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Duality between time and frequency domains for vibration serviceability analysis of floor structures

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Abstract

For vibration serviceability of floors, current design guidelines adopt different criteria to assess vibration levels due to human walking dynamic excitation. Whatever the adopted criterion is, it requires a quantified vibration response of the structure. This quantification could be achieved following either a time- or a frequency-domain approach to response analysis. Each approach has its advantages and disadvantages. For instance, when using the time-domain analysis, exact time-domain amplitudes of the response time histories could be quantified but the process could take time. On the other hand, a frequency-domain analysis approach could reduce the calculation time, but it is impossible to recover exact time-domain amplitudes of the response, which is essentially averaged by the process of calculation. In this paper, the theoretical duality between time and frequency domains is examined practically in the context of vibration serviceability of a floor structure. Weight-normalised vertical ground reaction force (GRF) measured on an instrumented treadmill due to walking is used for that purpose because it has realistic distribution of energy in the frequency domain. This GRF is applied on a finite element model of a reinforced concrete high-frequency floor and the responses are calculated via both time and frequency domain analyses. Comparison of these two methods reveals that time-domain analysis could introduce significant errors in the calculated vibration responses. This is due to the errors in the numerical solution of equation of motion.

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Keywords: Vibration Serviceability; Floor Structures; Ground Reaction Force; Time-domain analysis; Frequency-domain analysis

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1. Introduction

Vibration serviceability has become the dominant design criterion for modern lightweight, slender floor structures. Therefore, this issue is rapidly gaining attention in academia and industry [1]. Although some topics within the specific area of vibration serviceability are under-researched, the theoretical background of vibration engineering is well-established [2–4]. One of the well-researched areas is the methods to calculate the responses of a structure. To evaluate those responses, two approaches are available: time-domain analysis and frequency-domain analysis. For vibration serviceability assessment, some design guidelines follow the time-domain approach [5–7] while other guidelines adopts the frequency-domain counterpart [8]. In this paper, both analyses are used to evaluate the vibration response of a structure due to measured walking force which is a narrow band random function.

2. Prototype structure

A prototype reinforced concrete structure was modelled using the finite element package ANSYS® Academic Research, Release 17.1 [9]. The floor was modelled as an orthotropic shell structure with a thickness of 150 mm. It has three spans of 4.0 m length, three bays of 6.0 m width, and is supported by 300 mm × 600 mm reinforced concrete beams spanning across the 6.0 m wide bays. The beams are supported by 4.2 m high reinforced concrete columns with a cross section of 300 mm × 300 mm. For vibration serviceability considerations, the floor was modelled following the recommended techniques available in the state-of-the-art design guidelines [5,6] where columns above and below the floor were modeled with fixed supports introduced at the far end from the floor. An overview of the floor FE model is shown in Fig. 1 (Left). Modal analysis was performed and 38 modes of vibration up to 60 Hz were calculated. The first six modes of vibration are shown in Fig. 1 (Right).

