Nonlinear properties of the shear dynamo model

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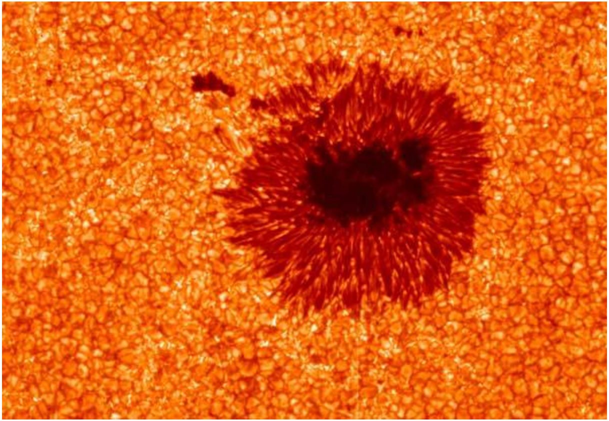
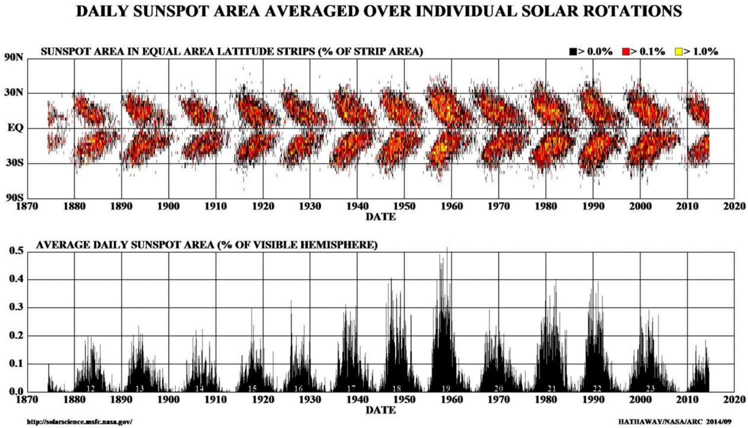
Solar surface features

- Solar surface features exist on a range of scales
- Convection is granulated: granules ($\sim 1,500$km), mesogranules ($5,000 - 10,000$km ?), supergranules ($\sim 30,000$km)



Solar dynamo

- Magnetic field also exists on a range of scales from the granular bright spots to global scales
- Sunspots, flares, prominences, etc.
- 11-year solar cycle evidenced by sunspot activity



Field generation

- Small-scale field: turbulent motions of plasmas amplify magnetic fluctuations via fluctuation dynamo effect

- Large-scale field: more complicated, traditionally modeled using mean-field theory
- A flow with global net helicity twists and stretches field lines
- Large-scale field generated by the 'α-effect'

Problems with mean-field theory

- Is mean-field theory valid in solar conditions?
- Mean-field theory should only apply when $Rm = UL/\eta$ is small, yet $Rm_{\odot} \gg 1$
- At large *Rm*s increased turbulence causes models to be dominated by small-scale fields
- Poorly correlated EMFs (due to turbulence) lead to a small α-effect in large domains relevant to the Sun

Alternatives to mean-field theory

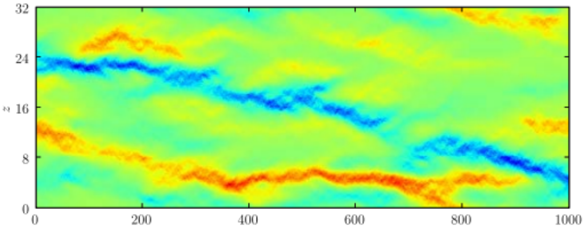
If the mean-field ansatz is not valid under solar conditions then a new mechanism for generating large-scale field is required

Several proposals have suggested a combination of turbulence and shear to produce large-scale field:

- enhancement of α via greater correlation of small-scale motions by the shear (Courvoisier et al., 2009)
- interaction with a fluctuating α-effect (Richardson & Proctor, 2012)
- *shear dynamo model* (Yousef et al., 2008)

