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# Measuring Marginal $q$

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## **Disciplines**

Finance and Financial Management

# Measuring Marginal $q$

Vito D. Gala\*

December 2014

## Abstract

Using asset prices I estimate the marginal value of capital in a dynamic stochastic economy under general assumptions about technology and preferences. The *state-space* measure of marginal  $q$  relies on the joint measurability of the value function, i.e. firm market value, and its underlying firm state variables. Unlike existing methodologies, the *state-space* marginal  $q$  requires only general restrictions on the stochastic discount factor and the firm investment technology, and it uses simple linear estimation methods. Consistently with a large class of neoclassical investment models, I construct the *state-space* marginal  $q$  using the firm capital stock and profitability shocks. I show that this new measure of real investment opportunities is substantially different from the conventional Tobin's  $Q$ , it yields more plausible and robust estimates of capital adjustment costs, it increases the correlation with investment and the sensitivity of investment to fundamentals.

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Tobin’s  $Q$ -theory of investment emphasizes a fundamental connection between financial markets and the real economy: marginal  $q$  - i.e. marginal value of capital - is a sufficient statistic to describe investment behavior (Hayashi, 1982). As any other shadow value in economics, however, the marginal value of capital is not directly observable. To overcome such empirical limitation, researchers have thus mainly used the “observable” (average) Tobin’s  $Q$  - i.e. ratio of market value of capital to its replacement cost - in empirical studies. Such common practice, however, relies on a set of very restrictive underlying assumptions under which marginal  $q$  is equal or proportional to (average) Tobin’s  $Q$ .<sup>1</sup> Despite the long-standing consensus that the restrictive underlying assumptions of perfect competition and homogeneity are misspecified, particularly at the firm level, the use of (average) Tobin’s  $Q$  remains still predominant in the empirical literature, primarily for lack of model-free and easy-to-compute alternatives.<sup>2</sup> In this paper, I provide a new measure of marginal  $q$  to fill this gap.

I propose a new methodology to measure marginal  $q$  under general assumptions regarding the nature of technology, markets, and preferences. This general procedure is both theoretically justified, and useful, empirically. Under general regularity conditions for the differentiability of the value function and the measurement of its underlying firm state variables, I show how marginal  $q$  can be easily estimated as market price elasticity of capital using a two-stage procedure, which I refer to as *state-space* approach. First, I project the observable market values - i.e. value function - onto the measurable firm-level state-space, which includes also the firm capital stock, and then I differentiate the projected market values with respect to the firm capital stock to obtain the marginal value of capital. The key insight underlying the *state-space* measure of marginal  $q$  rests on the *joint measurability* of the value function - i.e. market values - and its underlying set of firm state variables. Unlike the demand side of the economy, where one get

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<sup>1</sup>Hayashi (1982), and Abel and Eberly (1994) in a more general stochastic economy, showed that under the joint assumption of perfect competition and homogeneity of a firm production and adjustment cost technologies in investment and capital, marginal  $q$  is equal or proportional to (average) Tobin’s  $Q$ .

<sup>2</sup>Possible departures from perfect competition and homogeneity include market power or decreasing returns to scale in production (Gomes, 2001; Cooper and Ejarque, 2003; Abel and Eberly, 2011), and inhomogeneous costs of investment (Abel and Eberly, 1994, 1997; Cooper and Haltiwanger, 2006).

to observe *neither* the value function - i.e. indirect utility - *nor* its underlying state variables including above all household's wealth, in the production side of the economy one get to observe, or at least easily measure in most cases, *both* the value function - i.e. market values - *and* its underlying state variables including above all the stock of capital. Although some of the firm's state variables are not directly available, they can be readily constructed from observables in most cases.<sup>3</sup>

Consistently with a large class of neoclassical investment models *a la'* Abel and Eberly (1994), I identify empirically the firm capital stock and profitability shock as the key state variables for market values and investment. I construct the state-space measure of marginal  $q$ , which I then use to address several open questions in economics. In this paper, I focus on the empirical relationship between marginal  $q$  and (average) Tobin's  $Q$ , the estimation of capital adjustment costs, and investment- $q$  sensitivity.

I provide direct empirical evidence that marginal  $q$  is indeed statistically different from (average) Tobin's  $Q$ , with marginal  $q$  being substantially lower on average and less volatile than Tobin's  $Q$ . A statistical variance decomposition also shows that marginal  $q$  accounts only for about 25 percent of the variation in (average) Tobin's  $Q$ . Therefore, using Tobin's  $Q$  instead of marginal  $q$  may lead to incorrect inference.

Under the optimal investment policy, marginal  $q$  provides an upper bound on the total capital adjustment costs paid by the firm as a share of capital expenditure. Regardless of the specification of capital adjustment costs, I then show that using the empirical estimates of marginal  $q$  rather than observed Tobin's  $Q$  provides much tighter and more plausible bounds. If one were to use observed Tobin's  $Q$ , or equivalently, were to estimate adjustment costs under the assumptions of perfect competition and linear homogeneity, one would have estimated on average an upper bound about four times higher than estimated with marginal  $q$ .

Lastly, I investigate empirically the shape of the investment policy function and the magnitude of the investment- $q$  sensitivity using the new measure of marginal  $q$ . I found

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<sup>3</sup>Gala (2013) provides a general treatment of the state-space methodology for the estimation of shadow values of any measurable input of the value function in dynamic stochastic production economies.

that investment is substantially more responsive to changes in marginal  $q$  than (average) Tobin's  $Q$ . Specifically, the use of (average) Tobin's  $Q$  systematically under-estimates the sensitivity of investment to fundamentals by several orders of magnitude, ranging from a factor of seven for low levels of investment up to a factor of seventeen for high levels of investment. Most importantly the use of marginal  $q$  increases the correlation with investment. Furthermore, convex adjustment costs as a share of investment expenditure range from 16 percent when using marginal  $q$  up to 117 percent when using (average) Tobin's  $Q$ .

Abel and Blanchard (1986) and Gilchrist and Himmelberg (1995) offer an alternative methodology to estimate directly marginal  $q$ . They propose using VAR-based forecasts of the future expected marginal profit of capital. However, their approach requires explicit functional forms for the marginal revenue product of capital, the marginal adjustment cost, and the stochastic discount factor. Furthermore, their approach also imposes important restrictions on the formation of expectations which are outside of the structural model. Differently, the *state-space* approach only requires standard regularity conditions for the differentiability of the value function and for the measurement of the firm-level state variables. These conditions are generally satisfied in any well-behaved model of investment. In addition, the *state-space* approach imposes more discipline over the choice of firm-level state variables as implied by the structural model.

The Euler equation approach (e.g. Abel (1980), Shapiro (1986), and Whited (1992), among others) also provides an alternative methodology to estimate indirectly marginal  $q$ . Exploiting the first-order condition for investment, one can replace unobservable marginal  $q$  with a parameterized marginal investment cost in the Euler equation, and then estimate it using structural GMM. However, also this approach requires specific functional forms for the marginal profit of capital and the stochastic discount factor. In contrast, the *state-space* approach requires significantly fewer restrictions on the functional forms of technology and preferences, and can be implemented using fairly standard linear estimation methods rather than resorting to nonlinear GMM techniques.

More recently, Gala and Gomes (2013) propose a new methodology to estimate in-

vestment equations directly by approximating the optimal investment policy as function of a model’s underlying state variables. The policy function approximation is particularly useful to describe firm investment behavior and quantify, through a statistical variance decomposition, the importance of various state variables, and corresponding class of investment models. However, the policy function approximation does not use asset prices and, while requiring fewer structural restrictions than the *state-space* approach, does not provide direct identification of marginal  $q$ . The state-space approach, instead, provides direct identification and estimates of marginal  $q$  imposing only additional standard restrictions on the firm production technologies, and almost no restrictions on the functional forms of the stochastic discount factor and firm investment technologies.

Unlike these estimation methodologies, which do not rely on asset prices, the *state-space* measure of marginal  $q$  makes an efficient use of market values. In the construction of marginal  $q$ , I only use market values, which are maximally correlated with the underlying fundamentals or “instrumented” market values. As such, the *state-space* approach avoids measurement error problems induced, for instance, by potential stock market inefficiencies (Blanchard, Rhee and Summers (1993), Erickson and Whited (2000)).

Most importantly, unlike previous methods, which cannot disentangle the estimation of marginal  $q$  from the functional forms of adjustment costs and stochastic discount factor, the state-space approach allows for such separation. Therefore, the *state-space* measure of marginal  $q$  provides an estimate of the shadow value of capital robust to adjustment costs and stochastic discount factor misspecification. Furthermore, this separation allows to estimate easily structural parameters of adjustment cost technologies even in highly nonlinear models without resorting to simulation-based indirect inference methods.

While the focus in this paper is mainly on investment models only with frictions to capital adjustment, it is easy to modify the *state-space* approach to accommodate labor market frictions and financial market imperfections. Intuitively, any deviations from frictionless labor markets and/or from the Modigliani-Miller theorem imply that the value function and the optimal investment policy now depend on an augmented set of measurable state variables, which also include labor and/or (net) financial liabilities.

Those additional state variables would indeed accommodate investment models with labor market frictions as in Hall (2004), Merz and Yashiv (2007), and Bazdresch, Belo, and Lin (2014), and financial frictions as in Hennessy, Levy, and Whited (2004), Bustamante (2011), Bolton, Chen, and Wang (2011), Bolton, Schaller, and Wang (2013) and Hugonnier, Malamud, and Morellec (2014).

This paper contributes to the literature in three significant ways. First, and foremost, it provides a new general methodology to estimate the shadow price of capital in production economies. Unlike existing methodologies, the state-space measure of marginal  $q$  makes an efficient use of market values, and requires only standard regularity conditions for the differentiability of the value function. Imposing almost no restrictions on the functional forms of the stochastic discount factor and firm investment technologies makes the estimate of the shadow value of capital more robust to model misspecification. Second, I provide direct empirical evidence on the existence of a significant wedge between marginal and (average) Tobin's  $Q$ , and I obtain model-free estimates of capital adjustment costs, which are both more plausible and more robust to model misspecification than existing ones. Lastly, I show that investment is a convex function of fundamentals, and that the sensitivity of investment to fundamentals as measured by marginal  $q$  is substantially higher than estimated in conventional Tobin's  $Q$  regressions.

