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JEL classification: D61, D62, G01, G21, G28.

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1 Introduction

In the highly receptive world of today's financial markets populated with central banks' watchers, economic analysts, and various economic commentators, disclosure policies assume great importance. On any given day, many institutions with high public visibility such as government agencies, central banks, international organizations and rating agencies release news potentially affecting the allocative efficiency in the economy.

In this paper, we model the effect of public information on investment decisions and study whether the disclosure of more precise public information is socially beneficial. We show that public information can trigger systemic liquidity shortages, and hence be the source of allocative inefficiency, in an economy with uninsurable aggregate shocks to firms' production and limited commitment. We also show that increasing the quality of public information, not only may reduce social welfare, but also has redistributive effects in the economy. More precise public information makes entrepreneurs (weakly) better off and financiers (weakly) worse off.

We develop a general equilibrium model with three periods. At date 0 a continuum of wealthless risk-neutral entrepreneurs have access only to a risky investment technology with partially verifiable returns. Wealthy, risk-neutral financiers decide how much capital to provide to entrepreneurs and how much to invest in an alternative technology with non-verifiable payoffs. At date 1, entrepreneurs' technology is hit by an aggregate liquidity shock: with positive probability, *all* entrepreneurs need to raise new capital to meet their reinvestment needs. Only if they secure new funds, they produce a return at date 2. Their ability to raise new funds depends on the aggregate resources available at date 1, which in turn depend on the funds originally invested at date 0: more entrepreneurs' investment at date 0 leaves fewer resources for refinancing at date 1.

Compared with the constrained efficient solution (i.e. social planner economy with the same constraints as the private economy), the competitive equilibrium exhibits over-investment, as atomistic entrepreneurs do not internalize through prices the impact of their collective investment decisions on the equilibrium risk of liquidity shortage. The investment decisions of some entrepreneurs impose a negative externality on others because of capital rationing: their investments reduce the resources available for future refinancing and thus increase the equilibrium probability of liquidity shortage for all entrepreneurs.

The sources of inefficient investment are financial market and contract incompleteness. In particular, there are no *tradeable* technologies with payoffs independent of liquidity shocks, and both entrepreneurs and financiers have limited ability to commit to future payments. As such, entrepreneurs are unable to insure against liquidity shocks either via markets or contracts. If entrepreneurs could insure against liquidity shocks – for instance, by trading claims on financiers' technologies or purchasing lines of credit – they would fully internalize the risk of liquidity shortage via the insurance cost. However, market incompleteness and the non-verifiability of financiers' payoffs rule out such insurance possibilities. Similarly, if entrepreneurs' investment returns were fully contractible, then financiers would fully price the risk of liquidity shortage in the cost of financing, regardless of the existence of insurance possibilities.

The paper delivers three main contributions. First, we investigate the impact of public information on the competitive equilibrium. We show that investment inefficiency and liquidity shortages arise only in the presence of an informative, but imperfect, public signal. Furthermore, a more informative public signal can be welfare reducing because it exacerbates the negative externality in entrepreneurs' investment decisions. The non-monotonicity in the relationship between the informativeness of the public signal and social welfare arises because entrepreneurs' costly effort (in excess of their non-verifiable investment return) generates a trade-off in the investment demand, and makes aggregate investment strictly increasing in the quality of public information. As such, an improvement in the quality of public information, leads to more investment, which feedbacks into more capital rationing and lower welfare.

Second, we show that a change in the quality of public information has redistributive effects in the economy. Entrepreneurs always prefer high information quality as they benefit from the improved expected returns on investment. Conversely, financiers are better off with low information quality as they extract rents from liquidity shortages. As such, an increase in the quality of public information is not a Pareto improvement.

Third, we show that the constrained efficient allocation, which would be chosen by a social planner who can coordinate entrepreneurs' actions given their available information, can be achieved as a competitive market equilibrium outcome via investment restrictions and targeted disclosure of information. Investment restrictions on financiers prevent excessive risk taking and systemic liquidity shortages by optimally restricting aggregate investment in entrepreneurs' technology. With targeted disclosure of information, only informed entrepreneurs would invest, thus limiting welfare-reducing liquidity shortages. For those who access the information, a high informativeness about underlying fundamentals enhances efficiency of private decisions. Investment restrictions and the degree of disclosure should optimally vary with the informativeness of the public signal. No investment restrictions and full disclosure of public information are optimal only when the quality of the public signal is sufficiently high.

Our paper belongs to a large literature analyzing pecuniary externalities in incomplete markets. Most of this literature builds on Geanakoplos and Polemarchakis (1985)'s result that competitive equilibria may be constrained inefficient when markets are incomplete. One strand of this literature focuses on the fire-sale externalities induced typically by a collateral constraint (Kiyotaki and Moore, 1997; Gromb and Vayanos, 2002; Krishnamurthy, 2003; Lorenzoni, 2008; Acharya and Vishwanathan, 2010; and Jeanne and Korinek, 2010). Another strand of the literature (Shleifer and Vishny, 1992; Allen and Gale, 1994; Caballero and Krishnamurthy, 2001; Acharya and Yorulmazer, 2008; and He and Kondor, 2012) focuses on uninsurable shocks either to preferences (as in Diamond and Dybvig, 1983) or to firms' production technologies (as in Holmstrom and Tirole, 1998). Our model builds on Holmstrom and Tirole (1998) and investigates the importance of public information as a source of investment inefficiency and liquidity crises from positive and normative viewpoints.

This paper also contributes to the literature on the social value of information dating back to Hirshleifer (1971), who shows how disclosure of public information may preempt socially valuable risk-sharing opportunities. More recently, Morris and Shin (2002), Angeletos and Pavan (2004, 2007), Cornand and Heinemann (2008) examine the impact of public information when agents' payoffs exhibit exogenous externalities like in Keynesian beauty contests. Adding to this literature, we study the effect of public information in an economy with endogenous externalities.

The structure of the paper is as follows. In Section 2, we present the model, derive the main results, and discuss the sources of inefficiency. Section 3 considers the normative implications. Section 4 concludes. All proofs are in Appendix A.

2 The Model

We consider an economy with three periods, $t \in \{0, 1, 2\}$, a continuum of entrepreneurs indexed by $i \in [0, 1]$, and a continuum of financiers indexed by $j \in [0, 1]$. There is one (perishable) good used for both consumption and investment. All agents are risk-neutral and derive utility from consumption at date 1 and 2: $U = c_1 + c_2$.

Entrepreneurs have no initial capital endowment and they have access to a decreasing-returns-to-scale investment opportunity. Any capital investment $I \in [0, W]$ at date 0 produces a verifiable return, $R > 1$, and a non-verifiable return, or private benefit, $b > 0$, at date 2, per unit of capital. Any capital investment above W returns zero. The project also requires an initial effort cost per unit of invested capital $k > 0$ (in units of consumption). At date 1, all projects are subject to an aggregate liquidity shock: each entrepreneur must invest an additional amount $\widetilde{\lambda} \geq 0$ per unit of capital to realize the investment return at date 2, otherwise the project terminates and yields nothing. For simplicity, the liquidity shock takes only the value of 0 (no liquidity shock) or 1 (liquidity needs equal initial investment) with equal probabilities.

Financiers have an initial endowment of capital $W > 0$. They can use it either to finance entrepreneurs or to invest in their own technology, which produces a non-verifiable return $\widetilde{r}_j$ at date 1, taking the value of $1/\pi$, with probability $\pi \in (0, 1)$, or the value of 0 otherwise. The expected return on financiers' technology is thus 1. The returns on financiers' technology are independent and identically distributed across financiers, and independent from the aggregate liquidity shock, $\widetilde{\lambda}$.

The non-verifiability of the returns b and $\widetilde{r}_j$ is the source of limited commitment in the model, which leads to inefficient investment, as discussed in Section 2.2.3.

The timeline of events is described in Figure 1. At date 0, all agents observe the public signal $\theta \in \{L, H\}$ about the size of the liquidity shock and make investment and financing decisions. The signal is distributed as follows:

$$\Pr\left(\theta = H|\widetilde{\lambda} = 0\right) = \Pr\left(\theta = L|\widetilde{\lambda} = 1\right) = \sigma,$$

where $\sigma \in [1/2, 1]$ measures its informativeness, with $\sigma = 1/2$ being perfectly uninformative and $\sigma = 1$ being perfectly informative. Using Bayes' rule, entrepreneurs and financiers compute the conditional probabilities of the liquidity shock as

$$\mu_\theta \equiv \Pr\left(\widetilde{\lambda} = 0|\theta\right) = \begin{cases} \sigma & \text{if } \theta = H \\ 1-\sigma & \text{if } \theta = L \end{cases}$$

where $\Pr(\theta = H) = \Pr(\theta = L) = 1/2$. In order to finance their investment, entrepreneurs raise capital against the future risky investment proceeds at date 2. Each entrepreneur applies for funding in exchange for a fraction $\alpha_0 \in [0, 1]$ of the verifiable investment proceeds. Financiers choose whether to provide funding or to invest in their own technology.

At date 1, the liquidity shock $\widetilde{\lambda} \in \{0, 1\}$ hits the economy. When $\widetilde{\lambda} = 0$, there is no need of additional funds. Entrepreneurs continue the projects and their final payoffs are realized at date 2. When $\widetilde{\lambda} = 1$, entrepreneurs need an additional unit of capital per each unit of invested capital, which they can raise by selling a fraction $\alpha_1 \in [0, 1]$ of the project to new financiers. If the aggregate resources available are not sufficient to meet the aggregate reinvestment needs, i.e. there is a liquidity shortage, entrepreneurs are rationed. Entrepreneurs' projects continue and their final payoffs at date 2 are rescaled according to the amount of funds secured.

At date 2, financiers consume their profits and entrepreneurs consume their investment proceeds net of financiers' repayments.

We make three assumptions for the following analysis:

Assumption 1: $R+b \geq 1+k$. We require the project's net present value in the absence of a liquidity shock $(\widetilde{\lambda} = 0)$ to be positive. This assumption ensures the existence of a positive entrepreneurs' demand of investment.

Assumption 2: $k \geq b$. This is a sufficient condition to ensure the existence of a trade-off in entrepreneurs' demand of investment. If entrepreneurs' non-verifiable return b exceeds in expectation the effort cost k, entrepreneurs would always demand capital for investment regardless of the investment quality.

Assumption 3: $R \leq 3/2$. We assume that the project's expected pledgeable return is sufficiently low that there is no investment in the absence of public information since there is no supply of capital. This also implies no investment in the presence of public information with $\theta = L$. This restriction emphasizes the role of public information as a trigger for over-investment and simplifies the exposition of the analysis. Our main finding of information induced investment inefficiency extends also to the case with $R > 3/2$, which we present for completeness only in Appendix B.

