7-2006

Biases in Casino Betting: The Hot Hand and the Gambler's Fallacy

James Sundali

Rachel Croson
University of Pennsylvania

Biases in Casino Betting: The Hot Hand and the Gambler's Fallacy

Abstract
We examine two departures of individual perceptions of randomness from probability theory: the hot hand and the gambler's fallacy, and their respective opposites. This paper's first contribution is to use data from the field (individuals playing roulette in a casino) to demonstrate the existence and impact of these biases that have been previously documented in the lab. Decisions in the field are consistent with biased beliefs, although we observe significant individual heterogeneity in the population. A second contribution is to separately identify these biases within a given individual, then to examine their within-person correlation. We find a positive and significant correlation across individuals between hot hand and gambler's fallacy biases, suggesting a common (root) cause of the two related errors. We speculate as to the source of this correlation (locus of control), and suggest future research which could test this speculation.

Keywords
judgment and decision making, hot hand, gambler's fallacy, casino betting, field data, roulette

Disciplines
Other Business | Probability | Recreation Business

Judgment and Decision Making, Vol. 1, No. 1, July 2006, pp. 1–12

Biases in casino betting: The hot hand and the gambler's fallacy

James Sundali*
Managerial Sciences
University of Nevada, Reno

Rachel Croson
Operations and Information Management
Wharton School
University of Pennsylvania

Abstract

We examine two departures of individual perceptions of randomness from probability theory: the hot hand and the gambler's fallacy, and their respective opposites. This paper's first contribution is to use data from the field (individuals playing roulette in a casino) to demonstrate the existence and impact of these biases that have been previously documented in the lab. Decisions in the field are consistent with biased beliefs, although we observe significant individual heterogeneity in the population. A second contribution is to separately identify these biases within a given individual, then to examine their within-person correlation. We find a positive and significant correlation across individuals between hot hand and gambler's fallacy biases, suggesting a common (root) cause of the two related errors. We speculate as to the source of this correlation (locus of control), and suggest future research which could test this speculation.

Keywords: judgment and decision making, hot hand, gambler's fallacy, casino betting, field data, roulette

1 Introduction

Almost every decision we make involves uncertainty in some way. Yet research on decision making under uncertainty demonstrates that our judgments are often not consistent with probability theory. Intuitive ideas of randomness depart systematically from the laws of chance. This research suggests that we have developed a number of judgment heuristics for analyzing complex, real-world events. Although many decisions based on these heuristics are consistent with probability theory, there are also situations where heuristics lead to statistical illusions and suboptimal actions.

This paper investigates the existence and impact of two of these statistical illusions; the *gambler's fallacy* and the *hot hand*. Both of these illusions characterize individuals' perceptions of non-autocorrelated random sequences. Thus both involve perceptions of sequences of events rather than one-time events.

The gambler's fallacy is a belief in negative autocorrelation of a non-autocorrelated random sequence of outcomes like coin flips. For example, imagine Jim repeatedly flipping a (fair) coin and guessing the outcome before it lands. If he believes in the gambler's fallacy, then after observing three heads in a row, his subjective probability of seeing another head is less than 50%. Thus he believes a tail is "due," and is more likely to appear on the next flip than a head.

In contrast, the hot hand is a belief in positive autocorrelation of a non-autocorrelated random sequence of outcomes like winning or losing. For example, imagine Rachel repeatedly flipping a (fair) coin and guessing the outcome before it lands. If she believes in the hot hand, then after observing three correct guesses in a row her subjective probability of guessing correctly on the next flip is higher than 50%. Thus she believes that she is "hot" and more likely than chance to guess correctly.

Notice that these two biases are not simply opposites. The gambler's fallacy describes beliefs about outcomes *of the random process* (e.g., heads or tails), while the hot hand describes beliefs of outcomes *of the individual* (like wins and losses). In the gambler's fallacy, the *coin* is due; in the hot hand the *person* is hot. For purposes of our study, we will identify four possible biases that individuals could exhibit. The gambler's fallacy and its opposite, the hot outcome, are beliefs about the coin's outcomes involving negative versus positive autocorrelation of random outcomes. The hot hand and its opposite, the stock of luck, are beliefs about the individual's success involving positive versus negative autocorrelation of winning or losing.

*The authors thank Eric Gold for substantial contributions in earlier stages of this research. Thanks also to Jeremy Bagai, Dr. Klaus von Colorist, Bradley Ruffle, Paul Slovic, Willem Wagenaar, participants of the J/DM and ESA conferences, at the Conference on Gambling and Risk Taking and at seminars at Wharton, Caltech and INSEAD for their comments on this paper. Special thanks to the Institute for the Study of Gambling and Commercial Gaming for industry contacts which resulted in the acquisition of the observational data reported here. Financial support from NSF SES 98–76079–001 is also gratefully acknowledged. All remaining errors are ours. Address correspondence to Rachel Croson, 567 JMHH, The Wharton School University of Pennsylvania, 3730 Walnut Street, Philadelphia, PA 19104–6340, crosonr@wharton.upenn.edu. James Sundali's email is jsundali@unr.nevada.edu

Thus someone can believe both in the gambler's fallacy (that after three coin flips of heads tails is due) and the hot hand (that after three wins they will be more likely to correctly guess the next outcome of the coin toss). These biases are believed to stem from the same source, the *representativeness heuristic*, as discussed below (Gilovich, Vallone and Tversky 1985).

In this paper we use empirical data from gamblers in casinos to examine the existence, prevalence and correlation between gambler's fallacy and hot hand beliefs. A companion paper, Croson and Sundali (2005) uses the same data to examine the aggregate (market) impact of these biases. In contrast, here we will identify the biases at the individual level, and examine the within-participant correlation between the two.

Empirical data, while difficult to obtain and to code, can provide an important complement and robustness check on other methods in investigating biases. Participants in the casinos are making real decisions with their own money on the line. Further, the participants represent a more motivated sample than typical students at a university; gamblers have a very real incentive to learn the game they are playing and to make decisions in accordance with their beliefs.

