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Constrained Multiple Model Bayesian Filtering for Target Tracking in Cluttered Environment

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Abstract: This paper proposes a composite Bayesian filtering approach for unmanned aerial vehicle trajectory estimation in cluttered environments. More specifically, a complete model for the measurement likelihood function of all measurements, including target-generated observation and false alarms, is derived based on the random finite set theory. To accommodate several different manoeuvre modes and system state constraints, a recursive multiple model Bayesian filtering algorithm and its corresponding Sequential Monte Carlo implementation are established. Compared with classical approaches, the proposed method addresses the problem of measurement uncertainty without any data associations. Numerical simulations for estimating an unmanned aerial vehicle trajectory generated by generalized proportional navigation guidance law clearly demonstrate the effectiveness of the proposed formulation.

Keywords: Unmanned aerial vehicle, Trajectory estimation, Random finite set, Multiple model filtering, System state constraint, Sequential Monte Carlo implementation.

1. INTRODUCTION

The large scale of Unmanned Aerial Vehicle (UAV) applications has proliferated vastly within the last few years. The operational experience of UAVs has proven that their technology can bring a profound impact to the civilian and military arenas. This includes obtaining real-time, relevant situational awareness information before making contact; helping operators to lead appropriate decision making; and reducing risk to the mission and operation. The potential applications are wide, e.g surveillance, border patrol, search and rescue, convoy protection Grocholsky et al. (2006).

Despite of these advantages, UAV operations might pose serious risks: insertion of them to the non-segregated airspace could pose the risk of collision with other objects and hostile UAVs could raise a serious security concern. One of key technologies in reducing such a risk is the target tracking technology. The issue is that these UAVs could be operated in complex environments such as an urban environment.

Target tracking, in complex and or clustered environments is challenging since the sensor received measurements may include target-generated observation, clutters as well as spurious targets (decoys). Traditional way to address such measurement uncertainty is the well-known data association, such as nearest neighbour filter (NNF) Singer et al. (1974), probability data association (PDA) Bar-Shalom and Tse (1975), joint PDA Fortmann et al. (1983), multiple hypothesis tracking (MHT) Reid (1979). Unlike data association technique, the recently proposed random finite set (RFS) theory Mahler (2007); Vo et al. (2005); Ristic et al. (2016) results in elegant and rigorous mathematical formulation to solve this issue and provides a different view on filtering without data association. RFS-based algorithms, where system states and measurements are represented by RFSs, are joint decision and estimation approaches.

In the absence of measurement-origin uncertainty, UAV trajectory estimation faces another two interrelated issues: UAV manoeuvre mode uncertainty and system nonlinearity. The mainstream approach for target tracking under motion uncertainty is multiple model filtering Mazor et al. (1998); Li and Jilkov (2005). At each filtering cycle, multiple model approach runs a bank of nonlinear filters corresponding to each mode with the same measurement and fuses the output of these filters to find an overall estimate.

By combining the interacting multiple model approach and RFS theory, this paper proposes a new recursive Bayesian filtering algorithm for UAV trajectory estimation. Since the key of nonlinear filtering is to obtain the measurement likelihood function for the set-valued case, a simple but complete model is derived using RFS theory, where the target-generated observation and false alarms are represented by two different RFSs. Using the proposed measurement likelihood function obtained, a rigorous recursive multiple model Bayesian filtering is presented. Instead of generic Bayesian framework, system state constraints are also considered due to the existence of physical limits. Since analytic closed-form solution for such Bayesian filter is intractable due to nonlinearity, a Sequential Monte Carlo (SMC) implementation is presented to approximate the posterior density function. Simulation results show

that the estimation accuracy can be significantly improved compared with a classical PDA filter for highly cluttered environments

The remainder of this paper is organized as follows. Some backgrounds and preliminaries are presented in Sec. 2. In Sec. 3, the proposed constrained multiple model Bayesian filtering algorithm is derived in detail, followed by Sequential Monte Carlo implementation introduced in Sec. 4. Finally, a case study and some conclusions are offered.

