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Public Information and Heuristic Trade

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Public Information and Heuristic Trade

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Public Information and Heuristic Trade

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Abstract

We characterize the steady-state equilibrium in which informed traders who exhibit heuristic (i.e., representativeness, as opposed to Bayesian) and Bayesian behaviors achieve the same expected utility. Then, we show how the endogenous, steady-state proportion of heuristic traders is affected by the quality of public information and other exogenous features of our model. Finally, we discuss how the presence of heuristic traders potentially alters the link between improved public disclosure and: market liquidity, the variance in the change in price, and market efficiency.

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1 Introduction

A common assumption adopted in most models of securities trade is that traders form beliefs using Bayes rule. The assumption of Bayesian behavior, however, has been called into question by a significant body of experimental evidence in the psychology literature (see Kahneman, Slovic, and Tversky [1982]). In addition, the anomalies literature in accounting (see, for example, Ou and Penman [1989] or Bernard and Thomas [1990]) and finance (see, for example, De Bondt and Thaler [1985] or Chopra, Lakonishok, and Ritter [1992]) contains findings that are also consistent with the assertion that not all individuals are Bayesians. Nonetheless, Bayesian behavior remains the predominant behavioral assumption because of the compelling argument that Bayesian behavior should prevail in an evolutionary sense.¹ For example, unless they learn to follow Bayes rule, heuristic equity fund managers should perform poorly and be driven out of business.²

Recent research, however, has shown that, in a setting with imperfect competition among informed traders, informed *heuristic* traders who overreact to their private information are viable in an evolutionary sense because they achieve a higher expected utility than informed Bayesian traders.³ Heuristic traders who overreact to their private information can achieve a higher expected utility because their overreaction

¹ See Friedman [1953] for an example of an argument that rational behavior dominates in an evolutionary sense.

² In response to the behavioral evidence just discussed, those who rely on the logic of the evolutionary argument would assert that experimental markets do not contain the appropriate evolutionary forces to drive out those who deviate from Bayesian behavior. In response to the anomalies literature, they would argue that the experimental designs have not appropriately adjusted for risk. Furthermore, even if the anomaly persists in the presence of an acceptable adjustment for risk, they would argue that the anomalies are due to some other economic friction.

³ See, for example, Kyle and Wang [1996] and Palomino [1996]. De Long, et al. [1990] consider the viability of heuristic trade in a perfectly competitive market. They argue that heuristic behavior is viable because heuristic traders earn higher expected returns than Bayesian traders. As pointed out by Palomino [1996], heuristic traders continue to have lower expected utilities in the DeLong, et al. [1990] model, even though they have higher expected returns.

induces them to trade more aggressively than Bayesian traders. Such aggressive behavior, in turn, allows an informed trader who is heuristic to capture more information rents (i.e., achieve a higher expected utility) than an informed trader who is Bayesian.

We build off of this earlier work by considering the role played by public disclosure in a trading model with two types of informed traders, heuristic (i.e., those who overreact) and Bayesian. We undertake this extension of the earlier work for two reasons. First, we are interested in the effect of public disclosure on the viability of heuristic trade. Specifically, does improved disclosure work to drive heuristic trading behavior from the market? Second, we are interested in whether heuristic trading alters previously established relations between disclosure levels and metrics commonly employed to assess disclosure's economic impact: market liquidity, the variance of price change coincident with the disclosure, and price efficiency. We acknowledge that the introduction of heuristic behaviors could be exploited to generate all sorts of results about the economic impact of disclosure. In this paper, however, we introduce heuristic trade in a plausible manner by appealing to an evolutionary construct. By making heuristic behavior plausible, our results concerning the economic impact of disclosure in the presence of heuristic trade should be correspondingly plausible.

The model we employ is a standard Kyle [1985] model with the additional features of public disclosure of information and two types of identically informed traders: heuristic traders who overreact to their current information and Bayesian traders. We initially consider a static model in which the proportion of heuristic traders is exogenously fixed. In contrast to previous models in the literature, we then endogenize the proportion of informed traders who are heuristic by employing a simple evolutionary dynamic that is based upon relative expected utilities. We assume that the

population of informed traders shifts towards the type, Bayesian or heuristic, that earns higher expected utility.⁴ In our static model, heuristic traders achieve a level of expected utility that exceeds the level of Bayesian traders, provided that there are not too many of the former. Thus, our evolutionary dynamic implies that, when the number of heuristic traders is small their proportion of the population grows, and when the number is large their proportion declines. Consequently, there exists a stable, steady-state equilibrium proportion of heuristic traders. One benefit of our evolutionary dynamic, and the associated steady-state equilibrium, is that it is amenable to comparative static analysis.

With respect to the relation between public disclosure and heuristic trade, we show that improving the quality of public disclosure reduces the viability of heuristic trade. Specifically, we show that improving the quality of public disclosure causes the difference between the expected utilities of a heuristic and Bayesian trader to decline. This static equilibrium result, combined with our evolutionary dynamic, implies that heuristic traders comprise a smaller proportion of informed traders in the steady-state equilibrium. As a consequence, our model suggests that heuristic trading behavior is more likely to be observed in markets where public disclosure is poor.

With respect to previously established relations between disclosure levels and observable market characteristics, we show that heuristic trade weakens, but does not generally undo, these relations. For example, prior models with only Bayesian informed trade demonstrate that improving the quality of disclosure leads to more liquid markets, more efficient prices, and a greater price variance. These results arise

⁴ Kyle and Wang [1996] do not undertake any evolutionary analysis. Palomino [1996] considers an evolutionary dynamic based upon risk adjusted return realizations. While Palomino's choice of a dynamic is consistent with much of the literature in evolutionary economics, it seems inconsistent with his critique of De Long, et al. [1990], which suggests that viability should be a function of expected utility as opposed to return realizations.

because public disclosure reduces the information asymmetry in the market by increasing the amount of public information. Heuristic trade weakens results relating public disclosure to market liquidity and price efficiency, but does not overturn them. The result relating public disclosure to price variance, however, is overturned. To understand why these results are weakened or overturned, note first that market liquidity, market efficiency, and price variability are positively affected by the level of heuristic trade in some (market liquidity) or all (market efficiency and price variability) cases. This positive relation arises because heuristic trade makes aggregate demand more informative. Improved public disclosure, however, reduces the level heuristic trade. Thus, the reduction in the level of heuristic trade resulting from improved public disclosure militates against the force identified in prior models without heuristic trade. In our analysis of market liquidity and market efficiency, the countervailing force is never sufficient to surmount the force identified in the prior models. In contrast, the result for price variability is overturned because the countervailing force does more than compensate.

The remainder of the paper is organized as follows. In the next section, we present a static model of trade in which the proportion of informed, heuristic traders is *exogenously* fixed. In section 3 we *endogenize* the proportion of heuristic traders by introducing a simple evolutionary dynamic, and then derive a steady-state proportion of heuristic traders consistent with this dynamic. We discuss factors that affect the proportion of heuristic trade observed in equilibrium in Section 4. In Section 5, we discuss the relation between public disclosure and a variety of market phenomena in the presence of heuristic trade. We conclude by summarizing our results.

2 The Static Model

As discussed earlier, the model we employ is an extension of the Kyle [1985] model. A Kyle model depicts a security market in which market makers do not know the extent to which demands are driven by private information versus liquidity motivated reasons that have no information content. In addition, it characterizes a setting where informed traders are aware that their trade can move price (i.e., traders do not behave as price takers). The initial model we examine consists of one period, a Bayesian risk neutral market maker, liquidity traders, and N risk neutral informed traders. Among these N traders, we assume proportion p are heuristic and a proportion $1 - p$ are Bayesian.⁵ We will use the results for the static model in this section to endogenize p in Section 3.

Let \tilde{u} represent a firm's uncertain cash flow. At the beginning of the period, information about the firm's cash flow, \tilde{x}_p , is disclosed to the public, while private information, \tilde{x}_i , is revealed to all informed traders, where

$$\tilde{x}_p = \tilde{u} + \tilde{\epsilon}_i + \tilde{\epsilon}_p, \quad (1)$$

$$\tilde{x}_i = \tilde{u} + \tilde{\epsilon}_i, \quad (2)$$

$\tilde{\epsilon}_i$ is normally distributed with mean 0 and variance v_i , and $\tilde{\epsilon}_p$ is normally distributed with mean 0 and variance v_p . Common prior beliefs on \tilde{u} are that it is normally distributed with mean 0 and variance v .⁶

Note that the public information in our model is simply the private information plus some noise. This information structure is intended to capture the idea that

⁵ We treat the number of heuristic traders, Np , as a continuous variable. This permits us to employ a simple continuous evolutionary dynamic that yields a steady state equilibrium p . If Np is required to be an integer there may not be a steady state equilibrium.

⁶ Without loss of generality, we have assumed means of 0 to ease notation.

informed traders, such as financial analysts who cover a particular firm, have analysis advantages derived from their expertise or access to other information sources. These analysis advantages, in turn, allow them to eliminate noise from public disclosure. Within the context of our model, it follows that improvements in public disclosure can be interpreted as making information more accessible or more available to the market.

After the public and private information is received, all traders submit demand orders to the market maker. Liquidity demand is “noise” in our model: it is assumed to be normally distributed with mean 0 and variance v_l . Alternatively, informed trader demands are chosen strategically. After receiving the aggregate demand order, the market maker sets a price conditional upon aggregate demand and the public information. All orders are then cleared at that price. After trade takes place, the terminal value is realized and claims are paid.

We model heuristic traders’ behavior by assuming their posterior expectations deviate from true Bayesian expectations in a systematic manner. The posterior expectation of a Bayesian trader, b , is

$$E[\tilde{u}|x_p, x_i; b] = E[\tilde{u}|\tilde{x}_p = x_p, \tilde{x}_i = x_i] = \frac{v}{v + v_i}x_i. \quad (3)$$

Note that x_p has no weight because x_i is a sufficient statistic for x_i, x_p with respect to \tilde{u} . We assume that posterior expectation for a heuristic trader, h , is

$$E[\tilde{u}|x_p, x_i; h] = E[\tilde{u}] + r(E[\tilde{u}|\tilde{x}_p = x_p, \tilde{x}_i = x_i] - E[\tilde{u}]) = r\frac{v}{v + v_i}x_i, \quad (4)$$

where $r > 1$ represents the response of heuristic traders to the private information. Note that $r > 1$ implies that if a Bayesian trader’s posterior expectation is greater (less) than the prior expectation, then a heuristic trader’s posterior expectation is greater (less) than a Bayesian’s expectation. Thus, heuristic traders overreact to the

new or current information. Our characterization of heuristic trader beliefs is consistent with the findings in the psychology literature that some individuals exhibit *representativeness* (more specifically, the *base rate fallacy*) and overconfidence: for examples of this literature see Kahneman, Slovic, and Tversky [1982]. The assumption that $r > 1$ is consistent with the base rate fallacy because it implies heuristic traders place too much emphasis on their current information and too little on public information when forming posterior beliefs. It is also consistent with the notion of overconfidence because the heuristic traders are too confident in their private information.⁷

We assume that market making is a perfectly competitive industry. Consequently, we require that the market maker sets price equal to the conditional expectation of the asset's terminal value, plus an amount to cover the costs of supplying liquidity. Formally, let D represent the total net demand for firm shares and $c > 0$ some exogenous cost of supplying liquidity (e.g., inventory holding costs). The exogenous cost is a function of the demand orders that cannot be crossed, or total *net* demand, D . We assume that the price set to execute trades by the market maker, P , equals the expectation of the asset's terminal value based on total net demand plus an amount to cover costs:

$$P = E[\tilde{u} | \tilde{D} = D, \tilde{x}_p = x_p] + cD, \quad (5)$$

where $E[\cdot | \cdot; \cdot]$ is the conditional expectations operator. If total net demand is positive, for example, implying that the market maker must go short, the market maker sets price higher than the expectation of \tilde{u} in order to cover his holding costs. Demand

