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NUMERICAL SOLUTION OF UNSTEADY FREE CONVECTION FLOW IN A SECOND GRADE FLUID

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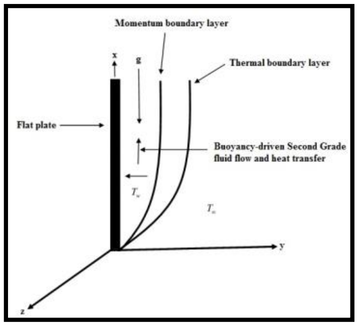
Graphical abstract



Abstract

In this paper, the problem of unsteady free convection flow moves along a vertical plate in a second grade fluid is studied. The vertical plate with constant temperature is considered. The dimensional governing equations are transformed into non dimensional equations using appropriate dimensionless variables and solved numerically using Finite Difference Method. Numerical results for velocity and temperature profiles are displayed graphically for viscoelastic parameter, Grashof number and Prandtl number and discussed in details. It is found that, increasing the values of Grashof number and time leads to increase in the velocity profiles. Increasing the values of the Prandtl number and viscoelastic parameter is found to decrease the velocity profile. It is further found that, increasing the values of Prandtl number tends to decrease the thermal boundary layer thickness.

Keywords: Unsteady; second grade fluid; free convection; heat transfer; finite difference method

Abstrak

Dalam kertas kerja ini, masalah aliran olakan bebas tak mantap bergerak disepanjang plat menegak di dalam bendalir gred dua dikaji. Plat menegak dengan suhu tetap dipertimbangkan. Persamaan menakluk yang bermatra diubah ke bentuk persamaan tak bermatra dengan menggunakan pembolehubah tak bermatra yang bersesuaian. Persamaan ini diselesaikan secara berangka dengan menggunakan Kaedah Beza Terhingga. Keputusan berangka bagi profil halaju dan suhu dipaparkan secara grafik untuk parameter viskoelastik, nombor Grashof dan nombor Prandtl dan dibincangkan secara terperinci. Keputusan menunjukkan, peningkatan nilai nombor Grashof dan masa menyebabkan profil halaju meningkat. Meningkatkan nilai nombor Prandtl dan parameter viskoelastik telah menurunkan profil halaju. Didapati juga, dengan nilai nombor Prandtl yang besar cenderung untuk mengurangkan ketebalan lapisan sempadan terma.

Kata kunci: Tak mantap; bendalir gred kedua; olakan bebas; pemindahan haba; kaedah perbezaan terhingga

1.0 INTRODUCTION

Recently, research in the boundary layer flow of non-Newtonian fluids have increased considerably due to the application in industrial and engineering particularly in the field of chemical engineering [1-3]. Viscoelastic fluids, which are known as one the sub-class of non-Newtonian fluids are the type of fluids which are more accurate than the first-order fluids with exponential dependence of viscosity and temperature [4]. Many models have been proposed to describe the rheological behaviour of non-Newtonian fluids due to its physical structure. Among the viscoelastic fluid models such as, Maxwell fluid, Oldroyd fluid, Walters-B fluid, and Burgers fluid, there is one simplest subfamily of viscoelastic fluids known as second grade fluids [5-6].

Most of the existing studies on convection flows of second grade fluid are concerned with analytical and numerical solutions due to its simplest model of viscoelastic fluid. Rajagopal et al. [7] illustrated their problem by considering the polymer processing application and presented a study of the flow of an incompressible second order fluid past a stretching sheet without heat transfer solved numerically using Runge Kutta Method. Later, Bujurke et al. [8] investigated the momentum and heat transfer in the flow of a second-order fluid over a stretching sheet using the momentum integral technique. Mustafa et al. [9] investigated the effects of free convection flow of a viscoelastic second-grade fluid along vertical flat surface with variable heat flux and presented the numerical solutions using Keller-box method. Very recently, Samiulhaq et al. [10] studied the free convection flow of second grade fluid with ramped wall temperature and obtained the exact solutions using Laplace transforms technique.

Motivated by the above investigations, the present study aims to investigate natural convection flow of a second grade fluid past a rigid vertical plate with constant temperature. In this paper, the coupled nonlinear partial differential equations have been solved numerically using a finite-difference method. Numerical results for velocity and temperature profiles are displayed graphically for embedded flow parameters including viscoelastic parameter, Prandtl number, Grashof number and dimensionless time. It is worth to mention that, the results obtained satisfy the imposed initial and boundary conditions.

