Active control of adiabatic soliton fission by external dispersive wave at optical event horizon

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Abstract: We show that the group-velocity-led optical event horizon (OEH) in optical fibers provides a convenient way to actively control the propagation property of higher-order solitons by a comparatively weak dispersive wave (DW) pulse. It has been found numerically that clean soliton breakup, a process by which a second-order soliton completely splits into a pair of constituent solitons with vastly different power proportions after interacting with the weak DW pulse, will occur while external DWs become polychromatic. The temporal separation between both constituent solitons can be controlled by adjusting the power of the external DW. The more energetic main soliton is advanced/trailed in time depending on the selected frequency of input DW pulse. We have developed an analytic formalism describing the external acting-force (AF) perturbation. These results provide a fundamental explanation and physical scaling of optical pulse evolution in optical fibers and can find applications in improved supercontinuum sources.

OCIS codes: (190.0190) Nonlinear optics; (060.4370) Nonlinear optics, fibers; (190.5530) Pulse propagation and temporal solitons.

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1. Introduction

Solitons are localized nonlinear waves formed by a balance between spreading and focusing effects in many different physical systems including nonlinear fiber-optics [1, 2], plasmas physics [3, 4] and Bose-Einstein condensates [5]. These pulses exist in diverse forms of matter yet exhibit similar properties, such as stability, periodic recurrence and particle-like trajectories. Yet it is fairly susceptible to perturbations [6-10]. One important property is soliton fission, a process where an energetic higher-order soliton breaks apart due to a sufficiently strong dispersive or nonlinear perturbation to the system [7]. This behavior intensively contrasts with the expected periodic recurrence for ideal higher-order solitons. The soliton fission dynamics was first numerically shown to take place resulting from the perturbations of the nonlinear Schrödinger equation including higher-order dispersion, self-steepening and stimulated Raman scattering [11, 12], and was experimentally observed very soon [13]. Since then, the higher-order soliton fission effect has been justified as a crucial mechanism in producing ultrashort frequency-shifted fundamental solitons [7] and ultra-broadband optical supercontinuum [1, 3].

The interactions between solitons and dispersive wave (DW) in optical waveguides [14, 15], one of the key physical mechanisms underlying the ultra-broadband supercontinuum generation [16, 17], are widely investigated in recent years. They have been proposed as a candidate to all-optical switching [18] and photonic transistor [19] owing to a close analogy with the event horizon in black holes. A fiber-optical analogue of an event horizon [20, 21] takes place when a weak linear wave, traveling at a slightly different group-velocity with respect to an energetic soliton, is unable to pass through the soliton during their collision process. The underlying nonlinear mechanism behind this phenomenon is that the nonlinear refractive index change induced by the soliton with high intensity alters the velocity of the linear DW pulse [22, 23], and leads to a reversible frequency conversion between the linear DW pulse and the newly generated wave in the regime of optical event horizon [24]. A weak DW packet trapped within a solitonic cavity formed by two solitons in an optical fiber has been numerically [25, 26] and experimentally [27] studied. Generally, these interactions were fundamentally studied in the context of a DW packet reflection at the group-velocity horizon of a bright fundamental soliton. However, it has been found recently that other sorts of solitons, such as dark [28] and higher-order solitons [29], can also interact with linear DW packet through this mechanism. Although light-by-light controlling has been well studied recently [22, 30, 31], the reported results focus mainly on the manipulation of fundamental solitons with a weak linear wave and the scattering of weak DWs on solitons. To the best of our knowledge, no particular attention has been paid to control the propagation property of higher-order solitons by a weak DW pulse.

In this paper, we provide both a numerical demonstration and a theoretical explanation of the physical mechanism underpinning soliton fission induced by the external DW pulse at optical event horizon. We find ways to manipulate the propagation of higher-order soliton via a weak DW pulse, leading to the occurrence of the clean soliton fission. The temporal separation between two constituent solitons can be controlled by adjusting the peak power of the external DW pulse. We derive an analytic formalism to reveal the physical parameters governing the system. These results provide a fundamental explanation and physical scaling of optical pulse evolution in nonlinear optical waveguide and can be applied to manipulate the formation of optical rogue waves.

