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Uncertainty Management for Rule-based Decision Support Systems

Quratul-ain Mahesar, Vania G Dimitrova, Derek R Magee, Anthony G Cohn
School of Computing, University of Leeds, Leeds, UK
Email: {Q.Mahesar|V.G.Dimitrova|D.R.Magee|A.G.Cohn}@leeds.ac.uk

***Abstract*—We present an uncertainty management scheme in rule-based systems for decision making in the domain of urban infrastructure. Our aim is to help end users make informed decisions. Human reasoning is prone to a certain degree of uncertainty but domain experts frequently find it difficult to quantify this precisely, and thus prefer to use qualitative (rather than quantitative) confidence levels to support their reasoning. Secondly, there is uncertainty in data when it is not currently available (missing). In order to incorporate human-like reasoning within rule-based systems we use qualitative confidence levels chosen by domain experts in urban infrastructure. We introduce a mechanism for the representation of confidence of input facts and inference rules, and for the computation of confidence in the inferred facts. We also present a mechanism for computing inferences in the presence of missing facts, and their effect on the confidence of inferred facts.**

Keywords: uncertainty, decision support systems, reasoning.

I. Introduction: Reasoning under Uncertainty

Rule-based Systems also widely known as Expert Systems [9], Knowledge Based Systems (KBSs) [1] and Intelligent Systems [10] were introduced in the 1970s [11] and have gained increasing popularity in various domains such as business, engineering, military, and medicine. Rule based systems allow us to capture the expert knowledge of humans in the form of rules and data in the form of facts. Given a set of facts in working memory, a forward chaining inference engine uses the rules to generate new facts until the desired goal is reached. The following steps are taken by a forward chaining inference engine: 1) match the condition patterns of rules against facts in working memory, 2) if there is more than one rule that could be used i.e. that could fire, select which one to apply (this is called conflict resolution), 3) apply the rule, maybe causing new facts to be added to working memory, 4) halt when some useful conclusion (or goal) is added to working memory or when all possible conclusions have been drawn.

Rule-based systems for decision making also known as intelligent decision support systems (DSS) [2], [18] require some additional functionality to emulate intelligent human behaviour. There is a degree of uncertainty [7] attached to such human reasoning. Whilst humans are adept at reasoning with uncertainty that may involve different confidence levels for different aspects of their knowledge, it is very challenging to incorporate this in rule-based systems for decision making that suitably reflects human usage. This type of uncertainty can be caused by problems with data, e.g., data might be missing or currently unavailable, data might be unreliable or ambiguous due to measurement errors, data representation might be imprecise or inconsistent etc. Furthermore, uncertainty may also be caused by represented knowledge, e.g., expert guesses that might be based on plausible or statistical associations they have observed, the same knowledge might not be appropriate for different situations etc. All these types of uncertainties need to be handled within reasoning in rule based systems that capture human expert knowledge. Some type of uncertainty management is crucial for such systems. There are some important issues that need to be dealt with when implementing an uncertainty management scheme, such as: (a) how to represent uncertain data and knowledge? (b) how to combine two or more pieces of uncertain data and knowledge? (c) how to draw inferences using uncertain data and knowledge?

Whilst certainty factors that indicate confidence of experts for data and rules have been previously implemented in systems such as [3], [5] based on the MYCIN approach [19], reasoning with missing information has only been performed with restrictions where all rules need to have the same number of antecedents [14], [13]. Moreover, identifying inference chains for conclusions that require minimum missing information and the effect of missing information on certainty factors does not appear to have been previously investigated. Other approaches have used certainty factors that are based on probability theory [16], [17]. However, our novel scheme is a result of discussions with experts in the field of urban infrastructure (which is the focus of our investigations). In particular, we present (in section II-A) a scheme in which confidence levels for data and rules take *qualitative* rather than numeric values as used in systems based on Fuzzy Logic [15], [23], [22], which better reflects the terminology and usage by experts in our chosen domain of reasoning about asset management in urban infrastructure [6], [4]. In section II-B, we present a mechanism that enables reasoning with confidence levels. We also present a mechanism to make inferences in the presence of missing information in section II-C. Furthermore, we combine both approaches in section II-D. Implementation details are presented in section III. Finally, we present conclusion and future work in section IV.

