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Optimised ambient vibration testing of long span bridges

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Abstract

Vibration testing of long span bridges is becoming a commissioning requirement. Long span bridges represent the extreme end of experimental capability, with challenges for logistics and access (due to length and location), instrumentation (due to frequency range, resolution and physical separation of accelerometers) and system identification (because of the extreme low frequencies).

Similar challenges apply to other extreme structures such as tall buildings, masts, offshore lighthouses and extended geotechnical structures stretching technology requirements for both instrumentation and signal interpretation. A solution for instrumentation is autonomous 'wireless' recorders. The problem with signal interpretation is the reliability of the modal parameter estimates that is particularly challenged with low frequency modes, which 'third generation' operational modal analysis procedures offer using Bayesian approaches.

The paper describes a preliminary exercise combining both these technologies in readiness for testing two very large bridges, one in China and one in Scotland.

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1. Introduction

The paper describes experimental research supporting development of new methodology for identification of modal parameters (MPs) from ambient vibration data, with most obvious direct application to extreme long span and tall structures (LSTS). The methodology will improve design of vibration field tests and structural health monitoring projects so that they can produce optimally reliable MP data with pre-defined precision and uncertainty.

Knowledge of structural MP, particularly modal frequencies and damping ratios is a fundamental requirement for design against dynamic loads such as wind, earthquakes and human excitation. MP estimation is central to finite element model updating and the dependent performance simulations. It is an effective means for uncertainty mitigation in structural dynamics, a vital prerequisite for design of vibration control devices and retrofits, and is fundamental to structural health monitoring systems where MP are classic indicators of structural state.

Reliable estimation of MPs such as damping is most critical in design for vibration serviceability, which governs design of tall buildings and for long span bridges where damping drives up flutter speeds. Damping estimates for design have mostly derived from operational modal analysis (OMA) of response data using techniques that over-estimate damping and do not quantify reliability, lead to under-conservative design.

Increasingly MP estimation is a requirement for structural commissioning tests to corroborate design. Forced vibration testing provides the best MP estimates but is technically unfeasible for LSTS which have to rely on less reliable MP from ambient vibration tests. OMA techniques such as stochastic subspace identification (SSI) and frequency domain decomposition (FDD) [1][2] are increasingly popular for such purposes.

The reliability issue of MP estimates from OMA are well recognised [3][4]. Methods for calculating the potential variability of MP estimates due to variability of data have been developed, e.g., for SSI [5][6][7][8]. Motivated by the need to quantify the uncertainty of MP, 'Third generation' Bayesian OMA methods [9] adopt a radically different philosophy to 'first' and 'second' generation OMA methods such as half power bandwidth and SSI respectively by posing the question 'what is known about MP based on the given data?'. Answers are expressed via the *posterior* probability density function (PDF) of MP implied fundamentally by Bayes' theorem [10]. The covariance matrix associated with the posterior PDF then naturally quantifies the identification uncertainty of MP for the given data set and modelling assumptions. This is in contrast to non-Bayesian methods where the uncertainty refers to the estimates of the MP rather than the MP itself, and is 'inherent' in nature, i.e., does not depend on data.

Algorithms for calculating uncertainty, be it Bayesian or non-Bayesian, only allow the value to be calculated in a 'point-wise' manner when the data are given. There has been no quantitative account for the origin of uncertainty, nor any quantitative guideline for test configuration to achieve a desired identification precision.

The BAYOMALAW research project will deliver 'uncertainty laws' using Bayesian inference to express MP uncertainties explicitly as functions of test configurations such as measurement noise, environmental load intensity and sensor configuration. The main objectives are to capture the effect of signal/noise ratio (SNR), close modes and multiple setups having more degrees of freedom (DOFs) than sensors. Hence BAYOMALAW will relate MP uncertainty directly to test configuration so that it can be *prescribed and managed*. Figure 1 provides an outline.

