2-2007

Strategic Technology Choice and Capacity Investment Under Demand Uncertainty

Manu Goyal

Serguie Netessine
University of Pennsylvania

Recommended Citation

Goyal, M., & Netessine, S. (2007). Strategic Technology Choice and Capacity Investment Under Demand Uncertainty. *Management Science, 53* (2), 192-207. http://dx.doi.org/10.1287/mnsc.1060.0611

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Abstract

This paper studies the impact of competition on a firm's choice of technology (product-flexible or product-dedicated) and capacity investment decisions. Specifically, we model two firms competing with each other in two markets characterized by price-dependent and uncertain demand. The firms make three decisions in the following sequence: choice of technology (technology game), capacity investment (capacity game), and production quantities (production game). The technology and capacity games occur while the demand curve is still uncertain, and the production game is postponed until after the demand curve is revealed.

We develop best-response functions for each firm in the technology game and compare how a monopolist and a duopolist respond to a given flexibility premium. We show that the firms may respond to competition by adopting a technology which is the same as or different from what the competitor adopts. We conclude that contrary to popular belief, flexibility is not always the best response to competition—flexible and dedicated technologies may coexist in equilibrium. We demonstrate that as the difference between the two market sizes increases, a duopolist is willing to pay less for flexible technology, whereas the decision of a monopolist is not affected. Further, we find that a firm that invests in flexibility benefits from a low correlation between demands for two products, but the extent of this benefit differs depending on the competitor's technology choice. Our results indicate that higher demand substitution may or may not promote the adoption of flexibility under competition, whereas it always facilitates the adoption of flexibility without competition. Finally, we show that contrary to intuition, as the competitor's cost of capacity increases, the premium a flexible firm is willing to pay for flexibility decreases.

Keywords

technology, flexible capacity, uncertainty, equilibrium, competition

Disciplines

Marketing | Organizational Behavior and Theory | Other Social and Behavioral Sciences

Strategic Technology Choice and Capacity Investment under Demand Uncertainty.*

Manu Goyal[†] Serguei Netessine[‡]
The Wharton School, University of Pennsylvania
Philadelphia, PA-19104

November, 2003, Revised July 2004

Abstract

The goal of this paper is to study the impact of competition on a firm's technology choice (product-flexible or product-dedicated) and capacity investment decisions. Specifically, we model two firms competing with each other in two markets characterized by price-dependent and uncertain demand. The firms make three decisions in the following sequence: choice of technology (technology game), capacity investment (capacity game) and production quantities (production game). The technology and capacity games occur while the demand curve is still uncertain and the production game is postponed until after the demand curve is revealed.

We formally characterize a Markov-perfect Nash equilibrium in the capacity and production games and under suitable assumptions solve the games in closed form. Further, we develop best-response functions for each firm in the technology game and compare how a monopolist and a duopolist respond to a given flexibility premium. We show that the cost premium which the duopolist is willing to accept, when investing in flexbile technology, is higher (smaller) than the premium which the monopolist is willing to accept, if the competitor invests in dedicated (flexible) technology. Finally, we characterize situations that give rise to each of the three possible equilibrium outcomes of the technology game: both firms may invest in dedicated technology, both may invest in flexible technology or one firm may invest in dedicated and the other in flexible technology. The last (asymmetric) outcome can arise even if firms are perfectly symmetric.

*We would like to thank Gerard Cachon, Gurhan Kok, Justin Ren, Mihkel Tombak, Fuqiang Zhang, three anonymous referees and the Associate Editor for helpful comments. Thorough and thoughtful criticism by Jiri Chod is gratefully acknowledged. Financial support was provided by the Mack Center for Technological Innovation.

[†]goyalm@wharton.upenn.edu

[‡]netessine@wharton.upenn.edu

1 Introduction

In the last decade, firms in a variety of industries have come under intense pressure to offer a large variety of products in response to highly variable and ever changing customer tastes. Consequently, in order to compete effectively in the marketplace, it is necessary to switch from manufacturing one product to another with ease. The answer to this challenge came in the form of flexible manufacturing systems (FMS). Apart from the obvious benefit of being able to hedge against uncertainty in demand (by cutting production of goods for which demand turns out to be much lower than forecasted and increasing production of goods that turn out to have high demand), some studies suggest that FMS also empowers the firm with other advantages such as a strategic weapon against competition (see Fine 1993). Our aim in this paper is to marry these two advantages of flexibility and study the strategic value of flexibility in an uncertain environment characterized by inter-firm competition.

Though it seems beyond doubt that FMS is a powerful competitive weapon, we find that in practice some companies still prefer to utilize dedicated technology (that can manufacture only a single product on a production line). While dedicated technology does not provide a hedge against uncertainty, it typically has the advantage of lower production costs. For instance, Upton (1995) studies 61 plants in the paper industry, an industry in which products are quite comparable across manufacturers (e.g., letter-size paper) and the same fundamental processes are used everywhere. Nevertheless, some firms have adopted flexible manufacturing technology while others have not. Therefore, in the market, products manufactured by different companies - and hence different technologies - compete directly.

The literature has showcased flexible manufacturing as a strategic competitive edge and as a hedge against uncertainty (Fine 1993, Roller and Tombak 1991). For example, Mackintosh (2003) notes that "... given all the benefits of flexibility, the surprise is that it has taken US manufacturers so long to start emulating their Japanese rivals". At the same time, normative models that actually quantify the benefits of product flexibility and aid in decision making under *both* uncertainty and competition are hard to come by. Moreover, given the evidence that dedicated and flexible technologies often co-exist, it is not at all clear that investment in flexibility is a universal competitive response. We thus address the following questions: What should the firm's best response be if the competitor invests in flexible (dedicated) technology? Does the technology investment depend on the competitor's choice of technology? Is the impact of problem parameters different with and without competition? Since answers to these questions are difficult to find in the extant literature, this paper makes an attempt to fill this void.

Using a stylized model, we analyze the technology choice, dedicated (D) or flexible (F), of a firm under competition in an environment with stochastic demand. We focus on product flexibility which entails the ability to produce several products on the same capacity without incurring major switch-over costs as a response to uncertainty in demand[1]. Specifically, two firms each manufacture two products that are sold in two markets. The firms can invest in either two dedicated (cheaper) production lines or one flexible (more expensive) production line. Independent of the technology choice, both firms manufacture both products and are in direct competition with each other in two different markets. The firms make three sequential decisions. The first is the choice of production technology (technology game). The second is capacity investment given the technology decision (capacity game). These two decisions are ex-ante before demand is revealed. Our setting, in which technology and capacity are decided before demand uncertainty is resolved, reflects the long lead time involved in capacity acquisition. The final decision concerns the quantities to be produced (production game) constrained by the earlier two decisions and is ex-post (responsive manufacturing). The market price is a function of the total amount of product offered to the market by the two firms (Cournot competition).

We formally characterize a Markov-perfect Nash equilibrium (MPNE) in the capacity and production games. For ex-ante symmetric firms and under appropriate assumptions on the demand distribution, we solve for the capacities in closed form and derive closed form expressions for expected prices and expected profits. Further, we develop best-response functions for each firm for a given strategic choice of technology by the rival as a function of the mean and the variance of the demand distribution and the costs of the two technologies. The effect of competition on the technology choice of firms is distilled by contrasting the actions of a duopolist with those of a monopolist. We show that the cost premium which the duopolist is willing to accept, when investing in flexible technology, is higher (smaller) than the premium which the monopolist is willing to accept, if the competitor invests in dedicated (flexible) technology. Thereafter, we establish the Nash Equilibrium in the technology game. We show that any of the two symmetric - (F, F) and (D, D) - or two asymmetric - $(D, F), (F, D)$ - equilibria can arise depending on the specific values of the problem parameters. We show that the bias towards flexibility is increasing with rising demand uncertainty, increasing cost of the dedicated technology, decreasing mean demand and increasing product substitutability. Somewhat surprisingly, even when two firms are completely symmetric, it is possible for an asymmetric equilibrium to emerge (i.e., flexible and dedicated technologies co-exist). This is because when both firms invest in flexible technology, the benefit of flexibility gets divided between them. Since flexible technology is expensive, one of the firm finds it profitable to invest in dedicated technology instead.

The rest of the paper is organized as follows. Section 2 surveys related literature while emphasizing the positioning of our work. In Section 3 we formulate the stochastic 3-stage game and, moving backwards, solve the last two stages using a Markov-perfect Nash equilibrium. In Section 4 the technology game is solved under appropriate assumptions on the demand distribution and our findings are summarized and discussed in Section 5.

[1]Hence, the flexible firm can adjust the allocation of capacity between products in response to demand uncertainty.

2 Literature Survey

Two streams of literature are relevant to our study: the first explores flexibility as a hedge against demand uncertainty and the second studies flexibility as a strategic weapon under competition. These are combined in our study for we believe they are equally important in practice. To the best of our knowledge, there are no other papers that model technology choice in a stochastic environment and under competition (Fine 1993 notes that there are only a few game-theoretic models that analyze competitive dynamics involving flexible manufacturing technology). Some of the relevant papers specifically focus on the FMS while others more generally analyze "flexible capacity" investment, either manufacturing or some other. We do not make a distinction between FMS and flexible capacity since our model applies to both.

Papers in the first stream consider investment in flexible vs dedicated capacity in the absence of competition and analyze the trade-off between the higher cost of flexible capacity and its ability to hedge against demand uncertainty by manufacturing multiple products. All papers in this stream consider a monopolistic firm. Fine and Freund (1990) model a firm manufacturing n products within two decision epochs. In the first stage, the firm must choose the capacity levels for the n dedicated resources as well as for one flexible resource that can manufacture all n products. In the second stage (after demand realization) the firm decides on production quantities given the capacity constraints. Fine and Freund (1990) show that the decision to invest in flexible technology is based on the cost differential between the dedicated and flexible technologies. Van Mieghem (1998) develops a similar model and finds that flexibility is beneficial even with perfect positive correlation if one product is more profitable than the other. Other works that have looked at similar issues are Harrison and Van Mieghem (1998) and Netessine et al. (2002). In all these papers, product prices are exogenous to the model. Chod and Rudi (2004) endogenize pricing decisions and analyze a firm manufacturing two products while investing in a flexible resource only. The capacity decision is ex-ante and the production decision is ex-post; moreover, the price of a product is a function of production quantity. Hence, Chod and Rudi (2004) look at responsive pricing and responsive manufacturing and categorize their impact on the management of a flexible resource. They conclude that the flexible capacity and expected profits are increasing in demand variance, and that positive correlation increases capacity investment while decreasing expected profits. Further, the benefits of flexibility are quantified by comparing a firm investing in flexible technology with a firm investing in two dedicated production lines under the assumption that investment costs for a flexible resource and dedicated resources are the same. The work of Chod and Rudi (2004) is close to ours since we utilize the same base model: the production/pricing decision is made after demand uncertainty is realized and capacity investment is made before. However, we account for the different costs of the two technologies and, more importantly, introduce competition. As opposed to Chod and Rudi where flexibility was always preferred, we provide conditions under which flexibility is not adopted in equilibrium or when it is adopted by one firm but not the other. Hence, the model of Chod and Rudi (2004) is a special case of ours. Bish and Wang (2003) consider a problem setting similar to Chod and Rudi (2004) but allow the firm to invest simultaneously into dedicated and flexible capacities.

The second stream of literature looks at the strategic value of flexibility in the absence of demand uncertainty. Hence, flexibility is shown in the light of economies of scope. Fine and Pappu (1990) and Roller and Tombak (1990, 1993) model two firms each manufacturing two products and competing with each other. The firm investing in flexible technology can enter both markets, while the firm investing in dedicated technology finds it economical to enter only one market. The technology choice is modeled as a 2x2 game in strategic form. The firms are trapped in a Prisoner's Dilemma like situation: while each can choose one market and make monopoly profit in it, both end up choosing flexible technology and hence intensifying competition under the threat that the rival might choose flexible technology and invade. As a result, these papers show that flexible technology makes firms worse off. In addition, Roller and Tombak (1990, 1993) show that both symmetric and asymmetric equilibria can exist and the prices are lowest when both firms choose flexible technology. They also show that decreasing product substitutability promote flexible technology. As opposed to Roller and Tombak, we model the situation in which the firm investing in the dedicated technology participates in both markets. Hence, in our model flexibility does not inherently fuel competition. Accordingly, prices in our model are not the lowest for the case in which both firms invest in flexible technology (they are, in fact, the highest). Moreover, decreasing product substitutability does not necessarily favor flexible technology in our work. The reason for this difference is that we model demand stochasticity as opposed to the deterministic case analyzed by Roller and Tombak. As detailed in section 4.4, the effect of an increase in the substitutability parameter is to decrease the "non-stochastic" component of the profit function (consistent with Roller and Tombak) and to increase the "stochastic component" (not modeled in Roller and Tombak). In another related work, Anand and Girotra (2003) analyze the benefits of delayed differentiation under competition. Delayed differentiation is similar to manufacturing flexibility since it allows the firm to hedge against demand uncertainty. However, in their model firms compete in one market only while being monopolists in the other market they serve. Hence, the setup and resulting insights are very different.

Several other papers that do not fit into the two streams above are, nevertheless, relevant to our work. In their seminal paper, Jordan and Graves (1995) look at total flexibility vs partial flexibility through the concept of chaining (a chain consists of product-plant links: more links correspond to higher flexibility). They find that adding limited flexibility in the right place can achieve nearly all the benefits of total flexibility in terms of hedging against demand uncertainty. Graves and Tomlin (2003) further extend this work to a multi-echelon supply chain setting. Finally, Parker and Kapuscinski (2003) study the role of flexible technology in entry deterrence (we do not model entry decisions).

To summarize, the prior literature has only partially addressed the question of technology choice under both demand uncertainty and competition: the focus has been on one or the other but not on both. Since the value of flexibility as a hedge against demand uncertainty has attracted significant attention in the literature, it seems imperative to understand how this value is affected by competition. Such an understanding will help us advise practicing managers on the strategic value of flexibility in a competitive environment. Hence, we contribute to the extant literature on manufacturing/capacity flexibility by *simultaneously* studying the impact of both demand uncertainty and competitive pressures on the technology choice of firms and attempting to bridge the gap between these two streams of literature. While our model is somewhat similar to Roller and Tombak (1990, 1993) and Fine and Pappu (1990) in which a "two-firm two-product" competitive scenario is modeled, we also incorporate demand uncertainty similar in spirit to the works of Fine and Freund (1990), Van Mieghem (1998) and particularly Chod and Rudi (2004).

3 The Model

There are two firms indexed by i and j, $i, j = 1, 2$. The firms are assumed to be risk neutral and are expected profit maximizers. Each firm manufactures two products indexed by $y = 1, 2$ and is engaged in competition in both markets with the other firm. By making the market entry decision exogenous to the model, we create a level playing field for the two technologies in terms of economies of scope and hence isolate flexibility as a hedge against uncertainty in a competitive environment. As detailed in the introduction, this is a three-stage sequential game: the technology game, the capacity game and the production game. Each stage is a simultaneous-move non-cooperative game with complete information. These are simplifying assumptions: in practice decisions might be neither simultaneous nor immediately observable by the competitors. Relaxing these assumptions is a valuable direction for future research but is outside of the scope of our paper. In the first stage, each firm can invest either in a flexible technology (F) that manufactures both products on the same line or in a dedicated technology (D) for each of the products separately. The firm cannot invest in flexible and dedicated technology simultaneously: this restriction may be imposed in practice due to the administrative costs associated with producing the same product in more than one production facility. Moreover, assuming that only one technology can be chosen allows us to emphasize the trade-off of main interest: dedicated vs flexible technology.

Depending on the technology choices in the first-stage game, three equilibria (which we refer to as markets) can potentially emerge. The superscripts refer to the type of market in which the firms operate: (m) refers to the mixed market in which one firm invests in flexible and the other in dedicated technology (also referred to as the D, F or F, D market), (f) refers to a pure flexible (F, F) market and (d) refers to a pure dedicated (D, D) market. The subscripts refer to the type of capacity, whether flexible (f) or dedicated (d), which can also be indexed by y for each of the products. If it is necessary to differentiate firms, the firm index i, j will appear in the subscript as well.

In the second stage (the capacity game), each firm invests in production capacity (one capacity if the firm pursues flexible technology and two capacities if the firm pursues dedicated technology) denoted by K. For instance, K^f_{fi} is the flexible capacity of firm i in the pure flexible market. Capacity investment is costly: let the cost of purchasing the flexible resource be c_f per unit and the cost of the dedicated resource be c per unit for each product with $c_f > c$, which is similar to Fine and Freund (1990) and several subsequent papers. Investment costs are linear in capacity and are the same for both firms, reflecting a common set of technologies available to the competitors. The expected optimal profit of the firm in this stage is denoted by Π so, for example, Π^m_{di} denotes the expected profit of firm i competing in the mixed market and investing in dedicated capacities K^m_{1i} and K^m_{2i}.

The last stage of the game is ex-post and is concerned with the quantities (denoted by q) to be put in the market given the first two decisions and demand realization. This decision is ex-post, reflecting that at the time of production the firm is better aware of market conditions. The inverse demand function for product y is $P_y(Q_y, Q_{3-y}) = A_y - Q_y - \beta Q_{3-y}$ for all $y = 1, 2$ where Q_y is the total quantity of product y put in the market by the two firms combined (Cournot competition model). The parameter $\beta \in (-1, 1)$ is the cross-elasticity[2] parameter where $\beta > 0$ ($\beta < 0$) signifies that the products are substitutes (complements) in a Cournot game. Note that substitutability implies that the demand for a product increases with the increase in price of the other product and vice versa for complementarity. The quantity of product y put in the market by firm i is q_{yi} so that $Q_y = q_{yi} + q_{yj}$. The demand intercepts, $A_y \in \Re_+$, are random draws from a bivariate continuous distribution function $F(.,.)$ with a density function $f(.,.)$. Whenever independence is assumed, the joint distribution simply becomes the product of the marginal distributions. Denote the mean of the marginal distribution by μ_y and the variance by σ^2_y. Profits in the production game are denoted by π.

