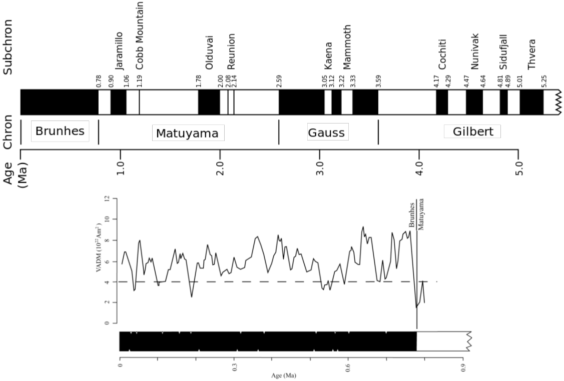
Geomagnetic field variations I

- The geomagnetic field is not steady: changes occur on a wide range of timescales (milliseconds to millions of years)
- For example: the rate of geomagnetic reversals; from $\sim$ 0.1 million years to over 10 million years (superchrons)



Constable & Korte, 2006

Geomagnetic field variations II

Variations also occur on timescales much shorter than those associated with reversals:

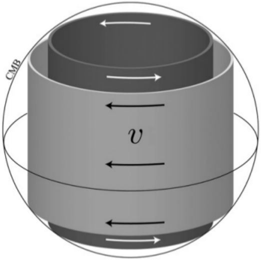
- <1 year mostly caused by currents in the ionosphere and magnetosphere
- >1 year (secular variation) mostly caused by changes in the Earth's core

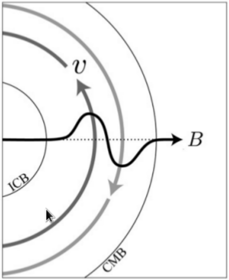
Waves in the core

- Changes in the geomagnetic field in the range 1-300 years can be observed in some detail.
- By identifying waves in this signal, we can potentially discover the strength and form of the magnetic field inside the liquid metal core.
- Waves on these timescales are affected by rotation, magnetic field and spherical geometry.
- Many types of wave are possible, but two are of particular interest as they have periods in the observable range: **Torsional Waves** and **Magnetic Rossby Waves**.

Torsional waves

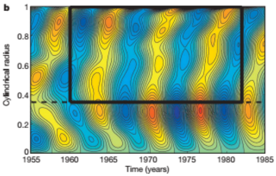
- Torsional waves (TWs) are azimuthal oscillations of rigid coaxial cylindrical surfaces
- Magnetostrophic balance between Lorentz, Coriolis and Archimedean forces
- Torsional waves occur about ‘Taylor state’ equilibrium
- Any resulting overshooting leads to the production of torsional waves.



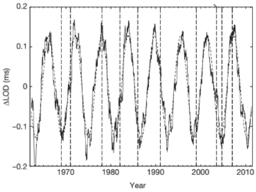


Observational evidence for torsional waves

- Torsional waves in the core have periods of about 6 years, the travel time of Alfvén waves.
- There is strong evidence these waves have been detected.
- A key question is how are they excited in the core.



Gillet et al., 2010



Holme & de Viron, 2013.

Magnetic Rossby waves

- These are non-axisymmetric, and have slower periods, typically a few hundred years.
- The secular variation has nonaxisymmetric and axisymmetric components and varies on this timescale.
- The difficulty in identifying these waves is that the core fluid speed is similar to the magnetic Rossby wave phase speed, so it is hard to separate wave motion from advection.
- It has been controversial whether secular variation is due to wave or core flow. It is likely a combination of both.
- The task is how can we use the improved data coming from satellites to separate the signal due to waves from the signal due to the flow? This requires an understanding of the dynamics of magnetic Rossby waves.

Physical Setup

- Spherical shell filled with electrically conducting fluid radially bounded above at $r = r_o$ and below at $r = r_i$
- Rotates about the vertical (z-axis) with rotation rate Ω and gravity acts radially inward so that $\mathbf{g} = g\mathbf{r}$.
- Fluid is assumed to have constant values of ρ, ν, κ and η
- Spherical polar coordinate system, (r, θ, ϕ), used for simulations.
- Cylindrical polar coordinate system, (s, ϕ, z), used for diagnostics.
  by an electrically insulating mantle and below at $r = r_i$ by an electrically insulating inner core.