2. Propagation model

The nonlinear pulse propagation in dispersive nonlinear media can be described by the generalized nonlinear Schrödinger equation (GNLSE) model [1, 14, 28], as follows:

$$\frac{\partial A(z,T)}{\partial z} = iD(i\partial_T)A(z,T) + i\gamma\,|A(z,T)|^2\,A(z,T) \tag{1}$$

where $A(z,T)$ is the envelope of the electric filed, the dispersion operator $D(i\partial_T)=\sum_{n\geq 2}\frac{\beta_n}{n!}(i\partial_T)^2$ takes into account the dispersion profile of fiber up to n=3, and γ is the nonlinear interaction coefficient of the fiber. Considering a standard single-mode fiber with zero dispersion wavelength (ZDW) λ_{ZDW} = 1311 nm where a soliton with a central wavelength of $\lambda_{sol} = 1391$ nm is launched, the following dispersion parameters are used in our model: $\beta_2 = -2.6$ ps^2/km and $\beta_3 = 0.005$ ps^3/km. The nonlinear coefficient is taken as $\gamma = 2.5$ W^{-1}km^{-1}. Aiming to demonstrate the propagation features of controlling light-by-light more clearly, the nonlinear dynamics here are dominated by the $\chi^{(3)}$ optical Kerr effect without considering the stimulated Raman scattering effect, which is reasonable as the Raman effect has little significance on the propagation dynamics of optical event horizon [19, 29, 30] and particularly can be justified in the hollow core fibers filled with Raman-free gas [4]. Apart from the dominant terms of Kerr and group velocity dispersion (GVD) that are required to form the soliton, we also include third-order dispersion term in the full model in order to obtain the regime of optical event horizon [19, 24]. It has been found that the self-steepening (SS) term and higher-order dispersion with n>3 do not play any significant role [21, 29, 33].

The injected light consists of two pulses with different frequencies namely the second-order soliton (2-soliton) and the DW. The overall electric field of the injected light is determined as following:

$$A(t) = A_{sol}(t) + A_{DW}(t), \tag{2}$$

where the 2-soliton has the initial form

$$A_{sol}(t) = 2\sqrt{P_0}\,\mathrm{sech}(t/T_0) \tag{3}$$

with temporal width of T_0 =85 fs and the peak power of corresponding fundamental soliton P_0=144 W. Although DW should be chirped due to propagation in the normal dispersion region after its formation [32], it was found that the Chirp does not play a significant role here. DW is given by

$$A_{DW}(t) = \sqrt{P_{DW}}\,\mathrm{sech}((t-t_1)/T_1)\exp(-i\Delta\omega t) \tag{4}$$

Here P_{DW} represents the peak power of DW, t_1 is the temporal delay of the DW with respect to the soliton, T_1=400 fs is the temporal width of DW, and $\Delta\omega$ is the angular frequency deviation of the DW from the soliton given by $\Delta\omega = 2\pi c(\lambda_{DW}^{-1} - \lambda_{sol}^{-1})$. Note that central wavelengths of the 2-soliton (λ_{sol}) and the DW packet (λ_{DW}) are selected to fall on either side of the ZDW where the GVD is negative for the 2-soliton and positive for the DW pulse [14].

3. Result and Discussion

3.1 Confirmation of external dispersive wave induced fission by modeling

In order to realize the interaction of the 2-soliton with the low-amplitude DW pulse, we need to make the group velocities reasonably appropriate for both of the soliton and the DW by choosing proper central wavelengths. If their group velocities are quite different, the pulses go through each other unchanged. In contrast, the pulses may never meet if they are too close [30]. When the low-amplitude DW reaches the trailing edge of the soliton, the intense optical soliton will produce a nonlinear perturbation of the refractive index, which then gives rise to an apparent 'reflection' of the DW on the soliton. This scenario is often described as a fiber-optical analog of the event horizon. Meanwhile, the reflection process can cause an abrupt change in the temporal properties of the soliton. This behavior suggests that we can use the external DW parameter as an efficient approach for all-optical manipulation, through adjusting the parameters such as frequency, peak power (amplitude) and width of external DW as well as the initial delay between both pulses [14]. Among all these parameters, the peak power (amplitude) of DW is considered as the most efficient one that can be simply adjusted.

