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## **NEGATIVE IMAGINARY THEOREM WITH AN APPLICATION TO ROBUST CONTROL OF A CRANE SYSTEM**

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### **Graphical abstract Abstract**



This paper presents an integral sliding mode (ISM) control for a case of negative imaginary (NI) systems. A gantry crane system (GCS) is considered in this work. ISM is a nonlinear control method introducing significant properties of precision, robustness, stress-free tuning and implementation. The GCS model considered in this work is derived based on the x direction and sway motion of the payload. The GCS is a negative imaginary (NI) system with a single pole at the origin. ISM consist of two blocks; the inner block made up of a pole placement controller (NI controller), designed using linear matrix inequality for robustness and outer block made up of sliding mode control to reject disturbances. The ISM is designed to control position tracking and anti-swing payload motion. The robustness of the control scheme is tested with an input disturbance of a sine wave signal. The simulation results show the effectiveness of the control scheme.

*Keywords:* Integral sliding mode control, negative imaginary systems, position tracking, payload sway motion control

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#### **1.0 INTRODUCTION**

Gantry crane systems (GCS) are industrial machines commonly used to transport substantial loads in both construction companies and heavy machine installations [1]. They comprise of a cart and trolley which moves in a horizontal direction and a payload attached to a cable. The trolley movement is considered as a one-dimensional pendulum having one-degree-of-freedom. The main target of crane operation is to transport a load from one place to another with fast response and stability. Throughout the running time, the payload is freely attached to the trolley and swings in a pendulum-like motion. A lot of effort has been put by researchers in controlling the

trolley movement and payload swing motion. However, in most cases the presented control schemes are not suitable for real time applications. Consequently, the operation of the crane is not automated and always needs a monitoring operator which, in most cases, cannot compensate for the payload swing motion [2].

Prior to the introduction of negative imaginary (NI) system theory, passivity theory was applied to analyze the properties of the system modeled using collocated force actuators and velocity sensors or negativevelocity feedback systems. However, the passivity theory cannot be applied directly if position or acceleration sensors are used [3, 4]. Thus, passivity theory cannot be applied in the field of

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nanotechnology, particularly in nano-positioning systems. Related problems also arise in application to the robotics field where acceleration and position sensors are widely used in an alternative approach to provide solution to the above problems, NI theorem was introduced by Lanzon and Petersen in [5, 6]. NI controllers such as the resonant controller, integral resonant controller and state feedback controller, guarantee internal stability when applied to NI systems.<sup>3</sup> Examples of NI systems are DC machines, electrical active filter circuits and lightly damped flexible structures [7].

NI theorems and lemmas are presented by different authors with diverse interpretations. In the first definition, a system is NI if and only if all the poles of *G(jω)* are in the open left half of the complex plane [4, 8]. The definition was improved in [6, 9-10] where the systems are allowed to have a simple pole on the imaginary axis except at the origin. Another modification in the definition of NI systems was presented in [11] in which the system is allowed to have poles at the origin. The definition was extended to include systems with non-rational transfer functions [12].

Numerous control algorithms have been developed to control load transportation using GCS. These include robust control of GCS as presented in [13, 14] and Sliding mode control [15-18] and H-infinity control proposed [19].

In this work, an integral sliding mode (ISM) control is proposed for robust control of GCS. The control scheme consists of two blocks: (1) the inner block is made up of a pole placement controller (PPC) which is NI, designed using linear matrix inequality (LMI). (2) The outer block is the sliding mode control designed to reject external disturbance. To remove the effects of chattering which is caused by the sliding mode switching function sign, a sigmoid function is used to replace the sign witching function. A sine wave disturbance input signal is used to test the robustness performance of the proposed ISM. The rest of the paper is organized as follows; section 2 presents NI preliminary theorems. Section 3 describes NI properties test on the GCS. The controller design is presented in Section 4. In Section 5, simulation results and discussion are presented and conclusions are drawn in Section 5.

#### **2.0 PRELIMINARY THEOREM**

Preliminary theorems that are important in the study of NI systems are presented in this section. Some of this theorem will be used in this study to show that GCS possesses the properties of NI systems and hence, can be classified as NI system with double poles at the origin.

