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Distinct Sums Over Subsets

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DISTINCT SUMS OVER SUBSETS

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ABSTRACT. A finite set of integers with distinct subset sums has a precisely bounded Dirichlet series.

Let $1 \leqslant a_1 < a_2 < \cdots < a_n$ be a set of integers for which all of the sums $\sum_{i=1}^n \varepsilon_i a_i$, $\varepsilon_i = 0$ or 1, are distinct. It was conjectured by P. Erdös and proved by C. Ryavec that

$$\sum_{i=1}^{n} \frac{1}{a_i} < 2.$$

We will show that for all real $s \geqslant 0$,

$$\sum_{i=1}^{n} \left(\frac{1}{a_i} \right)^s < \frac{1}{1 - 2^{-s}}.$$

The hypothesis clearly implies for $0 < x < 1$ that

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k > (1 + x^{a_1})(1 + x^{a_2}) \cdots (1 + x^{a_n})$$

and

$$-\log(1 - x) > \sum_{i=1}^{n} \log(1 + x^{a_i}),$$

as was observed in [1].

The crucial idea here is that the form $|\log x|^\beta \, dx/x$ is changed only by a constant factor under the substitution $y = x^a$, so integrating we have

$$\int_0^1 |\log(1 - x)| \, |\log x|^\beta \, \frac{dx}{x}$$
$$> \int_0^1 \log(1 + y) |\log y|^\beta \, \frac{dy}{y} \sum_{i=1}^{n} \left(\frac{1}{a_i} \right)^{1+\beta}.$$

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To calculate the first integral we substitute $x = e^{-u}$, expand $\log(1 - e^{-u})$, and integrate term-by-term to obtain $\Gamma(\beta + 1)\zeta(\beta + 2)$, where ζ is the Riemann zeta function. In the same way the second integral is found to be $\Gamma(\beta + 1)\zeta(\beta + 2)(1 - (\frac{1}{2})^{\beta+1})$. These calculations are valid for all $\beta > -1$, so the theorem follows.

Bibliography

1. S. J. Benkowski and P. Erdős, *On weird and pseudoperfect numbers*, Math. Comp. **28** (1974), 617–623.

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