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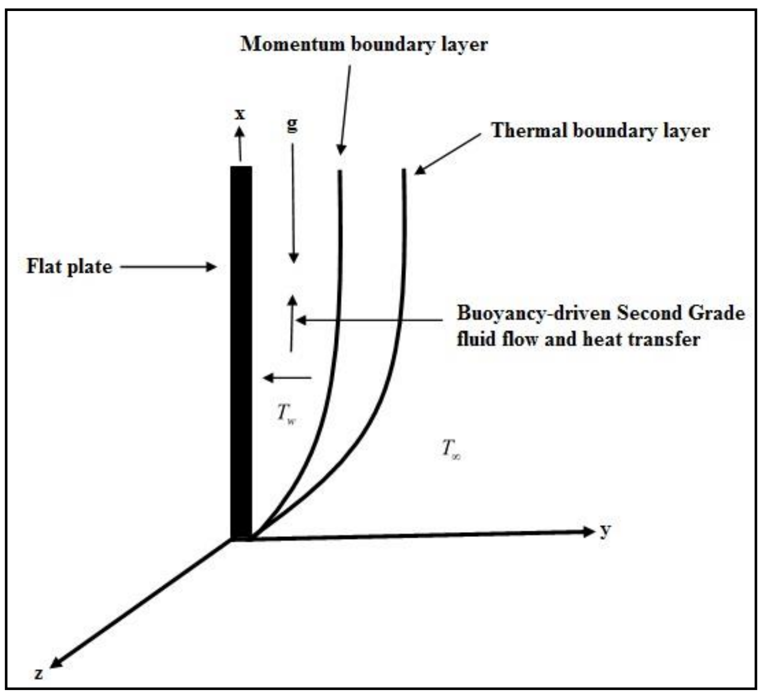

Figure 1 Physical model and coordinate system

2.0 MATHEMATICAL FORMULATION

An unsteady free convection flow of an incompressible second grade fluid over a vertical rigid plate at $y = 0$, driven by buoyancy force due to the temperature differences, which occur upward along the plate is considered for investigation.

We consider the x-axis to be parallel to the plate in upward direction while the y-axis is taken normal to it. Initially, the plate and the fluid are assumed at rest and at uniform temperature, T_∞. Then, at time $t^* > 0$, the temperature of the plate is raised or lowered to a constant temperature, T_w.

The governing equations for an unsteady free convection boundary layer flow of second grade fluid past a rigid vertical plate with Boussinesq's approximation are given by Ali et al. [11]

$$\frac{\partial u^*}{\partial t^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{k_0}{\rho} \frac{\partial^3 u^*}{\partial y^{*2} \partial t^*} + g\beta(T - T_\infty), \qquad (2.1)$$

$$\rho c_p \frac{\partial T}{\partial t^*} = k \frac{\partial^2 T}{\partial y^{*2}}. \qquad (2.2)$$

The initial and boundary conditions are prescribed as (Khan et al. [12])

$$\begin{aligned} t^* \le 0: &\quad u^* = 0, \quad T = T_\infty; \quad y^* \ge 0, \\ t^* > 0: &\quad u^* = 0, \quad T = T_w; \quad y^* = 0, \\ &\quad u^* = 0, \quad T = T_\infty; \quad y^* \to \infty. \end{aligned} \qquad (2.3)$$

Here u^* is the velocity in x-direction, υ is the kinematic viscosity, ρ is the fluid density, k_0 is the elastic parameter, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, T is the temperature, c_p is the specific heat of the fluid at constant pressure and k is the thermal conductivity of the fluid.

On introducing the following non-dimensional quantities

$$u = \frac{u^*}{U_0}, \quad y = \frac{U_0 y^*}{\upsilon}, \quad t = \frac{U_0^2 t^*}{\upsilon}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

effectively, equations (2.1), (2.2) and (2.3) are reduced to the following dimensionless system.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \Gamma \frac{\partial^3 u}{\partial y^2 \partial t} + Gr\theta, \qquad (2.4)$$

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2}. \qquad (2.5)$$

where $\Gamma = k_0 U_0^2 / (\rho \upsilon^2)$ is the dimensionless viscoelastic parameter, $Gr = [\upsilon g \beta (T_w - T_\infty)] / U_0^3$ is defines as Grashof number, $\Pr = \mu c_p / k$ is the Prandtl number and U_0 is the reference velocity. The corresponding dimensionless initial and boundary conditions are given by

$$\begin{aligned} t \le 0: &\quad u = 0, \quad \theta = 0; \quad y \ge 0, \\ t > 0: &\quad u = 0, \quad \theta = 1; \quad y = 0, \\ &\quad u = 0, \quad \theta = 0; \quad y \to \infty. \end{aligned} \qquad (2.6)$$

3.0 RESULTS AND DISCUSSION

In this study, the unsteady free convection flow of an incompressible second grade fluid which represented by equations (2.4) and (2.5) are solved subject to the initial and boundary layer conditions given by equation (2.6) using the Finite difference method.

