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On the effect of transport coefficient anisotropy on the plasma flow in heliospheric interface

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ABSTRACT

The plasma flow in the heliospheric interface is considered. The applicability of hydrodynamic description for this flow is studied. The effect of the magnetic field on the transport properties in the interface plasma is discussed and the dimensionless parameters related to the plasma flow are estimated. It is found that both resistivity and Hall effect can be neglected in Ohm's law, so that the classical induction equation of the ideal magnetohydrodynamic can be used. The Reynolds number is moderately large, so the approximation of inviscid plasma is fairly good. The most important dissipative process is thermal conduction along the magnetic field lines. This effect has to be definitely taken into account. The results obtained in the paper are used to outline the ways for advancing the existing models of the heliospheric interface.

Key words: hydrodynamics – plasmas – solar wind – ISM: general.

1 INTRODUCTION: MODEL OF SOLAR WIND–INTERSTELLAR MEDIUM INTERACTION WITH TWO SHOCKS

The Solar system is surrounded by a mixture of charged and neutral particles called the local interstellar cloud (LIC). The Sun is moving with a supersonic velocity with respect to LIC, so that there is a supersonic flow of the interstellar medium in the solar reference frame. Colliding this flow with the supersonic solar wind results in the interaction region called the heliospheric interface. It consists of the termination shock at which the solar wind is decelerated, the bow shock at which the interstellar medium flow is decelerated and the heliopause separating the two decelerated flows. A qualitative picture of interaction of the solar wind with the interstellar medium is shown in Fig. 1.

The model of the heliospheric interface with two shocks was first developed by Baranov, Krasnobaev & Kulikovski (1971). In this pioneering paper the heliospheric interface was considered as an infinitely thin layer. Baranov, Krasnobaev & Ruderman (1976) calculated the structure of the heliospheric interface using the method of asymptotic expansions, while Baranov, Lebedev & Ruderman (1979) studied the heliospheric interface structure using the direct numerical modelling.

In the first models of the heliospheric interface only the interaction of the solar wind with the plasma component of LIC was considered. The neutral component (which mainly consists of the H atoms) was ignored. After the importance of the charge exchange was realized, Baranov, Ermakov & Lebedev (1981) carried out the numerical study of the structure of the heliospheric interface taking the neutral particles into account. In their model Baranov et al. used the two-fluid description of the interstellar medium, one fluid consisting of charged and the other of neutral particles. This model has been then extended to the multifluid description (see e.g. Fahr, Kausch & Scherer 2000).

The multifluid models substantially advanced the study of the heliospheric interface structure. However they still do not provide its adequate description. The reason is that the fluid description of particle motion is applicable only when the mean free path is much smaller than the characteristic spatial scale of a problem. This condition is definitely violated for the neutrals. Baranov & Malama (1993) developed a model with the mixed description, hydrodynamic for the solar wind and the plasma component of LIC, and kinetic for the interstellar neutral atoms.

The self-consistent model developed by Baranov & Malama (1993) gave the estimate of the correct distance from the Sun to the termination shock. It also predicted many physical phenomena like the existence of the hydrogen wall near the heliospheric boundary and the presence of a few sorts of hydrogen atoms and, as a consequence, the existence of charge exchange in various regions of the heliosphere. These predictions were later confirmed by the observations onboard *Voyager 1* and *2*, *Hubble Space Telescope* (*HST*), *Ulysses* and *Interstellar Boundary Explorer* (*IBEX*). However further development of the kinetic-gasdynamic model became necessary for the interpretation of new observational data.

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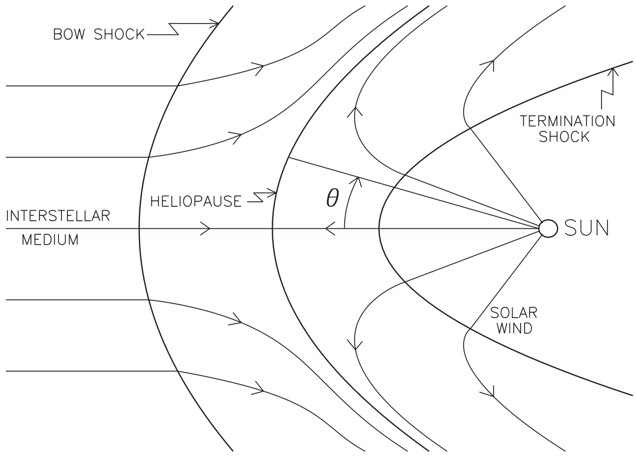

Figure 1. A qualitative picture of interaction of the solar wind with the interstellar medium.