The use of casino data does, however, involve some limitations. In particular, we were prevented from directly contacting the gamblers in the study, thus we cannot ask particular individuals why they bet how they did or about their beliefs at the time of placing the bet. Also, the gambling population, while motivated, is a selected subsample of the population at large. Thus we will have to be cautious in our claims of external validity from this study. Nonetheless, we believe that the demonstration of these biases in the field at the level of the individual is an important contribution in and of itself. We are also one of the very few papers to identify multiple biases within an individual and to characterize the correlation between them.

1.1 Definitions and previous research

1.1.1 Gambler's fallacy

The gambler's fallacy is defined as an (incorrect) belief in negative autocorrelation of a non-autocorrelated random sequence.[1] For example, individuals who believe in the gambler's fallacy believe that after three red numbers appearing on the roulette wheel, a black number is "due," that is, is more likely to appear than a red number.

Gambler's fallacy-type beliefs were first observed in the laboratory (under controlled conditions) in the literature on probability matching. In these experiments subjects were asked to guess which of two colored lights would next illuminate. After seeing a string of one outcome, subjects were significantly more likely to guess the other, an effect referred to in that literature as *negative recency* (see Estes, 1964, and Lee, 1971, for reviews). Ayton and Fischer (2004) also demonstrate the existence of gambler's fallacy beliefs in the lab when subjects choose which of two colors will appear next on a simulated roulette wheel. Gal and Baron (1996) show that gambler's fallacy behavior is not simply caused by boredom; participants in their experiments were asked how they would best maximize their earnings, and they responded with gambler's fallacy type logic.

The gambler's fallacy is thought to be caused by the representativeness heuristic (Tversky and Kahneman 1971, Kahneman and Tversky 1972). Here, chance is perceived as "a self-correcting process in which a deviation in one direction induces a deviation in the opposite direction to restore the equilibrium" (Tversky & Kahneman, 1974, p. 1125). Thus after a sequence of three red numbers appearing on the roulette wheel, black is more likely to occur than red because a sequence red-red-red-black is more representative of the underlying distribution than a sequence red-red-red-red. We test for the gambler's fallacy in our data by looking at the impact of previous outcomes on current bets at roulette. People who believe in the gambler's fallacy should be less likely to bet on a number that has previously appeared.

For purposes of this analysis, we will examine two separate definitions of hotness, *hot outcome* and *hot hand*. *Hot outcome* will simply be the opposite of the gambler's fallacy, that is, an (incorrect) belief in positive autocorrelation of a non-autocorrelated random sequence.[2] For example, individuals who believe in hot outcome believe that after three red numbers appearing on the roulette wheel, another red number is more likely to appear than a black number because red numbers are hot. Notice that here the *outcomes* are hot (e.g., red numbers), rather than individuals, as in the hot hand, below.

In the lab, the literature on probability matching also provides evidence favoring hot outcome beliefs. Edwards (1961), Lindman and Edwards (1961) and Feldman (1959) all found *positive recency* effects in probability matching tasks. In particularly long sequences of the probability matching game, participants were significantly more likely to guess the same outcome as had been observed previously.[3]

[1] Or, more generally, a belief in a more negative autocorrelation than is present. Thus when an individual overestimates the amount of negative autocorrelation in any sequence, we could say they were exhibiting gambler's fallacy beliefs as well.

[2] Or, more generally, a belief in a more positive autocorrelation than is present. Thus when an individual overestimates the amount of positive autocorrelation in any sequence, we could say they were exhibiting hot outcome beliefs as well.

[3] The hot outcome bias is related but not identical to the construct referred to by Keren and Lewis (1994) as the gambler's fallacy type II. They present results of a questionnaire study in the lab demonstrating

We will test for hot outcome beliefs in our data by looking at the impact of previous outcomes on current bets at roulette. If gamblers believe in hot outcomes, they should be more likely to bet on an outcome that has previously been observed. Thus a positive relationship between previously-observed outcomes and current bets is indicative of a belief in hot outcomes.[4]

1.1.2 Hot hand

Hot hand is different from hot outcome. Rather than believing that a particular *outcome* is hot, individuals who believe in the hot hand believe that a particular *person* is hot. For example, if an individual has won in the past, *whatever* numbers they choose to bet on are likely to win in the future, not just the numbers they've won with previously.

Gilovich, Vallone and Tversky (1985) demonstrated that individuals believe in the hot hand in basketball shooting, and that these beliefs are not correct (i.e., basketball shooters' probability of success is indeed serially uncorrelated). Other evidence from the lab shows that subjects in a simulated blackjack game bet more after a series of wins than they do after a series of losses, both when betting on their own play and on the play of others (Chau & Phillips, 1995). Further evidence of the hot hand in a laboratory experiment comes from Ayton and Fischer (2004). Participants exhibit more confident in their guesses of what color will next appear after a string of correct guesses than after a string of incorrect guesses.

Explanations for the hot hand are numerous. It is clearly related to the illusion of control (Langer, 1975), where individuals believe they can control outcomes that are, in fact, random. Gilovich et al., (1985) suggest that the hot hand also arises out of the representativeness heuristic, just as the gambler's fallacy. They write

> A conception of chance based on representativeness, therefore, produces two related biases. First, it induces a belief that the probability of heads is greater after a long sequence of tails than after a long sequence of heads — this is the notorious gambler's fallacy (see, e.g., Tversky and Kahneman, 1971). Second, it leads people to reject the randomness of sequences that contain the expected number of runs because even the occurrence of, say, four heads in a row — which is quite likely in a sequence of 20 tosses — makes the sequence appear nonrepresentative. (p. 296).

This second explanation is supported by data in which participants are asked to generate strings of random numbers. The strings generated produced significantly fewer runs of the same outcome than a truly random sequence would (see Wagenaar 1972 for a review, for an exception see Rapoport & Budesceu 1992).

We will test for hot hand beliefs in our data by looking at how betting behavior changes in response to wins and losses. In particular, hot hand beliefs predict that after winning, individuals will increase the number of bets they place and after losing, decrease them.