II. Proposed Approaches for Handling Uncertainty

We present two different approaches: one to compute confidence levels, i.e., likelihoods for both data and rules and the other that enables computing inferences in the presence of missing facts. We also present a mechanism to combine both approaches to provide an uncertainty management scheme that deals with both confidence levels and missing information.

A. Confidence Levels: Qualitative Likelihoods for Facts and Rules

We will allow rules to have a qualitative certainty factor associated with them; e.g. the conclusion is "very likely." The inferred facts will have that associated qualitative certainty factor. Since we propose to specify confidence factors qualitatively, we will find it convenient to record the different levels of certainty for the rules involved in the history of a derivation of an inferred fact. As a result, although we do not have a single numeric level of confidence in a single fact (as would be the case in a Mycin-like system) where confidences can be easily computed as rules are applied, it will still be possible to say which of two facts A and B, with confidence vectors C_A and C_B we are more confident in by comparing the values in the confidence vectors as will be seen below in section II-B. To reason under uncertainty, we therefore define a *confidence vector* for facts as follows. This vector records the number of rules with different certainty factors used in the derivation of a fact.

Definition II.1. *Let C_F denote the confidence vector of a fact F. Let U, L, V, D be the different confidence levels represented as follows in ascending order of confidence:*

1) *U: Unlikely.*
2) *L: Likely.*
3) *V: Very Likely.*
4) *D: Definite.*

Then, we define: $C_F = \langle U, L, V \rangle$, where U, L, V can have any non-negative (≥ 0) integer value. The confidence level D (Definite) is implicit in the definition of C_F when U, L, V are all 0.

We have chosen confidence levels U, L, V, D in the above Definition II.1. However, any finite set of values with a total order would also be possible.

Definition II.2. *We also need to be able to specify the confidence C_R an expert has given for a rule R. We denote the C_R as follows:*

$$C_R = \langle U, L, V \rangle$$

U, L, V are the different confidence levels given in Definition II.1 and $U + L + V \leq 1$. Thus, at most one of the U, L, V is 1. If all are 0, then the rule has confidence level D (Definite). This representation of the confidence in a rule will facilitate the calculation of the confidence of the consequent in a rule (see Formula II.2).

The confidence levels given in Definition II.1 were elicited as a result of discussions with domain experts following their difficulties in giving numeric certainties to rules.

B. Reasoning with Confidence Levels

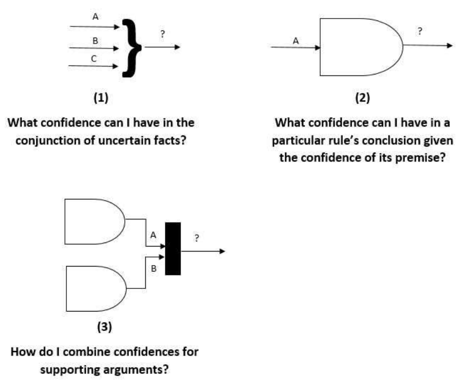

Figure 1. Handling uncertainty in rule-based systems

Three problems need to be addressed when reasoning with confidence levels, as shown in Figure 1. Each problem is defined below and a mechanism to solve the problem is presented as a formula which is followed by an example.