Shear dynamo model

- Periodic box MHD simulations performed in a long domain to reduce computing requirements
- Forced non-helical motion (no α-effect) in the presence of a uniform shear
- Large-scale structures in magnetic field can be generated (Yousef et al., 2008)
- Structures wander in time and space



Equations

- Solve the incompressible MHD equations in the presence of a uniform shear flow, $\mathbf{U} = -Sx\hat{\mathbf{y}}$
- Shear-periodic box subject to a white-noise nonhelical homogeneous isotropic body force, $\mathbf{f}$

$$\frac{d\mathbf{u}}{dt} = u_x S\hat{\mathbf{y}} - \frac{\nabla p}{\rho} + \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi\rho} + \nu\nabla^2\mathbf{u} + \mathbf{f}, \quad (1)$$

$$\frac{d\mathbf{B}}{dt} = -B_x S\hat{\mathbf{y}} + \mathbf{B}\cdot\nabla\mathbf{u} + \eta\nabla^2\mathbf{B}, \quad (2)$$

where $d/dt = \partial_t - Sx\partial_y + \mathbf{u}\cdot\nabla$

Box dimensions: L_x, L_y, L_z where $L_z \gg L_x, L_y$

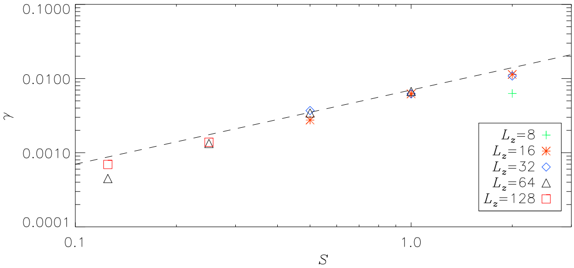
Input parameters

Use broadly the same parameter values as the previous work:

- $0.125 \leq S \leq 2$
- $L_x = 1 = L_y, \qquad 8 \leq L_z \leq 128$
- Energy injected in a shell centred at $k_f/2\pi = 3$ or, equivalently, $l_f = 1/3$
- Most cases have $\nu = 10^{-2} = \eta$ giving $Rm = Re = u_{\text{rms}}/k_f\nu \sim 5$

Kinematic regime

- Growth rate scales linearly with S

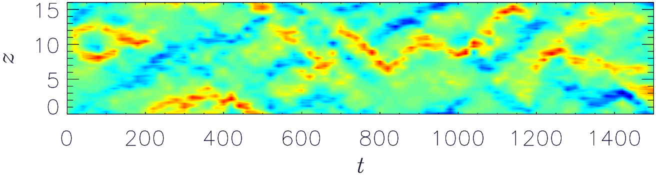


- Lengthscale, l_B, scales as $S^{-1/2}$
- Confirms results of Yousef et al., 2008

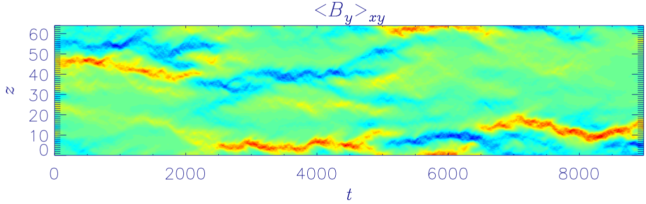
$$\frac{1}{l_B} = \left(\frac{\langle (\partial B_y^< / \partial z)^2 \rangle_z}{\langle (B_y^<)^2 \rangle_z} \right)^{1/2}$$

Wandering field

zt-plots of B_y averaged over x and y



$S = 2, L_z = 16$ (normalised by rms value)



$S = 0.5, L_z = 64$ (normalised by rms value)

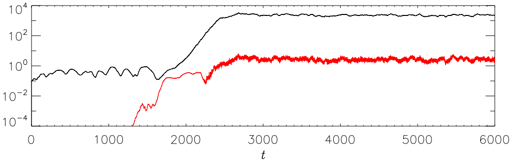
Large-scale field in y-direction wanders in space and time