Just as the gambler's fallacy and the hot outcome are opposing biases, the hot hand has an opposing bias, referred to here as "stock of luck" beliefs. Individuals believe they have a stock or fixed amount of luck and, once it's spent, their probability of winning decreases. Thus after a string of wins, individuals are less likely to win (rather than more likely as predicted by the hot hand) because they have exhausted their stock of luck. The effect has been demonstrated in the lab by Leopard (1978) who examines choice behavior in a series of gambles and demonstrates that subjects take more risk after losing than after winning, suggesting that their bad luck is about to change or their good luck about to run out.[5]

Stock of luck beliefs predict that after winning, individuals will decrease the number of bets they place and, after a loss, increase them. Thus a negative relationship observed between current betting behavior and previous wins/losses will provide evidence for this bias.

1.2 Individual differences

A large literature identifies individual differences in risk attitudes (e.g., Weber et al.,1992; Blais & Weber, 2006; Harris et al., 2006). In addition, previous work has identified individual heterogeneity in biased beliefs about sequences of gambles. Friedland (1988) uses a personality inventory to categorize individuals into luck-oriented and

that individuals underestimate the number of observations necessary to detect biased roulette wheels. Thus after seeing even a small streak of red numbers, gamblers might believe the wheel is biased and expect more red numbers. The number of spins participants believe they need to observe to detect a biased wheel, while significantly smaller than the true number of spins necessary, as derived in Ethier (1982), is significantly larger than the number of spins any individual in our data set will observe.

[4] One can construct other explanations for the behavior we here attribute to the hot outcome. For example, perhaps numbers that have recently hit on the roulette wheel are more *available* to the gambler than other numbers. This availability may cause the gambler to bet on numbers that have recently hit. Unfortunately in our empirical data we will not be able to distinguish between these alternative causes of this behavior, although previous lab research can and has done so.

[5] As with the hot outcome above, there are alternative explanations for these behaviors as well. For example, wealth effects or house money effects might cause an increase in betting after a win (hot hand) (Thaler & Johnson 1990). Prospect theory's assumption of increased risk-seeking in losses might cause an increase in betting after a loss (stock of luck). In the lab, these effects can be separated by eliciting beliefs directly as in Ayton and Fischer (2004). In our empirical data we will not be able to distinguish between these alternative explanations.

chance-oriented. In a questionnaire design, he finds gamblers' fallacy behavior in luck-oriented individuals but no such behavior, and in particular, no dependence of current bets on past outcomes, in chance-oriented individuals.

In the field, previous work has also found individual heterogeneity in biased beliefs. Keren and Wagenaar (1985) examine blackjack play of 47 individuals who played at least 75 hands and changed their bets over time. Of these, 25 had relationships between previous outcomes and bet changes (thus, exhibiting a bias of some sort). Fourteen of them increased their bets after they won and decreased them after they lost (consistent with the hot hand), while 11 decreased their bets after winning and increased them after losing (consistent with stock of luck). As in these studies, we will use our data to analyze individual differences in betting behavior.

Only two previous papers examine field behavior at roulette. The first is an observational sociological field study by Oldman (1974) which informally reports both the gambler's fallacy and the hot outcome. He writes that "[t]he bet on a particular spin tends to be placed on outcomes that are 'due' either because they have not occurred for some time or because that is the way 'things are running.'" (p. 418). The second source, Wagenaar (1988, Ch. 4), discusses data from 29 roulette players in a casino who stayed between 1 and 18 spins each. Of the 11 players who varied their bets most, he finds after a win 39% of bets involve increased risk (hot hand) and 61% involve decreased risk (stock of luck). After a loss, 43% of bets involve decreased risk (hot hand) and 57% of bets do not (stock of luck). However, Wagenaar does not present an analysis of how individuals differ on this dimension.

While previous papers have investigated the gambler's fallacy and hot hand biases, our work makes two important and original contributions. First, it provides a field setting in which it is possible to investigate both biases at once. These biases have been analyzed together only in the lab (Ayton and Fischer 2004). Second, our empirical data will allow us to identify individual differences in these biases. We will be able to examine the correlation between these biases within the individual.[6]

1.3 Field data

In this study we use observational data from the field; individuals betting at roulette in a casino. Roulette is a useful game for a number of reasons. First, it is serially uncorrelated, unlike other casino games like blackjack or baccarat where cards are dealt without replacement. Second, each player has his or her own colored chips, thus tracking an individual's betting behavior is feasible. Finally, roulette is an extremely popular and accessible game which requires relatively little skill to play (unlike craps, for example, which is perceived as a game for experts). Thus roulette is likely to suffer from less selection bias than craps, although we are already selecting participants from the casino gambling population, mentioned above as an unavoidable selection bias.

1.3.1 Roulette

Roulette involves a dealer (sometimes two), a wheel and a layout. The wheel is divided into 38 even sectors, numbered 1-36, plus 0 and 00. Each space is red or black, with the 0 and 00 colored green. The wheel is arranged as shown in Figure 1, such that red and black numbers alternate.

Figure 1: The wheel

Players arrive at the roulette table and offer the dealer money (either cash or casino chips). In exchange, they are given special roulette chips for betting at this wheel. These chips are not valid anywhere else in the casino, and each player at the table has a unique color of chips. Players bet by placing chips on a numbered layout, the wheel is spun and a small white ball rolled around its edge. The ball lands on a particular number in the wheel, which is the winning number for that round, and is announced publicly by the dealer. Next, the dealer clears away all losing bets, players who had bet on the winning number (in some configuration) are paid in their own-colored chips and a new round of betting begins.