2.1 Social Planner Allocation

In this section, we characterize the constrained first-best allocation useful to benchmark the competitive market equilibrium. In such a case, the social planner would choose the aggregate investment I to solve the following problem:

$$\max_{I \in [0,W]} W + \left[\mu_\theta \left(R+b\right) + \left(1-\mu_\theta\right)\rho\left(R+b-1\right) - \left(1+k\right)\right] I \tag{1}$$
$$\text{s.t. } \rho = \min\left\{\frac{W-I}{I}, 1\right\}.$$

The first term is financiers' endowment W, while the second term in brackets denotes the expected net present value (NPV) from entrepreneurs' investment inclusive of effort costs. The expected NPV from entrepreneurs' investment takes into account the proportion of investment securing new financing in the event of a liquidity shock. Such proportion, ρ, depends on the occurrence of capital rationing. Given that there is a continuum of financiers with mass 1 and i.i.d. $\widetilde{r}_j$ (with $E(\widetilde{r}_j) = 1$), by the law of large numbers, the aggregate supply of liquidity is $W - I$. Hence, the proportion of entrepreneurs' investment securing new financing at date 1 is $\rho = \min\left\{\frac{W-I}{I}, 1\right\}$.

The next proposition summarizes the social planner allocations and corresponding welfare.

Proposition 1 (Social Planner Allocation). *Let* $\widehat{\sigma} \equiv 2 + k - R - b$ and $\widehat{\widehat{\sigma}} \equiv \frac{R+b+k}{2(R+b)-1}$. *The social planner's investment decision is:*

$$I^{CFB} = \begin{cases} 0 & \text{if } \theta = L \text{ or } (\theta = H \text{ and } \sigma \in [1/2, \widehat{\sigma})) \\ \left[0, \frac{W}{2}\right] & \text{if } \theta = H \text{ and } \sigma = \widehat{\sigma} \\ \frac{W}{2} & \text{if } \theta = H \text{ and } \sigma \in \left(\widehat{\sigma}, \widehat{\widehat{\sigma}}\right) \\ \left[\frac{W}{2}, W\right] & \text{if } \theta = H \text{ and } \sigma = \widehat{\widehat{\sigma}} \\ W & \text{if } \theta = H \text{ and } \sigma \in \left(\widehat{\widehat{\sigma}}, 1\right] \end{cases}$$

with corresponding expected welfare:

$$E\left[U^W_{CFB}\right] = \begin{cases} W & \text{if } \sigma \in [1/2, \widehat{\sigma}] \\ W + \frac{W}{4}\left[R + b - (1-\sigma) - (1+k)\right] & \text{if } \sigma \in \left(\widehat{\sigma}, \widehat{\widehat{\sigma}}\right] \\ W + \frac{W}{2}\left[\sigma(R+b) - (1+k)\right] & \text{if } \sigma \in \left(\widehat{\widehat{\sigma}}, 1\right] \end{cases}.$$

When $\theta = L$, or $\theta = H$ and its informativeness is low, i.e. $\sigma \in [1/2, \widehat{\sigma}]$, there is no entrepreneurs' investment as the expected NPV inclusive of effort costs is negative. When $\theta = H$ and $\sigma \in \left(\widehat{\sigma}, \widehat{\widehat{\sigma}}\right]$, undertaking all available projects is suboptimal because it would lead to capital rationing if a liquidity shock hits the economy. Hence, the optimal constrained first-best investment is the highest possible amount that prevents capital rationing from happening, $I^{CFB} = W/2$. Only when the probability of a liquidity shock is sufficiently low, i.e. $\sigma \in \left(\widehat{\widehat{\sigma}}, 1\right]$, undertaking all available projects is optimal in the constrained efficient equilibrium ($I^{CFB} = W$) even if it may lead to capital rationing. Social welfare under the constrained efficient equilibrium is unaffected by σ when there is no investment, i.e. $\sigma \in [1/2, \widehat{\sigma}]$; and it is strictly increasing in σ otherwise.

2.2 Competitive Equilibrium

In this section, we characterize the competitive equilibrium by backward induction. We start from the equilibrium in the market for liquidity at date 1, and then proceed backwards to the equilibrium in the market for funding at date 0.

2.2.1 Market for Liquidity

When $\widetilde{\lambda} = 0$, there is no market for liquidity because there is no demand for new financing. When $\widetilde{\lambda} = 1$, all entrepreneurs need new funds. Therefore, the aggregate demand of liquidity is:

$$L^D = \begin{cases} 0 & \text{if } \widetilde{\lambda} = 0 \\ I^* & \text{if } \widetilde{\lambda} = 1 \end{cases},$$

where I^* is the equilibrium aggregate investment by entrepreneurs. Let α_1 be the share of the project's verifiable return R given to new financiers in exchange for additional financing of a unit of capital. At date 0, each financier j invested I_j^* in entrepreneurs' technology and $W - I_j^*$ in his own technology. Those financiers who realized a positive return on their own technology ($\widetilde{r}_j = 1/\pi$), choose whether to provide new funds $L_j^S \in \left[0, \left(W - I_j^*\right)/\pi\right]$ in exchange for a fraction α_1 of the project's verifiable return R so as to maximize their utility:

$$\max_{L_j^S \in \left[0, \left(W - I_j^*\right)/\pi\right]} L_j^S \alpha_1 R + \left(W - I_j^*\right)/\pi - L_j^S$$

subject to entrepreneurs' solvency constraint $\alpha_1 \leq 1$. Given that there is a continuum of financiers with mass 1 and i.i.d. $\widetilde{r}_j$, by the law of large numbers, the aggregate supply of liquidity is

$$L^S = \begin{cases} W - I^* & \text{if } \alpha_1 > 1/R \\ [0, W - I^*] & \text{if } \alpha_1 = 1/R \\ 0 & \text{if } \alpha_1 < 1/R \end{cases}$$

with $\alpha_1 \leq 1$. In equilibrium, α_1 is set to equate demand and supply of liquidity via an inter-financier market where competitive financiers can transfer liquidity among themselves.

If $L^D < W - I^*$, then $L^S = L^D$ and $\alpha_1^* = 1/R$ since there is excess supply of liquidity and competition among financiers drives the return on new financing down to their opportunity cost of 1. If instead $L^D > W - I^*$, then $L^S = L^D$ cannot be an equilibrium as financiers' resources are insufficient to meet the aggregate demand of liquidity. Therefore, there is capital rationing and $L^S = W - I^* < L^D$. Entrepreneurs compete for new funds thus transferring all the surplus to financiers, who can now provide new financing at the maximum possible rate satisfying entrepreneurs' solvency constraint, $\alpha_1^* = 1$. If $L^D = W - I^*$, the aggregate demand equals the aggregate supply of liquidity. In such case, $L^S = L^D$and the cost of liquidity is indeterminate, $\alpha_1^* \in [1/R, 1]$.

Therefore, when $\widetilde{\lambda} = 1$, the equilibrium in the market for liquidity is

$$\{L^*, \alpha_1^*\} = \begin{cases} \{I^*, \frac{1}{R}\} & \text{if } I^* < \frac{W}{2} \\ \{\frac{W}{2}, [\frac{1}{R}, 1]\} & \text{if } I^* = \frac{W}{2} \\ \{W - I^*, 1\} & \text{if } I^* > \frac{W}{2} \end{cases} . \tag{2}$$

When financiers net worth falls short of the liquidity needs, entrepreneurs will be capital rationed. The proportion of new financing, conditional on the liquidity shock, for each entrepreneur is:

$$\rho^* = \begin{cases} 1 & \text{if } I^* \leq \frac{W}{2} \\ \frac{W - I^*}{I^*} & \text{if } I^* > \frac{W}{2} \end{cases} . \tag{3}$$

2.2.2 Market for Funding

We can now proceed backwards to date 0 when the market for funding opens.

Demand of Funding. Competitive entrepreneurs take the cost of financing α_0 and the aggregate investment I^* as given. They are rational and can perfectly foresee the continuation of the game: they know the equilibrium α_1^* as in (2), and the proportion of new financing ρ^* as in (3).

Entrepreneurs' optimal investment policy is the solution to the following maximization problem:

$$\max_{I^D \in [0,W]} I^D \left\{ (1-\alpha_0) \left[\mu_\theta + (1-\mu_\theta) \rho^* (1-\alpha_1^*) \right] R + \left[\mu_\theta + (1-\mu_\theta) \rho^* \right] b - k \right\}. \tag{4}$$

Each entrepreneur retains a fraction $1 - \alpha_0$ of the project's expected verifiable value $[\mu_\theta + (1-\mu_\theta)\rho^*(1-\alpha_1^*)] R$, and consumes the project's non-verifiable value or private benefit b with probability $\mu_\theta + (1-\mu_\theta)\rho^*$. Regardless of the liquidity shock, entrepreneurs incur an effort cost k.

The aggregate demand of funds can then be characterized as:

$$I^D = \begin{cases} W & \text{if } \alpha_0 < 1 - \frac{k - [\mu_\theta + (1-\mu_\theta)\rho^*] b}{\left[\mu_\theta + (1-\mu_\theta)\rho^* \left(1-\alpha_1^*\right)\right] R} \\ [0, W] & \text{if } \alpha_0 = 1 - \frac{k - [\mu_\theta + (1-\mu_\theta)\rho^*] b}{\left[\mu_\theta + (1-\mu_\theta)\rho^* \left(1-\alpha_1^*\right)\right] R} \\ 0 & \text{if } \alpha_0 > 1 - \frac{k - [\mu_\theta + (1-\mu_\theta)\rho^*] b}{\left[\mu_\theta + (1-\mu_\theta)\rho^* \left(1-\alpha_1^*\right)\right] R} \end{cases}. \tag{5}$$

When the expected marginal return from investment exceeds its expected marginal cost, it is optimal to invest as much as possible, $I^D = W$. Otherwise, it is optimal not to invest. When the cost of financing $\alpha_0 = 1 - \frac{k - [\mu_\theta + (1-\mu_\theta)\rho^*] b}{\left[\mu_\theta + (1-\mu_\theta)\rho^* \left(1-\alpha_1^*\right)\right] R}$, entrepreneurs are indifferent among any values of $I^D \in [0, W]$.

Supply of Funding. Taking the equilibrium values of α_1^* in (2) and the proportion of new financing ρ^* in (3) as given, financiers choose the supply of capital $I^S \in [0, W]$ to maximize expected utility:

$$\max_{I^S \in [0,W]} I^S \alpha_0 \left[\mu_\theta + (1-\mu_\theta) \rho^* (1-\alpha_1^*) \right] R + \left(W - I^S \right) \left[\mu_\theta + (1-\mu_\theta) \alpha_1^* R \right] \tag{6}$$

subject to entrepreneurs' solvency constraint $\alpha_0 \leq 1$, where α_0 is the share of the project's verifiable return R given to financiers in exchange for one unit of capital.

With probability μ_θ there is no liquidity shock, and financiers consume the payoff from the funds supplied to entrepreneurs, $I^S \alpha_0 R$, plus any payoff from the investment in their own idiosyncratic technology, $\left(W - I^S\right)$, with expected return of 1. With probability $(1 - \mu_\theta)$ there is a liquidity shock, and financiers consume the payoff from the funds supplied to entrepreneurs net of dilution by new financiers, $I^S \alpha_0 \rho^* (1 - \alpha_1^*) R$, and the payoff from any capital used for new financing, $\left(W - I^S\right) \alpha_1^* R$.

