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COMPARISON OF WEIBULL AND NORMAL PROBABILITY DISTRIBUTION OF FLEXURAL STRENGTH OF DENSE AND POROUS FIRED CLAY

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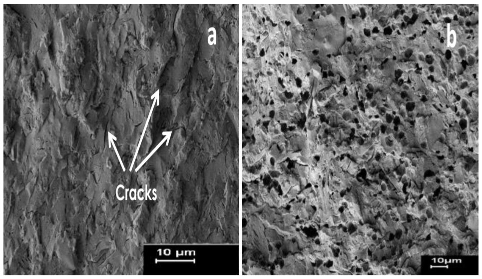
Graphical abstract



Abstract

In this research, dense and porous fired clay were produced at a firing temperature of 1300°C. The flexural strength data of the dense and the porous fired clay were determined using three point bending test. Two-parameter Weibull and normal probability distributions were used to estimate the reliability of the flexural strength data of the dense and the porous fired clay. From the result, the Weibull probability distribution scale parameter for the dense (36.31MPa) and Porous (18.85MPa) fired clay are higher than the mean strength value for the dense (33.84MPa) and the porous (17.87MPa) of the normal distribution. Distributions of flaws in the dense and the porous fired clay have a significant effect on the Weibull and normal distribution parameters. The fractured surface of the dense fired clay shows a random distribution of cracks while that of the porous fired clay shows a distribution of pores in the morphology. The normal distribution considers failure at 50% of the flexural strength data while Weibull probability distribution is failure at 62.3% of the strength data. Therefore, two-parameter Weibull is the suitable tool to model failure strength data of the dense and porous fired clay.

Keywords: Weibull distribution, normal distribution, fired clay, flexural strength

Abstrak

Tanah liat yang padat dan berliang dihasilkan dalam penyelidikan ini pada suhu pembakaran 1300 °C. Kekuatan lenturan untuk tanah liat bakar yang padat dan berlliang ditentukan menggunakan pengujian lenturan tiga titik. Dua parameter taburan kebarangkalian Weibull dan normal digunakan untuk menganggarkan kebolehpercayaan data kekuatan lenturan bagi tanah liat bakar yang padat dan berliang. Daripada keputusan eksperimen, skala parameter untuk taburan kebarangkalian Weibull bagi tanah liat bakar yang padat (36.31 MPa) dan yang berliang (18.85 MPa) adalah tinggi daripada nilai kekuatan purata untuk taburan kebarangkalian normal bagi tanah liat bakar yang padat (33.84 MPa) dan yang berliang (17.87 MPa). Taburan kecacatan bagi tanah liat bakar mempunyai kesan yang penting kepada parameter taburan Weibull dan normal. Morfologi permukaan patah tanah liat bakar yang padat menunjukkan taburan retak secara rawak, sementara tanah liat yang berliang menunjukan taburan liang-liang. Taburan kebarangkalian normal mempertimbangkan kegagalan 50% daripada data kekuatan lenturan manakala taburan kebarangkalian Weilbull mempertimbangkan kegagalan pada 62.3% daripada data kekuatan. Oleh itu, dua parameter Weibull lebih sesuai untuk model data kekuatan kegagalan bagi tanah liat bakar yang padat dan berliang.

Kata kunci: Taburan Weibull, taburan normal, tanah liat bakar, kekuatan lenturan

1.0 INTRODUCTION

Ceramic materials have gained general acceptance from more traditional applications to advanced technologies such as nozzles, membranes, biomedical implants, solid fuel cells, molten metal filters and catalyst supports. In addition, high cost of engineering ceramics has make researchers to use clay as a starting material to fabricate dense and porous ceramic materials for specific applications. Clay mineral can be a cheap source of ceramic materials such as mullite, which has good mechanical, thermal and chemical resistant properties.

In engineering materials requiring load bearing for structural applications, strength reliability is highly required [1]. Though ceramic materials have high hardness compared to metallic materials, the strength reliability of ceramic materials is very low. The poor in reliability strength normally arise due to distribution of flaws, cracks and porosity which are intrinsic properties of ceramic materials. As a result, ceramic materials under the same fabrication conditions tend to have different mechanical properties. The strength of ceramic materials is determined by the most critical flaw; therefore, the strength has to be presented by a distribution function, which exhibit the size distribution of flaws (in the form of cracks and porosity) in the specimen. A wide flaw distribution causes large scatter of strength[2]. While a uniformly distribution of flaws causes small scatter of the strength.