Problem II.1. ***Confidence level of conjunction of uncertain facts.***

$$If\ A_1\ \wedge\ ...\ \wedge\ A_n\ then\ ...$$

If my confidence in each A_i *is* C_i*, how confident am I in the conjunction (*$A_1\ \wedge\ ...\ \wedge\ A_n$*)?*

Formula II.1. *Let* $C_{1...n}$ *denote the confidence in the conjunction (*$A_1\ \wedge\ ....\ \wedge\ A_n$*) where the confidence of* A_i *is* $C_i = \langle U_i, L_i, V_i \rangle$*. Then,* $C_{1...n} = \langle U_{1...n}, L_{1...n}, V_{1...n} \rangle$*, where* $\langle U_{1...n}, L_{1...n}, V_{1...n} \rangle =$

$$max(\langle U_1, L_1, V_1 \rangle, ..., \langle U_n, L_n, V_n \rangle)$$

where $max(...)$ *returns the maximum value of the given arguments computed as follows.*

$$\langle U_1, L_1, V_1 \rangle > \langle U_2, L_2, V_2 \rangle \quad \textit{iff}$$
$$(U_1 > U_2) \vee (U_1 = U_2 \wedge L_1 > L_2) \vee$$
$$(U_1 = U_2 \wedge L_1 = L_2 \wedge V_1 > V_2)$$

The reason to take the maximum value is because an antecedent is only as likely as its most unlikely conjunct, and a higher value in U, L or V indicates greater uncertainty at that level of uncertainty.

Example II.1. *Suppose* $C_1 = \langle 0, 1, 0 \rangle$*,* $C_2 = \langle 0, 0, 1 \rangle$*,* $C_3 = \langle 1, 0, 0 \rangle$*, then confidence* $C_{1,2,3}$ *in the conjunction (*$A_1\ \wedge\ A_2\ \wedge\ A_3$*) can be computed by using Formula II.1 as follows:*

$$C_{1,2,3} = \langle U_{1,2,3}, L_{1,2,3}, V_{1,2,3} \rangle$$
$$= max(\langle U_1, L_1, V_1 \rangle, \langle U_2, L_2, V_2 \rangle, \langle U_3, L_3, V_3 \rangle)$$

Since, ($U_3 > U_1 \wedge U_3 > U_2$*), therefore, we get* $C_{1,2,3} = \langle 1, 0, 0 \rangle$.

Problem II.2. ***Confidence level of a rule's conclusion given the confidence level of its premise.***

$$If\ D\ then\ E$$

If my confidence in D *is* C_D *how confident can I be in* E*?*

Formula II.2. *Let* C_E *denote the confidence in rule conclusion E and* $C_D = \langle U_D, L_D, V_D \rangle$ *be the confidence in D, and* $C_R = \langle U_R, L_R, V_R \rangle$ *be the rule's confidence. Then* C_E *can be computed thus:*

$$C_E = C_D + C_R$$
$$C_E = \langle U_D + U_R, L_D + L_R, V_D + V_R \rangle$$

Example II.2. *If* C_E *denotes the confidence in rule conclusion* E *and* $C_D = \langle 0, 0, 1 \rangle$ *is the confidence in D, and* $C_R = \langle 1, 0, 0 \rangle$ *is the rule's confidence. Then* C_E *can be computed by using Formula II.2 as follows:*

$$C_E = C_D + C_R$$
$$C_E = \langle U_D + U_R, L_D + L_R, V_D + V_R \rangle$$
$$C_E = \langle 0 + 1, 0 + 0, 1 + 0 \rangle = \langle 1, 0, 1 \rangle$$

Problem II.3. ***Combining confidence levels of the same conclusion with two separate rules.***

If the same fact F *is deduced from two separate rules with confidences* C_1 *and* C_2*, how confident am I in* F*?*

Formula II.3. *Let* $C_F = \langle U_F, L_F, V_F \rangle$ *denote the confidence in the derived fact* F *deduced from two separate rules with confidences* $C_1 = \langle U_1, L_1, V_1 \rangle$*,* $C_2 = \langle U_2, L_2, V_2 \rangle$*, where* U, L, V *with subscripts* $1, 2$ *are the confidence levels defined earlier, then each element of* $C_F = \langle U_F, L_F, V_F \rangle$ *can be computed as follows:*

$$\langle U_F, L_F, V_F \rangle = min(\langle U_1, L_1, V_1 \rangle, \langle U_2, L_2, V_2 \rangle)$$

where $min(\langle U_1, L_1, V_1\rangle, \langle U_2, L_2, V_2\rangle)$ returns the lesser of the two arguments computed as follows.

$$\langle U_1, L_1, V_1\rangle < \langle U_2, L_2, V_2\rangle \quad \textit{iff}$$
$$(U_1 < U_2) \vee (U_1 = U_2 \wedge L_1 < L_2) \vee$$
$$(U_1 = U_2 \wedge L_1 = L_2 \wedge V_1 < V_2)$$

This formula can be applied in a similar way to a fact deduced from any number of separate rules by increasing the number of arguments. The reason to choose the minimum is that this represents the most likely of the confidences of C_1 and C_2.