The following standard conventions are used throughout. E denotes the expectation operator with respect to the random variables A_y. The state-space for realizations of (A_1, A_2) is divided into disjoint sets denoted by Ω_l. To avoid trivialities, we impose the assumption that $c_f < \mu_y$, $y = 1, 2$ (i.e., marginal cost is lower than expected maximum price – otherwise capacity investment is not profitable in expectation). The marginal cost of production is assumed to be the same for both technologies (Fine and Pappu 1990, Roller and Tombak 1990, 1993) and is normalized to zero. For expositional purposes, we now let $\beta = 0$ but this assumption will be relaxed in the last section.

We solve the problem backwards. The production game of the third stage is solved first and q_{yi} and

[2] We use "cross-elasticity" term loosely in the paper to mean the measure of product substitutability/complementarity. The mathematical definition of cross-elasticity is somewhat different.

q_{yj} are obtained for every demand intercept realization. Using MPNE solutions in each stage, we work our way backwards to the capacity decision and finally to the Nash Equilibrium in the technology game.

3.1 Problem formulation

Consider the technology game in Figure 1 that is schematically represented as a 2x2 matrix typical for strategic-form games (e.g., Prisoner's dilemma). Each firm is endowed with two strategies (D and F) and matrix entries signify second-stage profits.

Firm i \ Firm j	D	F
D	Π_i^d , Π_j^d	Π_{di}^m , Π_{fj}^m
F	Π_{fi}^m , Π_{dj}^m	Π_i^f , Π_j^f

Figure 1. The technology game.

The equilibrium technology choice in Figure 1 is a pure strategy MPNE of the 2x2 non-cooperative game in strategic form. As is typical for such games, the solution is obtained by considering the best-response functions of each firm given the technology choice of the other firm. Since we are unable to predict the equilibrium of the technology game up front, we proceed by analyzing capacity and production choices in all possible equilibrium outcomes of the technology game. The optimization problem for a firm i that invests in either a dedicated or a flexible technology in any market given some strategic choice by rival (firm j) is

Firm i invests in dedicated technology	Firm i invests in flexible technology
$\Pi_i = \max_{K_{1i},K_{2i}} \{E_A(\pi_i) - c(K_{1i} + K_{2i})\}$	$\Pi_i = \max_{K_{fi}} \{E_A(\pi_i) - c_f(K_{fi})\}$
$\pi_i = \max_{q_{1i},q_{2i}} \sum_{y=1}^{2} [(A_y - (q_{yi} + q_{yj}))\, q_{yi}]$	$\pi_i = \max_{q_{1i},q_{2i}} \sum_{y=1}^{2} [(A_y - (q_{yi} + q_{yj}))\, q_{yi}]$,
$s.t.\ 0 \le q_{yi} \le K_{yi},\ y = 1, 2.$	$s.t.\ q_{1i} + q_{2i} \le K_{fi}, q_{yi} \ge 0,\ y = 1, 2.$

Table 1. The general problem formulation.

We now proceed by solving each of the three optimization problems (since markets D, F and F, D are symmetric). Solutions for the quantity and capacity games are lengthy and tedious. Hence, we provide only an outline of the methodology below (full solutions to the quantity game and structural results concerning the existence/uniqueness of MPNE in capacities and production quantities as well as optimality conditions for equilibrium capacity choices are found in Goyal and Netessine 2004). We then impose certain assumptions on the distribution of demand intercepts to obtain solutions to the capacities and the firm profits in closed form. Thereafter, the technology game is analyzed in full.

3.2 The production and capacity games

In general, even though we assume that the two firms are ex-ante symmetric, it is possible that the equilibrium outcome of the technology game is asymmetric. Moreover, even if two firms end up in a pure (flexible or dedicated) market, i.e., a symmetric equilibrium, they might still select different capacities. In the Goyal and Netessine (2004), we solve production and capacity games for arbitrary (possibly asymmetric) capacity choices. However, we focus here on symmetric equilibria in the capacity game. This is a standard assumption in multi-stage Cournot games (see Salant and Shaffer 1999) and we are able to show that the symmetric equilibrium in the capacity game in these markets is unique. Moreover this restriction does not eliminate any equilibria in the technology game (i.e., the mixed market still arises) so the symmetry assumption does not reduce the richness of the game.

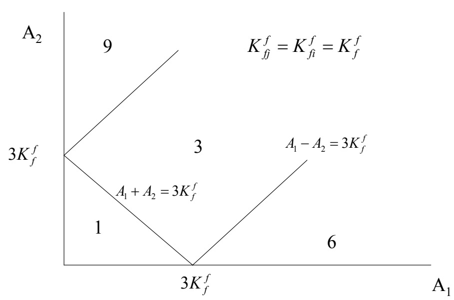

Figure 2. Pure flexible market

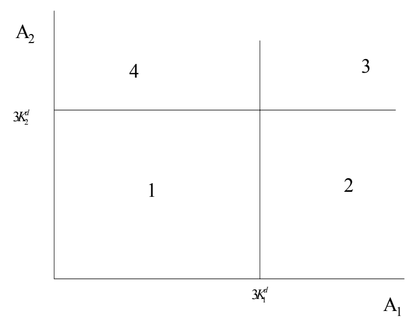

Figure 3. Pure dedicated market

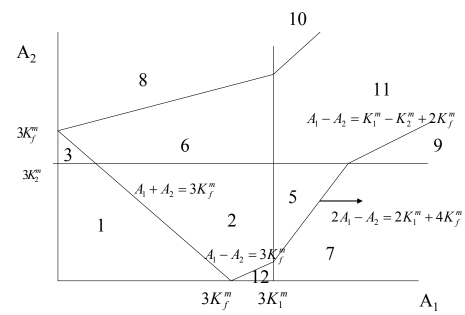

Figure 4. The mixed market.

We begin with the production game. Imagine that the firms have already played the earlier two stages of the game, the technology and the capacity games, i.e., the firms are endowed with a technology (D or F) and a capacity level. In the last stage, firms play a constrained Cournot duopoly game (with profits represented by π in Table 1). The visual state-space representation of the production game is given in Figures 2-4 for all the three markets. Along the axes we have all possible realizations of the demand intercepts[3]. Capacity constraints split the state-space into different areas. For instance, consider the mixed market (Figure 4): in area Ω_1 none of the firms is capacity constrained, in area Ω_{11} both firms are capacity constrained and in area Ω_7 the dedicated firm is capacity constrained for product 1 and the flexible firm manufactures only product 1 (due to a high enough realization of A_1 as compared to A_2), etc. In each of the areas in all the markets, firms compete on Cournot quantities and hence, from standard economic theory, we know that the equilibrium in the production game exists and is unique. The firms solve for the optimal production quantities in each of the areas. Price is determined as per Cournot competition and profits are gleaned. The expected profit Π_i for a given capacity choice in each market is calculated by weighing the profits in each of the areas by the probability of realizing that area. Differentiating this

[3]Remember that this is a game of complete information. Hence, each firm noiselessly observes the capacity and technology choice of the other firm as well as the demand intercept realizations.

expected profit gives us the first-order conditions for the capacity game.

4 The Technology game

Having analyzed the production and the capacity games, we are now ready to move ahead to the choice of technology. However, the problem in its current form is rather intractable. Although optimality conditions for capacities are available, they define capacities implicitly rather than explicitly since each integral in the first-order conditions has limits Ω_l, $\forall l$ which depend on capacity decisions themselves. In order to solve the technology game, we need to compare profits, which, needless to say, cannot be compared without first obtaining capacity investment decisions in closed form. In order to simplify the problem, while making no assumptions on the form of the marginal distributions for the demand intercepts, we do impose restrictions on the domain of the distribution. These assumptions A-1 through A-3 are summarized below.

A-1. The first simplification is to assume that the probability distributions of demand intercepts are independent, i.e., $dF(x_1, x_2) = f(x_1)f(x_2)\,dx_1dx_2$. The main qualitative insights are invariant to this assumption since correlation between products can easily be incorporated at the cost of added complexity (see Chod and Rudi 2004). The impact of correlation is no different in our model than what has been shown in earlier works: negative correlation favors flexible technology and the advantage of flexibility decreases as correlation rises.

A-2. We assume that both products are always manufactured by the flexible firm. This implies that it is never the case that the demand intercept realization is so high for one product so as to render the other product uneconomical. This is a plausible assumption from a practical perspective: it is highly unlikely that a capacity able to produce two products would be built without a high enough level of certainty that both products would actually be produced later on. As we will see shortly, such an assumption does not take away the essence of the problem (hedging against demand uncertainty under competition) as all the components of the problem (demand variability, cost differential, etc.) are still at play. Mathematically, this translates to the following: in the pure flexible market, we assume that $\Pr\{(A_1, A_2) \in \Omega_{6,9}\} = 0$ (see Figure 2) and in the mixed market we assume that $\Pr\{(A_1, A_2) \in \Omega_{7,8,9,10,12}\} = 0$ (see Figure 4).

A-3. We assume that each firm follows a *clearance* strategy as opposed to a *holdback* strategy in all markets. Clearance implies that firms always produce to capacity. This is a common assumption in the literature; for example, in addition to Chod and Rudi (2004), it is used by Deneckere et al. (1997), Anand and Girotra (2003). In practice, many firms find it difficult to cut back production below capacity in view of large fixed costs associated with production ramp-up and commitments to suppliers (these issues are not modeled explicitly here). For instance, as Mackintosh (2003) points out, car makers have been forced to slash prices to keep lines running as models fall out of favor with the public rather than keep plants idle. This implies that firms in the automotive industry follow a strategy close to clearance. Mathematically, $q_{1i} + q_{2i} = K_{fi}$, $i = 1, 2$ if a firm invests in flexible capacity and $q_{yi} = K_{yi}$, $y, i = 1, 2$ if a firm invests in dedicated capacity.

Assumptions A-2 and A-3 are in the spirit of Chod and Rudi (2004). The result of assumptions A-2 and A-3 is that the only area having a non-zero probability is Ω_3 in the pure flexible and the pure dedicated markets and Ω_{11} in the mixed market. Even though our assumptions may actually hold naturally in many problem settings, we nevertheless develop appropriate analytical restrictions on the domain of the probability distributions of demand intercepts. Hence, rather than assuming that firms behave sub-optimally according to assumptions A-2 and A-3, we restrict the distribution of (A_1, A_2) so that assumptions A-2 and A-3 always hold (see Lemma 1 after Propositions 1, 2 and 3 which develop some results necessary to prove the Lemma). Note also that Chod and Rudi (2004) test assumptions A-2 and A-3 numerically and find that they generally yield solutions that are very close to optimal.

A few more comments are in order here. First, note that the clearance assumption essentially renders trivial the last-stage (production) decisions for the firm investing in dedicated technology. However, the firm investing in flexible technology (in both the pure flexible and the mixed market) still has to allocate its capacity to each of the two products. Hence, production decisions are not trivial for a firm investing in flexible technology and the capacity game and the production game still need to be considered separately. Second, we need to worry about the non-negativity of prices in light of assumption A-3 (though negative prices may be tenable in many situations). Non-negativity of prices was ensured as we essentially solved a Cournot duopoly game in each of the areas for all three markets (this is shown formally in Goyal and Netessine 2004). But now, by imposing A-3, we break away from the standard Cournot game in areas where the capacity does not bind (such as area Ω_1 in Figure 2) and non-negativity of prices cannot be assured in these areas. However, by imposing Lemma 1, we ensure that no realization of demand intercepts is such that we fall in these areas where the capacity does not bind. Hence, in effect, by invoking the Lemma, we automatically rule out non-negative prices.

We now obtain closed-form solutions and profit expressions for each of the three markets under the above three assumptions. Note that since the solutions are unique, there is a unique equilibrium in the capacity game in each of the three markets under these assumptions. The next three Propositions provide the solution for the three markets (proofs are in Goyal and Netessine 2004).

Proposition 1 *For firms operating in the pure flexible market,*

(i) the MPNE in capacity is $K_f^f = (\mu_1 + \mu_2 - 2c_f)/3$,

(*ii*) *the expected profit at equilibrium is* $\Pi_f^f = (\mu_1 - c_f)^2/9 + (\mu_2 - c_f)^2/9 + \left(\sigma_1^2 + \sigma_2^2\right)/18$,

(*iii*) *the expected price for a product* $y = 1, 2$ *is* $P_y^f = (\mu_y + 2c_f)/3$.

Note that while equilibrium capacity and prices are functions of the mean of the demand intercepts and of the cost of flexible capacity only, profit is also an increasing function of the variance of the distribution of the demand intercepts. Naturally, a higher cost of flexible capacity leads to higher prices, lower capacity investment and lower profits. A higher mean of the demand intercepts leads to higher prices, capacities and profits. All these results are similar to Chod and Rudi (2004). We consider the pure dedicated market next.

Proposition 2 *For firms operating in a pure dedicated market,*

(*i*) *the MPNE in capacity is* $K_y^d = (\mu_y - c)/3$, $y = 1, 2$,

(*ii*) *the expected profit at equilibrium is* $\Pi_i^d = (\mu_1 - c)^2/9 + (\mu_2 - c)^2/9$, $i = 1, 2$,

(*iii*) *the expected price for a product* $y = 1, 2$ *is* $P_y^d = (\mu_y + 2c)/3$.

The game in the pure dedicated market simplifies to the standard Cournot competition results since the two markets are independent of each other. Finally, we turn to the mixed market.

Proposition 3 *For the firms operating in the mixed market:*

(*i*) *the MPNE in capacity for a*

(*a*) *flexible firm is:* $K_f^m = (\mu_1 + \mu_2 + 2c - 4c_f)/3$,

(*b*) *dedicated firm is:* $K_y^m = (5\mu_y - \mu_{3-y} + 4c_f - 8c)/12$, $y = 1, 2$,

(*ii*) *the expected profit at equilibrium for a*

(*a*) *flexible firm is:* $\Pi_f^m = (\mu_1 - \mu_2)^2/32 + (\mu_1 + \mu_2 + 2c - 4c_f)^2/18 + \left(\sigma_1^2 + \sigma_2^2\right)/8$,

(*b*) *dedicated firm is:* $\Pi_d^m = (\mu_1 - \mu_2)^2/16 + (\mu_1 + \mu_2 + 2c_f - 4c)^2/18$,

(*iii*) *the expected price for a product* $y = 1, 2$ *is* $P_y^m = (7\mu_1 + \mu_2 + 8c_f + 8c)/24$.

The solution for the mixed marked is evidently the most convoluted. Now capacity choices and profits of both firms depend on all problem parameters: e.g., optimal product-flexible capacity depends on the cost of dedicated capacity and vice versa. We see that, quite intuitively, as c rises (c_f falls), flexible capacity rises, dedicated capacity falls, profit of the firm investing in dedicated capacity falls and profit of the firm investing in flexible capacity rises. Prices, however, rise in both c and c_f. The effects of changing μs and σs are intuitive as well.

Having derived the solution for the capacity game under assumptions A-1 through A-3, we now take a step back and develop the lemma that specifies conditions under which assumptions A-2 and A-3 do not alter the solution. In the interest of simplicity, we henceforth assume that the two distributions are symmetric, i.e., $\mu_1 = \mu_2 = \mu$ and $\sigma_1 = \sigma_2 = \sigma$.

Lemma 1 *If there exist* $A_{\max}$ *and* $A_{\min}$ *such that* $\Pr\{A \in (A_{\min}, A_{\max})\} = 1$ *and* $A_{\max} - A_{\min} \leq \min[c, (4/3)(\mu + c - 2c_f)]$*, then A-2 and A-3 hold with probability one. In other words, A-2 and A-3 hold if* c_f *does not exceed value* $c_{f(crit)} = (\mu + c)/2 - (3/8)\min((A_{\max} - A_{\min}), c)$.

Proof. We prove the lemma in two stages. We first develop bounds on the realizations A_1 and A_2 such that A-2 holds with probability one for the pure flexible and the mixed markets. We then develop conditions under which A-3 holds. Combining the two completes the proof.

For the pure flexible market, A-2 holds if $\Pr\left\{|A_1 - A_2| \leq 3K_f^{fh}\right\} = 1$ (we add a superscript h to indicate that this is the optimal capacity under hold-back to distinguish it from the capacity derived under A-1 through A-3). It can be shown that $K_f^{fh} > K_f^f$. (Since Π_i^{fh} is concave, one can show that $\left(\partial\Pi_i^{fh}/\partial K_f^f\right)|_{K_f^f} \geq 0$; see Chod and Rudi 2004 for details). Hence, $\Pr\left\{|A_1 - A_2| \leq 3K_f^f\right\} = 1 \Rightarrow \Pr\left\{|A_1 - A_2| \leq 3K_f^{fh}\right\} = 1$. For A-2 to hold in the pure flexible market, we must have

$\Pr\{|A_1 - A_2| \leq 2(\mu - c_f)\} = 1$ by Proposition 1. Similarly for the mixed market, it is sufficient to show that $\Pr\left\{A_1 - A_2 < K_1^{mh} - K_2^{mh} + 2K_f^{mh}\right\} = 1$ and $\Pr\left\{A_2 - A_1 < K_2^{mh} - K_1^{mh} + 2K_f^{mh}\right\} = 1$. Since for a symmetric distribution $K_1^{mh} = K_2^{mh}$, we have $\Pr\left\{|A_1 - A_2| < 2K_f^{mh}\right\} = 1$. Again one can show that $K_f^m \leq K_f^{mh}$. Hence, the necessary condition for A-2 to hold in the mixed market is $\Pr\{|A_1 - A_2| < (4/3)(\mu + c - 2c_f)\} = 1$ by Proposition 3. Given that $c < c_f < \mu$, it can easily be shown that $(4/3)(\mu + c - 2c_f) < 2(\mu - c_f)$. Hence, we have $\Pr\{|A_1 - A_2| < (4/3)(\mu + c - 2c_f)\} = 1$ for A-2 to hold. This holds with probability one if $|A_1 - A_2|_{\max} < (4/3)(\mu + c - 2c_f)$ or in other words if $A_{\max} - A_{\min} < (4/3)(\mu + c - 2c_f)$.