The rest of our paper is organized as follows. The next section describes the general model, the implied value and optimal investment functions, and how to estimate empirically the marginal value of capital as function of the key state variables. In Section 3, I describe the data and the empirical implementations including the main findings. I conclude with a discussion of the potential extensions and generalization of the approach in Section 4. The Appendices contain additional estimation details, and a detailed review of the alternative methodologies available to estimate marginal  $q$ .



# I. A General Model of Investment

This section describes the general model of investment. I use a generalized version of the model in Abel and Eberly (1994), which allows for a weakly concave production technology and asymmetric, non-convex and possibly discontinuous capital adjustment costs. This model is flexible enough to include the large majority of investment models in the literature as special cases.

## A. Production and Investment Technologies

Consider a firm that uses capital and a vector of costlessly adjustable inputs, such as labor, to produce a nonstorable output. At each point of time, the firm chooses the amounts of costlessly adjustable inputs to maximize the value of its revenue minus expenditures on these inputs. Let  $\Pi(K_{it}, A_{it})$  denote the maximized value of this instantaneous operating profit at time  $t$ , where  $K_{it}$  is firm  $i$ 's capital stock at time  $t$  and  $A_{it}$  is a random variable representing uncertainty in technology, in the prices of costlessly adjustable inputs, and/or in the demand facing the firm.

**Assumption 1. Profit.** The function  $\Pi : K \times A \rightarrow R$  satisfies: (i)  $\Pi_K(K, A) > 0$  and  $\Pi_A(K, A) > 0$ ; and (ii)  $\Pi_{KK}(K, A) \leq 0$ .

This formulation of the profit function allows the firm to be either a price-taker or a price-setter. The random variable  $A_{it}$  evolves according to a diffusion process:

$$dA_{it} = \mu_A(A_{it}, \Psi_t) dt + \sigma_A(A_{it}, \Psi_t) dW_{it}^A \quad (1)$$

where  $dW_{it}^A$  is standard Wiener process. The vector of aggregate random variables,  $\Psi_t$ , summarizes the state of the economy and evolves as

$$d\Psi_t = \mu_\Psi(\Psi_t) dt + \sigma_\Psi(\Psi_t) dW_t^\Psi \quad (2)$$

with  $dW_t^\Psi$  being a vector of standard Wiener processes independent of  $dW_{it}^A$ . The general formulation in (1) allows for common systematic variation in the shocks to technology, prices of costlessly adjustable inputs, and demand facing the firm.

Capital is acquired by undertaking gross investment at rate  $I$ , and the capital stock depreciates at a fixed proportional rate  $\delta \geq 0$ , so that the capital stock evolves according to

$$dK_{it} = (I_{it} - \delta K_{it}) dt. \quad (3)$$

When the firm undertakes gross investment, it incurs costs, which reduce operating profits. The adjustment costs are summarized by the function  $\Phi(I, K)$ .

**Assumption 2. Adjustment Costs.** The function  $\Phi : I \times K \rightarrow R_+$  satisfies: (i) twice continuously differentiable for  $I \neq 0$ ; (ii)  $\Phi(0, K) = 0$ ; (iii)  $\Phi_I(\cdot) \times I \geq 0$ ; (iv)  $\Phi_K(\cdot) \leq 0$ ; and (v)  $\Phi_{II}(\cdot) \geq 0$ .

Conditions (ii) and (iii) imply that capital adjustment costs are non negative and minimized at  $I = 0$ . Assumption 2 allows for the possibility of very general non-convex and discontinuous adjustment costs. A general function satisfying all the conditions in Assumptions 2 is:

$$\Phi(I, K) = \begin{cases} f^+ K + p^+ I + \frac{\gamma^+}{\varphi} \left(\frac{I}{K}\right)^\varphi K & \text{if } I > 0 \\ 0 & \text{if } I = 0 \\ f^- K + p^- I + \frac{\gamma^-}{\varphi} \left(\frac{I}{K}\right)^\varphi K & \text{if } I < 0 \end{cases} \quad (4)$$

where the constants  $f^+$ ,  $f^-$ ,  $p^+$ ,  $p^-$ ,  $\gamma^+$ ,  $\gamma^-$ , and  $\varphi$  are all non-negative. The constant  $f^+ \gtrless f^- \geq 0$  denotes non-convex and discontinuous fixed cost of investment incurred whenever  $I \neq 0$ . The constant  $p^+ \geq p^- \geq 0$  denotes the purchase and sale price per unit of capital, and  $\gamma^- \geq \gamma^+ \geq 0$  allows for potentially asymmetric and convex ( $\varphi \geq 2$ ) adjustment costs reflecting costly reversibility. The standard smooth quadratic adjustment costs are obtained as special case of (4) with  $\varphi = 2$ ,  $p^+ = p^- > 0$ ,  $\gamma^- = \gamma^+ > 0$ , and  $f^+ = f^- = 0$ .

## B. Optimal Investment Decisions

Each firm chooses optimal investment by maximizing the expected present value of operating profit,  $\Pi(K, A)$  less total investment cost  $\Phi(I, K)$ . The value of the firm is

thus

$$V(K_{it}, A_{it}, \Psi_t) = \max_{\{I_{t+s}\}} E_t \int_0^\infty \frac{\Lambda_{t+s}}{\Lambda_t} [\Pi(K_{it+s}, A_{it+s}) - \Phi(I_{it+s}, K_{it+s})] ds \quad (5)$$

subject to the capital accumulation equation in (3), the firm shock process in (1), the dynamics for the vector of aggregate random variables in (2), and the pricing-kernel dynamics

$$\frac{d\Lambda_t}{\Lambda_t} = -r(\Psi_t) dt - \sigma_\Lambda(\Psi_t) dW_t^\Psi \quad (6)$$

where  $r_t$  denotes the instantaneous riskless rate, and  $\sigma_\Lambda(\Psi_t)$  denotes the market prices of risks associated with the vector of aggregate systematic shocks,  $\Psi_t$ .<sup>4</sup>

The firm value function  $V(K, A, \Psi)$  satisfies the following Hamilton-Jacobi-Bellman (HJB):<sup>5</sup>

$$0 = \max_I \{ \Lambda [\Pi(K, A) - \Phi(I, K)] + \mathcal{D}[\Lambda V] \} \quad (7)$$

with  $\mathcal{D}[\cdot]$  denoting the infinitesimal generator of the Markov processes  $A$  and  $\Psi$ , and the process  $K$

$$\mathcal{D}[M(\cdot)] = \mu_A(\cdot) M_A + \frac{\sigma_A^2(\cdot)}{2} M_{AA} + \mu_\Psi(\cdot) M_\Psi + \frac{\sigma_\Psi^2(\cdot)}{2} M_{\Psi\Psi} + (I - \delta K) M_K$$

applied to the discounted firm value  $\Lambda V$ , along with the transversality (“no bubble”) condition:

$$\lim_{T \rightarrow \infty} \mathbf{E}_t [|\Lambda_{t+T} V_{it+T}|] = 0.$$

Substituting for  $\mathcal{D}[\Lambda V]$  in (7), the optimal investment policy then satisfies

$$I^*(q, K) = \arg \max_I [qI - \Phi(I, K)] \quad (8)$$

where the marginal value of capital  $q \equiv V_K$ , by the Feynman-Kac Theorem, is equal to

$$q(K_{it}, A_{it}, \Psi_t) = E_t \int_0^\infty e^{-\delta s} \frac{\Lambda_{t+s}}{\Lambda_t} [\Pi_K(K_{it+s}, A_{it+s}) - \Phi_K(I_{it+s}^*, K_{it+s})] ds. \quad (9)$$

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<sup>4</sup>The vector  $\Psi_t$  summarizes the aggregate state of the economy, which potentially includes moments of the cross-sectional firm distribution, aggregate shocks to productivity, wages, relative price of investment goods, and household preferences.

<sup>5</sup>For simplicity of exposition, we have suppressed the firm and time subscripts  $i$  and  $t$ .

As shown in Abel and Eberly (1994), the marginal  $q$  is the present value of the stream of expected marginal profit of capital which consists of two components:  $\Pi_K$  is the marginal operating profit accruing to capital, and  $-\Phi_K$  is the reduction in the adjustment cost accruing to the marginal unit of capital.

## C. Measuring Marginal $q$

Marginal  $q$  in (9) does not yield an explicit closed form solution under the general conditions under consideration. Hence, we cannot directly test the optimal investment policies in (8), unless we can measure the unobservable marginal  $q$ .

### C.1 State-Space Approach

I propose a new methodology to measure marginal  $q$  that rests on the *joint measurability* of the firm value function,  $V(\cdot)$ , and its underlying state variables,  $\Omega = \{K_{it}, A_{it}, \Psi_t\}$ . Specifically, I can measure marginal  $q$  according to its definition as partial derivative of the *observable* value function - i.e. market value of the firm - with respect to its *observable* capital stock,  $q \equiv V_K(\Omega)$ .<sup>6</sup>

First, I approximate the (scaled) market value of firm  $i$  at time  $t$ ,  $Q_{it} \equiv V_{it}/K_{it}$ , using a tensor product polynomial in the state variables as

$$v_{it} \equiv \log Q_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} \sum_{j_\Psi=0}^{n_\Psi} c_{j_k, j_a, j_\Psi} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} \times [\Psi_t]^{j_\Psi} + \epsilon_{it} \quad (10)$$

where  $k_{it} \equiv \log K_{it}$ ,  $a_{it} \equiv \log A_{it}$ , and  $\epsilon_{it}$  captures measurement error in market values.<sup>7</sup> Given the state-space variables  $k_{it}$ ,  $a_{it}$  and  $\Psi_t$ , the coefficients  $c_{j_k, j_a, j_\Psi}$  are the subject of

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<sup>6</sup>The identification of marginal  $q$  rests on the ability to identify the *exogenous* state variables,  $A$  and  $\Psi$ . Therefore, the selection of the relevant state variables for the representation of the value function should always include the *exogenous* state variables implied by the model (or any one-to-one transformation).