2. BACKGROUNDS AND PRELIMINARIES

2.1 Bayesian Filtering

Suppose that the state vector $x_k \in \mathbb{R}^n$ provides the complete information of the system state of a target at time t_k, and let $z_k \in \mathbb{R}^m$ be the measured information. Typically, only partial system state can be observed, that is, $m < n$. Then, the general target dynamics can be formulated as

$$\begin{aligned} x_k &= f_{k-1}(x_{k-1}) + v_{k-1} \\ z_k &= h_k(x_k) + w_k \end{aligned} \tag{1}$$

where f_{k-1} is a nonlinear transition function governing the temporal evolution of first-order Markov target state, v_{k-1} the independent identically distributed (IID) process noise. Function h_k is used to define the relationship between system state and sensor measurement and w_k is the IID measurement noise. The term IID means that each variable that belongs to a collection of random variables has the same probability distribution as the others and all variables are mutually independent.

In the formulation of Bayesian filtering, two different probability density functions need to be specified, e.g. the transitional density function $\pi(x_k|x_{k-1})$ and the measurement likelihood function $g(z_k|x_k)$. Under these conditions, Bayesian filter propagates the posterior density $p(x_k|z_{1:k})$ according to

$$\begin{aligned} p(x_k|z_{1:k-1}) &= \int \pi(x_k|x)\, p(x|z_{1:k-1})\, dx \\ p(x_k|z_{1:k}) &= \frac{g(z_k|x_k)\, p(x_k|z_{1:k-1})}{\int g(z_k|x)\, p(x|z_{1:k-1})\, dx} \end{aligned} \tag{2}$$

where $z_{1:k} = [z_1, z_2, \ldots, z_k]$ stands for the measurement sequence.

2.2 Random Finite Set Probability Density

Recursion procedure (2) is formulated based on the assumption that at most one measurement can be generated at each time step. Typically, sensor detection is imperfect, leading to the fact that the target may not be observed at some time. Moreover, complex cases, such as electronic counter measures, multi-path effect, may generate unknown spurious measurements or decoys. In addition to decoys, clutters are also needed to be considered in severe conditions. Since each sensor measurement at any given time step has no physical importance, the unordered measurements at time t_k can be modelled by a RFS on Z, e.g. $Z_k \in \Xi(\mathrm{Z})$ with $\Xi(\mathrm{Z})$ being the set of finite subsets of Z. Obviously, the key to implement Bayesian recursion (2) is to extend the measurement likelihood function $g(z_k|x_k)$ to set-valued multiple measurement case. To make this paper self-contained, some preliminaries of RFS probability density are reviewed in this subsection.

A RFS is defined as that a random set that takes values as unordered finite sets, which means that both the number of elements and the individual state values of each element in Z are both random. To fully characterize the probability density of a RFS variable, it is necessary to define the discrete cardinality (the number of elements) distribution and a group of joint distributions conditioned on the cardinality. The cardinality distribution $\rho(n_z) = \Pr\{|Z_k| = n_z\}$ specifies the cardinality of the RFS, while the joint probability distributions $p_{n_z}(z_{k,1}, z_{k,2}, \ldots, z_{k,n_z})$ model the distribution of the elements. Naturally, the value of $p_{n_z}(z_{k,1}, z_{k,2}, \ldots, z_{k,n_z})$ remains unchanged for all $n!$ possible element permutations due to the unordered property of a RFS. With these two definitions, the rigorous mathematical representation of the probability density function of a RFS can be derived as Ristic et al. (2016)

$$\begin{aligned} f(\{z_{k,1}, z_{k,2}, \ldots, z_{k,n_z}\}) &= \rho(n_z) \\ &\quad \times p_{n_z}(z_{k,1}, z_{k,2}, \ldots, z_{k,n_z})\, n_z! \end{aligned} \tag{3}$$

3. CONSTRAINED MULTIPLE MODEL FILTERING IN A CLUTTERED ENVIRONMENT

This section details the proposed estimation algorithm in two parts: set-valued measurement likelihood function derivation, constrained multiple model Bayesian recursion.