Theorem 1 [6, 9]. A real-rational proper transfer function matrix *R(s)* is term NI if: *R(s)* has no poles at the origin and in *Re(s)>0; j(R(jω) - R(jω)\*) ≥* 0 for all *ω ≥ 0*

where *R(jω)\** is the conjugate of the *R(jω)* and *ω* is the frequency in rad/sec.

Theorem 2 [20]. A square transfer function matrix *G(s)* is NI if the following conditions are satisfied: *G(s)* has no pole in *Re(s) > 0.*

For all *ω ≥ 0* such that *jω* is not a pole of *G(s),*

*j[G(jω) - G(jω)\* ] ≥ 0*

If  $s = j\omega_0$ ,  $\omega_0 > 0$  is a pole of *G(s)* then it is a simple pole.

Theorem 3 [21]. A square transfer function matrix *P(s)* is NI if the following conditions are satisfied:

All of the poles of *P(s)* lie in the open loop half plane. For all  $\omega \geq 0$ ,  $j(P(j\omega) - P(j\omega)^*) \geq 0$ .

Theorem 4 [4]. Consider a minimal state-space system

$$
\begin{aligned}\n\dot{x} &= Ax + Bu \\
y &= Cx + Du\n\end{aligned}
$$

where  $A \in \mathfrak{R}^{n \times n}$ ,  $B \in \mathfrak{R}^{n \times m}$ ,  $C \in \mathfrak{R}^{m \times n}$ ,  $D \in \mathfrak{R}^{m \times m}$ . The system is NI if and only if *A* has no eigenvalues on the imaginary axis, *D* is symmetric, and there exists a positive-definite matrix  $Y \in R^{n \times n}$  satisfying

$$
AY + YA^T \le 0
$$
  

$$
B + AYC^T = 0
$$

Theorem 5 [7]. Let  $G(S) = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$  $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$  be a minimal realization with  $CB + B^T C^T > 0$ . Then G(s) is NI if and only if  $D = D<sup>T</sup>$  and there exist a matrix  $P \ge 0$  such that P is a solution to the algebraic Riccati equation;

 $PA_0 + A_0^T P + P B R^{-1} B^T P + Q = 0,$ where  $A_0 = A - BR^{-1}CA$ ,  $R = CB + B<sup>T</sup> C<sup>T</sup>$  and  $Q = A^T C^T R^{-1} C A.$ 

Lemma 1 [11]. Consider a square real rational proper transfer function matrix  $G(s)$  with the state space realization  $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$  $\begin{bmatrix} 1 & 0 \\ C & D \end{bmatrix}$  such that, the transfer function matrix  $\tilde{G}(s) = G(s) - D$ . Then the transfer function matrix  $G(s)$  is NI if and only if the transfer matrix  $H(s) = s\tilde{G}(s)$  is positive real assuming all pole-zero cancellation in  $s\tilde{G}(s)$  is taking care to obtain  $H(s)$ .  $\tilde{G}(s) = G(s) - D$  is in state space form, since  $G(s) = \begin{bmatrix} A & B \ C & D \end{bmatrix}$  $\begin{bmatrix} A & D \\ C & D \end{bmatrix}$ .

Explanation [11]. Suppose that H(s) is a positive real, then  $H(j\omega) + H(j\omega)^* \ge 0$  for all  $\omega \in (-\infty, \infty)$  such that  $j\omega$ is not a pole of  $H(s)$ . This shows that  $j\omega(\tilde{G}(j\omega) \tilde{G}(j\omega)^*) \ge 0$  for all  $\omega \ge 0$  such that jwis not a pole of  $G(s)$ . Then  $(\tilde{G}(j\omega) - \tilde{G}(j\omega)^*) \ge 0$  for all such that  $\omega \in (0, \infty)$  Furthermore, if  $j\omega_0$  is a pole of  $H(s)$ , then based on the positive real systems theorems the residual matrix  $\lim_{s\to j\omega_0}$  $(s - j\omega_0) H(s)$  is positive

semidefinite Hermitian. Similarly,

$$
\lim_{s \to j\omega_0} (s - j\omega_0) H(s) = \lim_{s \to j\omega_0} (s - j\omega_0) s\tilde{G}(s)
$$

$$
= \omega_0 \lim_{s \to j\omega_0} (j\omega_0) j\tilde{G}(s)
$$

Thus, it can be seen from Lemma 1,  $\tilde{G}(s)$  is NI and hence  $G(s)$  is NI.