⁷ This is not to suggest that there are not alternative characterizations of heuristic trader beliefs. An example of an alternative characterization is to weight the variances v_i and v_p by some amount less than one. We choose our characterization because it makes the mathematics facile. Finally, note that because traders are risk neutral, it is not necessary to consider how heuristic behavior affects beliefs about the posterior variance of \tilde{u} .

orders are then cleared at the market maker's price, terminal value is realized, and claims are settled.⁸

In the standard Kyle [1985] model, the only driver of trading cost is the information asymmetry between the informed traders and the market maker. All other drivers of trading cost, such as the order processing and inventory holding costs identified in the empirical literature (see Stoll [1989]), are assumed away. In our setting, the elimination of other drivers of trading cost corresponds to $c = 0$. The benefit of assuming away other trading costs in the standard model is that it permits an elegant, closed-form, linear equilibrium. This clean characterization facilitates the analysis of the implications of information asymmetry. Unfortunately, when heuristic trade is introduced into the standard model with $c = 0$, a linear equilibrium *may* fail to exist. Introducing other drivers of trading cost ensures the existence of a linear equilibrium, which, in turn, facilitates the analysis. An alternative to our approach would be to assume $c = 0$, and then assume that the parameters of the model are such that a linear equilibrium exists. With this alternative approach, all of our results continue to hold. The advantage of our approach is that it allows us to discuss comparative static results without continually qualifying our discussion with the assumption that an equilibrium exists over the comparative static range.⁹

An equilibrium for our model must satisfy the following two criteria.

- i) The pricing function must satisfy eqn. (5) given the equilibrium trading strategies of the informed traders.

⁸ We assume the market maker behaves in a Bayesian manner. One justification for this assumption is that Bayesian behavior is superior in perfectly competitive (i.e., price-taking) settings.

⁹ Of course, there are alternative ways to introduce a trading cost that are not associated with the adverse selection problem. For example, each trader could be charged a transaction cost that is increasing in his individual demand order (e.g., c times the absolute value of his demand order). We choose an inventory holding cost that is a function of total net demand because it makes the analysis considerably more facile than alternatives.

ii) The trading strategy for each informed trader must maximize his expected wealth given the equilibrium trading strategies of the other informed traders and the equilibrium pricing function.

In the following analysis, we conjecture and then confirm the existence of a unique linear equilibrium in which price is increasing in total net demand: that is, an equilibrium in which informed traders' demand functions are linear functions of their private information and the market maker's pricing function is an increasing linear function of total net demand. Specifically, the conjectured trading strategy of trader n is given by:

$$f_{in}x_i + f_{pn}x_p. \quad (6)$$

The market maker's pricing function in our conjectured equilibrium is:

$$P = (\lambda_D + c)D + \lambda_p x_p, \quad (7)$$

where $D = \sum_{n=1}^N (f_{in}x_i + f_{pn}x_p) + d_l$ is realized total net demand, d_l is the realization of liquidity demand, and λ_D and λ_p are determined from the requirement that $\lambda_D D + \lambda_p x_p = E[\tilde{u}|D, x_p]$. Note that in our conjectured equilibrium price is increasing in total net demand, which implies that $\lambda_D + c > 0$.

We begin our construction of the linear equilibrium by solving informed trader m 's optimization problem given conjectured demand functions for the other informed traders of the form in eqn. (6) and a conjectured pricing function of the form in eqn. (7). Let h and b represent a heuristic and a Bayesian-type trader, respectively. Given trader m 's beliefs about λ_D , other traders' demand coefficients, the realization x_i, x_p , and trader m 's type $j \in \{h, b\}$, m 's demand solves

$$\underset{\{d\}}{\text{Maximize}} \ d(E[\tilde{u}|x_p, x_i; j] - E[\tilde{P}|x_p, x_i; d]) \quad (8)$$

where

$$E[\tilde{u}|x_p, x_i; j] = R \frac{v}{v + v_i} x_i, \quad (9)$$

$$E[\tilde{P}|x_p, x_i; d] = (\lambda_D + c)(F_{ih}x_i + F_{ph}x_p + d), \quad (10)$$

$$R = r > 1 \text{ if } j = h \text{ and } R = 1 \text{ if } j = b, \quad (11)$$

$$F_{ih} = \sum_{\substack{n=1 \\ n \neq m}}^N f_{in}, \quad (12)$$

and

$$F_{ph} = \sum_{\substack{n=1 \\ n \neq m}}^N f_{pn}. \quad (13)$$

The trader's optimization problem is a strictly concave programming problem. Consequently, the first order condition provides a unique characterization of m 's optimal demand function, which is linear in the private and public information: specifically, $d = f_{im}x_i + f_{pm}x_p$, where

$$f_{pm} = \frac{-F_{ph}(\lambda_D + c) - \lambda_p}{2(\lambda_D + c)}, \quad (14)$$

and

$$f_{im} = \frac{R \frac{v}{v+v_i} - F_{ih}(\lambda_D + c)}{2(\lambda_D + c)}. \quad (15)$$

Eqn. (13) implies that trader m 's demand coefficient on public information is the same regardless of his type. Eqn. (14) implies that trader m 's demand coefficient on private information is greater if m is a *heuristic* type because a heuristic trader places more weight on his private information than a Bayesian trader. These two observations imply that heuristic traders react more aggressively to their information. In addition, like the standard Kyle model, m 's demand coefficients are decreasing in magnitude in the demand coefficients of other traders, and increasing in magnitude in market liquidity as represented by $\frac{1}{\lambda_D + c}$.

Before proceeding, it is useful to exploit the demand conditions for m to establish the result that all traders of the same type must have the same demand coefficient in any linear equilibrium. Note that eqns. (14) and (15) imply that, in any linear equilibrium,

$$f_{pm} = \frac{-\lambda_p}{(\lambda_D + c)} - F_p \quad (16)$$

$$f_{im} = \frac{R \frac{v}{v+v_i}}{\lambda_D + c} - F_i, \quad (17)$$

where

$$F_i = \sum_{n=1}^N f_{in}, \quad (18)$$

and

$$F_p = \sum_{n=1}^N f_{pn}, \quad (19)$$

represent the aggregate demand coefficients of informed traders. It follows from inspection of eqns. (15) and (16) that for any m and m' who are both of type $j \in \{h, b\}$, $f_{im} = f_{im'}$ and $f_{pm} = f_{pm'}$. Given this result, we let $\{f_{ih}, f_{ph}\}$ and $\{f_{ib}, f_{pb}\}$ denote the demand coefficients for each heuristic trader and each Bayesian trader, respectively. We can use the two equations of the form in eqn. (16) and the two equations of the form in eqn. (17) to characterize the equilibrium demand coefficients:

$$f_{ih} = \frac{(r + N[r - 1][1 - p]) \frac{v}{v+v_i}}{(1 + N)(\lambda_D + c)}, \quad (20)$$

$$f_{ib} = \frac{(1 - N[r - 1]p) \frac{v}{v+v_i}}{(1 + N)(\lambda_D + c)}, \quad (21)$$

and

$$f_{ph} = f_{pb} = \frac{-\lambda_p}{(1 + N)(\lambda_D + c)}. \quad (22)$$

In summary, we have shown that given a linear pricing function, demand functions for each informed trader type are linear. With this result in hand, we next consider the form of the pricing function given linear demand functions.

Assuming linear demand functions, the market maker's expectation of \tilde{u} conditional upon the public information and observed total net demand satisfies

$$E[\tilde{u}|D, x_p] = \frac{F_i v_p v}{F_i^2(v_p v_i + v_p v) + v_l(v + v_i + v_p)} D + \frac{-F_i F_p v_p v + v_l v}{F_i^2(v_p v_i + v_p v) + v_l(v + v_i + v_p)} x_p. \quad (23)$$

Eqn. (23) implies a linear pricing function with the following coefficients:

$$\lambda_D = \frac{F_i v_p v}{F_i^2(v_p v_i + v_p v) + v_l(v + v_i + v_p)} \quad (24)$$

and

$$\lambda_p = \frac{-F_i F_p v_p v + v_l v}{F_i^2(v_p v_i + v_p v) + v_l(v + v_i + v_p)}. \quad (25)$$

The expression for λ_D has the standard implications. First, assuming that $F_i > 0$ in equilibrium (which turns out to be the case), the market is more liquid when the variability of liquidity trade, v_l , increases. Second, the market is more liquid when private information is less precise (i.e., v_i is higher).

To prove that our conjectured linear equilibrium exists and is unique, we show in the appendix that there exists a unique solution $\{f_{ih}, f_{ib}, f_{ph}, f_{pb}, F_i, F_p, \lambda_D, \lambda_p\}$ that satisfies the eight linear equilibrium conditions captured by eqns. (18) to (22), (24), and (25). The solution has the characteristics presented in the following lemma.

Lemma 1. *For any (exogenous) proportion of heuristic traders, there exists a unique linear equilibrium with price increasing in total net demand that has the following properties: $F_i > 0$, $F_p < 0$, $\lambda_D > 0$, $\lambda_p \in (0, \frac{v}{v+v_i+v_p}]$, $f_{ih} > f_{ib}$, $f_{ih} > 0$, $f_{ph} = f_{pb} < 0$.*

An interesting implication of Lemma 1 concerns the trading strategies of each type. Lemma 1 states that heuristic traders always respond more aggressively than Bayesian traders to private information: that is, they go “longer” on good private news and “shorter” on bad private news ($f_{ih} > f_{ib}$). This aggressive behavior can be beneficial to heuristic traders in that it permits them to capture more of the rents from their private information.

The fact that the heuristic traders take more extreme positions in the same direction as the private news implies that heuristic demand may go in the opposite direction of Bayesian demand. Specifically, for some realizations of x_i, x_p , a heuristic trader may be long (short) while a Bayesian trader is short (long). Demands may go in opposite direction because excessive demand by heuristic traders can result in the expectation of price “overreacting” in the sense that $E[\tilde{P}|x_i, x_p] > (<) E[\tilde{u}|x_i, x_p]$, even though $E[\tilde{u}|x_p] < (>) E[\tilde{u}|x_i, x_p]$. In these cases, it becomes optimal for the Bayesian traders to take a position contrary to heuristic traders.

To close out our discussion of the static equilibrium, we turn to the role $c > 0$ plays in ensuring an equilibrium. Broadly stated, the equilibrium λ_D and F_i in our model can be characterized as the point at which two functions cross. These two functions are: the sensitivity of the market maker’s expectation to total net demand as a function of the aggregate demand coefficient on x_i , $\lambda_D(F_i)$; and the aggregate demand coefficient on x_i as a function of the sensitivity of the market maker’s expectation to total net demand, $F_i(\lambda_D)$. From eqn. (24), the former function is characterized by

$$\lambda_D(F_i) = \frac{F_i v_p v}{F_i^2 (v_p v_i + v_p v) + v_l (v + v_i + v_p)}. \quad (26)$$

Noting that $F_i > 0$ in any equilibrium in which $\lambda_D + c > 0$, this implies that the market maker’s sensitivity to total net demand is increasing in F_i when F_i is small

and decreasing in F_i when F_i is a large: that is, λ_D is unimodal in F_i . Using eqns. (20) and (21), the aggregate demand coefficient function can be characterized by

$$F_i(\lambda_D) = \frac{N(1 + [r - 1]p)v}{(\lambda_D + c)(1 + N)(v + v_i)}. \quad (27)$$

The aggregate demand coefficient on x_i is decreasing in the market maker's sensitivity to total net demand: that is, $F_i(\lambda_D)$ becomes smaller as λ_D increases. Note, however, that as the proportion of heuristic traders, p , increases, $F_i(\lambda_D)$ shifts to a larger number for each unit of λ_D . The intuition underlying this shift is that heuristic traders respond more aggressively to the private information for any λ_D . When there are no heuristic traders (i.e., $p = 0$) $\lambda_D(F_i)$ and $F_i(\lambda_D)$ always cross regardless of c . Thus, in this circumstance, there always exists an equilibrium. However, when $c = 0$, the existence of heuristic trade through p may shift $F_i(\lambda_D)$ to a position that precludes the possibility of $\lambda_D(F_i)$ and $F_i(\lambda_D)$ crossing.

