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Scott Condie

Jayant Ganguli

Philipp Karl Illeditsch
*University of Pennsylvania*

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**Keywords**

ambiguity aversion, Knightian uncertainty, informational efficiency, information inertia, inattention to news, public information, momentum, predictability

**Disciplines**

Finance and Financial Management

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Scott Condie[†]    Jayant Ganguli[‡]    Philipp Karl Illeditsch[§]

January 28, 2013

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**Keywords:**   Ambiguity Aversion, Knightian Uncertainty, Informational Effi-
ciency, Information Inertia, Inattention to News, Public Information, Momen-
tum, Predictability.

**JEL Classification:** D80, D81, G10, G11, G12.

[†]Department of Economics, Brigham Young University, Provo, UT 84602. Telephone: +1 801
422 5306, Fax: +1 801 422 0194, Email: `ssc@byu.edu`

[‡]Department of Economics, University of Essex, Wivenhoe Park, Colchester CO4 3SQ, UK.
Telephone: +44 (0) 1206 87 2723, Fax: +44 (0) 1206 872724, Email: `jayantvivek@gmail.com`

[§]Finance Department, The Wharton School, University of Pennsylvania, 3620 Locust Walk, 2426
SH-DH, Philadelphia, PA 19104-6367, Telephone: +1 215 898 3477, Fax: +1 215 898 6200, Email:
`pille@wharton.upenn.edu`

There is a vast amount of empirical research which studies the predictability of cash flows and discount rates for many asset classes around the world.[1] The economic and statistical significance of the predictability results vary from study to study and the strength of these results as well as the theoretical underpinnings and interpretations are widely debated. In this paper, we study how information about an asset affects optimal portfolios and equilibrium asset prices when investors are not sure about the model that predicts future asset values and thus treat the information as ambiguous. We show that this ambiguity (Knightian uncertainty) leads to optimal portfolios that do not react to news and prices do not reflect all available information about an asset in equilibrium. We refer to this phenomenon as information inertia.

Suppose investors receive information about the future payoff of an asset. Investors are averse to ambiguity and thus prefer situations in which they know the model that predicts future asset payoffs over situations where they do not. Specifically, they consider a set of models when processing information about an asset and evaluate the outcome of investment decisions under the model that yields the lowest expected utility. This "max-min" formulation of preferences is axiomatized in Gilboa and Schmeidler (1989) and is a commonly used representation of decision-making under ambiguity in asset markets, as discussed in Epstein and Schneider (2010).[2]

Ambiguity about the predictability of future asset returns has interesting implications for optimal portfolios. Specifically, we show that investors do not always act on information that is worse than expected and hence they do not trade as much as traditional models would predict in response to news. This is consistent with the household portfolio choice literature which documents that investors rarely rebalance their portfolios in retirement accounts.[3] Our explanation does not rely on information processing costs or other market frictions and it is different from the explanation in

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[1]For a review of this literature see Cochrane (2005) or Koijen and Nieuwerburgh (2011) and the references therein.

[2]These multiple prior preferences imply behavior that is consistent with experimental evidence (Ellsberg (1961)) and more recent portfolio choice experiments (Ahn, Choi, Gale, and Kariv (2011) and Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010)).

[3]See Bodie, Detemple, and Rindisbacher (2009), Campbell (2006), and the references therein for a review of this literature.

Epstein and Schneider (2010) and Illeditsch (2011) who show that aversion to ambiguity leads to risky portfolios that do not react to changes in the stock price—a phenomenon which they refer to as portfolio inertia.

We also study the effects of ambiguous public information about the future cash flows of an asset on its equilibrium price. We show that stock prices fail to incorporate all publicly available information in equilibrium. This informational inefficiency has an interesting asymmetry. While good news is always reflected in the stock price, some bad news is not. Moreover, this mispricing of news depends on the unconditional risk premium of the stock. Risky stocks are more likely to underreact to signals that convey bad news whereas stocks that are not very risky tend to overreact to this news. However, the most striking result is that stocks with intermediate risk show almost no reaction to signals that convey bad news even though there are no information processing costs or other market frictions.

This paper may also shed some light on the documented profitability of momentum strategies in the United States and other developed countries.[4] Specifically, assets that have performed well in the past tend do continue to perform well. Similarly stocks with high earnings momentum tend to outperform stocks with low earnings momentum. Most of the papers in the literature rely on behavioral explanations for this phenomenon (e.g. Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999)). We provide an explanation for the profitability of momentum strategies that is based on investors who are averse to ambiguity. Moreover, the economic significance of these strategies varies with the unconditional risk premium of the stock which may help distinguish our explanation from others in the literature.

We also study the effects of investor heterogeneity on optimal portfolios and asset prices in equilibrium. Specifically, we assume that all investors receive the same public signal about future cash flows but they may differ with respect to their aversion to risk

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[4]For a review of the literature on momentum strategies see Jegadeesh and Titman (2011), Moskowitz, Ooi, and Pedersen (2012), and the reference therein.

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and ambiguity. We show that when investors have common ambiguity, then the stock price is informational inefficient and investors hold risky portfolios in equilibrium that do not react to news.

There is a growing literature in macroeconomics that imposes an exogenous constraint or cost on the ability of investors to process information in order to explain why macroeconomic variables exhibit inertia.[5] These ideas have also been used in finance to explain information inertia of portfolios (Abel, Eberly, and Panageas (2007)), excess correlation (Peng and Xiong (2006)), financial contagion (Mondria (2010) and (Mondria and Quintana-Domeque 2012)), and portfolio under-diversification (Nieuwerburgh and Veldkamp 2010), among others.[6] We derive inertia from a rational choice model with multiple prior utility. Moreover, information inertia affects investors' utility and thus leads to a welfare loss that depends on risk aversion and the magnitude of the news surprise.

This paper complements recent work on optimal portfolios and equilibrium asset prices when investors process public signals. Epstein and Schneider (2008) show that investors react more to bad signals than to good signals when there is ambiguity about the precision of these signals. Illeditsch (2011) shows that this ambiguity leads to risky portfolios that are insensitive to changes in the stock price. However, these portfolios are sensitive to changes in the signal and thus prices always reflect all available information in equilibrium.

This paper is also related to a large literature that studies the informational efficiency of prices when there is asymmetric information. For instance, prices do not fully reveal private information in equilibrium, (i) if it is costly to acquire information (Grossman (1976) and Grossman and Stiglitz (1976)), (ii) if there are noise traders (Grossman and Stiglitz (1980)), (iii) if informed investors anticipate how their trades will impact prices (Kyle (1985) and Back, Cao, and Willard (2000)), (iv) if there is ambiguity (Caskey (2009), Condie and Ganguli (2011), and Condie and Ganguli

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[5] See Sims (2003), Sims (2010), and the references therein.
[6] See Veldkamp (2011) and the reference therein for an overview of this literature.

(2012)).[7] What is striking in this paper is that a costless informative public signal is not always incorporated in the price when an investor is averse to ambiguity.

This paper is also related to a large literature on optimal portfolio choice when there is Bayesian model uncertainty about the predictability of future returns (e.g. Keim and Stambaugh (1986), Barberis (2000), and Xia (2001) among others).[8] In all these papers investors hedge against model uncertainty but their portfolios react smoothly to new information. Balduzzi and Lynch (1999), Balduzzi and Lynch (2000), Lynch and Tan (2010), and Lynch and Tan (2011) study the effects of transaction cost on optimal portfolios when there is return predictability. While in these papers transaction costs lead to state dependent portfolio adjustment we derive state dependent adjustment of portfolios from a rational choice model.

This paper contributes to the literature on optimal portfolio choice with ambiguity. We know from Dow and Werlang (1992), Cao, Wang, and Zhang (2005), Epstein and Schneider (2007), Easley and O'Hara (2009), and Campanale (2011) that ambiguity leads to portfolio inertia of the risk-free portfolio. Epstein and Wang (1994), Epstein and Schneider (2010), and Illeditsch (2011) show that portfolio inertia can also arise for risky portfolios. Garlappi, Uppal, and Wang (2007) characterize optimal portfolios with multiple ambiguous assets. Uppal and Wang (2003), Benigno and Nistico (2012), and Boyle, Garlappi, Uppal, and Wang (2012) show that ambiguity leads to under-diversified portfolios. We show that if there is ambiguity about the predictability of future asset returns, then investors use the unconditional asset distribution when contemplating a long (short) position with moderate risk instead of relying on an ambiguous signal that conveys bad (good) news.

Our paper is also related to recent literature on portfolio choice and asset pricing when there is ambiguity about the predictability of future asset returns/cash flows. Hansen and Sargent (2010a) study the price of risk when investors who seek robust

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[7]Mele and Sangiorgi (2011), Ozsoylev and Werner (2011), and Tallon (1998) study the effects of ambiguity aversion on asset prices in the presence of private information and noise traders.

[8]For a comprehensive survey of static and dynamic portfolio choice models when returns are predictable see Wachter (2012).

decision rules find it difficult to differentiate between i.i.d. consumption growth and one with a persistent component (long run risk of Bansal and Yaron (2004)).[9] Chen, Ju, and Miao (2011) solve a dynamic consumption and portfolio choice problem when there is ambiguity about whether stock returns are IID or predictable. Ju and Miao (2012) and Collard, Mukerji, Sheppard, and Tallon (2011) explain many asset pricing puzzles by introducing ambiguity into a dynamic representative agent model in which consumption and dividends follow a hidden state regime-switching process and a hidden state model with a persistent latent state variable, respectively. The first paper considers the robust control approach and the other three papers consider the recursive smooth ambiguity model to describe preferences.[10] Our focus in this paper is on non-smooth preferences which are a good description of ambiguity averse behavior as shown by Ahn, Choi, Gale, and Kariv (2011) and Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010).[11]

The rest of the paper is organized as follows. In Section I, we introduce the model. In Section II, we solve for optimal portfolios and discuss the information inertia results. In Section, III we solve for the equilibrium stock price and discuss the mispricing results, in Section IV we show that our results are robust to aggregation, and in Section V we discuss momentum strategies. We conclude in Section VI.

# I  Ambiguous Information

Suppose there are two dates 0 and 1. Investors can invest in a risk-free asset and a risky asset. Let $p$ denote the price of the risky asset, $\tilde{d}$ the future value or dividend of the risky asset, and $\theta$ the number of shares invested in the risky asset. There is no consumption at date zero. The risk-free asset is used as numeraire, so the risk-free

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[9]For a survey of learning models when investors seek robust decision rules see Hansen and Sargent (2007).

[10]Strzalecki (2011) and Maccheroni, Marinacci, and Rustichini (2006) provide axiomatic foundations for the robust control model and Klibanoff, Marinacci, and Mukerji (2005), Nau (2006), Klibanoff, Marinacci, and Mukerji (2009), and Hayashi and Miao (2011) provide axiomatic foundations for the smooth ambiguity model and its dynamic extension.

[11]For a discussion of different preferences specifications that describe aversion to ambiguity see Backus, Routledge, and Zin (2004), Epstein and Schneider (2010), and Hansen and Sargent (2010b).

rate is zero. Hence, future wealth $\tilde{w}$ is given by

$$\tilde{w} = w_0 + \left(\tilde{d} - p\right)\theta, \tag{1}$$

in which $w_0$ denotes initial wealth.