The variability in strength of brittle materials is best modeled using Weibull and normal distributions [3]. The Weibull probability distribution is based on the "weakest link "theory concept. In other words, the largest flaw provides the crack initiation site. It is believed that the Weibull probability distribution is the model of distribution of the minimum failure stress characterized by flaws if a number of random flaws are present in a stressed volume[4]. There is no research on which of these two probability distributions will best fit to model the strength data of dense and porous fired clay.

In this work, we analyze the flexural strength data of dense and porous fired clay using two-parameter Weibull and normal probability distributions. Comparison of two-parameter Weibull and normal distribution is also considered. Also, the distribution of flaws in the dense and the porous fired clay will be discussed.

2.0 METHODOLOGY

The raw clay used in this research was obtained from local miners in Nigeria. The chemical composition of the clay and phases present in the clay and fired ceramic was determined using XRF and XRD respectively. The dense and the porous fired clay (33 samples each) were compacted at a pressure of 60MPa and fired at a temperature of 1300°C for two-hours. The flexural strength of the samples of dimension 4×80×30mm and span 40mm was determined using three-point bending test at a loading rate of 0.5mm/min using Instron 100-KN electro-mechanical testing machine according equation 1: The flexural strength values of the dense and porous fired clay lie in the range 24-46MPa and 13-23 MPa respectively,

$$\sigma = \frac{3PL}{2bd^2} \tag{1}$$

where P is the load, L is the span which is 40mm, b is the width and d is the thickness.

2.1 Two-Parameter Weibull

The two-parameter Weibull cumulative distribution is given by

$$P = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_o}\right)^m\right] \tag{2}$$

Where P is the probability of failure at stress σ, m is the Weibull modulus and σ_o is the scale parameter. Linearizing Equation 2, by taking double logarithm to obtain:

$$\ln(\ln(1/(1-P)) = m\ln\sigma - m\ln\sigma_o \tag{3}$$

The plot of $\ln(\ln(1/(1-P))$ vs $\ln\sigma$ gives a straight line graph with a slope (Weibull modulus) m and intercept $-m\ln\sigma_o$ the scale parameter was estimated using the intercept and the Weibull modulus

Modified Kaplan Meier (Hazan) estimator was used to determine the probability of failure since it gives the less bias of the Weibull modulus (m) for number of samples greater than 20 for estimating the Weibull parameters of ceramics and brittle materials [5], [6], [7].

$$P = \frac{r - 0.5}{N} \quad (4)$$

where N is the sample size and P is the probability of failure for the r[th] ranked position.

2.2 Descriptive Statistics

Normal distribution and boxplot are presented by minimum, first quartile, median, upper quartile maximum, mean and standard deviation (SD) of the flexural strength of the dense and the porous fired.

3.0 RESULTS AND DISCUSSION

The chemical composition from the XRF analysis was found to be 48.86 wt% SiO_2, 37.83 wt% Al_2O_3, 1.15 wt% K_2O, 0.27 wt% Fe_2O_3, 0.05wt% CaO, 0.04 wt% TiO_2, 0.04 wt% MgO, 0.01 wt% MnO, 0.01 wt% P_2O_5 and 11.81 wt% LOI [8]. The XRD result of the raw clay (Figure 1) shows kaolinite is the predominant phase in the raw clay with traces of illite and quartz [9]. In addition, the XRD of the fired clay shows predominant mullite phase at firing temperature of 1300°C [7]. The two-parameter Weibull probability plot Modified Kaplan Meier estimate of dense and porous fired clay using linear regression is shown in Figure 2. From the figure, it can be shown that the probability data points lie close to the line of best fit, with the points at the lower tail of the plot lying closer to the line of best fit. This is one characteristics of linear regression analysis, which chases the lower strength data points of Weibull probability plot [6]. From the plot, the line of best fit of the porous fired clay lies to the left of the plot while that of the dense fired clay lies to the right, this behavior can be attributed to the fact the strength range of the dense fired clay is higher than that of the porous fired clay.