Example II.3. *Suppose $C_1 = \langle 0, 0, 1\rangle$, $C_2 = \langle 0, 1, 0\rangle$ be the confidences of two separate rules to derive the fact F, then the confidence $C_F = \langle U_F, L_F, V_F\rangle$ combining the two confidences C_1 and C_2 can be computed by using Formula II.3 as follows:*

$$\langle U_F, L_F, V_F\rangle = min(\langle U_1, L_1, V_1\rangle, \langle U_2, L_2, V_2\rangle)$$

Since, $(U_1 = U_2 \wedge L1 < L2)$, therefore, we get $C_F = \langle 0, 0, 1\rangle$.

We now present a worked example for reasoning with confidence levels.

Example II.4. *Suppose we have the following initial facts with their confidence vectors:*

- $A_1 : C_1 = \langle 0, 0, 1\rangle$,
- $A_2 : C_2 = \langle 1, 0, 0\rangle$,
- $A_3 : C_3 = \langle 0, 0, 1\rangle$,
- $A_4 : C_4 = \langle 0, 0, 1\rangle$,

Suppose we have the following rules:

- *R_1: IF A_1 and A_3 THEN A_5, with confidence vector $C_{R_1} = \langle 0, 1, 0\rangle$*
- *R_2: IF A_1 and A_2 and A_4 THEN A_5, with confidence vector $C_{R_2} = \langle 0, 0, 1\rangle$*

Since A_1 and A_3 are input facts, and match the premise of rule R_1, therefore R_1 is fired by the inference engine. Suppose $C_{1,3}$ represents the confidence in the conjunction of facts A_1 and A_3, then $C_{1,3}$ can be computed by using Formula II.1 as follows:

$$C_{1,3} = \langle U_{1,3}, L_{1,3}, V_{1,3}\rangle$$
$$= max(\langle U_1, L_1, V_1\rangle, \langle U_3, L_3, V_3\rangle)$$

Therefore, $C_{1,3} = \langle 0, 0, 1\rangle$ Now, the confidence vector $C_5^{R_1}$ for the fact A_5 in the conclusion of rule R_1 can be computed by using Formula II.2 with $C_{R_1} = \langle 0, 1, 0\rangle$:

$$C_5^{R_1} = C_{1,3} + C_{R_1}$$
$$C_5^{R_1} = \langle U_{1,3} + U_{R_1}, L_{1,3} + L_{R_1}, V_{1,3} + V_{R_1}\rangle$$
$$C_5^{R_1} = \langle 0 + 0, 0 + 1, 1 + 0\rangle = \langle 0, 1, 1\rangle$$

Since A_1, A_2 and A_4 are input facts, and match the premise of rule R_2, therefore R_2 is fired by the inference engine. Suppose $C_{1,2,4}$ represents the confidence in the conjunction of facts ($A_1 \wedge A_2 \wedge A_4$), then $C_{1,2,4}$ can be computed by using Formula II.1 with $C_1 = \langle 0, 0, 1\rangle$, $C_2 = \langle 1, 0, 0\rangle$ and $C_4 = \langle 0, 0, 1\rangle$ as follows:

$$C_{1,2,4} = \langle U_{1,2,4}, L_{1,2,4}, V_{1,2,4}\rangle,$$
$$= max(\langle U_1, L_1, V_1\rangle, \langle U_2, L_2, V_2\rangle, \langle U_4, L_4, V_4\rangle)$$