Hence, the supply of funds can be summarized as:

$$I^S = \begin{cases} W & \text{if } \alpha_0 > \frac{\mu_\theta + (1-\mu_\theta)\alpha_1^* R}{\left[\mu_\theta + (1-\mu_\theta)\rho^*\left(1-\alpha_1^*\right)\right] R} \\ [0, W] & \text{if } \alpha_0 = \frac{\mu_\theta + (1-\mu_\theta)\alpha_1^* R}{\left[\mu_\theta + (1-\mu_\theta)\rho^*\left(1-\alpha_1^*\right)\right] R} \\ 0 & \text{if } \alpha_0 < \frac{\mu_\theta + (1-\mu_\theta)\alpha_1^* R}{\left[\mu_\theta + (1-\mu_\theta)\rho^*\left(1-\alpha_1^*\right)\right] R} \end{cases} \tag{7}$$

with $\alpha_0 \leq 1$. When the expected marginal benefit of financing exceeds its expected marginal cost, it is optimal to supply as much capital as possible, $I^S = W$. Otherwise, it is optimal to invest the capital in financiers' own technology and potentially use the proceeds for future financing. When $\alpha_0 = \frac{\mu_\theta + (1-\mu_\theta)\alpha_1^* R}{\left[\mu_\theta + (1-\mu_\theta)\rho^*\left(1-\alpha_1^*\right)\right] R}$, financiers are indifferent among any values of $I^S \in [0, W]$ as long as entrepreneurs' solvency constraint is not binding $\left(\frac{\mu_\theta + (1-\mu_\theta)\alpha_1^* R}{\left[\mu_\theta + (1-\mu_\theta)\rho^*\left(1-\alpha_1^*\right)\right] R} \leq 1\right)$.

Equilibrium Funding. In equilibrium, given perfect competition and enough resources to finance all available projects at date 0, financiers must be indifferent ex-ante between financing entrepreneurs and investing in their own technology (including the provision of future liquidity) as long as entrepreneurs' solvency constraint is met ($\alpha_0 \leq 1$). Hence, the equilibrium cost of financing at date 0 can be characterized as:

$$\alpha_0^* = \frac{\mu_\theta + (1 - \mu_\theta)\, \alpha_1^* R}{\left[\mu_\theta + (1 - \mu_\theta)\, \rho^* \,(1 - \alpha_1^*)\right] R} \tag{8}$$

provided that $\mu_\theta + (1 - \mu_\theta)\, \alpha_1^* R \leq \left[\mu_\theta + (1 - \mu_\theta)\, \rho^* \,(1 - \alpha_1^*)\right] R$. Otherwise, there is no financing ($\alpha_0^* = \{\varnothing\}$) because entrepreneurs' solvency constraint cannot be met.

Given the cost of liquidity provision α_1^* in (2), the proportion of new financing ρ^* in (3), and the cost of financing α_0^* in (8), market clearing requires aggregate demand of funds to equal aggregate supply: $I^* = I^D = I^S$ as given in (5) and (7), respectively. The next proposition characterizes the competitive market equilibrium.

Proposition 2 (Competitive Market Equilibrium). *Let* $\widehat{\sigma} \equiv 2 + k - R - b$, $\overline{\sigma} \equiv \frac{R+k-b}{2R-1}$ *and* $\overline{\overline{\sigma}} \equiv \frac{R+k}{2R+b-1}$. *The investment in the competitive equilibrium is:*

$$I^* = \begin{cases} 0 & \text{if } \theta = L \text{ or } \left(\theta = H \text{ and } \sigma \in \left[\frac{1}{2}, \widehat{\sigma}\right)\right) \\ [0, \frac{W}{2}] & \text{if } \theta = H \text{ and } \sigma = \widehat{\sigma} \\ \frac{W}{2} & \text{if } \theta = H \text{ and } \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ \frac{(1-\sigma)b}{R+k+b-\sigma(2R+2b-1)} W & \text{if } \theta = H \text{ and } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right) \\ W & \text{if } \theta = H \text{ and } \sigma \in \left[\overline{\overline{\sigma}}, 1\right] \end{cases}$$

with the corresponding cost of financing

$$\alpha_0^* = \begin{cases} \varnothing & \text{if } \theta = L \text{ or } (\theta = H \text{ and } \sigma \in [1/2, 2-R)) \\ \frac{1}{R+\sigma-1} & \text{if } \theta = H \text{ and } \sigma \in [2-R, \widehat{\sigma}] \\ \frac{R+\sigma-k+b}{R+\sigma+k-b} & \text{if } \theta = H \text{ and } \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ \frac{\sigma+(1-\sigma)R}{\sigma R} & \text{if } \theta = H \text{ and } \sigma \in (\overline{\sigma}, 1] \end{cases}.$$

To complete the description of the competitive equilibrium, we characterize the equilibrium in the market for liquidity at date 1 (conditional on $\theta = H$ and $\widetilde{\lambda} = 1$) from Proposition 2 as:

$$\{\alpha_1^*, \rho^*, L^*\} = \begin{cases} \left\{\frac{1}{R}, 1, 0\right\} & \text{if } \sigma \in [1/2, \widehat{\sigma}] \\ \left\{\frac{R+b-\sigma-k}{2(1-\sigma)R}, 1, \frac{W}{2}\right\} & \text{if } \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ \left\{1, \frac{R+k-\sigma(2R+b+1)}{(1-\sigma)b}, \frac{R+k-\sigma(2R+b-1)}{R+k+b-\sigma(2R+2b-1)} W\right\} & \text{if } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right] \\ \{1, 0, 0\} & \text{if } \sigma \in \left(\overline{\overline{\sigma}}, 1\right] \end{cases}.$$

Figures 2 and 3 provide graphical representations of the competitive equilibria as a function of the public signal's informativeness σ, conditional on $\theta = H$ in the market for funding, and conditional on $\theta = H$ and $\widetilde{\lambda} = 1$ in the market for liquidity, respectively. Each figure is further separated into three regions corresponding to no capital rationing, moderate and severe capital rationing if there is a liquidity shock.

When $\theta = L$, or $\theta = H$ and the public signal's informativeness is particularly low, i.e. $\sigma \in [1/2, 2-R)$, there is no supply of funding at any feasible rate ($\alpha_0 \leq 1$) because the expected pledgeable return is not sufficiently large to cover the investment cost. When $\theta = H$, and the public signal's informativeness is low, i.e. $\sigma \in [2 - R, \widehat{\sigma})$, there is also no investment as its expected NPV inclusive of effort costs is negative, even if the cost of financing entrepreneurs' investment, α_0^*, decreases with the likelihood of no liquidity shock, $\sigma = \Pr\left(\widetilde{\lambda} = 0 | \theta = H\right)$.

When the public signal's informativeness is in the low-medium range, i.e. $\sigma \in (\widehat{\sigma}, \overline{\sigma}]$, the strategy for all entrepreneurs to invest $I^* > W/2$ cannot be an equilibrium as, in such case, anticipating full dilution in the refinancing stage ($\alpha_1^* = 1$) because of capital rationing in the event of a liquidity shock, the cost of financing, α_0^*, would be sufficiently high to prevent entrepreneurs from investing. Similarly, $I^* < W/2$ cannot be an equilibrium as, in such case, the cost of financing, α_0^*, would be sufficiently low (in anticipation of no capital rationing) to induce all entrepreneurs to fully invest. Thus, in equilibrium $I^* = W/2$, and entrepreneurs are indifferent among any level of investment. This equilibrium is achieved with the cost of financing α_0^* and α_1^* increasing with σ, even if the likelihood of a liquidity shock decreases.

When the public signal's informativeness is in the medium-high range, i.e. $\sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right)$, there exist only equilibria for $I^* \in \left(\frac{W}{2}, W\right)$. $I^* = W/2$ can no longer be an equilibrium because the lower likelihood of a liquidity shock makes entrepreneurs' investment more attractive, and the cost of financing, which is already at the highest possible level ($\alpha_1^* = 1$), cannot increase any further. In this case, there exists an aggregate level of investment I^* for which entrepreneurs are indifferent on the investment decision. The higher the informativeness of the public signal, the larger the equilibrium investment because of its higher expected payoff. However, the larger the current investment, the lower the equilibrium proportion of new financing ρ^* because of the fewer resources available in the future. Therefore, if there is a liquidity shock, entrepreneurs will be rationed and financiers will extract the maximum possible rents from providing liquidity to the competing entrepreneurs at the maximum possible rate, $\alpha_1^* = 1$. In anticipation of full dilution in the refinancing stage ($\alpha_1^* = 1$) because of capital rationing, the current financing is overall more expensive reaching its maximum at $\sigma = \overline{\sigma}$. Furthermore, an increase in σ has (i) a negative direct effect on the equilibrium cost of financing because it decreases the probability of a liquidity shock, $1 - \sigma$, thus increasing the project's expected verifiable value, and (ii) a positive indirect effect through the decreased proportion of new financing ρ^*, which reduces the project's expected verifiable value. That is, an increase in σ, while making the liquidity shock less likely, conditional on the liquidity shock it increases the proportion of capital rationing. Overall, an increase in the informativeness of the public signal makes financing less expensive as capital rationing becomes unconditionally less likely.

Finally, when the public signal's informativeness is high, i.e. $\sigma \in \left(\overline{\overline{\sigma}}, 1\right]$, all entrepreneurs invest and capital rationing is at the highest level. The more informative the positive public signal, the lower the financing cost that in equilibrium competitive financiers can charge to competing entrepreneurs as capital rationing becomes unconditionally less likely. At the extreme, when the signal is perfectly informative, i.e. $\sigma = 1$, financiers cannot extract any rents, $\alpha_0^* = 1/R$, as the unconditional probability of capital rationing becomes zero since there is no liquidity shock for sure, i.e. $\Pr(\lambda = 0 | \theta = H) = 1$.

2.2.3 Inefficient Investment

Figure 4 compares the aggregate investment in the competitive equilibrium (as described in Proposition 2) with the constrained efficient allocation as in Proposition 1. The competitive equilibrium exhibits excessive investment when $\sigma \in \left(\overline{\sigma}, \widehat{\widehat{\sigma}}\right)$. The social planner invests only $W/2$ to avoid capital rationing in the refinancing market. Conversely, the investment in the competitive equilibrium exceeds $W/2$, thus making capital rationing likely.

The economic sources of over-investment are uninsurable aggregate liquidity shocks and entrepreneurs' limited commitment. Consider first the effect of insurance. If entrepreneurs could insure against liquidity shocks, the price of insurance would make them fully internalize the risk of liquidity shortage. For instance, insurance could be achieved by allowing *tradeable* assets with payoffs independent of liquidity shocks such as a riskless asset or traded claims on financiers' idiosyncratic technologies. Agents could then insure against liquidity by holding a portfolio of these claims. However, market incompleteness and the non-verifiability of financiers' payoffs rule out such possibilities. Similarly, entrepreneurs could also insure against liquidity shocks by purchasing insurance contracts such as lines of credit. However, the non-verifiability of financiers' payoffs also makes contracts incomplete as financiers cannot credibly write such contracts.

