x}\right)^{\beta}}}{\mathrm{e}^{-\left(\frac{\alpha}{x}\right)^{\beta}}}\right\}f(x) \quad (2)$$

$$f'(x)=\left\{-\frac{(\beta+1)}{x}+\frac{\alpha\beta}{x^2}\left(\frac{\alpha}{x}\right)^{\beta-1}\right\}f(x) \quad (3)$$

The condition necessary for the existence of equation is $\alpha,\beta,x>0$.

The first order ordinary differential equations can be obtained for the particular values of the parameters. The few cases are summarized in the **Table 1.**

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Table 1: Classes of differential equations obtained for the probability density function of the Frêchet distribution for different parameters.

α	β	ordinary differential equation
1	1	$x^2 f'(x)+(2x-1)f(x)=0$
1	2	$x^3 f'(x)+(3x^2-2)f(x)=0$
2	1	$x^2 f'(x)+(2x-2)f(x)=0$
2	2	$x^3 f'(x)+(3x^2-8)f(x)=0$

Equation (3) is differentiated in an attempt to obtain ordinary differential equations that are not dependent on the powers of the parameters.

$$f''(x)=\left\{\begin{aligned}&\frac{(\beta+1)}{x^2}-\frac{\alpha^2\beta(\beta-1)}{x^4}\left(\frac{\alpha}{x}\right)^{\beta-2}\\&-\frac{2\alpha\beta}{x^3}\left(\frac{\alpha}{x}\right)^{\beta-1}\end{aligned}\right\}f(x)$$
$$+\left\{-\frac{(\beta+1)}{x}+\frac{\alpha\beta}{x^2}\left(\frac{\alpha}{x}\right)^{\beta-1}\right\}f'(x) \quad (4)$$

The condition necessary for the existence of equation (4) is $\alpha,\beta,x>0$.

The following equations obtained from equation (3) are needed in the simplification of equation (4);

$$\frac{f'(x)}{f(x)}=-\frac{(\beta+1)}{x}+\frac{\alpha\beta}{x^2}\left(\frac{\alpha}{x}\right)^{\beta-1} \quad (5)$$

$$\frac{\alpha\beta}{x^2}\left(\frac{\alpha}{x}\right)^{\beta-1}=\frac{f'(x)}{f(x)}+\frac{\beta+1}{x} \quad (6)$$

$$\frac{2\alpha\beta}{x^2}\left(\frac{\alpha}{x}\right)^{\beta-1}=2\left(\frac{f'(x)}{f(x)}+\frac{\beta+1}{x}\right) \quad (7)$$

$$\frac{2\alpha\beta}{x^3}\left(\frac{\alpha}{x}\right)^{\beta-1}=\frac{2}{x}\left(\frac{f'(x)}{f(x)}+\frac{\beta+1}{x}\right) \quad (8)$$

$$\frac{\alpha^2\beta(\beta-1)}{x^2}\left(\frac{\alpha}{x}\right)^{\beta-1}=\alpha(\beta-1)\left(\frac{f'(x)}{f(x)}+\frac{\beta+1}{x}\right) \quad (9)$$

$$\frac{\alpha^2\beta(\beta-1)}{x^4}\left(\frac{\alpha}{x}\right)^{\beta-1}=\frac{\alpha(\beta-1)}{x^2}\left(\frac{f'(x)}{f(x)}+\frac{\beta+1}{x}\right) \quad (10)$$

$$\frac{\alpha^2\beta(\beta-1)}{x^4}\left(\frac{\alpha}{x}\right)^{\beta-2}=\frac{\beta-1}{x}\left(\frac{f'(x)}{f(x)}+\frac{\beta+1}{x}\right) \quad (11)$$

Substitute equations (5), (8) and (11) into equation (4) to obtain;

$$f''(x)=\frac{f'^2(x)}{f(x)}+\left\{\begin{aligned}&\frac{(\beta+1)}{x^2}-\frac{\beta-1}{x}\left(\frac{f'(x)}{f(x)}+\frac{\beta+1}{x}\right)\\&-\frac{2}{x}\left(\frac{f'(x)}{f(x)}+\frac{\beta+1}{x}\right)\end{aligned}\right\}f(x) \quad (12)$$

$$f''(x)=\frac{f'^2(x)}{f(x)}-\frac{(\beta+1)f'(x)}{x}-\frac{\beta(\beta+1)f(x)}{x^2} \quad (13)$$