Fig. 1 summarizes our obtained modelling results based on GNLSE which confirms that the fission event is triggered by the effective acting-force induced by the interaction between external DW pulse and 2-soliton. The collision process between a relatively weak DW (P_{DW} = 1 W) centered at relative frequency 160THz and the 2-soliton is presented in Figs. 1(a)-1(c). Considering their different group-velocities, the soliton precedes the external DW pulses temporally by 3 ps at the fiber input. Fig. 1(a) shows the temporal dynamics which demonstrates that the external DW pulse is substantially scattered and only partially transmitted through an effective periodic refractive index perturbation created by the 2-soliton. In the case of a moving inhomogeneity, the scattering is followed by a frequency conversion. The spectral evolution of transmitted and reflected radiation along the propagation distance is shown in Fig. 1(c), from which we can see that the resonant radiation becomes polychromatic resulting from the generation of unequally-spacing frequency comb like new spectral components. The newly appeared spectral components in DW are well predicted by the phase-matching resonance condition [24, 29]. However, as can be seen clearly from Figs. 1(a) and 1(b), the 2-soliton does not experience fission and maintain the soliton's oscillating structure with pulse breathing in both temproal and spectral domain as expected from soliton theory without perturbations. This is not surprising since it is not sufficiently perturbed by the effective acting-force induced by the collision, and also it is not emitting Cherenkov radiation due to the relatively small magnitude of β_3 at the soliton wavelength. We conclude that higher-order dispersion cannot be the fission mechanism here.

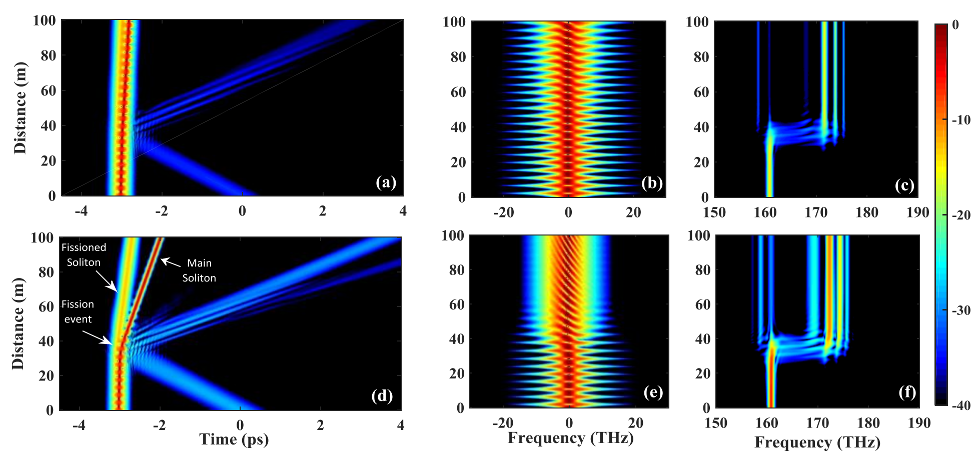

Fig. 1. Numerical results of the propagation of higher-order soliton control using the external dispersive wave with different power level (top row: P_{DW} =1 W and bottom row: P_{DW} = 4 W) and simulated spectrum against the propagation length. (a, d) Evolution of the temporal intensity of the collision of 2-soliton and external DW. (b, e) and (c, f) represents evolution of spectral intensity of soliton and external DW, respectively. The results demonstrate that the clean soliton breakup was induced by the interaction between 2-soliton and intensive DW pulse.