3.1 Multiple Measurements Likelihood Function

To distinguish the target-generated observation and false alarms, the measurement set Z_k is represented as

$$Z_k = \Theta_k \cup \Omega_k \tag{4}$$

where Θ_k denotes the target-generated measurement, and Ω_k stands for the false measurements, including decoys and clutters.

Since sensor measurement is usually imperfect, we model Θ_k as a Bernoulli RFS Ristic et al. (2016), which can either be empty or has target-generated measurement. Let $p_{D,k}(x_k)$ be the probability of target detection, then, the probability of miss detection is $1 - p_{D,k}(x_k)$, and in contrast, supposed that there exists a target-generated observation in the measurement set, the probability of obtaining such a measurement is $p_{D,k}(x_k)\, g_k(z_k|x_k)$. In conclusion, the probability density function of Θ_k can be obtained as

$$f_1(\Theta_k) = \begin{cases} 1 - p_{D,k}(x_k), & \Theta_k = \emptyset \\ p_{D,k}(x_k) \cdot g_k(z_k^t|x_k), & \Theta_k = \{z_k^t\} \end{cases} \tag{5}$$

where z_k^t denotes the target-generated observation.

Suppose that each element of Ω_k is independent of one another and is identically distributed according to probability density function $c_k(z_k|x_k)$, then the false alarm measurement Ω_k in complex environments can be modelled as a Poisson RFS and the cardinality distribution $\rho(|\Omega_k|)$ is Poisson with parameter λ, that is

$$\rho(|\Omega_k|) = \frac{e^{-\lambda}}{|\Omega_k|!}\lambda^{|\Omega_k|} \tag{6}$$

Then, the probability density function of Ω_k can be derived from (3) as

$$f_2(\Omega_k) = e^{-\lambda} \prod_{z_k \in \Omega_k} \lambda c_k(z_k|x_k) \tag{7}$$

For each time step k, consider the following events. D_1: target-generated observation detection and D_2: target-generated observation miss detection. Then, according to the law of total probability, the measurement likelihood function $g(Z_k|x_k)$ for the set-valued multiple measurement case can be obtained as

$$\begin{aligned} g(Z_k|x_k) &= \sum_{i=1}^{2} g(Z_k|x_k, D_i) P(D_i|x_k) \\ &= \left[1 - p_{D,k}(x_k)\right] f_2(Z_k) + \sum_{z_{k,j} \in Z_k} p_{D,k}(x_k) \\ &\quad \times g(z_{k,j}|x_k) f_2(Z_k \backslash \{z_{k,j}\}) \\ &= \left[1 - p_{D,k}(x_k)\right] e^{-\lambda} \prod_{z_k \in Z_k} \lambda c_k(z_k|x_k) \\ &\quad + p_{D,k}(x_k) e^{-\lambda} \sum_{z_{k,j} \in Z_k} g(z_{k,j}|x_k) \\ &\quad \times \prod_{z_k \in Z_k \backslash \{z_{k,j}\}} \lambda c_k(z_k|x_k) \end{aligned} \tag{8}$$

3.2 Constrained Multiple Model Filtering

For realistic scenarios, UAVs may have different manoeuvre modes and system states may also subject to some constraints. For instance, suppose that the UAV applies proportional navigation (PN) guidance law for trajectory tracking Yamasaki et al. (2007), the navigation gain usually has its lower and upper bounds due to physical limitations. To handle such cases, target model (1) is extended to

$$\begin{aligned} x_k &= f_{k-1}(x_{k-1}, m_k, C_k) + v_{k-1} \\ z_k &= h_k(x_k) + w_k \end{aligned} \tag{9}$$

where m_k denotes the manoeuvring mode at time k and C_k stands for the state constraints.

Invoking the fact that the nonlinearity and non-Gaussian distribution of UAV tracking system, a general recursive Bayesian filtering algorithm is required to propagate the posterior probability density function. More specifically, to improve the estimation performance, the concept of general multiple model filtering for hybrid systems Blom and Bloem (2007) is used in this paper. Each cycle of updating the posterior probability density from $p(x_{k-1}, m_{k-1}|Z_{k-1}, C_{k-1})$ at time step $k-1$ to $p(x_k, m_k|Z_k, C_k)$ at time step k consists of four main steps: mode switching, interaction, prediction and correction.