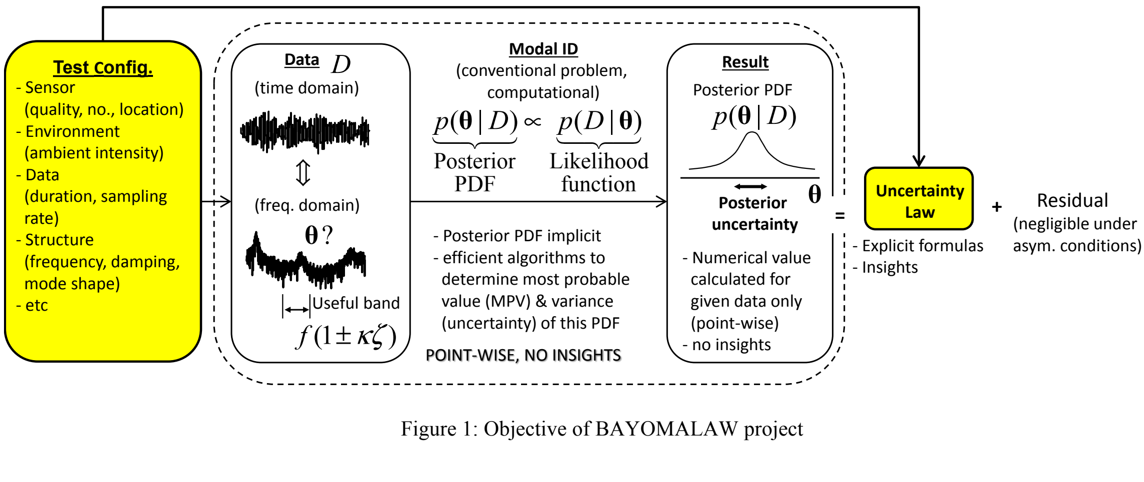


Figure 1: Objective of BAYOMALAW project

The project combines theoretical development with experimental trials and verification in a number of steps. Initially uncertainty laws are to be developed to define the effects of modal signal-to-noise ratio SNR. SNR quantifies test configurations such as measurement noise, environmental load intensities and the measured degrees of freedom. Next, uncertainty laws will be developed for close modes that are frequently encountered in modern structures, and are problematic due to the entangled frequency responses. Modes with close frequencies are frequently found in super-tall buildings or tower structures whose stiffness/mass properties along two horizontal principle directions are similar. Mode shapes with high modal assurance criteria can also be encountered due to limited measured DOFs, e.g., see Figure 2, whose modal testing is described in the last sections of the paper.

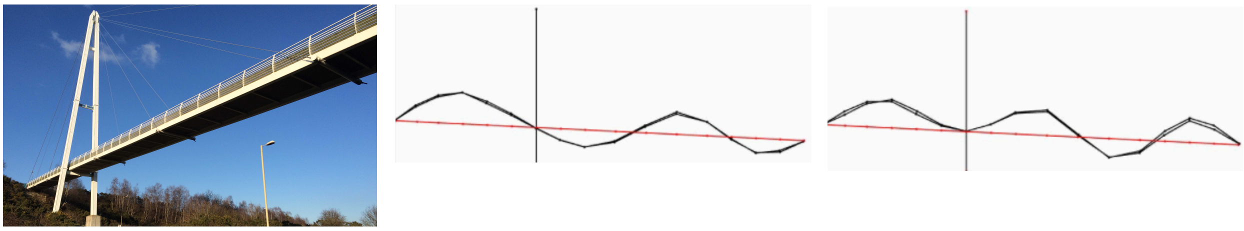


Figure 2: Similar modes: symmetry reverses at the support point.

Usually the number of measurement points on a structure exceeds the number of available sensors, so multiple setups are required. Resolving mode shapes this way requires 'gluing' pieces of data obtained from consecutive 'multiple setups' under different conditions, via a normalisation process with separate MP identification. Uncertainty laws for multiple setups are thus required.

The laws need to be tested on both historic and new data. Historic data represent a variety of configurations (sensor configurations, excitation intensities, numbers of modes). These data can be obtained from (inter alia) testing of Humber Bridge [11] and of offshore rock lighthouses (ibid).

With the aim of BAYOMALAW to advise design of ambient vibration testing for most reliable MP estimates, a series of tests planned for 2017 began with a short experimental campaign on Baker Bridge described in this paper.

2. Baker Bridge

Baker Bridge, shown in the Figure 2 and Figure 3, is a 109 m cable-stayed bridge that provides cycle and pedestrian access to Sandy Park Stadium, the home ground of Exeter Chiefs, a top-division English rugby club.

