

AUTOMATIC SHAPE OPTIMISATION OF THE TURBINE-99 DRAFT TUBE

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INTRODUCTION

The performance of a hydraulic reaction turbine is significantly affected by the efficiency of its draft tube. Factors which impede the tube's performance include the geometrical shape (profile), and velocity distribution at the inflow. So far, the design of draft tubes has been improved through experimental observations resulting in empirical formulae or 'rules of thumb'. The use of Computational Fluid Dynamics (CFD) in this design process has only been a recent addition due to its robustness and cost-effectiveness with increasing availability to computational power. The flexibility of CFD, allowing for comprehensive analysis of complex profiles, is especially appealing for optimising the design. Hence, there is a need for developing an accurate and reliable CFD approach together with an efficient optimisation strategy.

Flows through a turbine draft tube are characterised as turbulent with a range of flow phenomena, e.g. unsteadiness, flow separation, and swirling flow. With the aim of improving the techniques for analysing such flows, the turbomachinery community have proposed a standard test case in the form of the Turbine-99 draft tube [1]. Along with this standard geometry, with the aim of simulating the swirling inflow, an additional runner proposed by Cervantes [2] is included in the present work. The draft tube geometry is shown in Fig.1. The purpose of this abstract is to outline the framework developed to achieve the automated shape optimisation of this draft tube.

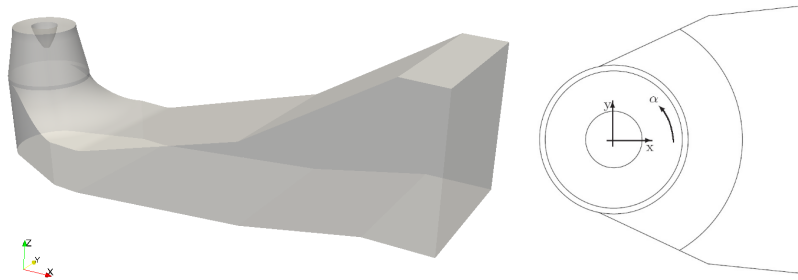


Figure 1: (left) Turbine-99 with inflow runner. (right) labelled schematic of the swirling inflow condition.

METHODOLOGY

Design (or shape) optimisation incorporates representing a shape and evaluating a measure of its quality using CFD simulations. To extremise the quality, the role of an optimisation method is to propose a range of promising solutions and thus locate a good approximation of the optimal design. Usually such quality measures (objective functions) induce a non-linear search landscape, and therefore Evolutionary Algorithms (EAs) may be used as an optimisation approach [3]. However, EAs generally require thousands of function evaluations, which renders impractical with computationally expensive simulations. An alternative is to use Bayesian Optimisation (BO) or an efficient global optimisation (EGO) method that has been proved to be an effective approach with limited budget on the number of function evaluations.

BO is a model-based global search strategy that sequentially samples the design space at likely locations of the global optimum. It starts with a space filling design (e.g. Latin Hypercube Sampling) of the design space. The initial set of shapes are then expensively evaluated with appropriate CFD simulations. Using this set of the initial shapes and the associated function values as data a stochastic regression model is trained with a Gaussian process (\mathcal{GP}). The benefit of using \mathcal{GP} s for regression is that they provide a posterior predictive distribution given the training data, and thus querying the surrogate model for any shape results in both a mean prediction and the uncertainty associated with the prediction. This often enables the closed form calculation of a utility function: the expected improvement in function value (with respect to the best function value observed so far) to be obtained by querying a solution. Therefore, a strategy for (expensively) evaluating the next solution is to select the shape that maximises the expected improvement. The newly evaluated shape is then added to the training database, and a retraining of the \mathcal{GP} model ensues. The process is repeated until the budget on