If A-3 holds, then no realization of A_1 and A_2 falls in area Ω_1 in Figures 2 and 4 and clearance is optimal since it coincides with holdback. For this to happen we need

$\Pr\left\{A_1 + A_2 > 3K_f^{mh}\right\} = \Pr\left\{A_1 + A_2 > 3K_f^{fh}\right\} = \Pr\left\{A_y > 3K_y^{dh}\right\} = 1$ for $y = 1, 2$. Take the case of a pure flexible market. Let K_f^{fu} be the optimal capacity in the deterministic case when $A_1 = A_2 = A_{\max}$. Then, $K_f^{fu} = (2/3)(A_{\max} - c_f) \geq K_f^{fh}$ and $\Pr\{A_1 + A_2 > 2(A_{\max} - c_f)\} = 1 \Rightarrow \Pr\left\{A_1 + A_2 > 3K_f^{fh}\right\} = 1$. For this to hold for all realizations of A, we must have $A_{\min} + A_{\min} > 2(A_{\max} - c_f)$ or $A_{\max} - A_{\min} < c_f$. Similarly for the mixed market, $A_{\max} - A_{\min} < 2c_f - c$ and for the dedicated market $A_{\max} - A_{\min} < c$. Hence, for clearance to be optimal we must have $A_{\max} - A_{\min} < c$. Taking the intersection of the conditions for A-2 and A-3, we finally obtain $A_{\max} - A_{\min} \leq \min[c, (4/3)(\mu + c - 2c_f)]$. ■

4.1 Comparison of prices and capacities

It is insightful at this point to compare the total capacity into which firms invest and the corresponding (expected) prices in all possible technology equilibria (we continue working with symmetric distributions of intercepts to minimize the number of variables). Under the clearance strategy, the total capacities translate directly into expected prices. Hence, we can compare prices/capacities across different outcomes of the technology game as follows:

Proposition 4 *The total capacities in each of the markets compare as follows:* $\left(K_{1i}^d + K_{2i}^d + K_{1j}^d + K_{2j}^d\right) > \left(K_f^m + K_1^m + K_2^m\right) > \left(K_{fi}^f + K_{fj}^f\right)$. *The expected prices compare as follows:* $P^f > P^m > P^d$.

The total capacity (of the two firms combined) is always lowest in the pure flexible market. This implies that expected price is highest in a pure flexible market. This result is due to the cost differential between the dedicated and flexible technologies (note that if $c = c_f$ all expected prices and capacities are the same). This result is different from some of the earlier works (see Roller and Tombak 1990, 1993 and Fine and Pappu 1990) that model flexibility of scope in the sense that the flexible firm has the ability to serve two markets while the dedicated firm serves a single market. Hence, in their models, flexibility enhances competition. However, in our case, given that both firms serve both markets, the pure flexible market is the least competitive (in the sense that prices are the highest). This result has found some support in the popular press. For example, Mackintosh (2003) points out: "Introducing flexible technology in factories should help moderate the fierce price wars under way in North America and Europe ..." which is consistent with our analytical results.

4.2 Best-response functions

We are ready to characterize the best responses of the firms in the technology game. We characterize the best response of firm i to a given technology choice for firm j and analogous results hold for the best response of firm j. However, before we do this, it will be helpful to first look at how a monopolist would behave under identical circumstances; that is, under which conditions would a monopolist choose dedicated or flexible technology. This will help us in distilling the effect of competition on the technology choice of the firm. Suppose that the monopolist manufactures the same two products and invests in either one flexible production line or two separate dedicated production lines. It can be shown that if the monopolist invests in flexible technology, then under assumptions A-1 through A-3, his optimal capacity investment is given by $K_f^M = (\mu - c_f)$ and he makes an expected profit of $\Pi_f^M = \left(\sigma^2/4\right) + (\mu - c_f)^2/2$ (see Chod and Rudi 2004 for details). If he invests in dedicated technology, then the capacity for the two products combined is $K_d^M = (\mu - c)$ and the total expected profit is $\Pi_d^M = (\mu - c)^2/2$. Comparing the two profits, it is straightforward to show that for $c_f < c_{fM}(\mu, \sigma, c) = \left(\mu - \sqrt{(\mu - c)^2 - \sigma^2/2}\right)$, the monopolist invests in flexible technology and otherwise invests in dedicated technology.[4]

[4]Notice that we choose to express the monopolist's decision in terms of a threshold on the cost of flexible capacity for we feel that this is the most intuitive comparison. Evidently, the same could be done for c, μ and σ.

For convenience, Table 2 below summarizes the profits for the firms for a given set of technology choices in the competitive environment (the first entry is for the row player and the second entry is for the column player).

	D	**F**
D	$(2/9)(\mu - c)^2, (2/9)(\mu - c)^2$	$(2/9)(\mu + c_f - 2c)^2, \sigma^2/4 + (2/9)(\mu + c - 2c_f)^2$
F	$\sigma^2/4 + (2/9)(\mu + c - 2c_f)^2, (2/9)(\mu + c_f - 2c)^2$	$\sigma^2/9 + (2/9)(\mu - c_f)^2, \sigma^2/9 + (2/9)(\mu - c_f)^2$

Table 2. Profit for each outcome of the technology game.

To characterize the best-response function for a firm i, we consider two possible choices of firm j: investment in either flexible or in dedicated technologies.

Proposition 5 *When firm j invests in dedicated technology*

(i) The best response of firm i is to invest in flexible technology as long as $c_f < \bar{c}_f(\mu, c, \sigma)$ where

$$\bar{c}_f(\mu, c, \sigma) = \left(\mu + c - \sqrt{(\mu - c)^2 - (9/8)\sigma^2}\right)/2.$$

Else firm i invests in dedicated technology.

(ii) $\bar{c}_f(\mu, c, \sigma)$ is convex increasing in σ^2, increasing in c and decreasing in μ.

(iii) $\bar{c}_f(\mu, c, \sigma) \geq c_{fM}(\mu, \sigma, c)$.

Proof. To show (i), define the incremental profit firm i will make by investing in flexible technology rather than in a dedicated technology given that firm j invests in dedicated technology as

$$\begin{aligned} \Delta\Pi_i^{F-Dj} &= \{\Pi_{fi}^m - \Pi_i^d \mid \text{firm } j \text{ invests in dedicated technology}\} \\ &= \sigma^2/4 + (2/9)(\mu + c - 2c_f)^2 - (2/9)(\mu - c)^2. \end{aligned}$$

The best response of firm i is to invest in flexible technology as long as $\Delta\Pi_i^{F-Dj} > 0$ or else it invests in dedicated technology. We will look for the best-response function vis-a-vis the cost of flexible capacity. $\Delta\Pi_i^{F-Dj}$ is quadratic convex in c_f. Roots can be found as $\left(\mu + c \pm \sqrt{(\mu - c)^2 - (9/8)\sigma^2}\right)/2$ and call the lower root c_{fL} and the upper root c_{fU}. It can be shown that the minima of the convex curve is achieved for $c_f = (\mu + c)/2 = c_{f\min} > c_{f(crit)}$. Also by geometry we know that $c_{fL} \leq c_{f\min} \leq c_{fU}$. Since the upper root is always above the critical value $c_{f(crit)}$ we can safely ignore it.[5] By convexity, $\Delta\Pi_i^{F-Dj} > 0$ for

[5] Observe that, without the restriction $c_f > c_{f(crit)}$, we would be left with an unsatisfying result that the flexible technology becomes more attractive as the cost of flexibility increases for some set of problem parameters. This observation further justifies our effort in developing a proper technical condition in Lemma 1 rather than assuming that the firm behaves suboptimally according to A-2 and A-3.

$c_f < c_{fL} = \overline{c}_f$ and the result follows. Result (ii) can be shown through differentiation. To show (iii), observe first that for $\sigma = 0, \overline{c}_f\left(\mu, c, \sigma\right) = c_{fM}\left(\mu, c, \sigma\right) = c$. For $\sigma > 0$,

$$\partial \overline{c}_f / \partial\left(\sigma^2\right) = (9/32)\left(\sqrt{(\mu - c)^2 - (9/8)\sigma^2}\right)^{-1} > (1/4)\left(\sqrt{(\mu - c)^2 - \sigma^2/2}\right)^{-1} = \partial c_{fM}\left(\mu, \sigma, c\right) / \partial\left(\sigma^2\right).$$

Hence, $\overline{c}_f\left(\mu, c, \sigma\right)$ increases faster in σ^2 than $c_{fM}\left(\mu, \sigma, c\right)$ does so that $\overline{c}_f\left(\mu, c, \sigma\right) \geq c_{fM}\left(\mu, \sigma, c\right)$. ■

The above proposition gives us the best response to a competitor investing in dedicated capacity in the form of a threshold value for c_f, $\overline{c}_f\left(\mu, c, \sigma\right)$. Hence, for $c < c_f < \overline{c}_f\left(\mu, c, \sigma\right)$ the best response of firm i to a choice of dedicated technology by firm j is to invest in flexible technology. For $c_f > \overline{c}_f$ firm i usually invests in dedicated technology, but not always since for $\sigma^2 > 8/9\left(\mu - c\right)^2$, $\overline{c}_f \notin \Re$ (in other words, the equation $\Delta\Pi_i^{F-Dj} = 0$ has no real roots). This implies that for high enough variance firm i will always prefer flexible technology if firm j invests in dedicated technology.

$\overline{c}_f\left(\mu, c, \sigma\right)$ is a function that is convex increasing in the variance of the distribution of the intercepts. Hence, the higher the variance, the higher the threshold and firm i prefers flexible technology for a wider range of costs c_f. Furthermore, the threshold also increases in the cost of dedicated technology and decreases in the mean of the demand intercepts. Finally, from part (iii) of the proposition we obtain the impact of the competitive presence on the behavior of a firm. Visual representation of this result is shown in Figure 5, where the cost threshold of the monopolist is juxtaposed against the cost threshold of a duopolist when the competitor invests in dedicated technology. Note that $\overline{c}_f\left(\mu, c, \sigma\right) > c_{fM}\left(\mu, \sigma, c\right)$ and $\overline{c}_f\left(\mu, c, \sigma\right)$ is more convex. Hence, when a firm faces competition and the competitor invests in dedicated technology, the firm invests in flexible technology for a wider range of costs than it would without any competition. In other words, competition raises the threshold beyond which flexible technology is not desirable. Yet another way to emphasize this effect is to say that for $c_{fM}\left(\mu, \sigma, c\right) < c_f < \overline{c}_f\left(\mu, c, \sigma\right)$, a firm facing no competition *would not* invest in a flexible resource but a duopolist *would* when there is a competitor who invests in dedicated technology. We now turn to analyzing the best response to a competitor investing in flexible technology.

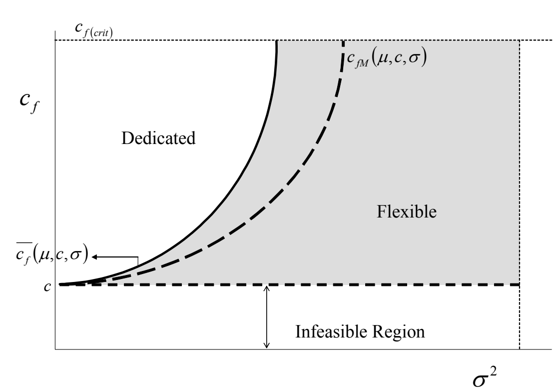

Figure 5. Best response to the dedicated technology.

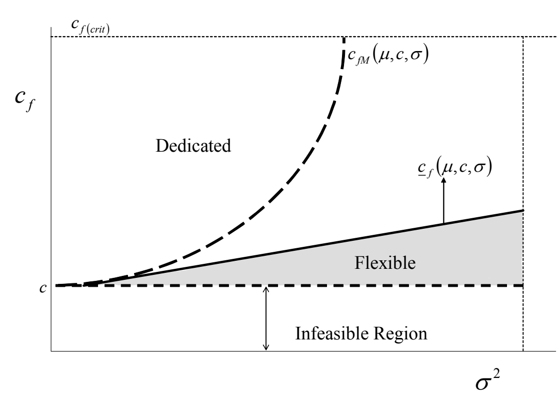

Figure 6. Best response to the flexible technology.

Proposition 6 *When firm j invests in flexible technology*

(i) The best response of firm i is to invest in flexible technology as long as $c_f < \underline{c}_f(\mu, c, \sigma)$ where

$$\underline{c}_f(\mu, c, \sigma) = c + \sigma^2 / (8(\mu - c)).$$

Else firm i invests in dedicated technology.

(ii) $\underline{c}_f(\mu, c, \sigma)$ is linearly increasing in σ^2, increasing in c and decreasing in μ.

(iii) $\underline{c}_f(\mu, c, \sigma) \leq c_{fM}(\mu, \sigma, c)$.

Proof. Similar to the previous proposition, define

$$\begin{aligned}\Delta\Pi^{F-Fj} &= \left\{\Pi_i^f - \Pi_{di}^m \mid \text{firm } j \text{ invests in flexible}\right\} \\ &= \sigma^2/9 + (2/9)(\mu - c_f)^2 - (2/9)(\mu + c_f - 2c)^2,\end{aligned}$$

which is the incremental profit that firm i makes by choosing flexible technology over dedicated technology. Firm i invests in flexible technology as long as $\Delta\Pi^{F-Fj} > 0$. It is easy to verify that $\Delta\Pi^{F-Fj}$ is linearly decreasing in c_f. The root of the equation $\Delta\Pi^{F-Fj} = 0$ is $\underline{c}_f(\mu, c, \sigma) = c + \sigma^2/8(\mu - c)$. Hence, $\Delta\Pi^{F-Fj} > 0$ for $c_f < \underline{c}_f(\mu, c, \sigma)$ and the result follows. To show (iii), observe that for $\sigma = 0$, $\underline{c}_f(\mu, c, \sigma) = c_{fM}(\mu, \sigma, c)$. For $\sigma > 0$, it is easy to see that

$$\partial \underline{c}_f(\mu, c, \sigma)/\partial(\sigma^2) = (1/8)(\mu - c)^{-1} < (1/4)\left(\sqrt{(\mu - c)^2 - \sigma^2/2}\right)^{-1} = \partial c_{fM}(\mu, \sigma, c)/\partial(\sigma^2)$$

and the result follows. ■

Similar interpretation can be attached to $\underline{c}_f(\mu, c, \sigma)$ with respect to the three variables as was done for $\overline{c}_f(\mu, c, \sigma)$. This time, however, comparing the decision of a duopolist with that of a monopolist gives an opposite interpretation. From Figure 6, we see that the duopolist's best response when the competitor invests in flexible technology lies *below* the threshold curve for a monopolist. This means that when a firm faces competition and the competitor invests in flexible technology, the firm invests in flexible technology over a smaller range of costs than it would if there were no competition. For example, for costs such that $\underline{c}_f(\mu, \sigma, c) < c_f < c_{fM}(\mu, c, \sigma)$, a monopolist invests in flexible capacity while a duopolist does not if there is a competitor investing in flexible technology.

The above two propositions show that a firm behaves differently under competition when making its technology choice than without competition. When the rival invests in dedicated technology, the firm is more insensitive to the cost of flexible technology than a monopolist and invests in it for higher costs. On the other hand, if the rival firm invests in flexible technology, the firm's cost threshold curve dips below that of a monopolist. The firm is now more cost sensitive and finds it economically viable to invest in flexible technology for lower costs than a monopolist. We provide the following intuition behind this result.

It is convenient to think about total market potential as consisting of two parts: one is deterministic and depends on the mean of the demand intercepts, while the other is stochastic and depends on the variance of the distribution of the demand intercepts. Recall further that profit of the firm investing in dedicated technology depends on the mean of the demand intercepts, but the profit of the firm investing in flexible technology also depends on (it is actually increasing in) the variance of the distribution of the demand intercepts. Hence, the firm investing in flexible technology is able to appropriate both deterministic and stochastic components of the market potential while the firm investing in dedicated technology is able to appropriate only the deterministic component. Further, given that the competitor invests in dedicated technology, the firm ought to anticipate that he will use all his capacity in each market. This reduces the deterministic component of the market potential but leaves the stochastic component unchanged. Hence, flexible technology is more attractive. If, in contrast, the competitor invests in flexible technology, the firm should anticipate that he will chase the larger market once demand intercepts are realized. This, in effect, dampens the variability that the firm perceives: a spike in the size of one market due to a favorable demand realization will be moderated by the competitor flooding that market. The effective variability falls and flexible technology is less valuable.

Hence, if only one firm in the market invests in flexible technology, this firm enjoys higher benefits from uncertainty (the threshold cost in Figure 5 is convex increasing). However, if two firms invest in flexible technology, the benefits to each firm from the stochastic component gets divided (the threshold cost in Figure 6 is linear increasing). Hence, we conclude that flexibility is more valuable when the competitor is not using it. Conversely, when the competitor is using flexible technology, the benefits of flexibility diminish.

4.3 The Nash Equilibrium of the technology game

The best-response functions derived in the previous section put us in a position to determine the equilibria for the technology game. We limit our analysis only to the pure strategy equilibria that are described in the following proposition.