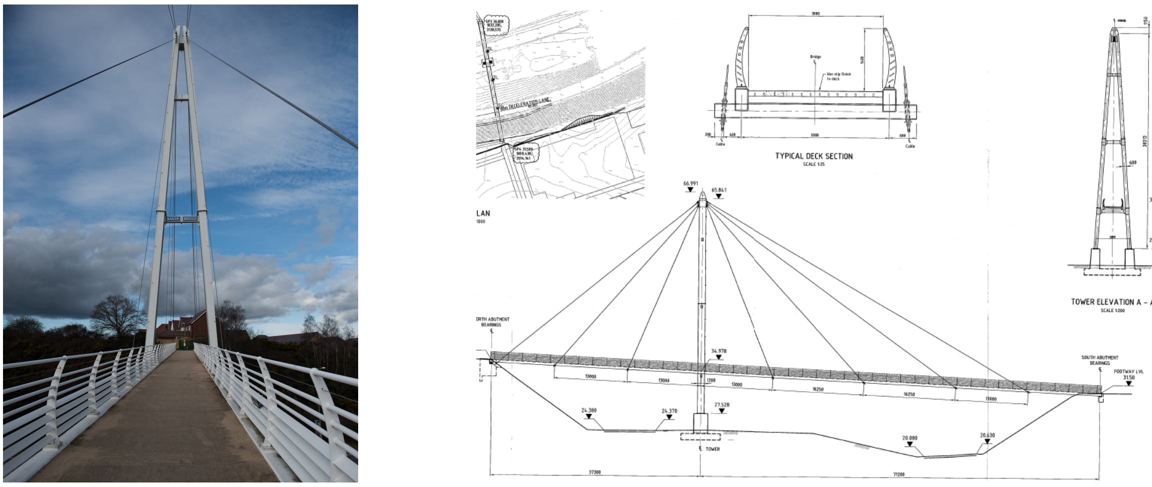


Figure 3: Baker Bridge, Exeter

The bridge comprises a single A-shaped 42 m tower supporting the deck via seven pairs of cables secured to the top of the tower; four cable pairs to a long front span on the (South) stadium side, two cable pairs to a short back span on the North side and one pair to a counterbalancing concrete mass at the North abutment. The twin uprights of the tower are steel 1200x600 mm rectangular hollow section sitting on a massive reinforced concrete bases. The deck comprises two 109 m Grade 50 steel 500x300x16 mm rectangular hollow section longitudinal beams with transverse 150x150x5 mm square hollow section beams at 3.1 m centres. 100x100x8 mm rolled steel angle sections welded to the rectangular hollow section beams provide support for the 120 mm in-situ reinforced concrete walkway, which is also secured to the square hollow section beams by ϕ19 x 74 mm shear studs at 150 mm centres. The six cables are each secured to circular hollow section that in turn supports the longitudinal beams. Parapets are bolted to the rectangular hollow section beams via curved, 15 mm steel plate uprights at 2.5 m centres. The estimated total mass of the bridge is 150 tonnes.

3. BAYOMALAW

In Bayesian OMA, the identification uncertainty of MP is quantified in terms of its variance associated with the posterior PDF. This depends on the data set, generally in an implicit manner, i.e., its value (e.g., in terms of variance) can be calculated for a given data set but there is little insight about how the value depends on the quality or characteristics of data. For sufficiently long data (typical in OMA) assumed to follow the distribution assumed by the identification model (i.e., classical damping, stochastic stationary data), however, it can be shown that the variance follows a statistical law analogous to the Laws of Large Numbers in classical statistics. It has a deterministic part that depends on the ‘information content’ of the data (related to test configuration) and a random part that depends on specific details of the data. On a relative basis, the latter is negligible and tends to zero as data length increases. The deterministic part tells the achievable precision of OMA and is what affects test-planning decisions. ‘Uncertainty laws’ are closed form asymptotic expressions for the deterministic part and they are the primary scientific target of the BAYOMALAW project. The mathematical derivation is generally tedious, involving asymptotics and leveraging on the special mathematical structure of the OMA problem in specific contexts (e.g., well-separated modes, close modes, multiple setups); but the discoveries so far show that the results can be remarkably simple [12]. Consider for example the damping ratio, which is the MP that has the highest uncertainty. For well-separated modes, small damping and long data in a single setup, it can be shown that the posterior COV δ (coefficient of variation = standard deviation/mean) is asymptotically given by

$$\delta \sim \delta_0 \sqrt{1+\frac{a(\kappa)}{\gamma}} \tag{1.}$$

where ‘~’ (read as ‘asymptotic to’) denotes that the ratio of the right hand side to left hand side tends to 1 under the stated asymptotic condition ($\zeta \to 0$, data length$\to\infty$);

$$\gamma = \frac{S}{4S_e\zeta^2} \tag{2.}$$

is the ‘modal SNR’, the ratio of acceleration data PSD ($S/4\varsigma^2$) to the noise PSD (S_e) at the natural frequency;

$$\delta_0 = \frac{1}{\sqrt{2\pi\zeta N_c B(\kappa)}} \tag{3.}$$

is the ‘zeroth order’ law that gives the COV when the modal SNR is infinite; N_c is the data duration expressed as a multiple of the natural period;

