On the formation of trapezium-like systems

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ABSTRACT

We investigate the formation and evolution of high-order massive star multiples similar to the trapezium in the Orion Nebula Cluster. We perform ensembles of N-body simulations of the evolution of $N = 1000$ Orion-like clusters with initial conditions ranging from cool and clumpy to relatively smooth and relaxed. We find that trapezium-like systems are frequently formed in the first 2 Myr in initially cool and clumpy clusters and can survive for significant amounts of time in such clusters. We also find that these systems are highly dynamical entities, constantly interacting with the surrounding cluster, changing their appearance and membership regularly. The eventual decay of trapezium-like systems can even destroy the host cluster. We argue that the current state of any trapezium-like system is transient and care should be taken when analysing and drawing conclusions from a single snapshot in the life of a highly dynamic object.

Key words: stars: formation – stars: kinematics and dynamics – stars: massive – galaxies: star clusters: general.

1 INTRODUCTION

Stars in our local environment (i.e. within 500 pc of our Sun) form in a continuous hierarchical distribution (Bressert et al. 2010) and dynamically evolve to form overdense groups of stars that we call 'clusters' (e.g. Gutermuth et al. 2005; Portegies Zwart, McMillan & Gieles 2010). The star formation process is thought to be driven by turbulence which imprints substructure in the clouds, thus star formation occurs in clumpy and filamentary regions of molecular clouds (e.g. Mac Low & Klessen 2004; Ballesteros-Paredes et al. 2007). Observations of molecular clouds have show that star formation in clumps and filaments is the norm in molecular clouds (e.g. Testi et al. 2000; Gutermuth et al. 2005; André et al. 2007, 2010; Sánchez, Alfaro & Pérez 2007; Goldsmith et al. 2008; di Francesco et al. 2010), a result which is also seen in hydrodynamic simulations (e.g. Klessen & Burkert 2000; Bonnell et al. 2001; Bate, Bonnell & Bromm 2003; Bonnell, Bate & Vine 2003; Bonnell, Clark & Bate 2008; Offner, Hansen & Krumholz 2009). Observations suggest that these newly born stars have subvirial velocities (e.g. Di Francesco, André & Myers 2004; Walsh, Myers & Burton 2004; Peretto, André & Belloche 2006; André et al. 2007; Kirk, Johnstone & Tafalla 2007; Walsh et al. 2007; Gutermuth et al. 2008), a result which is also found in simulations (e.g. Klessen & Burkert 2000; Bonnell et al. 2001, 2003, 2008; Bate et al. 2003; Offner et al. 2009). Dynamical simulations have also shown that initially subvirial conditions are able to reproduce observations of the Orion star-forming region (Adams et al. 2006; Allison et al. 2009b, 2010;

Proszkow et al. 2009), and erase initial substructure on short time-scales (Goodwin & Whitworth 2004; Allison et al. 2009b, 2010).

Both observations and theory indicate that the dynamical evolution of star clusters begins with clumpy and subvirial initial conditions. An early attempt to model young clusters (i.e. those with substructure) was made by Aarseth & Hills (1972), who simulated an initially structured cluster using a linear distribution of clumps with velocities set to be at rest. In this early model it was found that the dynamical evolution of the system was very rapid, and that the structure was destroyed within its first crossing time. It was also found that the formation of binary systems was increased and that stars were more likely to be ejected, and at higher velocities, with respect to a non-structured system. It was also found that initial structure could lead to the formation of high-order multiple systems [such as the trapezium system in the Orion Nebula Cluster (ONC); Aarseth 1977].

Following Allison et al. (2010) we investigate the ensemble of initially substructured and subvirial cluster simulations to study the formation and evolution of high-order, high-mass multiple systems; which we define as 'trapezium-like systems' after the canonical trapezium system in the ONC (Ambartsumian 1954). In these ONC-like clusters, trapezium-like systems contain the highest mass stars in the cluster, and as such have the potential to dominate the dynamical evolution of the host cluster. In this paper we investigate how and when these systems form dynamically and the affect these systems have on their host clusters. In Section 2 we describe our initial conditions. In Section 3 we define what we consider to be a trapezium-like system, and how our detection algorithm finds multiple star systems. In Section 4 we briefly discuss the results presented in Allison et al. (2010), and in Section 5 we present our

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results for this work. In Sections 6 and 7 we discuss the implications of this work and draw our conclusions.

2 INITIAL CONDITIONS

We perform 160 *N*-body simulations of cool, clumpy star clusters. We vary the level of substructure and initial virial ratio. We conduct ensembles of simulations with statistically identical initial conditions, varying only the initial random number seed used to initialize the simulations. These simulations are the same as are described in Allison et al. (2009b, 2010), but are reiterated here for clarity.

To create initial substructure in our simulations we use a fractal stellar distribution. Using a fractal distribution provides a parametrization of substructure using only a single number: the fractal dimension. (Note that we are not claiming that clusters are actually initially fractal, although they may be, just that this provides a simple descriptor of substructure that is easy to produce.)

The fractal stellar distributions were generated following the method of Goodwin & Whitworth (2004). The method begins by defining a cube of side N_{div}, inside of which the fractal will be built. A first-generation parent is placed at the centre of the cube, from which are spawned N_{div}^3 subcubes, each containing a first-generation child in its centre. The fractal is then built by determining which of the children themselves become parents, and spawn their own offspring. This is determined by the fractal dimension, *D*, where the probability that a child becomes a parent is $N_{\mathrm{div}}^{(D-3)}$. For a lower fractal dimension less children will mature and so the final distribution will contain more structure. Any children that do not become parents in a given step are removed, along with their parent. A small amount of noise is then added to the positions of the remaining children, preventing the final cluster from having a gridded appearance, and the children become parents of the next generation. Each new parent then spawns N_{div}^3 second-generation children in N_{div}^3 sub-subcubes, with each second-generation child having a $N_{\mathrm{div}}^{(D-3)}$ probability of becoming a second-generation parent. This process is then repeated until there are substantially more children than required. The children are pruned to produce a sphere from the cube and are then randomly removed (so maintaining the fractal dimension) until the required number of children are left. These children then become the stars in the cluster.