The model switching process characterizes the way that the conditional model probability evolves from time step $k-1$ to k. Based on the law of total probability and the assumption that the mode transition probability is independent of measurement Blom and Bloem (2007), we have

$$\begin{aligned} p(m_k|Z_{k-1}, C_{k-1}) &= \int p(x_{k-1}, m_k|Z_{k-1}, C_{k-1}) dx_{k-1} \\ &= \int \sum_{m_{k-1} \in \mathrm{M}} p(x_{k-1}, m_k, m_{k-1}|Z_{k-1}, C_{k-1}) dx_{k-1} \\ &= \sum_{m_{k-1} \in \mathrm{M}} \int p(m_k|m_{k-1}, x_{k-1}) \\ &\quad \times p(x_{k-1}, m_{k-1}|Z_{k-1}, C_{k-1}) dx_{k-1} \end{aligned} \tag{10}$$

where M represents the set of possible UAV manoeuvre modes.

The effect of model switching on the evolution of conditional state density is characterized by the interaction process. Again, it follows from the law of total probability such that

$$\begin{aligned} p(x_{k-1}|m_k, Z_{k-1}, C_{k-1}) &= \frac{p(x_{k-1}, m_k|Z_{k-1}, C_{k-1})}{p(m_k|Z_{k-1}, C_{k-1})} \\ &= \frac{\sum_{m_{k-1} \in \mathrm{M}} p(m_k|m_{k-1}, x_{k-1}) p(x_{k-1}, m_{k-1}|Z_{k-1}, C_{k-1})}{p(m_k|Z_{k-1}, C_{k-1})} \end{aligned} \tag{11}$$

The prediction step propagates the mode conditioned state density from time step $k-1$ to k. Given the interacted mode conditioned density in (11), one can obtain that

$$\begin{aligned} p(x_k|m_k, Z_{k-1}, C_k) &= \int p(x_{k-1}|m_k, Z_{k-1}, C_{k-1}) \\ &\quad \times p(x_k|x_{k-1}, m_k, Z_{k-1}, C_k) dx_{k-1} \end{aligned} \tag{12}$$

where the constrained state transition probability density function $p(x_k|x_{k-1}, m_k, Z_{k-1}, C_k)$ is obtained from the concept of constrained likelihood Simon (2010) as

$$\begin{aligned} &p(x_k|x_{k-1}, m_k, Z_{k-1}, C_k) = \\ &\qquad \begin{cases} \pi_{m_k}(x_k|x_{k-1}), x_k \in C_k \\ 0, \qquad\qquad otherwise \end{cases} \end{aligned} \tag{13}$$

where $\pi_{m_k}(x_k|x_{k-1})$ denotes the transitional density function of mode m_k.

Finally, the correction step is used to update the posterior joint probability density by Bayesian rule as

$$\begin{aligned} p(x_k, m_k|Z_k, C_k) &= \alpha g(Z_k|x_k) p(x_k, m_k|Z_{k-1}, C_k) \\ &= \alpha g(Z_k|x_k) p(x_k|m_k, Z_{k-1}, C_k) p(m_k|Z_{k-1}, C_k) \end{aligned} \tag{14}$$

where α is a normalizing scale factor. The multiple model Bayesian filtering cycle is now completed. Next, in order to extract state estimations, one needs to calculate the mode probability and the mode conditioned state probability as

$$\begin{aligned} p(m_k|Z_k, C_k) &= \int p(x_k, m_k|Z_k, C_k) dx_k \\ p(x_k|m_k, Z_k, C_k) &= \frac{p(x_k, m_k|Z_k, C_k)}{p(m_k|Z_k, C_k)} \end{aligned} \tag{15}$$

The mode with largest likelihood is regarded as the current UAV manoeuvre mode and the system states can be estimated via the well-known expected a posterior (EAP) or maximum a posterior (MAP) approaches from the mode conditioned state probability density $p(x_k|m_k, Z_k, C_k)$.

