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Abstract
This paper examines the demand and supply of annual and multi-year insurance contracts with respect to protection against a catastrophic risk in a competitive market. Insurers who offer annual policies can cancel policies at the end of each year and change the premium in the following year. Multi-year insurance has a fixed annual price for each year and no cancellations are permitted at the end of any given year. Homeowners are identical with respect to their exposure to the hazard. Each homeowner determines whether or not to purchase an annual or multi-year contract so as to maximize her expected utility. The competitive equilibrium consists of a set of prices where homeowners who are not very risk averse decide to be uninsured. Other individuals demand either single-year or multi-year policies depending on their degree of risk aversion and the premiums charged by insurers for each type of policy.

Keywords
insurance, multi-year policies, catastrophic risk, risk aversion

Disciplines
Insurance | Marketing | Other Business

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Single-Year and Multi-Year Insurance Policies in a Competitive Market

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ABSTRACT

This paper examines the demand and supply of annual and multi-year insurance contracts with respect to protection against a catastrophic risk, such as a hurricane or earthquake, in a competitive market. Households are identical with respect to their exposure to the hazard but with different degrees of risk aversion. They determine whether or not to purchase an annual or multi-year contract so as to maximize their expected utility. Insurers who offer annual policies can cancel policies at the end of each year and change the premium in the following year. Multi-year insurance has a fixed annual price for each year and no cancellations are permitted at the end of any given year.

The competitive equilibrium consists of a set of prices for a single year and multi-year policy that segments homeowners who are not very risk averse into the non-insurance category. Consumers who are not very risk averse decide not to purchase insurance. More risk averse individuals demand either single-year or multi-year policies depending on the premiums charged by insurers and their degree of risk aversion.

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1. Introduction

Insurance policies for property insurance are normally issued as annual contracts even though they are not precluded from offering coverage for a longer time period. If the risk of a loss is stable over time, then the insurer should be willing to offer a multi-year policy with a fixed annual premium reflecting risk. The variance of the losses to the insurer over time would be reduced in much the same way that an increase in the number of insured individuals reduces the variance. In this sense, offering a policy for more than one year is another form of risk diversification.

The insurer who offers a multi-year policy at a fixed premium per year is restricted by not being able to either raise the premium and/or cancel policies if it suffers a very large loss that reduces its surplus significantly. This feature is similar to guaranteed renewable policy with locked premiums for health insurance (Pauly, Kunreuther, and Hirth, 1995, Frick, 1998, Pauly, Kunreuther, Menzel, and Hirth, 2011). On the other hand, the administrative cost of marketing a policy is lower for a multi-year policy than for annual contracts that have to be renewed each year. A multi-year policy is attractive to a risk averse consumer because the premium is stable over time. Furthermore, the consumer knows that it will not have to incur search costs in finding another insurer if its policy is canceled that has occurred historically when major catastrophes cause insurers to adjust their underwriting criteria.

This paper examines from a theoretical perspective the relative attractiveness of multi-year policies and annual policies to insurers and to households facing a given risk in a competitive insurance market. We are particularly interested in pursuing this line of research because individuals are likely to find multi-year property insurance attractive relative to annual policies. Adding fixed-price multi-year property insurance policies to the menu of contracts offered by insurers should also lead to an increase in the demand for coverage.

Empirical evidence supporting these points comes from a web-based controlled experiment played for real money where individuals had the option of purchasing 1-year or 2-year insurance contracts to protect their property against damage from a hurricane With respect to the 1-year policy, the price would increase in year 2 if a hurricane occurred in year 1 to reflect an updating of the probability of a disaster occurring in the following year. The premium for a 2-year contract was kept stable over time; however, the insurer charged a higher premium than for an annual policy offered in year 1 because it could not increase its premium in year 2 if a hurricane occurred in the previous year.

The data reveal that when insurers offer individuals 1-year policies at an actuarially fair rate, approximately 62% of the subjects purchase insurance. When the same 1-year contract is offered along with a 2-year contract that has a loading cost of 5%, the demand for insurance increases to 73%---a statistically significant difference. Over 60% of those demand insurance preferred the 2-year policy even though its expected cost was 5% higher than buying two annual policies (Kunreuther and Michel-Kerjan, 2012).

In practice, insurers do offer multi-year contracts for life insurance where the losses are normally independent of each other. Term-life policies are typically offered with premiums “locked in” for five to ten years; buyers can choose whether they want to pay extra for such guarantees over annual contracts knowing that they may drop coverage at any time. Policyholders are then certain what their life insurance premiums will be over the next five or ten years, regardless of what happens to their health or the overall mortality rate of their insurer's portfolio.

Hendel and Lizzeri (2003) examine 150 term life insurance contracts, some of which have fixed premiums for 5, 10 or 20 years while others are 1-year renewable policies. They show that on average the extra prepayment of premiums to protect consumers against being reclassified into a higher risk category for a fixed period of time is more costly over the total period of coverage than a series of annual term policies that can be renewed but where premiums may fluctuate from year to year. Still, people buy those multi-year fixed-price life insurance policies, indicating that they view the stability of premiums as an important attribute in their utility function and are willing to pay more for it.

An important difference between property and life coverage is that insurers are concerned about catastrophic losses to property due to natural disasters such as hurricanes and earthquakes. They thus have to reserve capital in to protect themselves against these extreme events. There is an opportunity cost to them of being forced to keep this capital in relatively liquid form rather than investing the money in securities that earn a higher expected return.

On the demand side, consumers may want to purchase multi-year property insurance policies is to avoid the search costs of looking for another insurer should their annual policy be canceled. After the 2004 and 2005 hurricane seasons many insurers did not renew coverage for a significant number of homeowners in the Gulf Coast (Klein 2007). While most of those residents were able to find coverage with other insurers, they typically had to pay a higher price than prior to these disasters and they were required to have a higher deductible (Vitelo, 2007). Others obtained coverage from state insurance pools, which grew significantly after the 2004 and 2005 hurricane seasons (Grace and Klein, 2009).

Multi-year property insurance contracts can also improve social welfare by increasing the number of individuals whose homes are insured prior to a disaster. They will receive claim payments to restore property damaged from a hurricane or flood and have less need for federal disaster assistance. The cost to the general taxpayer is thus reduced.

In this regard, the number of uninsured homes facing losses from natural disasters is significant. The Department of Housing and Urban Development (HUD) revealed that 41% of damaged homes from the 2005 hurricanes were uninsured or underinsured. Of the 60,196 owner-occupied homes with severe wind damage from these hurricanes, 23,000 did not have insurance against wind loss (U.S. Government Accountability Office, 2007). With respect to risks from flooding, Kriesel and Landry (2004) and Dixon et al. (2006) found that only about half of the homes in high-risk areas had flood insurance. In California, despite the well-recognized risk of earthquakes, only 12% of homeowners in the state had coverage against earthquake damage at the end of 2010 (California Department of Insurance, 2011).

A principal reason for this lack of coverage is that many residents purchase coverage only after a disaster occurs and then allow their annual policy to lapse. Flood insurance in the U.S. provided by the federally run National Flood Insurance Program (NFIP) since its creation in 1968 highlights this point. Brown and Hoyt (2000) analyzed data from the NFIP between 1983 and 1993 found that flood insurance purchases were highly correlated with flood losses in the previous year. A recent analysis of all new policies issued by the NFIP from January 1, 2001 to 31 December 2009 revealed that the median length of time before these policies lapsed is 3 to 4 years. On average, only 74% of new policies were still in force 1 year after they were purchased; after 5 years, only 36% were still in place. The lapse rate is still high after correcting for migration and does not vary much across flood zones (Michel-Kerjan et. al. in press).

The paper is organized as follows. Section 2 models the multi-period insurance problem and develops conditions for determining a competitive equilibrium. Section 3 analyzes the case where the loss distribution facing each homeowner is Gaussian and where homeowner's risk preferences are of the CARA form using an exponential utility function. Section 4 provides illustrative examples based on the Gaussian distribution and CARA utility function to show how the demand for annual and multi-year insurance contracts is impacted by changes in the costs affecting the price of insurance and the correlation between risks. Section 5 summarizes the paper and suggests directions for future research.

2. Modeling Multi-period Insurance

We consider an insurance market operating over two periods to cover a set of households exposed to a common catastrophic risk such as earthquake or hurricane risk. The insurance market is assumed to be competitive with free entry and exit, but subject to solvency regulation. Risk bearing capital is obtained from premium income and reinsurance. The price of reinsurance in period 1 is known, but the price in period 2 is uncertain, and is specified by a binary random variable with a specified increase or decrease relative to the price in period 1 which depends on whether

Two types of products, single-period policies and multi-period policies, compete for consumer demand for insurance. Homeowners can purchase either no insurance, single-year policies in one or both periods, or a multi-year policy purchased at the beginning of the first period and covering both periods. The competitive insurance price is determined so that it covers expected losses, marketing and operating expenses, plus the cost of risk capital necessary to provide protection against insolvency, where the level of the required capital is determined exogenously by the insurance regulator.

We assume that households are identical in terms of their exposure to the hazard, but with some correlation in the losses, such as would be the case if a natural disaster damaged a number of insured homes. Denote the set of potential homeowners in the market by A. Homeowners are assumed to be risk averse with $a \in A$ being a scalar index of risk aversion and with two–period separable risk preferences given by

$$V(x_1, x_2, a) = U(x_1, a) + U(x_2, a) \qquad (1)$$

where $U(x, a)$ is concave increasing in x, and x_1 and x_2 are monetary outcomes in periods 1 and 2. Note that for simplicity we neglect discounting of utility in period 2. While A represents the set of potential homeowners, the actual distribution of homeowner risk aversion will be specified by the counting function defined for any $a \in A$ by

G(a)= Number of Homeowners with risk aversion less than or equal to "a".

To illustrate, suppose the number of homeowners is 100 of which 40 have $a = 0.1$ and 60 have $a=0.2$. Then, $G(a)=0$ for $a < 0.1$, $G(a)=40$ for $0.1 \le a < 0.2$, $G(a) = 100$ for $a \ge 0.2$.

We make the following assumptions concerning the hazard and the policies offered in the insurance market to homeowners.

A1. Only full insurance is offered and each household $a \in A$ faces the same annual/period risk $\tilde{X}(a)$ of loss, distributed according to the generic cdf $F_0(x, \mu, \sigma, \rho)$, with mean $E\{\tilde{X}(a)\} = \mu$ and variance $VAR\{\tilde{X}(a)\} = \sigma^2$, where $\rho \ge 0$ is a scalar index which is intended to measure the extent to which the loss distribution for a book of business (BoB) made up of properties from the set A is fat-tailed. The loss distribution is assumed to be identical for both periods, and statistically independent between the periods.