More specifically, when $c = 0$, a necessary and sufficient condition for $\lambda_D(F_i)$ and $F_i(\lambda_D)$ to cross is:

$$1 - N(r - 1)p > 0. \quad (28)$$

Once again, if all traders are Bayesian, $p = 0$, then eqn. (28) is always trivially satisfied. However, when $p > 0$, an equilibrium exists only in the event that the number of informed traders, N , is sufficiently small. When $c > 0$, eqn. (28) is no longer a necessary condition. Thus, requiring that $c > 0$ always ensures a linear equilibrium in which price is increasing in total demand.

3 Steady-State Heuristic Trade

Having characterized an equilibrium for our static model in which the proportion of heuristic trade is exogenous, we now utilize that static model within a dynamic

evolutionary framework to endogenize the extent of heuristic trade. In our discussion of the trading strategies adopted by each type, we alluded to the potential for heuristic traders to gain or lose relative to Bayesian traders as a result of their aggressive behavior. Whether heuristic traders gain or lose relative to Bayesian traders is a logical determinant of the viability of heuristic behavior in an evolutionary sense. Let $\pi_h(p)$ and $\pi_b(p)$ represent the (Bayesian) expected profits, which are also the expected utilities because all traders are risk neutral, of a heuristic and a Bayesian trader, respectively. Note that $\pi_h(p)$ and $\pi_b(p)$ are functions of the proportion of heuristic traders, p . To address the viability of heuristic behavior, we consider a simple continuous evolutionary dynamic that is a function of the difference in the expected profits of each type: $\pi_h(p) - \pi_b(p)$.

Formally, we let the instantaneous change in the proportion of heuristic traders when the economy is at state p be defined as

$$\Delta(\pi_h(p) - \pi_b(p); p), \tag{29}$$

where Δ is a continuous function,

$$\begin{aligned} \Delta &= 0 && \text{if } \pi_h(p) - \pi_b(p) = 0, \\ \Delta &> 0 && \text{if } \pi_h(p) - \pi_b(p) > 0 \text{ and } p < 1, \\ \Delta &= 0 && \text{if } \pi_h(p) - \pi_b(p) > 0 \text{ and } p = 1, \\ \Delta &< 0 && \text{if } \pi_h(p) - \pi_b(p) < 0 \text{ and } p > 0, \end{aligned}$$

and

$$\Delta = 0 \quad \text{if } \pi_h(p) - \pi_b(p) < 0 \text{ and } p = 0.$$

This evolutionary dynamic captures the idea that, over time, the population of informed traders shifts towards the type that achieves higher expected utility, assuming

that such a shift is possible. An interior steady-state equilibrium for this dynamic is an equilibrium proportion $p \in (0, 1)$ such that both types of traders do equally well, $\pi_h(p) = \pi_b(p)$. A corner steady-state equilibrium with only heuristic traders has the property that $\pi_h(p = 1) \geq \pi_b(p = 1)$. Conversely, a corner steady-state equilibrium with only Bayesian traders has the property that $\pi_h(p = 0) \leq \pi_b(p = 0)$.

Expected profit is the determinate of viability in our evolutionary dynamic. One justification for expected profit is that expected utility has been suggested as an appropriate measure of evolutionary fitness in the prior literature (see, e.g., Palomino [1996]), and the two are equivalent in our model with risk-neutral agents. Another justification arises from considering a setting where each generation of informed traders, whose types are fixed, plays our trading game an infinite number of times. Consistent with the notion of evolution, the behaviors of the next generation adapt towards those behaviors that performed well “on average” in the prior generation. In our model, “on average” is tantamount to expected profit.

In order to establish the existence and uniqueness of a steady-state, we need to understand what drives the difference in expected profits in order to ascertain how that difference behaves in p . Exploiting the static equilibrium conditions, the difference in expected profits can be written as

$$\begin{aligned} \pi_h(p) - \pi_b(p) &= (\lambda_D + c)E[(f_{ih} - f_{ib})x_i + (f_{ph} - f_{pb})x_p](f_{ib}x_i + f_{pb}x_p) \quad (30) \\ &= (\lambda_D + c)covariance(d_h - d_b, d_b) \end{aligned}$$

where d_j denotes the demand for a trader of type j .

The reason the difference in expected profits hinges on the sign of the covariance of $d_h - d_b$ and d_b is as follows. Note that a Bayesian trader has a positive (negative) demand only in cases where $E[\tilde{u}|x_i, x_p] > (<) E[\tilde{P}|x_i, x_p]$. If, on average, a heuristic

trader goes longer (shorter) than the Bayesian trader when the Bayesian trader is long (short), then one would expect the heuristic trader to have greater expected profits. If the converse were true, one would expect the heuristic trader to have lower expected profits. The covariance of $d_h - d_b$ and d_b is a measure of whether, on average, the heuristic trader is going longer (shorter) than the Bayesian trader when the Bayesian trader is long (short).

In the appendix, we show that $covariance(d_h - d_b, d_b)$ can equal 0 at only a single value of p and is strictly positive at $p = 0$. In addition, we show that the covariance of $d_h - d_b$ and d_b equals 0 for an interior p if N is large and is strictly positive for all p if N is small. This analysis yields our first proposition.

Proposition 1. *There exists a unique steady-state equilibrium. If the number of informed traders, N , is sufficiently large, both heuristic and Bayesian traders are observed in the steady-state: that is, the steady state equilibrium p, \bar{p} , satisfies $\bar{p} \in (0, 1)$. If the number of informed traders is small, then only heuristic traders survive in the steady-state: that is, $\bar{p} = 1$.*

Proposition 1 implies that heuristic traders who exhibit behavior consistent with representativeness or overconfidence always survive in an evolutionary sense. Furthermore, if there is a sufficient number of informed traders, Proposition 1 implies that both types of informed traders survive.

An issue of interest is what happens to the informed traders' expected profits at the steady-state. An informed Bayesian trader's expected profits are

$$(\lambda_D + c)([v + v_i][f_{ib} + f_{pb}]^2 + f_{pb}^2 v_p), \quad (31)$$

which are strictly positive. Because both trader types have equal expected profits in the steady-state, this implies that the heuristic traders also have positive expected

profits in the steady-state. The source of each trader’s expected profits, however, differs. Bayesian traders choose demands to yield positive, conditional expected profits for all x_i, x_p states. For some x_i, x_p states, however, the heuristic traders go long (short) when the Bayesian traders go short (long). Thus, for some x_i, x_p the heuristic traders earn negative conditional expected profits (i.e., they make some “bad deals”). Heuristic traders still have positive expected profits because for some x_i, x_p they take bigger positions in the same direction as the Bayesian traders (i.e., they make enough on “good deals” to compensate for the losses on “bad deals”).

In addition to our focus on steady-states, one might also be concerned with some notion of stability. A common stability condition requires that any perturbation from a steady-state is self-correcting in the sense that the evolutionary dynamic leads to movement back to the steady-state. Our steady-state satisfies such a condition. Recall that the evolutionary dynamic requires that $covariance(d_h - d_b, d_b) = 0$ at the steady-state \bar{p} . Because $covariance(d_h - d_b, d_b)$ is positive for $p < \bar{p}$, any negative perturbation from the steady would be self-correcting in that p would rise to \bar{p} . Similarly, because $covariance(d_h - d_b, d_b)$ is negative for $p > \bar{p}$, any positive perturbation from the steady would be self correcting in that p would fall to \bar{p} .

4 Determinants of the Viability of Heuristic Trade

The simplicity of our model permits us to generate some comparative static results that highlight how certain exogenous factors affect the viability of heuristic trade, as measured by \bar{p} . We then use these results to discuss the empirical relevance of our findings.

4.1 Comparative Static Results

Throughout our discussion of comparative static results, we assume that N is sufficiently large to ensure an interior value for \bar{p} . The steady-state equilibrium condition implies that:

$$\text{covariance}(d_h - d_b, d_b) = 0. \quad (32)$$

The comparative statics that follow are based on this condition.

Because a main focus of this paper is the relation between information and the presence of heuristic trade, we first consider the relation between the information structure in our model and the viability of heuristic trade. As has been pointed out, heuristic behavior can be a beneficial attribute because the aggressive trade of heuristic informed traders can earn them a greater proportion of the rents arising from information asymmetry. Thus, one might expect that improving the quality of the public information (i.e., reducing the variance of v_p) makes heuristic trade less viable. Corollary 1 confirms this intuition.

Corollary 1. *The proportion of heuristic traders sustained in the steady-state, \bar{p} , is decreasing in the quality of the public information: $\frac{\partial \bar{p}}{\partial v_p} > 0$.*

Extending the intuition underlying Corollary 1 to its limit, one might expect that heuristic behavior evolves away in cases where the public information is as good as the private information ($v_p = 0$). This extension is correct because the presence of inventory holding costs results in informed heuristic trader's always making "bad deals" and Bayesian trader's profiting by taking contrarian positions.

We now briefly turn our analysis to the effect of variables that do not pertain to the information structure. We first present a corollary that summarizes the results and then discuss the intuition underlying the results.

Corollary 2. *The proportion of heuristic traders sustained in the steady-state, \bar{p} , is decreasing in the degree of representativeness or overconfidence, r , decreasing in inventory holding costs, c , decreasing in the variance of liquidity trade, v_l , and decreasing in the number of informed traders, N : that is $\frac{\partial \bar{p}}{\partial r} < 0$, $\frac{\partial \bar{p}}{\partial c} < 0$, $\frac{\partial \bar{p}}{\partial v_l} < 0$, and $\frac{\partial \bar{p}}{\partial N} < 0$.*

The relation between the degree of representativeness or overconfidence, as formalized by r , and the viability of heuristic behavior is driven by the fact that heuristic traders take more aggressive trading positions as the degree of representativeness or overconfidence rises. Greater aggression, in turn, reduces the expected profits they earn in x_i, x_p states where they make “good deals” (i.e., states where they have positive conditional expected profits) and magnifies expected losses in x_i, x_p states where they make “bad deals” (i.e., states where they have negative conditional expected profits). Thus, heuristic behavior is likely to become less viable as the degree of representativeness or overconfidence rises.

The relation between inventory holding costs and the viability of heuristic trade is driven by similar reasoning. Specifically, because the inventory holding cost reduces the conditional expected profits in states where the heuristic traders make “good deals” and magnifies the losses in states where the heuristic traders make “bad deals,” the viability of heuristic trade is reduced by increases in inventory holding costs.