<sup>7</sup>Under the null of the model, the value function,  $V$ , depends only on the set of state variables  $\Omega$ . Therefore, I estimate the value function under the standard assumption that firm intrinsic values are observed only with error by the econometrician. The measurement error  $\epsilon_{it}$  (which can be serially correlated) does not affect firm optimal policies, and as such is orthogonal to the firm intrinsic value.

the estimation procedure. Then, I estimate the marginal  $q$  according to its definition of partial derivative of the value function as

$$\widehat{q}_{it} = \widehat{Q}_{it} \left( 1 + \frac{\partial \log \widehat{Q}_{it}}{\partial \log K_{it}} \right) = \widehat{Q}_{it} \left( 1 + \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} \sum_{j_\Psi=0}^{n_\Psi} \widehat{c}_{j_k, j_a, j_\Psi} \times j_k \times [k_{it}]^{j_k-1} \times [a_{it}]^{j_a} \times [\Psi_t]^{j_\Psi} \right) \quad (11)$$

Rather than imposing additional restrictive conditions concerning the functional forms of the stochastic discount factor and adjustment cost functions, the *state-space* approach only requires general regularity conditions for the existence and differentiability of the value function as well as for the measurement of the *firm-level* state variables.

*C.1.1 Measuring the State Variables* In order to estimate marginal  $q$  using the *state-space* approach, we need to measure the relevant state variables in  $\Omega$ . First, we focus on the firm-level state variables  $K$  and  $A$ . The firm capital stock,  $K$ , is directly observable. Differently, the firm shocks,  $A$ , are not directly observable, but can be easily estimated using the theoretical restrictions imposed by the model. As standard in the literature, and in accordance with Assumption 1, the operating profit evaluated at the optimal choice of the costlessly adjustable inputs of production can be represented as:

$$\Pi(K, A) = AK^\theta \quad (12)$$

where  $\theta$  denotes the share of capital in profits. I estimate the firm shocks,  $A$ , following the procedure in Cooper and Haltiwanger (2006) - see Appendix for more details. Having estimated a statistically significant  $\widehat{\theta} = 0.51$ , I then recover  $A$  from equation (12).

Given a large panel of firms, one can also account for unobserved time-invariant heterogeneity across firms by allowing the constant term  $c_{0,0,0}$  in (10) to be firm-specific.

The complete knowledge of the aggregate state variables in  $\Psi$  is not necessary for the purpose of estimating firm level marginal  $q$ . In fact, one can capture the impact of all unobserved aggregate state variables by allowing for time-specific polynomial coefficients in (10). Specifically, one can fit a separate cross-section of (scaled) firm market values

for each year as

$$v_{it} \equiv \log Q_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} b_{j_k, j_a, t} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} + \epsilon_{it} \quad (13)$$

where I have suppressed the direct dependence on the aggregate state variables,  $\Psi$ , and I have allowed the polynomial coefficients  $b_{j_k, j_a, t}$  to vary over time. For easy of exposition and comparison with the existing literature, I focus the empirical analysis on unobserved aggregate variation that affects only linearly the value function.<sup>8</sup>

## D. Discussion

In general, we cannot directly observe shadow prices. Therefore, most of the literature follows Hayashi (1982) and assumes perfectly competitive firms with homogeneous profit function,  $\Pi(\cdot)$ , and investment cost function,  $\Phi(\cdot)$ , to derive expression for  $q$  in terms of observable variables. Specifically, Abel and Eberly (1994) prove that if  $\Pi(\cdot)$  and  $\Phi(\cdot)$  are homogeneous of degree  $\rho$  in both  $I$  and  $K$ , then marginal  $q$  is proportional to (average) Tobin's  $Q$ :  $q = \rho \frac{V}{K}$ . Hayashi (1982) conditions of linear homogeneity follows as a special case with  $\rho = 1$ . The theoretical appeal of  $Q$ -theory lies on the fact that it is possible to summarize all relevant information about the state variables in a single (relative) market price. While this is often convenient and may well be a useful approximation in aggregate environments where homogeneity assumptions are more likely to hold, the identification and measurement of marginal  $q$  with (average) Tobin's  $Q$  under these restrictive homogeneity assumptions offer a fairly poor fit to the data at the firm level (e.g. Gala and Gomes (2013), among others).

I show instead how one can still use asset prices to estimate directly marginal  $q$  under general assumptions concerning technology and preferences. The *state-space* measure of marginal  $q$  rests on the *joint measurability* of the value function - i.e. market values - and the underlying set of state variables. As long as the observable value function depends on a set of measurable firm-level state variables, we can estimate marginal  $q$

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<sup>8</sup>Gala (2012) allows aggregate state variables to enter non-additively the value function and investigates empirically alternative state-space representations of asset prices.

by differentiating the projection of market values onto the firm level “state-space” with respect to the capital stock. The successful empirical identification of the marginal value of capital does not require the complete identification and measurement of *all* state variables affecting the value function, but only the identification of a subset affecting its partial derivative with respect to capital. As such, the state-space estimate of the marginal  $q$  is robust to the omission of any state variable that is independent of the firm capital stock. In addition, when using a large panel of firms, it is possible to allow for firm fixed effects to account for unobserved time-invariant firm heterogeneity. Similarly, one can allow for time-specific polynomial coefficients to fully account for the impact of any aggregate state variable.

Abel and Blanchard (1986) and Gilchrist and Himmelberg (1995) offer an alternative methodology to measure marginal  $q$ . They propose estimating marginal  $q$  using VAR-based forecasts of the future expected marginal profit of capital according to the definition in (9) - see Appendix for more details. However, their approach requires explicit simplifying assumptions, suitable for linear VAR, concerning the functional forms for the marginal revenue product of capital,  $\Pi_K$ , the marginal adjustment cost,  $\Phi_K$ , and the stochastic discount factor,  $\Lambda$ . Furthermore, their approach also imposes important restrictions on the formation of expectations which are outside of the structural model. In contrast, the state-space approach only requires standard regularity conditions for the differentiability of the value function as well as for the identification and measurement of the firm-level state variables (indeed, it can accommodate a larger class of functional forms beyond those suitable for linear VARs). These conditions are generally satisfied in any well-behaved model of investment. In addition, the state-space approach imposes more discipline over the choice of firm-level state variables as implied by the structural model. Within a neoclassical model of investment, I focus on firm size and profitability shocks.

The Euler equation approach (e.g. Abel (1980), Shapiro (1986), and Whited (1992), among others) also provides an alternative methodology to measure marginal  $q$ . Exploiting the first-order condition for investment, which requires the marginal benefit of

investment to equal its marginal cost, one can replace unobservable marginal  $q$  with a parameterized marginal investment cost in the Euler equation; and then use nonlinear GMM to estimate the underlying parameters of the model as well as marginal  $q$  - see Appendix for more details. However, also this approach requires specific functional forms for the marginal profit of capital and the stochastic discount factor. In addition, even though the estimation of the Euler equation allows to control for expectations without modelling them explicitly (under the assumption of rational expectations), in practice it still requires to make a (somewhat arbitrary) choice of variables among the set of valid instruments in the econometrician information set. In contrast, the state-space approach requires significantly fewer restrictions on the functional forms of technology and preferences, and can be implemented using standard linear estimation methods rather than nonlinear GMM techniques. Furthermore, the state-space approach identifies explicitly the set of relevant variables in the econometrician information set according to the structural model. Finally, and unlike the state-space approach, the measurement of marginal  $q$  in the Euler equation approach is subject to the occurrence of investment activity. Thus, the Euler equation approach is less suitable for the estimation of models with non-convex adjustment costs where investment activity is infrequent.

Unlike the existing estimation methodologies, which do not rely on asset prices, the *state-space* measure of marginal  $q$  makes an efficient use of market values. In the construction of marginal  $q$ , I only use the variation in market values driven by fundamental state variables, thus avoiding measurement error concerns induced, for instance, by potential stock market inefficiencies (Blanchard, Rhee and Summers (1993)).

Finally, an important point of departure from existing methodologies is that the state-space approach allows to estimate marginal  $q$  separately from adjustment cost technologies. This separation thus ensures a model-free estimate of the shadow value of capital. Importantly, given such a model-free estimate of marginal  $q$  one can then recover easily structural parameters of adjustment cost technologies, even in highly nonlinear and inhomogeneous models, and without resorting to computationally intensive simulation-based estimation methods.



## II. Empirical Implementation

I now describe the data used in the empirical analysis and additional issues concerning the state-space representation of marginal  $q$ . I then use the *state-space* measure of marginal  $q$  to estimate capital adjustment costs and investigate the shape of the investment policy function.

### A. Data

Our data comes from the combined annual research, full coverage, and industrial COMPUSTAT files. To facilitate comparison with much of the literature our sample is made of an unbalanced panel of firms for the years 1972 to 2010, that includes only manufacturing firms (SIC 2000-3999).