Figure 2 shows a typical layout, along with the types of bets that can be made. Unlike the wheel, the layout is arranged in numerical order. Players can place their bets on varying places on the layout. Bets of the type on the

[6] Our companion paper, Croson and Sundali (2005) has examined thee data at the aggregate level. There we provide evidence that the wheel is unbiased, that gambler's fallacy behavior is observed in outside bets after long streaks (5 and 6 observations of the same type), and that, in aggregate, individuals place more bets after they have won a previous bet than after they have lost one (or than on their first spin).

number 30 are called “straight up” bets. These are bets on a single number. If the number comes up on the wheel, this bet would pay the player 36 for 1 (35 to 1). That is, when 1 chip is bet, the dealer pays the player 35 chips directly, and the chip that was bet is not removed from the table. Bets of the type between the 8 and 11, “line bets” are bets on two numbers. If either of the numbers comes up, this bet pays the player 18 for 1. Players can also bet on combinations of 3 numbers (by the 13) which pay 12 for 1, combinations of 4 numbers (on the corner of 17-18-20-21) which pay 9 for 1, or combinations of 6 numbers (by the 22-25) which pay 6 for 1. Players can, of course, bet on “outside” bets like red/black, even/odd and low/high. These bets will not be included in our analysis, as they are not bet often enough to allow identification at the individual level, but are discussed in our companion paper on aggregate behavior, Croson and Sundali (2005).

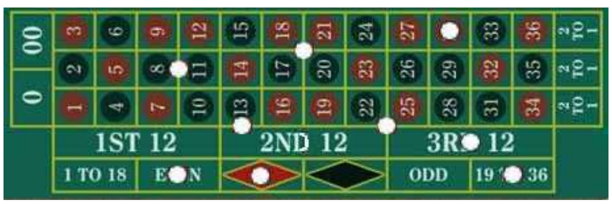

Figure 2: The layout

Notice that all these bets have the same expected value, -5.26% on a double-zero wheel.[7] Since the house advantage on (almost) all bets at the wheel is the same, there is no economic reason to bet one way or another (or for that matter, at all). In this paper, we will compare actual betting behavior we observe against a benchmark of random betting and search for systematic and significant deviations from that benchmark.

2 Method

The data were gathered from a large casino in Reno, Nevada, and were also used in Croson and Sundali (2005) to examine aggregate behavior.[8] Casino executives supplied the researchers with security videotapes for 18 hours of play of a single roulette table. The videotapes consisted of three separate six-hour time blocks over a 3-day period in July of 1998.[9] The videotapes provided an overhead view of the roulette area. The camera angle was focused on the roulette layout to allow the coding of bets placed and to protect player anonymity. Players were not directly visible, however individual bets could be tracked by the color of the chips being used. The videotape was subtitled with a time counter. Note that while many casinos employ electronic displays showing previous outcomes of the wheel, this casino had no such displays at the time the data was collected.

A research assistant was employed to view and record player bet data from these videotapes. Players were identified based upon the color of the chips being used to bet, the player’s location at the table, and any distinct characteristics of the player’s hand or arm such as jewelry, clothing, tattoos, etc. Players who ran out of chips and immediately bought more (of the same color) were coded as the same player. Players who ran out of chips and did not immediately buy more were coded as having left the table. When money was again exchanged for chips of that particular color, we assumed a new player had joined the table.[10]

The videotape methodology made it possible to view all of bets made by each player with a high degree of accuracy. However, while we could observe *if* a player bet on a particular number, given the angle of the camera (from above), we could not observe *how many* chips he or she bet on a particular number. Thus we simplified the data recording to include simply a bet being placed, without mention of how much the bet was. In order to be consistent in not recording the amount bet, we coded bets on multiple numbers (fractional bets like those in Figure 2) the same as we recorded bets on single numbers. For example, a player could place a single “corner bet” on 17, 18, 20, 21 by placing his chip at the intersection of these numbers. We recorded this bet as a bet placed on each of the four numbers. We limit our analysis in this paper to bets placed in the inside of the roulette layout, thus we do not count bets placed on black/red, even/odd, high/low, 1st, 2nd or 3rd 12 or columns in our data; the interested reader can find analysis of these outside bets in aggregate in Croson and Sundali (2005). After the assis-

[7] This statement is not strictly true. One bet has a house advantage of 7.89%. The bet involves placing a chip on the outside corner of the layout between 0 and 1. The bet wins if 0, 00, 1, 2 or 3 appears, but pays only 6 for 1 (as though the bet were covering 6 numbers instead of 5). We observed only 75 instances of this bet being placed (out of 22,527 bets). Only 11 different individuals placed this bet (out of 139 identifiable individuals in our data), and of them, only 6 placed this bet more than twice.

[8] At the time of data collection a casino in Washoe County, Nevada, was classified as “large” by the Nevada Gaming Control Board if total (yearly) gaming revenues for the property exceeds $36 million.

[9] The three time blocks were from 4:00 p.m. to 10:00 p.m., 8:00 p.m. to 2:00 a.m., and 10:00 p.m. to 4:00 a.m. These time blocks were appropriate since the majority of gaming business is done in the evening hours.

[10] This coding has the potential to introduce two possible errors; two different people could be counted as the same person, or the same person could be counted as two different people. We believe that the first of these errors is minimized; when chips were depleted and someone immediately purchased more, the coder could recognize from their hand characteristics if it was the same person. Additionally, this casino has many roulette tables, it was rare that this table was full or that people were waiting to buy in immediately after someone had gone bust. The second error may be somewhat more likely, here we rely on the observation that if an individual wants to rebuy, (s)he rarely waits to do so.

tant recorded all of the bets from the 18 hours of videotape, one of the principal investigators performed an audit check to insure accuracy.

3 Results

3.1 Descriptive statistics: The wheel and the bets

Nine hundred and four spins of the roulette wheel were captured in this data set (approximately 1 spin per minute). The expected frequency of a single number on a perfectly fair roulette wheel is 1/38 or 2.6%. In this sample the most frequent outcome was number 30 at 3.7%, the least frequent outcome was number 26 at 1.7%. These data provide no evidence that the wheel is biased.[11] Table 1 presents the outcomes and the bets placed during our sample.

If players bet randomly, we would expect them to bet on each number equally, thus 2.6% of the bets should fall on each number, independently of the history of numbers which have appeared. This independence is what we will test in our analyses.