Suppose investors receive a signal $\tilde{s}$ about the future value $\tilde{d}$ of the asset. The joint distribution of $\tilde{d}$ and $\tilde{s}$ is normal:

$$\begin{pmatrix} \tilde{d} \\ \tilde{s} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} \bar{d} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_d^2 & \beta \\ \beta & 1 \end{pmatrix}\right). \tag{2}$$

Investors are ambiguous about the covariance between $\tilde{d}$ and $\tilde{s}$ and consider a family of joint distributions described by $\beta \in [\beta_a, \beta_b]$ with $\beta_a > 0$ and $\beta_b < \sigma_d$.[12]

We follow Gilboa and Schmeidler (1993) and assume that investors update their beliefs model by model using Bayes rule.[13] Hence, standard normal-normal updating for each $\beta \in [\beta_a, \beta_b]$ leads to

$$\tilde{d} \mid \tilde{s} = s \sim \mathrm{N}_\rho \left(\bar{d} + \beta s, \sigma_d^2 - \beta^2\right). \tag{3}$$

We focus in this paper on ambiguity averse investors in the sense of Gilboa and Schmeidler (1989). Hence, the utility of an investor who holds $\theta$ shares of the risky asset is

$$\min_{\beta \in [\beta_a, \beta_b]} \mathrm{E}_\beta \left[u\left(w_0 + \left(\tilde{d} - p\right)\theta\right) \mid \tilde{s} = s\right], \tag{4}$$

where $u(\cdot)$ denotes the Bernoulli utility function of the investor. Investors are more averse to ambiguity if the interval $[\beta_a, \beta_b]$ is large and therefore the degree of aversion to ambiguous information can be measured by $\beta_b - \beta_a$.

Suppose investors have CARA utility over future wealth $\tilde{w}$ (i.e., $u(\tilde{w}) = -e^{-\gamma\tilde{w}}$ with $\gamma > 0$) and let $\mathrm{CE}(\theta)$ denote the certainty equivalent of an ambiguity averse

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[12]There is no ambiguity about the marginal distribution of the signal and hence there is no loss in generality by normalizing the mean and the variance of the signal to zero and one, respectively.

[13]See Epstein and Schneider (2003) for updating preferences in dynamic models.

investor. Then the investor's utility given in equation (4) is equal to $u\left(\text{CE}(\theta)\right)$ with

$$\text{CE}(\theta) = \min_{\beta \in [\beta_a, \beta_b]} \left( \text{E}_\beta \left[ \tilde{w} \mid \tilde{s} = s \right] - \frac{1}{2}\gamma \text{Var}_\beta \left[ \tilde{w} \mid \tilde{s} = s \right] \right). \tag{5}$$

The assumption of CARA utility and normal beliefs lead to mean-variance preferences over future wealth in which the posterior mean is a linear function of $\beta$ and the residual variance is a quadratic function of $\beta$. Ambiguity averse investors are worried about the effects of $\beta$ on the mean and variance of future wealth and thus consider for each portfolio $\theta$ and signal realization $s$ the minimum expected value of future utility. Hence, their worst case scenario belief will depend on the portfolio $\theta$ and signal $s$ as the next proposition shows.

**Proposition 1** (Preferences). *Let $\hat{\theta}_a \equiv -s/(\gamma\beta_a)$ and $\hat{\theta}_b \equiv -s/(\gamma\beta_b)$. The certainty equivalent of an investor with risk aversion $\gamma$ and aversion to ambiguity described by $[\beta_a, \beta_b]$ is*

$$CE(\theta) = \begin{cases} \text{E}_{\beta_a} \left[ \tilde{w} \mid \tilde{s} = s \right] - \frac{1}{2}\gamma \text{Var}_{\beta_a} \left[ \tilde{w} \mid \tilde{s} = s \right] & \text{if } \theta \leq \min\left(\hat{\theta}_a, 0\right) \\ \text{E}\left[\tilde{w}\right] - \frac{1}{2}\gamma \text{Var}\left[\tilde{w}\right] - \frac{s^2}{2\gamma} & \text{if } \min\left(\hat{\theta}_a, 0\right) < \theta \leq \min\left(\hat{\theta}_b, 0\right) \\ \text{E}_{\beta_b} \left[ \tilde{w} \mid \tilde{s} = s \right] - \frac{1}{2}\gamma \text{Var}_{\beta_b} \left[ \tilde{w} \mid \tilde{s} = s \right] & \text{if } \min\left(\hat{\theta}_b, 0\right) < \theta \leq \max\left(\hat{\theta}_b, 0\right) \\ \text{E}\left[\tilde{w}\right] - \frac{1}{2}\gamma \text{Var}\left[\tilde{w}\right] - \frac{s^2}{2\gamma} & \text{if } \max\left(\hat{\theta}_b, 0\right) < \theta \leq \max\left(\hat{\theta}_a, 0\right) \\ \text{E}_{\beta_a} \left[ \tilde{w} \mid \tilde{s} = s \right] - \frac{1}{2}\gamma \text{Var}_{\beta_a} \left[ \tilde{w} \mid \tilde{s} = s \right] & \text{if } \theta > \max\left(\hat{\theta}_a, 0\right). \end{cases}$$
$$\tag{6}$$

*The certainty equivalent $CE(\theta)$ is a continuous and concave function of the stock demand $\theta$. Moreover, it is continuously differentiable except for the portfolio $\theta = 0$ if $s \neq 0$.*

Investors who are contemplating a long position in the asset are worried about bad signals with a high $\beta$ and good signals with a low $\beta$ because informative bad signals significantly lower the posterior asset mean whereas good signals that are not very informative only moderately increase the posterior asset mean. On the other hand, investors always fear risk and thus are worried about signals with a low $\beta$.

Investors are more worried about the posterior mean for small risks and are more worried about the residual variance for big risks. Hence, investors treat bad signals

as informative for moderate long positions in the asset ($0 < \theta \leq \hat{\theta}_b$) and as not very informative for very large long positions ($\theta > \hat{\theta}_a$). There is a range of portfolio positions ($\hat{\theta}_b < \theta \leq \hat{\theta}_a$) for which investors' beliefs balance the counteracting mean and variance effects and thus

$$\beta^*(\theta) \equiv \operatorname*{argmin}_{\beta \in [\beta_a, \beta_b]} \mathrm{CE}^S(\theta, \beta) = -\frac{s}{\gamma\theta}, \tag{7}$$

where $\mathrm{CE}^S(\theta, \beta)$ denotes the certainty equivalent of a standard expected utility maximizer in the sense of Savage (1954) with belief $\beta$. Hence, investor revise their worst case scenario belief about $\beta$ downwards in response to an increase in risk and vice versa for an increase in the news surprise. We will show in the next section that this change in the worst case scenario belief will lead to risky portfolios that are insensitive to news.

## II    Information Inertia of Optimal Portfolios

In this section, we determine the optimal portfolio of investors who are ambiguous about the predictability of the future value of an asset. We show that these portfolios do not always react to news and the severity of this insensitivity to news depends on the unconditional risk premium of the asset.[14]

We are interested in the sensitivity of optimal portfolios to changes in the signal and hence for the remainder of this section we fix the stock price $p$ and determine the optimal demand for the risky asset as a function of the signal.[15]

It is well known that the optimal stock allocation for a Savage investor with belief $\beta$ is

$$\theta_\beta(s) = \frac{\mathrm{E}_\beta\left[\tilde{d} \mid \tilde{s} = s\right] - p}{\gamma \mathrm{Var}_\beta\left[\tilde{d} \mid \tilde{s} = s\right]}. \tag{8}$$

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[14]We conduct our analysis in the tractable CARA-normal framework which allows us to solve for optimal portfolios and equilibrium stock prices in closed form. All result in this section go through if we assume mean-variance preferences over excess returns.

[15]We endogenize the stock price in section IV.

An increase in the signal will always lead to an increase in an investor's stock position and hence optimal portfolio allocations always react to news. This is no longer true when investors are averse to ambiguity as the next theorem shows.[16]

**Theorem 1** (Optimal Portfolios). *Let $\lambda = \bar{d} - p$ denote the unconditional risk premium of the asset. The optimal stock allocation for an investor with risk aversion $\gamma$ and aversion to ambiguity described by $[\beta_a, \beta_b]$ is*

$$
\theta(s) = \begin{cases}
\theta_{\beta_a}(s) & s \geq s_1 \equiv -\frac{\beta_a}{\sigma_d^2}\max(\lambda, 0) - \frac{1}{\beta_a}\min(\lambda, 0) \\
\max\left(\theta_0(s), 0\right) & s_1 > s \geq s_2 \equiv -\frac{\beta_b}{\sigma_d^2}\max(\lambda, 0) - \frac{1}{\beta_b}\min(\lambda, 0) \\
\theta_{\beta_b}(s) & s_2 > s \geq s_3 \equiv -\frac{1}{\beta_b}\max(\lambda, 0) - \frac{\beta_b}{\sigma_d^2}\min(\lambda, 0) \\
\min\left(\theta_0(s), 0\right) & s_3 > s \geq s_4 \equiv -\frac{1}{\beta_a}\max(\lambda, 0) - \frac{\beta_a}{\sigma_d^2}\min(\lambda, 0) \\
\theta_{\beta_a}(s) & s < s_4.
\end{cases} \tag{9}
$$

The left graph of Figure 1 shows optimal stock allocations as a function of the signal when the unconditional risk premium is positive ($\lambda > 0$) and the right graph of Figure 1 shows it when the unconditional risk premium is negative ($\lambda < 0$). Suppose the unconditional risk premium is positive. If the signal conveys good news ($s > 0$), then ambiguity averse investors (black solid line) buy the asset and the worst case scenario for both the mean and the variance is always an unreliable signal. Hence, their demand coincides with a Savage investor with belief $\beta_a$ (blue dashed line). This is no longer true when the signal conveys bad news ($s < 0$). Specifically, investors are still long in the asset for a moderate bad news surprise. Hence, they behave like a Savage investor with belief $\beta_b$ (red chain-dotted line) if they are more worried about a low posterior mean and they behave like a Savage investor with belief $\beta_a$ if they are more worried about risk. On the other hand, if news is very bad then investors take on a short position in the asset and thus always behave like a Savage investor with belief $\beta_a$ because the worst case scenario for the posterior mean and variance is an unreliable signal.

There are two ranges of signal values for which optimal portfolios do not react to news and thus exhibit information inertia. The first range corresponds to the risk-free portfolio ($\theta = 0$) and the second range corresponds to risky stock allocations ($\theta \neq 0$).

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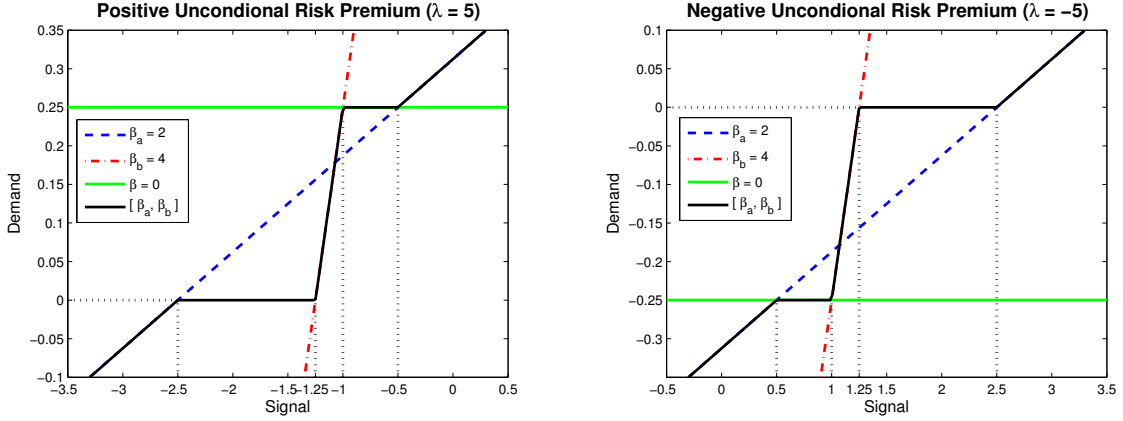[16]Optimal demand as a function of the stock price is given in Theorem 3 of the appendix.