Therefore, $C_{1,2,4} = \langle 1, 0, 0\rangle$ Now, the confidence vector $C_5^{R_2}$ for the fact A_5 in the conclusion of rule R_2 can be computed by using Formula II.2 with $C_{R_2} = \langle 0, 0, 1\rangle$ as follows:

$$C_5^{R_2} = C_{1,2,4} + C_{R_2}$$
$$C_5^{R_2} = \langle U_{1,2,4} + U_{R_2}, L_{1,2,4} + L_{R_2}, V_{1,2,4} + V_{R_2}\rangle$$
$$C_5^{R_2} = \langle 1 + 0, 0 + 0, 0 + 1\rangle = \langle 1, 0, 1\rangle$$

Since, $C_5^{R_1} = \langle 0, 1, 1\rangle$ and $C_5^{R_2} = \langle 1, 0, 1\rangle$ are the confidence vectors of two separate rules to derive the fact A_5, the confidence $C_5 = \langle U_5, L_5, V_5\rangle$ of the derived fact A_5 combining the two confidences $C_5^{R_1}$ and $C_5^{R_2}$ can be computed by using Formula II.3 as follows:

$$\langle U_5, L_5, V_5\rangle = min(\langle U_5^{R_1}, L_5^{R_1}, V_5^{R_1}\rangle, \langle U_5^{R_2}, L_5^{R_2}, V_5^{R_2}\rangle)$$

Therefore, we get $C_5 = \langle 0, 1, 1\rangle$.

C. Reasoning with Missing Facts

Just as SPARQL [20] allows rules to have optional preconditions, we also want to allow a rule to fire even if not all of its preconditions are present in the working memory. We propose a method to make inferences even if some of the premises of a rule are missing, by attaching these missing facts M as assumptions to the conclusion of the rule. This approach differs from that of SPARQL mentioned above since that does not remember the optional facts not found to be present; in our domain the experts want to know what facts were assumed to be present in the derivation of a conclusion, so that if the derived facts which depend on these are of interest/concern to them, then they can conduct further investigations to check whether these missing facts hold or not. We will also propagate the given facts G used in any inference forward to the conclusion so that the user can also see which facts were used in the derivation; this does not affect the reasoning at all, but is merely for user convenience.

As shown in Figure 2, three problems need to be addressed. Each problem is defined below and our proposed mechanism to solve the problem is presented as a formula which is followed by an example.

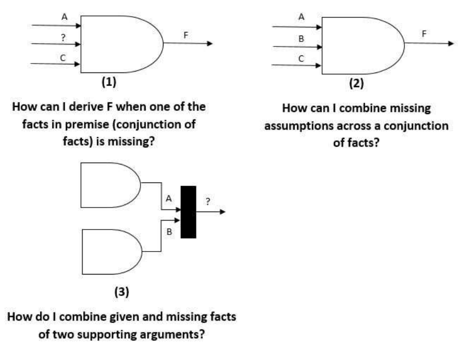

Figure 2. Handling missing information in rule-based systems

Problem II.4. ***Deriving a rule's conclusion given some missing facts in its premise.***

$$If\ A_1\ and\ A_2\ and\ A_3\ then\ F$$

If facts A_1 and A_2 are given and fact A_3 is missing, how can I derive fact F?

Formula II.4. *Let facts A_1 and A_2 be given and fact A_3 be missing in the premise (conjunction of facts) of the rule. Let $G_{1,2}$ denote the set of given facts in the conjunction of $(A_1 \ \wedge \ A_2)$ and let $M_{1,2}$ denote the set of missing facts in the conjunction of $(A_1 \wedge A_2)$ then we can derive fact F in the conclusion by attaching two sets consisting of given facts denoted by G and missing facts denoted by M. G and M can be computed by using the following formulae:*