One particular improvement of this model is related to the account of the magnetic field. The recent progress in the development of the kinetic-gasdynamic model is described in the review papers by Baranov (2009) and Izmodenov et al. (2009).

Up to now the ideal magnetohydrodynamic (MHD) equations have been used to describe the solar wind flow and the flow of changed component of LIC. Hence, all dissipative processes in the solar wind and LIC plasmas are neglected. In this paper we discuss the correctness of this approach and estimate various dimensionless parameters characterizing the plasma flow in the heliospheric interface. We also outline possible directions of improvement of existing models. The paper is organized as follows. In the next section we discuss the applicability of the hydrodynamic description to the plasma flow in the heliospheric interface. In Section 3 we present the equations of anisotropic MHD, discuss the magnetic field effect on the transport in the interface plasma and estimate various terms in the MHD equations. Section 4 contains the summary and our conclusions.

2 CRITERIA OF APPLICABILITY OF HYDRODYNAMIC DESCRIPTION

It is well known that the hydrodynamic equations can be obtained from the Boltzmann equation for the distribution function using the Chapman–Enskog method (Chapman & Cowling 1953). This method is based on using the asymptotic expansions with the Knudsen number Kn as a small parameter. Hence, the hydrodynamic description is only valid when

$$\mathrm{Kn} = \ell/L \ll 1, \tag{1}$$

where ℓ is the mean free path of particles and L is the characteristic spatial scale of the problem. In the first approximation of the Chapman–Enskog method the distribution function is locally Maxwellian, and the macroscopic equations are ideal, i.e. they do not describe any dissipative processes. The transport coefficients related to viscosity, thermal condition and finite resistivity are calculated in the next order approximation of the Chapman–Enskog method.

The inequality (1) is not satisfied for the solar wind plasma compressed at the termination shock. Hence, strictly speaking, we cannot use the hydrodynamic description for the plasma flow in the inner heliospheric interface, which is the region between the termination shock and heliopause. Nevertheless in all studies of the solar wind–interstellar medium interaction, including the gasdynamic-kinetic modelling, the hydrodynamic description is used in this region. The standard justification of this approach is as follows. The classical mean free path of charged particles in plasmas is calculated on the basis of their Coulomb collisions. Observations onboard space missions show that space plasmas are subject to various microinstabilities. As a result, there is a random wave ensemble in the plasma. Charged particle are scattered by waves. This scattering introduces the effective mean free path of charged particles. It is assumed that this effective mean free path, ℓ_{eff}, is much smaller than both the classical mean free path and the characteristic scale of a problem, so that

$$\mathrm{Kn}_{\mathrm{eff}} = \ell_{\mathrm{eff}}/L \ll 1. \tag{2}$$

It is worth noting that, at present, there is no rigorous theory of 'hydrodynamization' of collisionless plasmas due to charged particle scattering by waves. An attempt to describe this process has been made in the framework of quasi-linear theory (see e.g. Vedenov 1963). However, this theory failed to describe the full evolution of the distribution function. Hence, the possibility to use the hydrodynamic description of collisionless plasmas due to charged particle scattering by waves should be considered only as a conjecture.

The situation is quite different in the outer heliospheric interface, which is the region between the bow shock and heliopause. The mean free path of ions calculated using the Coulomb collisions is given by (e.g. Priest 1982)

$$\ell \approx 10^9 \frac{T^2}{n_{\mathrm{e}} \ln \Lambda}\ \mathrm{m}, \tag{3}$$

where T is the plasma temperature (measured in Kelvin), n_{e} the electron number density (measured in m^{-3}) and $\ln \Lambda$ the Coulomb logarithm with the typical values between 10 and 20. Since the typical values in the outer heliospheric interface are $T = 10^4$ K and $n_{\mathrm{e}} = 10^5\ \mathrm{m}^{-3}$, we obtain that the typical value for the ion mean free path is $\ell = 10^{11}\ \mathrm{m} \sim 1$ au. Since $L \sim 100$ au, we obtain $\mathrm{Kn} \sim 0.01$, so the hydrodynamic description of the plasma component in the outer heliospheric interface is perfectly correct.