3.2 Gambler's fallacy vs. hot outcome

We will use a general linear model to analyze the probability of a bet being placed on a number that has previously appeared, versus one which has not. Our dependent variable, P_{it} is binary; if a bet was placed on number i on spin t, we record a success (1). If no bet was so placed, we record a failure (0). Thus we will try to predict, on the basis of previous outcomes, whether a player will bet on a particular number.

Independent variables include an intercept, a measure of the hotness of a number, a control for the player's "favorite" numbers and a control for leaving a bet on the table. We measure a number's hotness by calculating a measure of how often the number i has appeared while the player was at the table in the spins *before* spin t. In particular, H_{it} is how many times number i has appeared while the player was at the table before round t minus the expected frequency of the number i appearing. This expected frequency is simply $(1-(37/38)^{t-1})$ where (t-1) is the number of trials observed by the player so far. This hotness measure thus calculates the actual frequency of a number appearing minus the expected frequency. If a number has appeared more than expected, this hotness measure is positive, otherwise it is negative.

[11] Based on the work of Ethier (1982), Keren and Lewis (1994) report that the number of observations necessary to detect a favorable number (bias) is generally quite large. For example, on a wheel with 37 numbers it would be necessary to view 30,195 spins in order to detect a bias of 1/33 with a 90% level of certainty.

If players bet according to the gambler's fallacy, the probability of their betting on a given number should be negatively related to its hotness measure; numbers which have come up more frequently while they were at the table are less likely to be bet on. In contrast, if players bet according to the hot outcome, the probability of their betting on a given number should be positively related to its hotness measure. Notice that this hotness measure is calculated separately for each individual in each period, based on what they have observed up to the point of placing their bets.

The second independent variable is an attempt to control for the baseline bets of individuals. Roulette players often bet the same numbers consistently and repeatedly; the bets don't vary with past outcomes. Thus, we need to control for these bets. Some players get lucky and hit those numbers (and others don't), which could cause the first type of players to look as though they were betting numbers which had come up before and the second, those which hadn't. Instead, we want to look at *deviations* from betting patterns as numbers come up. Thus in the model, we include F_{it}, the percentage of spins on which the player has bet on number i previously to period t.

We expect the coefficient on this variable to be significant and positive (if players bet on a number previously, they are more likely to bet on it again). However, our main reason for including it is to control for underlying personal preferences over numbers that might bias our coefficient of interest, the hotness measure. Thus a significant coefficient on the hotness measure measures a deviation from the expected betting pattern of an individual, given their bets up until now.

The final independent variable, L_{it}, controls for a behavioral anomaly particular to roulette. When a bet wins, the dealer pays the winnings directly to the player, but leaves the winning chip on the same spot on the table. Many players are reluctant to move this winning chip, claiming it is unlucky. If we were to count that unmoved chip as a bet, we would bias the results toward hot outcome, as players are often betting (by default) on numbers that have won in the previous round.[12] We control for this behavior by including an independent variable that equals one if an individual has bet on a number in the previous round and it has won, and a zero otherwise.

[12] This behavior is consistent with the status quo bias (Samuelson and Zeckhauser 1988) or the omission/commission bias (Ritov and Baron 1992, Baron and Ritov 2004), as this chip represents a bet that has been placed by default. Thus one can interpret a positive significant coefficient on this variable as evidence for these biases in this dataset. A positive coefficient on this variable is also consistent with Wagenaar (1988) who found 70 out of 75 winning bets in his data were not moved. However, as this is not the main focus of our paper, we do not provide a lengthy discussion of this finding. Interested readers are encouraged to contact the author for further discussion.

Table 1: Spin outcomes and player bets

Outcome	Frequency outcome	Percent outcome	Percent expected	Outcome −expected	Frequency bet	Percent bet
0/0	22	0.024	0.026	−0.002	354	0.016
0	25	0.028	0.026	0.001	442	0.020
1	23	0.025	0.026	−0.001	362	0.016
2	30	0.033	0.026	0.007	450	0.020
3	28	0.031	0.026	0.005	357	0.016
4	15	0.017	0.026	−0.010	375	0.017
5	28	0.031	0.026	0.005	636	0.028
6	20	0.022	0.026	−0.004	363	0.016
7	15	0.017	0.026	−0.010	682	0.030
8	26	0.029	0.026	0.002	633	0.028
9	23	0.025	0.026	−0.001	503	0.022
10	24	0.027	0.026	0.000	484	0.021
11	26	0.029	0.026	0.002	783	0.035
12	21	0.023	0.026	−0.003	360	0.016
13	21	0.023	0.026	−0.003	525	0.023
14	27	0.030	0.026	0.004	649	0.029
15	27	0.030	0.026	0.004	340	0.015
16	25	0.028	0.026	0.001	643	0.029
17	23	0.025	0.026	−0.001	1079	0.048
18	23	0.025	0.026	−0.001	518	0.023
19	30	0.033	0.026	0.007	595	0.026
20	24	0.027	0.026	0.000	983	0.044
21	26	0.029	0.026	0.002	447	0.020
22	32	0.035	0.026	0.009	576	0.026
23	24	0.027	0.026	0.000	746	0.033
24	18	0.020	0.026	−0.006	461	0.020
25	19	0.021	0.026	−0.005	521	0.023
26	15	0.017	0.026	−0.010	703	0.031
27	22	0.024	0.026	−0.002	490	0.022
28	25	0.028	0.026	0.001	827	0.037
29	23	0.025	0.026	−0.001	878	0.039
30	33	0.037	0.026	0.010	695	0.031
31	22	0.024	0.026	−0.002	664	0.029
32	29	0.032	0.026	0.006	925	0.041
33	17	0.019	0.026	−0.008	613	0.027
34	29	0.032	0.026	0.006	597	0.027
35	22	0.024	0.026	−0.002	627	0.028
36	22	0.024	0.026	−0.002	641	0.028

Table 2: Hot outcome results by individual

Coefficient	112 possible logistic models	39 logistic models w/o errors	112 possible linear models	93 linear models w/o errors
Negative Significant (GF)	17	9	19	19
Negative Nonsignificant (GF)	39	11	36	29
Positive Nonsignificant (HO)	37	10	34	25
Positive Significant (HO)	19	9	23	20

Thus our final model is

$$P_{it} = \alpha_0 + \alpha_1 H_{it} + \alpha_2 F_{it} + \alpha_3 L_{it} + \varepsilon$$

For each gambler we run two GLMs (one logistic and one linear). Of the 139 gamblers in our sample, not all had placed enough bets to allow us to estimate these models either with or without errors. Table 2 categorizes the results of the coefficient on the hotness measure (α_1) for each individual using a variety of techniques and error thresholds. Significant coefficients here represent estimates that are significant at the 5% level using a two-tailed test.