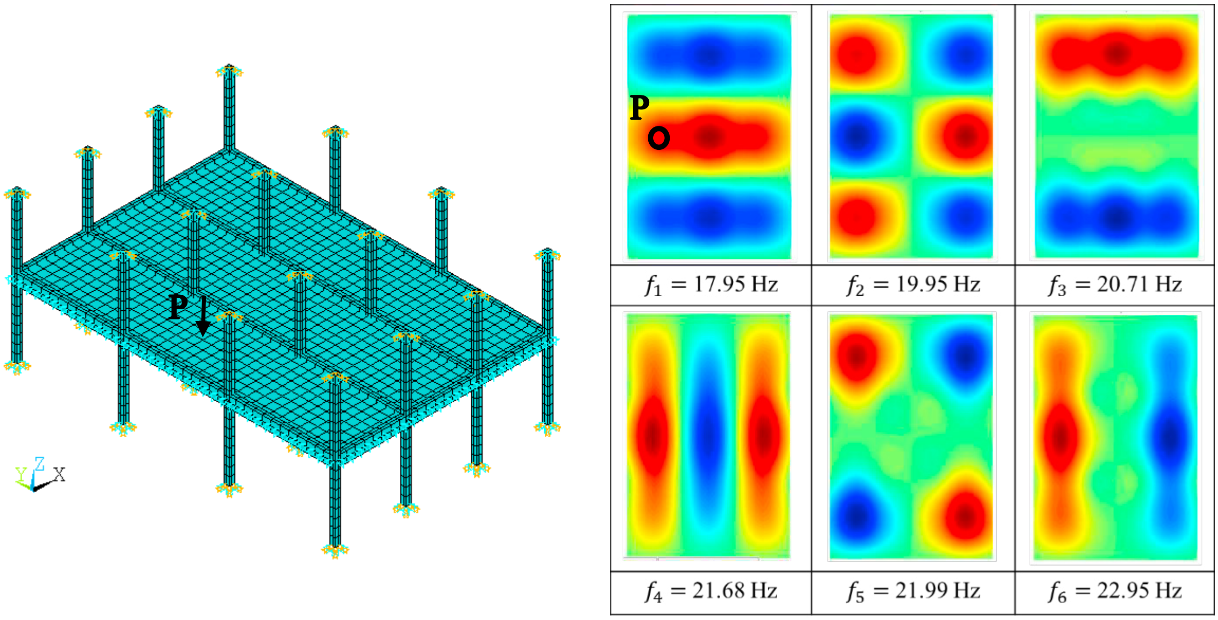

Fig. 1: Left – Overview of floor model showing excitation point. Right – First six modes of vibration. After [10]

Harmonic analysis using ANSYS® was conducted to estimate point-accelerance Frequency Response Function (FRF) at the excitation point shown in Fig. 1 (Left) [11]. The FRF modulus shown in Fig. 2 has a frequency resolution of 0.0488 Hz and it was calculated using mode superposition method and included all modes up to 60 Hz assuming damping of 2 % for each of them.

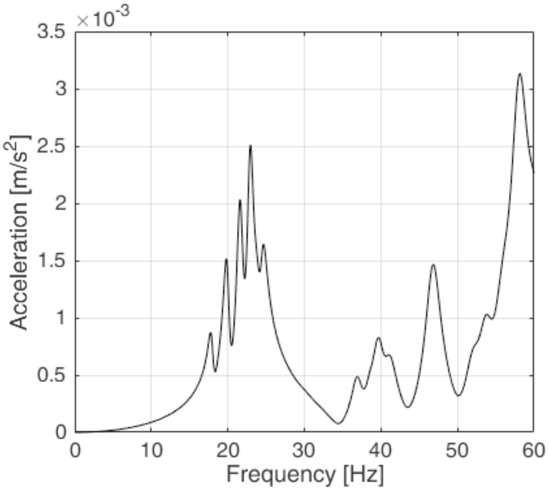

Fig. 2: Modulus of point-accelerance FRF at excitation point (P)

3. Analysis approach

Both time-domain and frequency-domain analyses were used to calculate the Auto Spectral Density (ASD) of the response, which is the basis for evaluating the criterion for assessing the vibration serviceability of floors. By utilising Discrete Fourier Transform (DFT), the *ASD* of a time history is given as:

$$ASD(f) = |DFT(f)|^2 \times 2df\ , \tag{1}$$

where f is the frequency, and df is the frequency resolution of the *DFT*. For all analyses in this paper, a real walking force measured using an instrumented treadmill was utilised. The walking force was sampled at 200 Hz and the test subject speed was controlled. The complete process of measuring the force is described in detail by Brownjohn et. al. [12]. Fig. 3 shows the force time history along with its normalized (by the weight of the test subject) discrete Fourier amplitudes where a data block of 20.475 s was used. The sampling frequency of the time history governs the size of the integration time-step in the time-domain analysis. Hence, it is required to resample the measured force in order to increase or decrease the integration time-step, which effect on the response calculations is investigated in this paper.