A generalize lemma is provided which allows for a simple pole or double poles at the origin. Consider a linear time invariant system as

$$
\dot{x}(t) = Ax(t) + Bu(t) \tag{1}
$$
\n
$$
y(t) = Cx(t) + Du(t) \tag{2}
$$

where  $A \in \mathbb{R}^{m \times m}$ ,  $B \in \mathbb{R}^{m \times n}$ ,  $C \in \mathbb{R}^{n \times m}$ ,  $D \in \mathbb{R}^{n \times n}$ ,  $m \times m$ and  $m \times n$  are the dimensions of the state space.  $m \times n$  $m$  and  $m \times n$  refers to dimension of  $A$  and  $B$ . The dimension of  $x(t)$  is  $m \times n$ , but the dimension of u depends on the number of control input signal to the system.

Lemma 2 [11]. Let  $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  be a minimal realization of the transfer function matrix  $G(s)$  for the system in Equations (1) and (2). Then  $G(s)$  is NI if and only if  $D =$  $D<sup>T</sup>$  and there exist a matrix  $P = P<sup>T</sup> \ge 0$  such that the LMI below satisfy the condition.

$$
P = P^T \ge 0 \tag{3}
$$

$$
\begin{bmatrix} PA + A^T P & PB - A^T C^T \\ B^T P - CA & -(CB + B^T C^T) \end{bmatrix} \le 0 \tag{4}
$$

#### **3.0 NI PROPERTIES TEST OF GANTRY CRANE**

This section describes the NI properties test on the GCS based on the lemma and theorems that allowed for systems with simple or double poles at the origin to be considered as NI systems. The GCS model used in this work is based on load movement along the x direction and the sway motion of the payload in the  $x$  direction only as obtained in [19].

The state space model is given by

$$
A = \begin{bmatrix} 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -9.81 & -0.11 \\ 0.00 & 0.00 & 0.00 & 1.00 \\ 0.00 & 21.76 & 21.33 & -10.03 \end{bmatrix}
$$

$$
B = [0 \ 0 \ 0 \ 6.91]^T
$$

$$
C = [1 \ 0 \ 0 \ 0]
$$
 and  $D = [0]$ 

given the state variable as  $x(t) = [x \dot{x} \theta \dot{\theta}]^T$  which representing cart movement in the x direction, derivative of  $x$  movement, payload swing angle in the direction of x and derivative of swing angle respectively. Subsequently, an investigation is conducted to test whether the GCS satisfies the conditions for NI systems. The test is done based on the conditions in Lemma 2. Using the LMI Matlab Toolbox, *P* is obtained as



The eigenvalues of  $P$  are  $[0.0067, 0.5225, 0.7263$  and 1.2444]. As all the eigenvalues are positive,  $P = P^T \ge 0$ and the conditions in Equations (3) and (4) are satisfied. In addition,  $D = D<sup>T</sup> = 0$ . The system is thus proven to be NI. Now, an NI controller can be design which will ensure internal stability of the closed-loop based on the dc gain condition.

#### **4.0 INTEGRAL SLIDING MODE CONTROL DESIGN**

This section describes the design of ISM control as proposed in [24-27]. The control scheme consists of two controllers: a PPC designed using LMI and a sliding mode controller. LMI approach is used to obtain optimal PPC gains. The block diagram of ISM is shown in Figure 1, where *K* is the state feedback control gain and  $\gamma$  is a positive scalar gain that guarantees the enforcing of the state motion on the sliding manifold.