The impact of the index ρ will be specified below; it may be thought of as an index of the cost of reinsurance cover for a BoB made up of properties from the set A. We use ρ to model the impact of correlated losses on standard reinsurance pricing models with constant or increasing loading factors. The reinsurance will be an Excess of Loss (XoL) contract with fixed upper and lower limits designated as $L_1(n)$, and $L_2(n)$ respectively.

A2. Firms offering single-year (SY) policies may cancel any policy at the end of the first period in response to increased cost of risk capital that leads them to want to reduce their BoB. Homeowners are aware of this possibility and assume that there is a probability $q \in (0, 1)$ of cancellation, with an ensuing transaction cost of $\tau \ge 0$ to search for new coverage alternatives for period 2 if their policy is canceled.

2.1 Demand for Single-Year (SY) and Multi-Year (MY) Policies

Homeowners face the choice of purchasing either two single-year policies or a multi-policy to cover the risks they face over the two–period horizon of interest. Of course, they may also elect to buy no insurance (NI) in one or both periods. At the beginning of period one, homeowners must either choose an MY policy covering both periods, or they must plan on some other sequence of SY policies or NI decisions. In doing so, we assume that homeowners have complete information on the prices they will face.

A3: Homeowners know the prices for all policies in all states of the world at the beginning of period 1. For the MY policy, the price per period price P_M is constant over both periods. For the SY policy, price in the first period is denoted P_{S1}, and the state-contingent price in the second period is denoted P_{S2}^{w}, where $w \in \{d, u\}$ reflects the state of the world in terms of reinsurance/capital costs with probability of state $d = \varphi \in [0,1]$ and probability of state $u = 1-\varphi$. The consequences of these alternative states for the insurer are described below, but their general import is that the cost of reinsurance in period 2 will decrease in state d and increase in state u relative to period 1.

At the beginning of period 1, there are three alternatives available to a homeowner:

i) Purchase an SY policy for period 1, at price P_{S1}, possibly facing cancellation of this policy at the end of period 1, with probability q and with resulting transactions costs τ.

ii) Purchase an MY policy at price per period of P_M covering both periods 1 and 2.

iii) Purchase No Insurance (NI).

At the beginning of period 2, after the state of the world w ∈ {d, u} is known, a homeowner faces the following choices (see Figure 1):

i) If the homeowner chose either an SY policy or NI in period 1, then the homeowner can either choose NI or purchase an SY policy for period 2 at price P_{S2}^{w} where the price can either decrease to P_{S2}^{d} or increase to P_{S2}^{u} depending on reinsurance capital costs.

ii) The homeowner who chose an MY policy in period 1 can continue to be covered under this MY policy or can cancel it with a cancellation fee $\psi \geq 0$. The cancellation fee is set by the insurer so that it breaks even at the end of period 2. If there were no cancellation costs associated with an MY policy, then all homeowners will want to switch to an SY policy if the following two conditions hold:

- the price of an SY policy decreased in period 2 so it was less than P_M; and
- the price differential between the MY and SY policy is greater than the transaction cost τ of purchasing a new policy.

In this case, the MY insurer would only offer coverage in the state of the world w = u. As we show in the Appendix, P_M is less than the MY insurer's average cost at w = u, so that the MY insurer would suffer a loss in the process. It will have priced its policy under the assumption that it would recover its costs from revenues generated in both periods, but could not recoup these costs (some of which will be sunk in period 1) because its policyholders abandoned ship at the end of period 1. Hence, for MY insurance to be viable, the cancellation fee ψ imposed by the MY insurer has to satisfy: $\psi \geq P_M - P_{s2}^{d} - \tau$.[5] In this case, all homeowners would maintain their MY policy in period 2.

For our benchmark analysis in this paper, we assume, per A3, that homeowners are perfectly informed about prices and the probability distribution on the states of the world w ∈ {d, u}, i.e., they know φ. We assume that their beliefs about the cancellation probability q are fixed and independent of the actual BoB changes by insurers.[6] Figure 1 shows the relevant choices for a homeowner in period 2.

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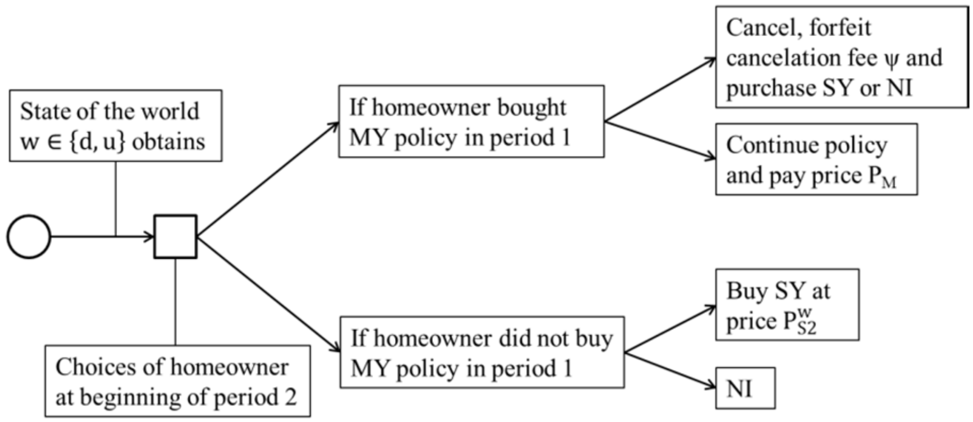
[5] Our numerical examples show for typical values of insurance cost that the lower bound $P_M - P_{s2}^{d} - \tau$ on the cancelation fee ψ will be only a small percentage of the MY premium. Indeed, the lower bound may even be negative if the MY insurer has significant marketing cost advantages and/or if τ is sufficiently large.

[6] Of course, q could be adjusted in rational expectation fashion so that it reflected the equilibrium outcomes of SY insurers between periods 1 and 2.

Figure 1: Homeowner's Choice Problem in Period 2



Given our assumption that homeowners will not cancel MY policies in period 2, the state-dependent insurance decision $I_2(a, w)$ of homeowner a ∈ A, w ∈ {d, u} at the beginning of period 2 will be the following:

D2-i) $I_2(a, w) = MY$ if $I_1(a) = MY$, with resulting period 2 utility of $U(-P_M, a)$

D2-ii) $I_2(a, w) = SY^w_2$ if $I_1(a) \neq MY$ and $U(-P^w_{S2}, a) \geq U(CE(NI, a), a)$ with resulting period 2 utility of $U(-P^w_{S2}, a)$

D2-iii) $I_2(a,w) = NI$ if $I_1(a) \neq MY$ and $U(-P^w_{S2}, a) < U(CE(NI, a), a)$ with resulting period 2 utility of $U(CE(NI, a), a)$

where the certainty equivalents (CEs) of the various policies offered, MY and SY^w_2, w {d, u}, are $CE(MY) = -P_M$ and $CE(SY^w_2) = -P^w_{S2}$. The CE(NI, a) is characterized by $U(CE(NI, a), a) = E\{U(-\tilde{X}(a), a)\}$.7

The above three conditions can be interpreted in the following manner. A homeowner will have an MY policy in period 2 only if he purchased an MY policy in period 1 (D2-i). A homeowner will purchase an SY policy in period 2 if he did not purchase an MY policy in period 1 and is sufficiently risk averse so that the utility of having full insurance is greater than the expected utility of having no insurance in period 2. (D2-ii). He will be in state NI if the expected utility of having no insurance in period 2 exceeds the utility of SY(D2-iii).

Demand in period 2 $[D_2(z, w)]$ for the policy options $Z_2 = \{MY, SY^w_2, NI\}$ follows directly from the above. Let $\Delta_2(z, a, w) = 1$ if $I_2(a, w) = z$ and $\Delta_2(z, a, w)=0$ otherwise, where $z \in \{MY, SY^w_2, NI\}$. Then,

$$D_2(z,w) = \int_A \Delta_2(z,a,w)dG(a), \; z \in \{MY, SY^w_2, NI\}. \tag{2}$$

Equation (2) just allocates homeowners in period 2 to an MY, SY or NI policy as a function of their degree of risk aversion and whether the reinsurance/capital costs are in state d or u.

7 For convenience, we suppress explicit dependence of certainty equivalents on initial wealth levels W(a).

Turning to period 1, recall that an SY insurer can cancel a policy at the end of period 1, and that homeowners believe this will occur with probability q and result in transactions costs τ. Then the certainty equivalent $CE(SY_1,a)$ of a first-period SY policy is characterized by

$$U(CE(SY_1, a), a) = qU(-(P_{S1}+ \tau), a)+(1-q)U(-P_{S1}, a). \tag{3}$$

Equation (3) indicates that higher values of q reduces the attractiveness of an SY policy because the homeowner is more likely to have her policy canceled and will have to incur a search cost τ to find an insurer who will be willing to sell her a policy in period 2.

The optimal period 1 decision (note that we ignore discounting) is then determined through dynamic programming as follows. First, define the expected utilities $V_1(z)$ associated with choosing each of the options $z \in Z_1 = \{MY, SY_1, NI\}$ in period 1 and the optimal state-dependent choice following this in period 2. Then

$$V_1(MY, a) = 2U(-P_M, a) \tag{4}$$

$$\begin{aligned} V_1(SY_1, a) &= U(CE(SY_1, a), a) \\ &+ \varphi Max[U(-P^d_{S2}, a),U(CE(NI, a), a)] + (1-\varphi)Max[U(-P^u_{S2}, a), U(CE(NI, a), a)] \end{aligned} \tag{5}$$

$$\begin{aligned} V_1(NI, a) &= U(CE(NI, a), a) \\ &+ \varphi Max[U(-P^d_{S2}, a),U(CE(NI, a), a)] + (1-\varphi)Max[U(-P^u_{S2}, a), U(CE(NI, a), a)] \end{aligned} \tag{6}$$

where, in view of the possibility of cancellation of the SY policy at the end of period 1, the period 1 expected utility of choosing an SY policy is given by (3).

Equation (4) states that a homeowner who purchases MY insurance in period 1 will continue to be insured by the same policy in period 2. Equation (5) states that a homeowner who purchases SY insurance in period 1 incurs the immediate cost of the premium P_{S1} and, with probability q, may incur an additional transactions cost τ if the policy is canceled at the end of period 1, as reflected in the CE of SY_1 given in (3). This same homeowner has the option of buying a second SY policy in period 2 or NI, and will choose the best of these two options in each state of the world $w \in \{d, u\}$. Equation (6) states that a homeowner choosing NI in period 1 can choose purchase an SY policy in period 2 or remain uninsured (NI) and will choose the best of these options in each state of the world $w \in \{d, u\}$.