$$B(\kappa)=\frac{2}{\pi}\left[\tan^{-1}\kappa+\frac{\kappa}{\kappa^2+1}-\frac{2}{\kappa}(\tan^{-1}\kappa)^2\right] \qquad a(\kappa)=\frac{4(\kappa^2+1)(3\tan^{-1}\kappa-3\kappa+\kappa^2\tan^{-1}\kappa)\tan^{-1}\kappa}{3[(\kappa^2+1)(\kappa-2\tan^{-1}\kappa)\tan^{-1}\kappa+\kappa^2]} \tag{4.}$$

and κ is a dimensionless 'bandwidth factor' that reflects usable bandwidth $f(1\pm\kappa\zeta)$ around the natural frequency f without incurring significant modeling error. Figure 4 plots $B(\kappa)$ and $a(\kappa)$ which are both increasing functions of κ. Data duration is considered long when the number of FFT points N_f in the usable band is large compared to 1. This number can be reasoned to be equal to $N_f = 2\kappa\varsigma N_c$.

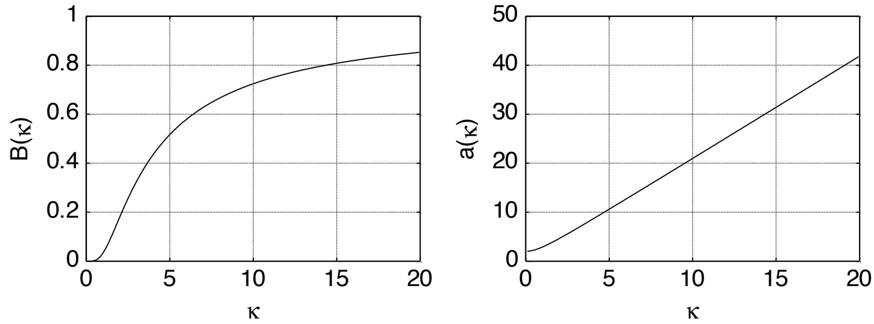


Figure 4: Plot of $B(\kappa)$ (left) and $a(\kappa)$ (right) with bandwidth factor κ.

The above results show that test configuration can be fundamentally quantified in terms of the modal SNR $\gamma = S/4S_e\varsigma^2$ and its effect of identification precision is encapsulated in the term a/γ. The definition of the modal SNR is motivated from the mathematical structure of the problem. The quality of sensor and data acquisition hardware comes into picture through the noise PSD S_e near the natural frequency. The number and location of sensors have their effect through the modal force PSD S, which is proportional to the square of the mode shape values at the measured DOFs. As far as the MPs of a single mode are concerned, this suggests a rather simple principle for sensor locations – maximise the sum of squares of mode shape values.

4. Test planning for optimal MP reliability

Figure 5 shows logical test point (TP) locations for Baker Bridge; at and between stay cable attachment points. Nodes 7 and 24 are at the pylon that divides the bridge into a short and a long span, nodes 1, 18 and 17, 34 are at the abutments. It is often assumed that modal ordinates are zero at supports but our experience shows this is not the case, so ideally these points should be measured.

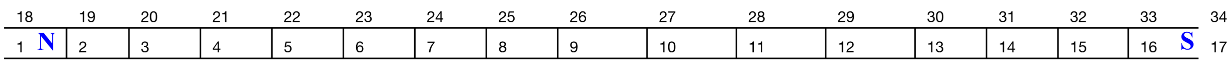


Figure 5: Test measurement points (TPs) for Baker Bridge. S=South, Sandy Park end

A full recipe for planning ambient vibration tests, especially in the multiple setup setting, has yet to be developed but here we demonstrate how the findings from the BAYOMALAW project so far can assist in making decisions on data length and allow test configurations to be factored in a quantitative manner. For discussion, the planning problem is formulated in a series of questions:

1. How many reference locations should be measured?
2. Where should the reference sensors be placed?
3. How should the remaining sensors be roved?
4. How long should the data of each setup be?

Although not mandatory, to simplify planning we assume that the reference locations are fixed in all setups. Generally, one aims at placing the reference sensors so that the target modes, or as many modes as possible, can be identified in all setups. The primary consideration is to avoid nodes of potential modes. Beyond this, the modal responses should be as large as possible for mode shapes estimated from experience or numerical simulations. For a single mode this means the sum of squares of mode shape values should be as large as possible, but a quantitative measure has yet to be formulated for multiple modes, which is in fact the question to be addressed. These also have to trade off with available resources – more reference sensors means less roving sensors and more setups required. Assuming that the reference sensors can provide sufficient information for identifying the mode, planning of the roving sensor schedule can be simplified to satisfy logistics constraints rather than identification precision.