4. SEQUENTIAL MONTE CARLO IMPLEMENTATION

Due to the nonlinearity and non-Gaussian distribution of UAV tracking system, closed-form solution of the proposed constrained multiple model Bayesian filtering is

intractable. To this end, a SMC implementation of (10)-(15) is presented in this section.

Suppose at time step $k-1$ the posterior joint probability density of each mode $\vartheta \in \mathrm{M}$ is represented by a set of N weighted particles $\left\{w_{k-1}^{\vartheta,i}, x_{k-1}^{\vartheta,i}\right\}$ as

$$p\left(x_{k-1}, m_{k-1}=\vartheta \,|Z_{k-1}, C_{k-1}\right) = \sum_{i=1}^{N} w_{k-1}^{\vartheta,i} \delta\left(x_{k-1}-x_{k-1}^{\vartheta,i}\right) \tag{16}$$

where $\delta(\cdot)$ denotes the Dirac delta function.

Then, the predicted mode probability $p\left(m_k=\gamma \,|Z_{k-1}, C_{k-1}\right)$ can then be approximated by

$$p\left(m_k=\gamma \,|Z_{k-1}, C_{k-1}\right) = \sum_{\vartheta \in \mathrm{M}} \sum_{i=1}^{N} p\left(m_k=\gamma \,\middle|\, m_{k-1}=\vartheta, x_{k-1}^{\vartheta,i}\right) w_{k-1}^{\vartheta,i} \tag{17}$$

Next, substituting (17) into (11) gives

$$\begin{aligned} & p\left(x_{k-1} \,|m_k=\gamma, Z_{k-1}, C_{k-1}\right) \\ & \quad = \sum_{\vartheta \in \mathrm{M}} \sum_{i=1}^{N} p\left(m_k=\gamma \,\middle|\, m_{k-1}=\vartheta, x_{k-1}^{\vartheta,i}\right) \\ & \quad \times \frac{w_{k-1}^{\vartheta,i} \delta\left(x_{k-1}-x_{k-1}^{\vartheta,i}\right)}{p\left(m_k=\gamma \,|Z_{k-1}, C_{k-1}\right)} \end{aligned} \tag{18}$$

Suppose the number of UAV manoeuvre modes is M, then it follows from (17) and (18) that the number of particles increases to $M \times N$ from N to approximate each model conditioned state probability density, which means that the total number of particles increases in an exponential manner as time goes. To address this problem, a resampling is performed on (18) to generate N new weighted particles $\left\{\bar{w}_{k-1}^{\gamma,i}, \bar{x}_{k-1}^{\gamma,i}\right\}$ as

$$\bar{x}_{k-1}^{\gamma,i} = \begin{cases} x_{k-1}^{\gamma,i}, & p\left(m_k=\gamma \,|Z_{k-1}, C_{k-1}\right)=0 \\ \sim p\left(x_{k-1} \,|m_k=\gamma, Z_{k-1}, C_{k-1}\right), otherwise \end{cases} \tag{19}$$

After resampling, the predicted particle states $x_{k-1}^{\gamma,i}$ are obtained by passing $\bar{x}_{k-1}^{\gamma,i}$ through the constrained transition probability density given in (13). Then, the prediction of mode conditioned state density can be approximated as

$$p\left(x_k \,|m_k=\gamma, Z_{k-1}, C_k\right) = \sum_{i=1}^{N} \bar{w}_{k-1}^{\gamma,i} \delta\left(x_{k-1}-x_{k-1}^{\gamma,i}\right) \tag{20}$$

Finally, given the measurement likelihood function $g\left(Z_k \,|x_k\right)$ for the set-valued multiple measurement case shown in (refeq:8), the particle filter approximates the posterior joint probability density function as

$$p\left(x_k, m_k=\gamma \,|Z_k, C_k\right) = \sum_{i=1}^{N} w_{k-1}^{\gamma,i} \delta\left(x_{k-1}-x_{k-1}^{\gamma,i}\right) \tag{21}$$

where $w_{k-1}^{\gamma,i} \propto g\left(Z_k \,|x_k\right) p\left(m_k=\gamma \,|Z_{k-1}, C_{k-1}\right)$ and is normalized to satisfy $\sum_{\gamma \in \mathrm{M}} \sum_{i=1}^{N} w_{k-1}^{\gamma,i} = 1$.