The viability of heuristic trade also declines in the variance of liquidity trade. To see why, note first that, in the imperfect competition setting we consider, heuristic behavior is viable because it commits heuristic informed traders to act aggressively on their private information. By acting aggressively, heuristic informed traders drive out demands by Bayesian informed traders, which, in turn, allows a heuristic trader to capture relatively more rents than a Bayesian informed trader. An increase in the

variance of liquidity trade lessens the effect of informed trade on total net demand, which, in turn, makes price less sensitive to total net demand. Because price is less sensitive to total net demand, the aggressive behavior of the heuristic traders is less effective at driving down Bayesian demands. Thus, the relative benefit of heuristic behavior declines and the proportion of heuristic traders falls.

Finally, we turn to the effect of the number of informed traders on the viability of heuristic trade. Recall again that heuristic behavior is viable because it commits heuristic informed traders to act aggressively on their private information which, in turn, drives out demands by Bayesian informed traders and allows the heuristic traders to capture relatively more rents. In a setting with perfect competition (i.e., price taking behavior), aggressive behavior is not rewarded because it does not affect the demands of other traders. Noting that traditional oligopoly settings converge to perfectly competitive settings as the number of participants becomes infinite, it is logical that the proportion of heuristic informed traders decreases in the number of informed traders. Indeed, one can show that the proportion of heuristic traders sustained in the steady-state approaches zero as the number of informed traders approaches infinity.

4.2 Empirical Relevance

At this stage it is useful to step back and discuss the empirical relevance of our findings. Our comparative static results provide explicit predictions regarding the extent of heuristic behavior. Direct tests of these predictions, however, are likely to be difficult because doing so would require a good proxy for heuristic trade. We are reluctant to suggest such a proxy.

Nonetheless, our model has some less explicit implications that do pertain to the

anomalies literature. In particular, if some of the anomalous empirical findings are driven by representativeness or overconfidence on the part of some traders, our model provides implicit predictions regarding the settings where the anomalous behavior should be most prominent. For example, Ou and Penman [1989] find evidence that prices overreact to transitory components of earnings. If these findings are driven by the presence of traders who overreact to their information, then one might expect overreaction to be more pronounced in markets where there are more heuristic traders. Our model, by suggesting some determinants of heuristic trade, provides some predictions regarding overreaction: namely, the overreaction to transitory components of earnings should be greater in markets or for firms where there is less public disclosure or less liquidity trade.

Our model's insights also apply to papers in the finance literature that suggest excess returns can be attained by adopting contrarian investment strategies (i.e., buying the winners and selling the losers).¹⁰ Again, if this finding is driven by the presence of heuristic traders who overreact, then our model provides some predictions as to settings where the returns to contrarian investing should be greatest: namely, the returns should be greatest in markets or for firms where there is less public disclosure and less liquidity trade.

Of course, there are anomalies that our particular behavioral construct cannot easily address. One that immediately comes to mind is the underreaction of prices to public information (see, for example, Bernard and Thomas [1989]). The framework for our analysis, however, does provide a useful discipline to those who would attribute this finding to some sort of heuristic behavior. Specifically, our framework suggests that there should be some rationale for why the heuristic is economically viable, and

¹⁰See, for example, De Bondt and Thaler [1985] or Chopra, Lakonishok, and Ritter [1992].

there should be predictions regarding the variables that determine the viability of the heuristic. These variables, in turn, can then be related to the extent of underreaction.

5 Disclosure and Market Phenomena

Three primary areas of interest in models of trade are the link between the quality of public disclosure and: market liquidity, the variance in the change in price, and the efficiency of prices. In this section, we discuss how heuristic trade introduces a force that potentially alters the standard link predicted in prior work.

5.1 Market Liquidity

Traditionally, market liquidity has been measured by the extent to which a unit of demand moves price. A more liquid market is one in which demand moves price less. Liquidity is captured in our model by $\frac{1}{\lambda_D + c}$. Because c is exogenous, however, the endogenous component of liquidity is captured by λ_D .

Because heuristic traders are more aggressive in their trading, one might expect that total net demand contains more information as the proportion of heuristic traders increases. A standard model with only Bayesian traders yields the insight that an increase in the information content of total net demand causes a decrease in liquidity (i.e., an increase in λ_D). Therefore, a logical conjecture is that price should become more sensitive to total net demand as the proportion of heuristic traders increases. However, in the presence of heuristic traders, this conjecture is not generally true: in some cases liquidity is increasing in p and in others it is decreasing in p .

Recall that $\lambda_D = \frac{F_i v_p v}{F_i^2 (v_p v_i + v_p v) + v_l (v + v_i + v_p)}$. As shown in the appendix, the static equilibrium aggregate demand coefficient, F_i , is increasing in p . Therefore, an increase in p affects both the numerator and the denominator through an increase in F_i . Thus,

it is the relative magnitude of the effects on the numerator versus the denominator that determines the impact of p on λ_D . For any value of p , it is possible to show that, when non-information costs, c , are sufficiently high, the numerator effect dominates the denominator effect. This implies that a marginal increase from p results in less liquid markets. When the converse occurs, the less intuitive outcome results — a marginal increase from p yields more liquid markets.

From an intuitive perspective, the positive relation between heuristic trade and liquidity can be explained as follows. Heuristic traders overreact to their information. The overreaction causes total net demand to convey more information. The market maker must take out the effect of this overreaction when she computes λ_D . This implies that λ_D may *decrease* in the presence of heuristic trade unless the heuristic traders' overreaction is sufficiently attenuated. One exogenous variable that attenuates heuristic trader behavior is the market maker's inventory holding cost parameter, c . If heuristic trading behavior is attenuated by a high c , then the market maker increases λ_D in response to more heuristic trade. If c is low, the market maker *decreases* λ_D in response to more heuristic trade. More formally, we have the following corollary.

Corollary 3. *There exist values for c , c_l and $c_h > c_l$, such that λ_D is globally decreasing in p if $c \leq c_l$, λ_D is increasing and then decreasing in p if $c \in (c_l, c_h)$, and λ_D is globally increasing in p if $c \geq c_h$.*

An implication drawn from static trading models with only Bayesian informed traders is that increasing the quality of the public disclosure (i.e., decreasing v_p) improves market liquidity (i.e., lowers λ_D).¹¹ This result arises because improved public disclosure reduces information asymmetry, which makes the price at which shares are executed less sensitive to the private information impounded in total net

¹¹See, for example, proposition 1 of Kim and Verrecchia [1994].

demand. Consequently, the response of price to demand, λ_D , falls. However, the presence of endogenous heuristic trade makes the relation between the quality of public disclosure and market liquidity potentially ambiguous.

To see why, recall that improving public disclosure reduces the steady-state proportion of heuristic traders (see Corollary 1). As we have just discussed, a reduction in the proportion of heuristic traders may decrease market liquidity (i.e., increase λ_D). Consequently, it is conceivable that the reduction in market liquidity arising from a decline in heuristic trade dominates the effect arising from a reduction in information asymmetry. This counterintuitive behavior does not arise in our model, however.

Observation 1. *Market liquidity increases in the quality of the public information despite the decline in heuristic trade: that is, $\frac{d\bar{\lambda}}{dv_p} > 0$, where $\bar{\lambda}$ is the steady-state value for λ_D .*

5.2 Variance of Price Change

A common variable examined by accounting researchers is the variance of price change around a public announcement. In our model, the expected price is zero. Consequently, the variance of price change is captured by the variance of \tilde{P} . Because heuristic traders trade more aggressively, an increase in heuristic trade is likely to increase the information content of total net demand. Consequently, we expect that price changes are more significant as the proportion of heuristic traders increases. Corollary 4 confirms this intuition.

Corollary 4. *The variance of the price change increases as the proportion of heuristic traders increases: that is, $\frac{d\text{Var}[\tilde{P}]}{dp} > 0$.*

The conventional wisdom in the theoretical literature is that the variance of price

change increases in the quality of disclosure.¹² The intuition underlying this result is that the higher the quality, the more disclosure moves market expectations. Consequently, as the market responds to the announcement, prices shift more dramatically. In rational models of trade where informed demand occurs concurrently with disclosure, this result continues to hold even though the existence of informed trade gives rise to a countervailing force. Specifically, as the quality of the disclosure increases, the magnitude of equilibrium informed demand decreases, which, in turn, implies less information is conveyed to the market through total net demand. The decline in information in total net demand, however, is dominated by the increase in information through public disclosure. Therefore, the variance in the price change continues to increase in the quality of the public disclosure.

If the proportion of heuristic trade is exogenously fixed in our model, the conventional wisdom also holds for our model: specifically, the variance of P decreases in v_p . However, when heuristic trade is endogenous, the variance of the price change decreases in the quality of disclosure. The explanation for a result of this type is that improved public disclosure decreases the proportion of heuristic traders in the steady-state, which dampens informed demand. This dampening of informed demand implies that the total net demand carries significantly less information. Therefore, despite the infusion of information that arises directly from public disclosure, price change is less dramatic. In summary, we have the following observation.¹³

¹²See, for example, the discussion in Holthausen and Verrecchia [1988].

¹³Kim and Verrecchia [1994] have a similar result. In their model, the number of informed traders endogenous, and they show that the variance of the price change is decreasing in the quality of the public information. The reason for their result is that better public information reduces the incentives for information acquisition, which, in turn, results in fewer informed traders in equilibrium. Fewer informed traders dampens informed demand. This leads to less information being conveyed to the market. In short, the Kim and Verrecchia [1994] result is attributable to an endogenous number of informed traders while ours is driven by an endogenous proportion of heuristic traders.

Observation 2. *The variance of the change in price decreases in the quality of the public information: that is, $\frac{d\overline{\text{Var}}[\tilde{P}]}{dv_p} > 0$, where $\overline{\text{Var}}[\tilde{P}]$ is the steady-state value for $\text{Var}[\tilde{P}]$.*

5.3 Price Efficiency

A final area of interest to be considered is the extent to which price reflects information, both public and private. We examine efficiency within the context of our model by considering the difference between the posterior variance of \tilde{u} conditional upon price, P , versus all public and private information, x_i, x_p : $\text{Variance}[\tilde{u}|P] - \text{Variance}[\tilde{u}|x_i, x_p]$.

As Kyle and Wang [1996] have observed, heuristic traders react more aggressively, which, in turn, implies that more private information is revealed as the proportion of heuristic traders increases, *ceteris paribus*. This result holds in our model as well.

Corollary 5. *Prices become more informationally efficient as the proportion of heuristic traders increases: that is, $\frac{d\{\text{Var}[\tilde{u}|P] - \text{Var}[\tilde{u}|x_i, x_p]\}}{dp} < 0$.*

Because heuristic traders exist in the steady-state equilibrium, Corollary 5 implies that the price is more informationally efficient in the steady-state than in an equilibrium with only Bayesian traders.

A standard implication of static trading models with only Bayesian informed traders is that increasing the quality of disclosure (i.e., decreasing v_p) improves price efficiency. This result arises because improved disclosure quality is tantamount to providing the market with more information. In contrast to static models with only Bayesian traders, however, a dynamic model with heuristic traders renders the relation between the quality of public information and price efficiency potentially ambiguous.

This potential ambiguity arises from the fact that improved disclosure, in addition to providing more information, reduces the steady-state proportion of heuristic traders (see Corollary 1). Because Corollary 5 implies that a decrease in heuristic trade results in less efficient prices, there is an effect on price efficiency that counters the effect arising from more disclosure. Nonetheless, the countervailing effect of less heuristic trade is dominated by improved public disclosure. In summary, we have the following observation.