From the estimates of the Weibull parameters, the Weibull modulus of the dense ceramic shows a value of 6.52 while that of the porous fired clay shows a value of 9.22, this shows that the Weibull modulus of the porous fired clay is higher than that of the dense fired clay. Also, this supports that the variability in the flexural strength of the porous fired clay is higher than that of the dense fired clay.

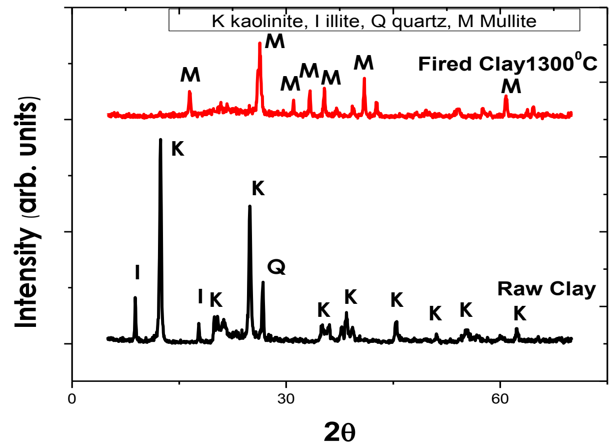


Figure 1 XRD analysis of raw and fired clay at 1300°C

The scale parameter of the dense fired clay have a value of 36.31 MPa while the porous fired clay has a value of 18.85 MPa; this shows that the strength at which 63.2% of the dense fired clay material will fail is higher than that of the porous fired clay. This reason is due to the fact that the porous fired clay has a pore former addition, which burns out during sintering this creates voids in the fired samples. These voids act as defects in the porous samples, thereby, decreasing the strength.

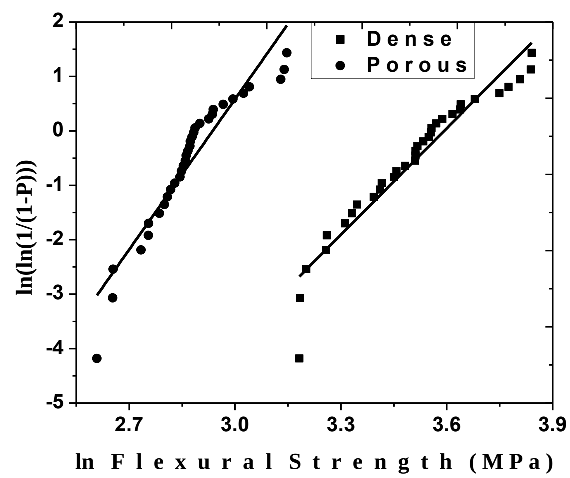


Figure 2 Weibull probability plot of dense and porous fired clay Hazen method

Table 1 gives the summary of standard deviation (SD), minimum, median, mean and maximum of the flexural strength computed from the boxplot. From the results, the mean strength of the dense and porous fired clay is found to be 33.84MPa and 17.87 MPa respectively. The values of the mean strength are lower than the scale parameter values of the dense and the porous fired clay using two-parameter Weibull probability distribution. The reason for this behavior is that the characteristic strength is the strength value at probability of failure equal 63.2%

while the mean strength value is at probability of failure equals to 50%. Similar observation was reported by [6]. The lower quartile and upper quartile of the dense and porous fired clay are computed to be 29.06MPa; 37.60 MPa and 16.51MPa; 18.87 MPa respectively. The interquartile range of the porous fired clay (2.36 MPa) is lower than that of the dense fired clay (8.54); this shows that the variability of the flexural strength of the dense fired clay is higher than that of the porous fired clay. This agrees with the values of the two-parameter Weibull modulus. The standard deviation, further confirmed less variability in the flexural strength of the porous fired clay with a value of 2.36 MPa as compared with the dense fired clay with a value of 6.28 MPa.