The second order ordinary differential equation for the probability density function of the Frêchet distribution is given by;

$$\begin{aligned}&x^2 f(x)f''(x)-x^2 f'^2(x)\\&+(\beta+1)xf(x)f'(x)+\beta(\beta+1)f^2(x)=0\end{aligned} \quad (14)$$

$$f(1)=\beta\alpha^\beta \mathrm{e}^{-\alpha^\beta} \quad (15)$$

$$f'(1)=\left(\alpha^\beta\beta-(\beta+1)\right)\beta\alpha^\beta \mathrm{e}^{-\alpha^\beta} \quad (16)$$

III. Quantile Function

The Quantile function of the Frêchet distribution is given as;

$$Q(p)=\frac{\alpha}{\left(\ln\left(\frac{1}{p}\right)\right)^{\frac{1}{\beta}}}$$

(17) To obtain the first order ordinary differential equation for the Quantile function of the Frêchet distribution, differentiate equation (17), to obtain;

$$Q'(p)=-\frac{\alpha}{\beta p\left(\ln\left(\frac{1}{p}\right)\right)^{\frac{1}{\beta}+1}} \quad (18)$$

Substitute equation (17) into (18);

$$Q'(p)=-\frac{Q(p)}{\beta p\left(\ln\left(\frac{1}{p}\right)\right)} \quad (19)$$

The condition necessary for the existence of equation is $\alpha,\beta>0, 0<p<1$.

Equation (17) can also be written as;

$$\ln\left(\frac{1}{p}\right)=\frac{\alpha^\beta}{Q^\beta(p)} \quad (20)$$

Substitute equation (20) into (19);

$$Q'(p)=-\frac{Q(p)Q^\beta(p)}{\beta p\alpha^\beta}=-\frac{Q^{\beta+1}(p)}{\beta p\alpha^\beta} \quad (21)$$

$$\beta p\alpha^\beta Q'(p)+Q^{\beta+1}(p)=0 \quad (22)$$

The first order ordinary differential equations can be obtained for the particular values of the parameters obtained from equation (22). The few cases are summarized in **Table 2**.

Table 2: Classes of differential equations obtained for the quantile function of the Frêchet distribution for different parameters.

α	β	ordinary differential equation
1	1	$pQ'(p)+Q^2(p)=0$
1	2	$2pQ'(p)+Q^2(p)=0$

1	3	$3pQ'(p)+Q^2(p)=0$
2	1	$2pQ'(p)+Q^3(p)=0$
2	2	$8pQ'(p)+Q^3(p)=0$
2	3	$18pQ'(p)+Q^3(p)=0$
3	1	$3pQ'(p)+Q^4(p)=0$
3	2	$24pQ'(p)+Q^4(p)=0$
3	3	$81pQ'(p)+Q^4(p)=0$

Equation (18) is differentiated in an attempt to obtain ordinary differential equations that are not dependent on the powers of the parameters.

$$Q''(p)=\left\{-\frac{\frac{1}{p^2}}{\frac{1}{p}}+\frac{\frac{1+\beta}{\beta p}\left(\ln\left(\frac{1}{p}\right)\right)^{-(\frac{1}{\beta}+2)}}{\left(\ln\left(\frac{1}{p}\right)\right)^{-(\frac{1}{\beta}+1)}}\right\}Q'(p) \quad (23)$$

The condition necessary for the existence of equation is $\alpha,\beta>0, 0<p<1.$

$$Q''(p)=\left\{\frac{1}{p}-\frac{1+\beta}{\beta p\left(\ln\left(\frac{1}{p}\right)\right)}\right\}Q'(p) \quad (24)$$

Equation (19) can also be written as;

$$\frac{Q'(p)}{Q(p)}=-\frac{1}{\beta p\left(\ln\left(\frac{1}{p}\right)\right)} \quad (25)$$

Substitute equation (25) into (24);

$$Q''(p)=\left\{\frac{1}{p}+\frac{(1+\beta)Q'(p)}{Q(p)}\right\}Q'(p) \quad (26)$$