Consider next the impact of limited commitment. If entrepreneurs' investment returns were fully verifiable ($b = 0$), then financiers would fully price the risk of liquidity shortage in the cost of financing, regardless of the existence of an insurance market for liquidity. In fact, with $b = 0$, the inefficient investment region disappears ($\overline{\sigma} = \overline{\overline{\sigma}} = \widehat{\widehat{\sigma}}$) and the competitive equilibrium coincides with the constrained efficient allocation. With $b > 0$, inefficient investment arises because the equilibrium cost of financing α_0^* is independent of the non-verifiable return b, and as such, it does not fully reflect the social cost of liquidity shortages.

2.3 Social Welfare

The equilibrium social welfare is defined as the sum of entrepreneurs' and financiers' expected utilities:

$$E\left[U^W\right] = E\left[U^E\right] + E\left[U^F\right] = \sum_{\theta\in\{L,H\}} \left[E\left(U^E|\theta\right) + E\left(U^F|\theta\right)\right] \Pr\left(\theta\right) \tag{9}$$

Alternatively, it can be computed as the sum of the financiers' endowment W and the expected net present value from investment:

$$E\left[U^W\right] = W + \frac{1}{2}\left\{\left[\sigma R + (1-\sigma)\,\rho^*\,(R-1)\right] + \left[\sigma + (1-\sigma)\,\rho^*\right] b - 1 - k\right\} I^* \tag{10}$$

where I^* and ρ^* are given in Proposition 2 and equation (3), respectively. The following proposition characterizes the social welfare under the competitive equilibrium.

Proposition 3 (Social Welfare). *The social welfare in the competitive market equilibrium is:*

$$E\left[U^W\right] = \begin{cases} W & \text{if } \sigma \in [1/2, \widehat{\sigma}] \\ W + \frac{W}{4}\left[R - (1-\sigma) + b - (1+k)\right] & \text{if } \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ W + \frac{W}{2}\left(1-\sigma\right)\left(R-1\right) & \text{if } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right] \\ W + \frac{W}{2}\left[\sigma\left(R+b\right) - (1+k)\right] & \text{if } \sigma \in \left(\overline{\overline{\sigma}}, 1\right] \end{cases}$$

with entrepreneurs' and financiers' expected utilities:

$$E\left[U^E\right] = \begin{cases} 0 & \text{if } \sigma \in \left[1/2, \overline{\overline{\sigma}}\right] \\ \frac{W}{2}\left[\sigma(R+b) - (1-\sigma)\left(R-1\right) - (1+k)\right] & \text{if } \sigma \in \left(\overline{\overline{\sigma}}, 1\right] \end{cases}$$

and

$$E\left[U^F\right] = \begin{cases} W & \text{if } \sigma \in [1/2, \widehat{\sigma}] \\ W + \frac{W}{4}\left(R + \sigma + b - k - 2\right) & \text{if } \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ W + \frac{W}{2}\left(1-\sigma\right)\left(R-1\right) & \text{if } \sigma \in (\overline{\sigma}, 1] \end{cases}.$$

Figure 5 provides a graphical representation of the social welfare, the entrepreneurs' and the financiers' expected utilities as functions of the public signal's informativeness σ. As the figure shows, an increase in the quality of public information, not only affects non monotically the social welfare, but also has redistributive effects between entrepreneurs and financiers.

When the signal informativeness is low, i.e. $\sigma \in [1/2, \widehat{\sigma}]$, the social welfare equals financiers' endowment as there is no investment. Hence, a marginal increase in σ has no effect on social welfare.

When the informativeness of the public signal is in the medium-low range, i.e. $\sigma \in (\widehat{\sigma}, \overline{\sigma}]$, aggregate investment is $W/2$. Entrepreneurs make no profit, while financiers extract a rent. The social welfare is strictly increasing in σ as the increase in the expected investment payoff outweighs the higher cost of financing necessary to keep the level of investment constant.

When the informativeness of the public signal is in the medium-high range, i.e. $\sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right]$, entrepreneurs have zero expected utility as they are indifferent among any level of investment. A marginal increase in σ leaves unaffected entrepreneurs' welfare as the increase in the investment's expected net verifiable value is exactly offset by the decrease in the expected non-verifiable value given the increased proportion of capital rationing. However, financiers' expected welfare decreases monotonically with the public signal's informativeness for any $\sigma \in (\overline{\sigma}, 1]$: financiers make profits only from liquidity provision in the event of capital rationing, whose likelihood decreases with σ. Therefore, an increase in σ, while leaving entrepreneurs' utility unaffected, reduces financiers' expected profits and affects negatively the social welfare, i.e. $\partial E\left[U^W\right]/\partial\sigma = -W\left(R-1\right)/2 < 0$.

When the public signal is highly informative, i.e. $\left(\overline{\overline{\sigma}}, 1\right]$, an increase in σ improves entrepreneurs' welfare as now there is not only an increase in the expected net verifiable value of investment (higher expected investment payoff and lower cost of financing), but also an increase in the expected non-verifiable value given that the proportion of capital rationing, already at its maximum, cannot increase any further. Therefore, the reduction in financiers' expected profits is now balanced against the increase in entrepreneurs' welfare, with a net positive effect on social welfare, i.e. $\partial E\left[U^W\right]/\partial\sigma = W(R+b)/2 > 0$.

The non-monotonic relationship between the public signal's informativeness, σ, and the social welfare, $E\left[U^W\right]$, exists as long as $\overline{\overline{\sigma}} > \overline{\sigma}$. A necessary condition is $b > 0$. However, this condition is not sufficient. In addition, we need Assumption 2 ($k \geq b$): entrepreneurs incur an effort cost k sufficiently large to offset the private benefit of investment b. This assumption ensures the existence of a trade-off in the entrepreneurs' demand for funding: because of the effort cost, wealthless entrepreneurs may find optimal not to invest if they expect a sufficiently high proportion of capital rationing.

2.3.1 Welfare Comparison

Figure 6 compares the social welfare under the constrained efficient and competitive equilibria as functions of the public signal's informativeness σ. The constrained efficient allocation increases welfare by

$$\Delta E[U^W] \equiv E[U^W_{CFB}] - E[U^W] = \begin{cases} \frac{W}{4}\left[\sigma\left(2R-1\right)+b-R-k\right] > 0 & \text{if } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right] \\ \frac{W}{4}\left[\left(R+b+k\right)-\sigma\left(2R+2b-1\right)\right] > 0 & \text{if } \sigma \in \left(\overline{\overline{\sigma}}, \widehat{\widehat{\sigma}}\right] \\ 0 & \text{otherwise} \end{cases}.$$

The welfare under the constrained efficient equilibrium strictly dominates the competitive equilibrium welfare for $\sigma \in \left(\overline{\sigma}, \widehat{\widehat{\sigma}}\right]$: unlike the social planner, atomistic entrepreneurs cannot coordinate their individual actions to avoid capital rationing in the competitive equilibrium. In the constrained efficient equilibrium, there is less investment than the competitive market equilibrium for $\sigma \in \left(\overline{\sigma}, \widehat{\widehat{\sigma}}\right]$. The difference arises because entrepreneurs do not internalize the negative externality their individual investment decisions have on the proportion of capital rationing and the cost of financing.

The increase in social welfare under the constrained efficient equilibrium benefits entirely entrepreneurs while making financiers worse off. The difference in entrepreneurs' welfare under the constrained efficient and the competitive equilibrium allocation is

$$\Delta E[U^E] = \begin{cases} \frac{W}{4}\left(R - 2 + \sigma + b - k\right) > 0 & \text{if } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right] \\ \frac{W}{4}\left[\left(R + b + k\right) - \sigma\left(2R + 2b - 1\right)\right] + \frac{W}{2}(1-\sigma)(R-1) > 0 & \text{if } \sigma \in \left(\overline{\overline{\sigma}}, \widehat{\widehat{\sigma}}\right] \\ 0 & \text{otherwise} \end{cases}$$

while the difference in financiers' welfare is

$$\Delta E[U^F] = \begin{cases} -\frac{W}{2}(1-\sigma)(R-1) < 0 & \text{if } \sigma \in \left(\overline{\sigma}, \widehat{\widehat{\sigma}}\right] \\ 0 & \text{otherwise} \end{cases} .$$

Therefore, moving from the competitive equilibrium to the constrained efficient equilibrium is not a Pareto improvement.

3 Normative Analysis

So far we have focused on the positive properties of the equilibrium. We now analyze its normative aspects by examining how the economy can achieve the constrained efficient outcome. We focus on optimal information disclosure and optimal investment restrictions.[1]

3.1 Targeted Disclosure of Information

In this section, we consider whether society can do better, relative to the equilibrium with public information, by making information available only to a subset of entrepreneurs.[2] Therefore, we solve for the competitive market equilibrium in an otherwise identical economy, which differs only for the information structure: the signal θ is given as private information to each entrepreneur with some probability γ. Since we have a continuum of identical

[1] While not the focus of the current paper, optimal taxation contingent on the informativeness of public information is also a mechanism to achieve the constrained efficient equilibrium. For instance, imposing on financiers a lump-sum tax, $T_0 = \frac{W}{2}1_{\left\{\sigma \in [\overline{\sigma}, \widehat{\widehat{\sigma}})\right\}}$ optimally restrict the equilibrium financing at date 0. Then, at date 1 the tax proceeds can be returned to financiers and potentially used for liquidity provision.

[2] For instance, central banks and international institutions like IMF and the World Bank use publications and press releases to disseminate information publicly to a wide audience; while they use speeches, interviews and private meetings to disclose only to a specific audience (see Cornand and Heinemann, 2008).

entrepreneurs, the fraction of entrepreneurs who receive information equals γ almost certainly. Without loss of generality, we may assume that entrepreneurs $i \in [0, \gamma]$ receive the signal θ and entrepreneurs $i \in (\gamma, 1]$ are uninformed. To allow for a direct comparison with the case of public information, we assume that the same signal θ is distributed to all informed entrepreneurs $i \in [0, \gamma]$. Uninformed entrepreneurs can neither observe informed entrepreneurs' individual actions nor aggregate outcomes to infer the signal θ; nor they can buy information about θ from informed entrepreneurs.

We find the subgame perfect equilibria by backwards induction, starting from the market for liquidity at date 1. Since the aggregate demand and supply of liquidity are identical to the public information case, the equilibrium outcomes conditional on $\widetilde{\lambda} = 1$ can be conveniently summarized as

$$\{\alpha_1^*, \rho^*\} = \begin{cases} \left\{\frac{1}{R}, 1\right\} & \text{if } I^* < \frac{W}{2} \\ \left\{\left[\frac{1}{R}, 1\right], 1\right\} & \text{if } I^* = \frac{W}{2} \\ \left\{1, \frac{W - I^*}{I^*}\right\} & \text{if } I^* > \frac{W}{2} \end{cases} . \tag{11}$$

When $\widetilde{\lambda} = 0$, there is no market for liquidity since there is no aggregate demand. At date 0 the market for funding opens. For a fraction γ of entrepreneurs with access to the signal, the individual demand for funding is exactly as in (5). The fraction $1 - \gamma$ of uninformed entrepreneurs instead do not invest because the expected net present value is negative as $R \leq 3/2$ by Assumption 3. Hence, the aggregate demand of funds is

$$I^D = \begin{cases} \gamma W & \text{if } \theta = H \text{ \& } \alpha_0 < 1 - \frac{k - [\sigma + (1-\sigma)\rho^*] b}{\left[\sigma + (1-\sigma)\rho^* \left(1 - \alpha_1^*\right)\right] R} \\ [0, \gamma W] & \text{if } \theta = H \text{ \& } \alpha_0 = 1 - \frac{k - [\sigma + (1-\sigma)\rho^*] b}{\left[\sigma + (1-\sigma)\rho^* \left(1 - \alpha_1^*\right)\right] R} \\ 0 & \text{if } \theta = H \text{ \& } \alpha_0 > 1 - \frac{k - [\sigma + (1-\sigma)\rho^*] b}{\left[\sigma + (1-\sigma)\rho^* \left(1 - \alpha_1^*\right)\right] R} \end{cases} . \tag{12}$$

Since only informed entrepreneurs with $\theta = H$ demand capital, financiers can always infer the information content of the signal, thus the supply of funding does not change from the case with public information. In equilibrium, competitive financiers must be indifferent ex-ante between financing entrepreneurs and investing in their own technology, implying that α_0^* is as given in (8) and $I^* = I^D$ as in (12). As shown in Appendix A, first we compute the social welfare corresponding to the competitive equilibrium with targeted disclosure of information, and then we maximize it to find the optimal information disclosure γ^*. The next proposition summarizes the main result.