The optimal first-period choice for homeowner $a \in A$ is then given by

$$I_1(a) = ArgMax_z\ [V_1(z, a) \mid z \in Z_1=\{MY, SY_1, NI\}]. \tag{7}$$

The demand in period 1$[D_1(z)]$ for the policy options $Z_1= \{MY, SY_1, NI\}$ follow directly from the above. Let $\Delta_1(z, a) = 1$ if $I_1(a) = z$ and $\Delta_1(z, a) = 0$ otherwise, where $z \in \{MY, SY_1, NI\}$. Then,

$$D_1(z) = \int_A \Delta_1(z,a)dG(a),\ z \in \{MY, SY_1, NI\}. \quad (8)$$

The above characterization of demand for MY and SY policies is general. Final demands for these policies in both periods depend on the distribution of homeowner risk preferences as reflected in their degree of risk aversion characterized by G(a). It also of course depends on the loss distribution $\tilde{X}$. In section 3 below we consider the special case where per period losses $\tilde{X}$ are normally distributed and the risk preferences of homeowners are of the CARA form, $U(x,a) = -e^{-ax}$.

2.2 Supply and Pricing of Insurance

We assume a competitive insurance market which consists of two types of firms, those offering SY policies and those offering MY policies. Firms offering SY policies can adjust the size of their BoB at the end of period 1 in response to changes in the cost of reinsurance (i.e. in response to the realized state of the world $w \in \{d, u\}$) . In a competitive equilibrium, the size of the insurer's BoB is determined by the minimum of the average cost curve for the insurer. SY insurers will therefore cancel some policies at the end of period 1 in the state of the world in which reinsurance rates increase from period 1 to period 2 and will expand their BoB when reinsurance rates decrease. MY insurers do not have this option and must provide coverage in both periods to all homeowners to whom they issued policies in period 1.

Reinsurance costs are assumed to depend on the non-negative scalar index $\rho > 0$, which may be thought of as an index of the fat-tailed nature of the distribution of a BoB of size n from the set A (see A1). A more specific model for reinsurance pricing is discussed in section 4. The following assumption summarizes the relationship of the regulated solvency risk level and reinsurance costs for insurers.

A4. Insurers are required to satisfy a regulatory solvency constraint.[8] It requires for each period their combined premium revenue plus reinsurance be sufficiently large to pay all claims with a probability of at least $1 - \gamma^*$.

This regulatory solvency constraint is similar to a safety first model that insurers often utilize to determine the optimal BoB and pricing decisions. It was first proposed by Roy (1952), examined in the context of the theory of the firm and profit maximization by Day, Aigner and Smith (1971) and applied to insurance by Stone (1973a and 1973b). Following the series of natural disasters that occurred at the end of the 1980s and in the 1990s, many insurers focused on the solvency constraint to determine the maximum amount of catastrophe coverage they were willing to provide. More specifically they were concerned that their aggregate exposure to a particular risk not exceed a certain level. Today rating agencies, such as A.M. Best, focus on insurers' exposure to catastrophic losses as one element in determining credit ratings, another reason for insurers to focus on the solvency constraint (Kunreuther and Michel-Kerjan 2011).

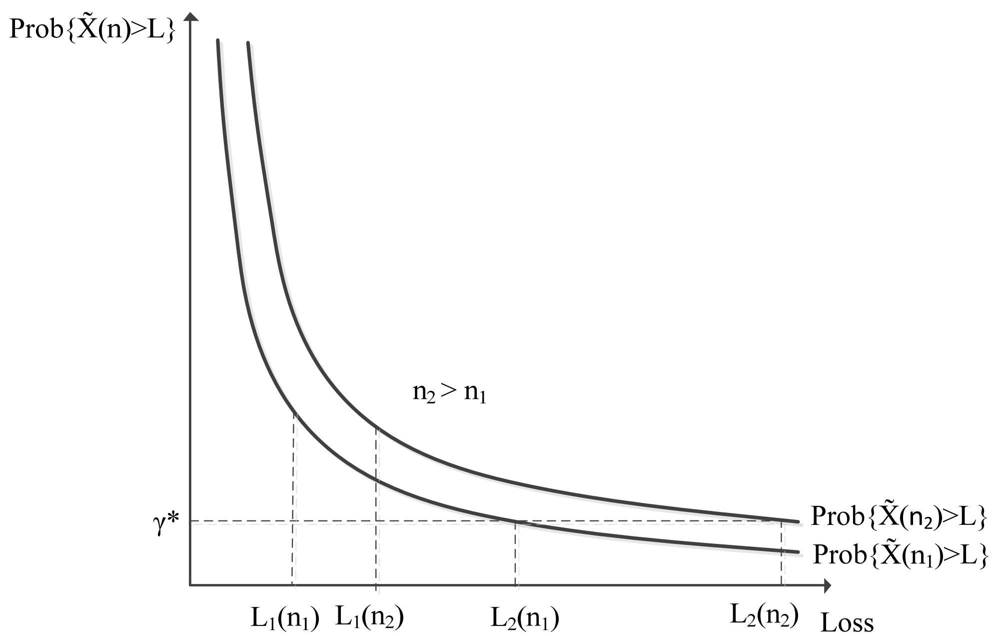
[8] We do not analyze the costs of insolvency here, but rather assume that any insolvencies are paid for by an independent mechanism that does not affect the supply and demand decisions modeled here.

For simplicity, we assume that insurers with a BoB of size n meet their solvency constraint by purchasing XoL reinsurance with limits $L_1(n)$, $L_2(n)$, with $L_1(n) = n\mu$ (the expected loss cost for a BoB of size n) and $L_2(n)$ set to meet the solvency constraint. Consider an insurer with BoB = $\{a_1, a_2, \ldots, a_n\}$ and the associated total loss distribution $\tilde{X}(n) = \sum_{i=1}^{n} \tilde{X}(a_i)$, with cdf $F\left(L; \tilde{X}(n)\right)$. Then, $L_2(n)$ is characterized by[9]

$$\gamma^* = 1 - F\left(L_2(n); \tilde{X}(n)\right) = \text{Prob}\{\tilde{X}(n) > L_2(n)\}.$$

Figure 2 illustrates the above assumption on reinsurance attachment points and the solvency constraint. Reinsurance contracts are on a per period basis, corresponding also to the solvency constraints that are required to be fulfilled in each period.

Figure 2: Illustrating reinsurance attachment points under assumption A4



The costs to insurers of providing coverage encompass administrative and selling expenses, loss costs and reinsurance costs, and depend on the size of the BoB (n). Formally, the expected total costs per period for SY and MY policies are given as:

$$C_{SY}(n;r,\zeta) = C_0(n) + C_m(n) + \mu n + C_s(n;r,\mu,\sigma,\rho) \quad (9)$$

$$C_{MY}(n;r,\zeta) = C_0(n) + \nu C_m(n) + \mu n + C_s(n;r,\mu,\sigma,\rho) \quad (10)$$

[9] In the insurance literature, the negative cdf $1-F\left(L;\tilde{X}(n)\right)$ is referred to as the exceedance probability (EP) curve. It is fundamental in reinsurance calculations since expected losses between any two attachment points can be calculated as the area under the EP curve between these attachment points. See Grossi and Kunreuther (2005) for details.

where the vector of parameter values is given by $\zeta = (\mu, \sigma, \rho, \gamma^*, \varphi, q, \tau)$ and where the elements of total expected cost are: $C_0(n)$ represents administrative and selling expenses; $C_m(n)$ represents marketing expenses; μn are total expected losses; and $C_s(n;r, \mu, \sigma, \rho)$ represents reinsurance cost. Note that, with the exception of marketing costs, all of the elements of total expected cost are identical for SY and MY insurers. Concerning marketing costs, it is assumed that $\nu \in (0, 1]$ so that these per period marketing costs are likely to be less for an MY insurer since its policyholders in the second period are locked in as a result of first period choices. Thus, if SY and MY insurers were to choose the same BoB "n" in both periods, and if $v = 1$, then both insurers would have identical total expected cost. However, SY insurers can adjust their BoB from period to period in response to changes in reinsurance costs, while the MY insurer is constrained to offer the same BoB in both periods. Thus, SY and MY insurers will in general face different total expected costs because of potential marketing cost advantages for the MY insurer and potential flexibility advantages of the SY insurer in responding to changing reinsurance costs.

Reinsurance costs are net of expected payoffs from reinsurance contracts, which are all of the XoL variety. In other words, the reinsurance cost reflects the additional premium above the expected loss paid by the insurer to the reinsurer for protection against a portion of the loss between the attachment points of the XoL contract. We assume that the average underwriting costs $[C_0(n)/n]$ are convex and decreasing as n increases to reflect the spreading of fixed costs over more policies. The average marketing costs $[C_m(n)/n]$ are convex and increasing in n, representing the higher marginal cost of marketing as the insurance territory increases. Losses to the insurer have a mean μ and standard deviation σ. Average reinsurance costs $[C_s(n;r, \mu, \sigma, \rho)/n]$ are convex in n and are dependent on whether the reinsurer is in state d or u so that $r = r(w)$, $w \in \{d, u\}$. These costs are also assumed to be increasing in ρ (the fat-tail index) reflecting the need for the reinsurer to hold higher reserves due to an increased probability of experiencing large losses.

The Appendix, Part 1, specifies the derivation of the average costs and the equilibrium conditions in a competitive market. Competitive equilibrium in both the SY and MY markets occurs where insurers of each type select a BoB that minimizes their average cost, $C_{SY}(n;r,\zeta)/n$, $C_{MY}(n;r,\zeta)/n$, with price given by the minimum of the respective average cost curve. The assumptions concerning the elements of average costs discussed in the Appendix, Part 1, assure the existence of the equilibrium for both SY and MY markets. These assumptions also imply a number of intuitively appealing results for the comparative statics of equilibrium prices and BoBs for both MY and SY insurers. For example, since reinsurance costs in period 2 increase (u) or decrease (d) relative to period 1, depending on the state of the world, $w \in \{d,u\}$, equilibrium prices in the SY market satisfy: $P^d_{S2} < P_{S1} < P^u_{S2}$ and the corresponding optimal BoBs satisfy: $n^d_{S2} > n_{S1} > n^u_{S2}$. Also, the average costs of the MY insurer satisfy: $AC^d_{M2} < P_M < AC^u_{M2}$ so that the MY insurer has positive profits in state $w = d$ and losses in state $w = u$.[10]

[10] In particular, as argued earlier, the cancelation fee needs to satisfy: $\psi \geq P_M - P^d_{s2} - \tau$ in order to assure that the MY insurer will have viable operations in both the profitable state of the world $w = d$ as well as the unprofitable state of the world $w = u$.