The second order ordinary differential equation for the Quantile function of the Frêchet distribution is given by;

$$pQ(p)Q''(p)-(1+\beta)pQ'^2(p)-Q(p)Q'(p)=0 \quad (27)$$

$$Q(0.1)=\frac{\alpha}{2.3026^{\frac{1}{\beta}}} \quad (28)$$

$$Q'(0.1)=\frac{10\alpha}{\beta(2.3026)^{\frac{1}{\beta}+1}} \quad (29)$$

Some cases can be considered such as:

When $\beta=1,$ equations(27)-(29) become;

$$pQ(p)Q''(p)-2pQ'^2(p)+Q(p)Q'(p)=0 \quad (30)$$

$$Q(0.1)=\frac{\alpha}{2.3026} \quad (31)$$

$$Q'(0.1)=\frac{10\alpha}{(2.3026)^2}=1.8860925\alpha \quad (32)$$

IV. Survival Function

The survival function of the Frêchet distribution is given as;

$$S(t)=1-\mathrm{e}^{-\left(\frac{\alpha}{t}\right)^{\beta}}$$

(33) To obtain the first order ordinary differential equation for the survival function of the Frêchet distribution, differentiate equation (33), to obtain;

$$S'(t)=-\frac{\alpha\beta}{t^2}\left(\frac{\alpha}{t}\right)^{\beta-1}\mathrm{e}^{-\left(\frac{\alpha}{t}\right)^{\beta}} \quad (34)$$

$$S'(t)=-\beta\alpha^{\beta}t^{-(\beta+1)}\mathrm{e}^{-\left(\frac{\alpha}{t}\right)^{\beta}} \quad (35)$$

The condition necessary for the existence of equation is $\alpha,\beta,t>0.$

Equation (33) can also be written as;

$$-\mathrm{e}^{-\left(\frac{\alpha}{t}\right)^{\beta}}=S(t)-1 \quad (36)$$

Substitute equation (36) into equation (35);

$$S'(t)=\beta\alpha^{\beta}t^{-(\beta+1)}(S(t)-1) \quad (37)$$

The first order ordinary differential equations can be obtained for the particular values of the parameters obtained from equation (37). The few cases are summarized in **Table 3.**

Table 3: Classes of differential equations obtained for the survival function of the Frêchet distribution for different parameters.

β	α	Ordinary differential equation
1	1	$t^2S'(t)-S(t)+1=0$
1	2	$t^2S'(t)-2S(t)+2=0$
1	3	$t^2S'(t)-3S(t)+3=0$
2	1	$t^3S'(t)-2S(t)+2=0$
2	2	$t^3S'(t)-8S(t)+8=0$
2	3	$t^3S'(t)-18S(t)+18=0$
3	1	$t^4S'(t)-3S(t)+3=0$
3	2	$t^4S'(t)-24S(t)+24=0$
3	3	$t^4S'(t)-81S(t)+81=0$

Equation (35) is differentiated in an attempt to obtain ordinary differential equations that are not dependent on the powers of the parameters.

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$$S''(t)=\left\{\frac{-(\beta+1)t^{-(\beta+2)}}{t^{-(\beta+1)}}+\frac{\frac{\alpha\beta}{t^2}\left(\frac{\alpha}{t}\right)^{\beta-1}\mathrm{e}^{-\left(\frac{\alpha}{t}\right)^{\beta}}}{\mathrm{e}^{-\left(\frac{\alpha}{t}\right)^{\beta}}}\right\}S'(t) \quad (38)$$

The condition necessary for the existence of equation is $\alpha,\beta,t>0$.

$$S''(t)=\left\{-\frac{(\beta+1)}{t}+\frac{\alpha\beta}{t^2}\left(\frac{\alpha}{t}\right)^{\beta-1}\right\}S'(t) \quad (39)$$

$$S''(t)=\left(-\frac{(\beta+1)}{t}+\frac{\beta\alpha^{\beta}}{t^{\beta+1}}\right)S'(t) \quad (40)$$

Equation (37) can also be written as;

$$\frac{S'(t)}{S(t)-1}=\frac{\beta\alpha^{\beta}}{t^{\beta+1}} \quad (41)$$

Substitute equation (41) into equation (40);

$$S''(t)=\left(\frac{S'(t)}{S(t)-1}-\frac{(\beta+1)}{t}\right)S'(t) \quad (42)$$

The second order ordinary differential equation for the survival function of the Frêchet distribution is given by;

$$t(S(t)-1)S''(t)-tS'^2(t)+(\beta+1)(S(t)-1)S'(t)=0 \quad (43)$$

$$S(1)=1-\mathrm{e}^{-\alpha^{\beta}} \quad (44)$$

$$S'(1)=-\beta\alpha^{\beta}\,\mathrm{e}^{-\alpha^{\beta}} \quad (45)$$

Some cases can be considered such as:

When $\beta=1$, equations(43)-(45) become;

$$t(S(t)-1)S''(t)-tS'^2(t)+2(S(t)-1)S'(t)=0 \quad (46)$$

$$S(1)=1-\mathrm{e}^{-\alpha} \quad (47)$$

$$S'(1)=-\alpha\,\mathrm{e}^{-\alpha} \quad (48)$$

V. Inverse Survival Function

The inverse survival function of the Frêchet distribution is given as;

$$Q(p)=\frac{\alpha}{\left(\ln\left(\frac{1}{1-p}\right)\right)^{\frac{1}{\beta}}} \quad (49)$$

To obtain the first order ordinary differential equation for the inverse survival function of the Frêchet distribution, differentiate equation (49), to obtain;

$$Q'(p)=\frac{\alpha}{\beta(1-p)\left(\ln\left(\frac{1}{1-p}\right)\right)^{\frac{1}{\beta}+1}} \quad (50)$$

Substitute equation (49) into (50);

$$Q'(p)=\frac{Q(p)}{\beta(1-p)\left(\ln\left(\frac{1}{1-p}\right)\right)} \quad (51)$$

The condition necessary for the existence of equation is $\alpha,\beta>0, 0<p<1$.

Equation (49) can also be written as;

$$\ln\left(\frac{1}{1-p}\right)=\frac{\alpha^{\beta}}{Q^{\beta}(p)} \quad (52)$$

Substitute equation (52) into (51);

$$Q'(p)=\frac{Q(p)Q^{\beta}(p)}{\beta\alpha^{\beta}(1-p)}=\frac{Q^{\beta+1}(p)}{\beta\alpha^{\beta}(1-p)} \quad (53)$$

$$\beta\alpha^{\beta}(1-p)Q'(p)-Q^{\beta+1}(p)=0 \quad (54)$$

The first order ordinary differential equations can be obtained for the particular values of the parameters obtained from equation (54). The few cases are summarized in **Table 4.**

Table 4: Classes of differential equations obtained for the inverse survival function of the Frêchet distribution for different parameters .

α	β	ordinary differential equation
1	1	$(1-p)Q'(p)-Q^2(p)=0$
1	2	$2(1-p)Q'(p)-Q^2(p)=0$
1	3	$3(1-p)Q'(p)-Q^2(p)=0$
2	1	$2(1-p)Q'(p)-Q^3(p)=0$
2	2	$8(1-p)Q'(p)-Q^3(p)=0$
2	3	$18(1-p)Q'(p)-Q^3(p)=0$
3	1	$3(1-p)Q'(p)-Q^4(p)=0$
3	2	$24(1-p)Q'(p)-Q^4(p)=0$
3	3	$81(1-p)Q'(p)-Q^4(p)=0$

VI. Hazard Function

The hazard function of the Frêchet distribution is given as;

$$h(t)=\frac{\beta\alpha^{\beta}t^{-(\beta+1)}\,\mathrm{e}^{-\left(\frac{\alpha}{t}\right)^{\beta}}}{1-\mathrm{e}^{-\left(\frac{\alpha}{t}\right)^{\beta}}} \quad (55)$$

$$h(t)=\frac{\beta\alpha^{\beta}t^{-(\beta+1)}}{\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}-1} \quad (56)$$

To obtain the first order ordinary differential equation for the hazard function of the Frêchet distribution, differentiate equation (56), to obtain;

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$$h'(t)=\left\{\begin{matrix}-\dfrac{(\beta+1)t^{-(\beta+2)}}{t^{-(\beta+1)}}\\ +\dfrac{\dfrac{\alpha\beta}{t^2}\left(\dfrac{\alpha}{t}\right)^{\beta-1}\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}(\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}-1)^{-2}}{(\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}-1)^{-1}}\end{matrix}\right\}h(t) \quad (57)$$

The condition necessary for the existence of equation is $\alpha,\beta,t>0$.