Figure 1: Optimal Stock Allocation

The left graph shows optimal stock allocations when the unconditional risk premium is positive and the right graph shows it when the unconditional risk premium is negative. The optimal stock allocations is plotted as a function of the signal for a Savage investor with belief $\beta_b$ (red chain dotted line), a Savage investor with belief $\beta_a$ (blue dashed line), a Savage investor with belief $\beta = 0$ (green solid line), and for an ambiguity averse investor with range of beliefs $[\beta_a, \beta_b]$ (black solid line). The risk-free and the risky portfolio do not always react to news when there is ambiguity. The parameters are $\bar{d} = 100$, $\sigma_d^2 = 20$, and $\gamma = 1$.

The risk-free portfolio does not react to news because a small increase (decrease) in the signal does not sufficiently raise (lower) the posterior asset mean to convince an investor to give up a portfolio that perfectly hedges against risk and ambiguity.[17]

For portfolios with intermediate risk investors are not sure whether they should be more worried about the posterior mean or the residual variance. There is no portfolio (that is independent of $\beta$) that perfectly hedges against ambiguity (by making utility independent of $\beta$) and thus investor chose not to rely on the signal when determining optimal demand.[18] Hence, investors demand coincides with the demand of a Savage

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[17]This form of inertia also appears in Condie and Ganguli (2011) and Illeditsch (2011).

[18]Illeditsch (2011) shows that there is a risky portfolio that perfectly hedges against ambiguity and thus causes portfolio inertia away from certainty. However, this portfolio is sensitive to changes in the signal.

investor who thinks the covariance between the asset and the signal is zero.[19] This is the case even though a higher signal is always good news for the asset ($\beta_a > 0$). However, investors are still worried about the ambiguous signal because it affects their utility. This utility cost is increasing in the news surprise and decreasing in risk aversion (see Proposition 1).

The size of both signal inaction regions is given in the next proposition.

**Proposition 2.** *The size of the signal region for which the two risky portfolios do not react to news is*

$$\frac{\beta_b - \beta_a}{\sigma_d^2} \mid \lambda \mid . \tag{11}$$

*The size of the signal region for which the risk-free portfolio does not react to news is*

$$\frac{\beta_b - \beta_a}{\beta_a \beta_b} \mid \lambda \mid . \tag{12}$$

Figure 2 shows optimal stock allocations as a function of the signal for different values of the unconditional risk premium $\lambda$. There is no information inertia when the unconditional risk premium is zero (black solid line) because in this case the ambiguity averse investor behaves like a Savage investor with belief $\beta_a$. Intuitively, investors will long the asset when news is good and they will short the asset when news is bad. But there is no confusion about the interpretation of the signal when news is good (bad) and investors are long (short) the asset because the worst case scenario for the posterior mean and the posterior variance is a signal that is not very informative. Information inertia is more severe when price deviates a lot from its expected future value. However, in this case risky portfolios only exhibit information inertia for very extreme news surprises. Hence, we determine the probability of the risk-free and risky portfolio exhibiting information inertia in the next proposition.[20]

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[19]It follows from the Envelope Theorem that

$$\frac{\partial \mathrm{CE}(\theta)}{\partial \theta} = \frac{\partial \mathrm{CE}^S(\theta, \beta^*(\theta))}{\partial \theta} + \frac{\partial \mathrm{CE}^S(\theta, \beta^*(\theta))}{\partial \beta} \frac{\partial \beta^*(\theta)}{\partial \theta}$$
$$= \frac{\partial \mathrm{CE}^S(\theta, \beta^*(\theta))}{\partial \theta} = \mathrm{E}\left[\tilde{d}\right] - p - \theta\gamma\mathrm{Var}\left[\tilde{d}\right], \tag{10}$$

for all $\theta \in [\min(\hat{\theta}_a, \hat{\theta}_b), \max(\hat{\theta}_a, \hat{\theta}_b)]$. Hence, marginal utility is not affected by changes in the signal.

[20]The probability of information inertia conditional on a specific news surprise is determined in
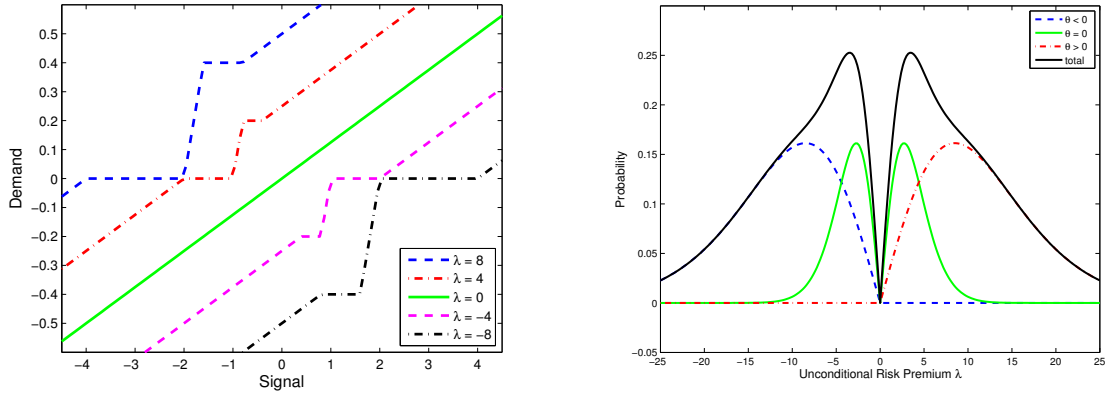
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Figure 2: Information Inertia of Optimal Portfolios

The left graph shows optimal demand as a function of the signal and the right graph shows the probability of optimal portfolios exhibiting information inertia as a function of the unconditional risk premium. Risky and risk-free portfolios do not always react to news and the probability of information inertia is non-monotonic in the risk of the stock. The parameters are $\bar{d} = 100$, $\sigma_d^2 = 20$, $\beta_a = 2$, $\beta_b = 4$, and $\gamma = 1$.

**Proposition 3.** *The unconditional probability of investors exhibiting information inertia for either a long ($\lambda > 0$) or a short ($\lambda < 0$) position in the asset is*

$$\left| \Phi\left( \beta_b \frac{\lambda}{\sigma_d^2} \right) - \Phi\left( \beta_a \frac{\lambda}{\sigma_d^2} \right) \right|, \tag{13}$$

*where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distributed variable.*

*The unconditional probability of investors exhibiting information inertia when holding the risk-free portfolio is*

$$\left| \Phi\left( \frac{\lambda}{\beta_a} \right) - \Phi\left( \frac{\lambda}{\beta_b} \right) \right|. \tag{14}$$

The probability of having information inertia depends on the unconditional risk premium and is plotted in the right graph of Figure 2. If the unconditional risk premium is zero, then there is no information inertia because investors behave like Savage investors with belief $\beta_a$. If the stock price deviates a lot form its expected future asset value, then investors take on very risky asset positions and hence are

the appendix.

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more worried about risk than a low posterior asset mean and thus behave like Savage investors with belief $\beta_a$. Hence, the probability of short sellers, asset buyers, and investors who do not participate in the stock market to exhibit information inertia is non monotonic in the unconditional risk premium of the asset.

We conclude this section with a summary of the information inertia results.

**Model Predictions 1** (Information Inertia of Optimal Portfolios). *If investors are ambiguous about the predictability of future asset values, then*

(i) *there is a range of bad signals over which investors do not adjust their long stock position when the unconditional risk premium is positive,*

(ii) *there is a range of good signals over which investors do not adjust their short stock position when the unconditional risk premium is negative,*

(iii) *there is a range of good and bad signals over which investors do not hold stocks when the unconditional risk premium is not zero, and*

(iv) *the probability of optimal portfolios exhibiting information inertia is non-monotonic in the unconditional risk premium of the asset.*

# III  Informational Inefficiency of Prices

In this section, we solve for the equilibrium stock price when a representative investor is ambiguous about the predictability of future cash flows.[21] We show that prices do not always incorporate public information that is worse than expected and the severity of this mispricing depends on the risk of the stock.

Suppose there is a representative investor (RI) with CARA utility who is averse to ambiguity. In equilibrium, the RI holds the stock and consumes the liquidating dividend $\tilde{d}$. Hence, the stock price at date 1 equals the liquidating dividend and $\theta = 1$. The price at date 0 depends on the signal and is determined below.

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[21]We discuss properties of the equilibrium stock price when the economy is populated by heterogenous investor in the next section.

The equilibrium stock price when the representative investor is a Savage investor with belief $\beta$ (standard expected utility maximizer in the sense of Savage (1954)) is

$$p(s) = \mathrm{E}_\beta \left[ \tilde{d} \mid \tilde{s} = s \right] - \gamma \mathrm{Var}_\beta \left[ \tilde{d} \mid \tilde{s} = s \right]. \tag{15}$$

The stock price is strictly increasing in the signal and hence prices fully incorporate all available public information. This is no longer true when the representative investor is averse to ambiguity as the next theorem shows.

**Theorem 2** (Equilibrium Stock Price). *There is a unique equilibrium stock price. Specifically,*

$$p(s) = \begin{cases} \mathrm{E}_{\beta_a} \left[ \tilde{d} \mid \tilde{s} = s \right] - \gamma \mathrm{Var}_{\beta_a} \left[ \tilde{d} \mid \tilde{s} = s \right] & if \quad s > -\gamma \beta_a \\ \mathrm{E} \left[ \tilde{d} \right] - \gamma \mathrm{Var} \left[ \tilde{d} \right] & if \quad -\gamma \beta_b \leq s \leq -\gamma \beta_a \\ \mathrm{E}_{\beta_b} \left[ \tilde{d} \mid \tilde{s} = s \right] - \gamma \mathrm{Var}_{\beta_b} \left[ \tilde{d} \mid \tilde{s} = s \right] & if \quad s < -\gamma \beta_b. \end{cases} \tag{16}$$

Figure 3 shows the equilibrium stock price as a function of the signal. The stock price reacts moderately to good news because in this case the worst case scenario for both the posterior mean and the residual variance is an unreliable signal. However, if news is very bad, then the RI is more worried about a low posterior mean than a high residual variance and thus the price strongly reacts to changes in the signal. There is a range of bad signal values for which the RI is inattentive to news and hence the stock price does not react to these signals.
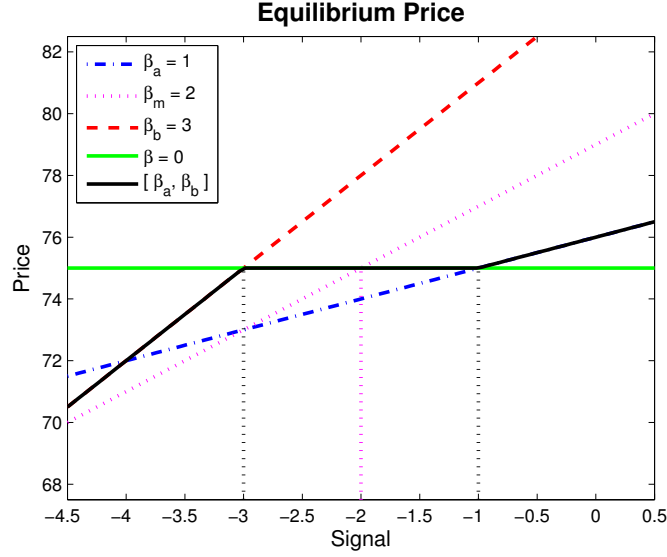
Figure 3: Equilibrium Price

This graph shows the equilibrium stock price as a function of the signal. The red dashed line represents an economy in which the representative investor (RI) is a Savage with belief $\beta_b$, the purple dotted line represents an economy in which the RI is a Savage with belief $(\beta_a + \beta_b)/2$, the blue chain-dotted line represents an economy in which the RI is a Savage with belief $\beta_a$, the green solid line represents and economy in which the RI is a Savage with belief $\beta = 0$, and the black solid line represents an economy in which the RI is ambiguity averse ($[\beta_a, \beta_b]$). There is a range of signals that are not priced and thus prices fail to incorporate all available public information. The parameters are $\bar{d} = 100$, $\sigma_d = 5$, and $\gamma = 1$.