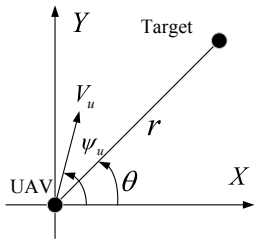

Fig. 1. Relative motion geometry

Based on the posterior joint probability density function, the mode probability can be obtained by the summation of the mode-related weights as

$$p\left(m_k=\gamma \,|Z_k, C_k\right) = \sum_{i=1}^{N} w_{k-1}^{\gamma,i} \tag{22}$$

And the model conditioned state probability is calculated as

$$p\left(x_k \,|m_k=\gamma, Z_k, C_k\right) = \frac{\sum_{i=1}^{N} w_{k-1}^{\gamma,i} \delta\left(x_{k-1}-x_{k-1}^{\gamma,i}\right)}{p\left(m_k=\gamma \,|Z_k, C_k\right)} \tag{23}$$

Then, the target state estimation at time step k can be extracted from $p\left(x_k \,|m_k=\gamma, Z_k, C_k\right)$ using either EAP or the MAP estimation approaches.

5. CASE STUDY

This section presents a case study for UAV trajectory prediction with unknown number of decoys in a cluttered environment using the proposed method.

5.1 System Model

This work assumes a fixed-wing UAV equipped with a high-performance low-level flight control system that provides roll, pitch and yaw stability of the UAV as well as velocity (or acceleration) tracking, heading and altitude hold functions. Under this assumption, the UAV kinematics in a two-dimensional plane can be represented by the following differential equations

$$\begin{aligned} \dot{x}_u &= V_u \cos\psi_u \\ \dot{y}_u &= V_u \sin\psi_u \\ \dot{\psi}_u &= \omega_u \end{aligned} \tag{24}$$

where (x_u, y_u) denotes the inertial position of the UAV, V_u the airspeed of the UAV, ψ_u the heading angle of the UAV, and ω_u UAV turning rate (also the control input) that guides the UAV to a pre-designated target.

Using the above command structure, the system model representing the guidance layer is a relative kinematic model between the UAV and the target position as shown in Fig.1, where θ denotes the line-of-sight (LOS) angle relative to the inertial frame; r denotes the relative distance between the UAV and the target position.

According to Fig.1, the differential equations, describing the relative kinematics, is formulated as

$$\begin{aligned} \dot{r} &= V_u \cos\left(\psi_u-\theta\right) \\ r\dot{\theta} &= V_u \sin\left(\psi_u-\theta\right) \end{aligned} \tag{25}$$

PN guidance law is a well-known guidance method and has been traditionally used in missile guidance during the last few decades due to its easy implementation and effectiveness against non-manoeuvrable or weakly manoeuvrable targets. The idea behind this benchmark guidance approach lies in that it issues a latax command proportional to the LOS rate to guide an interceptor to approach a collision triangle. Moreover, PN guidance law with navigation ratio three was proved to be an optimal law in the sense of minimization of both terminal miss distance as well as control effort. Recently, Lee et al. (2013) proposed a generalized weighted optimal PN guidance law that greatly extends the usage of PN law to different applications. In this study, we also assume that the UAV is performing PN guidance law, that is

$$\omega_u = N\dot{\theta} \tag{26}$$

where N is the navigation ratio, which can be time-varying or time-invariant.

Since one cannot know the exact type of the navigation ratio that the UAV is using, we regard it as a constant to construct the state transition model. Let $x = [r, \theta, N]^T$, a complete system model can be obtained as

$$\dot{x} = f(x) + v = \begin{bmatrix} V_u \cos(\psi_u - \theta) \\ V_u \sin(\psi_u - \theta)/r \\ 0 \end{bmatrix} + v \tag{27}$$

where v denotes the process noise.