The general form of LMI equation is given as

$$
F(x) = F_0 + \sum_{i=1}^{m} F_i x_i > 0
$$
  
  $i = 1, 2, 3, ..., ..., m.$  (6)

where  $F_i$  are symmetric matrices and  $x_i$  are the decision variables. The main property of LMIs is that the set of solutions of a variable positive definite matrix *F* is convex. The control purpose is to move the payload to a desired location with minimum payload sway motion. The external disturbances such as change in payload mass and effect of air resistance should also be compensated.



**Figure 1** ISM block diagram

Consider a nonlinear system given as

$$
\dot{x} = f(x, t) + Bu(x, t) + \phi(x, t) \tag{7}
$$

where  $x \in R^n$ ,  $u(x,t) \in R^m$  and  $\phi(x,t)$  is external disturbances or matched perturbation. It is assumed that the perturbation is a matched perturbation, which means it enters the system states together with control signals. However, in SMC this matched perturbation will be rejected by the  $u_1(x,t)$  which is in this case equal to the equivalent control signal.

The control law in an integral sliding mode is given as

$$
u(x,t) := u_0(x,t) + u_1(x,t)
$$
 (8)

where  $u_0(x,t)$  is the nominal control law and  $u_1(x,t)$  is the discontinuous control law that handles the rejection of matched perturbation to guarantee sliding mode. This discontinuous control law can be given by an equivalent control law. Therefore the control signal in the present of  $(x, t)$  is  $u(x, t) = u_0(x, t) - u_1(x, t) +$  $\phi(x,t)$ .  $u_1(x,t)$  is negative because of the negative feedback connection. In this work it is assumed that the perturbation is chosen as:

$$
\bar{\phi}(x,t) = B\phi_m(x,t). \tag{9}
$$

The equivalent control is given as exactly equal to the negative of the disturbance signals as

$$
u_{1eq} = -B\phi_m(x,t) - (G(x)B(x))^{-1}G(x)\phi_u(x,t)
$$
 (10)

where  $\phi_m(x,t)$  refers to matched perturbation and  $\phi_u(x,t)$  refers to unmatched perturbation. Since in this work, the perturbation is matched perturbation then,  $\phi_u(x,t) = 0$ , thus, the equivalent control action became as;

$$
u_{1eq} = -B\phi_m(x, t)
$$

$$
u_{1eq} = -\overline{\phi}(x, t)
$$

The sliding manifold is give as

$$
s(x,t) := W[x(t) - x(t_0) - \int_{t_0}^t (f(x,\tau)Bu(x,\tau))d\tau \qquad (11)
$$

where  $W \in R^{m \times n}$  is the projection matrix and in this work is considered to be equal to  $W = (B^TB)^{-1}B^T$  as derived in [22]. By simplifying Equation (11) the derivative of the sliding surface is used to derive the switching control action which is equal to the equivalent control action as given

$$
\dot{s}(x,t) = WBu_1(x,t) + WB\overline{\phi}(x,t) = 0
$$
  
\n
$$
WBu_1(x,t) = -WB\overline{\phi}(x,t)
$$
  
\n
$$
u_1(x,t) = -\overline{\phi}(x,t)
$$
  
\nbut, 
$$
u_1(x,t) = u_{1eq}
$$
  
\n
$$
u_{1eq} = -\overline{\phi}(x,t)
$$
\n(12)

The linearized GCS model is represented in controllable canonical in state space form for the implementation of the ISM controller as

$$
A = \begin{bmatrix} 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & -213.47 & -23.72 & -10.03 \end{bmatrix}
$$

$$
B = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T
$$

$$
C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}
$$

$$
D = \begin{bmatrix} 0 \end{bmatrix}
$$

The nominal control law  $u_0 = Kx + Hr$  where Kis the pole placement control gain matrix and

$$
H = -[C(A + BK)^{-1}B]^{-1}
$$
 (13)

The values of  $K$  and  $H$  are obtained by LMI optimization technique. The nominal GCS model without perturbation is given as

$$
\dot{x}(t) = Ax(t) + Bu(t)
$$
  
\n
$$
y(t) = Cx(t)
$$
\n(14)

The state feedback control law is given as

$$
u = Kx \tag{15}
$$

The closed-loop control equation as

 $(A^T + B^T K^T)X + X(A + BK) + 2\alpha X < 0, \quad X > 0$  (16)

Pre and post multiply by  $X^{-1}$  and let  $Q = K^{-1}$ 

$$
QAT + QBTKT + AQ + BKQ + 2\alpha Q < 0, \quad Q > 0 \tag{17}
$$

Equation (17) is still nonlinear (i.e not LMI). Let  $K = NQ^{-1}$ 

$$
QAT + AQ + BTNT + BN + 2\alpha Q < 0, \ Q > 0
$$
 (18)

hence, Equation (18) is in LMI form and can be solved using any LMI solver.