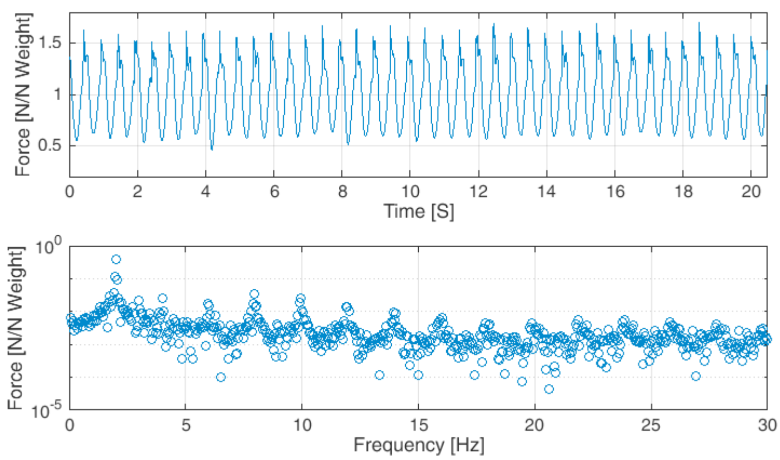

Fig. 3: Walking force time history and Fourier amplitudes.

3.1. Time-domain analysis

The dynamic response of a system to an applied excitation force *f(t)* can be obtained by solving the equation of motion given as:

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=f(t) \quad (2)$$

Where $\ddot{x}(t)$, $\dot{x}(t)$, and $x(t)$ are acceleration, velocity, and displacement of the system, respectively, m is the mass, c is the damping coefficient, and k is the system stiffness. To solve this equation, a numerical time integration method is usually utilised, such as the Newmark Integration. For such a technique, initial conditions must be introduced and those are given as initial velocity and displacement, $\dot{x}(0)$ and $x(0)$. Then, the solution at each step can be calculated based on the integration time step. This solution process is described in detail by Chopra [13] along with the guidance on how to implement it for programming. In this paper, four different time step values were used to evaluate the solution of the equation of motion given in Equation (2) and they are listed in Table 1.

Modal properties were imported from the ANSYS® modal analysis, then in MATLAB® the Newmark Integration method was used to solve the equation of motion and mode superposition was used to obtain the total response. The overall Root Mean Square (RMS) of each response time history is listed in Table 1 and it is given in Equation (3):

$$RMS=\sqrt{\frac{1}{N}\sum_{n=1}^{N}x_n^2}\,, \quad (3)$$

where x_n represent the samples, and N is the number of time-domain samples. N corresponds with the averaging time of the RMS which, for this analysis, was taken to represent the total time of the response time history (20.475 s). The RMS can also be calculated by calculating the area under the ASD curve [14].

Table 1: Summary of Newmark Integration results for different time integration steps

Integration time step [s]	Sampling Frequency [Hz]		RMS [m/s^2]
0.01	100	Half of the GRF original sampling frequency	3.31e-05
0.005	200	GRF original sampling frequency	3.54e-05
0.0025	400	Double the GRF original sampling frequency	3.58e-05
0.00083	1200	Recommended value of 1/20×maximum frequency of interest [9]	3.63e-05

The response time histories illustrating the effect of all four time steps on the analysis can be seen in Fig. 4. It can be seen that for smaller time step the peak accelerations are represented at points where larger time step fails to recover them. The ASD of each time history is presented in section 3.2, where each ASD is compared with its counterpart from the frequency-domain analysis involving FRFs and ASD of the forcing function.

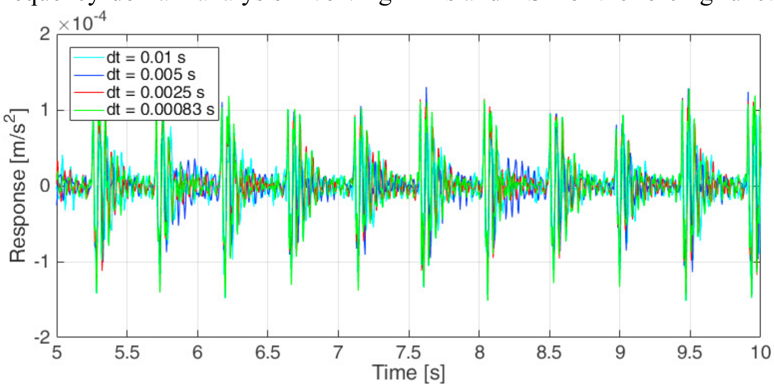

Fig. 4: Response time histories calculated using Newmark Integration with different integration time steps

3.2. Frequency-domain analysis

The dynamic response of a system due to random excitation can also be calculated in the frequency domain by applying the frequency response approach described by Newland [4] and given as:

$$ASD_y(f) = \left|H(f)\right|^2 ASD_x(f)\,, \tag{4}$$

where $ASD_y(f)$ and $ASD_x(f)$ are Auto Spectral Densities of the response and excitation, respectively. $H(f)$ is the complex Frequency Response Function (FRF) of the system.