5.2 Observation Model

In the considered scenario, it is assumed that the radar measures the bearing angle and the relative range. Suppose that the observer position is (x_o, y_o), then, the target-generated measurement model can be derived as

$$z = h(x) + w = \begin{bmatrix} \tan^{-1}\left(\dfrac{x_u - x_o}{y_u - y_o}\right) \\ \sqrt{(x_u - x_o)^2 + (y_u - y_o)^2} \end{bmatrix} + w \tag{28}$$

where w denotes the measurement noise.

5.3 Simulation Setup

To compensate for the loss of manoeuvrability of UAVs, an exponential type weighting function Lee et al. (2013) is used to generate the following time-varying navigation ratio as

$$N = \frac{3\left(\mu^3 e^{\mu} + b\mu^3\right)}{3e^{\mu}\left(\mu^2 - 2\mu + 2\right) - 6 + b\mu^3} \tag{29}$$

where $\mu = a(t_f - t)$ with t_f being the total flight time to reach to the target, which can be approximated by $t_f \approx t + r/V_u$, a, b are two design constants. In this study, $a = 0.1$, $b = -1$, which are suggested by Lee et al. (2013), are used for the simulation purpose. Generally, since the UAV is a mechanical system, subject to Newtons second law, its control input is always bounded. With this in mind, we impose a hard constraint on the navigation ratio as $0 < N < 18$. To accommodate the time-varying navigation ratio, a set of three models with constant navigation gains are used, that is, $N_1 = 4$, $N_2 = 8$ and $N_3 = 12$.

To evaluate the performance of the proposed algorithm, two different decoys are considered in the study. The extraneous decoy measurements are modelled as Poisson RFSs and each measurement of these two decoys is assumed to be IID according to

$$c_{k,d_1}(z|x) = \mathcal{N}\left(z; \begin{bmatrix} \tan^{-1}\left(\dfrac{x_u - x_o}{y_u - y_o}\right) \\ 1.5\sqrt{(x_u - x_o)^2 + (y_u - y_o)^2} \end{bmatrix}, R_k\right) \tag{30}$$

$$c_{k,d_2}(z|x) = \mathcal{N}\left(z; \begin{bmatrix} 2\tan^{-1}\left(\dfrac{x_u - x_o}{y_u - y_o}\right) \\ \sqrt{(x_u - x_o)^2 + (y_u - y_o)^2} \end{bmatrix}, R_k\right) \tag{31}$$

The birth time of decoy 1 is 0s while the birth time of decoy 2 is 8s.

As for the clutter measurement, it is assumed that the clutters are modelled as a RFS and each clutter measurement is uniformly IID over the observation region with the average $\lambda = 20$ clutter returns at one scan.

The probability of detection is assumed to be constant as $P_{D,k} = 0.98$. The process noise covariance Q_k is modelled by $Q_k = diag\left(\left[\sigma_r^2, \sigma_\theta^2, \sigma_N^2\right]\right)$, where $\sigma_r = 30m$, $\sigma_\theta = 2\pi/180rad$, $\sigma_N = 0.5$. The measurement noise covariance R_k is modelled as $R_k = diag\left(\left[\sigma_{\bar{\theta}}^2, \sigma_{\bar{r}}^2\right]\right)$, where $\sigma_{\bar{\theta}} = 2\pi/180rad$, $\sigma_{\bar{r}} = 10m$. The initial system state distribution for initialization is $x_0 \sim \mathcal{N}(x; \bar{x}_0, P_0)$, where the mean is $\bar{x}_0 = [4901m, -0.0383rad, 10]^T$, and the covariance matrix is $P_0 = diag\left(\left[100^2m^2,\ (2\pi/180)^2rad^2,\ 4\right]\right)$.The maximum number of particles for one mode is upper limited by 1000. The UAV velocity is constant as $V_u = 80m/s$.