The control purpose is to place the poles at the desired location to ensure internal stability. To achieve that, an LMI region in a complex plane is formulated. This region is defined as

$$
\mathcal{D} = \{ z \in \mathcal{C} : L + sM + \bar{s}M^T < 0 \} \tag{19}
$$

where L and M are real matrices and  $L = L^T$  and  $\bar{s}$ denotes complex conjugate of *s.*

$$
f_{\mathcal{D}}(z) = L + sM + \bar{s}M^{T}
$$
 (20)

The function in Equation (20) is called the characteristic function of Ɗ. The system matrix *A* will have all its eigenvalues in the region Ɗ, if there is existence of a positive definite, symmetric matrix *X*  that satisfies the condition such that

$$
L \otimes X + M \otimes (AX) + M^T \otimes (AX)^T < 0 \tag{21}
$$

where the symbol ⊗ stands for the Kronecker products.

LMI region is a region in the complex plane where the closed loop poles are located to achieve certain desired dynamic properties such as selecting the amount of overshoot and settling time. The LMI region in Figure 2 is used in this work. This region is formed by intersection of a-stability region  $R_e(s) \leq -\alpha$ , a disc centered at origin with radius r, a vertical strip and a conic sector with inner angle 2θ and apex at the origin. The region *S(θ,r,α)* is a set of complex numbers  $x \pm iy$  based on the following conditions:



**Figure 2** S(θ,r,α) region

$$
x < -\alpha < 0, \ |x \pm jy| < r \text{ , } \tan \theta \, x < -|y|.
$$

In this work, the desired closed loop poles are located in the region by setting a maximum overshoot of 3% and a settling time of 2 seconds to ensure a minimum damping ratio  $\zeta = \cos \theta$ , a minimum decay rate a and maximized undamped natural frequency  $w_d = r \sin \theta$ [23].

Solving the LMI in (16) the optimized and best values of *X, Q, K* and *N* with decay rate of 1.5 are obtained as

$$
X = \begin{bmatrix} 1.3840 & -5.0560 & -1.9837 & 7.3484 \\ -5.0560 & 18.9247 & 7.6764 & -29.7330 \\ -1.9837 & 7.6764 & 3.2792 & -13.7611 \\ 7.3484 & -29.7330 & -13.7611 & 66.8546 \end{bmatrix}
$$
  
\n
$$
Q = \begin{bmatrix} 394.14 & 206.97 & -305.38 & -14.13 \\ 206.97 & 112.27 & -172.88 & -8.40 \\ -305.38 & -172.88 & 282.48 & 14.82 \\ -14.13 & -8.40 & 14.82 & 0.88 \end{bmatrix}
$$
  
\n
$$
K = [-228.9712 - 136.698 \quad 256.2148 \quad 15.9642]
$$
  
\n
$$
N = [-16.6988 \quad 62.8613 \quad 25.3467 - 76.6174]
$$

#### $H = 228.9712$

#### **5.0 RESULTS AND DISCUSSION**

This section presents the simulation results and discussion of the proposed ISM control. The ISM is implemented on the GCS using Matlab/Simulink environment. The ISM control is required to move a payload to a desired location without payload sway motion. Load position, sway motion and control signal are observed to investigate the control performance with and without sine wave input disturbance, Figure 3 shows the reference input signal used. Figure 4 shows the sine wave input disturbance signal with maximum amplitude of 0.75.