The model proposed here suggests a number of trade-offs on both the demand and the supply side in evaluating the survival and efficiency of MY vs SY policies in competitive equilibrium. On the demand side, MY policies offer a constant price over both periods and therefore are attractive to risk-averse homeowners in avoiding intertemporal price volatility. MY policies also protect homeowners from the transactions costs of policy cancellation (represented by the parameters q, τ above) associated with changes in the equilibrium BoB for SY insurers between periods 1 and 2 that result from changes in the cost of capital and reinsurance.

On the supply side, there may be marketing and policy service cost savings associated with MY policies. However, MY policies expose the insurer to increased risk of reinsurance cost volatility, since the MY insurer cannot adjust his BoB in response to the realized state of the world $w \in \{d,u\}$ in contrast to the SY insurer. The ultimate outcome in terms of equilibrium prices and demands for MY vs SY policies is an empirical matter depending on which of these trade-offs dominates and on the risk preferences of homeowners. The results in the next sections illustrate this for the case of Gaussian loss distributions and CARA preferences.

3. The Case of Gaussian Losses and CARA Preferences

This section describes the case where $\widetilde{X}$, the generic loss distribution facing each homeowner, is Gaussian $N(\mu, \sigma^2)$ and where homeowner risk preferences are of the CARA form: $U(x, a) = -e^{-ax}$, where $a > 0$, i.e $A = \mathbb{R}^+$, the positive reals. We further assume that there is a non-negative correlation $\rho \in [0, 1]$ between the loss distributions facing two homeowners $\widetilde{X}(a)$ and $\widetilde{X}(b)$, identical for all $a, b \in A$ ($a \neq b$). Thus, an insurer with a BoB of n homes would face an annual loss distribution with mean $n\mu$ and variance $\sigma^2[n+n(n-1)\rho]$. These assumptions allow certainty equivalents (CEs) for the demand equations to be computed easily. Reinsurance costs for this case can be also readily computed for Excess of Loss (XoL) contracts with fixed limits $L_1(n)$, $L_2(n)$, and linearly increasing loading factors. These costs can be shown to satisfy all of the assumptions detailed in the Appendix Part 1, including increasing costs as ρ increases.

Equilibrium prices under competition are determined by (T1) and (T2) as shown in the Appendix. These determine, for any given cost functions and reinsurance market conditions, the price vector $\{P_M, P_{S1}, P^d_{S2}, P^u_{S2}\}$. This price vector and the parameter vector $\zeta = (\mu, \sigma, \rho, \gamma^*, \varphi, q, \tau)$ are assumed to be common knowledge for both insurers and homeowners. We calculate demand for each homeowner $a \in A$ from the demand equations (2)-(8), obtaining market demand by aggregation over G(a).

Note that, given the Gaussian and CARA assumptions, the Certainty Equivalent (CE) of No Insurance (NI) for homeowner $a \in A$ in either period 1 or 2 is given by: $CE(NI, a) = -(\mu+\frac{a\sigma^2}{2})$. Thus, conditions D2-i) – D2-iii) translate into the following decision rules for determining period 2 demand for homeowner $a \in A$:

D2N-i) $I_2(a, w) = MY$ if $I_1(a) = MY$, with resulting period 2 utility of $U(-P_M,a)$

D2N-ii) If $I_1(a) \neq MY$, then $I_2(a,w) = NI$ or SY whichever yields the maximum expected utility with a resulting period 2 utility of $\text{Max}\{U(-P_{S2}^w, a), U(CE(NI, a), a)\}$.

Period 1 demand for $a \in A$ is then specified by (4)-(6) which are summarized as follows:

D1N-i) $I_1(a) = MY$ if $V_1(MY, a) = 2U(-P_M, a) \geq \text{Max}\{V_1(SY, a), V_1(NI, a)\}$;

D1N-ii) Else $I_1(a) = SY$ or NI, whichever gives the highest period 1 expected utility $V_1(NI, a)$, $V_1(SY, a)$ as given in (5)–(6), noting from (7) that the CE of purchasing SY insurance in period 1 is $CE(SY_1, a) = -\frac{1}{a}\log\left[L(a, \zeta)\, e^{aP_{S1}}\right]$, with $L(a, \zeta) = qe^{a\tau}+(1-q)$.

The consequences of the above rules for period 2 choices are summarized as follows. A homeowner who chose an MY policy in period 1 will continue it in period 2. A homeowner who chose either NI or SY policy in period 1 will choose the option, NI or SY that has a higher expected utility. This gives rise to two critical cutoff values of homeowner risk preferences, $\hat{a}_2^d$ and $\hat{a}_2^u$, which are the cut off values for NI vs SY choices in period 2 given the state of the world $w \in \{d, u\}$ so that:

$$U(CE(NI, a), a) = \text{Max}\{U(-P_{S2}^w, a), U(CE(NI, a), a)\} \text{ for } a < \hat{a}_2^w$$

$$U(-P_{S2}^w, a) = \text{Max}\{U(-P_{S2}^w, a), U(CE(NI, a), a)\} \text{ for } a \geq \hat{a}_2^w. \tag{11}$$

The comparisons in (11) are equivalent to determining when $P_{S2}^w >$ or $\leq \mu+\frac{1}{2}a\sigma^2$, where the cutoff values are therefore given by: $\hat{a}_2^w=\frac{2(P_{S2}^w-\mu)}{\sigma^2}$, $w \in \{u, d\}$. Since, as noted above, $P_{S2}^d < P_{S2}^u$, it follows that $\hat{a}_2^d < \hat{a}_2^u$.

The cutoff values in Period 2 indicate that consumers who are more risk averse will want to purchase an SY policy in period 1. Furthermore if the price of insurance is lower because the insurer is in the d rather than u state, consumers who are less risk averse will still be willing to purchase insurance (i.e. $\hat{a}_2^d < \hat{a}_2^u$).

Given the period 2 choice, the optimal choice in period 1 is specified by D1N-i) - D1N-ii). To characterize this solution, define the critical threshold risk aversion $\hat{a}(P_{S1}) \in A$ as the solution to $V_1(NI, a) = V_1(SY_1, a)$. In the Appendix Part 2, $\hat{a}(P_{S1})$ is shown to exist and be unique. As we will see below, homeowners with risk aversion $a < \hat{a}(P_{S1})$ will prefer NI to SY in period 1 and homeowners with risk aversion $a > \hat{a}(P_{S1})$ will prefer SY to NI in period 1 (if these were the only choices available). From the definition of $V_1(NI, a)$, $V_1(SY_1, a)$ (see (5) and (6), we see that period 2 payoffs from choosing NI or SY in period 1 are identical. so that $\hat{a}(P_{S1})$ is the solution to $U(CE(NI, a), a) = U(CE(SY_1, a), a)$. This is equivalent to $CE(NI_1, a) = CE(SY_1, a)$.

For the CARA/Gaussian this yields the following identity characterizing $\hat{a}(P_{S1})$:

$$\hat{a}(P_{S1}): \quad \mu + \frac{1}{2} a\sigma^2 = \frac{1}{a}\log\left[L(a, \zeta)\, e^{aP_{S1}}\right]. \tag{12}$$

Similarly, define the functions $H_1(a, \text{NI-MY})$, respectively $H_1(a, \text{SY-MY})$, as the value of P_M for which $V_1(\text{NI}, a)=V_1(\text{MY}, a)$, respectively $V_1(\text{SY}_1, a)=V_1(\text{MY}, a)$. From (4)-(6), these functions are characterized by:

$$H_1(a, \text{NI-MY}): \quad 2U(-P_M,a) = U(CE(NI, a), a) + \varphi \text{Max}[U(-P^d_{S2}, a), U(CE(NI, a), a)] + (1-\varphi)\text{Max}[U(-P^u_{S2}, a), U(CE(NI, a), a)]; \tag{13}$$

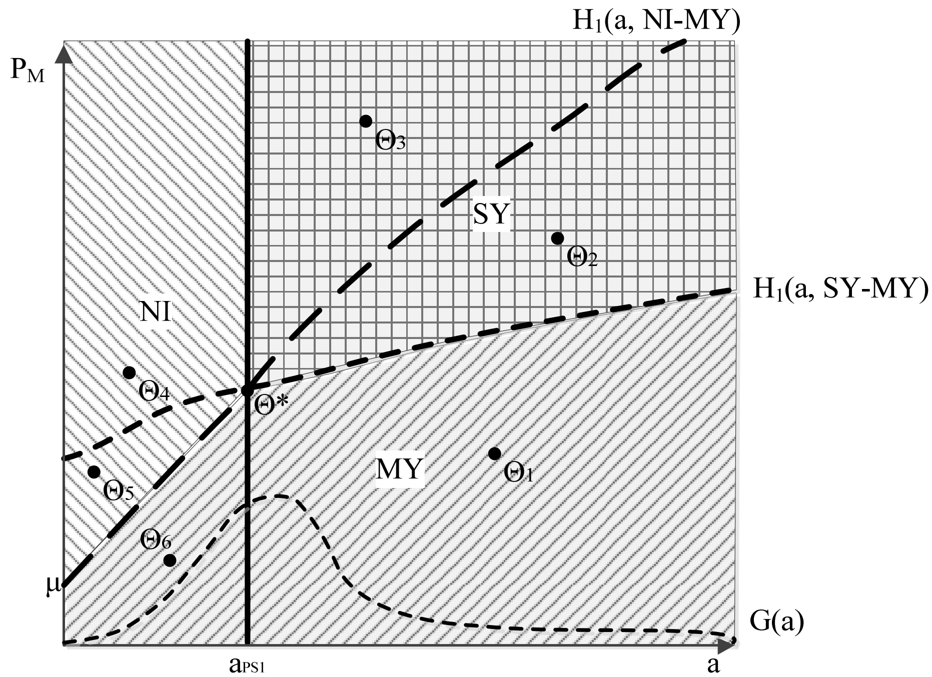
$$H_1(a, \text{SY-MY}): \quad 2U(-P_M,a) = U(CE(SY_1, a), a) + \varphi \text{Max}[U(-P^d_{S2}, a), U(CE(NI, a), a)] + (1-\varphi)\text{Max}[U(-P^u_{S2}, a), U(CE(NI, a), a)]\ . \tag{14}$$

In the Appendix, Part 2, we show that there is a unique solution P_M to (13)-(14) for every $a \in A$ so that these functions are well defined. We also show that both functions are increasing in $a \in A$. Intuitively, this reflects the fact that as homeowners become more risk averse, they are prepared to pay a higher premium P_M for an MY contract. They prefer this multi-period contract over the alternatives for the following reasons:

- Avoiding the risks of suffering a large loss from not being insured (NI)
- Having their policy cancelled or paying a higher premium in period 2 if they purchased an SY policy in period 1.