I keep only firm-years that have non-missing information required to construct the primary variables of interest, namely: investment,  $I$ , firm size,  $K$ , Tobin's  $Q$ , and sales revenues,  $Y$ . These variables are constructed as follows. Firm size, or the capital stock, is defined as gross property, plant and equipment. Investment is defined as capital expenditures in property, plant and equipment. Sales are measured by net sales revenues. These last two variables are scaled by the beginning-of-year capital stock. Finally, Tobin's  $Q$  is measured by the market value of capital (defined as market value of equity plus debt net of current assets) scaled by gross property, plant and equipment.<sup>9</sup>

The sample is filtered to exclude observations where total capital, Tobin's  $Q$  and sales are either zero or negative. To ensure that the measure of investment captures the purchase of property, plant and equipment, I eliminate any firm-year observation in which a firm made an acquisition. Finally, all variables are trimmed at the 1st and 99th percentiles of their distributions to reduce the influence of any outliers, which are common in accounting ratios. This procedure yields a base sample of 29,564 firm-years

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<sup>9</sup>Erickson and Whited (2006) show that using a perpetual inventory algorithm to estimate the replacement cost of capital and/or a recursive algorithm to estimate the market value of debt barely improves the measurement quality of the various proxies for  $Q$ .

observations. Table I reports summary statistics including mean, standard deviation and main percentiles for the variables of interest.

## **B. Findings**

I now describe our main findings. I first examine the variation of market values and investment rates across portfolios sorted by firm size,  $K$ , and profitability shock,  $A$ . I then proceed to estimate marginal  $q$ , and use it for the estimation of adjustment costs and investment- $q$  sensitivity.

## **C. Market Values and Investment by State-Variables Portfolios**

To gain some insights about the role of size and profitability shock in spanning the true underlying state-space for market values and investment rates, I sort all firms into 25 portfolios double-sorted on the empirical distribution of profitability shock conditional on firm size. Specifically, each firm is allocated annually first across five firm size quintiles, and then, within each size quintile, to five profitability shock quintiles. Table II reports the equally-weighted average market values and investment rates across the resulting 25 conditionally double-sorted portfolios.

Across all firm size quintiles the pattern in average market values and investment rates shows a monotonic increasing relation with the productivity shock. This relation is statistically and economically significant. In the smallest firm size decile, the equal-weighted average market value increases from \$0.02 billions for the lowest profitability shock quintile to \$0.11 billions for the highest. Similarly, the average investment rates increases from 0.12 percent for the lowest profitability shock quintile to 0.32 percent for the highest. In the highest firm size quintile, average market values and investment rates increase from \$1.83 billions and 0.10 percent for the lowest profitability shock quintile to \$23.11 billions and 0.14 percent for the highest, respectively. These relations are statistically and economically significant across these portfolios.

Thus, the double-sort portfolio analysis confirms that there is substantial variation in market values and investment rates as functions of the underlying state-variables. I now turn to estimate the marginal  $q$  using the *state-space* approach.

#### D. Empirical Value Function

I now turn to formally estimate firm market values (scaled by the capital stock) as function of the firm-level state variables in (10). The goal is to find a parsimonious polynomial representation in terms of order of approximation that provides the best overall fit for market values empirically and can be used to evaluate marginal  $q$ .

Several practical questions arise when implementing these approximations in empirical work. The first issue is whether to use natural or orthogonal terms in the estimation. In this paper, I opt for natural polynomials as they make easier the interpretation of the estimated coefficients. The second issue concerns the order of the polynomial. The choice of the polynomial order can be made according to standard model selection techniques based on a measure of model fit such as Akaike information criterion (AIC). Using stepwise regression analysis, I find that a second order complete polynomial in  $k$  and  $a$  is often sufficient, and higher order terms are generally not necessary to improve the quality of the approximation.

Table III reports the empirical estimates for various specifications of the value function polynomial regression:

$$v_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} c_{j_k, j_a} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} + \delta_i + \eta_t + \epsilon_{it} \quad (14)$$

where the left-hand side is the log market value scaled by capital,  $\ln V/K$ ,  $k$  is firm size,  $\ln K$ ,  $a$  is profitability shock,  $\ln A$ . As discussed above, all estimates use year- and firm-fixed effects to account for potential aggregate shocks and unobserved firm heterogeneity.

I find that first and second order terms are all strongly statistically significant. The complete second order polynomial in  $k$  and  $a$ , which also includes the interaction term, explains up to 64% (including fixed effects) of the total variation in log (scaled) market

values.<sup>10</sup> Based on the Akaike information criteria, I then choose the complete second order polynomial in firm size and productivity shock (column 3) as the best parsimonious state variable representation of market values empirically.<sup>11</sup>

### D.1 Variance Decomposition of Value Function

To better understand the relative importance of various variables in capturing variation in log (scaled) market values, I perform the analysis of covariance (ANCOVA) to the empirical value function specification in (14).

Table IV reports the results of this covariance decomposition for several polynomial specifications. Each column in the table corresponds to a different specification for the (scaled) value function. The numbers reported in the table, excluding the adjusted  $R^2$  reported in the last row, correspond to the fraction of the total Type III partial sum of squares for a particular specification.<sup>12</sup> That is, I normalize the partial sum of squares for each effect by the aggregate partial sum of squares across all effects in the model, so that each column sums to one. Intuitively, each value in the table corresponds to the fraction of the model sum of squares attributable to a particular effect - i.e. firm, year, and state variable polynomial.

As shown in Table IV, firm-fixed effects account for most of the explained variation in log Tobin's  $Q$  across all polynomial specifications, with values ranging from 76 to 78 percent. Such a large contribution of firm-fixed effects makes them good control variables to better identify empirically the partial derivative of the value function with respect to the capital stock, i.e. marginal  $q$ . Year-fixed effects, which capture unobserved aggregate variation, account instead for, at most, only 7 percent of the total explained variation in log Tobin's  $Q$ . Finally, the state variable polynomials in  $k$  and  $a$  provide sizable

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<sup>10</sup>I omit higher order terms because they are mostly insignificant and do not improve the overall quality of the approximation.

<sup>11</sup>Even if the quadratic and interaction terms do not increase substantially the overall fit of the value function, they are statistically significant and might be still important to explain variation in investment through marginal  $q$ .

<sup>12</sup>I use Type III sum of squares because is not sensitive to the ordering of the covariates.

contributions with values up to 18 percent of explained variation in log Tobin's  $Q$ .

### ***E. Marginal and Average Tobin's Qs***

Given the estimates of the (scaled) value function in (14), I can then compute the Fitted  $Q$  and marginal  $q$ . The Fitted  $Q$  is computed from the fitted values of the specification in (14), and it provides a measurement error-free measure of (average) Tobin's  $Q$ , which is maximally correlated with fundamental state variables. The firm marginal  $q$  is computed according to its definition as partial derivative of the value function with respect to the capital stock as in (11).

Table V reports summary statistics for the empirical distributions of estimated marginal  $q$ , Fitted  $Q$ , and observed (average) Tobin's  $Q$ . Tobin's  $Q$  is on average higher and more volatile than both Fitted  $Q$  and marginal  $q$ . Fitted  $Q$  is on average higher and more volatile than marginal  $q$ .

Figure 1 plots the empirical distributions of the logs of the ratio of (average) Tobin's  $Q$  to marginal  $q$  (left panel), and Fitted  $Q$  to marginal  $q$  (right panel). Marginal  $q$  can take on values substantially higher than observed (average) Tobin's  $Q$ . However, marginal  $q$  is always lower than Fitted  $Q$ . This is indeed consistent with the model-implied concavity of the firm value as a function of the firm capital stock (Hayashi, 1982).

In order to better understand the differences among these alternative  $Q$  measures and quantify the contribution of marginal  $q$  in explaining variation in both (average) Tobin's  $Q$  and Fitted  $Q$ , I perform the analysis of covariance.

Table VI reports the results of a variance decomposition of (average) Tobin's  $Q$  and Fitted  $Q$  using firm- and year-fixed effects and marginal  $q$  as covariates. Marginal  $q$  accounts only for 25 percent ( $0.47 \times 0.53$ ) of total variation in (average) Tobin's  $Q$ . Therefore, only this 25 percent of variation in Tobin's  $Q$  as identified by marginal  $q$  should be relevant in explaining variation in investment. Marginal  $q$  contributes for 67 percent ( $0.47 \times 0.53$ ) to the total variation in Fitted  $Q$ . As expected, marginal  $q$  has a

larger contribution in explaining variation in Fitted  $Q$  than (average) Tobin's  $Q$  because Fitted  $Q$  is measurement error-free.

### ***E.1 Equivalence Between Marginal and Average $Q$***

For each state variable representation of the value function in terms of polynomials in  $k$  and  $a$ , I can test directly the equivalence between marginal  $q$  and average  $Q$ . Testing such an equivalence requires that  $\partial \widehat{v}_{it} / \partial k_{it} = 0$ , or equivalently that all coefficients corresponding to terms involving  $k$  are jointly equal to zero:

$$c_{j_k, j_a} = 0 \quad \text{for } j_k = 1, \dots, n_k; \text{ and } \forall j_a. \quad (15)$$

The null hypothesis in (15) corresponds to a test of linear restrictions on the coefficient estimates. Such an hypothesis can be tested using a Wald statistic (“ $qQ$ -test”), which is distributed as  $\chi_r^2$  with degrees of freedom  $r$  equal to the number of restrictions.

The last row of Table III reports the p-values corresponding to the “ $qQ$ -test” for each polynomial specification. In all cases, I can *strongly* reject the null hypothesis that marginal  $q$  is equal to average  $Q$ .

## ***F. Estimating Adjustment Costs***

Under the optimal investment policy, the maximand in (8) - i.e.  $qI - \Phi(I, K)$  - is nonnegative. As such, the estimate of marginal  $q$  provides an upper bound on the total capital adjustment costs paid by the firm as a share of capital expenditure,  $\Phi(\cdot) / I$ .

Therefore, one can use the empirical distribution of marginal  $q$  to draw inference about the upper bound on the total capital adjustment costs as a share of investment. As shown in Table V, marginal  $q$  is on average much lower and less volatile than both Fitted  $Q$  and (average) Tobin's  $Q$ . As such, the total adjustment costs (including the purchase price) do not exceed on average 103 percent of the cost of investment for the average firm in the sample. In contrast, under the Hayashi (1982)'s assumptions of homogeneity and perfect competition, one would have estimated on average an upper bound of 364 percent of the

cost of investment based on the implied equivalence between observed (average) Tobin's  $Q$  and marginal  $q$ . A similar comparison holds true when considering the values for the median firm in the sample. Tobin's  $Q$  implies an upper bound of 139 percent for the total adjustment costs as share of investment expenditure, which is 2.6 times higher than the upper bound implied by marginal  $q$ .