The payload sway motion and load position with and without disturbance are shown in Figure 5 and Figure 6 respectively. It is observed that with an external disturbance signal, there is less significant effect on both load positioning and sway motion as the ISM controller is robust to external disturbance. However, it is also observed that, in the presence of input disturbance signal, the system consumed more control input to reject the effect of the external disturbance as shown in Figure 7.







**Figure 5** Payload sways motion with and without disturbance

The control input signal reduced to a value close to zero after 4 seconds when the system is running without disturbance. With disturbance signal it is observed that the control signal fluctuated around 0.2 to -0.2 throughout the time of simulation. This is because the system consumed more control input energy to be able to reject the disturbance signal.

#### **6.0 CONCLUSION**

The GCS is successfully tested to be NI. An ISM control has also been successfully designed and applied to control load positioning and sway motion in the *x* direction. The control performance is tested with a sine wave input disturbance signal. It is observed that the ISM control successfully rejected the effect of the external sine wave disturbance. The introduction of a sigmoid function removed the effect of chattering in the ISM. The load position and sway motion of the payload has been controlled to achieved the desired control goal. The LMI-based PPC optimized the control gains and choose the best value of *K* for robustness.



**Figure 6** Load position with and without disturbance



The study will be implemented on the real system in the future work.

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#### **References**

- [1] Eihab, M. A, and Masoud, Z. N. 2003. Dynamics And Control Of Cranes. *A Review Journal of Vibration and Control.* 9: 863-908.
- [2] Omar, H. M. and Nayfeh, A. H. 2005. Gantry Cranes Gain Scheduling Feedback Control With Friction Compensation. *Journal of Sound and Vibration.* 281: 1-20.
- [3] Petersen, I. R. 2011. Negative Imaginary Systems Theory In The Robust Control Of Highly Resonant Flexible Structures. *Proceedings of Australian Control Conference (AUCC)* 2011*,* Melbourne, Austrlia, 10-11 Nov. 2011. 1-6.
- [4] Petersen, I. R. and Lanzon, A. 2010. Feedback Control Of Negative Imaginary Systems. *IEEE Transaction on Control Systems*. 30: 54-72.
- [5] Lanzon, A. and Petersen, I. R. 2007. A Modified Positive-Real Type Stability Condition. *Proceedings of the European Control Conference* 2007. Kos, Greece, 2-5 July 2007. 3912- 3918.
- [6] Lanzon, A. and Petersen, I. R. 2008. Stability Robustness Of A Feedback Interconnection Of Systems With Negative Imaginary Frequency Response. *IEEE Transactions on Automatic Control*. 53: 1042-1046.
- [7] Mabrok, M. A., Kallapur, A. G., Petersen, I. and Lanzon, A. 2012. Stabilization Of Conditional Uncertain Negative-Imaginary Systems Using Riccati Equation Approach. *Proceedings of the 20th International Symposium on Mathematical Theory of Networks and Systems (MTNS)* 2012*.* Melbourne, Australia, 9-13 July, 2012. 9-13.
- Mabrok, M. A., Kallapur, A. G., Petersen, I. and Lanzon, A. 2011. A new Stability Result For The Feedback Interconnection Of Negative Imaginary Systems With A Pole At The Origin. *Proceedings of 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC) 2011*, Orlando, Florida, 12-15 Dec. 2011. 3753- 3757.
- [9] Junlin, X., Petersen, I. R. and Lanzon, A. 2010. A Negative Imaginary Lemma And The Stability Of Interconnections Of Linear Negative Imaginary Systems. *IEEE Transactions on Automatic Control*. 55: 2342-2347.