Using the above functions (12)-(14), we can summarize the solution to the two-period dynamic programming problem characterizing the optimal choice for homeowner $a \in A$ in period 1 as shown in Figure 3. This figure shows for arbitrary but fixed values of the prices P_{S1}, P^d_{S2}, P^u_{S2}, the optimal period 1 demand response for homeowners as P_M varies. Of course, P_M is also fixed at its equilibrium value (see (T2) in the Appendix), but one can think of this figure as illustrating alternative demand outcomes as the MY equilibrium price P_M varies, driven by the underlying costs in (10).

Figure 3. Period 1 Demand for NI, SY, MY



The logic of Figure 3 is as follows. Each of the functions in (12)-(14) separates the set of homeowners A into two subsets for each of the three sets of binary choices {NI, SY}, {NI, MY}, {SY, MY} into those who prefer the first of these binary choices and those who prefer the second. Thus, in evaluating the two choices NI and SY in period 1, any homeowner $a < \hat{a}(P_{S1})$ prefers NI to SY and in any $a > \hat{a}(P_{S1})$ prefers SY to NI. Similarly, in evaluating the two choices NI and MY in period 1, any homeowner $a \in A$ such that $P_M > H_1(a, \text{NI-MY})$ prefers NI to MY and prefers MY to NI if $P_M < H_1(a, \text{NI-MY})$. Finally, in evaluating the two choices SY and MY in period 1, any homeowner $a \in A$ such that $P_M > H_1(a, \text{SY-MY})$ prefers SY to MY and prefers MY to SY if $P_M < H_1(a, \text{SY-MY})$. This leads to a complete determination f consumer demand for any equilibrium price vector $P = \{P_M, P_{S1}, P^d_{S2}, P^u_{S2}\}$.

The six values of Θ_i i=1…6 in Figure 3 illustrate how one determines the optimal policy choice for homeowners in period 1.

- Consider the (a, P_M) pair Θ_1 Given its location relative to the three binary choice curves, homeowner $a(\Theta_1)$ prefers MY to SY, SY to NI and MY to NI: so that this homeowner's optimal period 1 choice at price $P_M(\Theta_1)$ is MY as shown.
- For the Θ_2 pair, homeowner $a(\Theta_2)$ prefers SY to MY, SY to NI and MY to NI: so that this homeowner's optimal period 1 choice at price $P_M(\Theta_2)$ is SY as shown.
- For the Θ_3 pair, homeowner $a(\Theta_3)$ prefers SY to MY, SY to NI and MY to NI: so that this homeowner's optimal period 1 choice at price $P_M(\Theta_2)$ is SY as shown.

- For the Θ_3 pair, homeowner $a(\Theta_3)$ prefers SY to MY, SY to NI and MY to NI: so that this homeowner's optimal period 1 choice at price $P_M(\Theta_3)$ is SY as shown.
- For the Θ_4 pair, homeowner $a(\Theta_4)$ prefers NI to MY, NI to SY and SY to MY: so that this homeowner's optimal period 1 choice at price $P_M(\Theta_4)$ is NI.
- For the Θ_5 pair, homeowner $a(\Theta_5)$ prefers NI to MY, NI to SY and MY to SY: so that this homeowner's optimal period 1 choice at price $P_M(\Theta_5)$ is NI.
- For the Θ_6 pair, homeowner $a(\Theta_6)$ prefers MY to NI, NI to SY, and MY to SY: so that this homeowner's optimal period 1 choice at price $P_M(\Theta_6)$ is MY.

All that is required to produce consistent preferences is that the intersection point Θ^* of $H_1(a, \text{NI-MY})$ and $H_1(a, \text{SY-MY})$ occur at $\hat{a}(P_{S1})$. This follows directly from the fact that $V_1(\text{MY}, a) = V_1(\text{NI}, a)$ on $H_1(a, \text{NI-MY})$ and $V_1(\text{MY}, a) = V_1(\text{SY}_1, a)$ on $H_1(a, \text{SY-MY})$, so that, at the intersection Θ^*, $V_1(\text{NI}, a) = V_1(\text{SY}_1, a)$. Since $\hat{a}(P_{S1})$ is the unique solution to (12), all three of the binary choice functions must intersect at the same point Θ^*.

The logic of Figure 3 determines period 1 demand for any homeowner $a \in A$. Aggregate demand for each policy type (NI, SY, MY) is then determined by the distribution G(a) of homeowner risk aversion in the population. A typical distribution function G(a) is shown in Figure 3. With period 1 choices determined as above, period 2 choices follow directly from D2N-i) - D2N-ii) following the logic of Figure 1. More specifically, if a homeowner bought an MY policy in period 1 then she will continue to have this policy in period 2. If the homeowner did not buy an MY policy in period 1, then she will decide to either buy an SY policy in period 2 or be uninsured (NI) depending on on whether reinsurance costs increase (u) or decrease (d) relative to period 1. Thus, the CARA/Gaussian case is completely solved.

To summarize, the price vector $P = \{P_M, P_{S1}, P^d_{S2}, P^u_{S2}\}$ is determined for the SY and MY markets from the conditions defining competitive equilibrium in these markets, as characterized by (T1)-(T2) in Appendix, Part 1. We define the critical values of risk aversion a_{NM} and a_{SM} as the unique solutions to:

$$P_M = H_1(a_{NM}, \text{NI-MY}) \; ; \; P_M = H_1(a_{SM}, \text{SY-MY}).$$

We thus have the following critical values with respect to risk aversion in determining the option in period 1 which maximizes the homeowner's expected utility when only two alternatives are available:

- a_{NM} when only NI and MY are available to the homeowner.
- a_{SM} when only SY and MY are available to the homeowner.
- $\hat{a}(P_{S1})$ when only NI and SY are available to the homeowner.

Given these critical values determining all possible binary choices, demand is then determined as follows:

i) There is a region from a = 0 to some $a_{NI}(P) = \min\{\hat{a}(P_{S1}), a_{NM}\}$, such that homeowners with $a < a_{NI}(P)$ all choose NI in period 1 (in period 2 they choose the best of the two options of SY or NI depending on the state-dependent prices that obtain).

ii) Defining $a_{SY}(P) = \text{Max}\{a_{SM}, \hat{a}(P_{S1})\}$, there is a region, possibly empty, between $\hat{a}(P_{S1})$ and $a_{SY}(P)$, such that homeowners with $a \in [\hat{a}(P_{S1}), a_{SY}(P)]$ choose SY in period 1 (in period 2 they choose the best of the two options of SY or NI depending on the state-dependent prices that obtain).

iii) Homeowners with $a \geq \max\{a_{NI}(P), a_{SM}\}$ all choose MY in period 1 (and they stick with this in period 2).

Total market demand is then determined by (2) and (8), by integrating over the distribution G(a) of homeowner risk preferences. We thus have a complete solution to the CARA/Gaussian case.

4. Illustrative Examples for the CARA/Gaussian Case

This section provides some numerical examples to illustrate the outcomes for the CARA/Gaussian case derived in section 3. The elements of the expected total cost functions [see (9)-(10)] are specified as follows:

Operating expenses $C_0(n)$ are represented as the sum of a fixed cost and a variable cost depending on the size of the BoB:

$$C_0(n) = c_0 n + K, \quad c_0 > 0, K > 0. \tag{15}$$

Marketing and selling expenses are assumed to be identical in both periods for the SY insurer and are specified by:

$$C_m(n) = c_m n^s, \quad c_m > 0, s > 2. \tag{16}$$

The marketing and selling expenses for the MY insurer are assumed to be allocated equally to both periods (although they are the same or lower in Period 2) and are represented as a fraction $\nu \leq 1$ of an SY insurer for the same BoB. Total marketing and selling costs for the MY insurer with a BoB of size n equals $2\, \nu\, C_m(n)$.

Reinsurance costs vary across periods 1 and 2. For period 1, they are specified in the form of a linearly increasing loading factor. This implies that reinsurance costs include a factor that is proportional to the expected value of the reinsurance coverage and a second factor that increases quadratically as the size $L_2(n)$ - $L_1(n)$ of the reinsurance tranche covered by the XoL treaty increases. Appendix, Part 3, characterizes the reinsurance costs facing the insurer.

For the base case the parameters for the total cost function and the reinsurance costs specified in the Appendix satisfy the stylized assumptions in Table 1 for a multi-line insurer offering catastrophe coverage:

Table 1: Assumptions Underlying Calibration for the Base Case

nμ- Re-Payout: Average Annual Loss net of Reinsurance Payout = 45% of the insurer's total cost
C_0: Operating/Underwriting Expenses (excluding marketing), including attritional losses and loss adjustment expenses = 25% of the insurer's total cost
C_m: Marketing and selling Expenses = 15 % of the insurer's total cost
C_s: Reinsurance premium = 15% of the insurer's total cost

We assume for the base case that capital costs vary by ± 20%. The distribution of risk aversion in the population of homeowners is assumed to be lognormal with mean and variance so that 20% of the homeowners choose NI at the base case equilibrium prices. The number of homeowners was set at N=1000. These assumptions, together with the calibration benchmarks in Table 1, give rise to the parameters in Table 2 for the base case corresponding to (15)-(16) and (T11)-(T17) in Appendix. Part 3. Tables 3 and 4 detail the base case outcomes. The number of homeowners who demand MY policies in period 1 is 580 with the remaining 220 purchasing an SY policy. Demand for SY policies in period 2 increases to 416 if w = d since almost all of the uninsured individuals in period 1 now decide to purchase coverage. When w = u all the individuals who were NI in period 1 will remain uninsured in period 2 and those who bought an SY policy in period 1 will decide not to renew, so there will be no demand for SY policies in period 2.