Full dynamo equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{Pm}{E}\left(\nabla p + 2\hat{\mathbf{z}}\times\mathbf{u} - (\nabla\times\mathbf{B})\times\mathbf{B}\right) + \frac{Pm^2 Ra}{Pr} T\mathbf{r} + Pm\nabla^2\mathbf{u}, \tag{1}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T = \frac{Pm}{Pr}\nabla^2 T + \epsilon, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla\times(\mathbf{u}\times\mathbf{B}) = \nabla^2\mathbf{B}, \tag{3}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{4}$$

$$\nabla\cdot\mathbf{B} = 0. \tag{5}$$

Input parameters:

$$E = \frac{\nu}{\Omega d^2}, \qquad E_{core} \sim 10^{-15}, \qquad E_{sim} \in [3\times 10^{-7}, 10^{-3}]$$

$$Ra = \frac{g\alpha|\epsilon|d^5}{\nu\kappa\eta}, \qquad Ra_{core} \sim 10^{30} \qquad Ra_{sim} \in [10^5 - 10^9]$$

$$Pr = \frac{\nu}{\kappa}, \qquad Pr_{core} \sim 0.1, \qquad Pr_{sim} = 1$$

$$Pm = \frac{\nu}{\eta}, \qquad Pm_{core} \sim 10^{-6}, \qquad Pm_{sim} \in [0.05, 5]$$

Output parameters: $Rm = \frac{UD}{\eta}$, $\Lambda = \frac{|B|^2}{\rho\mu\eta\Omega}$, $Ro = \frac{U}{\Omega D}$, etc.

Simulations

- Use the Leeds spherical dynamo code to perform dynamo and magnetoconvection simulations
- Various boundary conditions tested: stress-free, no-slip, fixed temperature, fixed flux
- Magnetic BCs are insulating but for magnetoconvection a dipolar magnetic field of strength B_0 is imposed at the CMB
- Integrate past transient state and analyse data over a short period of time, τ
- Identify waves as structures in u_ϕ travelling in s-direction (for TWs) or u_s travelling in ϕ-direction (for magnetic Rossby waves) moving with the correct speed

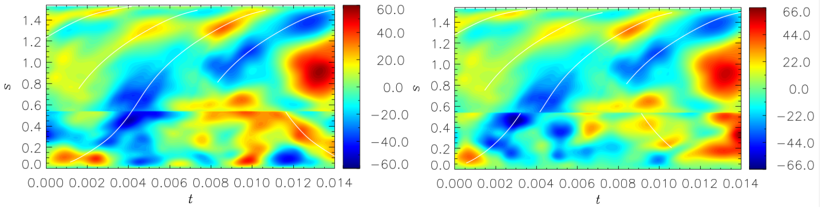
Detecting Torsional Waves

- Alfvén wave equation in spherical geometry is (Braginsky 1970)

$$\frac{\partial F_L'}{\partial t} = \frac{Pm}{E}\frac{1}{h}\frac{1}{s^2}\frac{\partial}{\partial s}\left(hs^3 \langle \overline{\tilde{B}_s^2} \rangle \frac{\partial}{\partial s}\left(\frac{\langle \overline{u_\phi'} \rangle}{s} \right)\right).$$

  where $U_A^2 = \langle \overline{hB_s^2/\mu\rho} \rangle$, the bar denoting ϕ average and angle brackets z-average. $h(s)$ is height of cylinder.
- Run the simulation, look at time segment, τ, and evaluate U_A from $\widetilde{B_s}$, the field averaged over τ.
- We make plots in the $t - s$ plane of the fluctuating part of u_ϕ, i.e. $u_\phi' = u_\phi - \widetilde{u_\phi}$ where $\widetilde{u_\phi}$ is the time-average over the whole τ segment.
- We then look for features in u_ϕ' propagating with the Alfvén speed.