As Table 2 shows, we observe significant heterogeneity in the population. Approximately half of the players in our data (depending which model the reader prefers) can be categorized as gambler's fallacy players; when a number has previously appeared, the probability of their betting on it decreases. The other half of the players in our data can be categorized as hot outcome players; when a number has previously appeared, the probability of their betting on it increases.

One concern with this analysis, raised by an astute referee, is that running so many regressions must result in some false positives (or false negatives). To test for whether simple chance is causing our results, we conducted two further analyses. First, we looked at the underlying p-values from the regressions in each column. If these values had been generated randomly, we would expect them to be uniformly distributed between 0 and 1. We compared the actual p-values to the uniform distribution using the Kolgoromov-Smirnov test. We confidently reject the null hypothesis that the p-values were generated by chance for each of the four columns in Table 2 ($p < .01$ for all four comparisons). Within each column, we run a similar test for the positive significant/nonsignificant individuals, and the negative significant/nonsignificant individuals. Again, we confidently reject the null hypothesis that the p-values were generated by chance for each ($p < .01$ for all eight comparisons).

A more discrete analysis examines the existing categorizations. If the results were due to chance, we would expect 5% of the observations to fall in the negative significant category, 45% in the negative nonsignificant category, 45% in the positive nonsignificant category and 5% in the positive significant category. We compare the actual observations with this expected distribution using a chi-squared test. We robustly reject the null that the p-values were observed by chance ($p < .0001$ for all four columns). A similar test on only the positive (negative) observations yields similar results ($p < .0001$ for all eight comparisons).

Results from this field study are consistent with previous lab studies demonstrating individual heterogeneity in gambler's fallacy/hot outcome beliefs. While some gamblers cannot be classified reliably; those that can are roughly equally split between betting in a fashion consistent with the gambler's fallacy and the hot outcome. In the next subsection we continue our analysis of roulette data by examining the hot hand and stock of luck biases.

3.3 Hot hand vs. stock of luck

There is an important conceptual difference between a belief in hot outcomes (e.g., hot numbers) and the hot hand (e.g., a hot person). Our second set of analyses investigates whether individual's behavior is consistent with hot hand beliefs. To do this, we analyze whether gamblers bet on more or fewer numbers in response to previous wins and losses. Thus, if I've won in the past, I am hot and more likely to be (more) in the future.

We first examine the average number of bets an individual places after winning on the previous spin and after losing on the previous spin. If the former is greater than the latter, we say this person bets consistently with the hot hand. If the reverse, we say this person bets consistently with stock of luck. Of our 139 gamblers, 62 bet consistently with the hot hand and 32 with the stock of luck bias. Of the remaining 45 gamblers, 31 of them either only won or only lost at the table in our sample while 14 played for only one spin of the wheel.

As a second, more formal analysis we run a general linear model for each individual. The dependent variable is the number of bets placed on spin t and the independent variables include an indicator variable describing whether the individual has won or lost on spin t-1. Table 3 reports the number of subjects whose parameter value falls into each category. Ninety-six subjects could be categorized in this way without errors.

As with the previous analysis of the gambler's fallacy versus the hot outcome, we find significant individual heterogeneity in the hot hand/stock of luck biases. Here, more subjects act consistently with the hot hand bias (which predicts a positive relationship between previous wins and number of bets placed) than with the stock of luck bias (which predicts a negative relationship). Similar reliability tests as those described above yield similar results ($p < .01$ for the Kolgoromov-Smirnov tests and $p < .001$ for the chi-squared tests).

Table 3: Hot hand results by individual

Coefficient	96 linear models without errors
Negative Significant (SL)	6
Negative Nonsignificant (SL)	37
Positive Nonsignificant (HH)	41
Positive Significant (HH)	12

3.4 Correlation of Biases

Our data allow us to independently characterize individuals as gambler's fallacy/hot outcome players and as hot hand/stock of luck players. A further analysis examines the distribution of players over those four types. Table 4 presents this distribution, categorizing players based on the general linear models at the individual level reported in Table 2 (the final column) and Table 3 including those categorized as directional.[13] We exclude 11 players who are categorized on one dimension and not on another.

Table 4: Relationship between the biases

	Hot outcome	Gambler's fallacy
Hot Hand	10	42
Stock-of-Luck	32	5

A chi-squared test strongly rejects the null hypothesis of no relationship between the biases ($p < .0001$). In particular, there appears to be a correlation; players who act

[13]Other possible categorizations yield qualitatively identical results (e.g., using those categorized both significantly and nonsignificantly regardless of error, restricting attention to those categorized significantly, either only without error or all and using the logistic models to categorize the Gambler's Fallacy/Hot Outcome subjects rather than the linear models).