Therefore, these estimates of marginal  $q$  provide much tighter (and plausible) bounds on the total adjustment costs as a share of investment, *regardless* of the specific assumptions concerning the investment technology.

### **F.1 Smooth Adjustment Costs**

With smooth adjustment costs, the optimality condition for investment requires the marginal cost of investment to equal marginal  $q$ . As such, the distribution of marginal  $q$  corresponds exactly to the distribution of marginal adjustment costs.

Under smooth adjustment costs, the average of the firm marginal adjustment cost of investment (including the purchase price) is only about 1.03 for each additional dollar of investment. This estimate is about 2.8 times smaller than the estimate under the Fitted  $Q$ . Thus, if I were to use the Fitted  $Q$  as a measurement error-free estimate of marginal  $q$  under the assumption of homogeneity, I would have substantially over-estimated the firm marginal adjustment costs. Even more so, if I were to use the observed (average) Tobin's  $Q$ . In such a case, I would have estimated the average marginal cost of investment at about 3.64, which is 3.6 times higher than under marginal  $q$ .

Therefore, while accounting for mismeasurement in marginal  $q$  - i.e. using Fitted  $Q$  - is important, it is accounting for misspecification - i.e. using marginal  $q$  - that substantially improves the estimates of firm capital adjustment costs.

## G. Investment- $q$ Sensitivity

In this section I investigate the shape of the investment policy function. I provide structural estimates of the investment- $q$  sensitivity and capital adjustment costs using alternative measures of Tobin's  $Q$  including the new measure of marginal  $q$ .

In line with the existing literature, I focus on a generalized adjustment cost function,  $\Phi(\cdot)$ , that is homogeneous of degree one in investment and capital. Specifically, I use the following polynomial specification for the adjustment cost function:

$$\frac{\Phi(I_{it}, K_{it}; \delta_i, \eta_t)}{K_{it}} = (\delta_i + \eta_t) \left( \frac{I_{it}}{K_{it}} \right) + \sum_{m=2}^M \frac{\gamma_m}{m} \left( \frac{I_{it}}{K_{it}} \right)^m \quad (16)$$

where the variables  $\delta_i$  and  $\eta_t$  allow for firm- and year-specific elements to the investment price. For example, the price of capital may systematically vary across firms due to tax considerations such as the value of investment tax credits and depreciation allowances. While this function is not restricted to be globally convex, I verify that the estimates imply convexity.

This functional form yields the following expression for the optimal investment policy in (8) and can be estimated as

$$q_{it} = \sum_{m=2}^M \gamma_m \left( \frac{I_{it}}{K_{it}} \right)^{m-1} + \delta_i + \eta_t + \varepsilon_{it} \quad (17)$$

where the error term  $\varepsilon$  captures measurement or estimation error in the alternative measures of  $Q$ . I use the different measures of  $Q$  as dependent variables to mitigate concerns about attenuation bias due to measurement error in market values.

### G.1 Quadratic Adjustment Costs

Table VII reports the estimates of the adjustment cost parameters  $\gamma_m$  in (17) obtained with the widely used (average) Tobin's  $Q$ , Fitted  $Q$ , and the *state-space* measure of marginal  $q$ . I concentrate first on the linear-quadratic adjustment costs specification. Panel A of Table VII shows that the positive and significant coefficient on investment



when using marginal  $q$  in specification (5) implies an adjustment cost parameter of only about 1.46. This is several orders of magnitude smaller than the value implied by the use of (average) Tobin's  $Q$  in specification (1), 12.18, and Fitted  $Q$  in specification (3), 6.65.

Figure 2 shows that the sensitivity of investment to fundamentals implied by these estimates differs substantially across alternative measures of  $Q$ .<sup>13</sup> As shown in Panel A, the sensitivity of investment to fundamentals as measured by marginal  $q$  is more than eight times higher than estimated in conventional (average) Tobin's  $Q$  regressions. Accounting for measurement error in (average) Tobin's  $Q$  - i.e. using Fitted  $Q$  - increases only slightly the investment sensitivity relative to observed Tobin's  $Q$ .

Following Abel and Eberly (2002), one could also obtain an estimate of the quadratic adjustment costs as a share of investment expenditure by multiplying  $\gamma/2$  by 0.15, the sample mean for the investment rate.<sup>14</sup> The magnitudes of quadratic adjustment costs as a share of investment expenditure implied by these coefficient estimates range on average from 11 percent of investment expenditure up to 91 percent.<sup>15</sup> These numbers imply that estimated quadratic adjustment costs are nearly 4.6 times higher under the Fitted  $Q$  specification, and 8.3 times higher under the observed (average) Tobin's  $Q$ . In light of the skewness in investment rates noted in Table 1, one could also evaluate the quadratic adjustment costs at the median investment rate, which is 0.10, rather than its sample mean. In such a case, the magnitudes of quadratic adjustment costs as a share

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<sup>13</sup>The investment sensitivity to fundamentals is defined as

$$\partial \left( \frac{I_{it}}{K_{it}} \right) / \partial q_{it} = 1 / \left( \sum_{m=2}^M \gamma_m (m-1) \left( \frac{I_{it}}{K_{it}} \right)^{m-2} \right).$$

<sup>14</sup>The total amount of quadratic adjustment costs is given by  $\frac{\gamma}{2} \left( \frac{I}{K} \right)^2 K$ . Therefore, the ratio of total adjustment costs to investment  $I$  is  $\frac{\gamma}{2} \frac{I}{K}$ .

<sup>15</sup>Given the quadratic adjustment cost specification, I can also compare these estimates with previous studies. For instance, Gilchrist and Himmelberg (1995) find estimates of  $\gamma = 20$  when using (average) Tobin's  $Q$  and  $\gamma = 5.46$  when using the VAR-based measure of marginal  $q$  (i.e. Fundamental  $Q$ ). These estimates, which correspond ( $\simeq \frac{\gamma}{2} \times \overline{I/K} = \frac{\gamma}{2} \times 0.17$ ) to 170 percent and 47 percent, respectively, are still an order of magnitude higher than the ones reported here.

of investment expenditure range from 7 percent of investment expenditure when using marginal  $q$  up to 61 percent when using observed (average) Tobin's  $Q$ . This last value is comparable with the lower estimate of 59 percent in the linear investment specification reported in Abel and Eberly (2002).

Importantly, investment is substantially more correlated with marginal  $q$  than with the measures of Tobin's  $Q$  including the measurement error-free Fitted  $Q$ .

Finally, as reported in Panel B of Table VII, I obtain similar results when I estimate the investment equation in (17) using first-differences rather than using the within-groups estimator. In such instance, the estimates of quadratic adjustment costs are even lower across alternative  $Q$  measures. Figure 2 Panel C displays the corresponding higher investment sensitivity to fundamentals.

While accounting for mismeasurement - i.e. using Fitted  $Q$  - helps to some extent, accounting for misspecification - i.e. using marginal  $q$  - substantially improves both the correlation with investment and the structural estimates of investment sensitivity to fundamentals.

This evidence still rests on the strong (though common) assumption of a linear relationship between investment and marginal  $q$ . Given that the state-space estimation of marginal  $q$  is independent of the specific assumptions concerning adjustment costs, a further investigation of nonlinearities in the relation between investment and marginal value of capital may improve our understanding of the shape of the optimal investment policy function.

## ***G.2 Polynomial Adjustment Costs***

Previous work has shown that the relationship between firms' investment and their (average) Tobin's  $Q$  is highly nonlinear (Barnett and Sakellaris, 1999; Abel and Eberly, 2002). This suggests that the responsiveness of investment to fundamentals may not be the same at all levels of the fundamentals as would be implied by the commonly used quadratic adjustment cost specification. I now revisit this empirical relationship using

the new measure of marginal  $q$ .

Table VII reports the adjustment cost parameters for the polynomial specification. The empirical results suggest to include only up to cubic terms ( $M = 3$ ). At all instances, I estimated this polynomial adjustment cost function to be convex for the range of investment rates observed in the sample.

Figure 2 demonstrates that the sensitivity of investment to fundamentals implied by these estimates exhibits substantial variation over the different levels of investment. It ranges from 0.06 to 0.14 for (average) Tobin's  $Q$ , from 0.12 to 0.22 for Fitted  $Q$ , and from 0.44 to 2.40 for marginal  $q$ . At the average level of investment, the sensitivity is 0.07, 0.13, and 0.50, correspondingly. Similarly, the corresponding sensitivity at the median level of investment is 0.07, 0.13, and 0.48, respectively. These figures show that the response of investment to marginal  $q$  is uniformly much higher than estimated in nonlinear (average) Tobin's  $Q$  regressions. For instance, using (average) Tobin's  $Q$  systematically underestimates the sensitivity of investment to fundamentals by several orders of magnitude, ranging from a factor of seven for low levels of investment up to a factor of seventeen for high levels of investment.

From Figure 2, it is evident that firm investment is more responsive to fundamentals as fundamentals improve - i.e. investment is a convex function of fundamentals. While this is true for all three alternative measures of  $Q$ , it is particularly pronounced when fundamentals are measured by marginal  $q$ .<sup>16</sup> According to the standard neoclassical model of investment, this rising sensitivity to fundamentals is due to the curvature of the convex adjustment costs. The estimated marginal cost of adjusting capital is increasing in the investment rate at a decreasing rate. Thus, as marginal  $q$  rises, the firm has to keep on increasing its investment rate at an ever-increasing rate in order to maintain equality between the marginal value and the marginal cost of investment.

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<sup>16</sup>Although based on different time periods and data samples, Barnett and Sakellaris (1999) and Abel and Eberly (2002) also find evidence that investment is a convex function of fundamentals. However, their investigation of the relationships between investment and fundamentals is performed under the conventional homogeneity assumptions and therefore limited to the use of (average) Tobin's  $Q$ .