Table 2: Base Case Parameter Values

Number of Homeowners $= N = \int_A dG(a)$ $G(a) = NG_A(a)$, $G_A(a) = \text{lognormal}(\mu_A, \sigma_A)$	$N = 1000$ $\mu_A = 0.0011847$ $\sigma_A = 0.00004$
$\mu, \sigma, \rho, \varphi, \tau, q$	$\mu = 20000$, $\sigma = 6000$, $\rho = 0.5$, $\varphi = 0.5$, $\tau = 100$, $q = 0.1$
$C_0(n) = c_0 n + K$	$c_0 = 150$, $K=265000$
SY: $C_m(n) = c_m n^s$, MY: $\nu\, C_m(n)$	$c_m = 235$, $s = 2.01$, $\nu = 1$
$C_{s1}(n; r_1, \zeta) = \int_{L_1(n)}^{L_2(n)} [\lambda_1 - 1 + \xi_1 x][1 - F(x, n, \zeta)]dx$	$\lambda_1 = 1.25$, $\xi_1 = 0.0000036$
$C_{s2}(n; r_2(w), \zeta) = \int_{L_1(n)}^{L_2(n)} [\lambda_2(w) - 1 + \xi_2(w) x][1 - F(x, n, \zeta)]dx$	$\lambda_2(d) = (1-\delta)\lambda_1$, $\xi_2(d) = (1-\delta)\xi_2$ $\lambda_2(u) = (1+\delta)\lambda_1$, $\xi_2(u) = (1+\delta)\xi_2$ $\delta = 0.2$

Table 3. Outcomes for the Base Case for the SY Insurer

	1st Period	2nd Period w = d	2nd Period w = u
1) Equilibrium prices P_{S1}, P_{S2}^{d}, P_{S2}^{u}	40,705	39,530	41,857
2) Size of the BoB n	26.00	27.00	25.00
3) Homeowner Demand for SY policies	220	416	0
4) Average Annual Losses	520,000	540,000	500,000
5) Expected Reinsurance Payouts	44,464	46,143	42,786
6) Reinsurance Premium	149,783	127,351	168,780
7) Operating/Underwriting Expenses	268,900	269,050	268,750
8) Marketing and selling Expenses	164,121	177,055	151,680
Total Expected Cost per period (4-5+6+7+8)	1,058,340	1,067,314	1,046,424
Total Expected Cost Column1+φColumn2 +(1-φ)Column3	2,115,208		

Table 4. Outcomes for the Base Case for the MY Insurer

	1st Period	2nd Period w = d	2nd Period w = u
1) Equilibrium price P_M	40,705		
2) Size of the BoB n	26.00		
3) Homeowner Demand for MY policies	580		
4) Average Annual Losses	520,000		
5) Expected Reinsurance Payouts	44,464		
6) Reinsurance Premium	149,783	119,826	179,739
7) Operating/Underwriting Expenses	268,900		
8) Marketing and selling Expenses	164,121		
Total Expected Cost per period (4-5+6+7+8)	1,058,340	1,028,383	1,088,296
Total Expected Cost Column1+φColumn2 +(1-φ)Column3	2,116,679		

As the BoB for the MY insurer is constant over both periods and all states of the world, the only outcomes that change between periods1 and 2 are the reinsurance premiums, and hence the Total Expected Cost. Note that the BoB changes for the SY insurer as a function of the state of the world with a slightly larger (BoB when reinsurance premium decreases under the state of the world w = d and a slightly lower BoB when reinsurance premiums increases when w = u. SY insurers choose the optimal BoB in each state of the world while the MY insurer must choose a constant BoB over both periods. This gives rise to lower Total Expected Costs over both periods for the SY insurer relative to the MY insurer since our base case assumes no marketing cost advantages for the MY insurer ($\nu = 1$). The results for Demand for the base case are intuitive and follow the logic of Figure 3.

We now consider some comparative results on SY vs MY policies as key parameters change. It should be noted from Figure 3 that in general neither policy type can be expected to dominate the other for all homeowners. The equilibrium outcomes will depend on the distribution of homeowner risk aversion. The more risk averse the population of homeowners, and the more significant are marketing cost advantages of MY insurers, the greater the demand for MY policies. Similarly, the greater the perceived probability q of policy cancellation, and the greater the transactions costs τ of finding a new insurer, the greater the advantage of MY over SY policies. These general advantages of MY policies are counterbalanced by the greater flexibility of SY insurers to adjust their BoB in the face of volatility in capital costs and reinsurance premiums. Tables 5-8 illustrate these results.

Table 5 shows the effects of increasing q, homeowners' belief of the probability of cancellation at the end of period 1 if they are insured under an SY policy. The effect of increases in q is to decrease $CE(SY_1, a)$ given in (3). With an eye on Figure 3, this increases $H_1(a, SY\text{-}MY)$ and increases $\hat{a}(P_{S1})$. Since q has no effect on equilibrium prices (see the equilibrium conditions (T1)-(T2) in the Appendix), it follows that an increase in q will decrease SY demand and increase MY demand. The negative impact of an increase in q on SY demand in period 1 is intuitively clear. An increase in q also has a negative impact on SY demand in period 2 for the following reason: The cut-off values $\hat{a}_2^d$, $\hat{a}_2^u$ for SY demand in period 2 are unaffected by q, so that the larger MY demand which is maintained in both periods, implies a decrease in SY demand.[11] Overall, we see that increases in q will make SY policies less attractive in both periods. The results for changes in transactions costs τ would be similar.

Table 6 shows the effects of changes in ν, reflecting the magnitude of the marketing cost advantage of the MY firm. As expected, as this advantage increases (i.e as ν decreases), price P_M decreases and the BoB of the MY firm increases. The result is increased demand for MY insurance at the expense of both NI and SY demand. In the case analyzed in Table 6 where marketing costs for MY firms are half those of SY firms (i.e. $\nu = 0.5$), SY policies are not viable in equilibrium.

Table 7 shows the effects of changes in correlation ρ. As ρ increases, the amount of reinsurance cover $L_2(n)$-$L_1(n)$ increases. The result is that for any fixed BoB, reinsurance premiums increase. In equilibrium, the optimal BoB decreases. As the SY firm is able to adjust its BoB in response to the changing state of the world, it is able to respond to the increased cost of reinsurance better than the MY firm by reducing its BoB in period 2 and increasing its price. The result is a decrease in demand for MY insurance and an increase in SY demand. Analogous results hold for the reinsurance volatility parameter δ. In general, the more costly reinsurance is, and the more volatile reinsurance premiums are under different states of the world, the larger the advantage of SY firms, relative to MY firms, in being able to adjust their BoB and prices in period 2.

Finally, Table 8 shows the effects of shifts in the mean risk aversion μ_A for the population of homeowners. Neither the functions defining the geometry of Figure 3, nor the equilibrium prices are affected by μ_A (see the equilibrium conditions (T1)-(T2) in the Appendix). Increases in μ_A increases demand for MY policies relative to both NI and SY given an increase in preferences for insurance protection and stable prices over time as homeowners become more risk averse. Indeed, by changing the distribution of homeowner risk preferences, one can obtain equilibrium outcomes varying from 100% NI to 100% SY to 100% MY insurance choices (one need only choose a probability distribution function G(a) with mass centered wholly in the NI, SY or MY region). The actual outcome for any market will depend on homeowner characteristics.

[11] This assumes positive SY demand in period 2.

5. Conclusions and Future Research

This paper shows that in a competitive insurance market it is feasible and efficient for insurers to offer both single and multi-year policies given that the degree of risk aversion will differ between consumers who face a given risk. The proportion of individuals who choose each of these types of policies or prefer to remain uninsured depends on the marketing and reinsurance costs incurred by the insurer, the correlation across risks and the likelihood that an insured individual purchasing an SY policy in period 1 will have it cancelled because the cost of risk capital increases as a result of catastrophic losses in period 1. The findings also imply that if insurers only offer one an SY or MY policy, total demand for coverage will either decrease or stay the same from what it would be if both types of coverage were offered to homeowners. This is because some individuals would prefer to be uninsured in a single-policy world but would purchase coverage when more than one type of policy is offered to them.

Future research on the tradeoffs between SY and MY policies on the demand and supply side could address the following issues:

- Extending the number of periods that an MY policy is offered to determine the impact this will have on the relative prices of MY and SY policies and the demand for each type of coverage. If MY policies cannot be canceled, then the results of the above two period model (i.e. $T = 2$) can be extended to $T > 2$ in a straightforward manner. This requires only that the valuation equations (4)-(6) be extended to account for the continuing free choice of NI or SY policies for $T > 2$.

  The state-dependent prices of an SY policy would be determined as in the two-period model and would depend on the state-dependent reinsurance price in each period. A simple model for reinsurance prices for a T-period problem would be a generalization of the binary up-down model for the two-period problem with reinsurance prices increasing or decreasing with fixed probabilities. Insurers and homeowners make decisions with respect to an SY policy in a highly myopic manner because they both know they have the freedom to change their decisions in the next period.

  In extending the two-period model to $T > 2$, the decisions by the insurer and the consumer with respect to an MY policy would be affected by the expected evolution of reinsurance prices in all periods as well as the probability of a policy being canceled at the end of any period $t < T$ (i.e. q), the costs of searching for a new policy should it be cancelled (i.e. τ), and other determinants of the average insurance cost [e.g. marketing costs, correlation of risks between individuals (ρ)].

- Incorporating the possibility of individuals investing in mitigation measures to reduce their losses where there is an upfront cost associated with the measure but the benefits of mitigation accrue over a number of periods. An insurer offering an MY policy can guarantee a premium reduction for each of the periods the policy is in place. If an individual purchases an SY policy, then she may be uncertain as to whether another insurer will give her the same premium discount should her current insurer cancel her policy.

- Examining the impact of structural changes in the risk over time on the premiums that insurers would need to charge for MY policies relative to SY policies as the number of periods the policy is offered increases. These issues are relevant when examining the impact of climate change and the possibility of global warming on future losses from natural disasters such as hurricanes and flooding.

More generally, this paper should be viewed as a first step in exploring the challenges and opportunities that multi-year contracts can play in providing protection against fat-tailed risks. We have used a competitive market environment with perfect information by both parties to provide a benchmark case for addressing these issues. A number of other issues could be explored related to real-world constraints such as enforcement of contracts and uncertainties associated with the risks coupled with behavioral models of insurer and consumer behavior.