To apply this approach, ANSYS® harmonic analysis option was used to obtain the FRF of the structure at the location (P) and then MATLAB® was used to apply Equation (4) using the ASD of the same walking force utilised in the time domain analysis. This process takes much less time than the Newmark Integration to perform. However, it is impossible to recover time-domain information from the resulting response ASD because it lacks phase information. For all frequency-domain analyses in this paper, an identical frequency resolution of 0.0625 Hz was used across all FRF and response ASD curves to guarantee fair comparison of the results.

Fig. 5, Fig. 6, Fig. 7, and Fig. 8 show a comparison between the results obtained from both time-domain analyses and the frequency-domain analysis where the trends of all ASD curves are similar. However, it is clearly shown that the time-domain approach is consistently failing to calculate correctly spectrum peaks corresponding to the resonances of the system. This behaviour is in line with the findings of other researchers [15,16], which suggest that the integration time step is not small enough to sample the response properly in the time-domain. Moreover, for a larger time step, not only the amplitude of the response at resonance is underestimated but also some frequency components are artificially introduced.

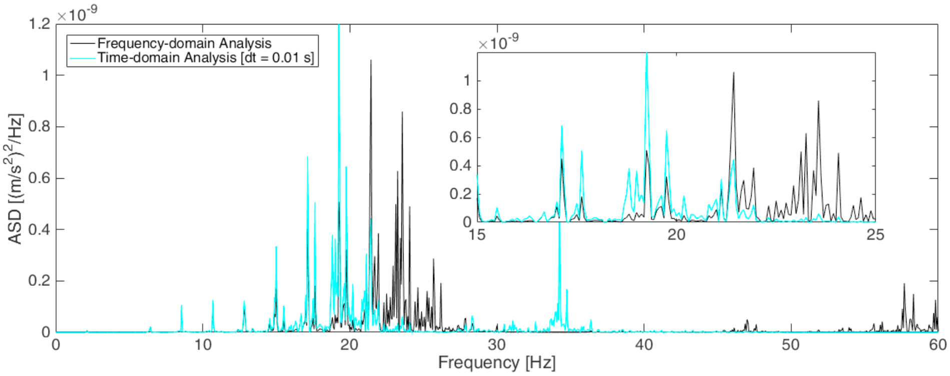

Fig. 5: Response ASD – Time-domain analysis (dt = 0.001 s) vs. Frequency-domain analysis

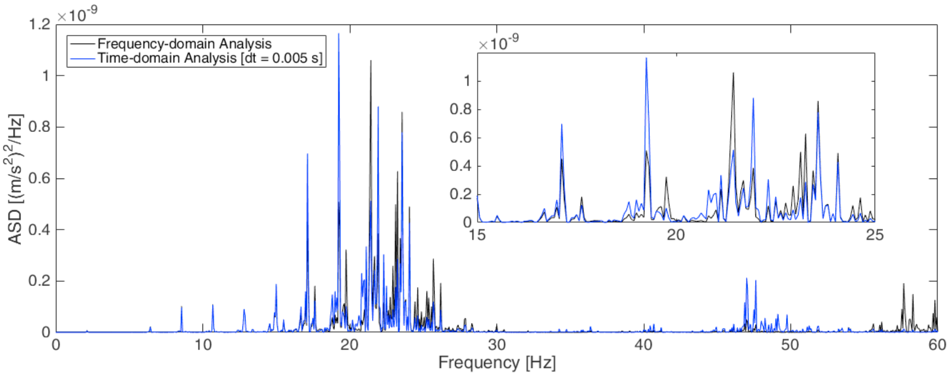

Fig. 6: Response ASD – Time-domain analysis (dt = 0.005 s) vs. Frequency-domain analysis