the number of expensive evaluations are exhausted. In BO, based on an initial set of M shapes, $D = \{(\mathbf{x}^m, f(x^m))\}_{m=1}^M$, a \mathcal{GP} model may be trained. Once trained, the predictive density from the model for a shape \mathbf{x} is: $P(\hat{f}(\mathbf{x})|D)$. Given the best evaluated shape $\mathbf{x}^* = \arg \min f(\mathbf{x}^m) \in D$, the expected improvement of an arbitrary feasible shape \mathbf{x}' is defined as:

$$\alpha(\mathbf{x}', \mathbf{x}^*) = \int_{-\infty}^{\infty} \max(f(x^*) - \hat{f}(\mathbf{x}'), 0) P(\hat{f}(\mathbf{x})|D) d\hat{f}(\mathbf{x}). \quad (1)$$

As the predictive distribution is Gaussian, this integral can be calculated in closed form. Thus, selecting the next shape to evaluate is the solution of the following sub-problem: $\mathbf{x}^{M+1} = \arg \max_{\mathbf{x}} \alpha(\mathbf{x}, \mathbf{x}^*)$. Bi-POP-CMA-ES was used to locate a good approximation of the optimum in the sub-problem [4]. The training data set was augmented with the newly evaluated shape $D \leftarrow D \cup \{(\mathbf{x}^{M+1}, f(\mathbf{x}^{M+1}))\}$, and the model is retrained. In this process, when the limit on the number of expensive function evaluations is reached, \mathbf{x}^* , i.e. the best shape evaluated so far, is the resulting optimised design.

An automated optimisation process was developed by combining a python-based framework (incorporating BO) with OpenFOAM 2.3.1. The communication of the python libraries with OpenFOAM was achieved using the pyFoam-0.6.5. PyFoam was used as an interface to control the OpenFOAM case set-ups for each proposed solution from the python code, and to post-process the data generated after each CFD evaluation. Catmull-Clark subdivision curves were used to construct an appropriate design space for the draft tubes [5]. Starting with the base draft tube (Fig.1), the idea is to deform the shape by adding or removing parts from within a predefined region using a subdivision curve. A two-dimensional curve was used, extruded in three dimensions, to achieve this. As a curve may be completely defined with a finite set of control points, the draft tube design optimisation may then be considered as manipulating the positions of the control points to locate the optimal shape. Therefore, a vector of control point coordinates $\mathbf{x} \in \mathbb{R}^{2n}$ was used to represent the design space for n control points. Thus the shape optimisation problem may be expressed as:

$$\min_{\mathbf{x}} f(\mathbf{x}) = C_p = \frac{2|p_i - p_o|}{\rho U_i^2}, \quad (2)$$

where the pressure difference between the inlet (P_i) and the outlet (P_o) is given by $f : \mathbb{R}^{2n} \rightarrow \mathbb{R}$. Additionally, the optimisation is subject to all control points residing within the predefined space, and the resulting curve being a simple (non-intersecting) polygon.