The above considerations have yet to be formulated quantitatively and here the decision is made based on qualitative judgment. We assume two well-separated reference sensors to minimize the chance of zero sum of squares of modal ordinates due to one reference being close to a nodal point for a mode. The remaining two (out of four) sensors are placed on either side of the bridge to rove along the bridge in different setups. With seventeen TPs on each side of the bridge, less the two references, sixteen setups are required. From the topology of the bridge, one reference sensor may be placed towards the abutment of the short span e.g. 2-4, and the other towards the abutment of the long span e.g. 14-17. Figure 6 suggests a possible scenario for reference (1,2) and rover (3,4) accelerometer.

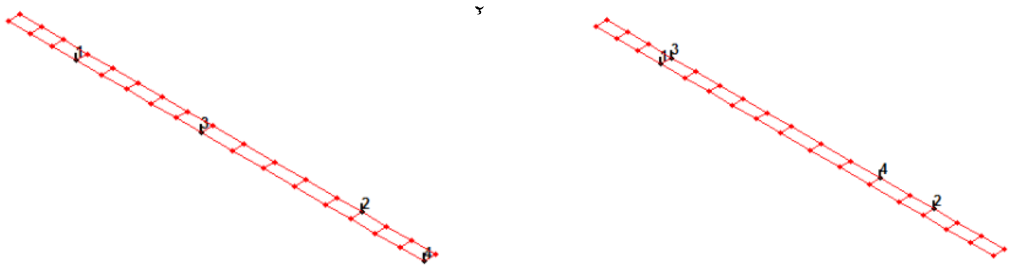


Figure 6: two setups for a possible reference and rover scenario..

The existing findings of the project allow the data length and quality in each setup to be factored quantitatively. Assumptions are inevitable and one needs to be aware of the robustness of planning decisions with respect to the assumptions. For planning purpose, the decision is made based on the mode with the lowest natural frequency (i.e., smallest N_c). A pre-test of the bridge with a single sensor revealed the lowest frequency of about 1 Hz and a damping ratio of about 0.5%. Take κ = 6, which is typical. The decision on data length can now be made by considering the two plots in Figure 7. The left plot shows the attainable posterior COV of damping ratio versus data length for infinite modal SNR, e.g., 'perfect equipment' (S_e = 0). The curve will be shifted higher/lower if the damping ratio is lower/higher. Assuming a higher/lower κ will shift the curve lower/higher, because of the data length factor $B(\kappa)$. Suppose one aims at a moderate precision in the damping ratio, e.g., a posterior COV of 30%. The plot shows that a data duration of at least N_c = 600 natural periods are needed, i.e., 600 sec for a 1 Hz mode.

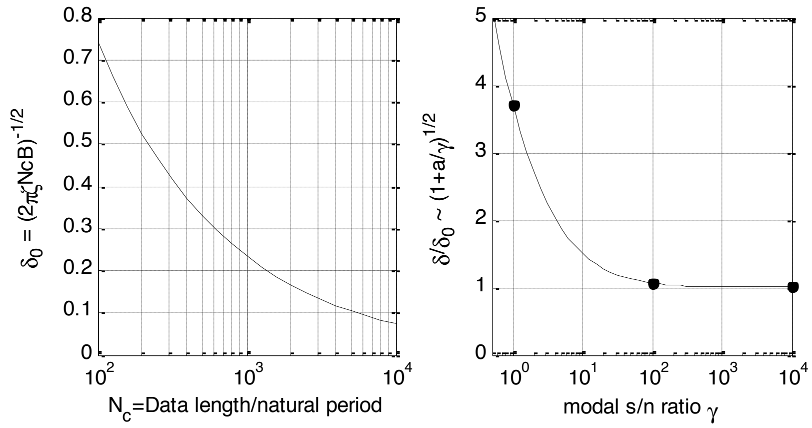


Figure 7: Uncertainty laws (ζ=0.5%, κ=6). Left: zeroth order law and data length effect. Right: 1st order law and SNR effect.

The right plot in Figure 7 shows the term $\sqrt{(1+a/\gamma)}$ in the uncertainty law in (1.). Multiplying the value in the left figure by the one on the right gives the posterior COV that accounts for the finite nature of the modal SNR, and hence test configuration. The flattening trend with increasing γ indicates a diminishing return on increasing equipment quality. For the usually deployed Honeywell QA750 servo-accelerometers and Data Physics or National Instruments data acquisition, with some conservatism one assumes $S_e = (10\ \mu g/\sqrt{Hz})^2$. Based on a pre-test, assume $S = (10\ \mu g/\sqrt{Hz})^2$ for a single sensor. This gives $\gamma = 10/4(10)(0.5\%)^2 = 10000$, which is marked as a dot in the right plot of Figure 7. The point is already well into the flattening part of the curve, suggesting that the test configuration is already good enough with one or two reference sensors. Further improving the quality of sensors or increasing their number has only a marginal effect on improving the identification precision of the damping ratio. The other two dots on the curve at $\gamma = 100$ and 1 show the cases when $S_e = (100\ \mu g/\sqrt{Hz})^2$ and $(1000\ \mu g/\sqrt{Hz})^2$, respectively. This indicates that the posterior COV can increase by almost four times in a substantially worse test scenario.