In contrast, Figs. 1(d)-1(f) demonstrate an outcome of the simulation in which an intensive external DW (the amplitude is increased twofold) collides with a 2-soliton, leading to a decomposition of the 2-soliton that is demonstrated by the pulse separation in the temporal domain. This is a non-trivial dynamics as the fission is an adiabatic process characterized as a continuous spectral and temporal walk-off of the two constituent solitons. As can be seen from Fig.1(d), at about 40 m robust pulse pairs (Main soliton and Fissioned soliton, respectively) have formed and separated in time for initially injected $N \approx 2$ soliton. It yields a profile in which the fissioned pulse with smaller amplitude advances in time with the main soliton trailing behind. This temporal separation is the essence of soliton fission. After the collision between the soliton and the external DW pulse occurred, well-defined spectral interference fringes, which are the characteristic optical spectrum for the 2-soliton fission, are observed over the region around the central frequency of soliton, as shown in Fig. 1(e). These fringes suggest the presence of a pulse pair separated in the temporal domain, with a temporal separation inversely proportional to the spectral fringe spacing [3, 26]. These results constitute an interesting effect of external DW-induced soliton fission at optical event horizon, which is, to our knowledge, the numerical demonstration for the first time. Unlike the case of soliton fission induced by third-order dispersion [1, 6], the fission event induced by the external DW pulse do not give rise to emission of Cherenkov radiation. The distance at which the fission occurs corresponds to the point where the collision between both pulses takes place, rather than the distance at which the bandwidth of the evolving N-soliton reaches its maximum. These underlying soliton-DW dynamics indicates that the effective acting-force induced by the collision between external DW pulse and 2-soliton is the dominant perturbation and the cause of the soliton fission.

In order to fully understand the manipulation of soliton fission induced by external DW pulse, we find that the dynamic process of the 2-soliton splitting into a pair of fundamental constituent solitons (the main soliton and fissioned soliton) can be investigated by simply adjusting the peak power of injected DW pulse. The results produced by simulations of Eq. (1) with three different power levels of DW are displayed in Fig. 2. The black curve in Fig.2 demonstrates the fission of the 2-soliton in the case of very small power (P_{DW} = 4 W). Clearly, distinct solitons have separated in time. This temporal separation is the essence of the soliton fission. However, larger P_{DW} can produce a significant increase in the temporal separation between the main soliton and fissioned soliton (the red and blue curves), along with the slight decrease of main soliton power. Adjusting the peak power of external DW pulse we can increase the frequency shift acting on the main soliton. Consequently, the main soliton frequency is red-shifted that leads to a higher value of β_1 and β_2, corresponding to a fundamental soliton with lower peak power and group-velocity. The soliton adiabatically changes its shape for maintaining the fundamental soliton due to the change of β_2 [19].

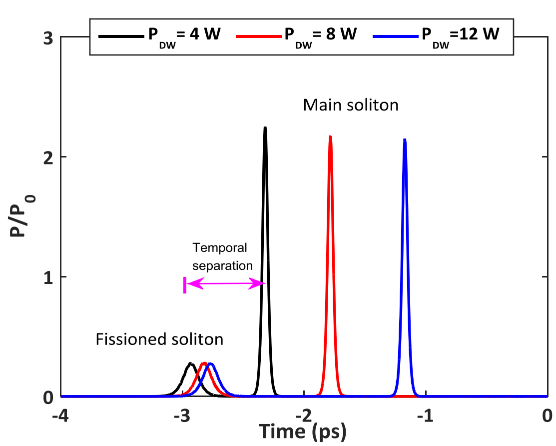

Fig. 2. Generation of frequency-tuned fundamental solitons for the 2-soliton input with relative delay -3 ps at three different values of P_{DW}. The output pulse shape demonstrated that the temporal separation between a pair of constituent solitons can be manipulated by adjusting the peak power of external DW pulse.