3 THE EFFECT OF MAGNETIC FIELD AND ANISOTROPIC TRANSPORT IN THE INTERFACE PLASMAS

The effect of the interstellar magnetic field on the heliospheric interface has been studied by many authors (e.g. Aleksashov et al. 2000; Izmodenov, Alexashov & Myasnkov 2005; Izmodenov & Alexashov 2006; Izmodenov et al. 2009). Recently Izmodenov & Alexashov (2013) studied the effect of the solar wind magnetic field on the heliospheric interface. All these studies concentrated on the effect of the magnetic field on such properties of the interface as its shape and size, and they used the ideal MHD equations for the plasma flow description.

In contrast, our aim is to study the effect of the magnetic field on the transport processes in the plasma. The MHD equations for collisional plasmas in the one-fluid approximation can be written as (e.g. Cowling 1960; Kulikovsky & Lyubimov 1965; Priest 1982; Landau, Lifshitz & Pitaevskii 1984; Goedbloed & Poedts 2004)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0, \tag{4}$$

$$\rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} \right) = -\nabla p + \boldsymbol{j} \times \boldsymbol{B} + \nabla \cdot \boldsymbol{\Pi} + \boldsymbol{Q}_1, \tag{5}$$

$$\frac{\partial p}{\partial t} + \boldsymbol{v}\cdot\nabla p + \gamma p \nabla\cdot\boldsymbol{v} = -(\gamma-1)(\mathcal{L} - Q_2), \tag{6}$$

$$p = \frac{k_B}{m}\rho T, \tag{7}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla\times\boldsymbol{E}, \tag{8}$$

$$\nabla\times\boldsymbol{B} = \mu_0 \boldsymbol{j}, \tag{9}$$

$$\boldsymbol{E} = \frac{\boldsymbol{j}}{\sigma} - \boldsymbol{v}\times\boldsymbol{B} - \frac{1}{en_e}(\nabla p_e - \boldsymbol{j}\times\boldsymbol{B}). \tag{10}$$

Here ρ is the plasma density, p the pressure, T the temperature (assumed the same for electrons and protons), $\boldsymbol{v}$ the velocity, $\boldsymbol{B}$ the magnetic field, $\boldsymbol{E}$ the electrical field, $\boldsymbol{j}$ the electrical current, $\boldsymbol{\Pi}$ the viscosity tensor and $\mathcal{L}$ the energy loss function; p_e is the electron pressure, n_e the electron number density, $\gamma(=5/3)$ the ratio of specific heats, k_B the Boltzmann constant, m the mean mass per particle, μ_0 the magnetic permeability of free space, e the elementary charge and σ the plasma electrical conductivity. When writing down equations (4)–(10) we have assumed that the plasma consists of electrons and protons with the same temperatures. Hence,

$$\rho \approx m_p n_e, \quad m = \frac{1}{2}m_p \quad p_e = \frac{1}{2}p, \tag{11}$$

where m_p is the proton mass.

The quantities $\boldsymbol{Q}_1$ and Q_2 on the right-hand sides of expressions (5) and (6) represent the sources of momentum and energy due to charge exchange of H atoms and protons. The expressions for these quantities are given by (Baranov & Malama 1993)

$$\boldsymbol{Q}_1 = m_H \iint u\sigma_{ex}^{Hp}(u)(\boldsymbol{w}_H - \boldsymbol{w}_p) \times f_H(\boldsymbol{w}_H) f_p(\boldsymbol{w}_p)\, d\boldsymbol{w}_H\, d\boldsymbol{w}_p, \tag{12}$$

$$Q_2 = \frac{m_H}{2}\iint u\sigma_{ex}^{Hp}(u)\left(w_H^2 - w_p^2\right) \times f_H(\boldsymbol{w}_H) f_p(\boldsymbol{w}_p)\, d\boldsymbol{w}_H\, d\boldsymbol{w}_p - \boldsymbol{v}\cdot\boldsymbol{Q}_1. \tag{13}$$

Here m_H is the atom mass, n_H the atom number density, $u = |\boldsymbol{w}_p - \boldsymbol{w}_H|$ the relative atom–proton velocity and $\sigma_{ex}^{Hp}(u)$ the charge exchange cross-section of an H atom with a proton. The proton velocity distribution function f_p is locally Maxwellian. It is given by

$$f_p(\boldsymbol{w}_p) = \frac{n_p}{\pi^{3/2}c_p^3}\exp\left(-\frac{(\boldsymbol{w}_p - \boldsymbol{v})^2}{c_p^2}\right), \quad c_p^2 = \frac{2k_B T}{m_p}, \tag{14}$$

where n_p is the proton number density and c_p the proton thermal speed. The atom distribution function f_H is determined by the kinetic equation which we do not write down because it is not used in what follows.