To gain some more intuition consider a two standard deviation bad news surprise; i.e. $s = -2$. In this case the equilibrium stock price is $p = 75$ when there is ambiguity and when there is no ambiguity $\beta_m = 2$ (see Figure 3). If the signal decreases slightly, then the Savage RI requires a lower stock price as compensation for the lower posterior mean in order to hold the market portfolio. However, the ambiguity averse RI is also worried about not having the right $\beta$ and hence she revises the worst case scenario belief about $\beta$ upwards if the signal drops. The stock price does not need to change because the lower posterior mean that would require a drop in the equilibrium price is exactly offset by the lower residual variance that would require an increase in the stock price.

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We determine the size of the inaction region and the probability of being in that region in the next proposition.

**Proposition 4.** *The size of the signal region for which the stock price does not react to news is $\gamma(\beta_b - \beta_a)$. The conditional probability of this informational inefficency is*

$$Prob\left(-\gamma\beta_b \leq \tilde{s} \leq -\gamma\beta_a \mid \tilde{s} \leq x\right) = \begin{cases} 0 & if \quad x < -\gamma\beta_b \\ \frac{\Phi(x) - \Phi(-\gamma\beta_b)}{\Phi(x)} & if \quad -\gamma\beta_b \leq x \leq -\gamma\beta_a \\ \frac{\Phi(-\gamma\beta_a) - \Phi(-\gamma\beta_b)}{\Phi(x)} & if \quad x > -\gamma\beta_a, \end{cases} \quad (17)$$

*where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distributed variable.*

The left graph of Figure 4 shows equilibrium stock prices as function of the signal for different unconditional risk premia. The blue chain-dotted line represents an economy in which the unconditional risk premium of the stock is 0, the red dashed line represents an economy in which the unconditional risk premium of the stock is 12.5, and the black solid line represents an economy in which the unconditional risk premium of the stock is 25. The figure shows that the inaction region increases with the risk of the stock. It also shows that increasingly worse signals will not be reflected in the price if the stock is more risky.

The right graph of Figure 4 shows that the probability of this mispricing of public information is not monotonic in the risk of the stock. Moreover, it also shows that conditional on increasingly bad news surprises this probability is very large for some stocks.
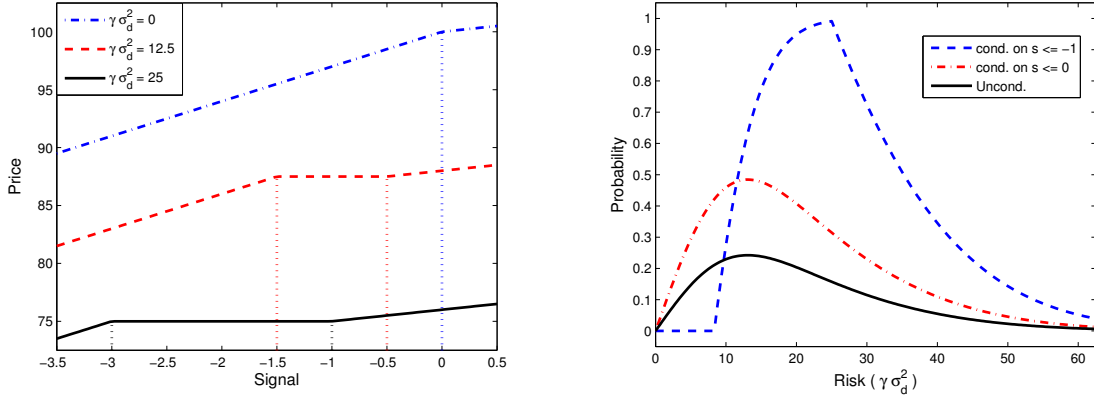
Figure 4: Mispricing of Public Information

The left graph shows the equilibrium stock price as a function of the signal for different unconditional risk premia. It shows that information inertia in prices is more severe for risky stocks. The right graph shows that the probability of having information inertia in prices is also less likely for risk stocks. The black solid line shows the unconditional probability, the red chain-dotted line shows the probability conditional on bad news, and the blue dashed line shows the probability conditional on having an at least one standard deviation bad news surprise. The parameters are $\bar{d} = 100$, $\sigma_d = 5$, $\beta_a = 1$, and $\beta_b = 3$.

Finally, we study the price reaction to news as a function of the unconditional risk premium of the stock. The graphs in Figure 5 show the probability of having a moderate price reaction (red area), a strong price reaction (blue area), and no price reaction (black area). The graphs in the last row show that stocks with moderate risk are more likely to react strongly to very bad news than expected than stocks with high risk. However, it is striking that stocks with intermediate risk show almost no reaction to bad ambiguous news.

We now summarize the predictions for the equilibrium stock price.

**Model Predictions 2** (Informational Inefficiency of Prices)**.** *If investors are ambiguous about the predictability of future cash flows, then*

(i) *prices do not always incorporate public signals that convey bad news and*

(ii) *this mispricing of news is more severe for stocks with intermediate risk and when news are worse than expected.*
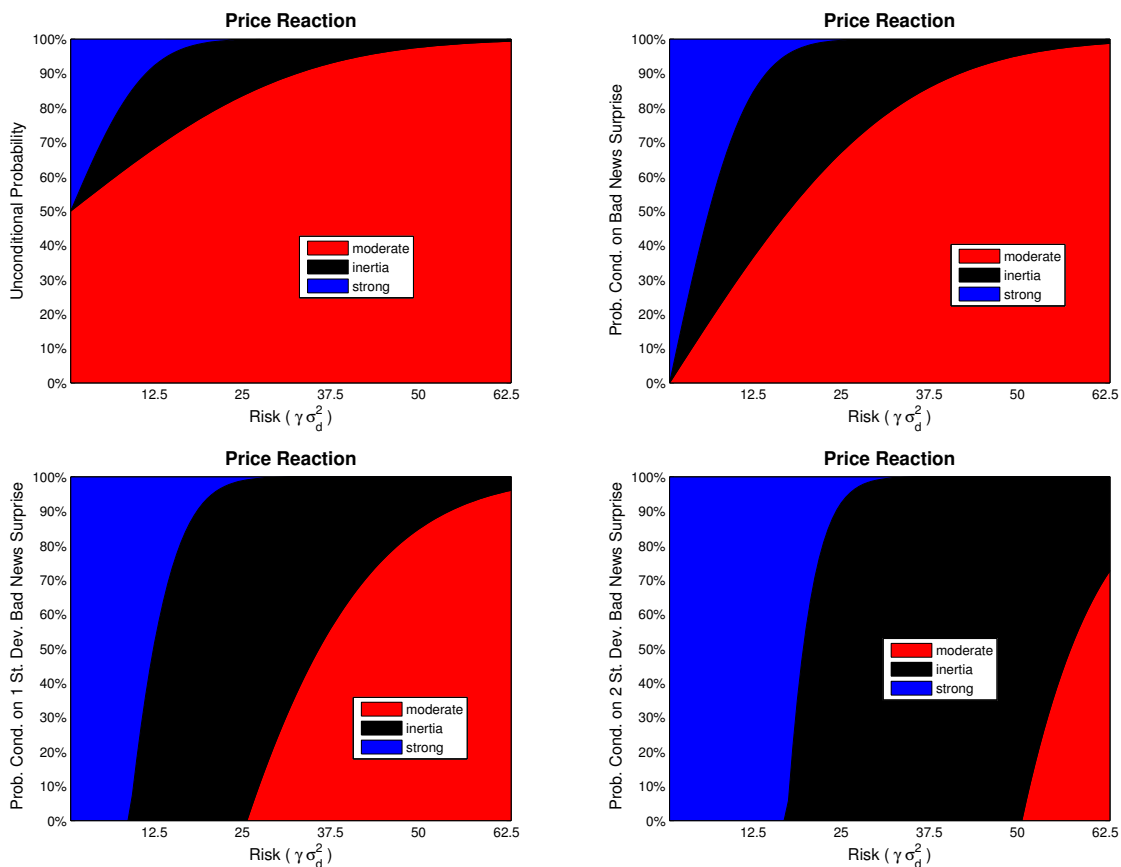
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Figure 5: Price Reaction to News

All four graphs show the probability of having a moderate price reaction (red area), a strong price reaction (blue area), and no price reaction (black area) as a function of the unconditional risk premium of the stock. The first graphs shows the unconditional probability, the second graphs shows the probability conditional on bad news, the third graphs shows the probability conditional on having an at least one standard deviation bad news surprise, and the last graph shows the probability conditional on having an at least two standard deviation bad news surprise. The parameters are $\bar{d} = 100$, $\sigma_d = 5$, $\beta_a = 1$ and $\beta_b = 3$.

# IV  Heterogenous Investors

In this section, we assume ambiguity about the predictability of future cash flows and study equilibrium prices and demands when the economy is populated by investors who may differ with respect to aversion to risk and ambiguity. We show that in equilibrium there are some investors who are inattentive to news and hence optimal portfolios exhibit information inertia. Moreover, the result that prices may not reflect all available public information in equilibrium is robust to this heterogeneity.

Suppose there are $H$ investors who all receive the same signal but may differ with respect to their initial wealth, and their aversion to risk and ambiguity. Let $w_{0h}$ denote investor $h$'s initial wealth, $\gamma_h > 0$ her risk aversion coefficient, and $[\beta_{ah}, \beta_{bh}]$ her set of beliefs with $0 < \beta_{ah} \leq \beta_{bh} < \sigma_d \, \forall \, h \in \{1, \ldots, H\}$.

An equilibrium in this economy is defined as follows:

**Definition 1** (Equilibrium). *The signal-to-price map $p(s)$ is an equilibrium $\forall s \in \mathcal{R}$ if and only if (i) each investor chooses a portfolio $\theta_h$ to maximize*

$$\min_{\beta_h \in [\beta_{ah}, \beta_{bh}]} \mathrm{E}_{\beta_h} \left[ u_h \left( w_{0h} + \left( \tilde{d} - p(s) \right) \theta_h \right) \mid \tilde{s} = s \right], \qquad \forall s \in \mathbb{R} \qquad (18)$$

*and (ii) markets clear; i.e. $\sum_{h=1}^{H} \theta_h = 1$ and investors consume the liquidating dividend $\tilde{d}$ at date 1.*

If all investors have the same ambiguity, then we know from Wakai (2007) and Illeditsch (2011) that there exists a representative investor with risk tolerance equal to the sum of the risk tolerances of all $H$ investors. Hence, the stock price is given in Theorem 2 and there is no trade in equilibrium. We show in the next proposition that equilibrium prices still fail to incorporate all available public information when investors are heterogeneous in their aversion to ambiguity and their ranges of beliefs overlap.[22]

**Proposition 5** (Aggregation). *Let $1/\gamma \equiv \sum_{h=1}^{H} 1/\gamma_h$ denote aggregate risk tolerance*

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[22]We do not report the equilibrium price outside of the inaction region but provide numerical examples in Figure 6 and 7.

and let $[\beta_a, \beta_b] \equiv \bigcap_{h=1}^{H} [\beta_{ah}, \beta_{bh}] \neq \emptyset$. Then the equilibrium stock price is

$$p(s) = \mathrm{E}\left[\tilde{d}\right] - \gamma \mathrm{Var}\left[\tilde{d}\right] \qquad \forall\, s \in [-\gamma\beta_b, -\gamma\beta_a]. \tag{19}$$

The size of the price inaction region is determined by the investors with common ambiguity and it is increasing with aggregate risk aversion $\gamma$. Hence, the size of the inaction region and the probability of the mispricing of public information is as given in Proposition 4 of the previous section.