The numerical solutions of the problem of unsteady free convection flow of second grade fluid over a vertical flat plate with heat transfer is analyzed. The effects of viscoelastic parameter Γ, Prandtl number $\Pr$, Grashof number Gr and dimensionless time t are presented graphically in this section. The influence of these parameters on velocity and temperature profiles are discussed.

The comparisons of present numerical result with exact solutions obtained by Samiulhaq et al. [10] are shown in Figure 2. It is observed that in the absence of free convection $Gr = 0$, the numerical solutions are matched well with the exact solutions with those of Samiulhaq et al. [10]. Hence, we can say that the results found to be in excellent agreement.

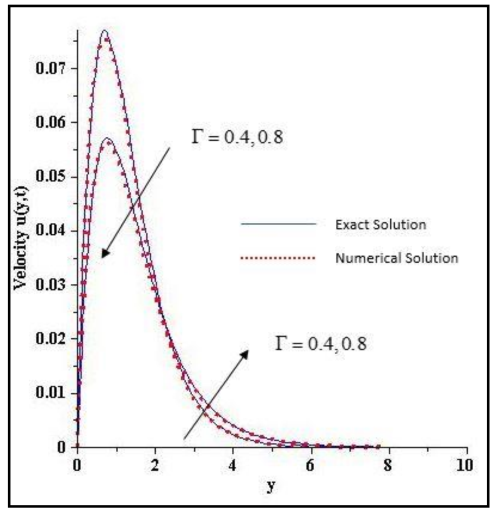

Figure 2 Comparison for velocity profiles with different values of $\Gamma = 0.4$ and $\Gamma = 0.8$ with $t = 0.5, \Pr = 0.71$.

The deviation in velocity profiles for different values of Γ, $\Pr$, Gr and t are demonstrated in Figures 3 - 6 whereas Figures 7 - 8 are drawn for temperature profiles. The influence of viscoelastic parameter Γ on velocity profiles is illustrated in Figure 3. Four different values of $\Gamma = 0.0,\ 0.1,\ 0.2$ and 0.3 are chosen according to Khan et al. [12]. It can be seen from this figure that the velocity tends to decrease with the increase of Γ. The large values of Γ, gives higher stability compared to the small value. Moreover, we can see that the Newtonian fluid which is $\Gamma = 0$ gives a higher velocity. The same behavior of Γ on the velocity are also obtained by Ali et al. [11]

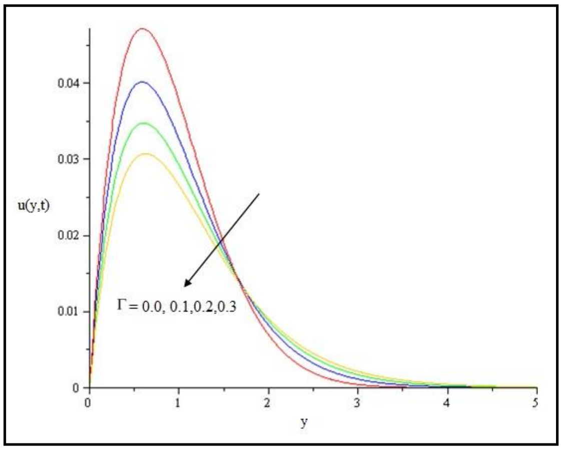

Figure 3 Velocity profiles for different values of Γ when $\Pr = 0.71,\ Gr = 0.5$ and $t = 0.4$.

The effect of Prandtl number, $\Pr$ for two values as $\Pr = 0.7$ and $\Pr = 7.0$ upon velocity is elucidated from Figure 3.3. Physically, these values are corresponds to air and water. It can be observed that, the velocity decreases when $\Pr$ increases. It is true because the Prandtl number is the ratio of viscous diffusion and thermal diffusion. Thus, it controls the relative thickness of the momentum and thermal boundary layers. The increase of viscosity will make the fluid thick and consequently decrease the velocity.