Observation 3. *Price efficiency increases in the quality of the public information despite the decline in heuristic trade: that is, $\frac{d\{\overline{\text{Var}}[\tilde{u}|P] - \text{Var}[\tilde{u}|x_i, x_p]\}}{dv_p} > 0$, where $\overline{\text{Var}}[\tilde{u}|P]$ is the steady-state value for $\text{Var}[\tilde{u}|P]$.*

6 Conclusion

Previous research has shown that informed heuristic traders who overreact to current information may do better than informed Bayesian traders. Consequently, heuristic behavior may be viable in an evolutionary sense. We extend this line of research by considering how public disclosure affects the proportion of heuristic trade in the steady-state equilibrium for an evolutionary model. We show that improved public disclosure, by reducing the rents available to informed traders, reduces steady-state heuristic trade.

We use results developed from our steady-state model to reassess the link between public disclosure and: market liquidity, the variance of the change in price that accompanies a public announcement, and the efficiency of market prices. In settings where all informed traders are Bayesian, improved (i.e., more precise) public disclosure is associated with greater market liquidity, increased variance of the change

in price that accompanies the announcement, and more efficient market prices. In our model, which posits an endogenous level of heuristic trade, there is the possibility that these results could be reversed. This reversion arises from the fact that improvements in the quality of public disclosure reduce the proportion of heuristic trade, and less heuristic trade may be associated with less market liquidity, less price efficiency, etc. In short, public disclosure in the presence of heuristic traders gives rise to a force that counters the impact of disclosure identified in models with only Bayesian traders. This countervailing force ensures that the variance of price change declines with improved public disclosure. Despite this countervailing force, we show that improved public disclosure continues to be positively related to market liquidity and strong-form efficiency in our model

On a broader level, we acknowledge that introduction of heuristic behaviors could be exploited to generate a variety of economic responses to information. One contribution of this paper is that *first* it establishes the *viability* of a behavior in an evolutionary sense, and *then* it considers the impact of the behavior on the link between information and economic outcomes. By adding the discipline imposed by an evolutionary construct, we avoid employing heuristics in an ad hoc manner to generate particular outcomes. As a consequence, the results attained in this paper, and others that utilize a similar approach, are likely to have a greater degree of plausibility.

Appendix

Proof of Lemma 1

Using eqns. (18)-(22), (24), and (25) we can write the endogenous variables, f_{ih} , f_{vp} , f_{ph} , f_{pb} , λ_D , and λ_p as functions of F_i :

$$f_{ih} = \frac{F_i(r + N[r - 1][1 - p])}{N(1 + [r - 1]p)} \quad (\text{A1})$$

$$f_{ib} = \frac{F_i(1 - N[r - 1]p)}{N(1 + [r - 1]p)} \quad (\text{A2})$$

$$f_{ph} = f_{pb} = -\frac{F_i v_l (v + v_i)}{F_i^2 v_p (v + v_i) N(r - 1)p + v_l (v + v_i + v_p) N(1 + [r - 1]p)} \quad (\text{A3})$$

$$\lambda_D = \frac{F_i v_p v}{F_i^2 v_p (v + v_i) + v_l (v + v_i + v_p)} \quad (\text{A4})$$

$$\lambda_p = \frac{v_l v (1 + [r - 1]p)}{F_i^2 v_p (v + v_i) (r - 1)p + v_l (v + v_i + v_p) (1 + [r - 1]p)}. \quad (\text{A5})$$

Furthermore, note that eqns. (18), (20) and (21) imply

$$F_i = \frac{N(1 + [r - 1]p) \frac{v}{v + v_i}}{(1 + N)(\lambda_D + c)}. \quad (\text{A6})$$

Note also that eqn. (A6) implies that $F_i > 0$ in any linear equilibrium in which $\lambda_D + c > 0$. Because all the endogenous variables can be expressed as functions of F_i and $F_i > 0$ in any equilibrium, the proof can be completed by showing that there exists a unique equilibrium $F_i > 0$. Using eqn. (A4) to substitute in for λ_D in eqn. (A6) and rearranging yields the following equilibrium condition for F_i :

$$\begin{aligned} & F_i^3 c(1 + N)v_p(v + v_i)^2 + F_i^2[1 - (r - 1)Np]v_p v(v + v_i) \\ & + F_i c(1 + N)v_l(v + v_i + v_p)(v + v_i) - N[1 + (r - 1)p]v_l v(v + v_i + v_p) = 0. \end{aligned} \quad (\text{A7})$$

For convenience, denote the function on the left and side of eqn. (A7) as J . If $F_i = 0$, $J < 0$, and as $F_i \rightarrow \infty$, $J \rightarrow \infty$. Therefore, there exists an interior equilibrium

$F_i > 0$. To show that the equilibrium is unique, one must show that J is increasing in F_i at any F_i which satisfies eqn. (A7). Differentiating J with respect to F_i yields

$$\begin{aligned} \frac{\partial J}{\partial F_i} &= 3F_i^2 c(1+N)v_p(v+v_i)^2 + 2F_i(1-[r-1]Np)v_p v(v+v_i) \\ &\quad + c(1+N)v_l(v+v_i+v_p)(v+v_i). \end{aligned} \quad (\text{A8})$$

If $1-(r-1)Np > 0$, then $\frac{\partial J}{\partial F_i}$ is clearly strictly positive. We show that $\frac{\partial J}{\partial F_i} > 0$ if $1-(r-1)Np < 0$ by contradiction. Assume $1-(r-1)Np < 0$ and $\frac{\partial J}{\partial F_i} < 0$ at an $F_i > 0$ that solves eqn. (A7). If $\frac{\partial J}{\partial F_i} < 0$, then $3F_i^2 c(1+N)v_p(v+v_i)^2 + 2F_i(1-[r-1]Np)v_p v(v+v_i) < 0$ by eqn. (A8). If $3F_i^2 c(1+N)v_p(v+v_i)^2 + 2F_i(1-[r-1]Np)v_p v(v+v_i) < 0$, eqn. (A7) is satisfied at F_i if and only if $F_i c(1+N)v_l(v+v_i+v_p)(v+v_i) - N[1+(r-1)p]v_l v(v+v_i+v_p) > 0$. Therefore, $F_i > \frac{N[1+(r-1)p]v}{c(1+N)(v+v_i)}$. We can rewrite eqn. (A8) as

$$\begin{aligned} \frac{\partial J}{\partial F_i} &= F_i v_p(v+v_i)[3F_i c(1+N)(v+v_i) \\ &\quad + 2(1-[r-1]Np)v] + c(1+N)v_l(v+v_i+v_p)(v+v_i). \end{aligned} \quad (\text{A9})$$

Note that eqn. (A9) implies that, if $3F_i c(1+N)(v+v_i) + 2(1-[r-1]Np)v > 0$, then $\frac{\partial J}{\partial F_i} > 0$. Furthermore, note that, if $3F_i c(1+N)(v+v_i) + 2(1-[r-1]Np)v > 0$ at any $F_i > 0$, it follows that it is strictly positive for all $F_i' > F_i$. Therefore, it follows that, if $\frac{\partial J}{\partial F_i} > 0$ at any $F_i > 0$, then $\frac{\partial J}{\partial F_i} > 0$ for all $F_i' > F_i$. We complete the contradiction by showing that when $F_i = \frac{N[1+(r-1)p]v}{c(1+N)(v+v_i)}$, $\frac{\partial J}{\partial F_i} > 0$. Assuming $F_i = \frac{N[1+(r-1)p]v}{c(1+N)(v+v_i)}$ we can write eqn. (A9) as

$$\frac{\partial J}{\partial F_i} = F_i v_p v(v+v_i)[3N + 2[r-1]Np + 2] + c(1+N)v_l(v+v_i+v_p)(v+v_i) > 0. \quad (\text{A10})$$

Therefore $\frac{\partial J}{\partial F_i} > 0$ at the F_i which satisfies eqn. (A7) if $1-(r-1)Np < 0$. Q.E.D.

The following two lemmas are used in the proof of Proposition 1. We will let a “ $\hat{\cdot}$ ” over a variable denote the static equilibrium value for that variable which is a

function of p as well as the other exogenous variables (e.g., \hat{F}_i is the static equilibrium F_i which is a function of p).

Lemma A1. The static equilibrium F_i, \hat{F}_i , is increasing in p : $\frac{d\hat{F}_i}{dp} > 0$.

Proof of Lemma A1

Using the static equilibrium condition provided by eqn. (A7) to perform standard comparative statics analysis yields:

$$\frac{d\hat{F}_i}{dp} = \frac{-\frac{\partial J}{\partial p}}{\frac{\partial J}{\partial F_i}}. \quad (\text{A11})$$

Recalling that the denominator is strictly positive, the sign of the numerator is the sign of $\frac{dF_i}{dp}$. The numerator can be written as

$$-\frac{\partial J}{\partial p} = \hat{F}_i^2(r-1)Nv_p v(v+v_i) + (r-1)Nv_l v(v+v_i+v_p) > 0. \quad (\text{A12})$$

Therefore $\frac{d\hat{F}_i}{dp} > 0$. Q.E.D.

Lemma A2. The static equilibrium F_i, \hat{F}_i , is increasing in N : $\frac{d\hat{F}_i}{dN} > 0$.

Proof of Lemma A2

Using the static equilibrium condition provided by eqn. (A7) to perform standard comparative statics analysis yields:

$$\frac{d\hat{F}_i}{dN} = \frac{-\frac{\partial J}{\partial N}}{\frac{\partial J}{\partial F_i}}. \quad (\text{A13})$$

Recalling that the denominator is strictly positive, the sign of the numerator is the sign of $\frac{dF_i}{dN}$. The numerator can be written as

$$\begin{aligned} -\frac{\partial J}{\partial N} = & -\hat{F}_i^3 cv_p(v+v_i)^2 + \hat{F}_i^2(r-1)pv_p v(v+v_i) \\ & -\hat{F}_i cv_l(v+v_i+v_p)(v+v_i) + [1+(r-1)p]v_l v(v+v_i+v_p). \end{aligned} \quad (\text{A14})$$

Exploiting the static equilibrium condition for $F_i, J = 0$, yields

$$-\frac{\partial J}{\partial N} = \frac{1}{N}(\hat{F}_i^3 cv_p[v+v_i]^2 + \hat{F}_i^2 v_p v[v+v_i] + \hat{F}_i cv_l[v+v_i+v_p][v+v_i]) > 0. \quad (\text{A15})$$

Therefore $\frac{d\hat{F}_i}{dN} > 0$. Q.E.D.