To gain some more intuition consider an economy in which both investors (Knights) are averse to ambiguity. The first Knight has the range of beliefs $[\beta_{a1}, \rho_{b1}] = [0.5, 2]$ and the second Knight has the range of beliefs $[\beta_{a2}, \beta_{b2}] = [1, 3]$. The black solid line in the left graph of Figure 6 shows the equilibrium stock price as a function of the signal. For comparison, we also show the equilibrium stock price for five economies that are populated by two Savages with different beliefs. For instance, the red dotted line shows the price in an economy where the first Savage has the belief $\beta_1 = 2$ and the second Savage has the belief $\beta_2 = 3$. The right graph of Figure 6 shows the equilibrium demand as a function of the signal when the economy is populated by the two Knights described above. There is a range of signals that are worse than expected for which both investors use their priors when computing demands and thus the equilibrium stock price does not react to changes in these signals.
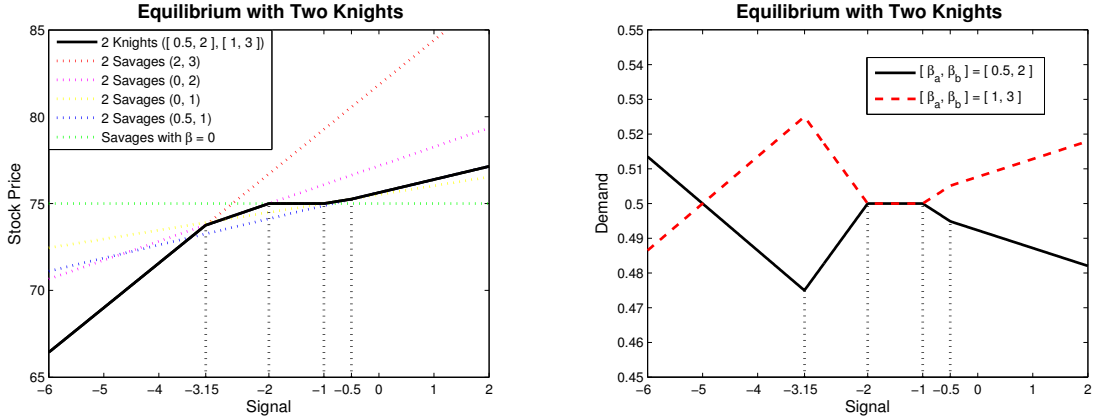
Figure 6: Investors with Different Ambiguity

The left graph shows the equilibrium stock price as a function of the signal for six different economies. The black solid line represents an economy that consists of two Knights with range of beliefs ($[\beta_{a1}, \beta_{b1}] = [0.5, 2]$, $[\beta_{a2}, \beta_{b2}] = [1, 3]$). The colored dotted lines represent economies that consist of two Savages with different beliefs. The left graph shows that if there is ambiguity, then there is a range of bad signals for which the price does not react much. The right graph shows equilibrium demand in an economy that consists of two Knights. The red dashed line shows demand of a Knight with range of beliefs $[\beta_{a1}, \beta_{b1}] = [0.5, 2]$ and the black solid line shows demand of a Knight with range of beliefs $[\beta_{a2}, \beta_{b2}] = [1, 3]$. If the signal lies in the interval $[-3.15, -2]$, then at least one of the investors ignores the signal and uses her prior information when determining demand. When both investors hedge against ambiguity ($[\hat{s}_b, \hat{s}_a] = [-2, -1]$), then demand is insensitive to changes in these signals and thus equilibrium prices fail to incorporate these signals. All investors have the same risk aversion $\gamma = 1$ and the remaining parameters are $\bar{d} = 100$, and $\sigma_d = 5$.

To discuss the properties of equilibrium demand and price given in Figure 6 we consider the five different signal regions (i) $(-\infty, -3.15]$, (ii) $[-3.15, -2]$, (iii) $[-2, -1]$, (iv) $[-1, -0.5]$, and (v) $[-0.5, \infty)$. Both Knights behave like Savages with beliefs $\beta_1 = 2$ and $\beta_2 = 3$ respectively for the first range of signals because if news is very bad ambiguity averse investors are more worried about a low posterior mean than a high residual variance and thus consider a high $\beta$. Hence, the equilibrium stock price reacts a lot to these signals. Equilibrium demand of the second Knight (red dashed line) is increasing in the signal because her worst case scenario belief ($\beta = 3$) is larger than the worst case scenario belief of the second Knight (black solid line) and thus she puts more weight on the signal. The analysis is similar for the fifth

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range because with good news the worst case scenario for both investors is a low $\beta$.

For the other three ranges of signals there is at least one investor who avoids the signal and uses her prior when forming optimal demand. In other words, there is at least one investor who behaves as if there is no correlation between the signal and the dividend even though a high signal is always good news about the asset. Consider the second signal range. The first Knight sill behaves like a Savage investor with belief $\beta = 2$ but the second Knight does not rely on the signal. Hence her demand which is increasing for the first range of signals is now decreasing because neither mean nor variance depends on the signal and the equilibrium price increases with it. The equilibrium price still reacts to signals in the second region because of the first investor but not as much as for the first range of signals. Both investors do not rely on the signals in the third region and hence the equilibrium does not reflect these signals. The intuition for the fourth signal range is similar to the second. In this case the first investor does not rely on the signal when forming demand and hence in equilibrium her demand decreases with the signal.

We conclude this section with a comparison of equilibrium demand for different economies. Specifically, the left graph of Figure 7 shows equilibrium demand as a function of the signal and the right graph shows it as a function of the equilibrium stock price. In both graphs the black solid line represents an economy consisting of two Knights, the red dashed line represents an economy consisting of two Savages with different beliefs, and the blue chain-dotted line represents an economy with one Knight and one Savage. The graphs show that if there is ambiguity, then equilibrium demand is neither monotone in the signal nor in the equilibrium stock price. Moreover, the left graph shows that this non monotone and seemingly erratic demand behavior only occurs for signals that are worse than expected. This is in stark contrast to an economy without ambiguity for which equilibrium demand is a smooth and monotone function of the signal.[23]

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[23]If everybody has the same belief (or ambiguity) then equilibrium demand is constant.
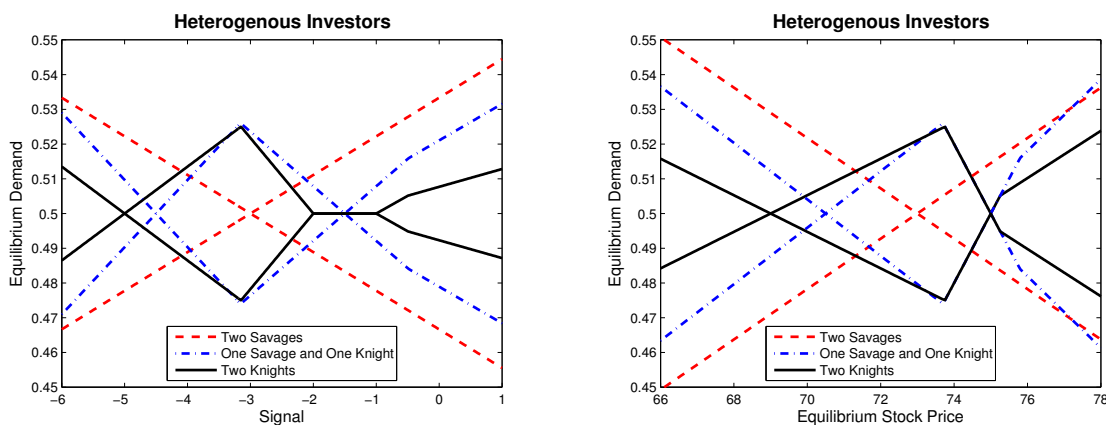
Figure 7: Equilibrium Demand

The left graph shows equilibrium demand as a function of the signal and the right graph shows it as a function of the equilibrium stock price. In both graphs the black solid line represents an economy consisting of two Knights with range of beliefs $[\beta_{a1}, \beta_{b1}] = [0.5, 2]$ and $[\beta_{a2}, \beta_{b2}] = [1, 3]$). The red dashed line represents an economy consisting of two Savages with beliefs $\beta_1 = 1$ and $\beta_2 = 2$ and the blue chain-dotted line represents an economy with one Knight with range of beliefs $[\beta_{a1}, \beta_{b1}] = [0.5, 3]$ and one Savage with belief $\rho_2 = 1.5$. Both graphs show that equilibrium demand is non-monotone if there is ambiguity. All investors have the same risk aversion $\gamma = 1$ and the remaining parameters are $\bar{d} = 100$, and $\sigma_d = 5$.

# V  Momentum Strategies

We show in this section that the failure of prices to incorporate all available public information leads to profitable trading strategies for investors who know the correct joint distribution of the signal and the dividend.

Suppose an econometrician observes a time series of dividends, signals, and stock prices and regresses future price changes on a constant and (i) the current signal $s$ or (ii) the current stock price $p(s)$. Let $\hat{\beta}$ denote the covariance between $\tilde{d}$ and $\tilde{s}$ that generates the data and assume that $\hat{\beta} = (\beta_a + \beta_b)/2$.[24]

The left and right graph of Figure 8 shows the slope of the first and second regression as a function of the unconditional risk premium of the stock for four different

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[24]This is a common assumption in the literature (e.g. Hansen and Sargent (2001))

representative agent economies. The purple dotted line represents an economy in which the belief of the Savage investor coincides with the data generating belief $\hat{\beta}$; i.e. the belief of the RI satisfies "rational expectations". In this case the equilibrium price incorporates all public information correctly and hence neither current news nor the current stock price predicts future price changes; i.e. the slope is zero in both regressions.

The blue dashed line represents an economy in which the Savage RI has a higher $\beta$ than the econometrician and the red chain-dotted line represent an economy in which the Savage RI has a lower beta than the econometrician. In both economies the stock price fully but incorrectly incorporates all available public information. Hence, the price overreacts in the first economy and underreacts in the second economy and hence public information predicts future price changes.