Table 1 Mean, median SD, interquartile ranges, maximum and minimum of flexural strength of dense and porous fired clay at 1300°C

Parameter	Dense	Porous
Mean (MPa)	33.84	17.87
SD (MPa)	6.28	2.36
Minimum (MPa)	24.11	13.58
Q1 (MPa)	29.06	16.51
Median (MPa)	33.50	17.59
Q3 (MPa)	37.60	18.87
Maximum (MPa)	46.56	23.26
IQ Range (MPa)	8.54	2.36

Boxplot of the flexural strength of both the dense and the porous fired clay at 1300°C sintering temperature is depicted in Figure 3. The dense fired clay gives a wider interquartile range while the porous fired clay shows a narrower interquartile range, the wider interquartile range of dense fired clay is due to the high variability of the flexural strength and the narrow range of the interquartile range of the porous fired clay can be attributed to the low variability in the flexural strength. This agrees with the two-parameter Weibull probability distribution, which shows a wide and narrow probability distribution for the dense and the porous fired clay respectively. In addition, the whisker of the dense fired clay to the right is longer than that to the left, these show there are extreme values (deviation from the median) towards the positive end and the distribution is positively skewed. However, the whisker of the porous fired clay to the left is longer than to the right, this means there are some extreme values from the median towards the negative end and the distribution is negatively skewed. Furthermore, the porous fired clay presents three outlier points (outer fence points) to the right end of the distribution; this means that the three outlier points are much higher than the other values of the flexural strength.

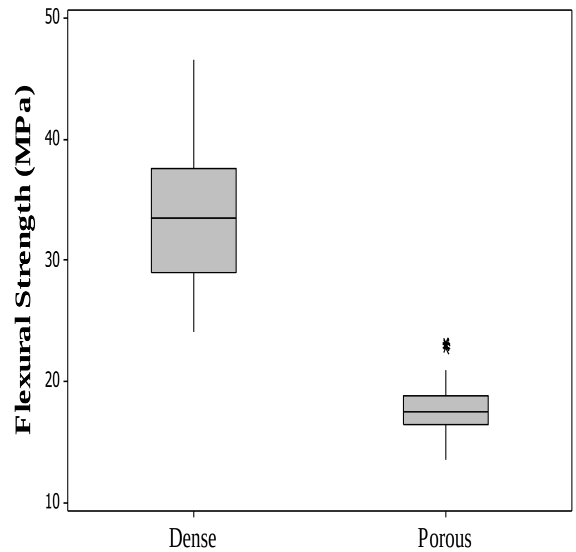


Figure 3 Boxplot of dense and porous fired clay at 1300°C

Figure 4 shows the Weibull and normal distribution probability density plot of the dense fired clay. The density plot of the Weibull distribution shows skewness to the right of the probability density graph. The skewness in the distributions has a population concentrated at a value close to the scale parameter. While in the normal distribution, there is no skewness in the probability density plot. The graph is distributed uniformly around the mean. Also, Figure 5 gives the Weibull and normal distribution probability density plot of the porous fired clay. Like the dense fired clay, the density plot shows skewness to the right, which is concentrated at the scale parameter value while the normal distribution density plot shows no skewness with the plot uniformly distributed around the mean. The skewness of Weibull distribution can be attributed to the fact that the scale parameter is strength at 63.2% while that of the normal distribution is strength at 50% [6]. This shows that normal distribution gives a conservative estimate of probability of failure compared with Weibull probability distribution.