Given the estimates in Table VII, one can also obtain structural estimates of the convex adjustment costs as share of investment expenditures.<sup>17</sup> I discuss adjustment costs at the mean investment rate of 0.15. The magnitudes of convex adjustment costs as a share of investment expenditure implied by the coefficient estimates range from 16 percent when using marginal  $q$  up to 117 percent when using (average) Tobin's  $Q$ . These numbers imply that estimated convex adjustment costs are nearly 3.7 times higher for the Fitted  $Q$  specification, and 7.0 times higher for the (average) Tobin's  $Q$  specification relative to the marginal  $q$  specification. In light of the skewness in investment rates, one could also evaluate the convex adjustment costs at the median investment rate, which is 0.10, rather than its sample mean. In such a case, the magnitudes of convex adjustment costs as a share of investment expenditure range from 11 percent when using marginal  $q$  up to 78 percent when using observed (average) Tobin's  $Q$ .

As shown in Panel B of Table VII, I obtain similar results when I estimate the investment equation in (17) using first-differences rather than using the within-groups estimator. In such instance, the estimates of convex adjustment costs are uniformly lower across alternative  $Q$  measures. Figure 2 Panel D displays the corresponding higher investment sensitivity to fundamentals.

Consistently with the empirical evidence based on quadratic adjustment costs, investment is substantially more correlated with marginal  $q$  than with the observed (average) Tobin's  $Q$  or the measurement error-free Fitted  $Q$ .

Based on this empirical evidence, one can conclude that accounting for misspecification - i.e. using marginal  $q$  - increases substantially the correlation with investment, the investment sensitivity to fundamentals, and yields plausible estimates of capital adjustment costs.

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<sup>17</sup>The total amount of convex adjustment costs is given by  $\frac{\gamma_2}{2} \left(\frac{I}{K}\right)^2 K + \frac{\gamma_3}{3} \left(\frac{I}{K}\right)^3 K$ . Therefore, the ratio of total adjustment costs to investment  $I$  is  $\frac{\gamma_2}{2} \frac{I}{K} + \frac{\gamma_3}{3} \left(\frac{I}{K}\right)^2$ .

### III. Concluding Discussion and Extensions

A new measure of firm marginal  $q$  - i.e. firm real investment opportunities - is surely a basis for a better understanding of corporate policies. Unlike much of the existing empirical literature that relies on counterfactual assumptions to replace unobservable marginal  $q$  with observable (average) Tobin's  $Q$ , I show instead how to estimate directly marginal  $q$  using asset prices under general assumptions concerning technology and preferences. A general measure of marginal  $q$  can then ultimately be applied to address several open questions in economics without making counterfactual assumptions. In this paper, I investigate the empirical relationship between marginal and average Tobin's  $Q$ , the estimation of capital adjustment costs, and investment- $q$  sensitivity. I provide direct empirical evidence that marginal  $q$  differs substantially from average Tobin's  $Q$ . I show how the use of marginal  $q$  rather than (average) Tobin's  $Q$  or Fitted  $Q$  (measurement error-free counterpart of observed Tobin's  $Q$ ) yields plausible estimates of capital adjustment costs, increases the correlation with investment and the response of investment to fundamentals.

In this paper, I focus only on two firm-level state variables shared by a large class of neoclassical investment models: firm capital and profitability shocks. While I show that these firm-level state variables capture already substantial variation in market values, the overall empirical value function approximation can be further improved by augmenting the firm-level state space with additional measurable variables including labor and (net) financial liabilities. Those additional state variables would indeed accommodate investment models with labor market frictions as in Hall (2004), Merz and Yashiv (2007), and Bazdresch, Belo, and Lin (2014), and financial frictions as in Bond and Meghir (1994), Hennessy, Levy, and Whited (2004), Bustamante (2011), Bolton, Chen, and Wang (2011), and Hugonnier, Malamud, and Morellec (2014). In such instances, the state-space methodology can be used not only to measure marginal  $q$ , but also to measure the marginal value of labor (as partial derivative of value function w.r.t. labor) and the marginal value of cash/debt (as partial derivative of value function w.r.t. cash/debt) so as to assess the overall contribution of technological versus financial market frictions

on real investment. Furthermore, given any set of firm-level state variables, the overall empirical value function approximation can also be improved by allowing for time-specific polynomial coefficients so as to account for the full impact of any aggregate state variable.

Unlike the existing methodologies, the *state-space* measure of marginal  $q$  is independent of the assumptions concerning capital adjustment costs. Therefore, it provides an ideal measure to investigate further the relationship between investment and the marginal value of capital in search of potential discontinuities to identify fixed costs of investment, fire sales, and investment irreversibility. In addition, this independence can also be exploited to validate empirically alternative class of adjustment costs including those depending on the growth rate of investment as in Eberly, Rebelo and Vincent (2012) rather than on its level. Even more so, the *joint measurability* requirement in the state-space methodology also allows for a fully nonparametric kernel density estimation of marginal  $q$ , which seems a new direction of promising research.

## IV. Appendix A: Estimating Profitability Shocks

I follow Cooper and Haltiwanger (2006) to measure profitability shocks. I assume that each firm has a Cobb-Douglas revenue function  $F(Z, K, N) = ZK^{\alpha_K}N^{\alpha_N}$ , where  $Z$  denotes the productivity shock,  $K$  is physical capital,  $N$  is the variable factor(s), and  $W$  is the price of the variable factor(s). The equations that follow are based on one variable factor for expositional purposes but extend easily to multiple variable factors. Maximization of operating profit,  $\Pi(Z, K, N) = F(Z, K, N) - WN$ , over the flexible factor,  $N$ , leads to a reduced form profit function,  $\Pi(K, A) = AK^\theta$ , where  $A = (1 - \alpha_N)[Z(\alpha_N/W)^{\alpha_N}]^{\frac{1}{(1-\alpha_N)}}$  includes shocks to productivity as well as variations in factor prices and in demand. The exponent on capital  $\theta$  is  $\alpha_K/(1 - \alpha_N)$ . Similarly, the revenue function evaluated at the optimal flexible factor takes the reduced form  $F(K, A) = \frac{A}{(1-\alpha_N)}K^\theta$ .

The coefficient on  $K$  measuring the degree of returns-to-scale in capital ( $\theta$ ) in both the revenue and profit functions is the same. Moreover, the properties of the shocks to revenue and profits are the same up to a factor of proportionality. Thus, the estimation strategy is to estimate  $\theta$  from either a quasi log-linear first-differenced profit or revenue regression on the capital stock. The latter seems preferred since there is potentially less measurement error involved.

Let  $a_{it} = \ln(A_{it})$  have the following structure

$$a_{it} = \gamma_t + \epsilon_{it}$$

where  $\gamma_t$  is a common shock, and  $\epsilon_{it}$  is a firm-specific shock, whose dynamics are given by

$$\epsilon_{it} = \eta_i + \rho_\epsilon \epsilon_{it-1} + \omega_{it}; \omega_{it} \sim MA(0)$$

where  $\eta_i$  is a firm-specific time-invariant effect capturing heterogeneity in the average firm profitability shocks. Taking logs and quasi-differencing yields

$$\pi_{it} = \rho_\epsilon \pi_{it-1} + \theta k_{it} - \rho_\epsilon \theta k_{it-1} + \gamma_t - \rho_\epsilon \gamma_{t-1} + \eta_i + \omega_{it}$$

or

$$\pi_{it} = \beta_1 \pi_{it-1} + \beta_2 k_{it} + \beta_3 k_{it-1} + \gamma_t^* + \eta_i + \omega_{it}$$

where  $k_{it} = \ln(K_{it})$  and  $\pi_{it} = \ln(\Pi_{it})$ ,  $\beta_1 = \rho_\epsilon$ ,  $\beta_2 = \theta$ ;  $\beta_3 = -\rho_\epsilon \theta$ ,  $\gamma_t^* = \gamma_t - \rho_\epsilon \gamma_{t-1}$ . Whenever, the standard assumption on the initial conditions hold ( $E[x_{i1}\omega_{it}] = 0$  for  $t = 2, \dots, T$ ), then by first differencing, we have

$$E[x_{it-s}\Delta\omega_{it}] = 0 \text{ where } x_{it} = (k_{it}, \pi_{it})$$

for  $s \geq 2$  if  $\omega_{it} \sim MA(0)$ . This allows the use of suitably lagged levels of the variables as instruments, after the equation has been first-differenced to eliminate the firm-specific effects (cf. Arellano and Bond, 1991) as:

$$\Delta\pi_{it} = \beta_1 \Delta\pi_{it-1} + \beta_2 \Delta k_{it} + \beta_3 \Delta k_{it-1} + \Delta\gamma_t^* + \Delta\omega_{it}.$$

I estimate this equation via GMM using a complete set of time dummies to capture the aggregate shocks and using twice and thrice-lagged capital and twice and thrice-lagged revenue as instruments. Table A reports both the unconstrained and constrained GMM estimates. All coefficient estimates are statistically significant and the test of over-identifying restrictions ( $J$ -test) does not reject the model. In addition, the GMM distance test ( $D$ -test) also does not reject the nonlinear constraint on the coefficient estimates (i.e.  $\beta_3 = -\beta_1\beta_2$ ). Having estimated a statistically significant  $\hat{\theta} = 0.51$  (0.13), I then recover  $a_{it}$  from  $\Pi(A_{it}, K_{it}) = A_{it}K_{it}^\theta$ .<sup>18</sup>

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<sup>18</sup>Although based on a different dataset - i.e. plant-level data - Cooper and Haltiwanger (2006) also obtain a similar estimate ( $\theta = 0.59$ ).



## V. Appendix B: Alternative Methodologies

I now review the alternative existing methodologies available to estimate marginal  $q$ : (i) VAR approach; and (ii) Euler equation approach.