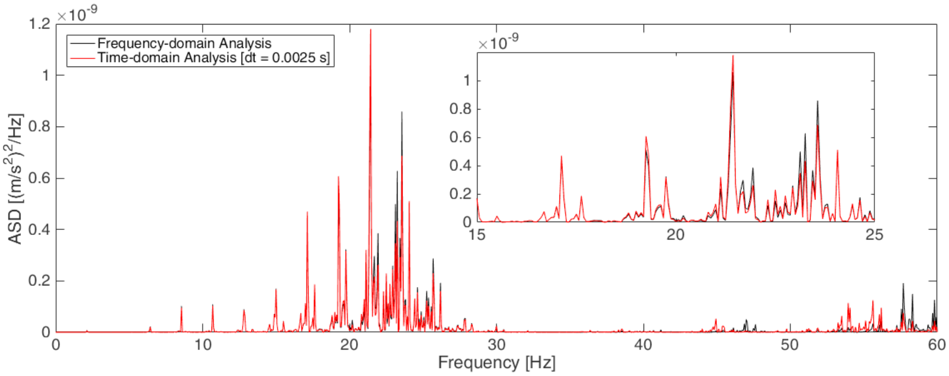

Fig. 7: Response ASD – Time-domain analysis (dt = 0.0025 s) vs. Frequency-domain analysis

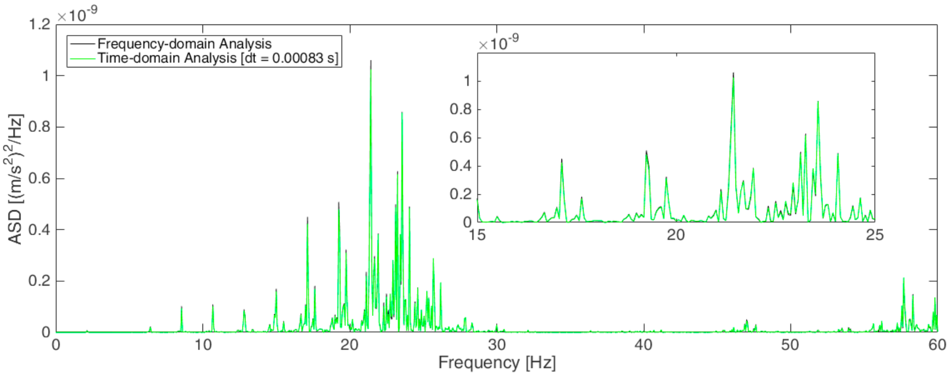

Fig. 8: Response ASD – Time-domain analysis (dt = 0.00083 s) vs. Frequency-domain analysis

To evaluate how much those differences could affect the assessment of the vibration serviceability of the floor, the RMS of each ASD obtained from time-domain analysis is compared with its counterpart obtained from frequency domain analysis. For this specific structure, the differences are presented in Table 2. The differences between the RMS values listed in Table 2 and their counterparts listed in Table 1 are due to numerical errors when resampling time histories. For larger time step, there is about 6% difference between the RMS calculated via the time- and frequency-domain approaches.

Table 2: RMS calculated based on ASD obtained from both time-domain analysis and frequency-domain analysis

Integration time step [s]	RMS [m/s^2] frequency-domain	RMS [m/s^2] time-domain	Error [%]
0.01	3.44e-05	3.22e-05	6.47
0.005	3.44e-05	3.42e-05	0.63
0.0025	3.44e-05	3.42e-05	0.47
0.00083	3.44e-05	3.43e-05	0.39

Looking at Table 2 it can be seen that the error is smallest for smaller time step, which corresponds to the recommended value of T/20 where T is the period of the highest mode of interest [9]. The difference for larger time step could potentially be misleading when assessing the vibration serviceability of the floor.

4. Conclusions

A response analysis was carried out on a prototype FE floor model using both time- and frequency-domain analyses. It is shown that the time-domain approach may fail to predict the responses of a linear system near resonances when compared with the frequency-domain approach. It is recommended that analysts should take extra care when utilising time-domain analysis for vibration serviceability assessment, as it may introduce errors when calculating RMS acceleration response.

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