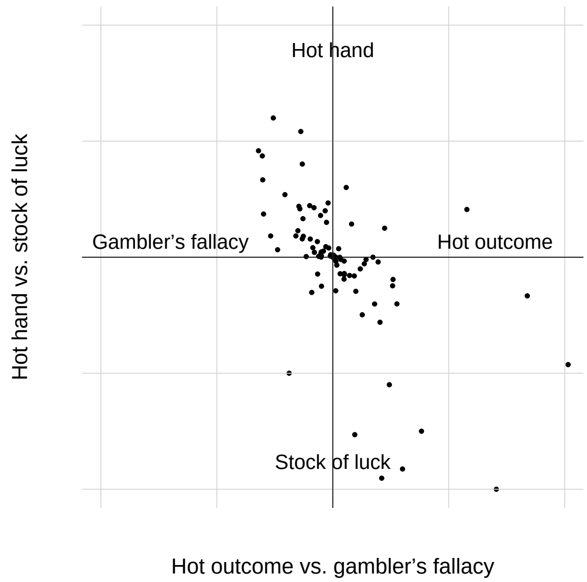

Figure 3: Relationship between biases

consistently with the gambler's fallacy (betting on numbers that haven't appeared previously), are more likely to act consistently with the hot hand (increasing the number of bets they place after a win). Almost half the subjects are in this first category, consistent with previous research demonstrating both biases in the lab. In contrast, players who act consistently with the hot outcome (betting on numbers that have appeared previously), are more likely to act consistently with the stock-of-luck bias (decreasing the number of bets they place after a win).

This relationship can be seen in Figure 3, below. Here we graph, for each of the 89 individuals characterized in Table 5, their regression parameters on the two biases.

What accounts for the pattern of individual beliefs found in Figure 3? While further research will be necessary to flesh out the variables underlying these patterns, we propose locus of control as an organizing explanation for this pattern. Originally developed by Rotter (1964), Zimbardo (1985) defines locus of control as: "...a belief about whether the outcomes of our actions are contingent on what we do (internal locus of control) or on events outside our personal control (external control orientation)" (p. 275).

Generally a person with an internal locus of control attributes outcomes to personal decisions and efforts while a person with an external locus of control attributes outcomes to chance or other external factors. Applying this concept to roulette, a person with an internal locus of control is likely to attribute previous wins to the decisions he made and thus to connect such winning with gambling skill. If a player has just won because of skill, then these skills should lead to more winning, which explains why players with such beliefs increase their bets after winning, exhibiting hot hand behavior. On the other hand, a person with an external locus of control attributes winning to simply luck. Thus a person with external locus of control concludes that winning again after a previous win is less likely and will decrease their bets after winning, exhibiting stock of luck behavior.

Remember that while the hot hand/stock of luck describes beliefs of outcomes of the individual (like wins and losses), the gambler's fallacy/hot outcome describes beliefs about outcomes of the random process (like heads or tails). So how would the beliefs of a person with an internal or external locus of control differ regarding random processes?

Consider first the person who has an external locus of control and thus attributes outcomes to luck (stock of luck). If one believes luck is in control of a random process and three heads in a row have appeared, then one should believe that luck will continue to control the outcomes and that another head will appear. Put another way, players who believe in luck are more likely to believe in streaks (hot outcomes) because luck produces streaks. Thus the external locus of control causes both stock of luck and hot outcome beliefs.

In contrast, a person with an internal locus of control who believes that winning is a result of skill is likely to reject the idea that the process producing the outcomes is random since this would mitigate the skill involved. A more plausible belief is that outcomes on the roulette wheel are controlled by some process that can be learned or discerned by the use of skill. When the internal person wins, it is confirmation that she has ascertained the pattern and this confidence leads her to bet more on the next spin of the wheel (hot hand). The most plausible cognitive explanation for her supposed pattern-detecting skill is representativeness, which explains why she bets consistently with the gambler's fallacy. Thus the internal locus of control causes both hot hand and gambler's fallacy beliefs.

Unfortunately we could not collect locus of control or other personality measures from our casino patrons, and thus cannot test our speculation of the underlying causes of the relationship between these two biases. Further lab testing will be necessary to address this question, and to compare this speculation with other candidate explanations for our results.

4 Conclusions and discussion

This paper uses observational data to demonstrate the existence and impact of the hot hand and gambler's fallacy biases. We demonstrate the existence of significant biases even in this, sophisticated, population, providing an important robustness check on previous laboratory data. Like this previous research, we observe significant individual heterogeneity in the population. Our participants are split almost evenly between betting in a way consistent with the gambler's fallacy and consistent with the hot outcome.

Importantly, however, our data allow us to investigate the correlation of these biases at the individual level. We find that gambler's fallacy players are more likely to also be hot hand gamblers. These relationships suggest there may be an underlying construct determining biased beliefs that further research might illuminate. Candidates for this construct have been suggested by us and others (locus of control, representativeness, cognitive reflection of Frederick [2005]), but further research in the lab will be need to identify these potential mediators.

These results are consistent with those previously observed in the lab (e.g., Ayton & Fischer, 2004; Chau & Phillips, 1995). That these observations are robust in the field with real money on the line and real participants is reassuring. However, the limitations inherent in field data admit of alternative interpretations of our results. For example, the hot outcome effect may be explained by an availability bias; individuals are more likely to bet on numbers that have recently won not because they believe these numbers more likely to win again but instead because they're easily called to mind. The hot hand effect may be explained by an income or house money effect; individuals bet on more numbers after they have won not because they believe that they (personally) are more likely to win again but because they're richer, or are playing with the house's money. While these alternative explanations can explain some results, they don't provide satisfactory explanations for the heterogeneity of the data at the individual level, nor for the correlation between the biases observed within the individual.

These limitations suggest further research combining empirical and questionnaire data in a way that we were prevented from accomplishing here. For example, a think-aloud protocol might provide evidence in favor or against these alternative explanations. Gathering psychological measures like locus of control as well as demographic information might help us to predict what type of biased beliefs an individual is likely to have. Finally, our data infers beliefs from observed actions; eliciting beliefs directly via a questionnaire, then observing actions would provide a useful check on our results. These combinations of field and lab data are attractive, but will require extreme cooperation from a casino, which is not currently available.

Other future projects might involve data from other non-autocorrelated casino games (e.g., craps, slot machines) both to replicate our current findings and to search for differences between the games. Finally, there are a number of other questions one might explore using the existing data including conformity (the correlation of betting across players as in Blank, 1968), the status quo bias (probability of leaving winning bets as they lie), the psychology of near misses (when an individual's bet almost wins), and when players leave the game (breaking even, busting out). While these data are not as targeted as that from the lab, we see empirical data as an opportunity to provide a robustness-check on (and external validity for) experimentally-observed biases.