Proof of Proposition 1

Using a variety of equilibrium conditions we can show that $covariance(d_h - d_b, d_b)$ is proportional to K , where

$$K = -(\hat{F}_i^3 c[1 + N]v_p[v + v_i]^2 + \hat{F}_i c[1 + N]v_l[v + v_i + v_p][v + v_i])(r - 1)p + v_p v_l v(1 + [r - 1]p), \quad (\text{A16})$$

and \hat{F}_i is the static equilibrium value given p . The proof of the proposition is completed by first showing that $K > 0$ at $p = 0$ and $\frac{dK}{dp} \leq 0$ if $K \leq 0$. We complete the proof by showing that there exists an N, \check{N} , such that $K \geq 0$ at $p = 1$ if and only if $N \leq \check{N}$. Note first that at $p = 0$, $K = v_p v_l v > 0$. Thus $p = 0$ is never a steady-state equilibrium value. Differentiating K with respect to p yields

$$\begin{aligned} \frac{dK}{dp} &= -(\hat{F}_i^3 c[1 + N]v_p[v + v_i]^2 + \hat{F}_i c[1 + N]v_l[v + v_i + v_p][v + v_i])(r - 1) + v_p v_l v(r - 1) \\ &\quad - \frac{d\hat{F}_i}{dp} (3\hat{F}_i^2 c[1 + N]v_p[v + v_i]^2 + c[1 + N]v_l[v + v_i + v_p][v + v_i])(r - 1)p. \end{aligned} \quad (\text{A17})$$

Assuming $K \leq 0$ implies that

$$\begin{aligned} &(\hat{F}_i^3 c[1 + N]v_p[v + v_i]^2 + \hat{F}_i c[1 + N]v_l[v + v_i + v_p][v + v_i])(r - 1) \\ &\geq \frac{1}{p} v_p v_l v(1 + [r - 1]p). \end{aligned} \quad (\text{A18})$$

It follows from eqns. (A17) and (A18) that

$$\begin{aligned} \frac{dK}{dp} &< -\frac{d\hat{F}_i}{dp} (3\hat{F}_i^2 c[1 + N]v_p[v + v_i]^2 \\ &\quad + c[1 + N]v_l[v + v_i + v_p][v + v_i])(r - 1)p - \frac{v_p v_l v}{p} < 0. \end{aligned} \quad (\text{A19})$$

Thus, $\frac{dK}{dp} \leq 0$ if $K \leq 0$. Finally, fix $p = 1$. At $N = 0$, $K > 0$ because $\hat{F}_i = 0$, and as $N \rightarrow \infty$, $K \rightarrow -\infty$ because $\hat{F}_i > 0$ if $N > 0$. Because $\frac{d\hat{F}_i}{dN} > 0$ for all N , it follows that $\frac{dK}{dN} < 0$ for all N when p is fixed at 1. Thus, there exists an N, \check{N} , such that $K \geq 0$ for all $N \leq \check{N}$ when $p = 1$. Thus, $p = 1$ is the unique steady-state equilibrium for all $N \leq \check{N}$, and the unique steady-state $p \in (0, 1)$ if $N > \check{N}$. Q.E.D.

For the remainder of the analysis, we assume that the steady-state is in the interior. We denote the interior steady-state values with a “ $\bar{\cdot}$ ” over the variable. Note that \bar{F}_i equals \hat{F}_i only when $p = \bar{p}$.

The following lemma is used to prove Corollary 1.

Lemma A3. The static equilibrium F_i, \hat{F}_i , is decreasing in v_p : $\frac{d\hat{F}_i}{dv_p} < 0$.

Proof of Lemma A3

Using the static equilibrium condition given by eqn. (A7) to perform standard comparative statics analysis yields:

$$\frac{d\hat{F}_i}{dv_p} = \frac{-\frac{\partial J}{\partial v_p}}{\frac{\partial J}{\partial \hat{F}_i}}. \quad (\text{A20})$$

Recalling that the denominator is strictly positive, the sign of the numerator is the sign of $\frac{\partial J}{\partial v_p}$. The numerator can be written as

$$\begin{aligned} -\frac{\partial J}{\partial v_p} &= -\hat{F}_i^3 c(1+N)(v+v_i)^2 - \hat{F}_i^2(1-[r-1]Np)v(v+v_i) \\ &\quad -\hat{F}_i c(1+N)v_l(v+v_i) + N(1+[r-1]p)v_l v. \end{aligned} \quad (\text{A21})$$

Exploiting the static equilibrium condition for $F_i, J = 0$ at \hat{F}_i , allows eqn. (A21) to be rewritten as

$$-\frac{\partial J}{\partial v_p} \propto \hat{F}_i c[1+N][v+v_i] - N(1+[r-1]p)v. \quad (\text{A22})$$

Thus, the sign of $\hat{F}_i c(1+N)(v+v_i) - N(1+[r-1]p)v$ determines the sign of $\frac{d\hat{F}_i}{dv_p}$. We show that $\hat{F}_i c(1+N)(v+v_i) - N(1+[r-1]p)v < 0$ by showing that $J > 0$ at

the value for F_i that sets $F_i c(1+N)(v+v_i) - N(1+[r-1]p)v = 0$, $\check{F} = \frac{N(1+[r-1]p)v}{c(1+N)(v+v_i)}$. At $F_i = \check{F}$ the value for $J = \frac{N^2(1+[r-1]p)^2 v_p v^3}{c^2(1+N)(v+v_i)} > 0$. Therefore, because J is increasing in F_i , $\hat{F}_i < \frac{N(1+[r-1]p)v}{c(1+N)(v+v_i)}$ which implies that $-\frac{\partial J}{\partial v_p} < 0$ and $\frac{d\hat{F}_i}{dv_p} < 0$. Q.E.D.

Proof of Corollary 1.

Using standard comparative statics analysis on the steady-state equilibrium condition, $K = 0$, yields:

$$\frac{d\bar{p}}{dv_p} = \frac{\frac{dK}{dv_p}}{-\frac{dK}{dp}}. \quad (\text{A23})$$

Recall that $-\frac{dK}{dp} > 0$ at \bar{p} so the sign of $\frac{d\bar{p}}{dv_p}$ is the same as the sign of $\frac{dK}{dv_p}$. $\frac{dK}{dv_p}$ satisfies

$$\begin{aligned} \frac{dK}{dv_p} &= -\frac{d\hat{F}_i}{dv_p} (3\hat{F}_i^2 c[1+N]v_p[v+v_i] + c[1+N]v_l[v+v_i+v_p][v+v_i])(r-1)\bar{p} \\ &\quad - (\hat{F}_i^3 c[1+N][v+v_i]^2 + \hat{F}_i c[1+N]v_l[v+v_i])(r-1)\bar{p} + v_l v(1+[r-1]\bar{p}). \end{aligned} \quad (\text{A24})$$

Exploiting the steady-state equilibrium condition, $K = 0$, allows eqn. (A24) to be rewritten as:

$$\begin{aligned} \frac{dK}{dv_p} &= -\frac{d\hat{F}_i}{dv_p} [3\hat{F}_i^2 c(1+N)v_p(v+v_i) \\ &\quad + c(1+N)v_l(v+v_i+v_p)(v+v_i)](r-1)\bar{p} + \frac{1}{v_p} \hat{F}_i c(1+N)v_l(v+v_i)^2(r-1)\bar{p} > 0 \end{aligned} \quad (\text{A25})$$

because Lemma A3 implies $\frac{d\hat{F}_i}{dv_p} < 0$. Therefore, $\frac{d\bar{p}}{dv_p} > 0$. Q.E.D.

The following lemmas are used to prove Corollary 2.

Lemma A4. At the interior steady-state p , $\bar{p} \in (0, 1)$, $[1 - N(r-1)\bar{p}] > 0$.

Proof of Lemma A4.

Using the equilibrium conditions we can show that $covariance(d_h - d_b, d_b)$ is proportional to $(1 - [r-1]Np)\frac{v}{v+v_i} - \lambda_p$. Noting that $\lambda_p > 0$ for all p and that the steady-state condition requires $(1 - [r-1]N\bar{p})\frac{v}{v+v_i} - \bar{\lambda}_p = 0$, it follows that $(1 - [r-1]N\bar{p}) > 0$. Q.E.D.

Lemma A5. The static equilibrium F_i , \hat{F}_i , is increasing in r : $\frac{d\hat{F}_i}{dr} > 0$.

Proof of Lemma A5

Using the equilibrium condition provided by eqn. (A7) to perform standard comparative statics analysis yields:

$$\frac{d\hat{F}_i}{dr} = \frac{-\frac{\partial J}{\partial r}}{\frac{\partial J}{\partial \hat{F}_i}} \quad (\text{A26})$$

Recalling that the denominator is strictly positive, the sign of the numerator is the sign of $-\frac{d\hat{F}_i}{dr}$. The numerator can be written as:

$$-\frac{\partial J}{\partial r} = \hat{F}_i^2 N p v_p v (v + v_i) + N p v_l v (v + v_i + v_p) > 0. \quad (\text{A27})$$

Therefore $\frac{d\hat{F}_i}{dr} > 0$. Q.E.D.

Proof of Corollary 2.

Using standard comparative statics analysis on the steady-state equilibrium condition, $K = 0$, yields: $\frac{d\bar{p}}{dr} = \frac{\frac{dK}{dr}}{-\frac{dK}{dp}}$, $\frac{d\bar{p}}{dc} = \frac{\frac{dK}{dc}}{-\frac{dK}{dp}}$, $\frac{d\bar{p}}{dv_i} = \frac{\frac{dK}{dv_i}}{-\frac{dK}{dp}}$, and $\frac{d\bar{p}}{dN} = \frac{\frac{dK}{dN}}{-\frac{dK}{dp}}$. Recall that $-\frac{dK}{dp} > 0$ at \bar{p} so the sign of $\frac{d\bar{p}}{dz}$ is the same as the sign of $\frac{dK}{dz}$ for variable z . $\frac{dK}{dr}$ satisfies:

$$\begin{aligned} \frac{dK}{dr} = & -\frac{d\hat{F}_i}{dr} (3\hat{F}_i^2 c[1+N]v_p[v+v_i]^2 + c[1+N]v_l[v+v_i+v_p][v+v_i])(r-1)\bar{p} \\ & -(\hat{F}_i^3 c[1+N]v_p[v+v_i]^2 + \hat{F}_i c[1+N]v_l[v+v_i+v_p][v+v_i])\bar{p} + v_p v_l v \bar{p}. \end{aligned} \quad (\text{A28})$$

Exploiting the steady-state equilibrium condition, $K = 0$, allows eqn. (A30) to be rewritten as:

$$\begin{aligned} \frac{dK}{dr} = & -\frac{d\hat{F}_i}{dr} (3\hat{F}_i^2 c[1+N]v_p[v+v_i]^2 + c[1+N]v_l[v+v_i+v_p][v+v_i])(r-1)\bar{p} \\ & -\frac{v_p v_l v}{r-1} < 0 \end{aligned} \quad (\text{A29})$$

because $\frac{d\hat{F}_i}{dr} > 0$. $\frac{dK}{dc}$ satisfies:

$$\frac{dK}{dc} = -\frac{d\hat{F}_i}{dc} (3\hat{F}_i^2 c[1+N]v_p[v+v_i]^2 + c[1+N]v_l[v+v_i+v_p][v+v_i])(r-1)\bar{p}$$

$$-(\hat{F}_i^3[1+N]v_p[v+v_i]^2 + \hat{F}_i[1+N]v_l[v+v_i+v_p][v+v_i])(r-1)\bar{p}. \quad (\text{A30})$$

Doing standard comparative statics analysis, we can use the static equilibrium condition, eqn. (A7), to derive $\frac{d\hat{F}_i}{dc}$. We then substitute in for $\frac{d\hat{F}_i}{dc}$ in eqn. (A32) and simplify to obtain:

$$\frac{dK}{dc} \propto -(1 - [r-1]N\bar{p}) < 0 \quad (\text{A31})$$

because $(1 - [r-1]N\bar{p}) > 0$ by Lemma A4. $\frac{dK}{dv_l}$ satisfies:

$$\begin{aligned} \frac{dK}{dv_l} = & -\frac{d\hat{F}_i}{dv_l}(3\hat{F}_i^2c[1+N]v_p[v+v_i]^2 + c[1+N]v_l[v+v_i+v_p][v+v_i])(r-1)\bar{p} \\ & -\hat{F}_i^3c[1+N][v+v_i+v_p][v+v_i](r-1)\bar{p} + v_pv(1 + [r-1]\bar{p}). \end{aligned} \quad (\text{A32})$$

Exploiting the equilibrium condition $K = 0$ allows eqn. (A32) to be rewritten as:

$$\begin{aligned} \frac{dK}{dv_l} = & -\frac{d\hat{F}_i}{dv_l}(3\hat{F}_i^2c[1+N]v_p[v+v_i]^2 + c[1+N][v+v_i+v_p][v+v_i])(r-1)\bar{p} \\ & +\frac{1}{v_l}\hat{F}_i^3c[1+N]v_p[v+v_i]^2(r-1)\bar{p}. \end{aligned} \quad (\text{A33})$$

Doing standard comparative statics analysis, we can use the static equilibrium condition, eqn. (A7), to derive $\frac{d\hat{F}_i}{dv_l}$. We then substitute in for $\frac{d\hat{F}_i}{dv_l}$ in eqn. (A33) and simplify to obtain:

$$\frac{dK}{dv_l} \propto -(1 - [r-1]N\bar{p}) < 0 \quad (\text{A34})$$

because $(1 - [r-1]N\bar{p}) > 0$ by Lemma A4. $\frac{dK}{dN}$ satisfies:

$$\begin{aligned} \frac{dK}{dN} = & -\frac{d\hat{F}_i}{dN}(3\hat{F}_i^2c[1+N]v_p[v+v_i]^2 + c[1+N]v_l[v+v_i+v_p][v+v_i])(r-1)\bar{p} \\ & -(\hat{F}_i^3cv_p[v+v_i]^2 + \hat{F}_i^2cv_l[v+v_i+v_p][v+v_i])(r-1)\bar{p} < 0 \end{aligned} \quad (\text{A35})$$

because $\frac{d\hat{F}_i}{dN} > 0$ by Lemma A2. Q.E.D.