Table 5. Outcomes as Cancellation Probability q Varies

	SY Outcomes		MY Outcomes	
	q = 0.1	q = 0.2	q = 0.1	q = 0.2
Average Equilibrium Price = Average Expected Cost= $\frac{P_1+\varphi P_2^d+(1-\varphi)P_2^u}{2}$	40,699	40,699	40,705	40,705
Average Size of the BoB $=\frac{n_1+\varphi n_2^d+(1-\varphi)n_2^u}{2}$	26.00	26.00	26.00	26.00
Average Homeowner Demand $=\frac{D_1+\varphi D_2^d+(1-\varphi)D_2^u}{2}$	220	207	580	586
Average Annual Losses $=\mu\frac{n_1+\varphi n_2^d+(1-\varphi)n_2^u}{2}$	520,000	520,000	520,000	520,000
Average Expected Reinsurance Payouts (See App. (T13)-(T14)) $=\frac{Payout_1+\varphi Payout_2^d+(1-\varphi)Payout_2^u}{2}$	44,464	44,464	44,464	44,464
Average Reinsurance Premium (See App. (T15)-(T16)) $=\frac{Prem_1+\varphi Prem_2^d+(1-\varphi)Prem_2^u}{2}$	148,924	148,924	149,783	149,783
Average Operating/Underwriting Expenses $=\frac{C_{01}+\varphi C_{02}^d+(1-\varphi)C_{02}^u}{2}$	268,900	268,900	268,900	268,900
Average Marketing and selling Expenses (multiply by ν for MY) $=\frac{C_{m1}+\varphi C_{m2}^d+(1-\varphi)C_{m2}^u}{2}$	164,244	164,244	164,121	164,121

Table 6. Outcomes as MY Marketing Costs Factor ν Varies

	SY Outcomes		MY Outcomes	
	ν = 0.5	ν = 1	ν = 0.5	ν = 1
Average Equilibrium Price = Average Expected Cost= $\frac{P_1+\varphi P_2^d+(1-\varphi)P_2^u}{2}$	40,699	40,699	37,192	40,705
Average Size of the BoB $=\frac{n_1+\varphi n_2^d+(1-\varphi)n_2^u}{2}$	26.00	26.00	32.00	26.00
Average Homeowner Demand $=\frac{D_1+\varphi D_2^d+(1-\varphi)D_2^u}{2}$	0	220	1,000	580
Average Annual Losses $=\mu\frac{n_1+\varphi n_2^d+(1-\varphi)n_2^u}{2}$	520,000	520,000	640,000	520,000
Average Expected Reinsurance Payouts (See App. (T13)-(T14)) $=\frac{Payout_1+\varphi Payout_2^d+(1-\varphi)Payout_2^u}{2}$	44,464	44,464	54,535	44,464
Average Reinsurance Premium (See App. (T15)-(T16)) $=\frac{Prem_1+\varphi Prem_2^d+(1-\varphi)Prem_2^u}{2}$	148,924	148,924	210,312	149,783
Average Operating/Underwriting Expenses $=\frac{C_{01}+\varphi C_{02}^d+(1-\varphi)C_{02}^u}{2}$	268,900	268,900	268,900	268,900
Average Marketing and selling Expenses (multiply by ν for MY) $=\frac{C_{m1}+\varphi C_{m2}^d+(1-\varphi)C_{m2}^u}{2}$	164,244	164,244	249,126	164,121

Table 7. Outcomes as Correlation ρ Varies

	SY Outcomes		MY Outcomes	
	$\rho = 0.5$	$\rho = 0.8$	$\rho = 0.5$	$\rho = 0.8$
Average Equilibrium Price = Average Expected Cost= $\frac{P_1+\varphi P_2^d+(1-\varphi)P_2^u}{2}$	40,699	41,812	40,705	41,822
Average Size of the BoB $=\frac{n_1+\varphi n_2^d+(1-\varphi)n_2^u}{2}$	26.00	25.00	26.00	25.00
Average Homeowner Demand $=\frac{D_1+\varphi D_2^d+(1-\varphi)D_2^u}{2}$	220	279	580	90
Average Annual Losses $=\mu\frac{n_1+\varphi n_2^d+(1-\varphi)n_2^u}{2}$	520,000	500,000	520,000	500,000
Average Expected Reinsurance Payouts (See App. (T13)-(T14)) $=\frac{Payout_1+\varphi Payout_2^d+(1-\varphi)Payout_2^u}{2}$	44,464	53,334	44,464	53,334
Average Reinsurance Premium (See App. (T15)-(T16)) $=\frac{Prem_1+\varphi Prem_2^d+(1-\varphi)Prem_2^u}{2}$	148,924	177,377	149,783	178,445
Average Operating/Underwriting Expenses $=\frac{C_{01}+\varphi C_{02}^d+(1-\varphi)C_{02}^u}{2}$	268,900	268,750	268,900	268,750
Average Marketing and selling Expenses (multiply by ν for MY) $=\frac{C_{m1}+\varphi C_{m2}^d+(1-\varphi)C_{m2}^u}{2}$	164,244	151,803	164,121	151,680

Table 8. Outcomes as Mean Homeowner Risk Aversion μ_A Varies

	SY Outcomes		MY Outcomes	
	$\mu_A = 0.0011847$	$\mu_A = 0.0012$	$\mu_A = 0.0011847$	$\mu_A = 0.0012$
Average Equilibrium Price = Average Expected Cost= $\frac{P_1+\varphi P_2^d+(1-\varphi)P_2^u}{2}$	40,699	40,699	40,705	40,705
Average Size of the BoB $= \frac{n_1+\varphi n_2^d+(1-\varphi)n_2^u}{2}$	26	26	26	26
Average Homeowner Demand $= \frac{D_1+\varphi D_2^d+(1-\varphi)D_2^u}{2}$	220	154	580	722
Average Annual Losses $= \mu \frac{n_1+\varphi n_2^d+(1-\varphi)n_2^u}{2}$	520,000	520,000	520,000	520,000
Average Expected Reinsurance Payouts (See App. (T13)-(T14)) $= \frac{Payout_1+\varphi Payout_2^d+(1-\varphi)Payout_2^u}{2}$	44,464	44,464	44,464	44,464
Average Reinsurance Premium (See App. (T15)-(T16)) $= \frac{Prem_1+\varphi Prem_2^d+(1-\varphi)Prem_2^u}{2}$	148,924	148,924	149,783	149,783
Average Operating/Underwriting Expenses $= \frac{C_{01}+\varphi C_{02}^d+(1-\varphi)C_{02}^u}{2}$	268,900	268,900	268,900	268,900
Average Marketing and selling Expenses (multiply by ν for MY) $= \frac{C_{m1}+\varphi C_{m2}^d+(1-\varphi)C_{m2}^u}{2}$	164,244	164,244	164,121	164,121

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Appendix

1. Average Insurer Costs and Prices in a Competitive Market

Competitive equilibrium in both the SY and MY markets occurs where insurers of each type select a BoB that minimizes their average cost, with price given by the minimum of the respective average cost curve. Thus, from (9), and noting that prices in the SY market are set after the state of the world w ∈ {d, u} is known, we have :

$$P_{S1} = \mathrm{Min}_{n\geq 0}\left\{\frac{C_{SY}(n;r_1,\zeta)}{n}\right\};\ P^{w}_{S2} = \mathrm{Min}_{n\geq 0}\left\{\frac{C_{SY}(n;r_2(w),\zeta)}{n}\right\},\ w \in \{d, u\} \qquad (T1)$$

with the optimal BoBs for the SY insurer being the corresponding solutions, $\hat{n}_{S1}$, $\hat{n}^{d}_{S2}$, $\hat{n}^{u}_{S2}$, to the indicated average cost minimization problems, where r_1 and $r_2(w)$ are reinsurance costs in periods 1 and 2, the latter being state dependent.

Similarly, from (10), the equilibrium price in the MY market is determined by the minimum of the total average cost for the two periods, so that:

$$2P_M = \mathrm{Min}_{n\geq 0}\left\{\frac{C_{MY}(n;\, r_1,\, \zeta)+\varphi C_{MY}(n;\, r_2(d),\, \zeta)+(1-\varphi)C_{MY}(n;\, r_2(u),\, \zeta)}{n}\right\} \qquad (T2)$$

with the optimal BoB for the MY insurer being the corresponding solution $\hat{n}_{MY}$ to (T2). Note that, for all n, $C_{MY}(n;\, r_2(d),\, \zeta) < C_{MY}(n;\, r_1,\, \zeta) < C_{MY}(n;\, r_2(u),\, \zeta)$, so that average costs also satisfy: $AC^{d}_{M2} < AC_{M1} < AC^{u}_{M2}$ and therefore $AC^{d}_{M2} < P_M < AC^{u}_{M2}$, verifying the need for imposing a cancelation fee $\psi \geq P_M - P^{d}_{s2} - \tau$ to assure expected breakeven operations for the MY insurer in period 2.

Given our assumptions a competitive equilibrium exists for both the SY and MY markets yielding the price vector $\{P_M, P_{S1}, P^{d}_{S2}, P^{u}_{S2}\}$ and the BoB vector $\{n_M, n_{S1}, n^{d}_{S2}, n^{u}_{S2}\}$. Some of these markets may be degenerate in the sense that there is no demand for one or other of these policies at the equilibrium prices. The assumptions on average costs imply a number of intuitively appealing results for the comparative statics of equilibrium prices and BoBs for both MY and SY insurers. For example, since reinsurance costs in period 2 increase or decrease relative to period 1 depending on the state of the world, equilibrium prices in the SaY market satisfy: $P^{d}_{S2} < P_{S1} < P^{u}_{S2}$ and the corresponding optimal BoBs satisfy: $n^{d}_{S2} > n_{S1} > n^{u}_{S2}$. Comparative statics with respect to the parameters in $\zeta = (\mu, \sigma, \rho, \gamma^*, \varphi, q, \tau)$ can be derived using (9) and (10) together with (A3) and (A4) for SY and MY equilibrium prices. Due to the assumption with respect to average reinsurance costs equilibrium prices increase and equilibrium BoBs decrease as ρ increases for both SY and MY insurers.

We record here one general comparative result between SY and MY policies. Suppose there are no marketing cost advantages for MY insurers (ν = 1 in (10)). Then the equilibrium price vector $\{P_M, P_{S1}, P^{d}_{S2}, P^{u}_{S2}\}$ satisfies[12]: $P_M \geq P_{S1} + \varphi P^{d}_{S2} + (1-\varphi) P^{u}_{S2}$. In

[12] This follows from (T1) - (T2) since ν = 1 implies $C_{SY} = C_{MY}$ and from the fact that for any real valued functions f, g, h, the following inequality is evidently true:

particular, a risk neutral homeowner would always prefer SY policies to MY policies when $\nu = 1$ and when there are no transactions costs for the homeowner from policy cancellation ($\tau = 0$). Of course, even when $\tau = 0$, risk averse homeowners might still prefer MY policies to avoid the risk of price volatility in period 2. Nonetheless, the above inequality expresses clearly one advantage of SY policies, namely the ability to adjust the BoB in the face of changing reinsurance costs.

2. Proofs of the CARA/Gaussian Case

This part of the appendix provides proofs of the basic properties of the CARA/Gaussian case shown in Figure 3. We assume a fixed parameter vector $\zeta = (\mu, \sigma, \rho, \gamma^*, \varphi, q, \tau)$ and a price vector $P = \{P_M, P_{S1}, P^d_{S2}, P^u_{S2}\}$. To avoid special cases we assume that $\sigma > \tau \geq 0$ so that the uncertainty associated with the hazard is larger than the transactions costs of finding a new policy if the policy is cancelled.