The black solid line represents an economy in which the representative investor is averse to ambiguity. In this case, both current signals and prices predict future changes in the stock price. However, the economic significance depends on the unconditional risk premium of the stock. Specifically, there is no predictability if the unconditional risk premium is zero because prices fully incorporate all available public information and the underreaction of prices to good news is offset by the overreaction to bad news. If the unconditional risk premium of the stock is large, then the slope in both predictive regressions is positive because the probability of having a moderate price reaction to news is close to one for very risky stocks (see the first graph of Figure 5). For stocks with intermediate risk, the economic significance of both predictability regression can be very large because in this case many bad signals are not reflected in the stock price.[25]

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[25]The results do not change qualitatively if we allow the trader with correct beliefs to affect the price in equilibrium.
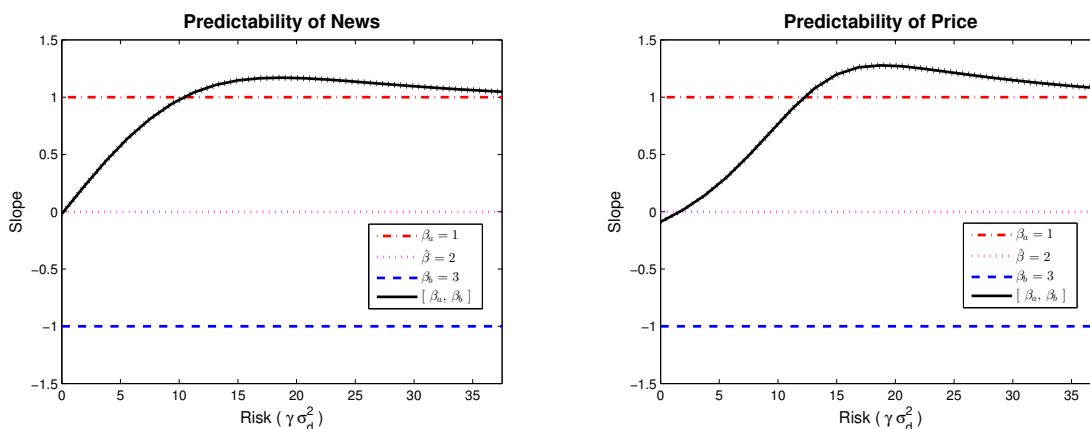
Figure 8: Predictive Regressions

The left graph shows the slope of an regression of price changes on the signal and the right graph shows the slope of an regression of price changes on the current stock price. Both graphs shows that aversion to ambiguity leads to profitable trading strategies based on public information. The economic significance of these predictability regressions depends on the unconditional risk premium of the stock. The parameters are $\bar{d} = 100$, and $\sigma_d = 5$.

**Model Predictions 3** (Momentum). *Ambiguity about the predictability of future cash flows leads to*

*(i) profitable trading strategies based on public information and*

*(ii) the economic significance of these momentum strategies depends on the unconditional risk premium of the stock.*

# VI   Conclusion

We study how information about an asset affects optimal portfolios and equilibrium asset prices when investors are not sure about the model that predicts future asset values and thus treat the information as ambiguous. We show that this ambiguity leads to optimal portfolios that are insensitive to news even though there are no information processing costs or other market frictions. In equilibrium, we show that stock prices may not react to public information that is worse than expected and this mispricing of bad news leads to profitable trading strategies based on public information.

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# A   Appendix

## A   Proofs

In this section, the following notation will be useful. For $\beta \in [0, \sigma_d]$, let $\text{CE}^S(\theta, \beta)$ denote the certainty equivalent for a standard Savage (1954) expected utility investor who considers covariance $\beta$ between $\tilde{s}$ and $\tilde{d}$, i.e.

$$\text{CE}^S(\theta, \beta) = \text{E}_\beta \left[ \tilde{w} \, | \tilde{s} = s \right] - \frac{1}{2} \gamma \text{Var}_\beta \left[ \tilde{w} \, | \tilde{s} = s \right]. \tag{20}$$

***Proof of Proposition 1***. The certainty equivalent $\text{CE}(\theta)$ of the ambiguity averse investor satisfies

$$\text{CE}(\theta) = \min_{\beta \in [\beta_a, \beta_b]} \text{CE}^S(\theta, \beta). \tag{21}$$

Note that

$$\frac{\partial \text{CE}^S(\theta, \beta)}{\partial \beta} = \theta s + \gamma \theta^2 \beta. \tag{22}$$

Consider three cases, (i) $s = 0$, (ii) $s > 0$, and (iii) $s < 0$.

(i) $s = 0 \Leftrightarrow \hat{\theta}_a = \hat{\theta}_b = 0$.

Then $\frac{\partial \text{CE}^S(\theta, \beta)}{\partial \beta} > 0$ for all $\beta \in [\beta_a, \beta_b]$. Thus the minimum of $\text{CE}^S(\theta, \beta)$ is attained at $\beta = \beta_a$ and hence,

$$\text{CE}(\theta) = \min_{\beta \in [\beta_a, \beta_b]} \text{CE}^S(\theta, \beta) = \text{CE}^S(\theta, \beta_a) \text{ for all } \theta \in \mathbb{R}. \tag{23}$$

$\text{CE}^S(\theta, \beta_a)$ is continuously differentiable and concave in $\theta$ for all $\theta \in \mathbb{R}$ and thus so is $\text{CE}(\theta)$.

(ii) $s > 0 \Leftrightarrow \hat{\theta}_a < \hat{\theta}_b < 0$.

Suppose $\theta \le \hat{\theta}_a < 0$ or $\theta \ge 0$. Then $\frac{\partial \text{CE}^S(\theta, \beta)}{\partial \beta} > 0$ for all $\beta \in [\beta_a, \beta_b]$. Thus, the minimum of $\text{CE}^S(\theta, \beta)$ is attained at $\beta = \beta_a$.

Suppose $\hat{\theta}_b < \theta < 0$. Then $\frac{\partial \mathrm{CE}^S(\theta,\beta)}{\partial \beta} < 0$ for all $\beta \in [\beta_a, \beta_b]$. Thus, the minimum of $\mathrm{CE}^S(\theta, \beta)$ is attained at $\beta = \beta_b$.

Suppose $\hat{\theta}_a < \theta \leq \hat{\theta}_b$. Then, since $\frac{\partial^2 \mathrm{CE}^S(\theta,\beta)}{\partial \beta^2} > 0$, the minimum is attained at $\frac{\partial \mathrm{CE}^S(\theta,\beta)}{\partial \beta} = 0$, i.e. $\beta^*(\theta) \equiv \underset{\beta \in [\beta_a, \beta_b]}{\mathrm{argmin}}\, \mathrm{CE}^S(\theta, \beta) = \frac{-s}{\gamma \theta}$. Note that $\beta^* \in [\beta_a, \beta_b]$ when $\hat{\theta}_a < \theta < \hat{\theta}_b < 0$ and that

$$\mathrm{CE}^S(\theta, \beta^*) = \mathrm{E}[\tilde{w}] - \frac{1}{2}\gamma \mathrm{Var}[\tilde{w}] - \frac{s^2}{2\gamma} = \mathrm{CE}^S(\theta, 0) - \frac{s^2}{2\gamma}. \tag{24}$$

Using the above, we get

$$\mathrm{CE}(\theta) = \begin{cases} \mathrm{CE}^S(\theta, \beta_a) & \text{if } \theta \leq \hat{\theta}_a \\ \mathrm{CE}^S(\theta, 0) - \frac{s^2}{2\gamma} & \text{if } \hat{\theta}_a < \theta \leq \hat{\theta}_b \\ \mathrm{CE}^S(\theta, \beta_b) & \text{if } \hat{\theta}_b < \theta \leq 0 \\ \mathrm{CE}^S(\theta, \beta_a) & \text{if } 0 < \theta. \end{cases} \tag{25}$$

as desired.

$\mathrm{CE}(\theta)$ is continuous for all $\theta \in \mathbb{R}$ and $\beta \in [\beta_a, \beta_b]$ and $\mathrm{CE}^S(0, \beta_a) = \mathrm{CE}^S(0, \beta_b)$. $\mathrm{CE}^S(\theta, \beta)$ is continuously differentiable for all $\theta \in \mathbb{R}$ and $\beta \in [\beta_a, \beta_b]$ and the $\frac{\partial^2 \mathrm{CE}^S(\theta,\beta)}{\partial \theta^2} \leq 0$ for all $\theta \in \mathbb{R}$ and $\beta \in [\beta_a, \beta_b]$. Thus, for any $\theta \neq 0$ there is an open neighborhood for such $\mathrm{CE}(\theta)$ is continuously differentiable and $\frac{\partial^2 \mathrm{CE}(\theta)}{\partial \theta^2}$ exists and is non-positive.

To verify non-differentiability at $\theta = 0$ and concavity of $\mathrm{CE}(\theta)$, we calculate the left derivative $\mathrm{CE}'^-(\theta)$ and the right derivative $\mathrm{CE}'^+(\theta)$ at $\theta = 0$.

$$\mathrm{CE}'^-(0) \equiv \lim_{\theta \uparrow 0} \frac{\partial \mathrm{CE}(\theta)}{\partial \theta} = \bar{d} + \beta_b s - p \tag{26}$$

$$\mathrm{CE}'^+(0) \equiv \lim_{\theta \downarrow 0} \frac{\partial \mathrm{CE}(\theta)}{\partial \theta} = \bar{d} + \beta_a s - p \tag{27}$$

Thus, $\mathrm{CE}'^-(0) > \mathrm{CE}'^+(0)$, so $\mathrm{CE}(\theta)$ is not differentiable at $\theta = 0$, but is concave for all $\theta \in \mathbb{R}$ and continuously differentiable at all $\theta \neq 0$.

(iii) $s < 0 \Leftrightarrow \hat{\theta} > \hat{\theta}_b > 0$.

Using reasoning similar to that for the above case, we get

$$
\text{CE}(\theta) =
\begin{cases}
\text{CE}^S(\theta, \beta_a) & \text{if } \theta \leq 0 \\
\text{CE}^S(\theta, \beta_b) & \text{if } 0 < \theta \leq \hat{\theta}_b \\
\text{CE}^S(\theta, 0) - \frac{s^2}{2\gamma} & \text{if } \hat{\theta}_b < \theta \leq \hat{\theta}_a \\
\text{CE}^S(\theta, \beta_a) & \text{if } \hat{\theta}_a < \theta
\end{cases}
\tag{28}
$$

and that $\text{CE}(\theta)$ is continuous and concave in $\theta \in \mathbb{R}$. Moreover, $\text{CE}(\theta)$ is continuously differentiable at all $\theta \neq 0$.

Finally, combining the above cases provides the desired expression and properties for $\text{CE}(\theta)$. $\qquad\square$

The following result provides the expression for optimal portfolio as a function of price and is of independent interest in addition to being useful for the proofs of other results.

**Theorem 3** (Optimal Demand). *Optimal demand at price $p$ for an investor with risk aversion $\gamma$ and aversion to ambiguity described by $[\beta_a, \beta_b]$ is*

$$
\theta(p, s) =
\begin{cases}
\theta_{\beta_a}(p) & p \leq p_1(s) \equiv \mu_{\beta_a}(s) - \gamma v_{\beta_a} \max\left(\hat{\theta}_a, 0\right) \\
\max\left(\theta_0(p), 0\right) & p_1(s) < p \leq p_2(s) \equiv \mu_{\beta_b}(s) - \gamma v_{\beta_b} \max\left(\hat{\theta}_b, 0\right) \\
\theta_{\beta_b}(p) & p_2(s) < p \leq p_3(s) \equiv \mu_{\beta_b}(s) - \gamma v_{\beta_b} \min\left(\hat{\theta}_b, 0\right) \\
\min\left(\theta_0(p), 0\right) & p_3(s) < p \leq p_4(s) \equiv \mu_{\beta_a}(s) - \gamma v_{\beta_a} \min\left(\hat{\theta}_a, 0\right) \\
\theta_{\beta_a}(p) & p > p_4(s),
\end{cases}
\tag{29}
$$

*where $\mu_\beta(s) = \bar{d} + \beta s$ and $v_\beta \equiv \sigma_d^2 - \beta^2$.*

***Proof of Theorem 3.*** Consider three cases: (i) $s = 0$, (ii) $s > 0$, and $s < 0$. For expositional simplicity, we suppress the dependence on $s$.