To conclude on required data length, to achieve a COV of 30% (a moderate precision) in the damping ratio of a 1 Hz mode requires at least 600 sec (based on δ_0). Based on the nominal assumption that $S = (10\ \mu g/\sqrt{Hz})^2$ and $S_e = (10\ \mu g/\sqrt{Hz})^2$, $\gamma = 10000$ is already high enough and this data duration should be sufficient. This decision is safe for a worse scenario with $\gamma = 100$. To accommodate a much worse scenario with $\gamma = 1$ would require $\delta_0 = 0.30/4 = 0.075$. From the left plot of Figure 7, this requires a data length in excess of 5000 natural periods, which is unrealistic and unnecessary.

The above exercise reveals a number of issues that are currently addressed empirically. Decisions are particularly affected by the damping ratio and the modal force PSD. A pre-test (e.g., with a single sensor) will give an idea of these quantities for planning. Otherwise, a damping value of 0.5% is a conservative choice for planning for (most) civil structures. It would be helpful to derive simple analytical expressions for the modal force PSD for typical structures (e.g., tall buildings, bridges) and scenarios (e.g., due to micro-tremor alone, or low wind). For conservatism, the assumed noise PSD should be higher than for the sensor/hardware to account for potential interference in the site or modeling error due to unaccounted contribution from other modes. Planning for multiple setups has only been addressed empirically and further work is much needed.

5. Baker Bridge modal test

Before embarking on modal tests of two major long span bridges in 2017 new hardware and software needed to be developed and tested. In particular the extreme length of the two bridges requires use of autonomous but synchronised recorders. Such a approach was used at the Humber Bridge [13] in 2008 but the synchronisation precision proved to be limited and reliant on visibility of GPS satellites, so a new system has been built for the BAYOMALAW project. The new system uses National Instruments C-Rio hardware with default oscillators replaced by oven controlled crystal oscillators (OCXOs). One recorder is used as a master, whose clock is used to synchronise all the slave units before dispersing them for the measurement sequence. At the time of writing the system was not ready for the Baker Bridge trial, so a set of four APDM Opal inertial measurement units was used for the initial measurement. These units were previously used at Baker Bridge for measuring human dynamic loading [14]. Hence the results from a very short campaign based on the analysis of Section 4 are presented here.

With the advice from Section 4 the measurements used a sequence of 16 measurement setups with at least 600 seconds of data each. The four Opals were set up in synchronised logging mode in the Vibration Engineering Section laboratory, undocked and carried to the bridge. Two Opals were taped to the bridge deck at the reference positions and the remaining pair roved over the other 32 TPs. At the end of the sequence the Opals were returned to the docking station and the four sets of triaxial acceleration signals downloaded and merged into a continuous time history. This was then divided into sixteen files corresponding to setups based on start and end times noted on site.

6. Modal test results and relevance for hindsight for planning

To visualize the modes of interest and the relevant frequency bands, cross-powers obtained from the time series were normalized and merged to generate singular value spectra shown in Figure 8. Figure 8 also shows singular value decomposition (SVD) spectra obtained from a previous single setup measurement with 16 high resolution QA-

750 accelerometers but with less spatial resolution. Opal accelerometers noise floor is quoted at 128 $\mu g/\sqrt{Hz}$ whereas QA-750 noise floor, conservatively taken here as 10 $\mu g/\sqrt{Hz}$ can be as good as 0.4 $\mu g/\sqrt{Hz}$ in the frequency range of Baker Bridge [15]. The figure shows the effect of the superior noise floor of the single setup QA measurement during which the response levels were higher.