To determine the velocity structure of the cloud, children inherit their parent's velocity plus a random component that decreases with depth in the fractal. The children of the first generation are given random velocities from a Gaussian of mean zero. Each new generation then inherits their parent's velocity plus an extra random component that becomes smaller with each generation. This results in a velocity structure in which neighbour stars have similar velocities, but distant stars can have very different velocities. Finally, the velocity of every star is scaled equally to obtain the desired total virial ratio for the cluster.

Each simulation contains 1000 stars, has an initial maximum radius of 1 pc, includes no primordial binaries or gas, and has a three-part power law is used to produce an initial mass function (IMF; Kroupa 2002):

$$N(M) \propto \begin{cases} M^{-0.3} & m_0 \leq M/\mathrm{M_{\odot}} < m_1, \\ M^{-1.3} & m_1 \leq M/\mathrm{M_{\odot}} < m_2, \\ M^{-2.3} & m_2 \leq M/\mathrm{M_{\odot}} < m_3, \end{cases} \quad (1)$$

with $m_0 = 0.08\,\mathrm{M_{\odot}}, m_1 = 0.1\,\mathrm{M_{\odot}}, m_2 = 0.5\,\mathrm{M_{\odot}}$ and $m_3 = 50\,\mathrm{M_{\odot}}$. No stellar evolution is included because of the short duration of the

Table 1. Notation for run identification where *D* is the initial fractal dimension, and *Q* is the initial virial ratio of each simulation. Within each ensemble only the random number seed used to generate the initial conditions is changed.

	D			
Q	1.6	2.0	2.6	3.0
0.3	a1.01–50	a2.01–10	a3.01–10	a4.01–10
0.4	b1.01–50	b2.01–10	b3.01–10	b4.01–10
0.5	c1.01–10	c2.01–10	c3.01–10	c4.01–10

simulations ($\sim$4 Myr). We use the STARLAB *N*-body integrator KIRA to run our simulations (Portegies Zwart et al. 2001).

In this study we explore a range of fractal dimensions and virial ratios. The fractal dimensions investigated are $D = 1.6, 2.0, 2.6$ and 3.0 (since these values correspond to the number of maturing children, 2^D, being an integer), where $D = 1.6$ produces a large amount of structure and $D = 3.0$ produces a roughly uniform sphere. We investigate virial ratios of $Q = 0.3, 0.4$ and 0.5, we define the virial ratio as $Q = T/|\Omega|$ (where T and $|\Omega|$ are the total kinetic and total potential energy of the stars, respectively), hence virial equilibrium is $Q = 0.5$. The 'sets' of initial conditions are tabulated in Table 1.

It is important to note that fractal initial conditions are inherently stochastic: statistically identical fractals (i.e. the same fractal dimension) can appear very different to the eye, and can evolve in very different ways (Allison et al. 2010). Therefore, it is vital to perform ensembles of simulations with different random number seeds. We have therefore simulated 50 $D = 1.6, Q = 0.3$ (a1) and $D = 1.6, Q = 0.4$ (b1) clusters (as they have the most interesting evolution, and to investigate anomalous results caused by low number statistics), and restricted our analysis of all other combinations of *D* and *Q* to 10 clusters each.

The trapezium-like systems described in this paper have been formed from the collapse and subsequent evolution of clusters, they are *not* primordial systems. Indeed, the high-mass stars that end up in a system may have started a significant distance apart from each other.

3 WHAT IS A 'TRAPEZIUM-LIKE SYSTEM'?

The classical idea of a trapezium system is the multiple star system in the ONC – the Orion Trapezium system. The trapezium system has a very complicated layout, comprising the four main OB stars (θ^1 Ori A, B, C and D) and their binary components (Preibisch et al. 1999). θ^1 Ori B is in fact a 'minitrapezium' on its own and has at least four lower mass companions (Weigelt et al. 1999), and θ^1 Ori A and C are known to have binary companions. The brightest star in the system is θ^1 Ori C whose companion star has a mass approximately half its own mass (making this a O star–O star or OB star binary system), this binary has a relatively short period of $\sim$11 yr (Kraus et al. 2007). Thus the 'classical' idea of the four OB star trapezium system is a misnomer, as the trapezium system is at the least an $N = 8$ system, if we ignore very close binaries (see e.g. the schematic of the trapezium; Moeckel & Bonnell 2009, their fig. 9). If θ^1 Ori E is included as a member of the trapezium system it would become an $N \sim 9$ system, as this star is also a close binary system (Zinnecker & Yorke 2007). Therefore, we use the term 'trapezium-like' (or simply 'trapezium') to identify any

multiple star system that contains several O stars with lower mass companions.

We define a trapezium system as a system of at least three high-mass stars (with possibly other massive stars and other low-mass stars present) that are 'observed' to be in a multiple system. These systems may not be gravitationally bound, although that some systems survive for extended periods of time does indicate that most systems we detect are bound in some sense. This differs from the usual definition, i.e. a trapezium system is a multiple star system whose pairwise separations are of the same order (e.g. Ambartsumian 1954; Pflamm-Altenburg & Kroupa 2006). While we have chosen to use a definition different from this, the method we use allows much useful information about any detected system to be gathered; such as the minimum spanning tree of the system, and a complete list of binary stars in the cluster. This extra information allows an in-depth study of the systems we find.

Our multiple system finding algorithm starts by finding the nearest and second nearest neighbours of each star. It also finds the local density from the volume enclosing the 40th nearest star.[1] Higher order multiples are found by looking for closed loops of first and second nearest neighbours in several steps.

First, mutual nearest neighbours are linked together as long as their separations are less than one-third of the typical separation for the local density. (We note that the algorithm is not very sensitive to changes such as using the 30th nearest neighbour and one-half of the typical local separation.) Pairs of mutual nearest neighbours are then used as a starting point to search for higher order multiple systems, and both members of the pair are registered as being potential members of a system.

In the second step, the nearest and second nearest neighbours of each potential member are examined and registered as being potential members of a system if their separations are less than one-third of the typical local separation (some might already be potential members, others may be new).