The expression for the viscosity tensor can be written as a sum of five terms (Braginskii 1965),

$$\boldsymbol{\Pi} = \eta_0\boldsymbol{\Pi}_0 + \eta_1\boldsymbol{\Pi}_1 + \eta_2\boldsymbol{\Pi}_2 - \eta_3\boldsymbol{\Pi}_3 - \eta_4\boldsymbol{\Pi}_4. \tag{15}$$

The ratios of the second and third terms on the right-hand side of this equation to the first one are of the order of $(\omega_p\tau_p)^{-2}$, while the ratios of the third and fourth terms to the first one are of the order of $(\omega_p\tau_p)^{-1}$, where ω_p is the proton gyrofrequency given by

$$\omega_p = \frac{eB}{m_p}, \tag{16}$$

and τ_p is the proton collisional time given by (e.g. Goedbloed & Poedts 2004)

$$\tau_p = 6\pi\sqrt{2\pi}\,\epsilon_0^2\frac{m_p^{1/2}(k_B T)^{3/2}}{e^4 n_e \ln\Lambda} \approx 6\times10^5 n_e^{-1}T^{3/2}, \tag{17}$$

where ϵ_0 is the permittivity of free space, n_e and T are measured in m^{-3} and K and we have taken $\ln\Lambda = 20$. In Tables 1 and 2 the typical plasma and magnetic field parameters in the inner and outer heliospheric interface are given. We see that $(\omega_p\tau_p)^{-1} \lesssim 10^{-5}$ in

Table 1. Plasma and magnetic field parameters in inner interface.

Plasma speed V	10^5 m s^{-1}
Electron number density n_e	2×10^3 m^{-3}
Proton temperature T	10^5 K
Magnetic field B	1.5×10^{-10} T = 1.5×10^{-6} G
Electron cyclotron frequency $\omega_e = eB/m_e$	30 s^{-1}
Electron collisional time $\tau_e \approx 1.4\times10^4 n_e^{-1}T^{3/2}$	2×10^8 s
Electron mean free path $\ell_e \approx 4\times10^3 T^{1/2}\tau_e$	3×10^{14} m
Hall parameter $\omega_e\tau_e$	7.5×10^9
Proton cyclotron frequency $\omega_p = eB/m_p$	0.02 s^{-1}
Proton collisional time $\tau_p \approx 6\times10^5 n_e^{-1}T^{3/2}$	10^{10} s
$\omega_p\tau_p$	2×10^8

Table 2. Plasma and magnetic field parameters in outer interface.

Plasma speed V	2.5×10^4 m s^{-1}
Electron number density n_e	10^5 m^{-3}
Proton temperature T	10^4 K
Magnetic field B	2×10^{-10} T = 2×10^{-6} G
Electron cyclotron frequency $\omega_e = eB/m_e$	40 s^{-1}
Electron collisional time $\tau_e \approx 1.4\times10^4 n_e^{-1}T^{3/2}$	1.4×10^5 s
Electron mean free path $\ell_e \approx 4\times10^3 T^{1/2}\tau_e$	8×10^{10} m
Hall parameter $\omega_e\tau_e$	6×10^6
Proton cyclotron frequency $\omega_p = eB/m_p$	0.03 s^{-1}
Proton collisional time $\tau_p \approx 6\times10^5 n_e^{-1}T^{3/2}$	6×10^6 s
$\omega_p\tau_p$	2×10^5

the outer interface, so the first term on the right-hand side of equation (15) strongly dominates all other terms, and we can take

$$\mathbf{\Pi} \approx \eta_0 \mathbf{\Pi}_0 = \eta_0 \left(\boldsymbol{bb} - \frac{1}{3}\boldsymbol{I} \right) [3\boldsymbol{b} \cdot \nabla(\boldsymbol{b} \cdot \boldsymbol{v}) - \nabla \cdot \boldsymbol{v}], \tag{18}$$

where $\boldsymbol{b} = \boldsymbol{B}/B$ is the unit vector in the magnetic field direction, $\boldsymbol{bb}$ is the dyadic product of two vectors, $\boldsymbol{I}$ is the unit tensor and we have used the expression for $\mathbf{\Pi}_0$ obtained by Braginskii (1965) (see also Goedbloed & Poedts 2004). As for the inner interface, as we have already mentioned, its MHD description is based on the conjecture that the scattering of charged particles by waves dramatically reduces the collision time and mean free path. The MHD description is possible if $\ell_{\rm eff} \lesssim 10\,$au. Even if we take $\ell_{\rm eff} = 1\,$au, we obtain $\tau_{\rm p,eff} \approx 3 \times 10^6\,$s and $(\omega_{\rm p}\tau_{\rm p,eff})^{-1} \lesssim 10^{-5}$, so the approximate expression (18) can be also used in the inner interface.