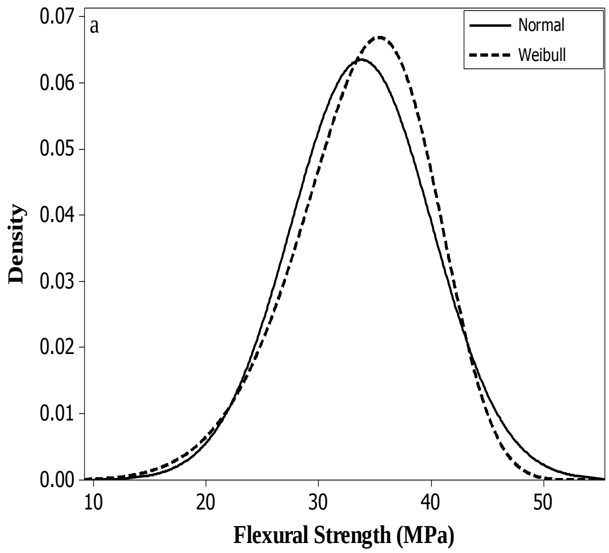


Figure 4 Density plot of dense fired clay Weibull and normal distributions

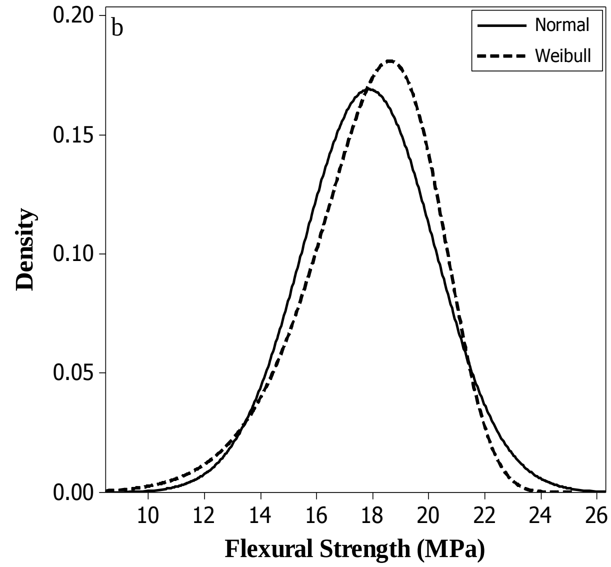


Figure 5 Density plot of porous fired clay Weibull and normal distribution

From Figures 4 and 5, narrow flexural strength distribution of the porous fired clay is due to its high value of the Weibull modulus and smaller standard deviation value for Weibull and normal distribution respectively while the dense material has a more spread distribution due to the low value of the Weibull modulus and high standard deviation of Weibull and normal distribution respectively. The wider spread of the probability density is due to random distribution of cracks which resulted in the scattering of the flexural strength. The narrow spread of the probability density is due relative distribution of porosity in the morphology of the porous fired clay which gives narrow distribution of the flexural strength. This makes the Weibull modulus of the porous fired clay higher and with higher density (Figure 3b) as reflected by the narrow distribution of the probability density plot. This observation has good agreement with previous studies [10],[11].

The narrower distribution curve suggests less variability in the flexural strength of the porous fired clay. In addition, predicting probability of failure using Weibull distribution is higher than normal distribution (Figures 4 and 5) for both the dense and the porous fired clay, as a result, more suitable to model the flexural strength data of the dense and the porous fired clay.

The micrographs of fractured surface of the dense and porous ceramic with 10wt% starch addition are shown in Figures 6a and 6b. From the micrographs, it can be depicted that the dense ceramic material contains random distribution of cracks while the porous ceramic contains relative distribution of porosity. Brittle materials under the same fabrication conditions tend to have variability in their mechanical properties due to porosity and cracks which are inherent in their microstructure [6]. This variability in mechanical properties needs to be model with Weibull probability distribution. The fractography analyses of dense and porous fired clay at 1300°C each show a single distinct flaw distribution (cracks and relative porosity).

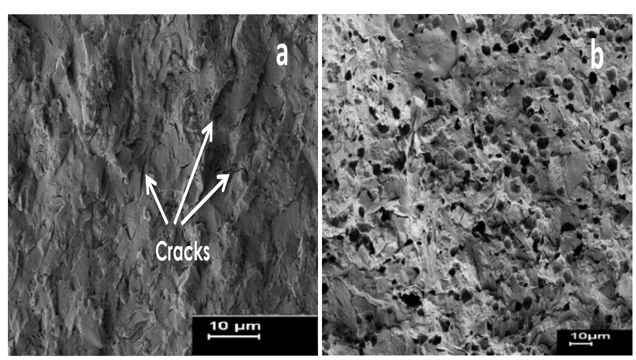


Figure 6 Fractured surface morphology of (a) dense and (b) porous fired clay at 1300°C

4.0 CONCLUSION

In this paper, two different probability distributions were tested in determining the reliability analysis of dense and porous fired clay. The scale parameter in Weibull distribution is higher than the mean strength in normal distribution, because normal distribution provides failure at 50% of the materials while Weibull modulus provides failure at 63.2%. The porous fired clay has higher reliability and failure can easily be predicted than the dense fired clay. The porous fired clay has relative distribution of porosity while the dense fired clay has random distribution of cracks. Weibull distribution will be a suitable tool to model the failure strength of dense and porous fired clay since it provides due to its high probability density.

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