Proof of Corollary 3.

Using eqn. (A4), the change in the static equilibrium λ_D , $\hat{\lambda}_D$, arising from a change in p is

$$\frac{d\hat{\lambda}_D}{dp} = \frac{\frac{d\hat{F}_i}{dp} v_p v (v_l [v + v_i + v_p] - \hat{F}_i^2 v_p [v + v_i])}{(\hat{F}_i^2 v_p [v + v_i] + v_l [v + v_i + v_p])^2}. \quad (\text{A36})$$

Because Lemma A1 implies that $\frac{d\hat{F}_i}{dp} > 0$, $\frac{d\hat{\lambda}_D}{dp}$ is of the same sign as $v_l(v + v_i + v_p) - \hat{F}_i^2 v_p(v + v_i)$. Because $\frac{d\hat{F}_i}{dp} > 0$, it follows that if $\frac{d\hat{\lambda}_D}{dp} = 0$ at some \check{p} , then $\frac{d\hat{\lambda}_D}{dp} > 0$ for $p < \check{p}$ and $\frac{d\hat{\lambda}_D}{dp} < 0$ for $p > \check{p}$. The proof is completed by deriving conditions under which $v_l(v + v_i + v_p) - \hat{F}_i^2 v_p(v + v_i) \leq 0$ at $p = 0$, and $v_l(v + v_i + v_p) - \hat{F}_i^2 v_p(v + v_i) \geq 0$ at $p = 1$. If $v_l(v + v_i + v_p) - \hat{F}_i^2 v_p(v + v_i) \leq 0$ at $p = 0$, it must be the case that at $F_i = \sqrt{\frac{v_l(v+v_i+v_p)}{v_p(v+v_i)}}$ and $p = 0$, $J \leq 0$. Evaluating J at $\sqrt{\frac{v_l(v+v_i+v_p)}{v_p(v+v_i)}}$ when $p = 0$ yields

$$\frac{v_l(v + v_i + v_p)}{v_p} (2c[1 + N] \sqrt{v_l v_p (v + v_i)(v + v_i + v_p)} - [N - 1] v_p v), \quad (\text{A37})$$

which is weakly negative if and only if

$$c \leq c_l = \frac{(N - 1) v_p v}{2(1 + N) \sqrt{v_l v_p (v + v_i)(v + v_i + v_p)}} \quad (\text{A38})$$

If $v_l(v + v_i + v_p) - \hat{F}_i^2 v_p(v + v_i) \geq 0$ at $p = 1$, it must be the case that at $F_i = \sqrt{\frac{v_l(v+v_i+v_p)}{v_p(v+v_i)}}$ and $p = 1$, $J \leq 0$. Evaluating J at $\sqrt{\frac{v_l(v+v_i+v_p)}{v_p(v+v_i)}}$ when $p = 1$ yields

$$\frac{v_l(v + v_i + v_p)}{v_p} [2c[1 + N] \sqrt{v_l v_p (v + v_i)(v + v_i + v_p)} - [\{2r - 1\}N - 1] v_p v), \quad (\text{A39})$$

which is weakly positive if and only if

$$c \geq c_h = \frac{([\{2r - 1\}N - 1] v_p v)}{2(1 + N) \sqrt{v_l v_p (v + v_i)(v + v_i + v_p)}} \geq c_l. \quad (\text{A40})$$

Therefore, $\hat{\lambda}_D$ is globally decreasing in p if $c \leq c_l$, $\hat{\lambda}_D$ is increasing and then decreasing in p if $c \in (c_l, c_h)$, and $\hat{\lambda}_D$ is globally increasing in p if $c \geq c_h$. Q.E.D.

Proof of Observation 1.

Using eqn. (A4) we can write the steady state value for $\lambda_D, \bar{\lambda}_D$, as

$$\bar{\lambda}_D = \frac{\bar{F}_i v_p v}{\bar{F}_i^2 v_p (v + v_i) + v_l (v + v_i + v_p)}, \quad (\text{A41})$$

where \bar{F}_i denotes the steady state equilibrium value for F_i (i.e., $\bar{F}_i = \hat{F}_i$ where \hat{F}_i is evaluated at \bar{p}). Total differentiation of the right hand side of eqn. (A41) with respect to v_p yields

$$\begin{aligned} \frac{d\bar{\lambda}_D}{dv_p} &\propto \bar{F}_i v_l (v + v_i) + \frac{d\bar{F}_i}{dv_p} v_p [v_l v (v + v_i + v_p) - \bar{F}_i^2 v_p (v + v_i)] \\ &\propto \bar{F}_i v_l (v + v_i) \left(\frac{\partial J}{\partial p} \frac{\partial K}{\partial F_i} - \frac{\partial J}{\partial F_i} \frac{\partial K}{\partial p} \right) + v_p [v_l (v + v_i + v_p) \\ &\quad - \bar{F}_i^2 v_p (v + v_i)] \left(\frac{\partial J}{\partial v_p} \frac{\partial K}{\partial p} - \frac{\partial J}{\partial p} \frac{\partial K}{\partial v_p} \right), \end{aligned} \quad (\text{A42})$$

where $\left(\frac{\partial J}{\partial p} \frac{\partial K}{\partial F_i} - \frac{\partial J}{\partial F_i} \frac{\partial K}{\partial p} \right)$ and $\left(\frac{\partial J}{\partial v_p} \frac{\partial K}{\partial p} - \frac{\partial J}{\partial p} \frac{\partial K}{\partial v_p} \right)$ are evaluated at the steady state values. Note that, at the steady state values, $\frac{\partial J}{\partial p} \frac{\partial K}{\partial F_i} > 0$, $-\frac{\partial J}{\partial F_i} \frac{\partial K}{\partial p} > 0$, $\frac{\partial J}{\partial v_p} \frac{\partial K}{\partial p} < 0$, and $-\frac{\partial J}{\partial p} \frac{\partial K}{\partial v_p} > 0$. Therefore, we complete the proof by showing that $-\bar{F}_i v_l (v + v_i) \frac{\partial J}{\partial F_i} \frac{\partial K}{\partial p} + v_p v_l (v + v_i + v_p) \frac{\partial J}{\partial v_p} \frac{\partial K}{\partial p} > 0$ and $\bar{F}_i v_l (v + v_i) \frac{\partial J}{\partial p} \frac{\partial K}{\partial F_i} + \bar{F}_i^2 v_p v (v + v_i) \frac{\partial J}{\partial p} \frac{\partial K}{\partial v_p} > 0$. Using the equilibrium conditions, it can be shown that

$$\begin{aligned} -\bar{F}_i v_l (v + v_i) \frac{\partial J}{\partial F_i} \frac{\partial K}{\partial p} + v_p v_l (v + v_i + v_p) \frac{\partial J}{\partial v_p} \frac{\partial K}{\partial p} &\propto \\ \bar{F}_i^3 c (1 + N) v_p (v + v_i)^2 + N [1 + (r - 1) \bar{p}] v_l v (v + v_i + v_p) &> 0, \end{aligned} \quad (\text{A43})$$

and

$$\begin{aligned} \bar{F}_i v_l (v + v_i) \frac{\partial J}{\partial p} \frac{\partial K}{\partial F_i} + \bar{F}_i^2 v_p v (v + v_i) \frac{\partial J}{\partial p} \frac{\partial K}{\partial v_p} &\propto 3\bar{F}_i^2 v_p (v + v_i) + v_l (v + v_i + v_p) \\ &> 0. \end{aligned} \quad (\text{A44})$$

Q.E.D.

Proof of Corollary 4.

The static equilibrium variance of the price can be expressed as

$$Var[\tilde{P}] = \frac{v^2}{v + v_i} \frac{Z_{11}}{Z_{12}} + 2cvZ_2 + c^2[(\hat{F}_i - Z_3)^2(v + v_i) + Z_3^2v_p + v_l] \quad (\text{A45})$$

where

$$\begin{aligned} Z_{11} &= \hat{F}_i^4 v_p^2 (v + v_i)^2 + 2\hat{F}_i^2 v_p v_l (v + v_i)^2 \\ &\quad + v_l^2 (v + v_i + v_p)(v + v_i) + \hat{F}_i^2 v_p^2 v_l (v + v_i) \end{aligned} \quad (\text{A46})$$

$$\begin{aligned} Z_{12} &= \hat{F}_i^4 v_p^2 (v + v_i)^2 + 2\hat{F}_i^2 v_p v_l (v + v_i)^2 \\ &\quad + v_l^2 (v + v_i + v_p)(v + v_i) + 2\hat{F}_i^2 v_p^2 v_l (v + v_i) + v_p v_l^2 (v + v_i + v_p), \end{aligned} \quad (\text{A47})$$

$$\begin{aligned} Z_2 &= \hat{F}_i(\hat{F}_i + \hat{F}_p) \\ &= \hat{F}_i - \frac{\hat{F}_i N v_l v (v + v_i)}{\hat{F}_i^3 c(1 + N)v_p(v + v_i)^2 + \hat{F}_i^2 v_p v (v + v_i) + \hat{F}_i c(1 + N)v_l(v + v_i + v_p)(v + v_i)}, \end{aligned} \quad (\text{A48})$$

and

$$Z_3 = \frac{N v_l v}{\hat{F}_i^2 c(1 + N)v_p(v + v_i) + \hat{F}_i v_p v + c(1 + N)v_l(v + v_i + v_p)}. \quad (\text{A49})$$

It is a straight forward exercise to show that Z_1 is increasing in \hat{F}_i and is not a function of p . Note that $1 > \frac{N v_l v (v + v_i)}{\hat{F}_i^3 c(1 + N)v_p(v + v_i)^2 + \hat{F}_i^2 v_p v (v + v_i) + \hat{F}_i c(1 + N)v_l(v + v_i + v_p)(v + v_i)}$ because of the static equilibrium condition given by eqn. (A7), $J = 0$. Thus, Z_2 is also increasing in \hat{F}_i and is not a function of p . Finally, because $\hat{F}_i > II$ and II is decreasing in \hat{F}_i , it can be shown that Z_3 is also decreasing in \hat{F}_i and is not a function of p . The proof is completed by noting that Lemma A1 states that \hat{F}_i is increasing in p . Q.E.D.