Proposition 4 (Optimal Information Disclosure). *The optimal information disclosure policy is*

$$\gamma^* = \begin{cases} [0,1] & \text{if } \sigma \in [1/2, \widehat{\sigma}] \\ \left[\frac{1}{2}, 1\right] & \text{if } \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ \frac{1}{2} & \text{if } \sigma \in \left(\overline{\sigma}, \widehat{\widehat{\sigma}}\right) \\ \left[\frac{1}{2}, 1\right] & \text{if } \sigma = \widehat{\widehat{\sigma}} \\ 1 & \text{if } \sigma \in \left(\widehat{\widehat{\sigma}}, 1\right] \end{cases}$$

where $\widehat{\sigma} \equiv 2 + k - R - b$, $\overline{\sigma} \equiv \frac{R+k-b}{2R-1}$ *and* $\widehat{\widehat{\sigma}} \equiv \frac{R+k+b}{2R+2b-1}$, *and the corresponding social welfare is* $E\left[U^W_{\gamma^*}\right] = E\left[U^W_{CFB}\right]$.

Intuitively, given that uninformed entrepreneurs do not invest, when $\gamma \leq 1/2$ there is no risk of capital rationing as all entrepreneurs can secure new funds if a liquidity shock occurs. If instead, γ is chosen above 1/2, the results are only qualitatively different from those obtained in Proposition 2. The equilibrium social welfare is defined as the sum of informed (I) and uninformed (U) entrepreneurs' and financiers' expected utilities. Evaluated at the optimal disclosure, the welfare function is identical to the one in Proposition 3. Hence, a regulator can perfectly replicate the constrained efficient solution by optimally choosing the information disclosure γ. The optimal disclosure policy increases with the quality of information. For low σ, a limited information disclosure prevents excess risk taking by effectively confining aggregate investment to informed entrepreneurs. For high σ, full information disclosure is optimal as the probability that risk taking reduces welfare becomes sufficiently low.

It is important to note that the effectiveness of the targeted information disclosure rests on the assumption that uninformed agents are unable to infer private information from observing aggregate outcomes or informed agents' individual actions. Allowing agents to condition their actions on aggregate outcomes would undo the effectiveness of such a policy.[3]

3.2 Investment Restrictions

In this section, we consider whether the constrained efficient solution can be replicated by imposing restrictions on financiers' investment in entrepreneurs' technology. For this purpose, we solve for the competitive market equilibrium in an otherwise identical economy, which differs for the fact that financiers can now allocate only up to a fraction χ of their capital endowment W to finance entrepreneurs. We then solve for the optimal $\chi \in [0,1]$.

We characterize the competitive market equilibrium by backward induction as in the case without investment restrictions, with the only exception that each financier's supply of capital at date 0 is restricted to $I_j^S \in [0, \chi W]$. The critical difference from the case without investment restrictions is that now there may be capital rationing at date 0 because of the constraint on the investment in the entrepreneurs' technology.

The next proposition characterizes the optimal policy.

Proposition 5 (Optimal Investment Restrictions). *The optimal choice of investment restrictions is*

$$\chi^* = \begin{cases} [0,1] & \text{if } \sigma \in [1/2, \widehat{\sigma}] \\ \left[\frac{1}{2}, 1\right] & \text{if } \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ \frac{1}{2} & \text{if } \sigma \in \left(\overline{\sigma}, \widehat{\widehat{\sigma}}\right) \\ \left[\frac{1}{2}, 1\right] & \text{if } \sigma = \widehat{\widehat{\sigma}} \\ 1 & \text{if } \sigma \in \left(\widehat{\widehat{\sigma}}, 1\right] \end{cases}$$

[3] Even though outside of the model, the ability of uninformed agents to condition their actions on contemporaneous aggregate outcomes or informed agents' individual actions, can indeed be limited in reality by the existence of observational lags.

where $\widehat{\sigma} \equiv 2+k-R-b$, $\overline{\sigma} \equiv \frac{R+k-b}{2R-1}$ *and* $\widehat{\widehat{\sigma}} \equiv \frac{R+k+b}{2R+2b-1}$, *and the corresponding social welfare is* $E\left[U^W_{\chi^*}\right] = E\left[U^W_{CFB}\right]$.

In the proof of Proposition 5, first we characterize the competitive market equilibrium with investment restrictions. Then, we maximize the corresponding social welfare to find the optimal investment restriction χ^*. The critical difference from Proposition 2 arises when $\chi \leq 1/2$ as now there is no capital rationing.

Evaluated at the optimal investment restriction χ^*, the welfare function is identical to the one in Proposition 1. Hence, a regulator can perfectly replicate the constrained efficient solution by limiting the size of financiers' investment in entrepreneurs' technology to χ^*: the worst the quality of the public information, the tighter the restriction on financiers' investment.

4 Conclusion

It is commonly believed that disclosure of more precise information by institutions such as government agencies, central banks, international organizations and rating agencies is socially valuable. For instance, among the various policy responses to the turbulence in international financial markets there has been a call for increased transparency through better disclosure from governments and other official bodies (International Monetary Fund, 1998, 2008). The International Monetary Fund has actively encouraged its members to be more transparent and made more of its own documents publicly available (Glennerster and Shin, 2003).[4]

[4] Among those there are IMF country documents and, in particular, Article IV reports, which evaluate the macroeconomic performance of all member countries; the production and publication of Reports on the Observance of Standards and Codes (ROSCs), which assess members' institutions; and the creation of the Special Data Dissemination Standard (SDDS), which sets common definitions for macroeconomic data as well as minimum frequency and timeliness standards.

This paper investigates the impact of public information in an economy with uninsurable aggregate liquidity shocks and limited commitment. In equilibrium, a negative externality in entrepreneurs' investment decisions may cause excessive risk taking in the presence of an informative public signal about the quality of the investment. Public information, while acting as "information equalizer" which reduces any information gaps among entrepreneurs, directs all entrepreneurs towards the same action and, thereby, may trigger systemic liquidity shortages. The negative externality arises endogenously from the competitive nature of entrepreneurs, who cannot internalize through prices the impact of their investment decisions on the equilibrium risk of liquidity shortages.

The quality of public information has also redistributive effects. Entrepreneurs always prefer high quality information to maximize their return on investment. Conversely, financiers prefer low information quality associated with liquidity crises because they extract rents from capital rationing.

How can a social planner tackle information sensitive inefficiencies? First, investment restrictions can achieve constrained efficiency, provided that they are based on the resources available at the aggregate rather than at the individual level, and that they are contingent on the quality of public information. Second, targeting information disclosure can also achieve constrained efficiency. Making the same information private and available only to a subset of entrepreneurs reduces over-investment.

With optimal investment restrictions or targeted disclosure of information, improving the quality of information is always welfare increasing. Hence, institutions affecting the allocative efficiency in the economy including central banks and government agencies can freely focus on the achievement of their social priorities without having to decide which information to disclose or withhold from the public as long as there are optimal investment restrictions in place. However, without optimal investment restrictions, disclosure policies become critical for the prevention of information-induced liquidity crises: targeting the disclosure of low-quality information (e.g. preliminary or incomplete data and noisy forecasts) is beneficial.

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Appendix A

Proof of Proposition 1

The social planner's objective is given in equation (1). If $\rho = \min\left\{\frac{W-I}{I}, 1\right\} = 1$, the social planner's objective simplifies to

$$\max_{I \in \left[0, \frac{W}{2}\right]} \quad W + \left[\mu_\theta \left(R+b\right) + \left(1-\mu_\theta\right)\left(R+b-1\right) - \left(1+k\right)\right] I .$$

Otherwise, if $\rho = \min\left\{\frac{W-I}{I}, 1\right\} = \frac{W-I}{I}$, the social planner's objective becomes

$$\max_{I \in \left[\frac{W}{2}, W\right]} \quad W + \left[\mu_\theta \left(R+b\right) + \left(1-\mu_\theta\right)\left(\frac{W-I}{I}\right)\left(R+b-1\right) - \left(1+k\right)\right] I \ .$$

The optimal solution, $I^{CFB} \in [0, W]$, follows from the first-order conditions and can be summarized as

$$I^{CFB} = \begin{cases} 0 & \text{if } \mu_\theta < \widehat{\sigma} \\ [0, \frac{W}{2}] & \text{if } \mu_\theta = \widehat{\sigma} \\ \frac{W}{2} & \text{if } \mu_\theta \in \left(\widehat{\sigma}, \widehat{\widehat{\sigma}}\right) \\ \left[\frac{W}{2}, W\right] & \text{if } \mu_\theta = \widehat{\widehat{\sigma}} \\ W & \text{if } \mu_\theta > \widehat{\widehat{\sigma}} \end{cases}$$

where $\widehat{\sigma} \equiv 2 + k - R - b$ and $\widehat{\widehat{\sigma}} \equiv \frac{R+b+k}{2(R+b)-1}$. Given that $\mu_H = \sigma \geq 1/2$ and $\mu_L = 1 - \sigma \leq 1/2$, and that $R \leq 3/2$ by Assumption 3, we have:

$$I^{CFB} = \begin{cases} 0 & \text{if } \theta = L \text{ or if } \left(\theta = H \text{ and } \sigma < \widehat{\sigma}\right) \\ [0, \frac{W}{2}] & \text{if } \theta = H \text{ and } \sigma = \widehat{\sigma} \\ \frac{W}{2} & \text{if } \theta = H \text{ and } \sigma \in \left(\widehat{\sigma}, \widehat{\widehat{\sigma}}\right) \\ \left[\frac{W}{2}, W\right] & \text{if } \theta = H \text{ and } \sigma = \widehat{\widehat{\sigma}} \\ W & \text{if } \theta = H \text{ and } \sigma > \widehat{\widehat{\sigma}} \end{cases} .$$

The social welfare corresponding to the constrained first-best (CFB) allocations is then computed as $E\left[U^W_{CFB}\right] = \sum_\theta E\left[U^W_{CFB}|\theta\right] \Pr\left(\theta\right)$, with $\Pr\left(\theta = H\right) = \Pr\left(\theta = L\right) = 1/2$.