Swapping the frequencies of the incident and reflected waves simply changes the dynamics property of optical event horizon [24]. When the initial relative frequency of the external DW pulse is shifted to 172 THz, corresponding to the central frequency of the generated wave (a reflected wave on the soliton; that is, a new spectral component generated by the collision between a higher-order soliton and an external DW) with highest intensity in Fig. 1, the soliton initially trails the external DW pulses temporally by 3 ps for the realization of the collision between them. We can see from Fig. 3 that the reflection process can cause such a strong change in the temporal properties of the soliton as the central frequency of the soliton is blue-shifted by the induced nonlinear interaction, and make the 2-soliton broken into a pair of pulses with different intensities. However, we can clearly identify in Fig. 3(a) that the observed dynamics is opposite to the well-known case of soliton fission induced by higher-order effect (such as higher-order dispersion, Raman scattering and self-steepening), in which the main soliton with larger peak power can acquire an additional acceleration, and then experiences a larger temporal advance with respect to the fissioned soliton with smaller amplitude. The notable qualitative result from the numerical model confirms that the interacting force of external DW on the soliton is the physical origin of the fission event and whether the main soliton leads or trails the fissioned soliton can be governed by changing the chosen initial frequency of the external DW. Given that we have established that its external acting-force is the dominant perturbation in our system, we develop an analytic description to obtain deeper insight.

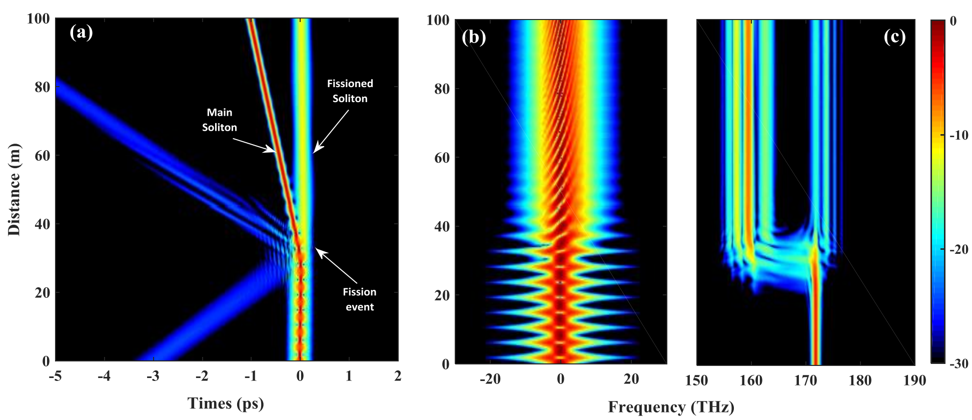

Fig. 3. Dynamics of higher-order soliton fission induced by the external DW (P_{DW} = 4 W) at optical event horizon when the initial central frequency of external DW pulse is shifted to 172 THz. (a) Evolution of the temporal intensity of the interaction between external DW and 2-soliton. (b) and (c) represent evolution of spectral intensity of soliton and external DW, respectively.

3.2 Derivation of the external acting-force perturbation

It is common to write the non-dimensional form of GNLSE to analyze the relative strengths of physical effects governing the optical pulse evolution. We introduce the following dimensionless variables:

$$\xi = \frac{z}{L_D}, \quad T = \frac{t-\beta_{11}z}{T_0}, \quad U(\xi,T) = A(z,T)/\sqrt{P_0} \tag{5}$$

where ξ,T and U correspond to the normalized propagation distance, time, and pulse envelope with respect to the dispersion length L_D, pulse duration T_0 of soliton, and peak power P_0, respectively. We characterize the higher-order soliton by its envelope $U(\xi,T)$, described by a NLSE that is centered at ω_0 [30]:

$$i\frac{\partial U}{\partial \xi} - \frac{\mathrm{sign}(\beta_2)}{2}\frac{\partial^2 U}{\partial T^2} + N^2 |U|^2 U = N^2 \cdot \frac{P_1}{P_0} |U_g|^2 U \tag{6}$$