Two saturated regimes

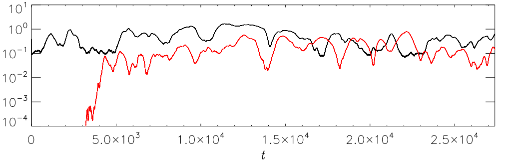
Saturated state appears to admit two rather different regimes (Teed & Proctor, 2016, 2017)

Clearly seen in different energy equilibration

$$E_M \ll E_K, \, (S = 2, \; L_z = 16, \; Pm = 1)$$



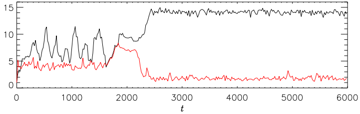
$$E_M \sim E_K, \, (S = 0.5, \; L_z = 16, \; Pm = 2)$$

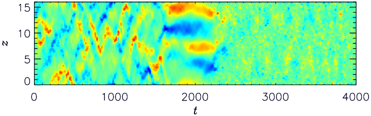


Lengthscales

Quenched state (Teed & Proctor, 2016)

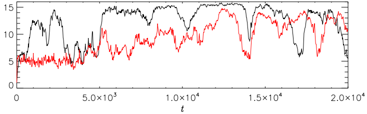
$l_B \ll l_u$, $(S = 2,\ L_z = 16,\ Pm = 1)$

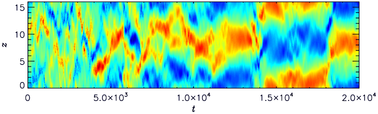




Quasi-periodic state (Teed & Proctor, 2017)

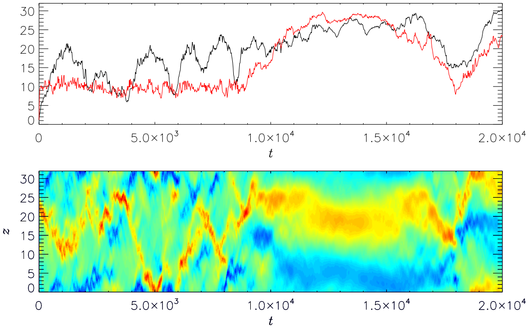
$l_B \sim l_u$, $(S = 2,\ L_z = 16,\ Pm = 1)$



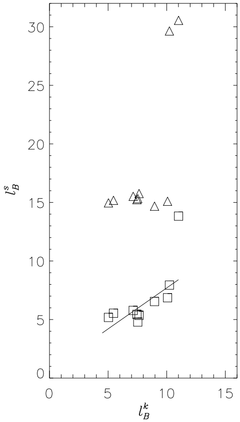


Quasi-periodic behaviour

- Two lengthscales: one on the size of the box and another on the intrinsic scale of the kinematic regime
- System moves between periods with $l_B^s \sim L_z$ and $l_B^s \sim l_B^k$



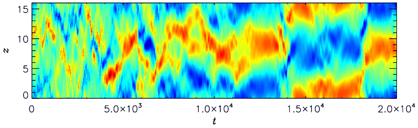
Linear dependence of l_B^s on l_B^k

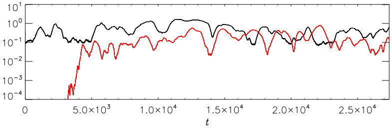


Triangles: values of l_B^s calculated during the periods when $l_B^s \sim L_z$ (box scale).
Squares: values of l_B^s calculated during the periods when $l_B^s \sim l_B^k$ (kinematic scale).