The third step is to see if the potential members form a closed loop. That is, are all of the nearest and second nearest neighbours (within one-third of the typical local separation) of potential members, also potential members of the system? If so we have a closed loop.

Finally, if a closed loop is not found we return to the second step and look for new potential members. Numerical experiments show that if a closed loop is not found in a few iterations (normally four or five) then a higher order multiple does not exist. Therefore, we restrict ourselves to eight iterations.

For example, the simplest higher order system is a triple system. If stars i and j are mutual nearest neighbours then they are the starting point for the search. If they both have the same second nearest neighbour, k, then this might be a triple system. It is a triple system if the nearest and second nearest neighbours of k are stars i and j (we have a closed loop). However, if one of the neighbours of k is not i or j then this new star becomes a potential member and we return to the second step.

There is one slight subtlety as outlying members of higher order systems can be missed by this method. For example, stars i and j are both members of a higher order system with a closed loop. Star k has i and j as its nearest and second nearest neighbours –

therefore it should be part of the system. However, it can happen that no members of the loop have k as a neighbour and therefore it is missed. To avoid this problem a final step is to check if any stars have both neighbours as members of the system but have not been included themselves.

As a very final step, trapeziums are found by searching for higher order multiples that contain at least three high-mass stars $>8\,M_\odot$ (e.g. Abt & Corbally 2000). It should be noted that in our definition of a trapezium a hard massive star–massive star binary orbited by a third massive star would be identified as a 'trapezium'. However, as we show in the results (in particular Fig. 2), most of our 'trapeziums' are higher order systems that we feel most people would agree are 'trapezium-like').

We note that this method can be applied in two or three dimensions and the boundness of the system in three dimensions can be found to locate 'real' higher order multiples.

4 THE DYNAMICAL EVOLUTION OF COOL, CLUMPY CLUSTERS

In this section we briefly discuss the work presented in Allison et al. (2010), which is a precursor to the work presented here.

Allison et al. (2010) show that clusters with clumpy and cool initial conditions dynamically evolve in a significant way on very short time-scales ($\sim$1 Myr), and that this rapid evolution allows the clusters to *dynamically* mass segregate at young ages. The initial conditions of cool and clumpy clusters places the clusters far from an equilibrium state, causing the cluster to enter a phase of violent relaxation (Lynden-Bell 1967). During the violent relaxation phase the cluster collapses, erasing its initial substructure, and approaches virial equilibrium. The collapse also causes the formation of a dense core which allows rapid dynamical mass segregation to occur. The cooler and more substructured the cluster is initially, the more it will collapse as it attempts to relax and reach virial equilibrium. This is shown by equation (2) (see also Allison et al. 2010):

$$\frac{R_0}{R_f} = \frac{\eta_0}{\eta_f} 2(1 - Q_0), \tag{2}$$

where R_0 and R_f are the initial and final radii of the system, respectively, η_0 and η_f are the initial and final structure parameters, respectively, and Q_0 is the initial virial ratio. The value of η, the structure parameter, depends on the structure of the cluster, it is therefore a measure of the distribution of potential energy in the cluster (Portegies Zwart et al. 2010). The more substructure the cluster has initially the greater η_0, and for a substructured cluster $\eta_0 > \eta_f$. This collapse leaves the cluster with a very short relaxation time in the core allowing mass segregation to occur on time-scales much shorter than usually expected (e.g. Bonnell & Davies 1998). The initial conditions of the cluster influences the ability of the cluster to mass segregate by determining the depth of the collapse of the cluster – the cooler and clumpier the cluster initially the denser the core it will form.

5 RESULTS

In this section we analyse and discuss the properties of the trapezium systems found in our simulations.

5.1 The nature of trapezium systems

We present here the details of the analysis of the trapezium systems formed in a fairly typical cool, clumpy cluster in run a1.11 (with