Proof of Observation 2.

Using the steady state conditions, the steady state variance of the price can be expressed as:

$$\overline{Var}[\tilde{P}] = \left[\frac{v}{(1+N)(v+v_i)} \right]^2 \left[(1+N)^2(v+v_i) + [1-(r-1)N\bar{p}]^2 v_p + \left(\frac{N[1+(r-1)\bar{p}]}{\bar{F}_i} \right)^2 v_l \right]. \quad (\text{A50})$$

Thus, to complete the proof we need to show that $L = [1-(r-1)N\bar{p}]^2 v_p + \left(\frac{N[1+(r-1)\bar{p}]}{\bar{F}_i} \right)^2 v_l$ is increasing in v_p . Differentiating L with respect to v_p yields:

$$\begin{aligned} \frac{dL}{dv_p} \propto & (r-1)\bar{p}[1-N(r-1)\bar{p}]^2 \bar{F}_i^3 (v+v_i) \\ & + \frac{d\bar{p}}{dv_p} (r-1) \{ 2N[1+(r-1)\bar{p}][1-(r-1)N\bar{p}]\bar{F}_i v_p v_l \} \\ & - \frac{d\bar{F}_i}{dv_p} [2N^2(r-1)\bar{p}[1+(r-1)\bar{p}]^2 v_l (v+v_i)]. \end{aligned} \quad (\text{A51})$$

The first term on the right hand side of eqn. (A51) is clearly positive. We complete the proof by showing that the last two terms is positive as well. Let M denote the sum of the last to terms in $\frac{dL}{dv_p}$. We can exploit the equilibrium conditions to show that

$$\begin{aligned} M \propto & \bar{F}_i c(1+n)[1+(r-1)\bar{p}]v_l \{ N(r-1)\bar{p}(v+v_i) - [1-(r-1)N\bar{p}]v_p \} (3N+2) \\ & + \{ 1 - [(r-1)\bar{p}]^2 \} N v_l \{ N[1+(r-1)v - \hat{F}_i c(1+N)(v+v_i)] \}. \end{aligned} \quad (\text{A52})$$

Using the two equilibrium equations, $J = 0$ [eqn. (A7)] and $K = 0$ [eqn. (A16)], it is possible to show that $N(r-1)\bar{p}(v+v_i) - [1-(r-1)N\bar{p}]v_p > 0$. The static equilibrium condition, $J = 0$, and the steady state equilibrium condition observation in Lemma A4 that $[1-(r-1)N\bar{p}] > 0$ imply that $N[1+(r-1)v - \bar{F}_i c(1+N)(v+v_i)] > 0$ and $1 - [(r-1)\bar{p}]^2 > 0$ respectively. Thus, the right hand side of eqn. (A52) is strictly positive. Q.E.D.

Lemma A5. The static equilibrium $\lambda_p, \hat{\lambda}_p$, is decreasing in p : $\frac{d\hat{\lambda}_p}{dp} < 0$.

Proof of Lemma A5

Using the static equilibrium condition for λ_p given by eqn. (A5) yields

$$\frac{d\lambda_p}{dp} = \frac{-v_p v_l v (v + v_i) \hat{F}_i^2 (r - 1) - \frac{d\hat{F}_i}{dp} 2v_p v_l v (v + v_i) (r - 1) p (1 + [r - 1]p)}{(\hat{F}_i^2 v_p [v + v_i] [r - 1] p + v_l [v + v_i + v_p] [1 + (r - 1)p])^2} < 0, \quad (\text{A53})$$

because, by Lemma A1, $\frac{d\hat{F}_i}{dp} > 0$. Q.E.D.

Proof of Corollary 5.

Note that the unconditional variance of \tilde{u} conditioned upon x_i, x_p is $\frac{vv_i}{v+v_i}$. Thus the change in efficiency is solely a function of the change in the static equilibrium variance of \tilde{u} conditioned upon P :

$$Var[\tilde{u}|P] = v - \frac{Cov[\tilde{u}, \tilde{P}]^2}{Var[\tilde{P}]}. \quad (\text{A54})$$

The proof proceeds by showing that $\frac{Cov[\tilde{u}, \tilde{P}]^2}{Var[\tilde{P}]}$ is increasing in p . We can use the static equilibrium conditions to show that

$$\frac{Cov[\tilde{u}, \tilde{P}]^2}{Var[\tilde{P}]} = \frac{v^2}{v + v_i + \left[\frac{\hat{\lambda}_p}{\hat{\lambda}_p (v + v_i) + N(1 + [r - 1]p)v} \right]^2 (v + v_i)^2 v_p + \left[\frac{N(1 + [r - 1]p)}{\hat{\lambda}_p \hat{F}_i (v + v_i) + N(1 + [r - 1]p) \hat{F}_i v} \right]^2 v^2 v_l}. \quad (\text{A55})$$

In order to show that $\frac{Cov[\tilde{u}, \tilde{P}]^2}{Var[\tilde{P}]}$ is increasing in p , we show that $\frac{\hat{\lambda}_p}{\hat{\lambda}_p (v + v_i) + N(1 + [r - 1]p)v}$ and $\frac{N(1 + [r - 1]p)}{\hat{\lambda}_p \hat{F}_i (v + v_i) + N(1 + [r - 1]p) \hat{F}_i v}$ are both decreasing in p . Noting that Lemma A5 implies that $\hat{\lambda}_p$ is decreasing in p it follows directly that $\frac{\hat{\lambda}_p}{\hat{\lambda}_p (v + v_i) + N(1 + [r - 1]p)v}$ is decreasing in p . We can rewrite $\frac{N(1 + [r - 1]p)}{\hat{\lambda}_p \hat{F}_i (v + v_i) + N(1 + [r - 1]p) \hat{F}_i v}$ as follows:

$$\frac{N(1 + [r - 1]p)}{\hat{\lambda}_p \hat{F}_i (v + v_i) + N(1 + [r - 1]p) \hat{F}_i v} = \frac{vE}{\hat{F}_i v_l v (v + v_i) + \hat{F}_i E} \quad (\text{A56})$$

where $E = \hat{F}_i^3 c(1+N)v_p(v+v_i)^2 + \hat{F}_i^2 v_p v(v+v_i) + \hat{F}_i c(1+N)v_l(v+v_i+v_p)(v+v_i)$.

Differentiation of $\frac{vE}{\hat{F}_i v_l v(v+v_i) + \hat{F}_i E}$ with respect to \hat{F}_i yields

$$\begin{aligned}
\frac{d \frac{vE}{\hat{F}_i v_l v(v+v_i) + \hat{F}_i E}}{d\hat{F}_i} &\propto -[\hat{F}_i^3 c(1+N)v_p(v+v_i)^2 + \hat{F}_i^2 v_p v(v+v_i) \\
&\quad + \hat{F}_i c(1+N)v_l(v+v_i+v_p)(v+v_i)] \\
&\quad [\hat{F}_i^2 v_p v(v+v_i)N(r-1)p + v_l v(v+v_i+v_p)(1+[r-1]p)] \\
&\quad + 2\hat{F}_i^3 c(1+N)v_p v_l v(v+v_i)^3 + \hat{F}_i^2 v_p v_l v^2(v+v_i)^2 \\
&= -\hat{F}_i^3 c(1+N)v_p v_l v(v+v_i)^2[v_p + (v+v_i+v_p)(r-1)p] \\
&\quad - \hat{F}_i^2 v_p v_l v^2(v+v_i)[v_p + (v+v_i+v_p)(r-1)p] \\
&\quad - \hat{F}_i c(1+N)v_p^2 v_l v(v+v_i)^2 \\
&\quad - [\hat{F}_i^3 c(1+N)v_p v_l v(v+v_i)^2 + \hat{F}_i^2 v_p v(v+v_i)][\hat{F}_i^2 v_p v(v+v_i)N(r-1)p] \\
&\quad - \hat{F}_i c(1+N)v_l v(v+v_i+v_p)(v+v_i) \\
&\quad [v_l(v+v_i+v_p)(1+[r-1]p) - \hat{F}_i v_p(v+v_i)(1-[r-1]p)] \\
&< 0
\end{aligned} \tag{A57}$$

because $v_l(v+v_i+v_p)(1+[r-1]p) - \hat{F}_i v_p(v+v_i)(1-[r-1]p) > 0$ by the static equilibrium condition given by eqn. (A7), $J = 0$. Noting that the term on the right hand side of eqn. (A56) is not directly a function of p and that \hat{F}_i is increasing in p by Lemma A1, it follows from eqn. (A57) that $\frac{N(1+[r-1]p)}{\lambda_p \hat{F}_i(v+v_i) + N(1+[r-1]p)\hat{F}_i v} = \frac{vE}{\hat{F}_i v_l v(v+v_i) + \hat{F}_i E}$ is decreasing in p . Therefore, $\frac{Cov[\tilde{u}, \hat{P}]^2}{Var[\hat{P}]}$ is increasing in p which implies that $Var[\tilde{u}|P]$ is decreasing in p . Q.E.D.

Proof of Observation 3.

Note that the steady state unconditional variance of \tilde{u} conditioned upon x_i, x_p is $\frac{vv_i}{v+v_i}$. Thus the change in efficiency is solely a function of the change in the steady state variance of \tilde{u} conditioned upon P :

$$\overline{Var}[\tilde{u}|P] = v - \frac{\overline{Cov}[\tilde{u}, \tilde{P}]^2}{\overline{Var}[\tilde{P}]}.$$
 (A58)

The steady state value for the covariance satisfies

$$\overline{Cov}[\tilde{u}, \tilde{P}] = \frac{v^2}{v + v_i}$$
 (A59)

which implies that the change in steady state price efficiency due to a change in v_p has the opposite sign of $\frac{d\overline{Var}[\tilde{P}]}{dv_p}$. From the proof of Observation 2 we know that $\frac{d\overline{Var}[\tilde{P}]}{dv_p} > 0$. Q.E.D.

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Table 1: Some Notation

N - the number of risk neutral informed traders

p - the proportion of informed traders who are heuristic

\tilde{u} - firm cash flow, which has a normal distribution with mean 0 and variance v

$\tilde{x}_p = \tilde{u} + \tilde{\epsilon}_i + \tilde{\epsilon}_p$ - public disclosure about firm cash flow, where:

$\tilde{\epsilon}_i$ has a normal distribution with mean 0 and variance v_i , and

$\tilde{\epsilon}_p$ has a normal distribution with mean 0 and variance v_p

$\tilde{x}_i = \tilde{u} + \tilde{\epsilon}_i$ - private information about firm cash flow

$r > 1$ - heuristic traders' response to private information

\tilde{D} - total net demand order

\tilde{d}_l - liquidity demand, which has a normal distribution with mean 0 and variance v_l

$c > 0$ - exogenous cost of supplying liquidity

h - a heuristic-type trader

b - a Bayesian-type trader

$\tilde{P} = (\lambda_D + c)\tilde{D} + \lambda_p\tilde{x}_p$ - price at which trades are executed, where:

λ_D is the weight the market maker places on total net demand, and

λ_p is the weight the market maker places on public information

f_{ih} - the weight a heuristic informed trader places on private information

f_{ib} - the weight a Bayesian informed trader places on private information

$f_{ph} = f_{pb}$ - the weight all informed traders place on public information

$F_i = \sum_{n=1}^N f_{in}$ - the sum of all informed traders' weights on private information

$F_p = \sum_{n=1}^N f_{pn}$ - the sum of all informed traders' weights on public information