where $N^2 = L_D/L_{NL}$ denotes the order of the soliton, representing the relative balance of the characteristic length scales for linear dispersion L_D and the nonlinear Kerr effect L_{NL} and thus determining the pulse propagation regime. The terms on the left-hand side of the Eq. (6) are related to soliton propagation. The right-hand side is reserved for perturbations, where the magnitude of the dimensionless perturbation parameter governs the conditions to trigger soliton fission. If the magnitude of these parameters exceeds a minimum threshold, a higher-order soliton will break apart. Otherwise, the higher-order soliton remains intact and maintains recurrent breathing behavior throughout when the parameter is below the threshold.

The envelope of the generated wave is described by a NLSE that is centered at ω_0. Assuming that the generated wave is spectrally narrow and small ($|U_g| << |U_{DW}|$) [33] and thus only the first-order dispersion is included, we obtain

$$i\frac{\partial U_g}{\partial \xi} + i(\beta_{12}-\beta_{11})\frac{\partial U_g}{\partial T} + \gamma P_0 \sqrt{\frac{P_{DW}}{P_1}} U^2 U_{DW}^* e^{i(\Delta\nu)T} = 0\,, \tag{7}$$

where the detuning $\Delta\nu$ is defined as $\omega_{DW} + \omega_g - 2\omega_0$, ω_g is the frequency of the generated pulse. Equation (7) can be easily solved as following:

$$U_g(T,\xi) = i\gamma P_0 \sqrt{\frac{P_{DW}}{P_1}} \int_0^z U^2 U_{DW}^* e^{i\Delta\nu[\tau+(\beta_{11}-\beta_{12})s]} \mathbf{d}s \tag{8}$$

with $\tau = T - (\beta_{11}-\beta_{12})z$. The integral in Eq. (8) can be calculated explicitly:

$$\begin{aligned} U_g(T,\xi) &= \frac{i\gamma P_0}{\beta_{11}-\beta_{12}} \sqrt{\frac{P_{DW}}{P_1}} \int_{T-(\beta_{11}-\beta_{12})z}^{T} \mathrm{sech}^2\left(\frac{t'}{T_0}\right) e^{i\Delta\nu t'} \mathbf{d}t' \\ &\approx \frac{i\gamma P_0}{|\beta_{11}-\beta_{12}|} \sqrt{\frac{P_{DW}}{P_1}} \int_{-\infty}^{\infty} \mathrm{sech}^2(t'/T_0) e^{i\Delta\nu t'}\, \mathrm{d}t' \\ &= \frac{\gamma P_0 \pi \Delta\nu T_0}{|\beta_{11}-\beta_{12}|} \sqrt{\frac{P_{DW}}{P_1}} \sinh^{-1}\left(\frac{\Delta\nu T_0 \pi}{2}\right). \end{aligned} \tag{9}$$

The generated wave energy is maximized when the condition of $\Delta\nu = 0$ is satisfied, i.e., $\omega_{DW} + \omega_g = 2\omega_0$, which indicates the energy conservation for degenerate four-wave-mixing (FWM) process involving the soliton, DW, and generated wave. This is called phase sensitive process, which generates new spectral bands the frequency of which directly depend on the soliton phase. Meanwhile, the phase-matched condition will maximize the nonlinear conversion during the scattering process.

We now focus on the role of external DW pulse for which we have introduced an external acting-force perturbation κ_{NL} :

$$\kappa_{NL} = \frac{L_{NL}}{L_{AF}} = \frac{4\gamma T_0^2 N^2 P_{DW}}{|\beta_{11} - \beta_{12}|^2} = 4\gamma N^2 P_{DW} L_W^2 \, , \tag{10}$$

where $L_W = T_0 / |\beta_{11} - \beta_{12}|$ denotes the walk-off length used as a measure of group-velocity mismatch between the soliton and the external DW pulse. Due to the cross-phase modulation, we have also defined the characteristic acting-force length L_{AF}, which is generated dynamically from the interaction between the N-order soliton (N-soliton) and the external DW pulse (peak power P_{DW}, the frequency detuning $\Delta\nu$ and traveling velocity β_{11}^{-1}). The physical interpretation of κ_{NL} is the relative nonlinear phase shift resulting from the Kerr effect compared with acting-force per unit length.