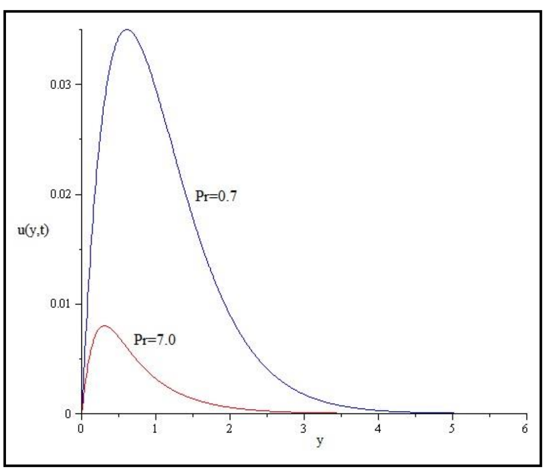

Figure 4 Velocity profile for different values of *Pr* when $\Gamma = 0.2,\ Gr = 0.5$ and $t = 0.4$.

The behavior of Grashof number, Gr on velocity profiles is shown in Figure 5. We noticed that, the velocity increases for large values of Grashof number. Essentially, an increase in the values of Grashof number implies a rise in velocity profiles. It is because the Grashof number defined as a ratio of buoyancy forces to viscous forces. Thus, it increase the velocity.

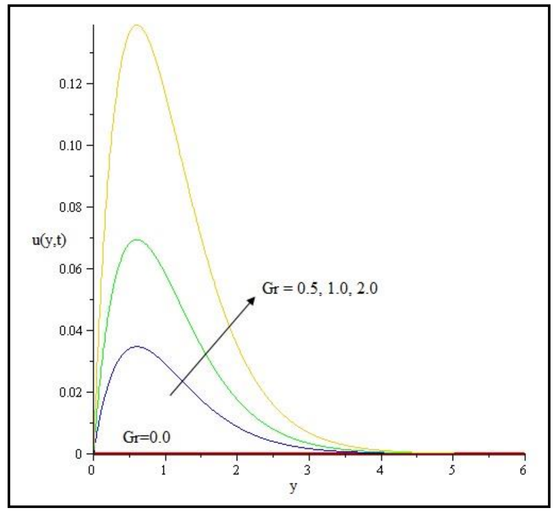

Figure 5 Velocity Profile for different values of Gr when $\Gamma = 0.2,\ \Pr = 0.71$ and $t = 0.4$.

Then from Figure 6, we clearly found that as time increase, the velocity increase.

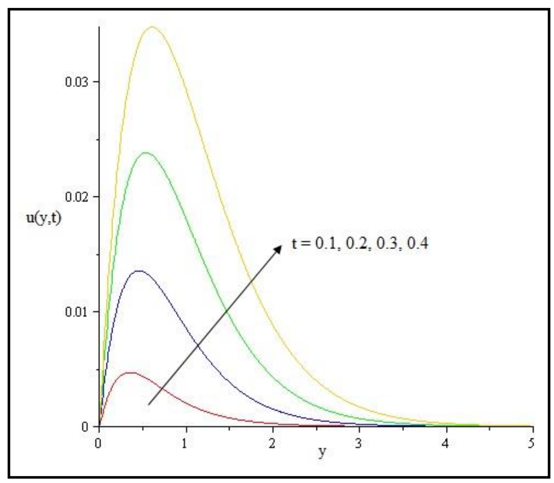

Figure 6 Velocity profile for different values of t when $\Pr = 0.71,\ \Gamma = 0.2$ and $Gr = 0.5$

Furthermore, a plotted temperature profiles with the effect of Prandtl number are shown in Figure 7. Different values of $\Pr = 0.015,\ 0.71$ and 7.0 which refer to mercury, air and water are chosen. The figure indicates that, temperature decrease rapidly when $\Pr$ is increased. As Prandtl number controls the relative thickness of momentum and thermal boundary layer in heat transfer so the decrease in $\Pr$ actually because the thermal diffusivity to be dominant and consequently increase the temperature profiles.

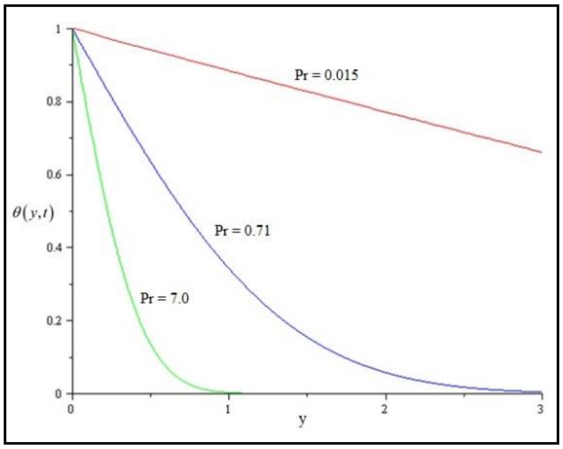

Figure 7 Temperature profile for different values of Pr when $\Gamma = 0.2,\ t = 0.4$ and $Gr = 0.5$.