(i) When $s = 0 \Leftrightarrow \hat{\theta}_a = \hat{\theta}_b = 0$, then it follows from the proof of Proposition 1 that $\text{CE}(\theta) = \text{CE}^S(\theta, \beta_a)$ for all $\theta \in \mathbb{R}$. Thus, it follows that $\theta(p) = \theta_a(p)$ for all $p \in \mathbb{R}$.

(ii) When $s > 0 \Leftrightarrow \hat{\theta}_a < \hat{\theta}_b < 0$, then it follows from the proof of Proposition 1 that $\text{CE}(\theta)$ is given by (25).

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Consider five sub-cases: (a) $p \leq p_1 = \mu_{\beta_a}(s)$, (b) $p_1 < p \leq p_2 = \mu_{\beta_b}(s)$, (c) $p_2 < p \leq p_3 = \mu_{\beta_b}(s) - \gamma v_{\beta_b}\hat{\theta}_b$, (d) $p_3 < p \leq p_4 = \mu_{\beta_a} - \gamma v_{\beta_a}\hat{\theta}_a$, and (e) $p_4 < p$.

(ii)(a) Suppose $p \leq p_1$. We show that $\theta(p) = \theta_a(p)$. First, note that

$$\theta_a(p) = \frac{\mu_{\beta_a}(s) - p}{\gamma v_{\beta_a}} \geq \frac{\mu_{\beta_a}(s) - p_1}{\gamma v_{\beta_a}} = 0. \tag{30}$$

Moreover, for any $\theta > 0$, $\mathrm{CE}(\theta) = \mathrm{CE}^S(\theta, \beta_a)$ from (25). Thus, since $\mathrm{CE}(\theta)$ is concave, $\theta_a(p)$ is the local and hence global maximizer of $\mathrm{CE}(\theta)$ for all $p \leq p_1$.

(ii)(b) Suppose $p_1 < p \leq p_2$. We show that $\theta(p) = 0$. First, note that since $\beta_a > 0$,

$$\theta_0(p) = \frac{\bar{d} - p}{\gamma \sigma_d^2} < \frac{\bar{d} - p_1}{\gamma \sigma_d^2} \leq \frac{\mu_{\beta_a}(s) - p_1}{\gamma \sigma_d^2} = 0. \tag{31}$$

Since $\mathrm{CE}(\theta)$ is concave, it suffices to show that $\theta = 0$ is a local maximizer. Given (25), there exists $\epsilon > 0$ such that

$$\mathrm{CE}(\theta) = \begin{cases} CE^S(\theta, \beta_b) & \text{if } -\epsilon < \theta \leq 0 \\ CE^S(\theta, \beta_a) & \text{if } 0 \leq \theta < \epsilon. \end{cases} \tag{32}$$

For $-\epsilon < \theta \leq 0$,

$$\mathrm{CE}(0) - \mathrm{CE}^S(\theta, \beta_b) = \theta\left(p - \bar{d} - \beta_b s\right) + \frac{1}{2}\gamma(\sigma_d^2 - \beta_b^2) \geq 0 \tag{33}$$

when $p \leq p_2$.
For $0 \leq \theta < \epsilon$,

$$\mathrm{CE}(0) - \mathrm{CE}^S(\theta, \beta_a) = \theta\left(p - \bar{d} - \beta_a s\right) + \frac{1}{2}\gamma(\sigma_d^2 - \beta_a^2) \geq 0 \tag{34}$$

when $p_1 \leq p$. Combining the above, shows that $\theta = 0$ is a local and hence global maximizer of $\mathrm{CE}(\theta)$ for $p_1 < p \leq p_2$.

(ii)(c) Suppose $p_2 < p \leq p_3$. We show that $\theta(p) = \theta_b(p)$. First, note that

$$\theta_b(p) = \frac{\mu_{\beta_b}(s) - p}{\gamma v_{\beta_b}} < \frac{\mu_{\beta_b}(s) - p_2}{\gamma v_{\beta_b}} = 0 \tag{35}$$

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when $p_2 < p$ and that

$$\theta_b(p) = \frac{\mu_{\beta_b}(s) - p}{\gamma v_{\beta_b}} \geq \frac{\mu_{\beta_b}(s) - p_3}{\gamma v_{\beta_b}} = \hat{\theta}_b \qquad (36)$$

when $p \leq p_3$.

From (25), $\mathrm{CE}(\theta) = \mathrm{CE}^S(\theta, \beta_b)$ when $\hat{\theta}_b < \theta \leq 0$. Thus, given concavity of $\mathrm{CE}(\theta)$, $\theta_b(p)$ is a local and hence global maximizer of $\mathrm{CE}(\theta)$ when $p_2 < p \leq p_3$.

(ii)(d) Suppose $p_3 < p \leq p_4$. We show that $\theta(p) = \theta_0(p)$. First, note that since $\beta_a > 0$,

$$\theta_0(p) = \frac{\bar{d} - p}{\gamma \sigma_d^2} < \frac{\bar{d} - p_3}{\gamma \sigma_d^2} < \frac{\bar{d} - p_2}{\gamma \sigma_d^2} \leq 0. \qquad (37)$$

Also, $p_3 = \mu_{\beta_b} - \gamma v_{\beta_b} \hat{\theta}_b = \bar{d} - \gamma \sigma_d^2 \hat{\theta}_b$ and $p_4 = \mu_{\beta_a} - \gamma v_{\beta_a} \hat{\theta}_a = \bar{d} - \gamma \sigma_d^2 \hat{\theta}_a$. Hence,

$$\hat{\theta}_a \leq \theta_0(p) < \hat{\theta}_b \qquad (38)$$

when $p_3 < p \leq p_4$.

From (25), $\mathrm{CE}(\theta) = \mathrm{CE}^S(\theta, 0) - \frac{s^2}{2\gamma}$ when $\hat{\theta}_a < \theta \leq \hat{\theta}_b < 0$. Thus, since $\mathrm{CE}(\theta)$ is concave, $\theta_0(p)$ is a local and hence global maximizer of $\mathrm{CE}(\theta)$ for $p_3 < p \leq p_4$.

(ii)(e) Suppose $p_4 < p$. We show that $\theta(p) = \theta_a(p)$. First, note that

$$\theta_a(p) = \frac{\mu_{\beta_a}(s) - p}{\gamma v_{\beta_a}} < \frac{\mu_{\beta_a}(s) - p_4}{\gamma v_{\beta_a}} \leq \hat{\theta}_a = 0. \qquad (39)$$

Moreover, for any $\theta < \hat{\theta}_a$, $\mathrm{CE}(\theta) = \mathrm{CE}^S(\theta, \beta_a)$ from (25). Thus, since $\mathrm{CE}(\theta)$ is concave, $\theta_a(p)$ is the local and hence global maximizer of $\mathrm{CE}(\theta)$ for all $p > p_4$.

Using the above, we get

$$\theta(p) = \begin{cases} \theta_a(p) & \text{if } p \leq p_1 \\ 0 & \text{if } p_1 < p \leq p_2 \\ \theta_b(p) & \text{if } p_2 < p \leq p_3 \\ \theta_0(p) & \text{if } p_3 < p \leq p_4 \\ \theta_a(p) & \text{if } p_4 < p. \end{cases} \qquad (40)$$

as desired.

(iii) When $s < 0 \Leftrightarrow \hat{\theta}_a > \hat{\theta}_b > 0$, then it follows from the proof of Proposition 1 that $CE(\theta)$ is given by (28). Moreover, $p_1 = \mu_{\beta_a}(s) - \gamma v_{\beta_a}\hat{\theta}_a$, $p_2 = \mu_{\beta_b}(s) - \gamma v_{\beta_b}\hat{\theta}_b$, $p_3 = \mu_{\beta_b}(s)$, and (d) $p_4 = \mu_{\beta_a}(s)$. Thus, using similar reasoning as above, we get

$$\theta(p) = \begin{cases} \theta_a(p) & \text{if } p \le p_1 \\ \theta_0(p) & \text{if } p_1 < p \le p_2 \\ \theta_b(p) & \text{if } p_2 < p \le p_3 \\ 0 & \text{if } p_3 < p \le p_4 \\ \theta_a(p) & \text{if } p_4 < p. \end{cases} \tag{41}$$

as desired.

Combining the three cases above provides the desired expression for $\theta(p)$. $\qquad\square$

**Proof of Theorem 1.** Consider three cases: (i) $\lambda = 0$, (ii) $\lambda > 0$, and (iii) $\lambda < 0$.

(i) Suppose $\lambda = 0$. Then $s_1 = s_2 = s_3 = s_4 = 0$ and from theorem 3, $\theta(p) = \theta_{\beta_a}(p)$ if $p \le p_1 \Leftrightarrow s \ge 0$ and if $p > p_4 \Leftrightarrow s < 0$.

(ii) $\lambda > 0$. Then $s_1 = -\frac{\beta_a}{\sigma_d^2}\lambda > s_2 = -\frac{\beta_b}{\sigma_d^2}\lambda > s_3 = -\frac{1}{\beta_b}\lambda > s_4 = -\frac{1}{\beta_a}\lambda$.

Then from theorem 3 the following holds.

$$\theta(p) = \begin{cases} \theta_{\beta_a}(p) & \text{if } p \le p_1 \Leftrightarrow s \ge s_1 \\ \theta_0 & \text{if } p_1 < p \le p_2 \Leftrightarrow s_1 > s \ge s_2 \\ \theta_{\beta_b}(p) & \text{if } p_2 < p \le p_3 \Leftrightarrow s_2 > s \ge s_3 \\ 0 & \text{if } p_3 < p \le p_4 \Leftrightarrow s_4 \le s < s_3 \\ \theta_{\beta_a}(p) & \text{if } p > p_4 \Leftrightarrow s < s_4. \end{cases} \tag{42}$$

(iii) $\lambda < 0$. Then $s_1 = -\frac{1}{\beta_a}\lambda > s_2 = -\frac{1}{\beta_b}\lambda > s_3 = -\frac{\beta_b}{\sigma_d^2}\lambda > s_4 = -\frac{\beta_a}{\sigma_d^2}\lambda$.

Then from theorem 3 the following holds.

$$\theta(p) = \begin{cases} \theta_{\beta_a}(p) & \text{if } p \le p_1 \Leftrightarrow s \ge s_1 \\ 0 & \text{if } p_1 < p \le p_2 \Leftrightarrow s_1 > s \ge s_2 \\ \theta_{\beta_b}(p) & \text{if } p_2 < p \le p_3 \Leftrightarrow s_2 > s \ge s_3 \\ \theta_0 & \text{if } p_3 < p \le p_4 \Leftrightarrow s_4 \le s < s_3 \\ \theta_{\beta_a}(p) & \text{if } p > p_4 \Leftrightarrow s < s_4. \end{cases} \tag{43}$$

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Combining the above cases provides the desired expression.