<u>Claim 1</u>: The solution $\hat{a}(P_{S1})$ in (12) is unique

Proof: Consider the function arising from (12) defined as:

$$g(a) = \mu+\frac{1}{2}a\sigma^2 - \frac{1}{a}\log\left[L(a, \zeta)\, e^{aP_{S1}}\right] \text{ where } L(a, \zeta) = qe^{a\tau}+(1-q). \qquad \text{(T3)}$$

To establish the claim, it suffices to show that the following properties for the function $h(a) = ag(a) = a\mu+\frac{1}{2}a^2\sigma^2 - \log\left[L(a, \zeta)\, e^{aP_{S1}}\right]$.

P1: $h(0) = 0$; $h'(0) = \mu-(P_{S1}+q\tau)$;

P2: $h(a) > 0$ for $a > \underline{a} = \frac{2(P + \tau - \mu)}{\sigma^2}$;

P3: $h''(a) > 0$ (i.e., h is strictly convex for $a > 0$).

Assume P1-P3. Then $h(0) = 0$ and $h'(0) < 0$. Since, by P1, $h(a) > 0$ for a sufficiently large (viz., for $a > \underline{a}$), it must be that the continuous function $h(a)$ has a minimum in the interval $[0, \underline{a}]$. However, given P3, this minimum is unique and (again by P3) the value “a” at which $h(a)$ crosses zero is also unique. We need therefore only show that P1-P3 hold (under the assumptions that $\sigma > \tau \geq 0$).

P1 is obvious from direct calculation. P2 follows by noting that log( ) is monotonic increasing and $e^{a\tau} > 1$, so that

$$\ln\left[qe^{a\tau}+(1-q)\right] < \ln\left(e^{a\tau}\right) = a\tau. \qquad \text{(T4)}$$

Thus, for $a > \underline{a}$, we have

$$h(a) = ag(a) > a(\mu+\frac{1}{2}a\sigma^2-P-\tau) > \underline{a}(\mu+\frac{1}{2}a\sigma^2-P-\tau) = 0. \qquad \text{(T5)}$$

Concerning P3, it can be calculated that

$\mathrm{Min}\{f(x)+\varphi g(x)+(1-\varphi)h(x) | x\geq 0\} \geq \mathrm{Min}\{f(x) | x\geq 0\}+\varphi \mathrm{Min}\{g(x) | x\geq 0\}+(1-\varphi)\mathrm{Min}\{h(x) | x\geq 0\}$ assuming that all relevant minima exist.

$$h''(a) = \sigma^2 - (1-q)\delta^2\left[\frac{qe^{a\delta}}{qe^{a\delta}+(1-q)}\right]\left[\frac{1}{qe^{a\delta}+(1-q)}\right]. \qquad (T6)$$

Both fractions in (T6) are clearly < 1. Thus, since $\sigma > \tau \geq 0$, $h''(a) > \sigma^2 - (1-q)\tau^2 \geq \sigma^2 - \tau^2 > 0$, completing the proof of Claim 1.

For the next claim, we need the following property of CARA risk preferences: Let $\tilde{Y}$ be any random variable with positive variance. The Certainty Equivalent CE($\tilde{Y}$, a) under CARA preferences is decreasing in "a". This follows directly from Theorem 1 of Pratt (1964).

<u>Claim 2</u>: The functions $H_1(a, \text{NI-MY})$, $H_1(a, \text{SY-MY})$ are increasing in $a \in A$.

Proof: Consider $H_1(a, \text{NI-MY})$. Divide both sides of (13) by 2. Then, Claim 2 is equivalent to the assertion that the solution P_M to the following equation is increasing in $a \in A$:

$$U(-P_M, a) = 0.5U(CE(NI, a), a) + 0.5\varphi Max[U(-P^d_{S2}, a), U(CE(NI, a), a)]$$
$$+ 0.5(1-\varphi)Max[U(-P^u_{S2}, a), U(CE(NI, a), a)]. \qquad (T7)$$

With an eye on (12), there are three relevant intervals in A associated with (T7): $0 < a < \hat{a}^d_2$: $\hat{a}^d_2 \leq a < \hat{a}^u_2$: $\hat{a}^u_2 \leq a$. Observe first that the solution $P_M(a) = H_1(a, \text{NI-MY})$ to (T7) is unique and continuous (since the solution to $U(-P_M,a)=K(a)$ is $P_M = \frac{\log[-K(a)]}{a}$). Thus, it suffices to show the Claim for each of the three relevant intervals separately.

Consider the first interval, $0 < a < \hat{a}^d_2$. In this interval, NI is always superior to SY in period 2, so that (T7) can be expressed as

$$U(-P_M, a) = 0.5E\{U(-\tilde{X}_1, a)\} + 0.5\varphi E\{U(-\tilde{X}_2, a)\} + 0.5(1-\varphi)E\{U(-\tilde{X}_2, a)\} \qquad (T8)$$

where $\tilde{X}_1$, $\tilde{X}_2$ are the loss distributions for periods 1 and 2 respectively. We see that the rhs of (T8) is the expected utility of the random variable which yields $\tilde{X}_1$ with probability 0.5, and $\tilde{X}_2$ with probability 0.5. The solution P_M to (T8) is clearly just the negative of the CE of this random variable, so that Pratt's result cited above, establishes the claim for this interval.

Consider the second interval $\hat{a}^d_2 \leq a < \hat{a}^u_2$. In this interval, SY is always superior to NI in period 2, when w = d and inferior to NI when w = u, so that (T7) can be expressed as

$$U(-P_M,a) = 0.5E\{U(-\tilde{X}_1, a)\} + 0.5\varphi U(-P^d_{S2}, a) + 0.5(1-\varphi)E\{U(-\tilde{X}_2, a)\}. \qquad (T9)$$

From (T9) P_M is the negative of the CE of the random variable equal to $\tilde{X}_1$ with probability 0.5, $-P^d_{S2}$ with probability 0.5φ, and $\tilde{X}_2$ with probability 0.5(1-φ). Pratt's result implies that P_M is increasing in "a".

Finally, consider the interval $\hat{a}^u_2 \leq a$, for which SY is superior to NI for all states of the world $w \in \{u, d\}$. In this case, (T7) is equivalent to

$$U(-P_M, a) = 0.5E\{U(-\tilde{X}_1, a)\} + 0.5\varphi U(-P^d_{S2}, a) + 0.5(1-\varphi)5\varphi U(-P^u_{S2}, a). \qquad (T10)$$

Following the same logic as above establishes the claim for this interval.
Thus, $H_1(a, \text{NI-MY})$ is strictly increasing in "a" as asserted in Claim 2.

A similar argument establishes Claim 2 for $H_1(a, \text{SY-MY})$.

We finally note that $H_1(a, \text{NI-MY}) < H_1(a, \text{SY-MY})$ for "a" sufficiently small since from (T8) $H_1(a, \text{NI-MY}) \rightarrow \mu$ as $a \rightarrow 0$, whereas $H_1(a, \text{SY-MY}) > \mu$ since the price P_M that would make a homeowner indifferent between an MY and SY policy is certainly no lower than the lowest SY policy price P^d_{S2} , which is clearly greater than the mean of the loss distribution μ. We see, therefore, that $H_1(a, \text{NI-MY})$ is below $H_1(a, \text{SY-MY})$ for "a" small. On the other hand, as explained in the text, $H_1(a, \text{NI-MY})$ and $H_1(a, \text{SY-MY})$ have a unique intersection at $a = \hat{a}(P_{S1})$. As both functions are monotonic increasing, it must be that $H_1(a, \text{NI-MY}) > H_1(a, \text{SY-MY})$ for $a > \hat{a}(P_{S1})$. These facts establish the basic geometry of Figure 3.

3. Reinsurance Costs for the CARA/Gaussian Case

Reinsurance costs in period 1 for the CARA/Gaussian case are given by:

$$C_{s1}(n;r_1, \zeta) = \int_{L_1(n)}^{L_2(n)} [\lambda_1\text{-}1+\xi_1 x][1\text{-}F(x, n, \zeta)]dx \tag{T11}$$

where $r_1 = (\lambda_1, \xi_1)$ with $\lambda_1 > 1$, $\xi_1 > 0$, and where $F(x, n, \zeta)$ is the cdf of the normal distribution with mean $n\mu$ and variance $\sigma^2[n+n(n\text{-}1)\rho]$ corresponding to the loss distribution $\widetilde{X}(n) = \sum_{i=1}^{n} \widetilde{X}(a_i)$ for a BoB of size n.

Period 2 reinsurance costs are state dependent and are given by:

$$C_{s2}(n;r_2(w), \zeta) = \int_{L_1(n)}^{L_2(n)} [\lambda_2(w)\text{-}1+\xi_2(w)x][1\text{-}F(x, n, \zeta)]dx. \tag{T12}$$

The reinsurance costs (T11) and (T12) are net of expected reinsurance payments (this is the effect of subtracting 1 from the respective loading factors, λ_1 and $\lambda_2(w)$, in the first term under the integral). Thus, the total expected reinsurance payouts and premiums for this XoL treaty are given in a standard linearly increasing loading factor form by:

$$\text{Payout}_{s1}(n;r_1, \zeta) = \int_{L_1(n)}^{L_2(n)} [1\text{-}F(x, n, \zeta)]dx; \tag{T13}$$

$$\text{Payout}_{s2}(n;r_2(w), \zeta) = \int_{L_1(n)}^{L_2(n)} [1\text{-}F(x, n, \zeta)]dx; \tag{T14}$$

$$\text{Premium}_{s1}(n;r_1, \zeta) = \int_{L_1(n)}^{L_2(n)} [\lambda_1+\xi_1 x][1\text{-}F(x, n, \zeta)]dx; \tag{T15}$$

$$\text{Premium}_{s2}(n;r_2(w), \zeta) = \int_{L_1(n)}^{L_2(n)} [\lambda_2(w)+\xi_2(w)x][1\text{-}F(x, n, \zeta)]dx; \tag{T16}$$

where $r_2(w)= (\lambda_2(w), \xi_2(w))$ with $\lambda_2(w) > 1$, $\xi_2(w) > 0$, $w \in \{d, u\}$.

In line with our assumption that the state d (respectively u) represents a decrease (respectively increase) in capital cost relative to period 1, we assume:

$$\lambda_2(d) \leq \lambda_1 \leq \lambda_2(u); \ \xi_2(d) \leq \xi_1 \leq \xi_2(u). \tag{T17}$$

Note that the cdf $F(x, n, \zeta)$ is identical in both periods, since we assume that the hazard distribution is identical in both periods (of course the BoB may change for the SY insurer between periods 1 and 2). For the same reason, for any fixed BoB of size n, the attachment points $L_1(n)$, $L_2(n)$ are also unchanged between periods 1 and 2.