Proposition 7 *The Nash equilibrium in the technology game is characterized as follows:*

Case 1: $c_f < \underline{c}_f(\mu, c, \sigma)$ *then (F,F) is the unique MPNE.*

Case 2: $c_f > \overline{c}_f(\mu, c, \sigma)$ *then (D,D) is the unique MPNE.*

Case 3: $\underline{c}_f(\mu, c, \sigma) < c_f < \overline{c}_f(\mu, c, \sigma)$ *then (F,D) and (D,F) are the two MPNE.*

The Nash Equilibrium in the technology game is illustrated in Figure 7, which is obtained by merging Figures 5 and 6. Roughly speaking, low uncertainty and high cost of flexibility lead to an equilibrium in which both firms prefer dedicated technology, and high uncertainty and low cost of flexibility lead to an equilibrium in which both firms invest in flexible technology. These results are expected given the knowledge of choices that the monopolist makes. What is more interesting is that the interplay between the two upper bounds for the cost of flexibility, each for a different strategy by the other firm, results in the interesting case of an asymmetric equilibrium even though the two firms are entirely symmetric. This observation suggests that different technologies may co-exist in a competitive market and no firm will want to deviate from its choice, which is consistent with Upton (1995). Of course, in the asymmetric equilibrium, a natural question arises as to which firm would choose dedicated technology and which firm would choose flexible technology. One possible solution to resolve this indeterminacy is mixed strategies: each firm would select either F or D with some probability such that in equilibrium, each is indifferent towards choosing F or D. Finally note that the response of a monopolist as measured by the threshold cost $c_{fM}(\mu, \sigma, c)$ is shown against the two best responses of a duopolist in Figure 7. In some sense, a monopolist's behavior averages out the limiting behavior of a duopolist.

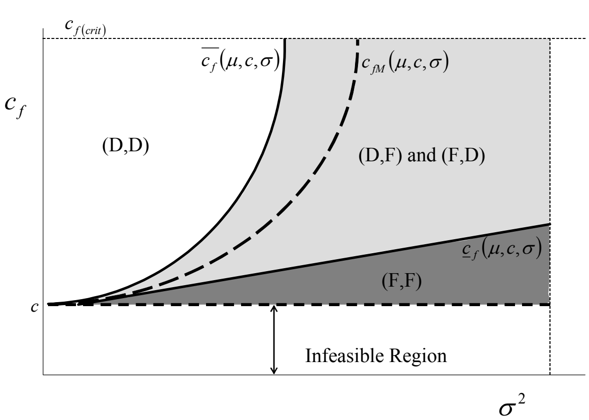

Figure 7. Nash Equilibrium in the technology game.

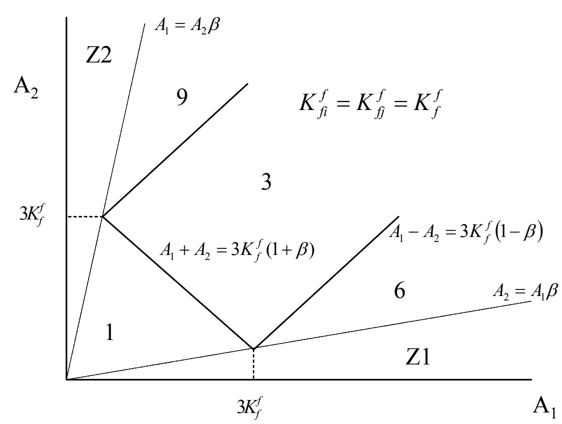

Figure 8. Pure flexible market, $\beta > 0$.

4.4 Dependent products: cross-elasticity of demand

In the previous analysis we have made an assumption that the two products manufactured by firms are entirely independent, i.e., the price of one product does not depend on the price/amount put on the market of the other product. In practice, however, we would expect some level of complementarity or substitutability between the products. Hence, we now relax the independence assumption between the two products and allow the demand to be linked by a cross-elasticity parameter.

In this section, we look into a pure flexible market in some detail. Specifically, we look at how the cross-elasticity parameter affects the production game and how the state space of the problem is modified. The intuition gained here applies equally well to the other two markets and hence we do not reproduce the modified state-space representation for them. Under assumptions A-1 through A-3 we solve the capacity game explicitly. Thereafter, we show how our results are modified by the cross-elasticity parameter.

Consider the case of the pure flexible market. To gain intuition with regard to the impact of the cross-elasticity parameter, consider the production game under the assumption that there is no capacity constraint. It is easy to show that the profit-maximizing quantity for a firm in any of the markets for product y is $q_y^{un} = (A_y - \beta A_{3-y})/3(1-\beta^2)$. Evidently, for $A_y < \beta A_{3-y}$, $q_y^{un} < 0$, production quantity is negative. Hence, given $\beta > 0$, there is a set of demand intercept realizations for which it is never optimal to produce product y and consequently modification of the state space is needed to ensure quantities are non-negative. This is shown for the pure flexible market in Figure 8 (for expositional convenience, we restrict our attention to a symmetric case). By this, we also ensure that prices are non-negative since in effect we solve a Cournot Game with non-negative optimal quantities in each of the areas (see Goyal and Netessine (2004) for details).

Areas Z1 and Z2 do not appear without cross-elasticity. In these areas, it is never optimal to manufacture products 2 and 1, respectively. Contrast this with the cases in which $\beta = 0$ and $\beta < 0$. For the former, unless the demand intercept realization for product y, A_y, is zero, the unconstrained production quantity is strictly positive. However, for $\beta > 0$, even though $0 < A_y < A_{(3-y)}\beta$, it is optimal not to manufacture product y. For $\beta < 0$, even though $A_y = 0$, it is still optimal to manufacture product y ($q_y^{un} = A_{(3-y)}\beta/3(1-\beta^2)$). Here is the intuition: when $\beta < 0$, the quantity demanded for product y increases as the quantity of product $(3-y)$ increases since the products are complements. Hence, even though the demand for a product is zero, due to its positive impact on the sale of the other product, it may still be beneficial to manufacture it. For instance, companies manufacture and sell printers at a loss and make money on the cartridges.

To summarize, the effect of $\beta > 0$ is to *squish* the state space by bringing the two vertical axes closer together. This gels well with the notion of substitutability: because the products are *substitutable* (or in other words are *similar),* the state space is no longer defined by a set of perpendicular boundaries. Rather, the two axes come closer together as β increases. Hence, as $\beta \to 1$, the production area in the pure flexible market approaches zero (there is no need for flexible capacity). Note that the equations that define the various boundaries of the state space are now a function of β. It is easy to see that Figure 8 merges into Figure 2 for $\beta = 0$. Similar conclusions follow for the pure dedicated and the mixed markets. It is worthwhile to note that the products being substitutable has no connection to the dependence/independence of the distributions for the demand intercepts.

In Goyal and Netessine (2004) we show that the symmetric equilibrium in the capacity game for the pure dedicated market is unique for $\beta < 1/3$ and for the pure flexible market it is unique for all β. We now move on to the solution of the game under assumptions A-1 through A-3 and $\beta < 1/3$. In Tables 3 and 4 below we summarize optimal capacities and profits.

	D	F
D	$2(\mu-c)/3\,(1+\beta)\,,2(\mu-c)/3\,(1+\beta)$	$2(\mu+c_f-2c)/3\,(1+\beta)\,,2(\mu+c-2c_f)/3\,(1+\beta)$
F	$2(\mu+c-2c_f)/3\,(1+\beta)\,,2(\mu+c_f-2c)/3\,(1+\beta)$	$2(\mu-c_f)/3\,(1+\beta)\,,2(\mu-c_f)/3\,(1+\beta)$

Table 3. Total capacities for each outcome of the technology game.

	D	F
D	$\frac{2(\mu-c)^2}{9\,(1+\beta)},\frac{2(\mu-c)^2}{9\,(1+\beta)}$	$\frac{2(\mu+c_f-2c)^2}{9\,(1+\beta)},\frac{\sigma^2}{4\,(1-\beta)}+\frac{2(\mu+c-2c_f)^2}{9\,(1+\beta)}$
F	$\frac{\sigma^2}{4\,(1-\beta)}+\frac{2(\mu+c-2c_f)^2}{9\,(1+\beta)},\frac{2(\mu+c_f-2c)^2}{9\,(1+\beta)}$	$\frac{\sigma^2}{9\,(1-\beta)}+\frac{2(\mu-c_f)^2}{9\,(1+\beta)},\frac{\sigma^2}{9\,(1-\beta)}+\frac{2(\mu-c_f)^2}{9\,(1+\beta)}$

Table 4. Profit for each outcome of the technology game.

The following observations are in order. The capacity investment decreases in β for all markets. Hence, the more substitutable the products, the lower the investment in capacity. This is consistent with the findings of earlier papers modeling cross-elasticity (e.g.,. Roller and Tombak 1993). However, more interesting is the behavior of the profit function for each of the markets. It is easy to see that the profit unambiguously decreases in β for the firms investing in dedicated technology in both the pure dedicated and the mixed markets. For the firm investing in flexible technology, however, there are two different directions in which the profit can move with an increase in cross-elasticity.

The effect of an increase in β is to amplify the profit contribution of the *stochastic* term and reduce the profit contribution of the *non-stochastic* term (i.e., terms depending on variance and the mean of demand intercepts, correspondingly). That the non-stochastic portion of the profit decreases in β is consistent with previous findings (Roller and Tombak 1993). But the stochastic portion was not modeled in earlier works. One possible explanation for the increase in the stochastic component with β is that for $\beta > 0$, the products are strategic substitutes resulting in negative externality ($\partial^2\pi_i/\partial q_y \partial q_{3-y} = -\beta < 0$). This implies that the demand for product y falls with the rise in demand for product $(3-y)$ all else held constant. Hence, the demands for the two products move in opposite directions. With high variance, one therefore gets troughs and crests in demand for the two products, which the flexible capacity is better able to cope with. With complementary products, demand falls and rises in tandem for the two products. Hence, the benefit of flexibility is less with the rise in variance. On a more analytical note, the profit for the flexible firm is convex in β. Hence, there exists a $\beta^*(\mu, c_f, \sigma)$ such that for $\beta > \beta^*$, the profit for the flexible firm is increasing in β and for $\beta < \beta^*$ it is decreasing in β. For instance, in a pure flexible market, $\beta_f^*(\mu, c_f, \sigma) = \left((\mu - c_f) - \sigma/\sqrt{2}\right) / \left((\mu - c_f) + \sigma/\sqrt{2}\right)$. Observe that β_f^* decreases in σ. Similar observations can be made for the mixed market.

Let us now focus on the form of the best-response thresholds, $\overline{c}_f$ and $\underline{c}_f$, and the way they depend on

β. It can be shown that

$$\begin{aligned}\overline{c}_f(\mu,\sigma,c,\beta) &= \left(\mu+c-\sqrt{(\mu-c)^2-(9/8)\,\sigma^2\,(1+\beta)\,/\,(1-\beta)}\right)/2,\\ \underline{c}_f(\mu,\sigma,c,\beta) &= c+\left((1/8)\,\sigma^2\,(1+\beta)\,/\,(\mu-c)\,(1-\beta)\right).\end{aligned}$$

We suggest the following way to think about the impact of β on the equilibrium of the technology game. The impact of β is to modify the variance from σ^2 to $\widetilde{\sigma}^2 = (1+\beta)\,\sigma^2/\,(1-\beta)$. Note that $\widetilde{\sigma}^2$ is convex increasing in β. Hence, the effect of β on the two thresholds reinforces the effect of variance even further. Recall that $\overline{c}_f$ was convex increasing in σ^2. Hence, with increasing β the convexity is amplified and bias towards flexible technology for high variance increases with increasing and positive β. The same holds for $\underline{c}_f$. In fact, this threshold was linear in the variance for $\beta = 0$ but now it increases in a convex manner with rising β.

To summarize our discussion of the impact of cross-elasticity, it is safe to say that as products become more substitutable, firms are more favorably inclined towards investing in flexible technology.

5 Conclusion

In this paper we have looked at the technology choice and capacity investment of firms facing stochastic price-dependent demand in a competitive market. Each firm makes three decisions: technology choice, capacity choice and production quantity choice. Hence, we cover all three levels of firms' decisions: strategic, tactical and operational. We proved that the equilibrium exists and that the symmetric equilibrium is unique for the capacity game under rather general conditions, and we derived optimal production quantities and capacities. After simplifying the problem appropriately, we were able to solve the entire game in closed form. We developed the best responses of the firms in the technology game and compared the best response of a duopolist with the behavior of a monopolist. We concluded that flexibility becomes more valuable when the rival firm invests in dedicated technology. On the other hand, if the rival invests in flexible technology, the value of flexibility diminishes. The intuition behind this result is as follows. The total market potential consists of two parts: one is deterministic and depends on the mean demand, while the other is stochastic and depends on the variance of the demand distribution. The firm investing in flexible technology is able to appropriate both deterministic and stochastic components of the market potential while the firm investing in dedicated technology is able to appropriate only the deterministic component. We showed that all four equilibria - $(F,F)\,,(D,D)\,,(D,F)$ and (F,D) - could result depending on the specific values of the problem parameters. Specifically, asymmetric equilibria can result even if the two firms are completely symmetric and hence different technologies might co-exist in the market. We generalized our findings to incorporate cross-elasticity and showed that as products become more substitutable, the value of flexibility rises.

Our findings provide systematic answers to questions regarding the value of flexibility as a competitive weapon. Anecdotal evidence from the popular press suggests that flexibility is universally "good" in a competitive environment. Our results, however, point out that a variety of equilibrium outcomes are possible, including some in which both firms invest only in dedicated capacity or two different production technologies may co-exist. We also identify technology choice as an important dimension of competition that can alleviate other dimensions of competition (e.g., on price). Overall, we show that flexible technology is not a panacea for all evils – there are conditions under which dedicated technology emerges in equilibrium.

Our results come with several limitations. In our work we did not endogenize economies of scope. Further, we did take a restrictive view of flexibility as product flexibility only and not, for example, volume flexibility. By incorporating reduction in lead times, economies of scope, and the advantage for new product development, the benefits of flexibility would definitely be increased further. It would also be interesting to study flexibility as an entry deterrence tool under stochastic demand. Moreover, allowing the flexible firm to enter another market, i.e., allowing it to manufacture a third product would influence the choice of technology for the rival firm. Many firms have focused on the role of flexible technology on developing prototypes thereby drastically shortening the time to market for a new product development. However, analytic models to this effect are few. This should prove to be an interesting problem for further research.

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technical appendix to:

Strategic Technology Choice and Capacity Investment under Demand Uncertainty.

Manu Goyal* Serguei Netessine†

November 2003, Revised July 2004

In this technical appendix, we detail the solutions to the production game. Further, we formally show the existence of equilibrium in the capacity game for all three markets and prove that the symmetric equilibrium in capacity choice is unique for the pure flexible and the pure dedicated markets. Finally, we formally derive optimal capacities, profits and prices under assumptions A-1 through A-3 for all three markets for a general case of $\beta \neq 0$ and formally show non-negativity of ex-post prices in the pure flexible market[1].

Recall that the price for product y is $P_y(Q_y, Q_{3-y}) = A_y - Q_y - \beta Q_{3-y}$. Profit expressions for the last-stage production game can be calculated as follows:

$$\widehat{\pi}_i^x = \sum_{y=1}^{2} P_y\left(\widehat{Q}_y, \widehat{Q}_{3-y}\right)\widehat{q}_{yi}^x = \sum_{y=1}^{2}\left(A_y - \left(\widehat{q}_{yi}^x + \widehat{q}_{yj}^x\right) - \beta\left(\widehat{q}_{(3-y)i}^x + \widehat{q}_{(3-y)j}^x\right)\right)\widehat{q}_{yi}^x \tag{1}$$

where $x = f, d, m$ depending on the type of market in which the firm operates (pure flexible, pure dedicated or mixed). The superscript $\hat{}$ denotes the optimal values of profits/decision variables.

Bold letters denote vectors. All vectors are column vectors and the superscript T denotes the transpose. For example, $\mathbf{A}^T$ represents the vector (A_1, A_2). All vectors are compared component-wise.

For the production game, we assume that the firms follow the optimal *holdback* strategy, i.e., the firm produces the optimal profit maximizing quantity in the production game subject to capacity constraints.

*goyalm@wharton.upenn.edu

†netessine@wharton.upenn.edu

[1]The non-negativity of prices is shown in the pure flexible market for illustrative purposes only. Proofs for the other markets are along similar lines.

If necessary, the firm *holds back* some capacity in case the optimal production quantity is less than installed capacity.

1 The pure flexible market

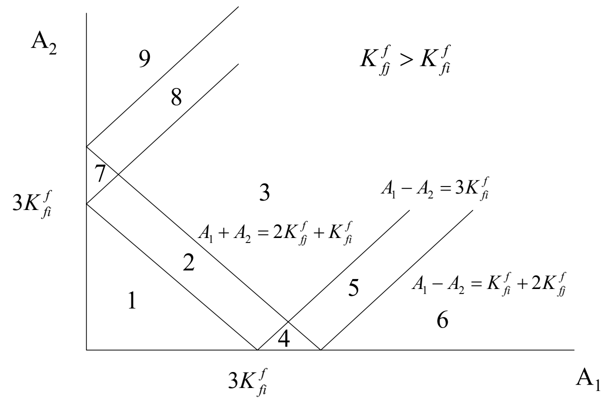


Figure 1. Pure flexible market - asymmetric solution, $\beta = 0$.