□

**Proof of Proposition 3.** From the proofs of Proposition 1 and Theorem 3, it follows that investors are short in the stock, but demand does not react to changes in the signal when

$$\theta(p) = \theta_0(p) \leq 0, \tag{44}$$

i.e. when $p_3 = \mu_{\beta_b}(s) - \gamma v_{\beta_b}\hat{\theta}_b < p \leq p_4 = \mu_{\beta_a}(s) - \gamma v_{\beta_a}\hat{\theta}_a$, which corresponds to

$$\mathrm{CE}(\theta_0(p)) = \mathrm{CE}^S(\theta_0(p), 0) - \frac{s^2}{2\gamma}, \tag{45}$$

i.e. when $\hat{\theta}_a \leq \theta_0(p) \leq \hat{\theta}_b < 0$. Using the expressions for $p_3$ and $p_4$ (or those for $\hat{\theta}_a$, $\hat{\theta}_b$, and $\theta_0(p)$) shows that the signal value $s$ must satisfy

$$-(\bar{d} - p)\frac{\beta_a}{\sigma_d^2} \leq s \leq -(\bar{d} - p)\frac{\beta_b}{\sigma_d^2}. \tag{46}$$

Thus, the size of the signal region is

$$\frac{\beta_a - \beta_b}{\sigma_d^2}(\bar{d} - p) \tag{47}$$

where $\bar{d} - p \leq 0$ since $\theta_0(p) \leq 0$.

Similarly, the size of the signal region when investors are long in the stock, but demand does not react to changes in the signal is

$$\frac{\beta_b - \beta_a}{\sigma_d^2}(\bar{d} - p) \tag{48}$$

where $\bar{d} - p \geq 0$ since $\theta_0(p) \geq 0$.

Combining the results of the two cases yields the desired expression for the size of the signal region.

Since $s \sim \mathrm{N}(0, 1)$, using the bounds for the signal region identified above and the properties of the standard normal distribution provides the desired expression for the probability of investors exhibiting information inertia for either a long or a short position in the risky asset.

From the proofs of Proposition 1 and Theorem 3, it follows that investors are

neither long nor short in the stock and demand does not react to changes in the signal when $\theta(p) = 0$, i.e. when $p_1 = \mu_{\beta_a}(s) < p \le p_2 = \mu_{\beta_b}(s)$ (corresponding to good news $s > 0$) or $p_3 = \mu_{\beta_b}(s) < p \le p_4 = \mu_{\beta_a}(s)$ (corresponding to bad news $s < 0$). Using the expressions for $p_1$ and $p_2$ shows that the signal value $s$ must satisfy

$$-(\bar{d} - p)\frac{1}{\beta_b} \le s < -(\bar{d} - p)\frac{1}{\beta_a}. \tag{49}$$

Thus, the size of the signal region when investors are neither long nor short in the stock and demand does not react to changes in the signal is

$$\left(\frac{1}{\beta_a} - \frac{1}{\beta_b}\right)(\bar{d} - p). \tag{50}$$

Similarly, using the expressions for $p_3$ and $p_4$ shows that signal region is

$$\left(\frac{1}{\beta_a} - \frac{1}{\beta_b}\right)(p - \bar{d}). \tag{51}$$

Combining the results of the two cases provides the desired expression for the size of the signal region.

Since $s \sim \mathrm{N}(0, 1)$, using the bounds for the signal region identified above and the properties of the standard normal distribution provides the desired expression for the probability of the investors exhibiting information inertia when holding the risk-free portfolio. $\qquad \square$

***Proof of Theorem 2.*** Market clearing requires that $\theta(p) = 1$ since there is one unit of the risky asset in aggregate.

Consider three cases: (i) $s > -\gamma\beta_a$, (ii) $-\gamma\beta_b \le s \le -\gamma\beta_a$, and (iii) $s < -\gamma\beta_b$.

(i) Suppose $s > -\gamma\beta_a$. Then $\hat{\theta}_a < 1$. We need to verify that markets clear when $p(s) = \mu_{\beta_a}(s) - \gamma v_{\beta_a}$. From Theorem 3, it follows that

$$\theta(p(s)) = \theta_a(p(s)) = \frac{\mu_{\beta_a}(s) - p(s)}{\gamma v_{\beta_a}} = 1 \tag{52}$$

if and only if

$$p(s) = \mu_{\beta_a}(s) - \gamma v_{\beta_a} \le p_1 = \mu_{\beta_a}(s) - \gamma v_{\beta_a} \max\left\{\hat{\theta}_a, 0\right\} \tag{53}$$

or

$$p(s) = \mu_{\beta_a}(s) - \gamma v_{\beta_a} > p_4 = \mu_{\beta_a}(s) - \gamma v_{\beta_a} \min \left\{ \hat{\theta}_a, 0 \right\}. \tag{54}$$

Since $\hat{\theta}_a < 1$, $p(s) \le p_1$ and the result follows.

(ii) Suppose $-\gamma \beta_b \le s \le -\gamma \beta_a$. Then $\hat{\theta}_b \le 1 \le \hat{\theta}_a$. We need to verify that markets clear when $p(s) = \bar{d} - \gamma \sigma_d^2$. From Theorem 3, it follows that

$$\theta(p(s)) = \theta_0(p(s)) = \frac{\bar{d} - p(s)}{\gamma \sigma_d^2} = 1 \tag{55}$$

if and only if

$$p(s) = \bar{d} - \gamma \sigma_d^2 > p_1 = \mu_{\beta_a}(s) - \gamma v_{\beta_a} \max \left\{ \hat{\theta}_a, 0 \right\} \tag{56}$$

and

$$p(s) = \bar{d} - \gamma \sigma_d^2 \le p_2 = \mu_{\beta_b}(s) - \gamma v_{\beta_b} \max \left\{ \hat{\theta}_b, 0 \right\}. \tag{57}$$

Since $\hat{\theta}_a \ge 1$ and $\mu_{\beta_a}(s) - \gamma v_{\beta_a} \hat{\theta}_a = \bar{d} - \gamma \sigma_d^2 \hat{\theta}_a$, we have $p(s) > p_1$.

If $\hat{\theta}_b \le 0$, then $s \ge 0$. So, $p(s) = \bar{d} - \gamma \sigma_d^2 \le \bar{d} + \beta_b s = \mu_{\beta_b}(s) = p_2$. If $0 < \hat{\theta}_b$, then since $\hat{\theta}_b \le 1$ we $p(s) = \bar{d} - \gamma \sigma_d^2 \le \bar{d} - \gamma \sigma_d^2 \hat{\theta}_b = \mu_{\beta_b}(s) - \gamma v_{\beta_b} \hat{\theta}_b = p_2$. So, $p_1 < p(s) \le p_2$

(iii) Suppose $s < -\gamma \beta_b$. Then $\hat{\theta}_b > 1$. We need to verify that markets clear when $p(s) = \mu_{\beta_b}(s) - \gamma v_{\beta_b}$. From Theorem 3, it follows that

$$\theta(p(s)) = \theta_b(p(s)) = \frac{\mu_{\beta_b}(s) - p(s)}{\gamma v_{\beta_b}} = 1 \tag{58}$$

if and only if

$$p(s) = \mu_{\beta_b}(s) - \gamma v_{\beta_b} > p_2 = \mu_{\beta_b}(s) - \gamma v_{\beta_b} \max \left\{ \hat{\theta}_b, 0 \right\} \tag{59}$$

and

$$p(s) = \mu_{\beta_b}(s) - \gamma v_{\beta_b} \le p_3 = \mu_{\beta_b}(s) - \gamma v_{\beta_b} \min \left\{ \hat{\theta}_b, 0 \right\}. \tag{60}$$

Since $\hat{\theta}_b > 1$, $p_2 < p(s) \le p_3$ and the result follows.

Combining the above cases provides the desired result. □

**_Proof of Proposition 5_**. We know from Theorem 3 that demand for investor $h$ is

$$
\theta_h(p) = \begin{cases}
\frac{\mu_{ah}-p}{\gamma_h v_{ah}} & p \;\leq p_{1h} \equiv \mu_{ah} - \gamma_h v_{ah} \max\left(\hat{\theta}_{ah}, 0\right) \\[2mm]
\max\left(\frac{\bar{d}-p}{\gamma_h \sigma_d^2}, 0\right) & p_{1h} < \;\; p \;\leq p_{2h} \equiv \mu_{bh} - \gamma_h v_{bh} \max\left(\hat{\theta}_{bh}, 0\right) \\[2mm]
\frac{\mu_{bh}-p}{\gamma_h v_{bh}} & p_{2h} < \;\; p \;\leq p_{3h} \equiv \mu_{bh} - \gamma_h v_{bh} \min\left(\hat{\theta}_{bh}, 0\right) \\[2mm]
\min\left(\frac{\bar{d}-p}{\gamma_h \sigma_d^2}, 0\right) & p_{3h} < \;\; p \;\leq p_{4h} \equiv \mu_{ah} - \gamma_h v_{ah} \min\left(\hat{\theta}_{bh}, 0\right) \\[2mm]
\frac{\mu_{ah}-p}{\gamma_h v_{ah}} & p \;\; > p_{4h},
\end{cases}
$$

(61)

where $\hat{\theta}_{ah} \equiv -s/(\gamma_h \beta_{ah})$ and $\hat{\theta}_{bh} \equiv -s/(\gamma_h \beta_{bh})$.

We first show that there exists an equilibrium. Individual demand given in equation (61) is continuous and non-increasing in $p$ with $\lim\limits_{p\to-\infty} \theta_h(p) = \infty$ and $\lim\limits_{p\to\infty} \theta_h(p) = -\infty$ for all $h \in \{1,\ldots,H\}$. Hence, aggregate demand $\theta(p) = \sum_{h=1}^{H} \theta_h(p)$ is continuous and non-increasing in $p$ with $\lim\limits_{p\to-\infty} \theta(p) = \infty$ and $\lim\limits_{p\to\infty} \theta(p) = -\infty$. Hence, there exists an equilibrium because the market clearing condition $\theta(p) - 1 = 0$ has always a solution.

We next determine the equilibrium stock price $p(s)$ for all $s \in [\hat{s}_b, \hat{s}_a]$. By assumption we have that $\beta_a = \max\{\beta_{a1},\ldots,\beta_{aH}\}$ and $\beta_b = \min\{\beta_{b1},\ldots,\beta_{bH}\}$. Hence,

$$
p_1 \equiv \max_{h\in\{1,\ldots,H\}} p_{1h} = \max_{h\in\{1,\ldots,H\}} \left\{\bar{d} + \frac{\sigma_d^2}{\beta_{ah}}s\right\} = \bar{d} + \frac{\sigma_d^2}{\beta_a}s
\tag{62}
$$

$$
p_2 \equiv \min_{h\in\{1,\ldots,H\}} p_{2h} = \min_{h\in\{1,\ldots,H\}} \left\{\bar{d} + \frac{\sigma_d^2}{\beta_{bh}}s\right\} = \bar{d} + \frac{\sigma_d^2}{\beta_b}s.
\tag{63}
$$

We have that $\beta_b \geq \beta_a$ and thus (i) $[\hat{s}_b, \hat{s}_a] \neq \emptyset$ and (ii) $p_2(s) \geq p_1(s)$ for all $s \in [\hat{s}_b, \hat{s}_a]$.

It follows from equations (61)-(63) that

$$
\theta_h(p) = \frac{\bar{d} - p}{\gamma_h \sigma_d^2} \qquad \forall\, p_1 \leq p \leq p_2, \quad \text{and} \quad \forall h \in \{1,\ldots,H\}.
\tag{64}
$$

Summing over all investors leads to

$$
\theta(p) = \sum_{h=1}^{H} \theta_h(p) = \frac{\bar{d} - p}{\sigma_d^2} \sum_{h=1}^{H} \frac{1}{\gamma_h} = \frac{\bar{d} - p}{\gamma \sigma_d^2} \qquad \forall\, p_1 \leq p \leq p_2.
$$

Imposing the market clearing condition $\theta(p){=}1$ leads to the price $p = \bar{d} - \gamma\sigma_d^2$. $\qquad\square$