$$G = G_{1,2} \cup A_1 \cup A_2$$
$$M = M_{1,2} \cup A_3$$

Example II.5. *If facts A_1 and A_2 are given and fact A_3 is missing in the premise (conjunction of facts) of the rule. $G_{1,2} = \{B_1, B_3\}$ is the set of given facts in the conjunction of $(A_1 \ \wedge \ A_2)$ and $M_{1,2} = \{B_2\}$ is the set of missing facts in the conjunction of $(A_1 \wedge A_2)$ then we can derive fact F in the conclusion by attaching two sets consisting of given facts denoted by G and missing facts denoted by M. G and M can be computed by using Formula II.4 as follows:*

$$G = G_{1,2} \cup A_1 \cup A_2$$
$$= \{B_1, B_3\} \cup A_1 \cup A_2 = \{A_1, A_2, B_1, B_3\}$$
$$M = M_{1,2} \cup A_3 = \{B_2\} \cup A_3 = \{A_3, B_2\}$$

Problem II.5. ***Combining assumptions within a conjunction of facts.***

$$If\ A_1 \ \wedge \ .... \ \wedge \ A_n\ then\ F$$

Let fact A_i be derived from given facts G_i and missing facts M_i, then how can I compute set of given facts $G_{1..n}$ and set of missing facts $M_{1...n}$ in the conjunction of $(A_1 \ \wedge ... \ \wedge \ A_n)$

Formula II.5. *If fact A_i is derived from given facts G_i and missing facts M_i, then we can compute the set of given facts $G_{1...n}$ and the set of missing facts $M_{1...n}$ in the conjunction of $(A_1 \ \wedge \ ... \ \wedge \ A_n)$ thus:*

$$G_{1...n} = G_1 \cup ... \cup G_n$$
$$M_{1...n} = M_1 \cup ... \cup M_n$$

Example II.6. *Suppose fact A_1 is derived from a set of given facts $G_1 = \{B_1, B_2\}$ and a set of missing facts $M_1 = \{B_3\}$, fact A_2 is derived from a set of given facts $G_2 = \{B_4\}$ and a set of missing facts $M_2 = \{B_5\}$, and fact A_3 is derived from a set of given facts $G_3 = \{B_7, B_8\}$ and a set of missing facts $M_3 = \{B_3\}$, then we can compute the set of given facts $G_{1,2,3}$ and the set of missing facts $M_{1,2,3}$ in the conjunction of $(A_1 \wedge A_2 \wedge A_3)$ using Formula II.5 as follows:*

$$G_{1,2,3} = G_1 \cup G_2 \cup G_3$$
$$= \{B_1, B_2\} \cup \{B_4\} \cup \{B_7, B_8\}$$
$$= \{B_1, B_2, B_4, B_7, B_8\}$$
$$M_{1,2,3} = M_1 \cup M_2 \cup M_3$$
$$= \{B_3\} \cup \{B_5\} \cup \{B_3\} = \{B_3, B_5\}$$

Problem II.6. ***Combining given and missing facts of the same conclusion derived by two***

separate rules. *If the same fact F is deduced from two separate rules with the attached sets of given and missing facts G_1, M_1 and G_2, M_2, how can I conclude F by combining the attached sets of given and missing facts?*

Formula II.6. *If there is a fact F deduced from two separate rules with the attached sets consisting of given and missing facts G_1, M_1 and G_2, M_2 respectively, then we can compute the combined sets of given and missing facts $G_{1,2}, M_{1,2}$ for F using the following formula:*

$$\langle G_{1,2}, M_{1,2}\rangle = min(\langle G_1, M_1\rangle, \langle G_2, M_2\rangle)$$

where $min(\langle G_1, M_1\rangle, \langle G_2, M_2\rangle$ returns the vector with the fewer missing facts computed as follows.
$\langle G_1, M_1\rangle \leq \langle G_2, M_2\rangle$ iff $length(M_1) \leq length(M_2)$