The ratio of the last term on the right-hand side of equation (5) to the left-hand side of this equation is of the order of the inverse Reynolds number $\mathrm{Re} = \rho LV/\eta_0$, where V is the characteristic plasma speed. The dynamic viscosity η_0 is given by (Braginskii 1965)

$$\eta_0 \approx n_{\rm e} k_{\rm B} T \tau_{\rm p} \approx 1.4 \times 10^{-23} n_{\rm e} T \tau_{\rm p}. \tag{19}$$

Then, once again taking $L \sim 100\,$au, we obtain $\mathrm{Re} \approx 750$ in the outer interface. Taking $\ell_{\rm eff}$ between 1 and 10 au in the inner interface gives Re between 35 and 350. Hence, although the Reynolds number is not very large, it seems to be reasonable to neglect viscosity when studying the plasma flow in the heliospheric interface.

It is also instructive to compare the term describing the viscosity on the right-hand side of equation (5) with the term describing the effect of charge exchange with atoms. Although, in general, the atom distribution function can substantially deviate from locally Maxwellian, to obtain the estimate for $\boldsymbol{Q}_1$ we take it to be locally Maxwellian. Hence, we assume that it is given by equation (14) with $n_{\rm H}$ substituted for $n_{\rm p}$ and $c_{\rm H}$ substituted for $c_{\rm p}$. We also substitute $\sigma_{\rm ex}^{\rm Hp}(u)$ by its typical value. Then, making a proper variable substitution in the integral in equation (12), we obtain

$$\begin{aligned} |\boldsymbol{Q}_1| &\simeq \frac{m_{\rm H} n_{\rm H} n_{\rm p}}{\pi^3} \sigma_{\rm ex}^{\rm Hp} \int \mathrm{e}^{-y^2} \mathrm{d}\boldsymbol{y} \int |c_{\rm H}\boldsymbol{x} - c_{\rm p}\boldsymbol{y}|^2 \mathrm{e}^{-x^2} \mathrm{d}\boldsymbol{x} \\ &= 2 m_{\rm H} n_{\rm H} n_{\rm p} \sigma_{\rm ex}^{\rm Hp} \left(c_{\rm p}^2 + c_{\rm H}^2 \right). \end{aligned} \tag{20}$$

For $n_{\rm H}$ and $c_{\rm H}$ we take their values in the interstellar medium before the bow shock. Then $n_{\rm H} \approx 2 \times 10^5\,\mathrm{m}^{-3}$ and, taking the temperature of H atoms equal to 6500 K, we obtain $c_{\rm H} \approx 10\,\mathrm{km\,s}^{-1}$. The typical value for the charge exchange cross-section is $\sigma_{\rm ex}^{\rm Hp} = 5 \times 10^{-19}\,\mathrm{m}^2$. Using the values given in Tables 1 and 2 we get $c_{\rm p} \approx 40\,\mathrm{km\,s}^{-1}$ in the inner interface and $c_{\rm p} \approx 13\,\mathrm{km\,s}^{-1}$ in the outer interface. Then we obtain $|\boldsymbol{Q}_1| \simeq 6 \times 10^{-28}\,\mathrm{kg\,m^{-2}\,s^{-2}}$ in the inner interface and $|\boldsymbol{Q}_1| \simeq 4.5 \times 10^{-27}\,\mathrm{kg\,m^{-2}\,s^{-2}}$ in the outer interface. The ratio of the term describing viscosity to the term related to the effect of charge exchange with atoms in equation (5) is

$$\frac{|\nabla \cdot \mathbf{\Pi}|}{|\boldsymbol{Q}_1|} \simeq \frac{\rho V^2}{L\,\mathrm{Re}|\boldsymbol{Q}_1|}. \tag{21}$$

Substituting the numbers in this formula we obtain that this ratio is approximately between 0.01 and 0.1 in the inner interface, and it is approximately equal to 0.002 in the outer interface. Hence, the momentum exchange between the plasma and atoms due to charge exchange strongly dominates viscosity. This result gives an additional reason to neglect viscosity.