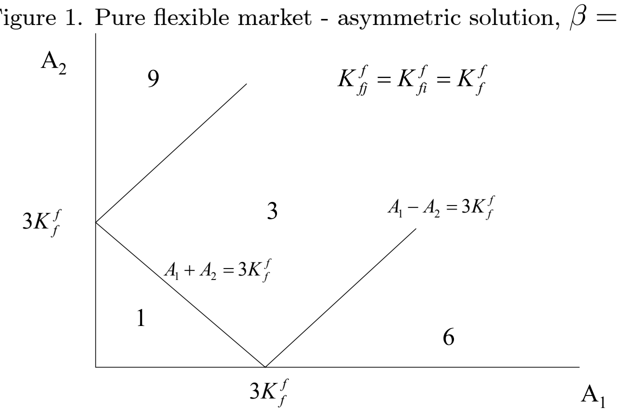


Figure 2. Pure flexible market - symmetric solution, $\beta = 0$.

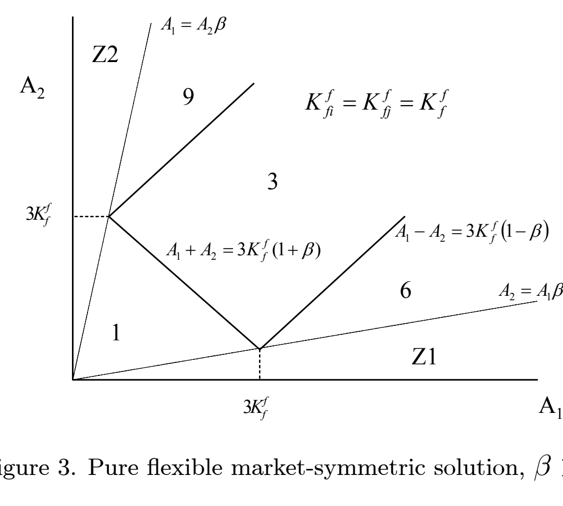


Figure 3. Pure flexible market-symmetric solution, $\beta > 0$.

Suppose both firms decide to invest in flexible technology that can produce both products and consider the last stage of the game (the production game). Assume without any loss of generality that firm j has a higher capacity than firm i, i.e., the outcome of the capacity game is such that $K^f_{fi} \leq K^f_{fj}$. Given these capacities and a vector of demand realizations $\mathbf{A}^T$, firms decide upon production quantities. The decision for one firm in isolation has been obtained by Chod and Rudi (2004). For two firms, the last-stage optimization problem can be formulated using Lagrange multipliers as follows:

$$\max\, L^f_i(u_i, q^f_{1i}, q^f_{2i}) = \sum_{y=1}^{2} \left(A_y - (q^f_{yi} + q^f_{yj})\right) q^f_{yi} - u_i \left(q^f_{1i} + q^f_{2i} - K^f_{fi}\right),\ i = 1, 2. \tag{2}$$

Combinations of the Lagrange multipliers and the slack variables give rise to 9 different optimization problems. It is convenient to represent the possible outcomes of the production game using the state-space diagram in Figure 1[2]. The various areas have an intuitive explanation. For instance, area Ω_1 represents the set of demand realizations such that no firm is capacity constrained. Similarly, area Ω_3 represents the case in which both firms are capacity constrained. For area Ω_2, only firm i is capacity constrained. Areas Ω_6 and Ω_9 arise when the demand for one product is so high that, when the firms are capacity constrained, they prefer to manufacture only one product. In areas Ω_4 and Ω_7, firm i is capacity constrained while firm j is not, whereas in areas Ω_5 and Ω_8 both firms are capacity constrained. Moreover, in these last four areas, firm i finds it economical to produce only one product but firm j produces both products. A mathematical description of the areas follows (we assume $A_y \geq \beta A_{3-y}, y = 1, 2$ to ensure non-negativity of quantities):

$$\begin{aligned}
\Omega_1 &= \left\{\mathbf{A} : \mathbf{1}^T\mathbf{A} \leq 3K^f_{fi}(1+\beta)\right\}, \\
\Omega_2 &= \left\{\mathbf{A} : \mathbf{1}^T\mathbf{A} \geq 3K^f_{fi}(1+\beta), \mathbf{1}^T\mathbf{A} \leq \left(2K^f_{fj} + K^f_{fi}\right)(1+\beta), |A_1 - A_2| \leq 3K^f_{fi}(1-\beta)\right\}, \\
\Omega_3 &= \left\{\mathbf{A} : \mathbf{1}^T\mathbf{A}\ \geq \left(2K^f_{fj} + K^f_{fi}\right)(1+\beta), |A_1 - A_2| \leq 3K^f_{fi}(1-\beta)\right\}, \\
\Omega_{4,7} &= \left\{\mathbf{A} : |A_1 - A_2| \geq 3K^f_{fi}(1-\beta); \mathbf{1}^T\mathbf{A} \geq 3K^f_{fi}(1+\beta), \mathbf{1}^T\mathbf{A} \leq \left(2K^f_{fj} + K^f_{fi}\right)(1+\beta)\right\}, \\
\Omega_{5,8} &= \left\{\mathbf{A} : 3K^f_{fi}(1-\beta) \leq |A_1 - A_2| \leq \left(2K^f_{fj} + K^f_{fi}\right)(1-\beta); \mathbf{1}^T\mathbf{A} \geq \left(2K^f_{fj} + K^f_{fi}\right)(1+\beta)\right\}, \\
\Omega_{6,9} &= \left\{\mathbf{A} : |A_1 - A_2| \geq \left(2K^f_{fj} + K^f_{fi}\right)(1-\beta)\right\}.
\end{aligned}$$

In each area, the production game can be solved in closed form (and the MPNE is trivially unique). The first-order KKT conditions are (it is straightforward to verify that the objective function is concave so these conditions are also sufficient):

$$\begin{aligned}
A_1 - 2q^f_{1i} - q^f_{1j} - \beta\left(q^f_{2i} + q^f_{2j}\right) - \beta q^f_{2i} - u_i + v_{1i} &= 0, \\
v_{1i} q^f_{1i} &= 0,
\end{aligned}$$

[2]Note that though Figure 1 is for $\beta = 0$, the solutions are expressed for $\beta \neq 0$. Figure 2 represents a symmetric flexible market with $\beta = 0$ and Figure 3 represents a symmetric market with $\beta > 0$.

$$
\begin{aligned}
A_2 - 2q_{2i}^f - q_{2j}^f - \beta\left(q_{1i}^f + q_{1j}^f\right) - \beta q_{1i}^f - u_i + v_{2i} &= 0, \\
v_{2i} q_{2i}^f &= 0, \\
q_{1i}^f + q_{2i}^f + v_{3i} &= K_{fi}^f, \\
u_i v_{3i} &= 0,
\end{aligned}
$$

where v_{li} are the slack variables. For firm j we have similar expressions with the Lagrange multipliers and the slack variables labelled as u_j, v_{lj}, $l = 1, 2, 3$. The expressions for optimal quantities and profits for the various areas of the state-space diagram are obtained by taking appropriate values of the various Lagrange multipliers and the slack variables. Unless specified otherwise, all quantities will be assumed to be positive and hence the slack variables $v_{li} = v_{lj} = 0$ for $l = 1, 2$.

Capacity is not binding (area Ω_1).

$u_i = u_j = 0$ and $v_{3i,} v_{3j} > 0$ by complementary slackness. Solving for quantities we get

$$
\begin{aligned}
\widehat{q}_{yi}^f &= \widehat{q}_{yj}^f = \frac{A_y - A_{3-y}\beta}{3\left(1-\beta^2\right)}, \\
P_y &= \frac{A_y}{3}.
\end{aligned}
$$

The quantities are non-negative as long as $A_y \geq A_{3-y}\beta$. As shown in Figure 3, this is true as long as we are outside areas Z_1 and Z_2. The prices are, of course, positive.

Capacity is binding for firm i but not for firm j (area Ω_2).

$u_i > 0$ while $u_j = 0$. From complementary slackness, we have,

$$
\begin{aligned}
\widehat{q}_{yi}^f &= \frac{A_y - A_{3-y}}{6\left(1-\beta\right)} + \frac{K_{fi}^f}{2},\ \widehat{q}_{yj}^f = \frac{\left(\left(5-\beta\right)A_y + \left(1-5\beta\right)A_{3-y}\right)}{12\left(1-\beta^2\right)} - \frac{K_{fi}^f}{4}, \\
P_y &= \frac{5A_y + A_{3-y} - 3\left(1+\beta\right)K_{fi}^f}{12} \text{ for } y = 1, 2.
\end{aligned}
$$

It can also be shown that $u_i = (1/4)\left(A_1 + A_2 - 3K_{fi}^f\left(1+\beta\right)\right) > 0$ and $v_{3j} > 0 \Rightarrow (A_1 + A_2) \leq \left(2K_{fj}^f + K_{fi}^f\right)\left(1+\beta\right)$, which gives the defining equation for Ω_2 when $\beta = 0$ in Figure 1. The price is non-negative if $5A_y + A_{3-y} \geq 3\left(1+\beta\right)K_{fi}^f$. However, we know that $\left(A_1 + A_2 - 3K_{fi}^f\left(1+\beta\right)\right) > 0$ since $u_i > 0$. Hence prices are non-negative.

Capacity is binding for both firms (area Ω_3).

$u_i, u_j > 0$. Solving for quantities we get

$$
\widehat{q}_{yi}^f = \frac{A_y - A_{3-y}}{6\left(1-\beta\right)} + \frac{K_{fi}^f}{2},\ \widehat{q}_{yj}^f = \frac{A_y - A_{3-y}}{6\left(1-\beta\right)} + \frac{K_{fj}^f}{2},
$$

$$P_y = \frac{4A_y + 2A_{3-y} - 3(1+\beta)\left(K_{fi}^f + K_{fj}^f\right)}{6}, y = 1, 2.$$

$u_j = (1/2)\left(A_1 + A_2 - \left(K_{fi}^f + 2K_{fj}^f\right)(1+\beta)\right) > 0$ gives the defining equation for Ω_3 in Figure 1. Quantities are non-negative if $A_{3-y} - A_y \leq 3K_{fi}^f(1-\beta)$. To show that prices are non-negative is a bit more involved. We shall show the non-negativity of prices for $\beta \geq 0$. It may be noted that if prices are non-negative for $\beta \geq 0$, then they are non-negative everywhere. The prices are non-negative if $4A_y + 2A_{3-y} \geq 3(1+\beta)\left(K_{fi}^f + K_{fj}^f\right)$. We know that $A_y + A_{3-y} \geq 3(1+\beta) K_{fi}^f$ and $A_y + A_{3-y} \geq \left(K_{fi}^f + 2K_{fj}^f\right)(1+\beta)$ from $u_j > 0$. Adding these two we get,

$$2(A_y + A_{3-y}) \geq (1+\beta)\left(4K_{fi}^f + 2K_{fj}^f\right). \tag{3}$$

Also, we know from the geometry of the state space (Figure 1) that the minimum value of A_y (call it $A_y^{\min}$) is obtained from the intersection of lines $A_{3-y} - A_y^{\min} = 3K_{fi}^f(1-\beta)$ and $A_{3-y} + A_y^{\min} = \left(K_{fi}^f + 2K_{fj}^f\right)(1+\beta)$. From these two equations we get $A_y^{\min} = (1+\beta) K_{fj}^f - (1-2\beta) K_{fi}^f > 0$. Hence,

$$2A_y \geq (1+\beta) K_{fj}^f - (1-2\beta) K_{fi}^f. \tag{4}$$

Adding inequalities (3) and (4), we get

$$4A_y + 2A_{3-y} \geq 3(1+\beta)\left(K_{fi}^f + K_{fj}^f\right) + 3\beta K_{fi}^i \geq 3(1+\beta)\left(K_{fi}^f + K_{fj}^f\right), \tag{5}$$

since $\beta \geq 0$. This proves the non-negativity of prices in the two markets in Ω_3.

Difference in demand states is very large with capacity constraint for firm i (areas $\Omega_{4,7}$). We solve in Ω_4 first. Firm i has a capacity constraint and the difference in the demand for the two products is so large that firm i manufactures only one product. Firm j has no capacity constraint and can manufacture both products. The values of various variables for firm i are as follows: $q_{2i}^f = v_{3i} = v_{1i} = 0$ with $q_{1i}^f, u_i, v_{2i} > 0$. For firm j, $u_j = v_{1j} = v_{2j} = 0$ with the corresponding duals being positive. Solving with the above parameters gives us $v_{2i} = \left(A_1 - A_2 - 3K_{fi}^f(1-\beta)\right)/2 > 0$. From this condition we get the defining equation for the region as $A_1 - A_2 > 3K_{fi}^f(1-\beta)$. Solving similarly for area Ω_7, we get

$$\begin{aligned}
\widehat{q}_{yi}^f &= K_{fi}^f,\ \widehat{q}_{(3-y)i}^f = 0,\ \widehat{q}_{yj}^f = \frac{A_y - \beta A_{3-y}}{2(1-\beta^2)} - \frac{K_{fi}^f}{2},\ \widehat{q}_{(3-y)j}^f = \frac{A_{3-y} - \beta A_y}{2(1-\beta^2)}, \\
P_y &= \frac{A_y - K_{fi}^f}{2},\ P_{3-y} = \frac{A_{3-y} - \beta K_{fi}^f}{2}.
\end{aligned}$$

and $\mathbf{A} \in \Omega_{4,7}$ where $y = 1$ for $\mathbf{A} \in \Omega_4$ and $y = 2$ for $\mathbf{A} \in \Omega_7$. It is relatively straightforward to see that the quantities and prices are non-negative.

Firm i manufactures one product and firm j has a capacity constraint (areas $\Omega_{5,8}$).

We solve in Ω_5 first. Firm i has a capacity constraint and manufactures only one product while firm j, though capacity constrained, manufactures both products. The values of various variables for firm i are as follows: $q_{2i}^f = v_{3i} = v_{1i} = 0$ with $q_{1i}^f, u_i, v_{2i} > 0$. For firm j, $v_{3j} = v_{1j} = v_{2j} = 0$ with the corresponding duals being positive. Again by forcing $v_{2i} > 0$ we get the defining equation similar to the one derived above. The complete solution for $\Omega_{5,8}$ is

$$\begin{aligned}
\widehat{q}_{yi}^f &= K_{fi}^f,\ \widehat{q}_{3-y}^i = 0,\ \widehat{q}_y^j = \frac{A_y - A_{3-y}}{4(1-\beta)} - \frac{K_{fi}^f}{4} + \frac{K_{fj}^f}{2},\ \widehat{q}_{3-y}^j = \frac{A_{3-y} - A_y}{4(1-\beta)} + \frac{K_{fi}^f}{4} + \frac{K_{fj}^f}{2}, \\
P_y &= \frac{3A_y + A_{3-y} - (3+\beta) K_{fi}^f - 2(1+\beta) K_{fj}^f}{4}, P_{3-y} = \frac{A_y + 3A_{3-y} - (1+3\beta) K_{fi}^f - 2(1+\beta) K_{fj}^f}{4}.
\end{aligned}$$

and $\mathbf{A} \in \Omega_{5,8}$ where $y = 1$ for $\mathbf{A} \in \Omega_5$ and $y = 2$ for $\mathbf{A} \in \Omega_8$. It is straightforward to show that the quantities are non-negative (follows from the boundary equations for these areas). Now $P_y \geq 0$, if $3A_y + A_{3-y} \geq (3+\beta) K_{fi}^f + 2(1+\beta) K_{fj}^f$. We know that for Ω_5, $A_y - A_{3-y} \geq 3K_{fi}^f (1-\beta)$ and $A_y + A_{3-y} \geq \left(K_{fi}^f + 2K_{fj}^f\right)(1+\beta)$. Adding these two inequalities, we get

$$2A_y \geq K_{fi}^f (4 - 2\beta) + K_{fj}^f (2 + 2\beta). \tag{6}$$

Now, adding (6) and $A_y + A_{3-y} \geq \left(K_{fi}^f + 2K_{fj}^f\right)(1+\beta)$ we get

$$3A_y + A_{3-y} \geq K_{fi}^f (5 - \beta) + 2K_{fj}^f (2 + 2\beta) \geq (3+\beta) K_{fi}^f + 2(1+\beta) K_{fj}^f. \tag{7}$$

To show $P_{3-y} \geq 0$, we need $A_y + 3A_{3-y} \geq (1+3\beta) K_{fi}^f + 2(1+\beta) K_{fj}^f$. This follows from $A_y + A_{3-y} \geq \left(K_{fi}^f + 2K_{fj}^f\right)(1+\beta)$ and $2A_{3-y} \geq 2\beta K_{fi}^f$.

Both firms are capacity constrained and manufacture one product (areas $\Omega_{6,9}$).

We solve in Ω_6 first. Both firms are capacity constrained and the difference in demands is so high that they manufacture only one product. The values of the parameters for firm i are $q_{2i}^f = v_{3i} = v_{1i} = 0$ and for firm j are $q_{2j}^f = v_{3j} = v_{1j} = 0$ with the corresponding duals being positive. The optimal quantities and prices are

$$\begin{aligned}
\widehat{q}_y^i &= K_{fi}^f,\ \widehat{q}_y^j = K_{fj}^f,\ \widehat{q}_{3-y}^i = \widehat{q}_{3-y}^j = 0, \\
P_y &= A_y - \left(K_{fi}^f + K_{fj}^f\right),\ P_{3-y} = A_{3-y} - \beta\left(K_{fi}^f + K_{fj}^f\right),
\end{aligned}$$

where $\mathbf{A} \in \Omega_{6,9}$ with $y = 1$ for $\mathbf{A} \in \Omega_6$ and $y = 2$ for $\mathbf{A} \in \Omega_9$. Moreover,

$v_{2i} = \left(A_1 - A_2 - (1-\beta)\left(2K_{fi}^f + K_{fj}^f\right)\right) > 0$ gives us $A_1 - A_2 > \left(2K_{fi}^f + K_{fj}^f\right)(1-\beta)$ and forcing $v_{2j} > 0$ gives us $A_1 - A_2 > \left(K_{fi}^f + 2K_{fj}^f\right)(1-\beta)$. Since $K_{fj}^f > K_{fi}^f$ the defining equation for the areas is $A_1 - A_2 > \left(K_{fi}^f + 2K_{fj}^f\right)(1-\beta)$.