Almost every decision we make involves uncertainty in some way, both over individual events and over sequences of events. Previous research has demonstrated a number of biases in how individuals perceive and react to this uncertainty, but the demonstrations have been primarily in the lab, using undergraduate student participants. This paper uses data from individuals gambling with their own money in a casino to test for the presence of these biases in a naturally-occurring environment. The behavior we observe is indeed consistent with previously-observed biases, providing an important robustness check on the previous research. We observe significant individual heterogeneity among the population in their direction and strength of each bias.

In addition, our data allows us to identify these biases separately within each individual, and to examine the correlation between them. We find a significant and positive correlation between individuals who act in accordance with gambler's fallacy beliefs and with hot hand beliefs, suggesting a unifying cause for the two illusions. Further research will be needed to identify this cause, and to help us predict an individual's biases and their resulting actions.

References

Ayton, P. & Fischer. I. (2004). The gambler's fallacy and the hot-hand fallacy: Two faces of subjective randomness? *Memory and Cognition, 32*, 1369–1378.

Baron, J. & Ritov, I. (2004). Omission bias, individual differences, and normality. *Organizational Behavior and Human Decision Processes, 94*, 74–85.

Blais, A-R., & Weber, E. U. (2006). A domain-specific risk-taking (DOSPERT) scale for adult populations. *Judgment and Decision Making, 1*, 33–47.

Blank, A. D. (1968). Effects of group and individual conditions on choice behavior. *Journal of Personality and Social Psychology, 8*, 294–298.

Chau, A. & Phillips, J. (1995). Effects of Perceived Control Upon Wagering and Attributions in Computer Blackjack. *The Journal of General Psychology, 122*, 253–269.

Croson, R. & Sundali, J. (2005). The gambler's fallacy and the hot hand: Empirical data from casinos. *Journal of Risk and Uncertainty, 30*, 195–209.

Edwards, W. (1961). Probability learning in 1000 trials. *Journal of Experimental Psychology, 62*, 385–394.

Estes, W. K. (1964). Probability learning. In A. W. Melton (ed.), *Categories of Human Learning*. New York: Academic Press.

Ethier, S. N. (1982). Testing for favorable numbers on a roulette wheel. *Journal of the American Statistical Association, 77*, 660–665.

Feldman, J. (1959). On the negative recency hypothesis in the prediction of a series of binary symbols. *American Journal of Psychology, 72*, 597–599.

Friedland, N. (1998). Games of luck and games of chance: The effect of luck- versus chance-orientation on gambling decisions. *Journal of Behavioral Decision Making, 11*, 161–179.

Frederick, S. (2005). Cognitive reflection and decision making. *Journal of Economic Perspectives, 19*, 25–42.

Gal, I. & Baron. J. (1996). Understanding repeated choices. *Thinking and Reasoning, 2*, 81–98.

Gilovich, T., Vallone, R. & Tversky, A. (1985). The hot hand in basketball: On the misperception of random sequences. *Cognitive Psychology, 17*, 295–314.

Harris, C. R., Jenkins, M., & Glaser, D. (2006). Gender differences in risk assessment: Why do women take fewer risks than men? *Judgment and Decision Making, 1*, 48–63.

Kahneman, D. & Tversky, A. (1972). Subjective probability: A judgment of representativeness. *Cognitive Psychology, 3*, 430–454.

Keren, G. & Lewis, C. (1994). The two fallacies of gamblers: Type I and Type II. *Organizational Behavior and Human Decision Processes, 60*, 75–89.

Keren, G. & Wagenaar, W.. (1985). On the Psychology of Playing Blackjack: Normative and Descriptive Considerations with Implications for Decision Theory. *Journal of Experimental Psychology: General, 114*, 133–158.

Langer, E. (1975). The illusion of control. *Journal of Personality and Social Psychology, 32*, 311–328.

Lee, W. (1971). *Decision theory and human behavior*. New York: Wiley.

Leopard, A. (1978). Risk preference in consecutive gambling. *Journal of Experimental Psychology: Human Perception and Performance, 4*, 521–528.

Lindman, H., & Edwards, W. (1961). Supplementary report: Unlearning the gambler's fallacy. *Journal of Experimental Psychology,* 62, 630.

Oldman, D. (1974). Chance and skill: A study of roulette. *Sociology, 8*, 407–426.

Rapoport, A. & Budesceu, D. (1992). Generation of random series in two-person strictly competitive games.

Journal of Experimental Psychology: General, 121, 352–363.

Ritov, I. & Baron, J. (1992). Status-quo and omission bias. *Journal of Risk and Uncertainty, 5*, 49-61.

Rotter, J. B. (1966). Generalized expectancies for internal vs. external control of reinforcement. *Psychological Monographs, 80*, 1–28.

Samuelson, W. & Zeckhauser, R. (1988). Status quo bias in decision making. *Journal of Risk and Uncertainty, 1*, 7–59.

Thaler, R. H., & Johnson, E. J. (1990). Gambling with the house money and trying to break even: The effects of prior outcomes on risky choice, *Management Science 36*, 643–660.

Tversky, A. & Kahneman, D. (1971). Belief in the law of small numbers. *Psychological Bulletin, 76*, 105–110.

Tversky, A. & Kahneman, D. (1974). Judgement under uncertainty: Heuristics and biases. *Science, 185*, 1124–1131.

Wagenaar, W. (1972). Generation of random sequences by human subjects: A critical survey of literature. *Psychological Bulletin, 77*, 65–72.

Wagenaar, W. (1988). *Paradoxes of gambling behavior*. Lawrence Erlbaum Associates, London, UK.

Weber, E. U., Anderson, C. J. & Birnbaum, M. H. (1992). A theory of perceived risk and attractiveness. *Organizational Behavior and Human Decision Processes, 52*, 492–523.

Zimbardo, P. G. (1985). *Psychology and Life*. Glenview, IL: Scott Foreman and Company.