Q.E.D.

Proof of Proposition 2

Given the expressions for α_1^* in (2), ρ^* in (3) and α_0^* in (8), consider first $I^* = 0$. In this case, $\rho^* = 1$, $\alpha_1^* = 1/R$ and $\alpha_0^* = \frac{1}{(R-1+\mu_\theta)}$. Given (5), this is an equilibrium only if $\mu_\theta < \widehat{\sigma} = 2 + k - R - b$. Consider next $I^* = W$. In this case, $\rho^* = 0$, $\alpha_1^* = 1$ and $\alpha_0^* = \frac{\mu_\theta + (1-\mu_\theta)R}{\mu_\theta R}$. This is an equilibrium only if $\mu_\theta > \overline{\overline{\sigma}} \equiv \frac{R+k}{2R+b-1}$. In all remaining cases, the equilibrium must be such that entrepreneurs are indifferent between investing and not investing. Consider then $I^* \in (W/2, W)$. In such case, $\alpha_1^* = 1$, $\alpha_0^* = \frac{\mu_\theta + (1-\mu_\theta)R}{\mu_\theta R}$, and $\rho^* = \frac{W-I^*}{I^*}$. For this to be an equilibrium, entrepreneurs must be indifferent among any level of investment, that is,

$$\mu_\theta R - [\mu_\theta + (1-\mu_\theta) R] + \left[\mu_\theta + (1-\mu_\theta)\left(\frac{W-I^*}{I^*}\right)\right] b = k$$

or

$$I^* = \frac{(1-\mu_\theta) b}{R + k + b - \mu_\theta (2R + 2b - 1)} W.$$

This equilibrium exists only if $I^* \in \left(\frac{W}{2}, W\right)$, which corresponds to $\overline{\sigma} < \mu_\theta < \overline{\overline{\sigma}}$, where $\overline{\sigma} \equiv \frac{R+k-b}{2R-1}$. Consider next $I^* = W/2$. In such case, $\rho^* = 1$, $\alpha_0^* = \frac{\mu_\theta + (1-\mu_\theta)\alpha_1^* R}{\left[\mu_\theta + (1-\mu_\theta)\left(1-\alpha_1^*\right)\right] R}$ and α_1^* is such that entrepreneurs are indifferent among any level of investment, that is,

$$\frac{\mu_\theta + (1-\mu_\theta)\alpha_1^* R}{[\mu_\theta + (1-\mu_\theta)(1-\alpha_1^*)] R} = 1 - \frac{k - [\mu_\theta + (1-\mu_\theta)] b}{[\mu_\theta + (1-\mu_\theta)(1-\alpha_1^*)] R}$$

or

$$\alpha_1^* = \frac{R + b - \mu_\theta - k}{2(1-\mu_\theta) R}.$$

This equilibrium exists only if $\alpha_1^* \in \left(\frac{1}{R}, 1\right)$, which corresponds to $\widehat{\sigma} < \mu_\theta < \overline{\sigma}$. Finally, consider $I^* \in (0, W/2)$. In such case, $\alpha_1^* = 1/R$, $\alpha_0^* = 1/(R - 1 + \mu_\theta)$, and $\rho^* = 1$. This equilibrium exists only if entrepreneurs are indifferent among any level of investment, which corresponds to $\mu_\theta = \widehat{\sigma}$. Hence, we can summarize the equilibrium investment as:

$$I^* = \begin{cases} 0 & \text{if } \mu_\theta < \widehat{\sigma} \\ [0, \frac{W}{2}] & \text{if } \mu_\theta = \widehat{\sigma} \\ \frac{W}{2} & \text{if } \mu_\theta \in (\widehat{\sigma}, \overline{\sigma}] \\ \frac{(1-\mu_\theta)b}{R+k+b-\mu_\theta(2R+2b-1)}W & \text{if } \mu_\theta \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right) \\ W & \text{if } \mu_\theta \geq \overline{\overline{\sigma}} \end{cases}.$$

Similarly, the equilibrium cost of financing is

$$\alpha_0^* = \begin{cases} \varnothing & \text{if } \mu_\theta < 2-R \\ \frac{1}{R+\mu_\theta-1} & \text{if } \mu_\theta \in [2-R, \widehat{\sigma}] \\ \frac{R+\mu_\theta-k+b}{R+\mu_\theta+k-b} & \text{if } \mu_\theta \in (\widehat{\sigma}, \overline{\sigma}] \\ \frac{\mu_\theta+(1-\mu_\theta)R}{\mu_\theta R} & \text{if } \mu_\theta > \overline{\sigma} \end{cases}.$$

Notice that when $\mu_\theta < 2 - R$ there is no supply of funding at any feasible rate ($\alpha_0 \leq 1$) as entrepreneurs' solvency constraint cannot be met. Hence, $\alpha_0^* = \{\varnothing\}$. In all other cases, entrepreneurs' solvency constraint is always satisfied as $\alpha_0^* \leq 1$ under Assumption 2.

Given that $\mu_H = \sigma \geq 1/2$ and $\mu_L = 1 - \sigma \leq 1/2$, the equilibrium investment under Assumptions 1-3 can be characterized as in Proposition 2.

Q.E.D.

Proof of Proposition 3

The social welfare is defined as the sum of entrepreneurs' and financiers' expected utilities:

$$E\left[U^W\right] = \sum_{\theta \in \{L,H\}} \left[E\left(U^E|\theta\right) + E\left(U^F|\theta\right)\right] \Pr(\theta)$$

where

$$\begin{aligned} E\left(U^E|\theta\right) &= \{\mu_\theta (R-1+b) + (1-\mu_\theta)\left[\rho^*(1-\alpha_1^*)R - \alpha_1^* R + \rho^* b\right] - k\} I^* \\ E\left(U^F|\theta\right) &= W\left[\mu_\theta + (1-\mu_\theta)\alpha_1^* R\right] \end{aligned}$$

with I^*, ρ^* and α_1^* as given in Proposition 2. There are four cases to consider:

(1) if $\sigma \leq \widehat{\sigma}$, there is no investment regardless of θ: $E\left(U^E\right) = 0$, $E\left(U^F\right) = W$ and $E\left[U^W\right] = W$;

(2) if $\sigma \in (\widehat{\sigma}, \overline{\sigma}]$, there is investment ($I^* = \frac{W}{2}$) only when $\theta = H$: $E\left(U^E\right) = 0$, $E\left(U^F\right) = W + W\frac{R+\sigma+b-k-2}{4}$, and

$$E\left[U^W\right] = W + \frac{W}{4}\left[R - (1-\sigma) + b - (1+k)\right];$$

(3) if $\sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right]$, there is moderate capital rationing when $\theta = H$: $E\left(U^E\right) = 0$, $E\left(U^F\right) = W + \frac{W}{2}(1-\sigma)(R-1)$, and

$$E\left[U^W\right] = W + \frac{W}{2}(1-\sigma)(R-1);$$

(4) if $\sigma \in \left(\overline{\overline{\sigma}}, 1\right]$, there is severe capital rationing when $\theta = H$: $E\left(U^E\right) = \frac{W}{2}\left[\sigma(R+b) - (1-\sigma)(R-1) - (1+k)\right]$, $E\left(U^F\right) = W + \frac{W}{2}(1-\sigma)(R-1)$, and

$$E\left[U^W\right] = W + \frac{W}{2}\left[\sigma(R+b) - (1+k)\right].$$

Q.E.D.

Proof of Proposition 4

The social welfare if $\gamma > 1/2$ is as described in Proposition 3, with the only difference that the aggregate entrepreneurs' investment I^* equals γW when $\sigma \in \left(\overline{\overline{\sigma}}, 1\right]$ rather than W:

$$E\left[U^W\right] = \begin{cases} W & \text{if } \sigma \in [1/2, \widehat{\sigma}] \\ W + \frac{W}{4}\left[R - (1-\sigma) + b - (1+k)\right] & \text{if } \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ W + \frac{W}{2}(1-\sigma)(R-1) & \text{if } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right] \\ W + \frac{\gamma W}{2}\left[\sigma(R+b) - (1+k)\right] & \text{if } \sigma \in \left(\overline{\overline{\sigma}}, 1\right] \end{cases}.$$

This expression is weakly increasing in γ and thus it is optimal to disclose information to everybody ($\gamma = 1$) when $\gamma > 1/2$.

If instead $\gamma \leq 1/2$, the economy is never liquidity constrained and thus $\alpha_1^* = 1/R$ and

$\rho^* = 1$. Hence, the social welfare becomes:

$$E\left[U^W\right] = \begin{cases} W & \text{if } \sigma \in [1/2, \widehat{\sigma}] \\ W + \frac{\gamma W}{2}\left[R - (1-\sigma) + b - (1+k)\right] & \text{if } \sigma \in (\widehat{\sigma}, 1] \end{cases}.$$

Given that $E\left[U^W\right]$ is strictly increasing in γ, the optimal choice of γ conditional on $\gamma \leq 1/2$ is to set $\gamma = 1/2$.

Comparing the two cases above, it is optimal to disclose information only to a fraction 1/2 of entrepreneurs ($\gamma^* = 1/2$) if and only if $\left(\overline{\sigma}, \widehat{\widehat{\sigma}}\right)$ and disclose the information to everybody ($\gamma^* = 1$) when $\sigma \in \left(\widehat{\widehat{\sigma}}, 1\right]$; while the choice of γ is irrelevant otherwise.

Q.E.D.

Proof of Proposition 5

The structure of the proof is as follows: (i) we derive the competitive market equilibrium under capital restrictions; (ii) we derive the social welfare; and (iii) we solve for the optimal capital requirements.