5.4 Results and Discussions

Figure 2 presents the UAV trajectory prediction performance in the presence of two decoys in $x - y$ plane. It follows from this figure that the proposed algorithm accurately identifies the target and tracks its ground truth trajectories. Figure 3 shows the response curves of the ground truth navigation gain and its estimated value. By using multiple constant navigation gain models, the estimated signal rapidly and precisely converges to the truth time-varying navigation gain. The position estimates in inertial frame are provided in Fig. 4, where the grey stars are measurements. From this figure, it is clear that the proposed algorithm is suitable for target identification and tracking even in a highly cluttered environment. The reason for this is quite obvious, since all measurements, including target-generated observation, clutters as well as decoy measurements, are all modelled in the measurement likelihood function and a mathematically rigorous Bayesian filtering is synthesized based on this probability model.

To further demonstrate the effectiveness, the performance of the proposed method is compared with that of traditional interactive multiple model PDA (IMM-PDA) filter. For the fair comparison, the IMM-PDA filter is also performed through SMC implementation. Table 1 summarizes the root-mean-squared (RMS) range estimation errors during different time durations of 50 runs Monte Carlo simulations for different average clutter returns at one scan. From the table, it is clear that the performance

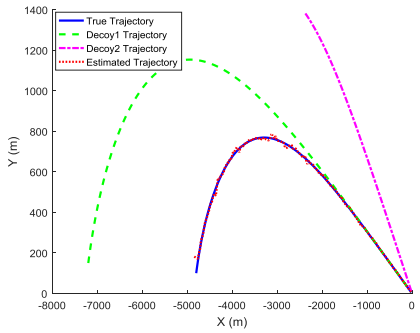

Fig. 2. Trajectory estimation performance

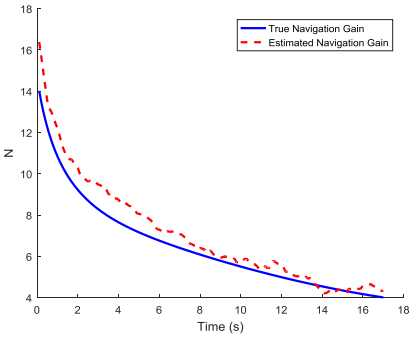

Fig. 3. Time-varying navigation gain estimation performance

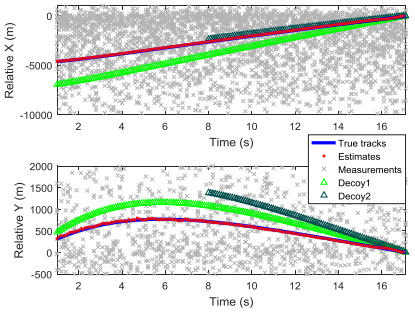

Fig. 4. UAV position estimation performance

of the proposed algorithm for different clutter rates is almost comparable while IMM-PDA filter is quite sensitive to clutter rates in terms of estimation performance. Its because all measurement information is used in the proposed formulation while IMM-PDA filter only used validated measurements by gating and therefore is prone to lost target in a highly cluttered environment.

Table 1. RMS range errors

Time window	Filter	Average clutter returns at one scan		
		$\lambda = 10$	$\lambda = 20$	$\lambda = 30$
$0 < t < 8s$	Proposed	51.78m	53.01m	60.54m
	IMM-PDA	67.17m	87.65m	128.77m
$8s < t < 17.1s$	Proposed	15.11m	15.98m	17.34m
	IMM-PDA	23.81m	29.33m	45.74m

6. CONCLUSIONS

This paper proposes a constrained multiple model Bayesian filtering algorithm for UAV trajectory prediction. The proposed algorithm is derived using the random finite set theory and interactive multiple model filter approach, where the random finite set theory is adopted to obtain a rigorous measurement likelihood function. Compared with traditional approaches, the current work can accurately identify the target UAV and estimate its trajectory without any data associations, which makes the proposed algorithm less sensitive to the clutter rate. A case study of estimating the position of a manoeuvring UAV with time-varying proportional navigation guidance confirms the validity of the proposed algorithm.

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