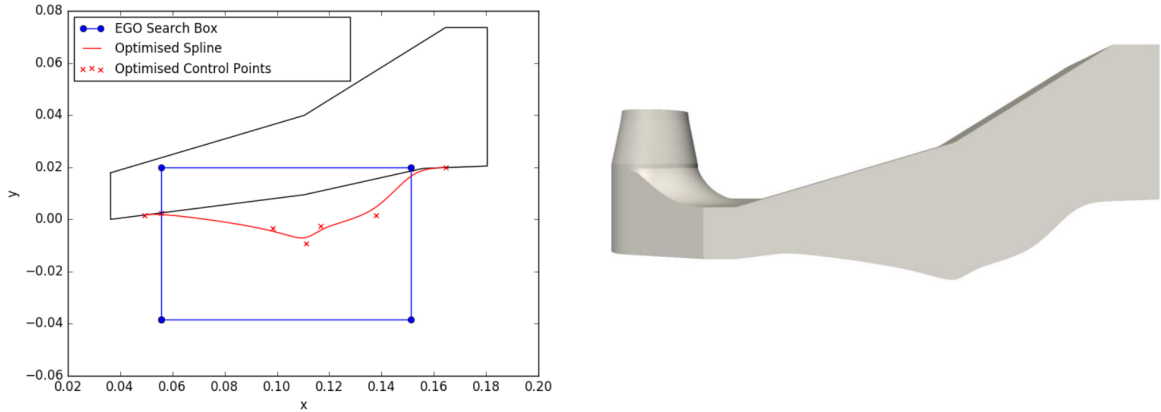


Figure 2: Demonstration of the shape distortion method using Catmull-Clark splines and cfMesh. (left) Schematic of the bounding box and formation of the Catmull-Clark splines. (right) Resulting geometry after cfMesh

Fig.2 demonstrates the methodology for changing the shape of the geometry. The red crosses indicate the control points that may be altered by the BO, and thus change the curvature of the spline (red). The blue lines indicate the bounding box (search space) for the BO. After the new positions of the control points are proposed by the BO, a Stereolithography (STL) file is generated and passed to OpenFOAM. Using this file, the meshing utility cfMesh-v1.1.2 was used to alter the computational domain (and if required re-mesh the altered region). Subsequently, the case was run using the steady-state solver simpleFoam with $k - \epsilon$ turbulence model. After this, the cost function was obtained and the BO determines a new position for the coordinates of the control points.

The swirling flow structure at the inflow is paramount for the overall machine efficiency. Although the swirl is quite complex, analytical representations for the tangential and axial velocity profiles have been proposed in the literature. With the aid of Laser-Doppler-Anemometry, [6] have provided the velocity profiles for the mean axial (U) and tangential (W) velocity components. To the best of the authors' knowledge, no measured data is available for the radial velocity (V). Therefore, in the literature, a general assumption for this component is made with the expressions proposed by [7]:

$$V(r) = U(r) \tan(\theta), \quad (3)$$

$$\theta = \theta_{cone} + \left(\frac{\theta_{wall} - \theta_{cone}}{R_{wall} - R_{cone}} \right) (r - R_{cone}), \quad (4)$$

with $R_{cone} \leq r \leq R_{wall}$, and $\theta_{cone} = -12.8$ and $\theta_{wall} = 2.8$ after [2]. The unknown turbulent quantities at the inflow are assumed $\overline{v'} = \overline{w'}$, and $\overline{u'v'} = \overline{v'w'} = \overline{u'w'}$. Fig.3 demonstrates the effects of the inflow condition on the flowfield with a ‘vortex-rope’ forming past the runner. It is envisioned that this will give a suitable representation of a realistic industrial case for the present work.

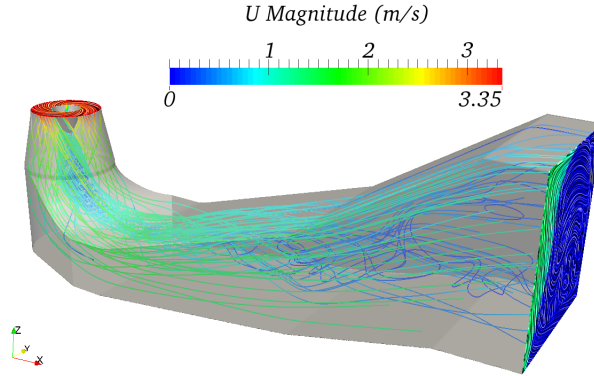


Figure 3: Streamlines of the flow under the swirling inflow condition.

FUTURE WORK

A methodology in optimising the shape of turbine draft duct is presented here. This abstract outlines the use extruded spline across the lower wall of the exhaust. However, this work aims to expand this approach to altering the remaining sides. Furthermore, currently pending, a multi-objective approach to BO has been developed [8] to consider an additional cost-function for the energy efficiency:

$$\zeta = \frac{\frac{1}{A_i} \int_i P_{total,i} dA_i - \frac{1}{A_o} \int_o P_{total,o} dA_o}{1/2\rho U_i^2} \quad (5)$$

which will be considered to future work.

Acknowledgments

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