(i) There are three cases to distinguish. If $\chi = 1$, the market equilibrium is as described in Proposition 2. If instead $\chi \leq 1/2$, there is a financial constraint at date 0, but not at date 1. Hence, $\alpha_1^* = 1/R$ and $\rho^* = 1$. When $I^D \leq \chi W$, the financial constraint at date 0 does not bind and $\alpha_0^* = \frac{\mu_\theta + (1-\mu_\theta)\alpha_1^* R}{\left[\mu_\theta + (1-\mu_\theta)\rho^*\left(1-\alpha_1^*\right)\right]R} = \frac{1}{R-1+\mu_\theta}$. However, when $I^D > \chi W$, the financial constraint binds, and the shortage of capital at date 0 implies that the cost of capital raises to its highest level that makes entrepreneurs indifferent among any level of investment: $\alpha_0^* = 1 - \frac{k - [\mu_\theta + (1-\mu_\theta)\rho^*]b}{\left[\mu_\theta + (1-\mu_\theta)\rho^*\left(1-\alpha_1^*\right)\right]R} = 1 - \frac{k-b}{R-1+\mu_\theta}$. The market equilibrium in such case is:

$$I^* = \begin{cases} 0 & \text{if } \theta = L \text{ or } (\theta = H \;\&\; \sigma \in [1/2, \widehat{\sigma})) \\ \chi W & \text{if } \theta = H \;\&\; \sigma \in [\widehat{\sigma}, 1] \end{cases}.$$

If $\chi \in [1/2, 1)$, there are financial constraints at both date 0 and date 1. When $I^D \leq W/2 < \chi W$, no financial constraints bind and $\alpha_0^* = \frac{\mu_\theta + (1-\mu_\theta)\alpha_1^* R}{\left[\mu_\theta + (1-\mu_\theta)\rho^*\left(1-\alpha_1^*\right)\right]R} = \frac{1}{R-1+\mu_\theta}$. When

$W/2 < I^D \le \chi W$, the financial constraint at date 0 does not bind, but there is a financial constraint at date 1. Hence, $\alpha_1^* = 1$, $\rho^* = \frac{W-I^*}{I^*}$, and $\alpha_0^* = \frac{\mu_\theta+(1-\mu_\theta)\alpha_1^* R}{\left[\mu_\theta+(1-\mu_\theta)\rho^*\left(1-\alpha_1^*\right)\right]R} = \frac{\mu_\theta+(1-\mu_\theta)R}{\mu_\theta R}$. Following the proof in Proposition 2, this is an equilibrium with $I^* = \frac{(1-\mu_\theta)b}{R+k+b-\mu_\theta(2R+2b-1)}W$, which exists only if $I^* \in (W/2, \chi W]$ or $\overline{\sigma} < \mu_\theta < \overline{\overline{\sigma}}(\chi)$, where $\overline{\overline{\sigma}}(\chi) \equiv \frac{\chi(R+k+b)-b}{\chi(2R+2b-1)-b}$. Finally, when $I^D > \chi W$, both financial constraints at date 0 and date 1 bind. Hence, $\alpha_1^* = 1$, $\rho^* = \frac{1-\chi}{\chi}$, and $\alpha_0^* = 1 - \frac{k-\left[\mu_\theta+(1-\mu_\theta)\frac{(1-\chi)}{\chi}\right]b}{\mu_\theta R}$. The market equilibrium in such case is:

$$I^* = \begin{cases} 0 & \text{if } \theta = L \text{ or } (\theta = H \ \& \ \sigma \in [1/2, \widehat{\sigma})) \\ [0, W/2] & \text{if } \theta = H \ \& \ \sigma = \widehat{\sigma} \\ W/2 & \text{if } \theta = H \ \& \ \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ \frac{(1-\sigma)b}{R+k+b-\sigma(2R+2b-1)}W & \text{if } \theta = H \ \& \ \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}(\chi)\right] \\ \chi W & \text{if } \theta = H \ \& \ \sigma \in \left(\overline{\overline{\sigma}}(\chi), 1\right] \end{cases}.$$

(ii) The social welfare if $\chi = 1$ is as described in Proposition 3; if instead $\chi \le 1/2$:

$$E\left[U^W\right] = \begin{cases} W & \text{if } \sigma \in [1/2, \widehat{\sigma}) \\ W + \frac{\chi W}{2}\left[R - (1-\sigma) + b - (1+k)\right] & \text{if } \theta = H \ \& \ \sigma \in [\widehat{\sigma}, 1] \end{cases};$$

while if $\chi \in [1/2, 1)$, the social welfare is:

$$E\left[U^W\right] = \begin{cases} W & \text{if } \sigma \in [1/2, \widehat{\sigma}] \\ W + \frac{W}{4}\left[R - (1-\sigma) + b - (1+k)\right] & \text{if } \sigma \in (\widehat{\sigma}, \overline{\sigma}] \\ W + \frac{W}{2}(1-\sigma)(R-1) & \text{if } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}(\chi)\right] \\ W + \frac{W}{2}\left\{ \begin{array}{l} \left[\chi\sigma + (1-\chi)(1-\sigma)\right](R+b) \\ -(1-\sigma)(1-\chi) - (1+k)\chi \end{array} \right\} & \text{if } \sigma \in \left(\overline{\overline{\sigma}}(\chi), 1\right] \end{cases}.$$

(iii) Notice that $E\left[U^W\right]$ is strictly increasing in χ for $\sigma \in (\widehat{\sigma}, 1]$ when $\chi \le 1/2$. Hence, it is optimal to choose $\chi^* = 1/2$ over that range of values. If instead, $\chi \in [1/2, 1)$, then $E\left[U^W\right]$ is strictly decreasing in χ if $\sigma < \widehat{\widehat{\sigma}}$ and strictly increasing in χ if $\sigma > \widehat{\widehat{\sigma}}$.

Hence, when $\sigma \in [1/2, \widehat{\sigma}]$, the choice of capital requirement is indeterminate: $\chi^* \in [0, 1]$. When $\sigma \in (\widehat{\sigma}, \overline{\sigma})$, the optimal choice is $\chi^* = 1/2$ as the social welfare is strictly increasing in χ for $\chi \le 1/2$ and is indeterminate if $\chi > 1/2$. When $\sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}(\chi)\right]$, the optimal choice is $\chi^* = 1/2$ as $\left[R - (1-\sigma) + b - (1+k)\right]/2 > (1-\sigma)(R-1)$ for $\sigma > \overline{\sigma}$. When $\sigma \in \left(\overline{\overline{\sigma}}(\chi), 1\right]$, the comparison is between $\chi^* = 1/2$ with welfare $W + \frac{W}{4}\left[R - (1-\sigma) + b - (1+k)\right]$, and $\chi^* = 1$

with welfare $W + \frac{W}{2}\left[\sigma\left(R+b\right) - \left(1+k\right)\right]$, which depends on whether σ is smaller or greater than $\widehat{\widehat{\sigma}}$. If $\sigma < \widehat{\widehat{\sigma}}$, the optimal choice is to set $\chi^* = 1/2$; if instead $\sigma > \widehat{\widehat{\sigma}}$, the optimal choice is to set $\chi^* = 1$.

Q.E.D.

Appendix B

The positive analysis above on the role of public information quality as a trigger of allocative inefficiency is based on Assumption 3. To simplify the exposition of the analysis, we assume that the investment opportunity is not good enough to be financed without public information. We show here that our findings are robust to removing Assumption 3.

We now perform the same analysis as in Propositions 1-3 under the assumption $R > 3/2$.

Consider first the constrained first-best equilibrium. From the Proof of Proposition 1 we have:

$$I^{CFB} = \begin{cases} 0 & \text{if } \theta = L \text{ and } \sigma > 1 - \widehat{\sigma} \\ [0, \frac{W}{2}] & \text{if } \theta = L \text{ and } \sigma = 1 - \widehat{\sigma} \\ \frac{W}{2} & \text{if } (\theta = L \text{ and } \sigma < 1 - \widehat{\sigma}) \text{ and } \left(\theta = H \text{ and } \sigma < \widehat{\widehat{\sigma}}\right) \\ \left[\frac{W}{2}, W\right] & \text{if } \theta = H \text{ and } \sigma = \widehat{\widehat{\sigma}} \\ W & \text{if } \theta = H \text{ and } \sigma > \widehat{\widehat{\sigma}} \end{cases}$$

given that $\widehat{\widehat{\sigma}} > 1/2$. The corresponding welfare is

$$E\left[U^W_{CFB}\right] = \begin{cases} W + \frac{W}{2}\left(R + b - \frac{3}{2} - k\right) & \text{if } \sigma \in [1/2, 1 - \widehat{\sigma}] \\ W + \frac{W}{4}\left[R + b - (1-\sigma) - (1+k)\right] & \text{if } \sigma \in \left(1 - \widehat{\sigma}, \widehat{\widehat{\sigma}}\right] \\ W + \frac{W}{2}\left[\sigma\left(R+b\right) - (1+k)\right] & \text{if } \sigma \in \left(\widehat{\widehat{\sigma}}, 1\right] \end{cases}$$

if $k > R + b + \frac{1}{2(R+b)} - 2$ so that $1 - \widehat{\sigma} < \widehat{\widehat{\sigma}}$; and

$$E\left[U^W_{CFB}\right] = \begin{cases} W + \frac{W}{2}\left(R + b - \frac{3}{2} - k\right) & \text{if } \sigma \in \left[1/2, \widehat{\widehat{\sigma}}\right] \\ W + \frac{W}{4}\left[R + b - \sigma - (1+k)\right] + \frac{W}{2}\left[\sigma\left(R+b\right) - (1+k)\right] & \text{if } \sigma \in \left(\widehat{\widehat{\sigma}}, 1 - \widehat{\sigma}\right] \\ W + \frac{W}{2}\left[\sigma\left(R+b\right) - (1+k)\right] & \text{if } \sigma \in (1 - \widehat{\sigma}, 1] \end{cases}$$

if instead $k < R + b + \frac{1}{2(R+b)} - 2$.

Consider next the competitive equilibrium. From the Proof of Proposition 2, we have:

$$I^{*} = \begin{cases} 0 & \text{if } \theta = L \text{ and } \sigma > 1 - \widehat{\sigma} \\ [0, \frac{W}{2}] & \text{if } \theta = L \text{ and } \sigma = 1 - \widehat{\sigma} \\ \frac{W}{2} & \text{if } (\theta = L \text{ and } \sigma < 1 - \widehat{\sigma}) \text{ and } (\theta = H \text{ and } \sigma \leq \overline{\sigma}) \\ \frac{(1-\sigma)b}{R+k+b-\sigma(2R+2b-1)} W & \text{if } \theta = H \text{ and } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right) \\ W & \text{if } \theta = H \text{ and } \sigma > \overline{\overline{\sigma}} \end{cases}$$

given that $\overline{\sigma} > 1/2$. The corresponding welfare is:

$$E\left[U^W\right] = \begin{cases} W + \frac{W}{2}\left(R + b - \frac{3}{2} - k\right) & \text{if } \sigma \in [1/2, 1 - \widehat{\sigma}] \\ W + \frac{W}{4}\left[R + b - (1-\sigma) - (1+k)\right] & \text{if } \sigma \in (1 - \widehat{\sigma}, \overline{\sigma}] \\ W + \frac{W}{2}(1-\sigma)(R-1) & \text{if } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right] \\ W + \frac{W}{2}\left[\sigma(R+b) - (1+k)\right] & \text{if } \sigma \in \left(\overline{\overline{\sigma}}, 1\right] \end{cases}$$

if $k > R + b + \frac{1}{2R} - 2$ so that $1 - \widehat{\sigma} < \overline{\sigma}$;

$$E\left[U^W\right] = \begin{cases} W + \frac{W}{2}\left(R + b - \frac{3}{2} - k\right) & \text{if } \sigma \in [1/2, \overline{\sigma}] \\ W + \frac{W}{4}\left[R + b - \sigma - (1+k)\right] + \frac{W}{2}(1-\sigma)(R-1) & \text{if } \sigma \in (\overline{\sigma}, 1 - \widehat{\sigma}] \\ W + \frac{W}{2}(1-\sigma)(R-1) & \text{if } \sigma \in \left(1 - \widehat{\sigma}, \overline{\overline{\sigma}}\right] \\ W + \frac{W}{2}\left[\sigma(R+b) - (1+k)\right] & \text{if } \sigma \in \left(\overline{\overline{\sigma}}, 1\right] \end{cases}$$

if $k \in \left(R + b + \frac{1}{2R+b} - 2, R + b + \frac{1}{2R} - 2\right)$; and

$$E\left[U^W\right] = \begin{cases} W + \frac{W}{2}\left(R + b - \frac{3}{2} - k\right) & \text{if } \sigma \in [1/2, \overline{\sigma}] \\ W + \frac{W}{4}\left[R + b - \sigma - (1+k)\right] + \frac{W}{2}(1-\sigma)(R-1) & \text{if } \sigma \in \left(\overline{\sigma}, \overline{\overline{\sigma}}\right] \\ W + \frac{W}{4}\left[R + b - \sigma - (1+k)\right] + \frac{W}{2}\left[\sigma(R+b) - (1+k)\right] & \text{if } \sigma \in \left(\overline{\overline{\sigma}}, 1 - \widehat{\sigma}\right] \\ W + \frac{W}{2}\left[\sigma(R+b) - (1+k)\right] & \text{if } \sigma \in (1 - \widehat{\sigma}, 1] \end{cases}$$

if $k < R + b + \frac{1}{2R+b} - 2$.