The energy loss function is given by (e.g. Priest 1982)

$$\mathcal{L} = -\nabla \cdot \boldsymbol{q} + \rho^2 Q(T) - \frac{j^2}{\sigma} - \mathbf{\Pi} : \nabla \boldsymbol{v}. \tag{22}$$

Here $\boldsymbol{q}$ is the heat flux, $Q(T)$ the optically thin radiative loss function and the colon indicates the double contraction of two tensors. The third and fourth terms on the right-hand side of equation (22) describe the resistive and viscous heating, respectively. The heat flux is given by

$$\boldsymbol{q} = -\kappa_\parallel \boldsymbol{b}(\boldsymbol{b} \cdot \nabla T) - \kappa_\perp[\nabla T - \boldsymbol{b}(\boldsymbol{b} \cdot \nabla T)] - \kappa_\wedge \boldsymbol{b} \times \nabla T. \tag{23}$$

The electron gyrofrequency is given by

$$\omega_{\rm e} = \frac{eB}{m_{\rm e}}, \tag{24}$$

and the electron collisional time by (e.g. Goedbloed & Poedts 2004)

$$\tau_{\rm e} = 6\pi\sqrt{2\pi}\,\epsilon_0^2 \frac{m_{\rm e}^{1/2}(k_{\rm B}T)^{3/2}}{e^4 n_{\rm e} \ln\Lambda} \approx 1.4 \times 10^4 n_{\rm e}^{-1} T^{3/2}, \tag{25}$$

where $m_{\rm e}$ is the electron mass. For $\omega_{\rm e}\tau_{\rm e} \gg 1$ we have the estimates $\kappa_\perp/\kappa_\parallel \sim (\omega_{\rm e}\tau_{\rm e})^{-2}$ and $\kappa_\wedge/\kappa_\parallel \sim (\omega_{\rm e}\tau_{\rm e})^{-1}$. Since, in accordance with Table 2, $(\omega_{\rm e}\tau_{\rm e})^{-1} < 10^{-6}$, it follows that we can neglect the second and third term on the right-hand side of equation (23) in the outer interface. A similar result is valid in the inner interface.

For the typical plasma temperature in the heliospheric interface the main radiative losses are related to the emission in resonance lines of ionized heavy atoms (bound–bound emission), and the radiative recombinations (free–bound radiation). There are practically no heavy atoms in the interface plasma, and the process of the recombination of electrons with protons is very weak. Hence, the term $\rho^2 Q(T)$ in equation (22) can be safely neglected.

The coefficient of the parallel thermal conduction is given by (Spitzer 1962; Priest 1982)

$$\kappa_\parallel \approx 10^{-11} T^{5/2}\,\mathrm{W\,m^{-1}\,K^{-1}}. \tag{26}$$

However, we can use this expression only in the outer interface. The point is that $\kappa_\parallel$ is proportional to $\tau_{\rm e}$. Since we use $\tau_{\rm e,eff}$ instead of $\tau_{\rm e}$ in the inner interface, we have to use $\kappa_{\parallel\rm eff} = (\tau_{\rm e,eff}/\tau_{\rm e})\kappa_\parallel$ instead of $\kappa_\parallel$. The plasma electrical conductivity is given by (e.g. Priest 1982)

$$\sigma = \frac{e^2 n_{\rm e} \tau_{\rm e}}{m_{\rm e}} \approx 3 \times 10^{-8} n_{\rm e} \tau_{\rm e}. \tag{27}$$

Using equations (9), (23) and (26) we obtain that, in the outer interface, the ratio of the third term on the right-hand side of equation (22) to the first one is of the order of

$$\frac{B^2}{\mu_0^2 \sigma \kappa_\parallel T} \approx 2 \times 10^{22} \frac{B^2}{n_{\rm e} \tau_{\rm e} T^{7/2}}. \tag{28}$$

To obtain this ratio in the inner interface we have to multiply this expression by $\tau_{\rm e}/\tau_{\rm e,eff}$ and substitute $\tau_{\rm e,eff}$ for $\tau_{\rm e}$. This ratio is approximately equal to 6×10^{-22} in the outer interface, and it is approximately between 10^{-19} and 10^{-17} in the inner interface for $\ell_{\rm eff}$ between 1 and 10 au. Hence, it is obvious that we can neglect the third term on the right-hand side of equation (22) in comparison with the first one.