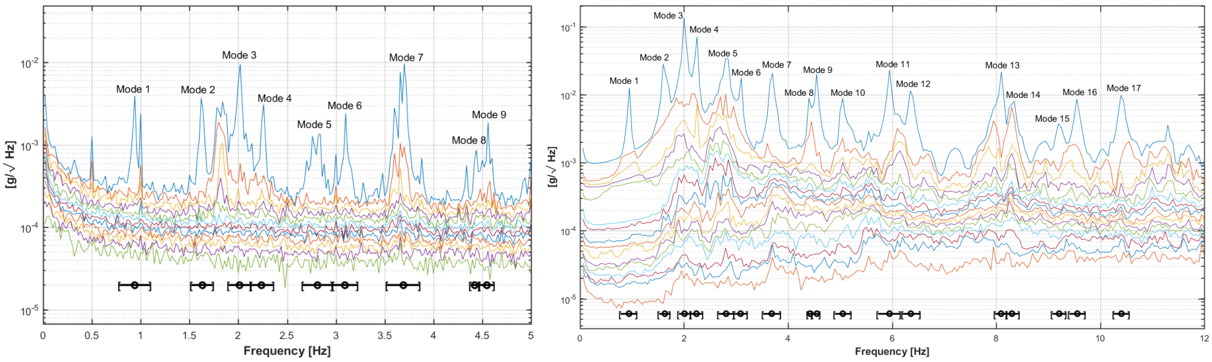
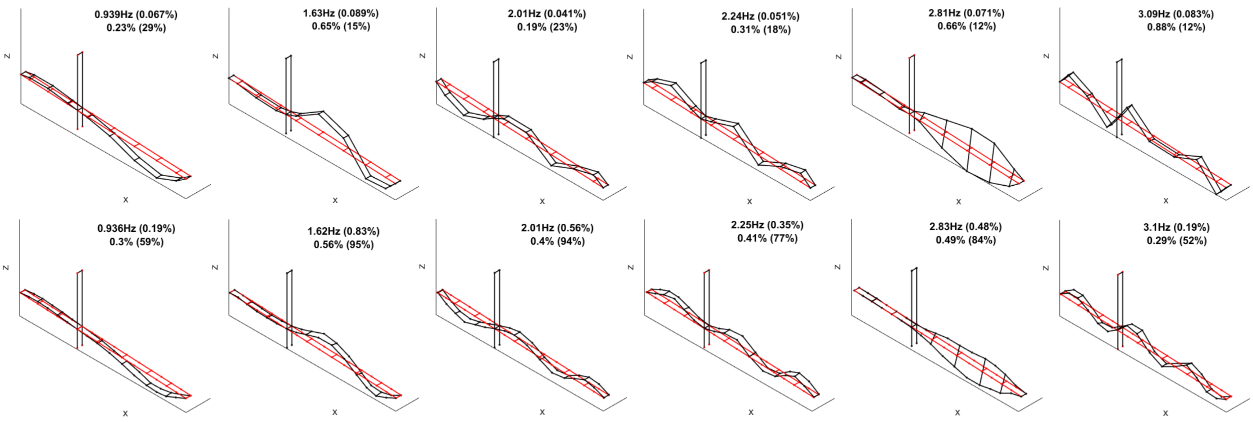


Figure 8: SVD spectra from measurement setups 4 (left) using Opals and from single setup using force balance accelerometers (right).

The modal parameters using both multi-setup Opals and single setup QA are summarised in Figure 9 including COVs for all frequency and damping estimates. For the single setup results (upper figures), the COV values indicated in parenthesis are the values corresponding to the posterior PDF of the modal parameters, and hence reflect identification uncertainty. For the multiple setup results (lower figures), the COV values indicated are ‘representative’ statistics contributed by both the identification uncertainty (posterior COV in each setup) and variability of MPV over different setups [16]. The COV values from the single and multiple setup results cannot be directly compared for identification precision, but they represent in an aggregate sense our uncertainty about the modal parameters based on the data collected. Modal damping is as low as 0.19% most probable value for this structure, a typical figure for steel footbridges. With such low damping and frequencies aligned with comfortable jumping or walking footfall rates Baker Bridge is easy to excite well enough to obtain good quality estimates from free or forced vibration testing. Using curve fitting to the exponentially decaying sinusoid (‘logdec’) and circle fitting to force/response data from such a test provides values usually expected to be more reliable than from operational modal analysis, but obtained using larger vibration levels, at least for free vibration response.



Logdec	0.16%	0.135%	0.19%	0.15%	0.22%	0.18%
Circle fit	0.16%	0.18%	0.29%	0.36%		

Figure 9: Summary of modal parameters. Upper: using single setup and 16 QA -750s; Lower: using multiple setups and four Opals. Estimates from free vibration and forced vibration testing at larger amplitude are also provided

The COVs for the multiple setups hide the considerable variations within setups. Figure 10 shows the variation for two modes where a few high estimates (with high COV) lead to the high overall estimates. Selectively ignoring these could achieve estimates closer to the free and forced vibration estimates, but this is not a strategy available with no prior knowledge of the structure and/or larger structures not amenable to these forms of testing.