Torsional waves in dynamo runs



- $\langle \overline{u'_\phi} \rangle$ inside Tangent Cylinder North (left) and inside Tangent Cylinder South (right) for $E = 10^{-4}$, $Pm = 5$. White curves have gradient U_A. Similar pictures for $E = 10^{-5}$.
- Plus points: TWs found in dynamo simulations. Mostly (but not exclusively) travel outwards. When field is scaled to observed B_r at the CMB, travel time from tangent cylinder to equator is about 3 years, similar to Gillet et al. data.
- Minus points: Origin inside the tangent cylinder rather than at the TC. Waves not as periodic as suggested by the Gillet et al. data.

Driving terms

Take the ϕ and z average of the ϕ-component of equation of motion so $\langle\overline{u_\phi}\rangle$ is the geostrophic part of the azimuthal flow:

$$\left\langle\overline{\frac{\partial u_\phi}{\partial t}}\right\rangle = -\langle\overline{\hat{\phi}\cdot(\mathbf{u}\cdot\nabla)\mathbf{u}}\rangle + PmE^{-1}\langle\overline{\hat{\phi}\cdot((\nabla\times\mathbf{B})\times\mathbf{B})}\rangle + Pm\langle\overline{\hat{\phi}\cdot\nabla^2\mathbf{u}}\rangle$$
$$\equiv \quad F_R + F_L + F_V$$

for Reynolds force, F_R, Lorentz force, F_L and viscous force, F_V.
Look at fluctuating part only:

$$\left(\frac{\partial\langle\overline{u_\phi}\rangle}{\partial t}\right)' = F'_{LR} + F'_{LD} + F'_R + F'_V$$

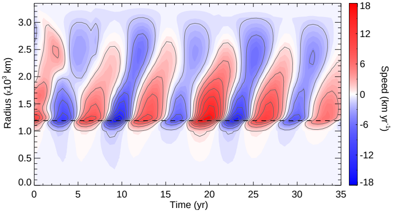
In the core, we expect $F_{LR}, F_{LD} \gg F_R, F_V$.

Magnetoconvection simulations

- In dynamo simulations F_R is found to be overestimated
- Kinetic and magnetic energy is typically similar, whereas magnetic energy is much larger than kinetic energy in the core
- The need to generate a magnetic field places severe restrictions on the accessible parameter space (e.g. very expensive to lower Pm)
- Can be alleviated with magnetoconvection runs (imposed dipolar field, stength B_0, at the outer boundary)
- Saves CPU time: don't have to wait for field to build up
- Allows lower E, lower Pm and can get into dominant magnetic energy regime

Torsional waves in magnetoconvection runs

In some magnetoconvection runs, TWs are very clear!



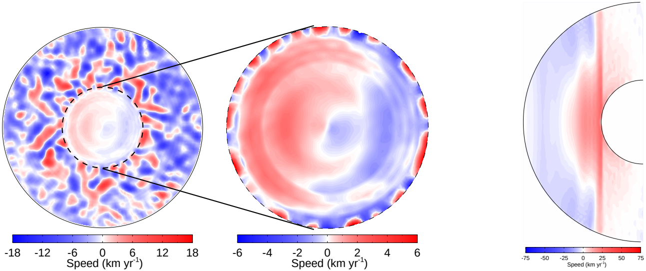
$\langle \overline{u_\phi} \rangle$ contours for $E = 5 \times 10^{-6}, Pm = 0.1, Ra = 5Ra_c, \Lambda \sim 100$.
In this strong field, low E case, TWs are much more periodic, and originate from the tangent cylinder and propagate outwards.
No significant reflection.

Excitation I

equation?

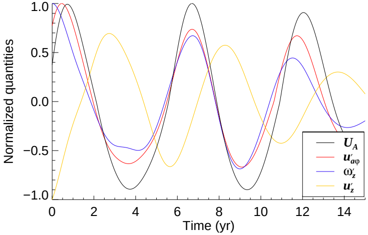
- Lorentz force is found to be dominant in these runs (over Reynolds, viscous forcing)
- Many terms contribute to the forcing, but the dominant terms are ageostrophic part of u_ϕ, combined with the ageostrophic part of B_s^2
- Combine to give a geostrophic forcing
- This is how ageostrophic convection, $u'_{\phi a}$, can drive a torsional oscillation, even though a TO itself cannot convect heat.