Now we compare the last and first term on the right-hand side of equation (22). Using equations (15), (18), (19), (23) and (26) we obtain that, in the inner interface, this ratio is of the order of

$$\frac{\eta_0 V^2}{\kappa_\parallel T} \approx 1.4 \times 10^{-12} n_{\rm e} \tau_{\rm p} V^2 T^{-5/2}. \tag{29}$$

Once again, to obtain this ratio in the inner interface we have to multiply this expression by $\tau_e/\tau_{e,\mathrm{eff}}$ and substitute $\tau_{p,\mathrm{eff}}$ for τ_p. Then we obtain that this quantity is approximately equal to 0.05 in the outer interface, and to 0.075 in the inner interface for ℓ_{eff} between 1 and 10 au. While the ratio of these two terms is not very small, the first term still dominates. Hence, we conclude that we can use the approximate expression

$$\mathcal{L} = \nabla \cdot [\kappa_\parallel \boldsymbol{b}(\boldsymbol{b} \cdot \nabla T)] \tag{30}$$

in the whole heliospheric interface.

The ratio of the right-hand side of equation (6) to its left-hand side is of the order of the inverse Peclet number given by

$$\mathrm{Pe} = \frac{k_\mathrm{B} n_\mathrm{e} L V}{\kappa_\parallel} \approx 1.4 \times 10^{-12} n_\mathrm{e} L V T^{-5/2} \tag{31}$$

in the outer interface. To obtain the expression for the Peclet number in the inner interface we have to multiply this expression by $\tau_e/\tau_{e,\mathrm{eff}}$. We obtain Pe ≈ 5 in the outer interface, and Pe is approximately between 0.4 and 4 in the inner interface for ℓ_{eff} between 1 and 10 au. These results imply that the account of plasma thermal conduction in the heliospheric interface is very important.

Now we compare the term describing the heat conduction in equation (6) with that related to the energy exchange between the plasma and atoms due to charge exchange. To estimate Q_2 we, for the sake of simplicity, assume that the two terms on the right-hand side of equation (13) are of the same order. As we do not know the angle between $\boldsymbol{v}$ and $\boldsymbol{Q}_1$, we now use as an estimate $|Q_2| \simeq |\boldsymbol{v} \cdot \boldsymbol{Q}_1|/2$. Then we obtain $|\boldsymbol{Q}_2| \simeq 3 \times 10^{-23}$ kg m^{-1} s^{-1} in the inner interface and $|\boldsymbol{Q}_2| \simeq 5 \times 10^{-23}$ kg m^{-1} s^{-1} in the outer interface. The ratio of the energy loss function to the term related to the energy flux due to the charge exchange with atoms in equation (6) is

$$\frac{|\mathcal{L}|}{|Q_2|} \simeq \frac{k_\mathrm{B} n_\mathrm{e} T V}{L\, \mathrm{Pe} |Q_1|}. \tag{32}$$

Substituting the numbers in this formula we obtain that this ratio is approximately between 0.2 and 2 in the inner interface, and it is approximately equal to 0.1 in the outer interface. These estimates show that the energy flux due to thermal conduction is not small in comparison with that due to charge exchange.

Finally, we compare the terms on the right-hand side of equation (10). The ratio of the first term on the right-hand side of this equation to the second one is of the order of the inverse magnetic Reynolds number given by

$$R_\mathrm{m} = \mu_0 \sigma V L \approx 4 \times 10^{-14} n_\mathrm{e} \tau_\mathrm{e} V L. \tag{33}$$

We obtain $R_\mathrm{m} \approx 2 \times 10^{14}$ in the outer interface, while R_m is approximately between 1.3×10^{13} and 1.3×10^{14} in the inner interface. We see that we can neglect the first term on the right-hand side of equation (10) in comparison with the second one. The ratio of last term on the right-hand side of equation (10), which describes the Hall effect, to the second one is of the order of $\omega_\mathrm{e}\tau_\mathrm{e}/\mathrm{R_m}$. Hence, it is of the order of 3×10^{-8} in the outer interface, and of the order of 2×10^{-7} in the inner interface, which implies that the Hall term also can be neglected. Then, eliminating $\boldsymbol{E}$ from equations (8) and (10), we obtain the induction equation of the ideal MHD:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}). \tag{34}$$

Note, however, that this conclusion may be incorrect if we take the effect of charge exchange into account. We do not discuss this problem here because it has been already addressed by Baranov & Fahr (2003, 2006).