Example II.7. *Suppose fact F is deduced from two separate rules with the attached sets consisting of given and missing facts $G_1 = \{A, B, C\}, M_1 = \{D\}$ and $G_2 = \{A, E\}, M_2 = \{B, G\}$, then we can compute the vector consisting of sets of given and missing facts $\langle G_F, M_F\rangle$ for F using Formula II.6 as follows:*

$$\langle G_F, M_F\rangle = min(\langle G_1, M_1\rangle, \langle G_2, M_2\rangle)$$
$$\langle G_F, M_F\rangle = min(\langle\{A, B, C\}, \{D\}\rangle, \langle\{A, E\}, \{B, G\}\rangle)$$
$$\langle G_F, M_F\rangle = \langle\{A, B, C\}, \{D\}\rangle$$

D. Reasoning with Confidence Levels and Missing Facts

To reason with uncertainty, in particular to show the effect of missing facts on the confidence of an inferred fact, we define the confidence vector for facts as follows. (Note: here we modify our previous Definition II.1 to take into account the number of missing facts in the inference path for a deduced fact.)

Definition II.3. *Let C_F denote the confidence vector of a fact F. Let U, L, V, D be the different confidence levels as given in Definition II.1 and N be the number of missing facts accumulated in the inference path of fact F. (Note: N will be 0 for initial input facts)*

Then, we define C_F as follows:

$$C_F = \langle U, L, V, N\rangle$$

The confidence vectors for facts computed in Formulae II.1, II.2 given in section II-B can now be modified to add the number of missing facts N for a set of missing facts M, where $N = length(M)$.

Problem II.7. ***Combining confidence levels of the same conclusion (having missing facts) with two separate rules.*** *If the same fact F is deduced from two separate rules with sets of missing facts M_1 and M_2, how confident am I in F?*

The are two approaches to this problem: (1) we assume the user is prepared to assume that the missing facts are in fact true with high confidence; in this case we should compute the confidence of F using Formula II.3, and propagate the missing facts from those derived from the chosen rule according to that computation. (2) we assume that the user is rather uncertain about the missing facts, even more so than facts derived with a low $\langle U, L, V\rangle$ confidence vector; in this case we should compute the confidence of F according to which derivation has the fewer missing facts, using Formula II.6. For space reasons we omit the formal definition of this here but present an example.

Example II.8. *Suppose fact F is deduced from two separate rules with the attached sets of given and missing facts $G_1 = \{A, B, C\}$, $M_1 = \{D\}$ and $G_2 = \{A, E\}$, $M_2 = \{B, G\}$. Let C_F denote the confidence in the concluded fact F deduced from two separate rules with confidences C_1 and C_2, and $C_1 = \langle 2, 1, 2, 1\rangle$, $C_2 = \langle 2, 1, 1, 3\rangle$.*

If we follow approach (1) above then we can compute that $C_2 < C_1$ since $U_1 = U_2$ and $L_1 = L_2$ but $V_2 < V_1$, so $C_F = C_2 = \langle 2, 1, 1, 3\rangle$, with the set of missing facts and given facts as M_2 and G_2.

However, if we follow approach (2) then we compute that $M_1 < M_2$ and hence $C_F = C_1 = \langle 2, 1, 2, 1\rangle$, with the set of missing facts and given facts as M_1 and G_1.

III. Implementation Details

Our techniques are not written for any specific languages or tools and can be implemented in any off the shelf tool of choice that is based on a forward chaining rule inference engine. We use Jess [12] for implementing our proposed approaches for evaluation purpose. Jess is a forward chaining rule inference engine implemented in Java. It is a partial reimplementation of the CLIPS [21] Expert System shell. Jess uses an enhanced version of the Rete algorithm [8] to process rules. Rete has a very efficient mechanism for solving the difficult many-to-many matching problem, i.e., comparing a large collection of patterns to a large collection of objects.

We have implemented our three different reasoning approaches:

Reasoning with confidence levels: The confidence vector defined for facts in Definition II.1 is attached with each fact and confidence vector defined for rules in Definition II.2 is attached with each rule. Whenever, each rule is fired the confidence vector for the conjunction of premise is computed by using Formula II.1 and the confidence vector for the newly deduced fact (i.e., conclusion) is computed by using Formula II.2. During the execution of the inference engine, the confidence vectors for all deduced facts that are same but computed by using different rules are combined by using Formula II.3.