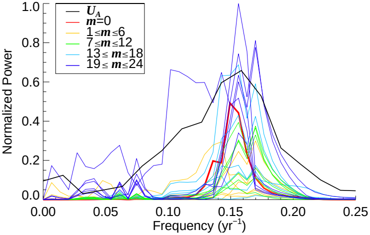
Excitation II



- Left plot shows a shapshot of $u'_{\phi a}$ in the equatorial plane
- Right plot shows the azimuthal and time average of u_ϕ in a meridional plane
- This ageostrophic part of the convective flow has large azimuthal wavenumbers, $m \sim 20$ focussed at the tangent cylinder

Excitation III





- The frequency of the convection at the TC matches that of the torsional wave
- Power concentrated in $m \simeq 20$ modes ($\simeq$ 400km) with frequency $\simeq 0.16$ corresponding to a period of $\simeq 6$ years
- Other simulations show similar results but periodic wave excitation ceases if convective and TW frequencies become mismatched

Detecting magnetic Rossby waves

Magnetic Rossby waves

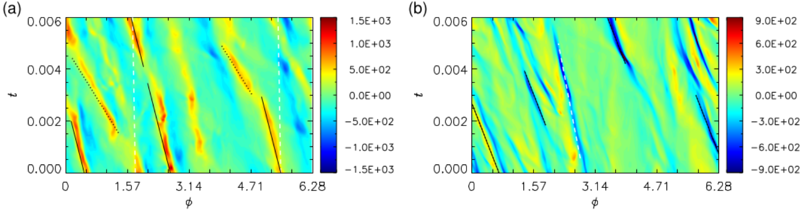
- Having found torsional waves, can we find other types of waves in simulations?
- Magnetic Rossby waves are a good candidate; they operate on observable time scales
- Wave speed derived from governing equations is:

$$u_{MR} = -\frac{m^2(r_0^2 - s^2)\langle\overline{B_\phi^2}\rangle}{2\Omega\mu\rho s^4}$$

- Oscillations in u_s travel westward (in ϕ direction) with speed dependent on the azimuthal magnetic field
- Possible cause of westward drift in geomagnetic features seen in observations
- Search for these waves in simulations as before

Magnetic Rossby waves detected

$\phi - t$ plots of u_s from Hori, Jones & **Teed**, 2015

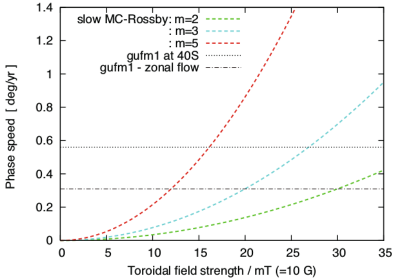


$s = 0.5r_o$, i.e. latitude 60° $s = 0.766r_o$, i.e. latitude 40°

White dashed: advection speed. Black dashed: wave speed + advection speed.

- At higher latitudes, waves + flow correlate well
- At low latitudes, hard to distinguish

Estimate of the Toroidal Field Strength



At 40°S, black dotted is drift speed at CMB from observations.

Black solid is drift speed with core flow removed (Hulot et al. 2002).

Difference could be drift due to waves!

- Coloured dashed lines are the expected wave speeds from the simulations
- Intersection of lines gives estimates for the (unobservable) **toroidal** field strength
- Indicates that the azimuthal field is a lot stronger than the (observable) **poloidal** field of 3 mT

Summary

- Observed torsional oscillations (TOs) in simulations
- Evidence of periodic excitation of waves
- As the Ekman number is lowered (more Earth-like):
  - Short timescale flow more dominated by TOs
  - Realistic Earth-like period of wave excitation and core travel times
  - Driving controlled by Lorentz force and delivered at the TC
- Current work: finalising identification of the excitation mechanism

For further reading

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