Table 3. Dimensionless parameters in inner interface.

Reynolds number Re $= \rho V L/\eta_0$	35/350
Peclet number Pe $= k_\mathrm{B} n_\mathrm{e} V L/\kappa_\parallel$	0.4/4
Magnetic Reynolds number $R_\mathrm{m} = \mu_0 \sigma V L\, T$	$1.3 \times (10^{13}/10^{14})$
$\omega_\mathrm{e}\tau_\mathrm{e}/R_\mathrm{m}$	2×10^{-7}

Table 4. Dimensionless parameters in outer interface.

Reynolds number Re $= \rho V L/\eta_0$	750
Peclet number Pe $= k_\mathrm{B} n_\mathrm{e} V L/\kappa_\parallel$	5
Magnetic Reynolds number $R_\mathrm{m} = \mu_0 \sigma V L\, T$	2×10^{14}
$\omega_\mathrm{e}\tau_\mathrm{e}/R_\mathrm{m}$	3×10^{-8}

The main dimensionless parameters related to the plasma flow in the heliospheric interface are presented in Tables 3 and 4. In Table 3 for each dimensionless parameter a range of its variation corresponding to ℓ_{eff} between 1 and 10 au is given. There is only one value for $\omega_\mathrm{e}\tau_\mathrm{e}/R_\mathrm{m}$ in the inner interface because this quantity is independent of ℓ_{eff}.

4 SUMMARY AND CONCLUSIONS

In this paper we considered the plasma flow in the heliospheric interface. We discussed the applicability of the hydrodynamic description for the plasma flow in the interface. We found that the Knudsen number Kn is small in the outer interface, which is the region between the bow shock and the heliopause. This warranties the applicability of the hydrodynamic description for the plasma flow in the outer interface. However, in the inner interface, which is the region between the termination shock and the heliopause, Kn $\gg$ 1, at least if we calculate the mean free path ℓ of charged particles using Coulomb collisions. Hence, to approve the application of the hydrodynamic description to the plasma flow in the inner interface we assumed that there is scattering on charged particles on plasma waves which reduces the effective mean free part, so that the effective mean free path ℓ_{eff} becomes of the order or smaller than 10 au. We noted that, at present, there is no rigorous derivation of the hydrodynamic equations for plasmas based on the charged particle scattering by waves. Hence, we conclude that there is also no firm mathematical ground for the application of hydrodynamic description of the plasma flow in the inner interface. Even if the charged particle scattering by waves can result in a kind of hydrodynamization of the plasma flow, the distribution functions of the ions and electrons in this flow can be quite different from the local Maxwellian. Recently Chalov & Fahr (2013) have shown that, in order to explain the properties of the solar wind flow behind the termination shock one needs to assume that the electron temperature is about 10 times higher than the ion temperature. Hence, even if the plasma flow in the inner interface can be described by the MHD equations, it should be a modified hydrodynamics with different temperatures of the ion and electron gases.

We presented the MHD equations with anisotropic transport coefficients and estimated various terms in these equations. The estimates for the inner interface are based on the assumption that ℓ_{eff} is between 1 and 10 au. We found that the first term in the Braginskii viscosity tensor strongly dominates all other terms. The Reynolds number is moderate, however, it is still sufficiently large to enable neglecting viscosity. The resistive and Hall term in Ohm's equation can be safely neglected, so that the classical induction equation of the ideal MHD can be used. The dominant term in the energy loss function is the one describing thermal conduction, and thermal conduction along the magnetic field is by far larger than that in the directions orthogonal to the magnetic field. The Peclet number determining the importance of thermal conduction is quite small, which means that the account of thermal conduction along the magnetic field lines in the energy equation is very important.

On the basis of the results obtained in the paper we can suggest how to advance the existing models of the heliospheric interface. It is obvious that using the approximation of ideal MHD is inappropriate, and the thermal conduction along the magnetic field lines should definitely be included in the energy equation. Since the magnetic field lines cross both the termination and bow shock, these shocks are not classical shocks considered in the ideal MHD, but isothermal shocks with the temperature not changing across the shock.

The model can be advanced further by using the kinetic description of plasma flow in the inner interface. The charged particle gyroradius is much smaller than the characteristic spatial scale of the problem. This makes possible mixed description of the plasma flow in the inner interface, hydrodynamic in the directions perpendicular to the magnetic field and kinetic along the magnetic field.

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