In all these cases, there exist a range of values of σ for which the social welfare decreases with σ. Hence, the result that there is a non-monotonic relation between social welfare and the informativeness of the public information is robust to removing Assumption 3. Similarly, across all cases, there exist values of σ for which the welfare associated with the competitive equilibrium is strictly lower than the welfare in the constrained efficient equilibrium.

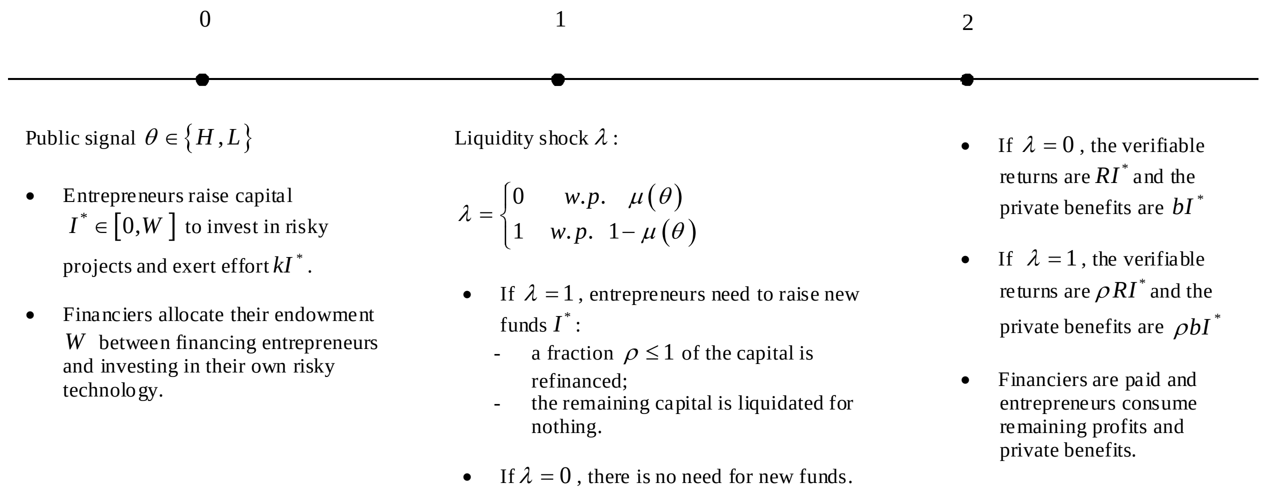

Figure 1: Timeline, Actions and Payoffs

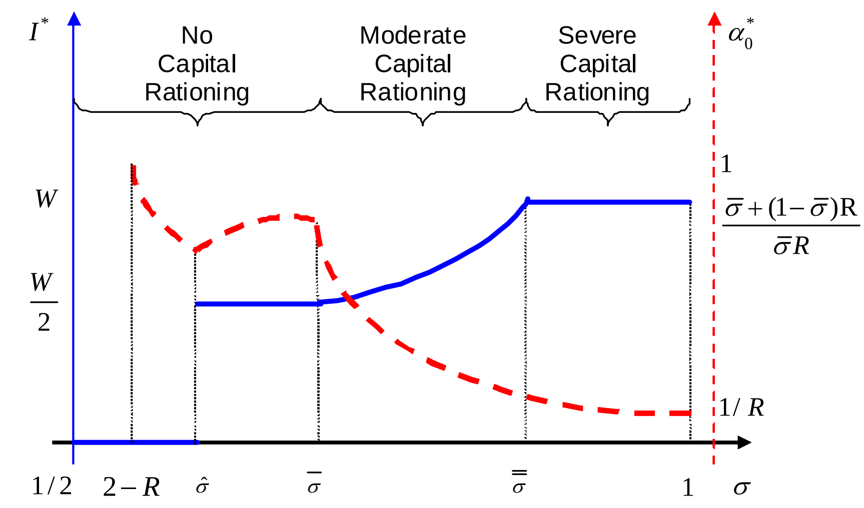

Figure 2: Market for funding conditional on $\theta = H$. Equilibrium α_0^* (dashed line) and I^* (solid line) as a function of signal's informativeness σ.

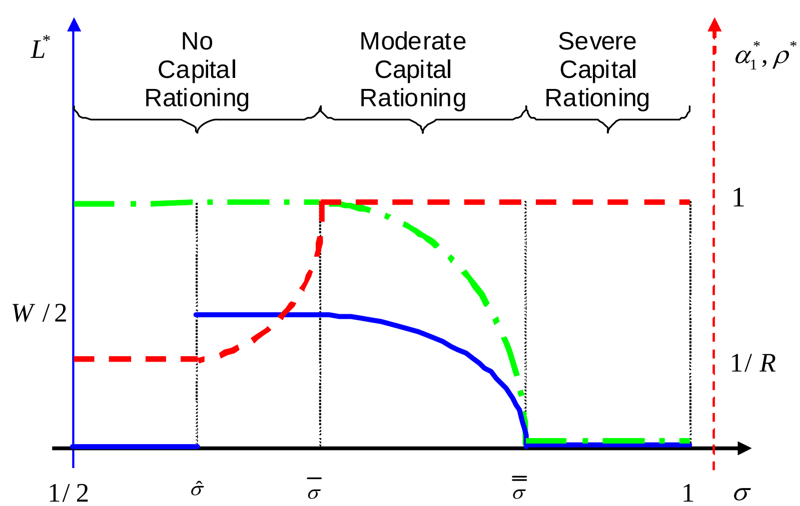

Figure 3: Market for liquidity conditional $\theta = H$ and $\lambda = 1$. Equilibrium α_1^* (dashed line), ρ^* (dashed-dotted line), and L^* (solid line) as functions of signal's informativeness σ.

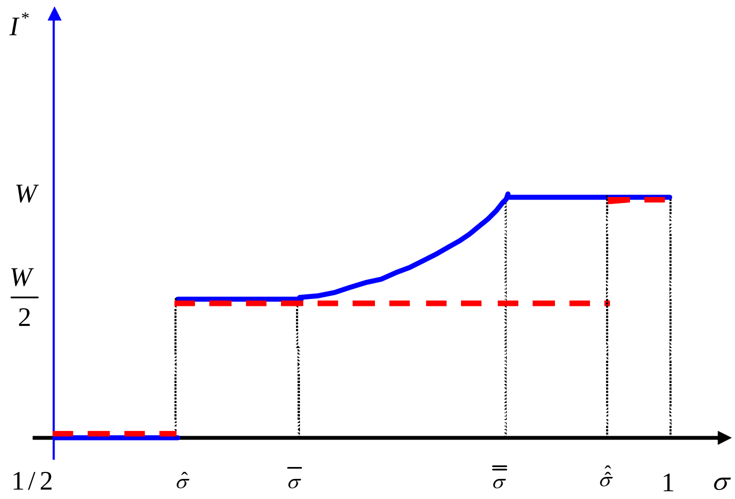

Figure 4: Optimal investment in the competitive equilibrium (solid line) and in the constrained efficient equilibrium (dashed line) as functions of signal's informativeness σ.

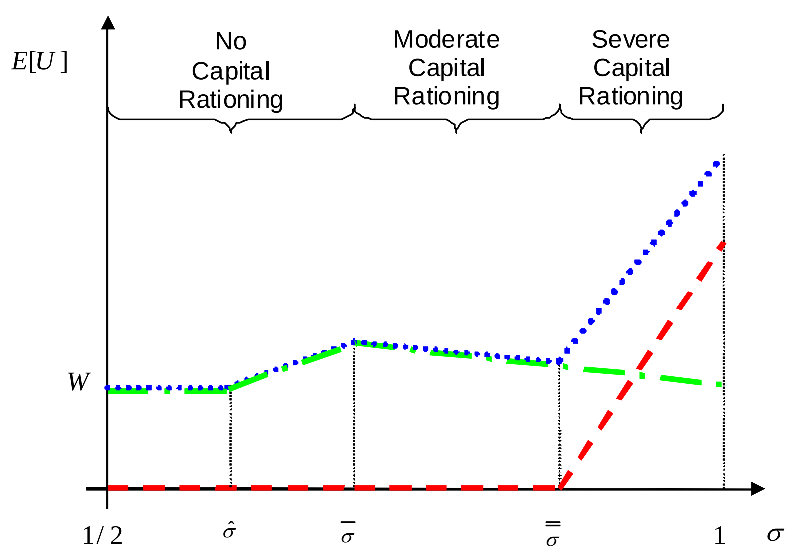

Figure 5: Ex-ante social welfare (dotted line), entrepreneurs' expected utility (dashed line) and financiers' expected utility (dashed-dotted line) in the competitive equilibrium as functions of signal's informativeness σ.

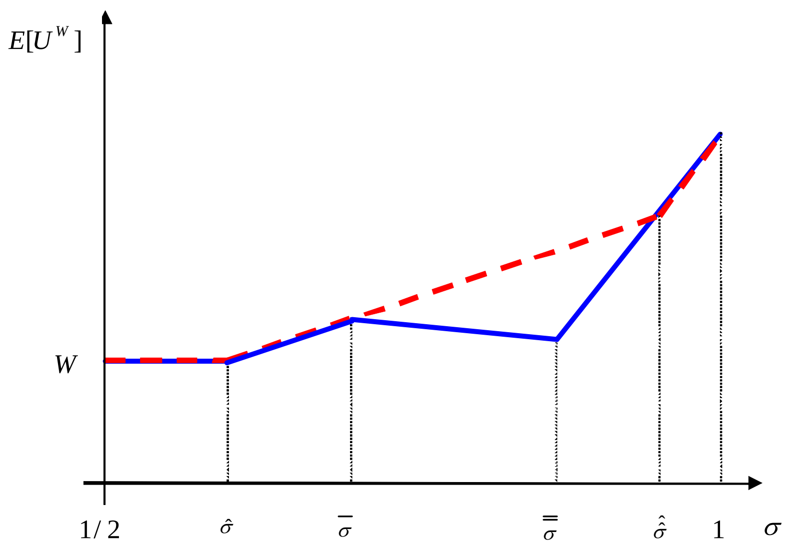

Figure 6: Social welfare in the competitive equilibrium (solid line) and in the constrained efficient equilibrium (dashed line) as functions of signal's informativeness σ.