It is straightforward to see that the quantities and the prices are non-negative.

The FOC in the capacity game.

The first-order condition for firm i in the capacity game can be expressed as[3]

$$E\frac{\partial \pi_i^f}{\partial K_{fi}^f} = c_f \Rightarrow \sum_l \iint_{\Omega_l} \frac{\partial \pi_i^f}{\partial K_{fi}^f} dF(x_1, x_2) = c_f.$$

Differentiating the profit function w.r.t. K_{fi}^f in each area and using Leibniz' Rule gives us the first-order condition for firm i:

$$\begin{aligned}
c_f \;=\; & \iint_{\Omega_2} \frac{1}{4}\left(x_1 + x_2 - 2K_{fi}^f(1+\beta)\right) dF(x_1, x_2) \qquad (8)\\
& + (1/2)\iint_{\Omega_3} \left(x_1 + x_2 - \left(2K_{fi}^f + K_{fj}^f\right)(1+\beta)\right) dF(x_1, x_2)\\
& + (1/2)\iint_{\Omega_4} \left(x_1 - 2K_{fi}^f\right) dF(x_1, x_2)\\
& + (1/4)\iint_{\Omega_5} \left(3x_1 + x_2 - 2(3+\beta)K_{fi}^f - 2K_{fj}^f(1+\beta)\right) dF(x_1, x_2)\\
& + \iint_{\Omega_6} \left(x_1 - 2K_{fi}^f - K_{fj}^f\right) dF(x_1, x_2) + (1/2)\iint_{\Omega_7} \left(x_2 - 2K_{fi}^f\right) dF(x_1, x_2)\\
& + (1/4)\iint_{\Omega_8} \left(x_1 + 3x_2 - 2(3+\beta)K_{fi}^f - 2K_{fj}^f(1+\beta)\right) dF(x_1, x_2)\\
& + \iint_{\Omega_9} \left(x_2 - 2K_{fi}^f - K_{fj}^f\right) dF(x_1, x_2),
\end{aligned}$$

and similarly for firm j:

$$\begin{aligned}
c_f \;=\; & (1/2)\iint_{\Omega_3} \left(x_1 + x_2 - \left(K_{fi}^f + 2K_{fj}^f\right)(1+\beta)\right) dF(x_1, x_2)\\
& + (1/2)\iint_{\Omega_{5,8}} \left(x_1 + x_2 - (1+\beta)\left(2K_{fj}^f + K_{fi}^f\right)\right) dF(x_1, x_2)\\
& + \iint_{\Omega_6} \left(x_1 - 2K_{fj}^f - K_{fi}^f\right) dF(x_1, x_2)\\
& + \iint_{\Omega_9} \left(x_2 - 2K_{fj}^f - K_{fi}^f\right) dF(x_1, x_2).
\end{aligned}$$

[3] Note that we interchanged differentiation and integration. This is justified if the function under differentiation/integration is Lipschitz-continuous of order one (see Glasserman 1994). This is easily verified since the first derivative is clearly bounded.

In the following proposition, we show uniqueness of a symmetric equilibrium. Proving uniqueness for an asymmetric capacity investment is difficult because one needs to differentiate the first-order conditions and these are not continuous at the boundaries of the various regions when capacities are asymmetric. Since the boundaries are themselves functions of the capacities of the two firms, differentiating the first-order conditions does not result in tractable expressions. However, above we did obtain the optimality conditions for the capacity game without assuming symmetry. Therefore, all asymmetric equilibria can be found numerically (or in closed form for some probability distributions). The same comment applies for the pure dedicated market analyzed in the next section.

Proposition TA 1 *Equilibrium in the capacity game for the pure flexible market exists and the symmetric equilibrium is unique* $\forall \beta \in (-1, 1)$.

Proof. The concavity of the objective functions was demonstrated by Chod and Rudi (2004), which immediately implies existence of the equilibrium. Uniqueness is proved by showing that the slope of the best-response function for each firm is less than one (Cachon and Netessine 2004). Using implicit differentiation, the absolute value of the slope of the best-response function for, say firm j, is found as $\left|\partial^2 \Pi_j^f / \partial K_{fi}^f \partial K_{fj}^f\right| / \left|\partial^2 \Pi_j^f / \partial \left(K_{fj}^f\right)^2\right|$.

Note that for a symmetric case, Figure 1 simplifies into Figure 2 (which is a special case of Figure 3 for $\beta = 0$) and hence areas 4, 5, 7 and 8 disappear. It is easy to check that the integrands in equation (8) are continuous at the boundaries of the areas once we assume symmetry. For instance, the boundary of areas 3 and 6 is given as $A_1 - A_2 = \left(2K_{fj}^f + K_{fi}^f\right)(1-\beta)$. Evaluating the integrands of Ω_3 and Ω_6 (say for firm j) at this boundary gives $(1/2)\left(2x_2 + \left(2K_{fj}^f + K_{fi}^f\right)(1-\beta-1-\beta)\right) = \left(x_2 - \beta\left(2K_{fj}^f + K_{fi}^f\right)\right)$ for Ω_3 and $\left(x_2 - \left(2K_{fj}^f + K_{fi}^f\right)(1-1+\beta)\right) = \left(x_2 - \beta\left(2K_{fj}^f + K_{fi}^f\right)\right)$ for Ω_6. Hence, we can safely ignore differentiating the limits of the integrals when we apply Leibniz' Rule as the corresponding terms get cancelled.

Now,

$$\left|\frac{\partial^2 \Pi_j^f}{\partial \left(K_{fj}^f\right)^2}\right| = \iint_{\Omega_3} (1+\beta)\, dF(x_1, x_2) + 2 \iint_{\Omega_{6,9}} dF(x_1, x_2),$$

$$\left|\frac{\partial^2 \Pi_j^f}{\partial K_{fi}^f \partial K_{fj}^f}\right| = \frac{1}{2} \iint_{\Omega_3} (1+\beta)\, dF(x_1, x_2) + \iint_{\Omega_{6,9}} dF(x_1, x_2).$$

Clearly, $\left|\partial^2 \Pi_j^f / \partial K_{fi}^f \partial K_{fj}^f\right| / \left|\partial^2 \Pi_j / \partial \left(K_{fj}^f\right)^2\right| = 1/2 < 1$. Similarly, for firm i.

■

Hence, the symmetric equilibrium in the capacity game is unique for all $\beta \in (-1,1)$. The first-order conditions simplify to ($K^f_{fi} = K^f_{fj} = K^f_f$):

$$c_f = (1/2) \iint_{\Omega_3} \left(x_1 + x_2 - 3K^f_f (1+\beta)\right) dF(x_1,x_2) + \iint_{\Omega_6} \left(x_1 - 3K^f_f\right) dF(x_1,x_2) + \iint_{\Omega_9} \left(x_2 - 3K^f_f\right) dF(x_1,x_2). \quad (9)$$

We now solve for the optimal capacity and profit for a firm in the pure flexible market under assumptions A-1 through A-3.

Proposition TA 2 *Under assumptions A-1 through A-3, for firms operating in the pure flexible market,*

(i) the MPNE in capacity is

$$K^f_f = \frac{(\mu_1 + \mu_2 - 2c_f)}{3(1+\beta)},$$

(ii) the expected profit at equilibrium is

$$\Pi^f_f = \frac{(\sigma_1^2 + \sigma_2^2)}{18(1-\beta)} + \frac{(\mu_1 - c_f)^2 + (\mu_2 - c_f)^2}{9(1+\beta)},$$

(iii) the expected price for product $y = 1,2$ is $P^f_y = (\mu_y + 2c_f)/3$.

Proof. The first-order condition from (9) under symmetry (i.e. $K^f_{fi} = K^f_{fj} = K^f_f$) can be written using assumption A-3 as

$$\begin{aligned} c_f = {} & (1/2) \iint_{\overline{\Omega}} \left(x_1 + x_2 - 3K^f_f (1+\beta)\right) dF(x_1,x_2) + \\ & + (1/2) \iint_{\Omega_6} \left(2\left(x_1 - 3K^f_f\right) - \left(x_1 + x_2 - 3K^f_f (1+\beta)\right)\right) dF(x_1,x_2) \\ & + (1/2) \iint_{\Omega_9} \left(2\left(x_2 - 3K^f_f\right) - \left(x_1 + x_2 - 3K^f_f (1+\beta)\right)\right) dF(x_1,x_2), \end{aligned}$$

where $\overline{\Omega}$ represents the entire state space. Using A-2 and A-1 we can further simplify it

$$(1/2) \iint_{\overline{\Omega}} \left(x_1 + x_2 - 3K^f_f (1+\beta)\right) f_1(x_1) f_2(x_2) dx_1 dx_2 = c_f,$$

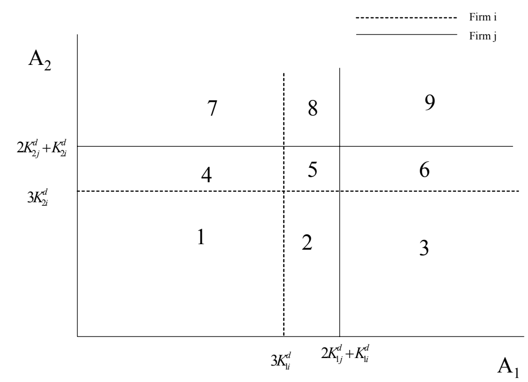
and the result for the capacity follows. Further, under assumptions A-1 through A-3 the expression for the firm's profit becomes

$$\Pi_i^f = \left(\frac{1}{6}\right) \iint\limits_{\Omega} \left[\sum_{y=1}^{2} \left(4x_y + 2x_{3-y} - 6\left(1+\beta\right) K_f^f\right) \left(\frac{(x_y - x_{3-y})}{(1-\beta)} + 3K_f^f\right) f_y(x_y) f_{3-y}(x_{3-y}) dx_1 dx_2 \right] - c_f K_f^f.$$

After substituting the expression for the equilibrium capacity into the equation for profit, the result follows after tedious algebra. ■

2 The pure dedicated market.

Figures 4 and 5 below represent the state-space for a pure dedicated market. The figures are drawn for $\beta = 0$. In the interest of simplicity, the modified state-space representation for $\beta \neq 0$ is omitted here. However, for $\beta \neq 0$, the same precautions and comments apply in the pure dedicated market as are applicable in the pure flexible market (see Section 4.4 of the main paper).



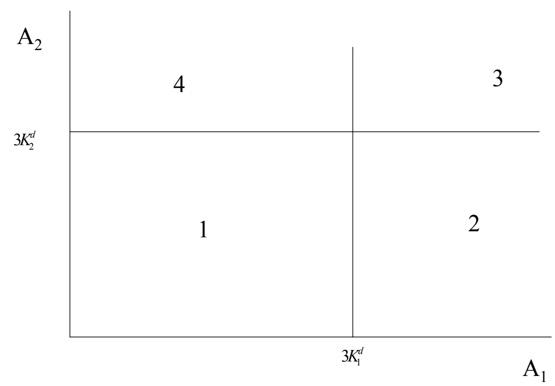


Figure 4. Pure dedicated market - asymmetric solution. Figure 5. Pure dedicated market - symmetric solution.

Suppose that both firms invest in dedicated technology, i.e., there is a dedicated production line for each product (see Figures 4 and 5). Note the assumption that the capacity investment for firm j is higher than that of firm i in both markets. It needs to be emphasized that this is in not a unique representation. For instance, the capacity of firm j could be higher than firm i in one market but lower in another. We could solve the production game in closed form for any of these scenarios. However, the above assumption is for expositional convenience only. Later we restrict our attention to the symmetric equilibrium, which is shown to be unique.

Compared to the pure flexible market, the interpretations of the areas in Figure 4 are much simpler. For instance, area Ω_1 represents no capacity constraint for either firm. In areas Ω_2 and Ω_4, firm i has capacity constraint for products 1 and 2 respectively while firm j has no capacity constraint for either. Since in each area the production game can be solved uniquely, the MPNE in the production game is trivially unique.

The Lagrangian can be written as:

$$\max L_i^d(q_{1i}^d, q_{2i}^d, \mathbf{u}) = \sum_{y=1}^{2} \left(A_y - (Q_y + \beta Q_{3-y})\right) q_{yi}^d - u_{yi}(q_{yi}^d - K_{yi}^d).$$

A similar expression can be obtained for firm j with the Lagrange multipliers u_{1j} and u_{2j}. The KKT conditions for firm i are:

$$\begin{aligned}
A_1 - 2q_{1i}^d - q_{1j}^d - \beta\left(q_{2i}^d + q_{2j}^d\right) - \beta q_{2i}^d - u_{1i} + v_{1i} &= 0,\\
q_{1i}^d v_{1i} &= 0,\\
A_2 - 2q_{2i}^d - q_{2j}^d - \beta\left(q_{1i}^d + q_{1j}^d\right) - \beta q_{1i}^d - u_{2i} + v_{2i} &= 0,\\
q_{2i}^d v_{2i} &= 0,\\
q_{1i}^d + v_{3i} &= K_{1i}^d,\\
v_{3i}u_{1i} &= 0,\\
q_{2i}^d + v_{4i} &= K_{2i}^d,\\
v_{4i}u_{2i} &= 0,
\end{aligned}$$

where v_{li}, $l = 1, 2, 3, 4$ are the slack variables. We now provide closed-form solutions for the optimal quantities, while the equilibrium profits can be obtained from (1). All quantities are positive and hence, unless specified otherwise, $v_{li} = v_{lj} = 0$ for all l.

Capacity is not binding (area Ω_1).

$u_{li} = u_{lj} = 0$ for $l = 1, 2$. The optimal production quantities are

$$\widehat{q}_{yi}^d = \widehat{q}_{yj}^d = \frac{A_y - A_{3-y}\beta}{3\left(1-\beta^2\right)}.$$

Capacity is binding for one product for firm i (areas $\Omega_{2,4}$).

For Ω_2, $u_{2i} = 0$ and $u_{kj} = 0$ for $k = 1, 2$. However, since capacity binds for product 1 for firm i, $u_{1i} > 0$. The corresponding duals are positive by complementary slackness. Solving for quantities we get

$$\begin{aligned}
\widehat{q}_{yi}^d &= K_{yi}^d,\ \widehat{q}_{(3-y)i}^d = \frac{A_{3-y} - 3\beta K_{yi}^d}{3},\ \widehat{q}_{yj}^d = \frac{A_y - \beta A_{3-y} - K_{yi}^d\left(1-\beta^2\right)}{2\left(1-\beta^2\right)},\\
\widehat{q}_{(3-y)j}^d &= \frac{A_{3-y}\left(2+\beta^2\right) - 3\beta A_y}{6\left(1-\beta^2\right)} + \frac{\beta K_{yi}^d}{2}.
\end{aligned}$$

and $\mathbf{A} \in \Omega_{2,4}$ where $y = 1$ for $\mathbf{A} \in \Omega_2$ and $y = 2$ for $\mathbf{A} \in \Omega_4$.

Capacity is binding for both products for firm i (area Ω_5).

We have $u_{1i}, u_{2i} > 0$, $u_{1j} = u_{2j} = 0$. Other variables are positive by complementary slackness and production quantities are

$$\widehat{q}_{1i}^{d} = K_{1i}^{d},\ \widehat{q}_{2i}^{d} = K_{2i}^{d},\ \widehat{q}_{1j}^{d} = \frac{A_1 - \beta A_2 - \left(1-\beta^2\right) K_{1i}^{d}}{2\left(1-\beta^2\right)},\ \widehat{q}_{2j}^{d} = \frac{A_2 - \beta A_1 - \left(1-\beta^2\right) K_{2i}^{d}}{2\left(1-\beta^2\right)}.$$

Capacity is binding for both firms for the same product (areas $\Omega_{3,7}$).

For Ω_3 we have $u_{1i}, u_{1j} > 0$, $u_{2i} = u_{2j} = 0$ and more generally:

$$\widehat{q}_{yi}^{d} = K_{yi}^{d},\ \widehat{q}_{(3-y)i}^{d} = \frac{A_{3-y} - 3\beta K_{yi}^{d}}{3},\ \widehat{q}_{yj}^{d} = K_{yj}^{d},\ \widehat{q}_{(3-y)j}^{d} = \frac{A_{3-y} - 3\beta K_{yj}^{d}}{3},$$

and $\mathbf{A} \in \Omega_{3,7}$ where $y = 1$ for $\mathbf{A} \in \Omega_3$ and $y = 2$ for $\mathbf{A} \in \Omega_7$.

Capacity is binding for one product for firm j and both products for firm i (areas $\Omega_{6,8}$).

For Ω_6 we have $u_{1i}, u_{2i} > 0$, $u_{2j} = 0, u_{1j} > 0$. Other variables are non-zero by complementary slackness and we obtain

$$\widehat{q}_{yi}^{d} = K_{yi}^{d},\ \widehat{q}_{(3-y)i}^{d} = K_{(3-y)i}^{d},\ \widehat{q}_{yj}^{d} = K_{yj}^{d},\ \widehat{q}_{(3-y)j}^{d} = \frac{A_{3-y} - K_{(3-y)i}^{d} - \beta(K_{yi}^{d} + 2K_{yj}^{d})}{2},$$

and $\mathbf{A} \in \Omega_{6,8}$ where $y = 1$ for $\mathbf{A} \in \Omega_6$ and $y = 2$ for $\mathbf{A} \in \Omega_8$.

Capacity is binding for both products for both firms (area Ω_9).