$$h'(t)=\left\{-\frac{(\beta+1)}{t}+\frac{\beta\alpha^{\beta}t^{-(\beta+1)}\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}}{(\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}-1)}\right\}h(t) \quad (58)$$

$$h'(t)=\left(-\frac{(\beta+1)}{t}+h(t)\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}\right)h(t) \quad (59)$$

Equation (56) can be simplified as;

$$\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}=\frac{\beta\alpha^{\beta}t^{-(\beta+1)}}{h(t)}+1=\frac{\beta\alpha^{\beta}t^{-(\beta+1)}+h(t)}{h(t)} \quad (60)$$

Substitute equation (60) into equation (59);

$$h'(t)=\left(-\frac{(\beta+1)}{t}+\beta\alpha^{\beta}t^{-(\beta+1)}+h(t)\right)h(t) \quad (61)$$

$$th'(t)-h^2(t)-(\beta\alpha^{\beta}t^{-\beta}-(\beta+1))h(t)=0 \quad (62)$$

Differentiation is carried out again in order to obtain an ordinary differential equation that does not contain the powers of the parameters.;

$$h''(t)=\left\{-\frac{(\beta+1)}{t}+h(t)\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}\right\}h'(t)+\left\{\frac{(\beta+1)}{t^2}+h'(t)\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}-\frac{\alpha\beta}{t^2}\left(\frac{\alpha}{t}\right)^{\beta-1}\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}h(t)\right\}h(t) \quad (63)$$

The condition necessary for the existence of equation is $\alpha,\beta,t>0$.

The following equations obtained from equation (59) are needed to simplify equation (63);

$$\frac{h'(t)}{h(t)}=-\frac{(\beta+1)}{t}+h(t)\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}} \quad (64)$$

$$h(t)\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}=\frac{h'(t)}{h(t)}+\frac{(\beta+1)}{t} \quad (65)$$

$$\mathrm{e}^{\left(\frac{\alpha}{t}\right)^{\beta}}=h'(t)+\frac{\beta+1}{th(t)} \quad (66)$$

Substitute equations (64) and (66) into equation (63);

$$h''(t)=\frac{h'^2(t)}{h(t)}+\frac{(\beta+1)}{t^2}h(t)+\left(h'(t)+\frac{\beta+1}{th(t)}\right)h(t)h'(t)-\frac{\beta\alpha^{\beta}}{t^{\beta+1}}\left(h'(t)+\frac{\beta+1}{th(t)}\right)h^2(t) \quad (67)$$

When $\beta=1$, equation (67) becomes;

$$h''(t)=\frac{h'^2(t)}{h(t)}+\frac{2h(t)}{t^2}+\left(h'(t)+\frac{2}{th(t)}\right)h(t)h'(t)-\frac{\alpha}{t^2}\left(h'(t)+\frac{2}{th(t)}\right)h^2(t) \quad (68)$$

VII. Reversed Hazard Function

The reversed hazard function of the Frêchet distribution is given as;

$$j(t)=\beta\alpha^{\beta}t^{-(\beta+1)} \quad (69)$$

$$j'(t)=-(\beta+1)\beta\alpha^{\beta}t^{-(\beta+2)}=-\frac{(\beta+1)\beta\alpha^{\beta}t^{-(\beta+1)}}{t} \quad (70)$$

The condition necessary for the existence of equation is $\alpha,\beta,t>0$.

Substitute equation (69) into equation (70) to obtain;

$$j'(t)=-\frac{(\beta+1)}{t}j(t) \quad (71)$$

The first order ordinary differential equation for the reversed hazard function of the Frêchet distribution is given by; $tj'(t)+(\beta+1)j(t)=0$

(72) $j(1)=\beta\alpha^{\beta}$

(73)

The ODEs of all the probability functions considered can be obtained for the particular values of the distribution. Several analytic, semi-analytic and numerical methods can be applied to obtain the solutions of the respective differential equations [26-40]. Also comparison with two or more solution methods is useful in understanding the link between ODEs and the probability distributions.

VIII. Concluding Remarks

In this work, differentiation was used to obtain some classes of ordinary differential equations for the probability density function (PDF), quantile function (QF), survival function (SF), inverse survival function (ISF), hazard function (HF) and reversed hazard function (RHF) of the Frêchet distribution. The work was simplified by the application of simple algebraic procedures. Interestingly, the ODE of the RHF yielded simple result compared with others. In all, the parameters that define the distribution determine the nature of the respective ODEs and the range determines the existence of the ODEs.

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