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[1] This is a larger volume than is usually considered, which would normally only contain a handful of stars (see Aarseth 2003, p. 265). This is because we want a density estimator that includes all of the members of a high-order multiple system and surrounding stars as well.
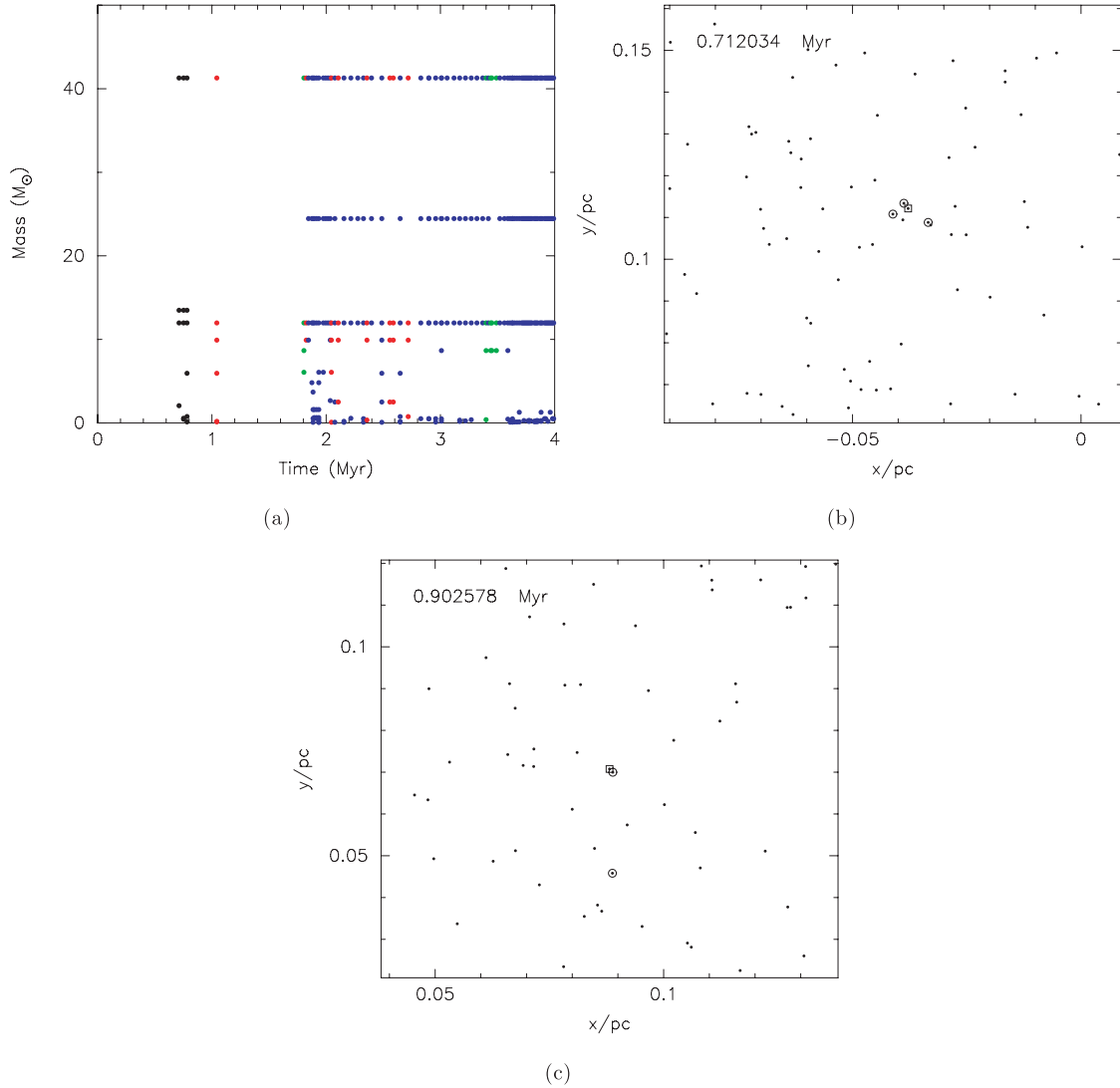
Figure 1. Run a1.11: $D = 1.6, Q = 0.3$. (a) Members of trapezium systems for the run a1.11. The horizontal axis is measured in Myr, and the vertical axis shows stellar mass. The points show the mass of the member stars in the detected trapezium systems, and different colours show different trapezium systems (black: 41.3, 13.5, 11.9 $M_\odot$ ∼ 0.7 Myr; red: 41.3, 12.0, 9.9 $M_\odot$ ∼ 1 Myr; green: 41.3, 12.0, 8.7 $M_\odot$ ∼ 1.8 Myr; blue: 41.3, 24.5, 12.0 $M_\odot$ ∼ 1.8 Myr). (b) Projection of spatial positions for the first trapezium system (black) identified in (a), at the time it was detected. (c) The members of the 'black' trapezium shown at a time ∼0.1 Myr after its last detection, when it is not detected by our algorithm. It is clear that the original system has now evolved to a point where it no longer represents an 'overdense' group of neighbour stars. Trapezium system members are marked with outlines. The square outline identifies the most massive member.

$D = 1.6$ and $Q = 0.3$). Fig. 1(a) shows how the existence and members of the trapezium systems in this cluster evolve. Groups of points that occur at the same time on the horizontal axis indicate that a trapezium system has been identified at that particular time, and the points themselves show the mass of the member stars. The different colours indicate the detection of different trapezium systems, defined by a change in the three highest mass stars in the system.

For example, our algorithm first finds a trapezium at $t \approx 0.7$ Myr, which contains four stars of masses of around 41, 13, 12 and 2 $M_\odot$ (black points). There are two important things to note about this system. First, this trapezium system only lasts a very short time, around 0.1 Myr, before decaying. Secondly, the low-mass members of the multiple system change even in this short time (e.g. a ∼5 $M_\odot$ star is very briefly a member just before the system decays). Fig. 1(b) shows the spatial distribution of this trapezium at the time of its

first detection, the trapezium members are identified by an outlined symbol. It is clear from the figure that the quadruple system is at a higher spatial density than the local stars and therefore should be detected as a high-order multiple system, and because this system contains at least three high-mass stars it is further defined as a trapezium system. Fig. 1(c) shows the system ∼0.2 Myr later, when it is not detected by our algorithm. The system has now dissolved, and is spread over a much larger area, thus no longer satisfying the local density criteria.

A 'new' trapezium (red points) then appears just after 1 Myr but lasts for only one snapshot and contains two of the members of the first trapezium.

Then just before 2 Myr a long-lived trapezium forms which survives for the rest of the simulation (blue points). This trapezium contains the *same* ∼41 and ∼12 $M_\odot$ stars that were present in the first and second systems, and also a new ∼24 $M_\odot$ member.

Similarly to the first system to form this system contains several lower mass stars which come-and-go (for example a $\sim 10\,M_\odot$ star is present for much of the first Myr of this system but is then ejected). While the system is not detected at all times it has an almost constant presence until the termination of the simulation, lasting at least 2.2 Myr.

Fig. 2 shows how the 'blue' trapezium identified in Fig. 1 evolves spatially. The progression from Fig. 2(a) (1.8 Myr) to Fig. 2(f) (4 Myr) shows that the trapezium system frequently adds and removes members, and that the spatial distribution of the member stars changes often. For example in Fig. 2(b) the member stars are all fairly evenly separated, with no obvious hierarchy; this is in comparison to Fig. 2(f), in which a hard binary (with the ~ 42 and $\sim 12\,M_\odot$ stars at a separation of 75 au)[2] is in a system with two other stars. Trapezium systems have often been defined as 'hierarchical' or 'trapezium' systems based on the separations of the member stars (e.g. Allen & Poveda 1974). However, this maybe potentially misleading as these systems can evolve between the states many times in their lifetime.

The features shown in the simulation presented above are generic to the simulations that produce trapezium systems, the Supporting Information (with the online version of the article) contains figures similar to Fig. 1 for all of the simulations. Examination of all of the simulations shows that the initial conditions of the simulations do play a significant role in the formation and evolution of trapezium systems.