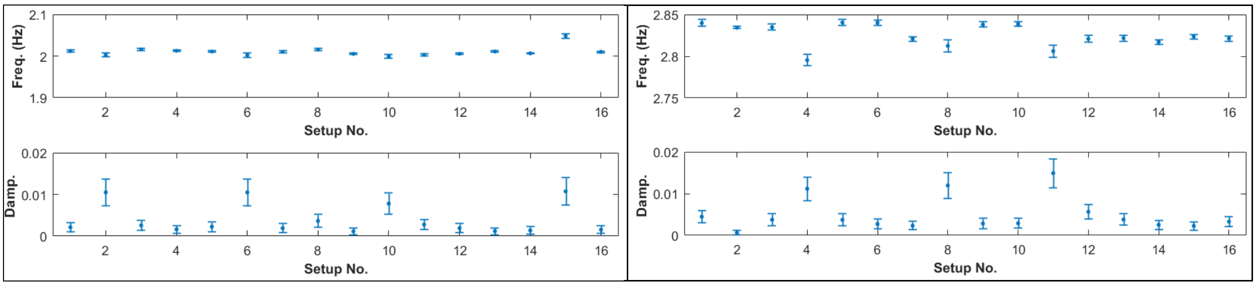


Figure 10: MPV ranges across setups for modes 3 and 5.

Finally the values of posterior COV in each setup are plotted with the modal SNR in Figure 11, in a manner analogous to the right plot of Figure 7. COVs for one specific setup (multiple setups) or one 600 second segment of data (single setup) are indicated in the legends. In both cases the estimates are well along the flat part of the curve, consistent with the predictions, confirming the effectiveness of the planning.

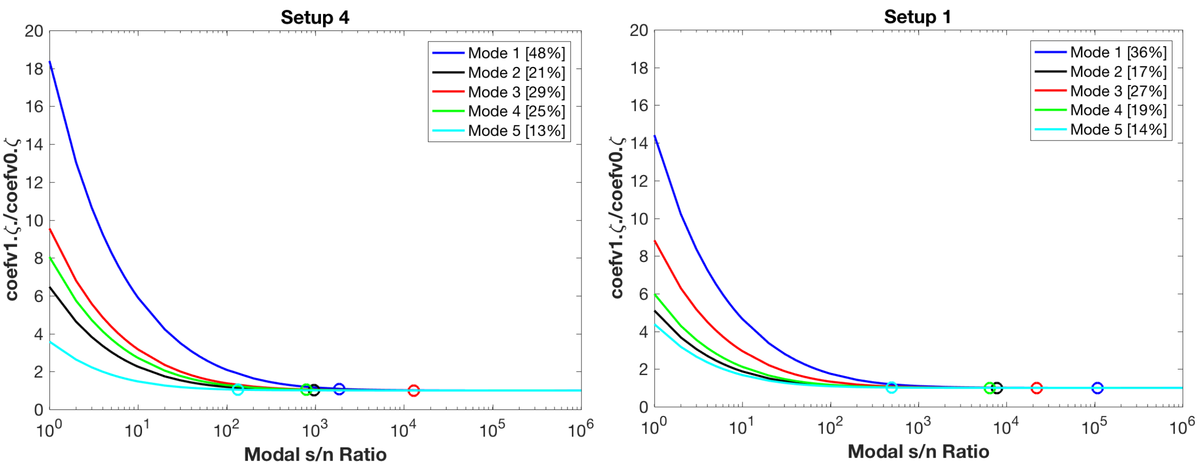


Figure 11: SNR effect with damping COVs for multiple set ups and Opals (left) and single setups and QA-750s (right).

7. Conclusions

A process for planning modal tests of civil structures to provide data for operational modal analysis is outlined, then evaluated using data from a well-studied lively cable-stayed footbridge. The study shows that with good planning, good results can be obtained with a small set of sensors not of the best quality.

The COVs and MPVs of the modal parameters are compared with estimates using the ideal situation of a long record with a complete set of high quality sensors. COVs of frequency estimates are ten times better with the ideal sensor arrangement, with COVs of the damping estimates up to six times better.

Acknowledgements

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Nomenclature

COV	Coefficient of variation
DOF	Degree(s) of freedom
FFT	Fatst (discrete) Fourier transform
LSTS	Long span and tall structure
MP	Modal parameter
MPV	Most probably value
OMA	Operational modal analysis
PDF	Probability density function
PSD	Power spectral density
SNR	Signal to noise ratio
SVD	Singular value decomposition
TP	Test point

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