- [10] Junlin, X., Petersen, I. R. and Lanzon, A. 2012. Finite Frequency Negative Imaginary Systems. *IEEE transactions on Automatic Ccontrol.* 57: 2917-2922.
- [11] Mabrok, M. A., Kallapur, A. G., Petersen, I. and Lanzon, A. 2012. A Stability Result On The Feedback Interconnection Of Negative Imaginary Systems With Poles At The Origin. *Proceedings of the 2nd Australian Control Conference (AUCC)* 2012*,* Sydney, Australia, 15-16 Nov. 2012. 98-103.
- [12] Ferrante, A. and Ntogramatzidis, L. 2014. Some Newresults In The Theory Of Negative Imaginary Systemswith Symmetric Transfermatrix Function. *Automatica.* 49: 1- 9.
- [13] Karkoub, M. A. and Zribi, M. 2001. Robust Control Schemes For An Overhead Crane. *Journal of Vibration and Control.* 7: 395-416.
- [14] Tangwe, Y., Qin, Y. and Jianda, H. 2011. Robust Control Of Gantry Crane System With Hoisting A New Solution Based On Wave Motion. *Journal of Central South University*. 42: 288-292.
- [15] Almutairi, N. B. and Zribi, M. 2009. Sliding Mode Control Of A Three-Dimensional Overhead Crane. *Journal of Vibration and Control*. 15: 1679-1730.
- [16] Diantong, L., Jianqiang, Y., Dongbin, Z., and Wang, W. 2004. Swing-free transporting Of Two-Dimensional Overhead Crane Using Sliding Mode Fuzzy Control. *Proceeding of the American Control Conference.* Boston, Massachusetts June 30-July 2. 1764-1769.
- [17] Le, A. T., Jae-Jun, K., Lee, S. G., Lim, T. G. and Nho, L. C. 2014. Second-order sliding Mode Control Of A 3d Overhead Crane With Uncertain System Parameters. *International Journal of Precision Engineering and Manufacturing*. 15(5): 811-819
- [18] Liu, D., Yi, J., Zhao, D. and Wang, W. 2005. Adaptive Sliding Mode Fuzzy Control For A Two-Dimensional Overhead Crane. *Mechatronics*. 15: 505-522.
- [19] Mohd Tumari, M. Z., Saealal, M S., Ghazali, M. R. and Wahab, Y. A. 2012. H∞ Controller With Graphical LMI Region Profile For Gantry Crane System. *Proceedings of the IEEE SCIS-ISIS* 2012 Kobe, Japan, 20-24 Nov. 2012. 1397-1402.
- [20] Mabrok, M. A., Kallapur, A. G., Petersen, I. and Lanzon, A. 2013. Generalizing Negative Imaginary Systems Theory To Include Free Body Dynamics: Control Of Highly Resonant Structures With Free Body Motion. *IEEE Transaction on Automatic Control.* 59(10): 2692-2707.
- [21] Junlin, X., Petersen, I. R. and Lanzon, A. 2009. Stability Analysis Of Positive Feedback Interconnections Of Linear Negative Imaginary Systems. *Proceedings of the American Control Conference (ACC 2009)*, St. Louis, Missouri, USA, 10- 12 June, 2009. 1855-1860.
- [22] Castanos, F. and Fridma, L. 2006. Analysis And Design Of Integral Sliding Manifolds For Systems With Unmatched Perturbations. *IEEE Transactions on Automatic Control.* 51: 853-858.
- [23] Chilali, M. and Pascal, G. 1996. H-infinity design With Pole Placement Constraints: An LMI Approach. *IEEE Transactions on Automatic Control.* 41: 358-367.
- [24] andyopadhyay, B., Deepak, B., F. and Kim, K. S. 2009. Sliding Mode Control Using Novel Sliding Surfaces. Textbook Springer. 392.
- [25] Bera, M. K., Bandyopadhyay, B., and Paul, A. K. 2013. Integral Sliding Mode Control For GMAW Systems. Proceedings of the 10th International Symposium on Dynamics and Control of Process Systems (DYCOPS) 2013, Mumbai, India 18-20 December 2013. 337-342.
- [26] Kamal, S., Chalanga, A., Ramesh, K. P. and Bandyopadhyay, B. 2015. Multivariable Continuous Integral Sliding Mode Control. *Proceedings of IEEE International Workshop on Recent Advances in Sliding Modes (RASM)* 2015, Istanbul, Turkey, 9-11 April 2015. 1-5.
- [27] Rubagotti, M., Estrada, A., Castanos, F., Ferrara, A. and Fridman, L. 2011. Integral Sliding Mode Control For Nonlinear Systems With Matched And Unmatched Perturbations. *IEEE Transactions on Automatic Control.* 56: 2699-2704.