5.1.1 Trapezium frequency, lifetimes and sizes

Table 2 shows the fraction of simulations that form a trapezium system (at least for one snapshot) and its dependence on the virial ratio, Q, and fractal dimension, D, of the initial cluster. An analysis of Table 2 shows that trapezium systems are much more likely to be found in clusters that initially have more substructure and are cooler. As we look across the table, from more ($D = 1.6$) to less ($D = 3.0$) substructured, we can see that there is a drop in the number of systems that show a trapezium system beyond $D = 2.0$. Two-thirds of the clusters with $D \leq 2.0$ form a trapezium system, compared to one-fifth with $D > 2.0$. The same trend is seen as we look down the table from cooler ($Q = 0.3$) to virialized clusters, although this effect is much less pronounced. Clusters that are initially cool and substructured collapse to a much denser state, allowing the massive stars to mass segregate and form trapezium systems (Allison et al. 2009a, 2010).

At what age trapezium systems form is also dependent on the initial conditions of the simulation. In Table 3 we define the first detection of a trapezium system in a particular run as the 'formation time', we find that in clusters that are initially cool and more substructured trapezium systems tend to form at earlier times. In the lowest Q and D clusters trapeziums tend to form early (< 2 Myr), whilst in high-Q and high-D clusters trapeziums form close to the end of the simulations at around 3–4 Myr (the simulations end at 4 Myr). This can be explained by the initial collapse of the cluster – clusters that are initially further from an equilibrium state go through a more dramatic violent relaxation event. The depth of the collapse in these clusters allows the massive stars to encounter each other much sooner in the evolution of the cluster, compared to clusters that are initially close to equilibrium.

The length of time a trapezium can be 'observed' in any cluster also depends on the initial conditions. Note that as seen above each cluster may have several different trapeziums with different members, here we discuss the time over which *any* trapezium is in existence. Fig. 3 shows how likely it is that a trapezium-like system will be observed (i.e. is detected by our algorithm) in a particular cluster type at any time during the life of those clusters. This figure shows the plots for the 'a1-type' ($Q = 0.3$, $D = 1.6$, top) and 'b1-type' ($Q = 0.4$, $D = 1.6$, bottom) clusters, the plots for all the simulations can be found in the Supporting Information.

The low numbers of high-D and high-Q clusters found to have trapeziums in Table 2 (typically 1 or 0) is partly due to the longevity of their trapeziums. Therefore, the fractions found in Table 2 may be unreliable as there may be very short-lived trapeziums that exist between snapshots that we miss. However, if this were to occur it would still be extremely unlikely to observe a trapezium in such clusters as their lifetimes even if they do form are extremely short (< 0.1 Myr).

This point is emphasized in Table 4 that shows the different numbers of trapezium systems typically found in different simulations. We identify different trapezium systems by a change in the three most massive members. The dynamical nature of many trapezium systems is shown here as the decay, re-formation and swapping of members can frequently significantly change the trapezium system observed in any one cluster. We should note that the longevity of the trapeziums in our simulations is limited by the 4 Myr duration of the simulations.

The size of a trapezium is a rather difficult property to quantify. We define the size of a trapezium to be the length of the minimum spanning tree (MST)[3] connecting all of the members. This provides a unique length with which the size of the trapeziums can be described. For reference, the trapezium system in the ONC is an $N \approx 8$ system, and has an MST length $\approx 13\,600$ au (assuming a distance of 440 pc; Jeffries 2007). In Table 5 we show the average MST lengths for all $N = 3$–9 systems. This data were taken for every snapshot of every simulation as the size and membership of trapeziums changes even within a single simulation. As can be seen the ONC trapezium is smaller than an average $N \approx 8$ system, although this is likely due to the fact that the trapezium system in the ONC contains a 'minitrapezium', which is included in this MST analysis. The hierarchical nature of the trapezium probably occurs due to the initial primordial binary population of the ONC. Primordial binaries are not included in these simulations, which probably explains why we do not see the levels of hierarchy seen the ONC trapezium. The ONC trapezium could also be smaller than the average lengths presented here because it is currently in a dense phase of dynamical evolution. One of the main conclusions of this work is that trapezium-like systems are extremely dynamical entities.

Fig. 4 shows how the average size of a trapezium system changes with the initial conditions of the simulation. To recap Table 1, simulation letters correspond to initial virial ratio a: $Q = 0.3$, b: $Q = 0.4$ and c: $Q = 0.5$, and numbers to the initial fractal dimension 1: $D = 1.6$, 2: $D = 2.0$, 3: $D = 2.6$ and 4: $D = 3.0$. Fig. 4 shows that trapeziums tend to be smaller when clusters are initially low D and low Q (e.g. a1 and b1), and larger when initially high D and high Q (e.g. c3, b4 and c4). However, it is interesting to note that clusters that begin as 'b1-type' ($D = 2.0$, $Q = 0.4$) seem to have the smallest trapezium systems, which may indicate that these initial

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[2] For stars of this mass in a core with a velocity dispersion of $\sim 2\,\mathrm{km\,s^{-1}}$ the hard–soft boundary is at around 1000 au.