Finally, Figure 8 shown the temperature profiles is increased as the dimensionless time t increase.

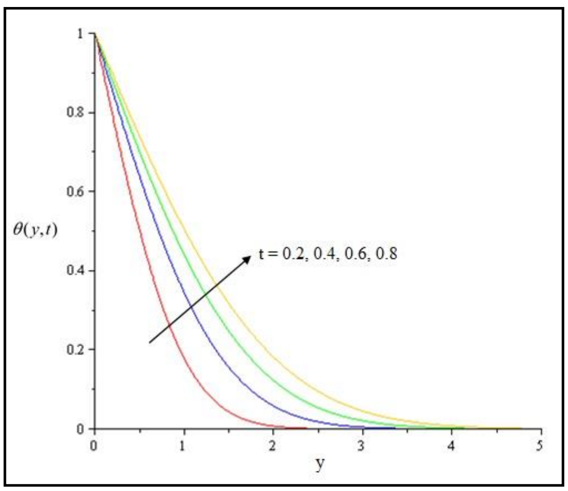

Figure 8 Temperature profile with different values of t when $\Gamma = 0.2,\ \Pr = 0.71$ and $Gr = 0.5$.

It is worth to mention that, from all the figures plotted the solutions obtained satisfies the corresponding boundary conditions given by equations (2.6).

4.0 CONCLUSION

The problem of heat transfer on unsteady free convection flow in a second grade fluid past a rigid vertical plate is investigated. A parameters of interest in this study are the viscoelastic parameter Γ , Prandtl number $\Pr$ and Grashof number Gr . The transformed conservation equations were solved by using Finite difference method. The results shown that, behavior of velocity profiles are reduced when Γ and $\Pr$ are increased. However, increasing *Gr* boost the velocity which close to the plate surface. Further, the temperature of the fluid decreases for large values of $\Pr$.

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References

[1] Goerke, A.R., Leung, J. and Wickramasinghe, S.R. 2002. Mass and Momentum Transfer in Blood Oxygenators. *Chemical Engineering Science*. 57: 2035-2046.
[2] Vrentas, J.S. and Vrentas, C.M. 1996. Axial Moment Analyses of Convective Heat and Mass Transfer Processes. *Chemical Engineering Science*. 51: 921-929.
[3] Elperin, T. and Fominnykh, A. 2001. Effect of Solute Concentration Level on the Rate of Coupled Mass and Heat Transfer during Solid Sphere Dissolution in a Uniform Fluid Flow. *Chemical Engineering Science*. 56: 3065-3074.
[4] Kassim, A.R.M., Mohammad, N. F., Shafie, S. and Pop, I. 2013. Constant Heat Flux Solution for Mixed Convection Boundary Layer Viscoelastic Fluid. *Heat and Mass Transfer*. 49:163-171.
[5] Fosdick, R. L. and Rajagopal, K. R. 1979. Anomalous Feature in the Model of Second order Fluids. *Archieve for Rational Mechanics and Analysis*. 70: 145-152.
[6] Dunn, J. E. and Fosdisk, R. L. 1974. Thermodynamics, Stability and Boundedness of Fluids of Complexity 2 and Fluids of Second-grade. *Archieve for Rational Mechanics and Analysis*. 56: 191-252.
[7] Rajagopal, K. R., Na, T. Y. and Gupta, A. S. 1984. Flow of a Viscoelatic Fluid over a Stretching Sheet. *Rheological Acta*. 23: 213-215.
[8] Bujurke, N.M., Biradar, S.N. and Hiremath, P. S. 1987. Second-order Fluid Flow past a Stretching Sheet with Heat Transfer. *Journal of Applied Mathematics and Physics*. 38:653-657.
[9] Mustafa, N., Asghar, S. and Hossain, M. A. 2010. Natural Convection Flow of Second-grade Fluid along a Vertical Heated Surface with Variable Heat Flux. *International Journal of Heat and Mass Transfer*. 53: 5856-5862.
[10] Samiulhaq, Khan, I., Ali, F. and Shafie, S. 2014. Free Convection Flow of a Second Grade Fluid with Ramped Wall Temperature. *Heat Transfer Research*. 45(7): 579-588.
[11] Ali, F., Khan, I. and Shafie, S. 2014. Closed Form Solutions for Unsteady Free Convection Flow of a Second Grade Fluid over an Oscillating Vertical Plate. *Plos One*. 9(2).
[12] Khan, I., Ali, F. Shafie, S. and Qasim, M. 2014. Unsteady Free Convection Flow in a Walter's-B Fluid and Heat Transfer Analysis. *Bulletin of the Malaysian Sciences Society*. 37: 437-448.