### A. VAR Approach

In their seminal work, Abel and Blanchard (1986) and Gilchrist and Himmelberg (1995) propose to estimate marginal  $q$  using VAR-based approximations of marginal  $q$  computed according to its definition as the expected present value of the future marginal profit of capital:

$$q_{it} = E \left[ \int_0^{\infty} e^{-\delta s} \frac{\Lambda_{t+s}}{\Lambda_t} D_K(K_{it+s}, A_{it+s}) ds \mid \Theta_{it} \right] \quad (18)$$

where  $D_K \equiv \Pi_K - \Phi_K$  denotes the marginal profit of capital, and  $\Theta_{it}$  denotes the time  $t$  information set.

Estimation of marginal  $q$  then requires the specification of functional forms for the marginal profit of capital (i.e.  $\Pi$  and  $\Phi$ ), the stochastic discount factor,  $\Lambda$ , together with a method of evaluating the expected discounted stream of marginal profit of capital. To this end, one can assume that the marginal profit of capital and the stochastic discount factor are each generated as linear combinations of the elements of some observable vector which evolves according to a vector autoregressive process. The choice of some vector of fundamentals observable to the econometrician then identifies the relevant variables in the time  $t$  information set  $\Theta_{it}$  for the computation of expectations.

In summary, the VAR-based approximation of marginal  $q$  requires: 1) (usually linear) approximations of marginal  $q$  (as function of  $\Lambda$  and  $D_K$ ) to facilitate the computation of expectations of long products of stochastic variables; 2) specific functional forms for  $\Pi$ ,  $\Phi$  and  $\Lambda$  (usually,  $\Pi_K = \Pi/K$ ;  $\Phi_K = 0$ ; and  $\Lambda \equiv$  weighted average of equity and debt discount factors); 3) the specification of the relevant observable variables (often not implied by the model) in the time  $t$  information set  $\Theta_{it}$ ; 4) assumptions on the dynamics of the relevant variables observable to the econometrician (usually linear VAR); 5) no

direct use of asset prices (unless potentially included in the time  $t$  information set  $\Theta_{it}$ ).

## B. Euler Equation Approach

An alternative methodology available to estimate structural investment equations is based on the Euler equation (e.g. Abel (1980), Shapiro (1986), Whited (1992), and Bond and Meghir (1994)). We can rewrite the equation defining marginal  $q$  in (18) as

$$q_{it} = E \left[ \int_0^\Delta e^{-\delta s} \frac{\Lambda_{t+s}}{\Lambda_t} D_K(K_{it+s}, A_{it+s}) ds + e^{-\delta\Delta} \frac{\Lambda_{t+\Delta}}{\Lambda_t} q_{it+\Delta} \mid \Theta_{it} \right] \quad (19)$$

for any time interval  $\Delta > 0$ . This is the standard (continuous-time) Euler equation for investment: the shadow value of capital at time  $t$  equals its discounted expected value at time  $t + \Delta$  plus any marginal profit flow generated over the time interval  $\Delta$ .

Since marginal  $q$  is unobservable, one can exploit the optimality condition for investment in (8) - i.e.  $q = \Phi_I$  at any times  $t$  and  $t + \Delta$  when capital adjustment occurs - and substitute marginal  $q$  in the Euler equation (19) with the parameterized marginal cost of investment  $\Phi_I$ :<sup>19</sup>

$$\Phi_I(I_{it}^*, K_{it}) = E \left[ \int_0^\Delta e^{-\delta s} \frac{\Lambda_{t+s}}{\Lambda_t} D_K(K_{it+s}, A_{it+s}) ds + e^{-\delta\Delta} \frac{\Lambda_{t+\Delta}}{\Lambda_t} \Phi_I(I_{it+\Delta}^*, K_{it+\Delta}) \mid \Theta_{it} \right]. \quad (20)$$

Equation (20) is now only a function of “observables” and its estimation then follows the procedure introduced by Hansen and Singleton (1982). The expectation operator in (20) is replaced with an expectational error uncorrelated with any information known at time  $t$ .<sup>20</sup> Ex-post errors based on specific functional forms for the marginal profit of capital (i.e.  $\Pi$  and  $\Phi$ ) and the stochastic discount factor,  $\Lambda$ , are calculated using

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<sup>19</sup>In models without non-convex adjustment costs, investment activity occurs each period. However, with non-convex adjustment costs, investment activity becomes infrequent, and the equivalence between marginal  $q$  and the marginal cost of investment holds only when investment activity takes place. Cooper, Haltiwanger, and Willis (2010) consider the Euler equation estimation in the presence of non-convex adjustment costs.

<sup>20</sup>If expectations are formed rationally, the difference between the realized and expected value should be orthogonal to information available at time  $t$ .

observations on capital and profit flows. Then structural parameters are estimated using Hansen's (1982) GMM based on appropriate orthogonality conditions. The choice of some vector of fundamentals observable to the econometrician then identifies the relevant variables in the time  $t$  information set  $\Theta_{it}$  which can be used as instruments for the computation of orthogonality conditions.

In summary, the Euler equation approach requires: 1) specific functional forms for  $\Pi$ ,  $\Phi$  and  $\Lambda$ ; 2) the specification of the relevant observable variables (often not implied by the model) in the time  $t$  information set  $\Theta_{it}$ ; 3) no direct use of asset prices (unless potentially included in the time  $t$  information set  $\Theta_{it}$ ); 4) (usually) nonlinear structural estimation methods.

Unlike the VAR-based approach, the Euler equation approach circumvents the direct estimation of marginal  $q$  by replacing it with the parameterized marginal cost of investment whenever investment activity takes place. One can then compute the implied marginal  $q$  from the optimality conditions for investment using the estimated structural parameters.

In models without non-convex adjustment costs, one can always compute the implied marginal  $q$  as investment activity takes place each period. However, in models with non-convex adjustment costs, where investment activity is infrequent, one can only recover marginal  $q$  from the optimality conditions for investment when investment activity takes place. As such the measurement of marginal  $q$  in the Euler equation approach is subject to the occurrence of investment activity.

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Table I: Summary Statistics

This table reports summary statistics for the primary variables of interest from Compustat over the period 1972-2010. Investment rate,  $I/K$ , is defined as capital expenditures in property, plant and equipment scaled by the beginning-of-year capital stock. The capital stock,  $K$ , is defined as gross property, plant and equipment. Firm size,  $\ln(K)$ , is the natural logarithm of the beginning-of-year capital stock. The sales-to-capital ratio,  $\ln(Y/K)$ , is computed as the natural logarithm of end-of-year sales scaled by the beginning-of-year capital stock. Tobin's  $Q$  is defined as the market value of capital (market value of equity plus debt net of current assets) scaled by gross property, plant and equipment.

	Obs.	Mean	Std. Dev.	25th	50th	75th
$I/K$	29,564	0.15	0.16	0.06	0.10	0.18
$\ln(K)$	29,564	4.30	2.26	2.58	4.23	5.96
$\ln(Y/K)$	29,564	0.97	0.74	0.50	0.95	1.42
$Q$	29,564	3.64	7.09	0.55	1.39	3.54

Table II: Market Value and Investment by State-Variable Portfolios

This table reports equal-weighted averages of market values and investment rates for portfolios based on conditional sorts on firm size,  $K$ , and profitability shock,  $A$ . The sample period is 1972 to 2010.

Market Value, $V$ (\$, bn)		Profitability Shock ( $A$ )				
		Q1	2	3	4	Q5
Firm Size ( $K$ )	Q1	0.02	0.02	0.03	0.04	0.11
	2	0.07	0.08	0.12	0.19	0.34
	3	0.18	0.27	0.34	0.44	0.90
	4	0.43	0.66	1.04	1.48	2.59
	Q5	1.83	3.35	5.54	11.20	23.11

Investment Rate, $I/K$		Profitability Shock ( $A$ )				
		Q1	2	3	4	Q5
Firm Size ( $K$ )	Q1	0.124	0.132	0.183	0.225	0.318
	2	0.111	0.138	0.148	0.205	0.264
	3	0.110	0.131	0.143	0.170	0.222
	4	0.106	0.119	0.128	0.136	0.169
	Q5	0.098	0.107	0.116	0.120	0.139



Table III: Empirical Value Function

This table reports estimates from the empirical value function:

$$v_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} c_{j_k, j_a} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} + \delta_i + \eta_t + \epsilon_{it}$$

where the left-hand-side is the log market value scaled by capital,  $\ln V/K$ ,  $k$  is firm size,  $\ln K$ ,  $a$  is profitability shock,  $\ln A$ ,  $\delta_i$  is a firm fixed effect, and  $\eta_t$  is a year fixed effect. Robust standard errors are clustered by firm and reported in parenthesis.  $\bar{R}^2$  denotes adjusted  $R$ -square and  $AIC$  is the Akaike Information Criterion. “ $qQ$ -test” is a Wald test of the equivalence between marginal  $q$  and average  $Q$  as described in (15). P-values are reported. The sample period is 1972 to 2010.

	(1)	(2)	(3)
$\ln K$	-0.60 (0.02) <sup>***</sup>	-0.76 (0.04) <sup>***</sup>	-0.67 (0.04) <sup>***</sup>
$\ln A$	1.01 (0.02) <sup>***</sup>	0.50 (0.05) <sup>***</sup>	0.42 (0.05) <sup>***</sup>
$\ln K^2$		0.02 (0.00) <sup>***</sup>	0.03 (0.00) <sup>***</sup>
$\ln A^2$		0.09 (0.01) <sup>***</sup>	0.15 (0.01) <sup>***</sup>
$\ln A \times \ln K$			-0.07 (0.01) <sup>***</sup>
$\bar{R}^2$	0.63	0.64	0.64
$AIC$	77,507	76,919	76,861
$qQ$ -test	0.00	0.00	0.00

Table IV: Variance Decomposition of Value Function

This table presents a variance decomposition for several polynomial specifications of the log (scaled) value function in (14). I compute the Type III partial sum of squares for each effect in the model and then normalize each estimate by the sum across the effects, forcing each column to sum to one. For example, in specification (2), 6% of the explained sum of squares captured by the included covariates can be attributed to year-fixed effects. Firm FE are firm fixed effects. Year FE are calendar year fixed effects. Poly(1) denotes the polynomial of degree one in  $k$  and  $a$ , Poly(2) denotes the polynomial of degree two in  $k$  and  $a$ , Poly(2C) denotes the complete polynomial of degree two in  $k$  and  $a$  including the interaction term.  $\overline{R}^2$  denotes adjusted  $R$ -square. The sample period is 1972 to 2010.