[3] The MST is the shortest path which connects all of the vertices in a sample with simple edges, and no closed loops (Prim 1957).
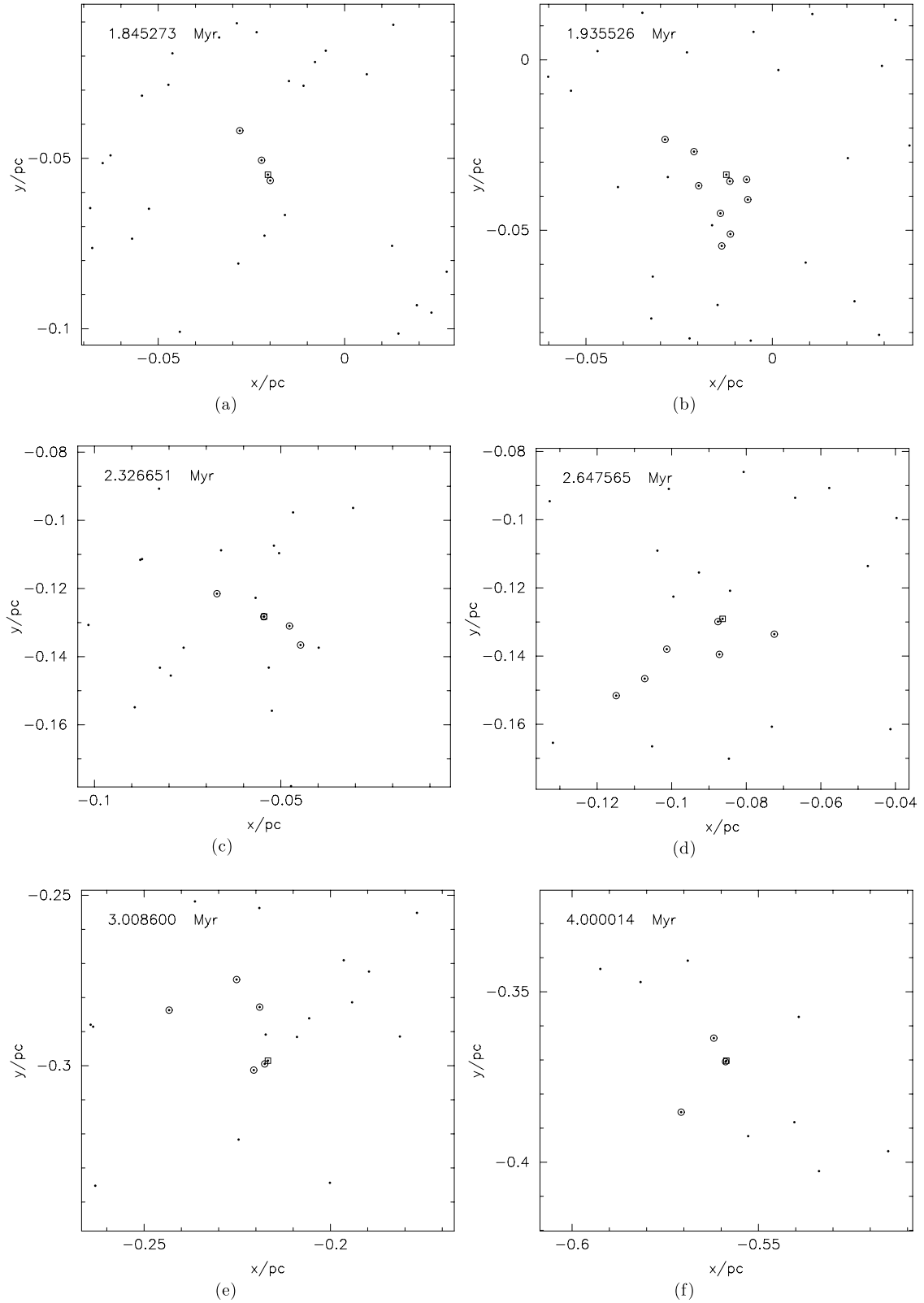
Figure 2. Run a1.11: $D = 1.6, Q = 0.3$. Projection of spatial positions for the 'blue' (∼1.8 Myr) trapezium system identified in Fig. 1. Trapezium system members are identified marked with outlines. The square outline identifies the most massive member. The plots (a)–(f) show how the trapezium changes over the course of its life.

Table 2. The number of simulations which produce a trapezium system and its dependence on the virial ratio Q and fractal dimension D of the initial cluster.

Q	1.6	2.0	2.6	3.0
		D		
0.3	45/50	4/10	2/10	4/10
0.4	32/50	8/10	4/10	1/10
0.5	5/10	5/10	0/10	1/10

Table 3. The average formation time for trapezium systems in Myr in simulations with different fractal dimensions (D) and virial ratios (Q). Note that these averages are over the simulations which form trapezium systems, and in the $D = 3.0$, $Q = 0.4$ and 0.5 ensembles marked by a ⋆ only one system forms and so this is the formation time of those single systems rather than an average.

| Q	1.6	2.0	2.6	3.0
		D		
0.3	1.52	2.13	2.50	3.13
0.4	2.00	2.13	3.63	3.25⋆
0.5	2.05	3.45	–	3.25⋆

parameters provide a good environment to form tightly bound high-order multiple systems. As is shown in Tables 2 and 4 clusters that begin with warmer and smoother initial distributions form only a few trapezium systems. Therefore, the average size of trapeziums in these clusters is statistically less reliable than in the cooler, clumpier clusters.
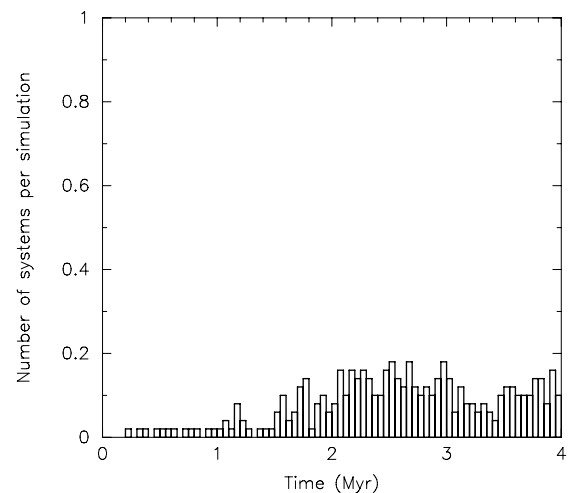
5.2 The formation of trapeziums

We have seen that in initially low-D and low-Q clusters many small trapeziums are formed, they tend to form early (<2 Myr), and can survive for a significant fraction of the cluster's early life (~2 Myr). However, in initially high-D and high-Q clusters far fewer and larger trapeziums form, they form later, and they have shorter lifetimes. We have also seen that trapeziums are highly dynamic entities that constantly change their low-mass members and also decay, re-form and swap higher mass members.

These properties come about because in our simulations trapeziums are most often formed during the dense phase of the collapse of a young star cluster. As described in detail in Allison et al. (2009a, 2010) clusters which are out-of-equilibrium undergo a rapid (<1 Myr) collapse and violent relaxation phase as they attempt to virialize. In clusters with a low Q and low D their radii may shrink initially by a factor of several leading to a short-lived but extremely dense phase. In this phase the two-body relaxation time can be extremely short (i.e. <1 Myr) and clusters can dynamically mass segregate their most massive stars (in the case of Orion-like clusters this is stars more massive than a few solar masses).