We have $u_{li}, u_{lj} > 0$ for $l = 1, 2$. The optimal solution is simply

$$\widehat{q}_{yi}^{d} = K_{yi}^{d};\ \widehat{q}_{yj}^{d} = K_{yj}^{d}.$$

The FOC in the capacity game.

For firm i, the first-order condition can be expressed as $\partial\left(E\pi_i^d\right)/\partial K_{yi}^d = c$ which translates into

$$\begin{aligned}
c \;=\; & (1/2)\iint_{\Omega_2}\left(x_1 - x_2\beta - 2\left(1-\beta^2\right)K_{1i}^{d}\right)dF(x_1,x_2) \\
& + \iint_{\Omega_3}\left(x_1 - \beta x_2 - \left(1-\beta^2\right)\left(2K_{1i}^{d} + K_{1j}^{d}\right)\right)dF(x_1,x_2) \\
& + (1/2)\iint_{\Omega_5}\left(x_1 - 2K_{1i}^{d} - 2\beta K_{2i}^{d}\right)dF(x_1,x_2) \\
& + \iint_{\Omega_6}\left(x_1 - \left(2K_{1i}^{d} + K_{1j}^{d}\right) + \beta\left(-x_2 + 2\beta\left(K_{1i}^{d} + K_{1j}^{d}\right) - 2K_{2i}^{d}\right)/2\right)dF(x_1,x_2) \\
& + (1/2)\iint_{\Omega_8}\left(x_1 - 2\beta K_{2i}^{d} - 2K_{1i}^{d}\right)dF(x_1,x_2) \\
& + \iint_{\Omega_9}\left(x_1 - \left(2K_{1i}^{d} + K_{1j}^{d}\right) - \beta\left(2K_{2i}^{d} + K_{2j}^{d}\right)\right)dF(x_1,x_2).
\end{aligned}$$

Analogously, $\partial E\left(\pi_i^d\right)/\partial K_{2i}^f = c$ yields:

$$\begin{aligned}
c &= (1/2)\iint\limits_{\Omega_4}\left(x_2 - x_1\beta - 2\left(1-\beta^2\right)K_{2i}^d\right)dF(x_1,x_2) \\
&\quad + (1/2)\iint\limits_{\Omega_5}\left(x_2 - 2K_{2i}^d - 2\beta K_{1i}^d\right)dF(x_1,x_2) \\
&\quad + (1/2)\iint\limits_{\Omega_6}\left(x_2 - 2\beta K_{1i}^d - 2K_{2i}^d\right)dF(x_1,x_2) \\
&\quad + \iint\limits_{\Omega_7}\left(x_2 - \beta x_1 - \left(1-\beta^2\right)\left(2K_{2i}^d + K_{2j}^d\right)\right)dF(x_1,x_2) \\
&\quad + \iint\limits_{\Omega_8}\left(x_2 - \left(2K_{2i}^d + K_{2j}^d\right) + \beta\left(-x_1 + 2\beta\left(K_{2i}^d + K_{2j}^d\right) - 2K_{1i}^d\right)/2\right)dF(x_1,x_2) \\
&\quad + \iint\limits_{\Omega_9}\left(x_2 - \left(2K_{2i}^d + K_{2j}^d\right) - \beta\left(2K_{1i}^d + K_{1j}^d\right)\right)dF(x_1,x_2).
\end{aligned}$$

For firm j, $E\left(\partial\pi_j^d/\partial K_{1j}^d\right) = c$ translates into:

$$\begin{aligned}
c &= \iint\limits_{\Omega_{3,6}}\left[x_1 - \beta x_2 - \left(1-\beta^2\right)\left(2K_{1j}^d + K_{1i}^d\right)\right]dF(x_1,x_2) \\
&\quad + \iint\limits_{\Omega_9}\left[x_1 - \left(2K_{1j}^d + K_{1i}^d\right) - \beta\left(2K_{2j}^d + K_{2i}^d\right)\right]dF(x_1,x_2),
\end{aligned} \tag{10}$$

and finally from $E\left(\partial\pi_j^d/\partial K_{2j}^d\right) = c$ we get

$$\begin{aligned}
c &= \iint\limits_{\Omega_{7,8}}\left[x_2 - \beta x_1 - \left(1-\beta^2\right)\left(2K_{2j}^d + K_{2i}^d\right)\right]dF(x_1,x_2) \\
&\quad + \iint\limits_{\Omega_9}\left[x_2 - \left(2K_{2j}^d + K_{2i}^d\right) - \beta\left(2K_{1j}^d + K_{1i}^d\right)\right]dF(x_1,x_2).
\end{aligned}$$

The following proposition states the existence and the uniqueness of equilibrium in the capacity game.

Proposition TA 3 *Equilibrium in the capacity game for the pure dedicated market exists* $\forall\beta \in (-1,1)$ *and the symmetric equilibrium is unique for* $\beta \in (-1, 1/3)$.

Proof. It can be easily verified that each objective function is concave so a pure strategy Nash Equilibrium exists. The Hessian for this game can be written as:

$$H^d = \begin{pmatrix} \frac{\partial^2 \Pi_i^d}{\partial (K_{1i}^d)^2} & \frac{\partial^2 \Pi_i^d}{\partial K_{1i}^d \partial K_{2i}^d} & \frac{\partial^2 \Pi_i^d}{\partial K_{1i}^d \partial K_{1j}^d} & \frac{\partial^2 \Pi_i^d}{\partial K_{1i}^d \partial K_{2j}^d} \\ \frac{\partial^2 \Pi_i^d}{\partial K_{1i}^d \partial K_{2i}^d} & \frac{\partial^2 \Pi_i^d}{\partial (K_{2i}^d)^2} & \frac{\partial^2 \Pi_i^d}{\partial K_{2i}^d \partial K_{1j}^d} & \frac{\partial^2 \Pi_i^d}{\partial K_{2i}^d \partial K_{2j}^d} \\ \frac{\partial^2 \Pi_j^d}{\partial K_{1j}^d \partial K_{1i}^d} & \frac{\partial^2 \Pi_j^d}{\partial K_{1j}^d \partial K_{2i}^d} & \frac{\partial^2 \Pi_j^d}{\partial (K_{1j}^d)^2} & \frac{\partial^2 \Pi_j^d}{\partial K_{1j}^d \partial K_{2j}^d} \\ \frac{\partial^2 \Pi_j^d}{\partial K_{2j}^d \partial K_{1i}^d} & \frac{\partial^2 \Pi_j^d}{\partial K_{2j}^d \partial K_{2i}^d} & \frac{\partial^2 \Pi_j^d}{\partial K_{2j}^d \partial K_{1j}^d} & \frac{\partial^2 \Pi_j^d}{\partial (K_{2j}^d)^2} \end{pmatrix}.$$

Following Cachon and Netessine (2004), a condition sufficient for the uniqueness of the Nash equilibrium is the diagonal dominance that translates into

$$\left| \frac{\partial^2 \Pi_i^d}{\partial \left(K_{yi}^d\right)^2} \right| > \left| \frac{\partial^2 \Pi_i^d}{\partial K_{yi}^d \partial K_{(3-y)i}^d} \right| + \left| \frac{\partial^2 \Pi_i^d}{\partial K_{yi}^d \partial K_{yj}^d} \right| + \left| \frac{\partial^2 \Pi_i^d}{\partial K_{yi}^d \partial K_{(3-y)j}^d} \right|, \ y = 1, 2, \tag{11}$$

$$\left| \frac{\partial^2 \Pi_j^d}{\partial \left(K_{yj}^d\right)^2} \right| > \left| \frac{\partial^2 \Pi_j^d}{\partial K_{yj}^d \partial K_{(3-y)j}^d} \right| + \left| \frac{\partial^2 \Pi_j^d}{\partial K_{yj}^d \partial K_{yi}^d} \right| + \left| \frac{\partial^2 \Pi_j^d}{\partial K_{yj}^d \partial K_{(3-y)i}^d} \right|, y = 1, 2. \tag{12}$$

Because of symmetry assumption, we show the analysis for one firm only (say firm j). We re-write the first-order conditions (10) for firm j for the symmetric case (note that Figure 4 changes to Figure 5). These are

$$\begin{aligned} c &= \iint_{\Omega_2} \left[x_1 - \beta x_2 - \left(1-\beta^2\right)\left(2K_{1j}^d + K_{1i}^d\right)\right] dF(x_1, x_2) \\ &\quad + \iint_{\Omega_3} \left[x_1 - \left(2K_{1j}^d + K_{1i}^d\right) - \beta\left(2K_{2j}^d + K_{2i}^d\right)\right] dF(x_1, x_2), \end{aligned} \tag{13}$$

and

$$\begin{aligned} c &= \iint_{\Omega_4} \left[x_2 - \beta x_1 - \left(1-\beta^2\right)\left(2K_{2j}^d + K_{2i}^d\right)\right] dF(x_1, x_2) \\ &\quad + \iint_{\Omega_3} \left[x_2 - \left(2K_{2j}^d + K_{2i}^d\right) - \beta\left(2K_{1j}^d + K_{1i}^d\right)\right] dF(x_1, x_2). \end{aligned}$$

The numbering of the areas is now with respect to Figure 5. We again verify that the integrands are continuous across the areas. To show a specific case, take areas 2 and 3 of Figure 5. The boundary condition is $A_2 = \beta\left(K_{1i}^d + 2K_{1j}^d\right) + \left(K_{2i}^d + 2K_{2j}^d\right)$. The integrand of Ω_2 in equation (13) reduces to $x_1 - \left(2K_{1j}^d + K_{1i}^d\right) - \beta\left(2K_{2j}^d + K_{2i}^d\right)$ which is the same as the integrand of Ω_3. Hence, we ignore the limits while differentiating using Leibniz' Rule.

Next we derive conditions for (12) to hold. We first show the result for $y = 1$:

$$\begin{aligned}
\left|\frac{\partial^2 \Pi_j^d}{\partial \left(K_{1j}^d\right)^2}\right| &= \iint\limits_{\Omega_2} 2\left(1-\beta^2\right) dF(x_1,x_2) + \iint\limits_{\Omega_3} 2dF(x_1,x_2), \\
\left|\frac{\partial^2 \Pi_j^d}{\partial K_{1j}^d \partial K_{2j}^d}\right| &= \iint\limits_{\Omega_3} 2\beta dF(x_1,x_2), \\
\left|\frac{\partial^2 \Pi_j^d}{\partial K_{1j}^d \partial K_{1i}^d}\right| &= \iint\limits_{\Omega_2} \left(1-\beta^2\right) dF(x_1,x_2) + \iint\limits_{\Omega_3} 1dF(x_1,x_2), \\
\left|\frac{\partial^2 \Pi_j^d}{\partial K_{1j}^d \partial K_{2i}^d}\right| &= \iint\limits_{\Omega_3} \beta dF(x_1,x_2).
\end{aligned}$$

It is straightforward to show that the inequality holds for each of the areas except for Ω_3. For the inequality to hold in Ω_3 we need $\beta < 1/3$. The same result for $y = 2$ can be shown analogously. Hence, there is a unique symmetric Nash Equilibrium in the capacities for $\beta \in (-1, 1/3)$. ■

The symmetric first-order conditions can now be represented as (see Figure 5 with $K_{yi}^d = K_{yj}^d = K_y^d$)

$$\begin{aligned}
c &= \iint\limits_{\Omega_2} \left(x_1 - \beta x_2 - \left(1-\beta^2\right)\left(3K_y^d\right)\right) dF(x_1,x_2) + \iint\limits_{\Omega_3} \left(x_1 - 3K_y^d(1+\beta)\right) dF(x_1,x_2), \qquad (14) \\
c &= \iint\limits_{\Omega_4} \left(x_2 - \beta x_1 - \left(1-\beta^2\right)\left(3K_y^d\right)\right) dF(x_1,x_2) + \iint\limits_{\Omega_3} \left(x_2 - 3K_y^d(1+\beta)\right) dF(x_1,x_2).
\end{aligned}$$

Proposition TA 4 *Under assumptions A1-A3, for firms operating in a pure dedicated market,*

(i) *the MPNE in capacity is*

$$K_y^d = \frac{(\mu_y - c)}{3(1+\beta)}, y = 1, 2,$$

(ii) *the expected profit at equilibrium is*

$$\Pi_i^d = \frac{(\mu_1 - c)^2 + (\mu_2 - c)^2}{9(1+\beta)},\ i = 1, 2,$$

(iii) *the expected price for a product* $y = 1, 2$ is $P_y^d = (\mu_y + 2c)/3$.

Proof. From equation (14), the symmetric first-order conditions in a pure dedicated market can be written as

$$\begin{aligned}
c &= \iint\limits_{\Omega_3} \left(x_1 - 3K_y^d(1+\beta)\right) dF(x_1,x_2), \\
c &= \iint\limits_{\Omega_3} \left(x_2 - 3K_y^d(1+\beta)\right) dF(x_1,x_2).
\end{aligned}$$

The expressions for capacities and profits follow after tedious algebra. ■

3 The mixed market.

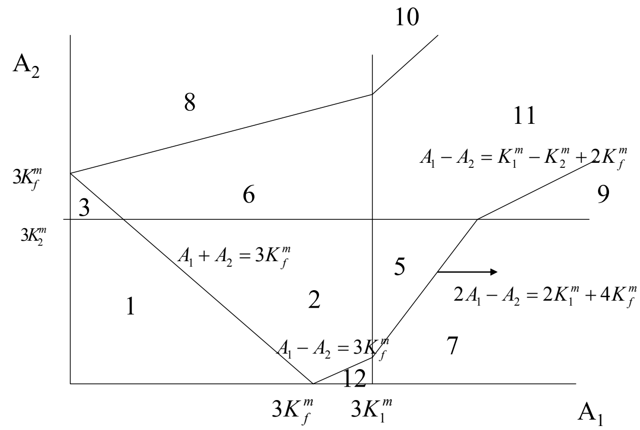


Figure 6. The mixed market.

Suppose that firm i decides to invest in dedicated capacity and firm j decides to invest in flexible capacity. We alter our notation for the purpose of this section only: for clarity we remove the subscripts i and j (subscripts d and f will be used if necessary for the flexible and the dedicated firm). For instance, K_1^m represents dedicated capacity for product 1 and q_{1f}^m represents the quantity of product one produced by the flexible firm. Similar to the previous two cases, a number of areas arise due to the capacity constraints of both firms (see Figure 6)[4] and the production game can be solved uniquely for each area. However, this is not a unique representation (i.e., there are other possibilities that could give rise to a different set of areas). Figure 6 is based on the assumption that $K_1^m > K_f^m > K_2^m$. For instance, there could be an area 4 similar to area 3 instead of area 12 in the figure. The presence of these areas depends on the assumptions on the capacity levels for the firms. Hence, Figure 6 is only a schematic representation of how the areas are placed with respect to each other. However, because there are multiple representations of the state-space, we suspect that there might be multiple equilibria in the capacity game in the mixed market associated with each such representation.

The areas in Figure 6 have intuitive explanations. For instance, in area Ω_1 no firm has a capacity constraint, while in area Ω_2 the flexible firm has a capacity constraint. In area Ω_3, the dedicated firm has a capacity constraint for product 1, etc.

Using the methodology below, one can solve for all possible ways of representing the mixed market. For ease of understanding, some of the areas can be referenced back to Figure 6. Others (like area Ω_4) which do not find representation in Figure 6, can be understood from the text detailing what they stand

[4]Figure 6 represents the state-space for a mixed market for $\beta = 0$. The modified state-space representation for $\beta \neq 0$ is omitted for simplicity.

for.

The Lagrangian formulation for the flexible firm is

$$\max L_f^m(u_f, q_{1f}^m, q_{2f}^m) = \sum_{y=1}^{2} \left(A_y - (Q_y + \beta Q_{3-y})\right) q_{yf}^m - u_f(q_{1f}^m + q_{2f}^m - K_f^m).$$

The KKT conditions are (since the objective function is concave these are also sufficient):

$$\begin{aligned}
A_1 - 2q_{1f}^m - q_{1d}^m - \beta\left(q_{2f}^m + q_{2d}^m\right) - \beta q_{2f}^m - u_f + v_{1f} &= 0, \\
v_{1f} q_{1f}^m &= 0, \\
A_2 - 2q_{2f}^m - q_{2d}^m - \beta\left(q_{1f}^m + q_{1d}^m\right) - \beta q_{1f}^m - u_f + v_{2f} &= 0, \\
v_{2f} q_{2f}^m &= 0, \\
q_{1f}^m + q_{2f}^m + v_{3f} &= K_f^m, \\
u_f v_{3f} &= 0,
\end{aligned}$$

where v_{lf} are the slack variables for $l = 1, 2, 3$. For the dedicated firm, the Lagrangian is

$$\max L_d^m(q_{1d}^m, q_{2d}^m, \mathbf{u}) = \sum_{y=1}^{2} \left(\left(A_y - (Q_y + \beta Q_{3-y})\right) q_{yi}^d - u_{yd}(q_{yd}^m - K_y^m)\right).$$

The KKT conditions for the firm employing dedicated technology are:

$$\begin{aligned}
A_1 - 2q_{1d}^m - q_{1f}^m - \beta\left(q_{2d}^m + q_{2f}^m\right) - \beta q_{2d}^m - u_{1d} + v_{1d} &= 0, \\
q_{1d}^m v_{1d} &= 0, \\
A_2 - 2q_{2d}^m - q_{2f}^m - \beta\left(q_{1d}^m + q_{1f}^m\right) - \beta q_{1d}^m - u_{2d} + v_{2d} &= 0, \\
q_{2d}^m v_{2d} &= 0, \\
q_{1d}^m + v_{3d} &= K_1^m, \\
v_{3d} u_{1d} &= 0, \\
q_{2d}^m + v_{4d} &= K_2^m, \\
v_{4d} u_{2d} &= 0,
\end{aligned}$$

where v_{ld}, $l = 1, 2, 3, 4$ are the slack variables. We proceed by finding the optimal production quantities. Unless otherwise specified, the quantities are all positive and hence $v_{lf} = v_{ld} = 0$, $l = 1, 2$. Similar in spirit to the case of a pure flexible market, the Lagrange multipliers often define the boundary conditions for the various areas of integration. For the sake of simplicity, we do not show the values of the Lagrange multipliers whenever they are positive. We do, however, show some interesting cases below in which the slack variables are positive.