Reasoning with missing facts: All initial input facts have empty sets of missing and given facts (as they are not deduced by an inference chain). When a rule is fired the sets of missing and given facts are computed in the conjunction of the premise by using Formula II.5 and the sets of missing and given facts for the newly deduced fact in the conclusion are calculated by using Formula II.4. During the execution of the inference engine, the sets of missing and given facts for all deduced facts that are same but computed by using different rules are combined by using Formula II.6.

Combined reasoning with confidence levels and missing facts: We combine both our approaches, and compute the confidence vector given in Definition II.3 for each newly deduced fact by giving the count of missing facts in the inference chain of the deduced fact. This allows us to see the effect of missing facts on the confidence of the conclusion. We use all formulae used in the previous approaches and additionally use the reasoning exemplified in Example II.8 for combining the confidence vector of the conclusion with two supporting arguments.

The following are the steps that occur when our system is used: 1) The user inputs the data in the form of facts with an associated confidence. 2) The confidence vectors of all the derived conclusions are calculated. 3) If there are multiple conclusions (inferring the same fact), the combined confidence vector is calculated. 4) The final confidence vector is transformed into a textual representation. 5) The conclusion and its confidence vector (textual representation) are displayed.

An example rule from the domain of urban infrastructure [6] is as follows: **If** "Soil wetting is large" **and** "Soil Moisture Content is medium" **then** "Soil Moisture Content goes to wet". In the above rule, "Soil wetting is large" and "Soil Moisture Content is medium" are the conditions in the conjunction of the premise and "Soil Moisture Content goes to wet" is the inferred fact in the conclusion. For making inferences in the presence of missing facts, we impose a condition that the rule can fire when at least one of the facts (in the conjunction) of the premise is given and the rest can be optional. Similarly, we can also impose a condition that the rule can fire when only one fact (in the conjunction) of the premise is optional and the rest of the facts must be given, having an additional condition that if there is a single fact in the premise then it needs be given (i.e., not optional).

IV. Conclusion and Future Work

We have presented an uncertainty management scheme for rule-based systems that enables reasoning with qualitative confidence levels for data and rules to emulate the reasoning of domain experts for decision making in the urban infrastructure. Moreover, our proposed scheme allows us to make inferences even in the presence of missing information and its effect on the certainty factors. The novelty of our scheme with respect to previous research is threefold: (1) unlike Mycin-like or fuzzy rule based methods which represent uncertainty using a numeric scheme, our method is entirely symbolic (though counts of different levels of uncertainty are still maintained); (2) whilst previous systems have allowed optional or missing antecedents in rules, these have not been retained in the subsequent inference system for subsequent analysis/reporting (e.g. identifying inference chains for conclusions that require minimum number of missing antecedents or to facilitate abductive reasoning); (3) these two aspects are integrated into a single representation and method to compare uncertain inferred facts. We have implemented our methods in an off the shelf forward chaining inference engine.

In the future, we aim to use a backward chaining inference engine for a goal-based approach; we expect the representations and mechanisms we have presented here to be applicable to this. We will also include data from various sources and extend our uncertainty management scheme to deal with the issues such as data inconsistency and reliability that arise when integrating data from different sources. Although the main focus of our investigation was for decision making in the urban infrastructure, our proposed scheme is likely to be applicable to many other domains.

Another extension we envisage is to change the treatment of what happens when a fact is inferred in two different ways with two different sets of missing facts. At present, we just take the derivation with the smallest set of missing facts, and allow the possibility that one could backtrack and explore the other derivation if needed later (e.g. if a missing fact in the smaller set turns out to be false). Alternatively, both derivations could be propagated forward simultaneously, disjunctively, which would eliminate the need to backtrack but at the expense of a possible blowup in the number of disjunctive derivations (note that the number of derived facts would not explode, only the length of the associated uncertainty descriptions). Another avenue to explore is to allow costs to be associated with the difficulty of verifying the truth of an unknown fact; this would be particularly useful when combined with the extension mentioned immediately above of propagating all derivations forward.

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