This leads to a situation where the most massive stars are centrally concentrated and in which the formation of trapezium systems is relatively easy. The densest phase is usually reached at around 1 Myr
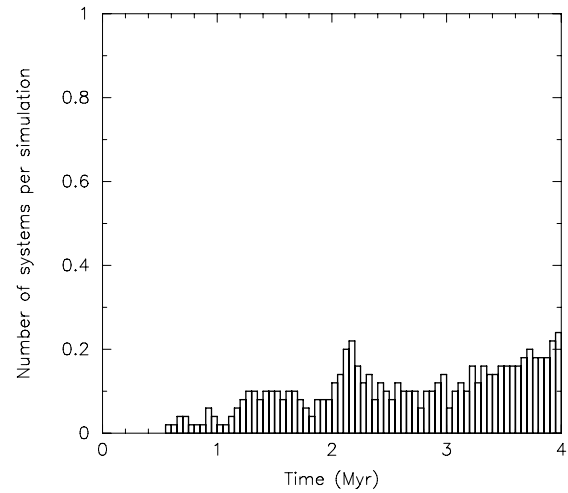
(a) a1

(b) b1

Figure 3. Plots show the fraction of clusters with trapezium-like systems changes as a function of time in the 'a1-type' (top) and 'b1-type' (bottom) simulations. The plots for all simulations types can be found in the Supporting Information.

(about an initial free-fall time) explaining why trapeziums in low-Q and low-D clusters tend to first form on such a time-scale.

The higher both D and Q become the deeper the depth of the collapse and so the lower the degree of mass segregation and trapezium

Table 4. Average number of different trapezium systems present in simulations with different fractal dimensions (D) and virial ratios (Q). Again note that these averages are over the simulations which form trapezium systems, and the ensembles marked with a ⋆ have only one system that briefly appears and so do not represent an average.

Q	1.6	2.0	2.6	3.0
		D		
0.3	3.7	3.8	2.5	2.0
0.4	2.0	2.6	1.8	1.0⋆
0.5	2.4	1.4	–	1.0⋆

Table 5. The average MST size of all trapezium systems with N members in all simulations with 1σ errors, as well as the total number of snapshots in which systems of each N are found.

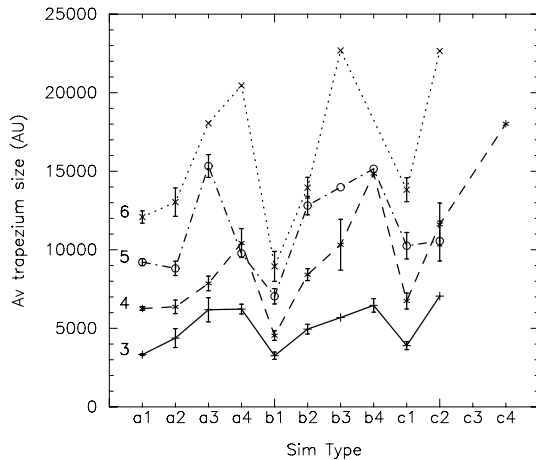| N	Length (au)			# Systems
3	5143	$\pm$	469	776
4	9575	$\pm$	1085	739
5	11 279	$\pm$	1157	418
6	16 169	$\pm$	2346	196
7	16 890	$\pm$	2290	70
8	18 864	$\pm$	2253	23
9	18 613	$\pm$	3085	4

Figure 4. The average size of trapeziums of $N = 3, 4, 5$ and 6 (bottom to top, indicated by numbers on left of plot) for the different initial conditions letters correspond to initial virial ratio a: $Q = 0.3$, b: $Q = 0.4$ and c: $Q = 0.5$, and numbers to the initial fractal dimension 1: $D = 1.6$, 2: $D = 2.0$, 3: $D = 2.6$ and 4: $D = 3.0$. Error bars show standard error.

formation. Low D is the dominant factor as the degree of collapse is more sensitive to this parameter than the virial ratio Q. This is because initially clumpy distributions have a larger initial net potential energy ($|\Omega|$), which increases during the collapse of the cluster, therefore allowing the collapse to form a smaller final cluster compared to less clumpy initial conditions. In initially virialized and smooth clusters there may be some collapse and relaxation (especially as the cluster moves from initially uniform density to a more Plummer-like density distribution), but it is far less extreme and violent. Trapeziums that form in such clusters are freak events and tend to decay on time-scales of a few of their own crossing times.

Formation during collapse also explains the difference in the size of trapezium systems. In low-Q and low-D clusters, the collapse is deeper producing smaller trapeziums, whilst in high-Q and high-D clusters if a trapezium does form it is more by chance and will tend to be much larger as the chance of a very close encounter between several of the most massive stars is fairly low.

5.2.1 *The dissolution of trapezium systems and their host clusters*

We have shown that trapezium systems are frequently formed through dynamical interactions in cool and clumpy clusters, but how and at what ages would we expect these systems to decay? There are three main decay paths for the massive star multiples that

are formed in our simulations: the supernova of the most massive member; the dynamical decay of the trapezium system itself possibly, destroying the host cluster in the process and the dissolution of the cluster around the system (most likely caused by gas expulsion) leaving an unstable few-body system that will dynamically decay in a few crossing times.

The dynamical decay of trapezium systems occurs due to the inherent instability of the few-body multiple systems. The decay occurs when at least three of the massive stars in the system approach closely to each other. The encounter leads to the ejection of massive stars from the system, leaving a hard massive-star binary. The binary can increase its binding energy by as much as the entire potential energy of the cluster. The decay of multiple systems does not appear to occur with any regularity or obvious trigger, indicating that this decay mechanism is chaotic. It is dependant only on the chance encounter between massive stars.

Dynamically decaying trapezium systems can lead to the dissolution of the host clusters they reside in; but the converse is also true – the dissolution of a cluster around a trapezium can lead to the decay of the massive multiple. The cluster can dissolve around the multiple system because of a rapid change in the cluster's potential due to the expulsion of gas – the group of luminous massive stars in the core will most likely drive out the natal gas from the cluster. When this occurs the rapid change in potential causes the cluster to attempt to re-virialize itself, leading the cluster to rapidly expand on a time-scale of $\sim$2–5 Myr (Goodwin & Bastian 2006). This will leave the trapezium as a lone, and highly unstable, high-order multiple with no host cluster to replace ejected stars.