Capacity is not binding (area Ω_1).

$u_f = u_{1d} = u_{2d} = 0$. The unconstrained solution is

$$\widehat{q}_{yf}^m = \widehat{q}_{yd}^m = \frac{A_y - A_{3-y}\beta}{3\left(1-\beta^2\right)},\ y = 1,2.$$

Only the flexible firm is capacity constrained (area Ω_2).

For the flexible firm $u_f > 0$. For the dedicated firm $u_{kd} = 0$ for $k = 1,2$. The optimal production quantities are:

$$\begin{aligned}
\widehat{q}_{yf}^m &= \frac{A_y - A_{3-y}}{6\left(1-\beta\right)} + \frac{K_f^m}{2} \text{ for } y = 1,2, \\
\widehat{q}_{yd}^m &= \frac{\left(5-\beta\right)A_y + A_{3-y}\left(1-5\beta\right)}{12\left(1-\beta^2\right)} - \frac{K_f^m}{4} \text{ for } y = 1,2.
\end{aligned}$$

The flexible firm is not capacity constrained while the dedicated firm is capacity constrained for both products (area $\Omega_{\tilde{2}}$).

Note that this area is not represented in Figure 6. Here, $u_f = 0$ and $u_{kd} > 0$ for $k = 1,2$. The optimal production quantities are

$$\begin{aligned}
\widehat{q}_{yf}^m &= \frac{A_y - \beta A_{3-y} - K_y^m\left(1-\beta^2\right)}{2},\ \text{for } y = 1,2, \\
\widehat{q}_{yd}^m &= K_y^m.
\end{aligned}$$

The flexible firm is not capacity constrained while the dedicated firm is constrained for product 2 for area Ω_3 and product 1 for area Ω_4.

Area Ω_4 is not in Figure 6. For the flexible firm $u_f = 0$ with the corresponding duals being non-zero. For the dedicated firm we have $u_{yd} = 0$ and $u_{(3-y)d} > 0$. In what follows, $y = 1$ for area Ω_3 and $y = 2$ for area Ω_4. Solving for quantities we get:

$$\begin{aligned}
\widehat{q}_{yf}^m &= \frac{A_y\left(2+\beta^2\right) - 3\beta A_{3-y} + 3\beta K_{(3-y)}^m\left(1-\beta^2\right)}{6\left(1-\beta^2\right)}, \\
\widehat{q}_{(3-y)f}^m &= \frac{A_{3-y} - \beta A_y - K_{(3-y)}^m\left(1-\beta^2\right)}{2\left(1-\beta^2\right)},\ \widehat{q}_{yd}^m = \frac{A_y - 3\beta K_{(3-y)}^m}{3}, \widehat{q}_{(3-y)d}^m = K_{(3-y)}^m.
\end{aligned}$$

Capacity is binding for the flexible firm and for one product for the dedicated firm (area $\Omega_{5,6}$).

For the flexible firm $u_f > 0$. For the dedicated firm, $u_{yd} > 0$ and $u_{(3-y)d} = 0$ where $y = 1$ for $\mathbf{A} \in \Omega_5$ and $y = 2$ for $\mathbf{A} \in \Omega_6$. The solution is:

$$\begin{aligned}
\widehat{q}_{yf}^m &= \frac{2A_y - A_{(3-y)}(1+\beta) - 2K_y^m(1-\beta^2) + 3K_f^m(1-\beta)}{7-\beta(6+\beta)}, \\
\widehat{q}_{(3-y)f}^m &= \frac{-2A_y + A_{3-y}(1+\beta) + 2K_y^m(1-\beta^2) + K_f^m(4-3\beta-\beta^2)}{7-\beta(6+\beta)}, \\
\widehat{q}_{yd}^m &= K_y^m,\ \widehat{q}_{(3-y)d}^m = \frac{A_y + 3A_{3-y} - K_y^m(1+7\beta) - 2K_f^m(1+\beta)}{7+\beta},
\end{aligned}$$

and $\mathbf{A} \in \Omega_{5,6}$ where $y = 1$ for $\mathbf{A} \in \Omega_5$ and $y = 2$ for $\mathbf{A} \in \Omega_6$.

Capacity is binding for the flexible firm and for one product for the dedicated firm. The flexible firm manufactures one product (areas $\Omega_{7,8}$).

In area Ω_7 the difference in demand realizations is so high that the flexible firm manufactures only product 1. Hence, for the flexible firm, $u_f > 0$ and $v_{2f} > 0$ so that $\widehat{q}_{2f}^m = 0$. For the dedicated firm $u_{1d} > 0$. Upon solving we get $v_{2f} = (1/2)\left(2A_1 - A_2(1+\beta) - 2K_1^m(1-\beta^2) - K_f^m(4-3\beta-\beta^2)\right) > 0$. For $\beta = 0$ this reduces to the boundary condition $2A_1 - A_2 > 2K_1^m + 4K_f^m$ as is evident in Figure 6. The optimal quantities are:

$$\widehat{q}_{yf}^m = K_f^m,\ \widehat{q}_{(3-y)f}^m = 0,\ \widehat{q}_{yd}^m = K_y^m,\ \widehat{q}_{(3-y)d}^m = \left(A_{3-y} - 2\beta K_y^m - \beta K_f^m\right)/2,$$

and $\mathbf{A} \in \Omega_{7,8}$ where $y = 1$ for $\mathbf{A} \in \Omega_7$ and $y = 2$ for $\mathbf{A} \in \Omega_8$.

Capacity is binding for the flexible firm and for both products for the dedicated firm. The flexible firm manufactures one product (areas $\Omega_{9,10}$).

The only change from the preceding case is that for the dedicated firm we have $u_{1d}, u_{2d} > 0$. Solving in area Ω_9 we get $v_{2f} = A_1 - A_2 - (K_1^m - K_2^m + 2K_f^m)(1-\beta) > 0$. From here we obtain the boundary condition for these areas as shown in Figure 6. Solving for the optimal quantities, we get:

$$\widehat{q}_{yf}^m = K_f^m,\ \widehat{q}_{(3-y)f}^m = 0,\ \widehat{q}_{yd}^m = K_y^m,\ \widehat{q}_{(3-y)d}^m = K_{(3-y)}^m,$$

and $\mathbf{A} \in \Omega_{9,10}$ where $y = 1$ for $\mathbf{A} \in \Omega_9$ and $y = 2$ for $\mathbf{A} \in \Omega_{10}$.

Both firms are capacity constrained (area Ω_{11}).

All slack variables are zero for both firms. Solving for quantities we get

$$\widehat{q}_{yf}^m = \frac{A_y - A_{3-y}}{4(1-\beta)} + \frac{\left(K_{(3-y)}^m - K_y^m\right)}{4} + \frac{K_f^m}{2},\ \widehat{q}_{yd}^m = K_y^m,\ y = 1, 2.$$

The flexible firm is capacity constrained and manufactures one product: product 1 for area Ω_{12} and product 2 for area Ω_{13}.

Area Ω_{13} is not in Figure 6. Take $y = 1$ for area Ω_{12} and $y = 2$ for area Ω_{13}. For the flexible firm, $u_f > 0$ and $v_{(3-y)f} > 0$ so that $\widehat{q}^m_{(3-y)f} = 0$. For the dedicated firm, $u_{ld} = 0$ for $l = 1, 2$. Solving we get $v_{(3-y)f} = (1/2)\left(A_y - A_{3-y} - 3K^m_f(1-\beta)\right) > 0$ which gives us the boundary condition for this area. The optimal production quantities are:

$$\widehat{q}^m_{yf} = K^m_f,\ \widehat{q}^m_{(3-y)f} = 0,\ \widehat{q}^m_{yd} = \frac{A_y - A_{(3-y)}\beta - K^m_f\left(1-\beta^2\right)}{2\left(1-\beta^2\right)},\ \widehat{q}^m_{(3-y)d} = \frac{A_{3-y} - A_y\beta}{2\left(1-\beta^2\right)}.$$

The FOC in the capacity game.

For the flexible firm, the FOC is given by $E\left(\partial\pi^m_f/\partial K^m_f\right) = c_f$, which translates into:

$$\begin{aligned}
c_f &= \iint\limits_{\Omega_2} \frac{1}{4}\left(x_1 + x_2 - 2K^m_f\left(1+\beta\right)\right) dF(x_1,x_2) \\
&+ \iint\limits_{\Omega_5} \frac{1}{\left(7+\beta\right)^2}\left(\begin{array}{c}(17-\beta)x_1 + (16-\beta\left(1-\beta\right))x_2 \\ +\left(-17+\beta\right)\left(1+\beta\right)\left(\left(1-\beta\right)K^m_1 + 2K^m_f\right)\end{array}\right) dF(x_1,x_2) \\
&+ \iint\limits_{\Omega_6} \frac{1}{\left(7+\beta\right)^2}\left(\begin{array}{c}(17-\beta)x_2 + (16-\beta\left(1-\beta\right))x_1 \\ +\left(-17+\beta\right)\left(1+\beta\right)\left(\left(1-\beta\right)K^m_2 + 2K^m_f\right)\end{array}\right) dF(x_1,x_2) \\
&+ \iint\limits_{\Omega_7} \left(x_1 - (\beta x_2/2) - \left(1-\beta^2\right)K^m_1 - (2-\beta^2)K^m_f\right) dF(x_1,x_2) \\
&+ \iint\limits_{\Omega_8} \left(x_2 - (\beta x_1/2) - \left(1-\beta^2\right)K^m_2 - (2-\beta^2)K^m_f\right) dF(x_1,x_2) \\
&+ \iint\limits_{\Omega_9} \left(x_1 - K^m_1 - \beta K^m_2 - 2K^m_f\right) dF(x_1,x_2) \\
&+ \iint\limits_{\Omega_{10}} \left(x_2 - K^m_2 - \beta K^m_1 - 2K^m_f\right) dF(x_1,x_2) \\
&+ (1/2)\iint\limits_{\Omega_{11}} \left(x_1 + x_2 - (1+\beta)\left(K^m_1 + K^m_2 + 2K^m_f\right)\right) dF(x_1,x_2) \\
&+ \iint\limits_{\Omega_{12}} \frac{1}{2}\left(x_1 - 2K^m_f\right) dF(x_1,x_2) + \iint\limits_{\Omega_{13}} \frac{1}{2}\left(x_2 - 2K^m_f\right) dF(x_1,x_2).
\end{aligned}$$

For the dedicated firm we have $\partial E\left(\pi^m_d\right)/\partial K^m_1 = c$, which translates into

$$\begin{aligned}
c &= (1/2)\iint\limits_{\Omega_2} \left(x_1 - 2\left(K^m_1 + \beta K^m_2\right)\right) dF(x_1,x_2) + (1/2)\iint\limits_{\Omega_4} \left(x_1 - x_2\beta - 2\left(1-\beta^2\right)K^m_1\right) dF(x_1,x_2) \\
&+ \frac{1}{\left(7+\beta\right)^2}\iint\limits_{\Omega_5} \left(\begin{array}{c}\left(33-\beta\left(2-\beta\right)\right)x_1 + \left(1-\beta\left(34-\beta\right)\right)x_2 \\ -\left(17-\beta\right)\left(1-\beta^2\right)\left(4K^m_1 + K^m_f\right)\end{array}\right) dF(x_1,x_2)
\end{aligned}$$

$$+\iint\limits_{\Omega_7}\left(x_1-\beta x_2-\left(1-\beta^2\right)\left(2K_1^m+K_f^m\right)\right)dF(x_1,x_2)$$

$$+\iint\limits_{\Omega_9}\left(x_1-\left(2K_1^m+2\beta K_2^m+K_f^m\right)\right)dF(x_1,x_2)$$

$$+\iint\limits_{\Omega_{10}}\left(x_1-\left(2K_1^m+2\beta K_2^m+\beta K_f^m\right)\right)dF(x_1,x_2)$$

$$+(1/4)\iint\limits_{\Omega_{11}}\left(3x_1+x_2-2\left(\left(3+\beta\right)K_1^m+\left(1+3\beta\right)K_2^m+\left(1+\beta\right)K_f^m\right)\right)dF(x_1,x_2),$$

and $\partial E\left(\pi_d^m\right)/\partial K_2^m=c$ yields:

$$\begin{aligned}
c &= (1/2)\iint\limits_{\Omega_2}\left(x_2-2\left(\beta K_1^m+K_2^m\right)\right)dF(x_1,x_2)+(1/2)\iint\limits_{\Omega_3}\left(x_2-x_1\beta-2\left(1-\beta^2\right)K_2^m\right)dF(x_1,x_2) \\
&+\frac{1}{(7+\beta)^2}\iint\limits_{\Omega_6}\begin{pmatrix}\left(33-\beta\left(2-\beta\right)\right)x_2+\left(1-\beta\left(34-\beta\right)\right)x_1 \\ -\left(17-\beta\right)\left(1-\beta^2\right)\left(4K_2^m+K_f^m\right)\end{pmatrix}dF(x_1,x_2) \\
&+(1/4)\iint\limits_{\Omega_{11}}\left(3x_2+x_1-2\left(\left(3+\beta\right)K_2^m+\left(1+3\beta\right)K_1^m+\left(1+\beta\right)K_f^m\right)\right)dF(x_1,x_2) \\
&+\iint\limits_{\Omega_8}\left(x_2-\beta x_1-\left(1-\beta^2\right)\left(2K_2^m+K_f^m\right)\right)dF(x_1,x_2) \\
&+\iint\limits_{\Omega_9}\left(x_2-2K_2^m-2\beta K_1^m-\beta K_f^m\right)dF(x_1,x_2) \\
&+\iint\limits_{\Omega_{10}}\left(x_2-2K_2^m-2\beta K_1^m-K_f^m\right)dF(x_1,x_2).
\end{aligned}$$

Proposition TA 5 *Equilibrium in the capacity game for the mixed market exists for all* $\beta\in(-1,1)$.

Existence follows from the concavity of the objective functions, which can be easily verified. Uniqueness is analytically difficult to show in this case as there is no symmetry argument that we can invoke. In fact, we conjecture that for a holdback strategy, the equilibrium in the mixed market may not be unique. This follows from the fact that there is more than one way to represent the capacity of the two firms as detailed by Figure 6 and, as we pointed out in the body of the paper, Figure 6 is not a unique representation.

Proposition TA 6 *For the firms operating in the mixed market:*

(i) *the MPNE in capacity for a*

- (a) *flexible firm is:* $K_f^m=\left(\mu_1+\mu_2+2c-4c_f\right)/3\left(1+\beta\right),$
- (b) *dedicated firm is:* $K_y^m=\left(5\mu_y-\mu_{3-y}+4c_f-8c\right)/12\left(1+\beta\right),\ y=1,2,$

(*ii*) *the expected profit at equilibrium for a*

(*a*) *flexible firm is:* $\Pi_f^m = \left(\sigma_1^2 + \sigma_2^2\right)/8\left(1-\beta\right) + (\mu_1 - \mu_2)^2/32\left(1+\beta\right) + \left(\mu_1 + \mu_2 + 2c - 4c_f\right)^2/18\left(1+\beta\right)$,

(*b*) *dedicated firm is:* $\Pi_d^m = (\mu_1 - \mu_2)^2/16\left(1+\beta\right) + \left(\mu_1 + \mu_2 + 2c_f - 4c\right)^2/18\left(1+\beta\right)$,

(*iii*) *the expected price for product* $y = 1, 2$ is $P_y^m = \left(7\mu_1 + \mu_2 + 8c_f + 8c\right)/24$.

Proof. For the mixed market under assumptions A-2 and A-3, the optimality condition of the firm investing into flexible technology is:

$$(1/2)\iint_{\Omega}\left(x_1 + x_2 - (1+\beta)\left(K_1^m + K_2^m\right) - 2K_f^m\right)dF(x_1, x_2) = c_f,$$

while the profit is

$$\begin{aligned}\Pi_f^m &= \frac{1}{16\left(1-\beta\right)^2}\iint_{\Omega}\left[\sum_{y=1}^{2}\left(\left(3-4\beta\right)x_y + x_{3-y} - \left(1-\beta\right)\left(3K_y^m + K_{3-y}^m + 2K_f^m\right)\right)\right.\\ &\quad \times\left.\left(x_y - x_{3-y} + \left(1-\beta\right)\left(K_{3-y}^m - K_y^m\right) + 2K_f^m\right)\right]dF(x_1, x_2) - c_f K_f^m.\end{aligned}$$

The optimality conditions of the firm investing into dedicated technology are:

$$(1/4)\iint_{\Omega}\left(3x_y + x_{3-y} - 2\left(\left(3+\beta\right)K_y^m + \left(1+3\beta\right)K_{3-y}^m + \left(1+\beta\right)K_f^m\right)\right)dF(x_1, x_2)dF(x_1, x_2) = c,\ y = 1, 2$$

while the profit is

$$\Pi_d^m = \sum_{y=1}^{2}\left[\left(\frac{1}{4\left(1-\beta\right)}\right)\iint_{\Omega}\left(\left(3-4\beta\right)x_y + x_{3-y} - \left(1-\beta\right)\left(3K_y^m + K_{3-y}^m + 2K_f^m\right)\right)K_y^m dF(x_1, x_2) - cK_y^m\right].$$

The results follow after some algebra. ■

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