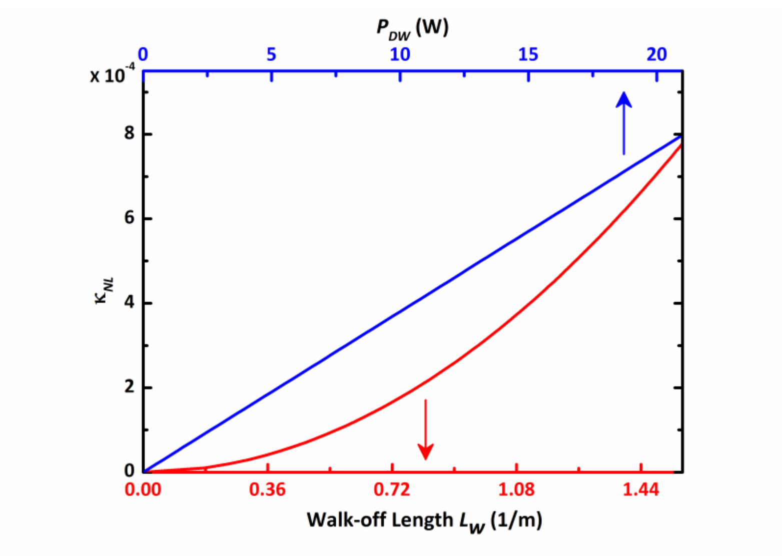

Fig. 4. Analysis of the external acting-force perturbation generated from the collision between the 2-soliton and the external DW pulse. Here we show the external DW pulse power PDW and the walk-off length L_W versus the κ_{NL} perturbation indicating the different scaling for each ($\kappa_{NL} \propto P_{DW}$ and $\kappa_{NL} \propto L_W^2$).

In terms of characteristic physical scaling, we see that $\kappa_{NL} \propto N^2$, with the external DW pulse acting as constants. This means that larger soliton numbers require smaller perturbations to break up the higher-order soliton. Correspondingly, the minimum threshold to break a soliton decreases with soliton number N. First of all, it is worth highlighting that the exact scaling of κ_{NL} depends on the magnitude of group-velocity mismatch between both of the interacting pulses (that is, the soliton and the external DW pulse) (see red pentagram curve in Fig. 4). This feature leads to a walk-off effect that plays an important role in the description of the nonlinear phenomena involving two closely spaced optical pulses. More specifically, the nonlinear interaction between two optical pulses disappears during the period when the pulse travelling faster completely walks through the one moving slowly. However, we know that in a dispersive medium, such as an optical fiber, a change in velocity must be associated with a change in frequency [19, 21]. This behavior suggests that we can use the frequency of the external DW pulse as an effective controlling variable for all-optical manipulation of optical light pulses. Secondly, it should be noting that κ_{NL} depends on the peak power of the input DW pulse, as can be seen from blue curve in Fig. 4. Changing the amplitude of the external DW pulse can directly affect the magnitude of perturbation, and then can manipulate the fission event for the higher-order soliton. However, among all these parameters, the peak power of the external DW pulse can be simply adjusted and is the most efficient controlling variable. Besides, it is worth noting that any change of the external DW pulse parameters that can be induced to manipulate the higher-order soliton fission is constrained by the requirement to enable the scattering process at optical event horizon.