If these other mechanisms have not destroyed the trapezium system by the time the most massive member becomes a supernova, the trapezium system could be destroyed by the supernova of the most massive member. The supernova will cause a huge loss of mass from the system (and quite possibly eject the remnant due to a large kick velocity). Obviously, the time-scale that this decay path would occur on is dependant on the mass of the most massive member. Even if the supernova of the most massive member is not catastrophically destructive, all of the massive stars in a trapezium will go supernovae within a few tens of Myr at most meaning that the system is no longer a trapezium (as it has no massive stars).

6 DISCUSSION

We have analysed N-body simulations of initially cool and clumpy star clusters, and of the formation and evolution of trapezium-like systems. We find that trapezium system formation is dependant on the initial conditions of the host cluster – with clumpier and cooler clusters more likely to form trapezium systems. We also find that the physical properties of trapezium systems, such as size, are also dependant on the initial state of the cluster. Importantly we find that trapezium systems are highly dynamical, and often transient, objects that add, remove and swap members often.

The formation of trapezium systems is an almost ubiquitous process in the collapse of cool and clumpy clusters, with around 75 per cent of clusters with $Q = 0.3$–0.4 and $D = 1.6$–2.0 forming a trapezium system during the first 4 Myr of their dynamical evolution. In contrast, <10 per cent of clusters with relatively little substructure ($D = 2.6$–3.0) and close to virial equilibrium form trapezium systems. From this analysis, if the formation of stars is a cool and clumpy process we should expect to see trapezium systems in many young clusters ($\sim$2–4 Myr), that have recently aggregated into a newly formed 'cluster' (Zinnecker & Yorke 2007; Zinnecker

2008). The trapezium formation processes described in this work and by Zinnecker are qualitatively similar to each other in that both processes involve the merger of a clumpy stellar distribution, which leads to the formation of a trapezium system. The main difference between the models is that Zinnecker proposes that the clumpy distribution of stars is one in which there are distinct subclusters, each containing a massive star. The merger of these subclusters then brings the massive stars together and a trapezium system is formed. The process described in this work instead proposes a hierarchical distribution of stars, a random distribution of massive stars and it is this global collapse and subsequent dynamical mass segregation which brings the massive stars together. Both models propose that trapezium systems would be formed by the dynamical evolution of the merging cluster stars.

The constant changing of trapezium systems (e.g. Fig. 2) suggests that trapezium systems are highly dynamical objects. From this analysis, it appears very likely that the trapezium system in the ONC that we observe today is only a snapshot in its lifetime; *the system very probably appeared different in the past, and will likely change in the future* by incorporating new members into the system, discarding current members, changing its spatial distribution and eventually dissolving – possibly destroying the ONC at the same time. Recent investigations have in fact already indicated that the ONC has ejected some member stars (Tan 2004; Poveda, Allen & Hernández-Alcántara 2005; Pflamm-Altenburg & Kroupa 2006).

Observations of the 'canonical' trapezium system located in the ONC find that it is hierarchical; the member stars in the trapezium system are all multiple, found to be in either binary systems or in 'minitrapeziums' of their own (Preibisch et al. 1999; Kraus et al. 2007; Moeckel & Bonnell 2009). It should be mentioned here that we do not form such hierarchical systems, and usually find that only the most massive stars in the trapezium systems that are formed appear to be in lasting binary systems. As we begin our simulations with only single stars this may not be surprising, but the result does show that to produce trapezium systems with the multiplicity seen in the ONC primordial multiplicity is important.

Observations of two high-mass star-forming regions have found possible examples of trapezium systems in embedded clusters. Megeath, Wilson & Corbin (2005) find a deeply embedded $N = 5$ system in W3. This system has a maximum projected separation between the five sources of $\sim$5600 au, which would likely indicate that this system would be detected as a trapezium system by our algorithm. Observations of the star-forming region NGC 7538 find a possible system consisting of nine submillimetre cores, with masses between 20 ± 11 and $6 \pm 3 \, M_\odot$ within a $0.3 \times 0.3 \, pc^2$ area (Qiu, Zhang & Menten 2011). This system has a large MST length ($\sim$160 000 au), and so is substantially larger than the $N = 9$ systems we find in our simulations (which have a MST length $\sim$18 000 $\pm$ 3000 au) and is therefore unlikely to be classified as a trapezium system through our definition. However, because of the age of this cluster, the potential system could be in the process of forming. Kinematical information would be of great use for determining the future evolution of this system.

7 CONCLUSIONS

We have simulated the early N-body dynamical evolution of Orion-like ($N = 1000$) star clusters with a range of initial conditions from cool and clumpy to smooth and relatively relaxed. We find that the formation of higher order massive star multiples similar to the trapezium in the ONC to be fairly common in cool and clumpy star clusters, but rare in those that start smooth and relatively relaxed.

Our main conclusions can be summarized as follows.

(i) High-order massive multiple (trapezium) systems can form dynamically during the dense collapse of cool, clumpy clusters. *Therefore, trapezium systems have no need to be primordial.*

(ii) Trapezium systems change their size, structure and membership (especially low mass) frequently with trapeziums rarely looking the same over time-scales of only a few $\times 10^5$ yr. *Therefore, drawing conclusions from the current appearance of a trapezium system must be done with great care.*

(iii) Trapeziums can be very long lived (up to 2–3 Myr in our simulations that only last for 4 Myr) only in the sense that a trapezium system is observable, however, the dynamical nature of the trapeziums means it is difficult (to say the least) to say that the 'same' trapezium is always present.

Like the trapezium in the ONC, the trapezium systems that we find have, by definition, at least three massive ($>8 \, M_\odot$) stars, but also generally have several low-mass members as well. However, unlike the trapezium system, our trapeziums do not display the close binaries and 'minitrapeziums' that are observed. As the initial conditions of the simulations do not include primordial binaries, this is a likely reason why we do not see such a phenomenon.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article:

Figures a–u. Plots similar to Fig. 1 for all of the simulations.

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