	(1)	(2)	(3)
Firm FE	0.78	0.77	0.76
Year FE	0.07	0.06	0.06
Poly(1)	0.15		
Poly(2)		0.17	
Poly(2C)			0.18
$\overline{R}^2$	0.63	0.64	0.64

Table V: Distribution of Marginal  $q$

This table reports summary statistics for (average) Tobin's  $Q$ , and the estimated marginal  $q$  and Fitted  $Q$ . Fitted  $Q$  is computed as the fitted value of the value function approximation in (14). Marginal  $q$  is computed according to its definition as partial derivative of the value function approximation with respect to the capital stock. The sample period is 1972 to 2010.

	Obs.	Mean	Std. Dev.	25th	50th	75th
Tobin's $Q$	29,564	3.64	7.09	0.55	1.39	3.54
Fitted $Q$	29,564	2.85	5.14	0.62	1.28	2.93
Marginal $q$	29,564	1.03	1.64	0.26	0.53	1.13

Table VI: Variance Decomposition of Tobin's  $Q$ s

This table presents a variance decomposition of (average) Tobin's  $Q$  and Fitted  $Q$  using firm- and year-fixed effects and marginal  $q$  as covariates. I compute the Type III partial sum of squares for each effect in the model and then normalize each estimate by the sum across the effects, forcing each column to sum to one. For example, in the variance decomposition of Tobin's  $Q$ , 47% of the explained sum of squares captured by the included covariates can be attributed to marginal  $q$ . Firm FE are firm fixed effects. Year FE are calendar year fixed effects.  $\bar{R}^2$  denotes adjusted  $R$ -square. The sample period is 1972 to 2010.

	Tobin's $Q$	Fitted $Q$
Firm FE	0.49	0.25
Year FE	0.05	0.02
Marginal $q$	0.47	0.73
$\bar{R}^2$	0.53	0.92

Table VII: Investment- $q$  Sensitivity

This table reports estimates from the following regression:

$$Y_{it} = \sum_{m=2}^M \gamma_m \left( \frac{I_{it}}{K_{it}} \right)^{m-1} + \delta_i + \eta_t + \varepsilon_{it}$$

where the left-hand side variable  $Y_{it}$  is either marginal  $q$ , Fitted  $Q$  or (average) Tobin's  $Q$ , and the right-hand side variables include the investment rate ( $I/K$ ),  $\delta_i$  is a firm fixed effect,  $\eta_t$  is a year fixed effect. Specifications (1)-(2) report estimates using (average) Tobin's  $Q$  as dependent variable. Specifications (3)-(4) report estimates using Fitted  $Q$  as dependent variable. Specifications (5)-(6) report estimates using marginal  $q$  as dependent variable. Panel A reports estimates based on fixed-effect estimators (Within-Groups), while Panel B reports estimates based on first-difference estimators. Robust standard errors are clustered by firm and reported in parenthesis.  $\bar{R}^2$  denotes adjusted  $R$ -square. The sample period is 1972 to 2010.

	Tobin's $Q$		Fitted $Q$		Marginal $q$	
A: Within-Groups						
	(1)	(2)	(3)	(4)	(5)	(6)
$\gamma_2$	12.18 (0.73) <sup>***</sup>	15.97 (1.58) <sup>***</sup>	6.65 (0.54) <sup>***</sup>	8.25 (0.83) <sup>***</sup>	1.46 (0.13) <sup>***</sup>	2.29 (0.23) <sup>***</sup>
$\gamma_3$		-4.27 (1.29) <sup>***</sup>		-1.81 (0.71) <sup>***</sup>		-0.94 (0.17) <sup>***</sup>
$\bar{R}^2$	0.12	0.12	0.13	0.14	0.16	0.17
B: First-Difference						
$\gamma_2$	5.56 (0.46) <sup>***</sup>	6.42 (1.25) <sup>***</sup>	4.17 (0.39) <sup>***</sup>	4.38 (0.57) <sup>***</sup>	0.94 (0.10) <sup>***</sup>	1.26 (0.13) <sup>***</sup>
$\gamma_3$		-0.90 (1.12)		-0.22 (0.47)		-0.33 (0.08) <sup>***</sup>
$\bar{R}^2$	0.05	0.05	0.14	0.15	0.21	0.21

Table A: Estimating Productivity Shocks

This table reports GMM estimates of the following specification:

$$\Delta\pi_{it} = \beta_1\Delta\pi_{it-1} + \beta_2\Delta k_{it} + \beta_3\Delta k_{it-1} + \Delta\gamma_t + \Delta\omega_{it}$$

using a complete set of time dummies, twice and thrice-lagged capital and twice and thrice-lagged profits as instruments. Standard errors are reported in parenthesis. *CRS*-test is a test of constant returns to scale hypothesis  $\beta_2=1$ . *J*-test denotes the test of overidentifying restrictions, and *D*-test denotes the GMM distance test concerning the constraints on the coefficient estimates as described in Appendix. P-values are reported. The sample period is 1972 to 2010.

	Unconstrained GMM	Constrained GMM
$\beta_1$	0.54 (0.13) <sup>***</sup>	0.63 (0.10) <sup>***</sup>
$\beta_2$	0.47 (0.12) <sup>***</sup>	0.51 (0.13) <sup>***</sup>
$\beta_3$	-0.29 (0.09) <sup>***</sup>	-0.32 (0.09) <sup>***</sup>
<i>J</i> -test	0.18	0.20
<i>CRS</i> -test	0.00	0.00
<i>D</i> -test		0.25

Figure 1: Empirical Distribution of Q Wedges

This figure plots the empirical distributions of the log ratio of (average) Tobin's  $Q$  to marginal  $q$  (left panel) and Fitted  $Q$  to marginal  $q$  (right panel). The sample period is 1972 to 2010.

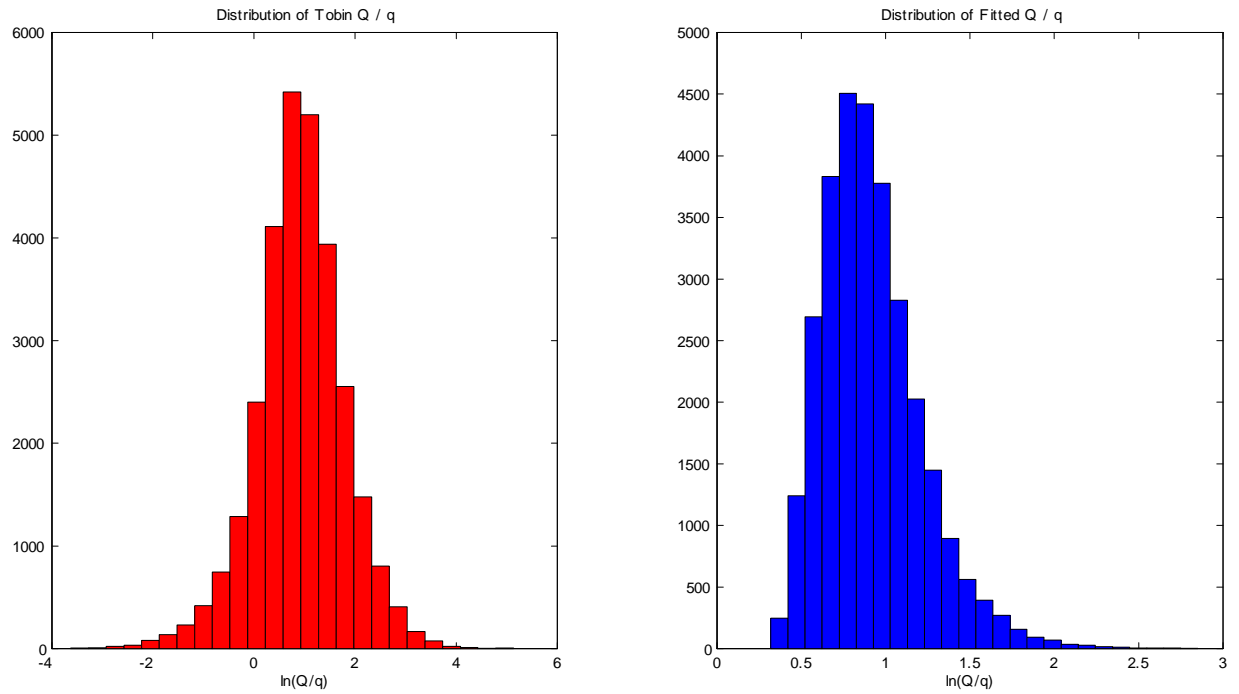


Figure 2: Investment Sensitivity to Fundamentals

This figure plots the investment sensitivity to alternative measures of Tobin's  $Q$  - i.e.  $\partial(I/K)/\partial Q$  - as a function of investment ( $I/K$ ). Each panel displays the investment sensitivity to marginal  $q$  (solid line), Fitted  $Q$  (dashed line), and (average) Tobin's  $Q$  (dotted line). Panel A and C display investment sensitivities under quadratic adjustment costs ( $M = 2$ ), while Panel B and D display investment sensitivities under polynomial adjustment costs ( $M = 3$ ). Panel A and B display investment sensitivities based on fixed-effect estimators (Within-Groups), while Panel C and D display investment sensitivities based on first-difference estimators. The sample period is 1972 to 2010.