Relaxation oscillations I

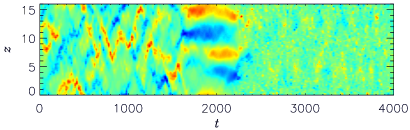
- Possible explanation for quasi-cyclic is relaxation oscillations between a ‘mean-vorticity dynamo’ (Elperin, Kleeorin, and Rogachevskii, 2003) and a shear dynamo (for the magnetic field).
- Large z-dependent shearing flow generated by a vorticity dynamo when field is weak
- Stronger magnetic field suppresses this mechanism $\rightarrow$ weaker vertical shear and operational shear dynamo (Käpylä & Brandenburg, 2009)
- Only occurs if the kinetic and magnetic energies are of a similar order (quasi-cyclic state below)

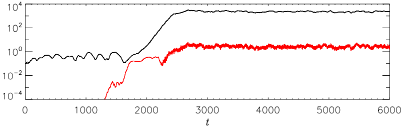




Relaxation oscillations II

- If vorticity dynamo greatly dominates, no large-scale field can be generated by a shear dynamo mechanism
- In this case the (weak) magnetic field is generated by a fluctuation dynamo mechanism
- Hence lengthscale is reduced to that of the imposed forcing (quenched state below)

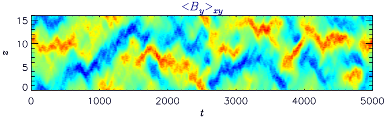




Tweaking the model

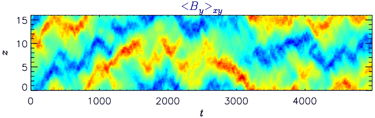
- Only basic linear shear (dependent on x) considered thus far
- Generates large-scale field with some cyclic properties but not solar-like
- Altering shear and/or forcing may promote more cyclic behaviour similar to the solar cycle
- Two main tweaks considered:
  - Changing shear profile; sinusoidal dependence, z-dependence
  - Adding a small amount of helicity into the forcing

Tweaking the model - preliminary results

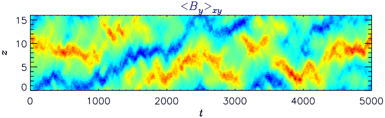


$$\mathbf{U} = (Sx + S_2 \cos(2\pi z/L_z))\hat{\mathbf{y}} \qquad \mathbf{f} = \mathbf{f_{nh}} + \mathbf{f_h}$$

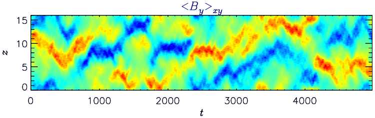
$S_2 = 0.1$



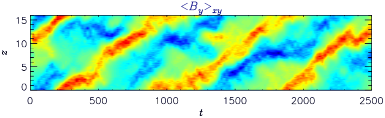
$|\mathbf{f_h}| = 0.01$



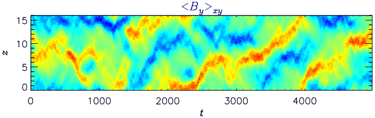
$S_2 = 1$



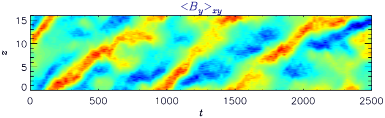
$|\mathbf{f_h}| = 0.05$



$S_2 = 4$



$|\mathbf{f_h}| = 0.1$



Conclusions

- Pure linear shear case shows that the shear dynamo could form the basis for a model of the solar dynamo
- Saturated state admits two regimes: i) quenched state with small-scale field (not solar-like); ii) quasi-periodic state (possibly solar-like)
- Quasi-periodic state displays times of differing field length scale, proportional to the imposed shear rate
- Tweaking the purely linear shear case (z-dependent shear/small amount of helicity) could promote cyclic behaviour in the kinematic phase
- Analysis of further parameter regimes and larger boxes required
- Effects of rotation, compressibility?

Meeting to celebrate Mike Proctor's retirement!

- Abstract submissions welcome on dynamo theory, MHD, convection, magnetoconvection and other relevant topics.
- Dates: September 11-12, 2017
- Venue: Centre for Mathematics Sciences, Cambridge with conference dinner at King's College, Cambridge
- Organisers: Rob Teed & Valeria Shumaylova