3.3 Temporal analog of multimode waveguide structure

We now ask what happens when a moderate weak external DW pulse is located between two second-order solitons that both move at the same speed and separate in time. The input in this case will consist of two well separated 2-solitons and DW pulse launched between them, such as

$$A(t) = 2\sqrt{P_0}\,\mathrm{sech}[(t+t_1)/T_0] + \sqrt{P_{DW}}\,\mathrm{sech}(t/T_1)\exp(-i\Delta\omega t) + 2\sqrt{P_0}\,\mathrm{sech}[(t-t_1)/T_0]\ . \tag{11}$$

If the central frequency of DW is located well within the resonance band, we will be able to observe its multiple reflections from the 2-solitons. As shown from Fig. 5(a), when the peak power of the DW is relatively weak, the trajectories and the spectral characteristics of the two solitons remain almost the same, but the injected DW pulse can break up into multiple pulses during the collision process and get almost completely trapped within the solitonic well. This phenomenon is strikingly similar to an optical beam undergoing total internal reflection at a dielectric interface [34]. Therefore, a pair of second-order solitons with the same velocity but separated in time act as the temporal analog of planar dielectric multimode waveguides, including having a finite number of supported modes.

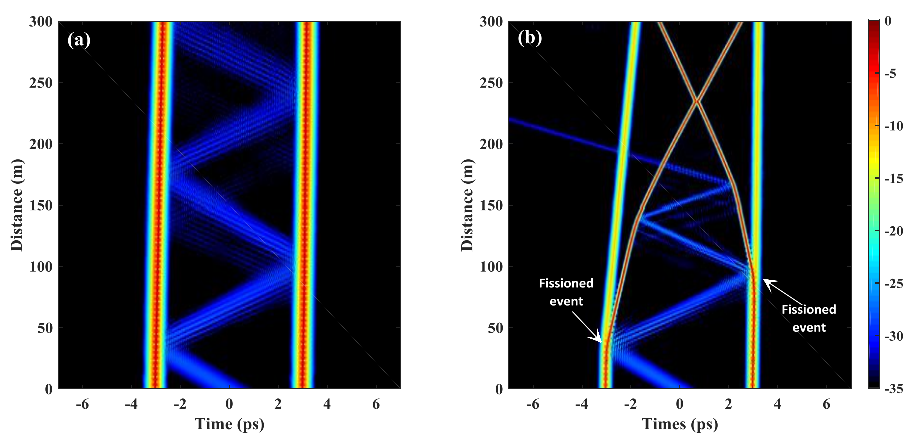

Fig. 5. Temporal evolution of solitonic well created by two co-propagating 2-solitons and external DW pulse bouncing between them. (a) DW with relative weak amplitude (P_{DW} = 1 W); (b) DW of considerable intensity (P_{DW} = 4 W).

A moderate increase in the amplitude of the external DW pulse (the amplitude is increased twofold) leads to a decomposition of the 2-soliton that manifests itself by a frequency continuum in the spectral domain. The temporal analog of multimode waveguides structure will be damaged. Figure 5(b) demonstrates an outcome of the simulation in which an intense DW with its amplitude 2 times larger than that of DW in Fig. 5(a) collides with a 2-soliton, causing the breakup of the 2-soliton. There is a major difference from the case of solitonic well where the relative weak DW bounces between the 2-solitons: both of the more energetic main solitons, originating from the external-DW-induced higher-order-soliton fission, can acquire an additional deceleration and acceleration, respectively. The collision between them can eventually be caused by the effective attraction due to multiple scattering of external DW pulse.

4. Conclusion

In conclusion, we have demonstrated that the effective acting-force formed by the interaction of higher-order soliton with external DW at optical event horizon can induce soliton fission with both numerical and analytical approaches. During the collision between second-order soliton and external DW pulse, the adiabatic fission of second-order soliton (i.e. completely splitting into a pair of constituent solitons with very different power levels), will take place. Meanwhile, the injected DW pulse becomes polychromatic due to an effective periodical potential provided by the periodic recurrence structure of higher-order solitons. The temporal separation between both fundamental constituent solitons, which is the essence of soliton fission, can be controlled by adjusting the power of the external DW. The more energetic main soliton is advanced/trailed in time depending on the chosen initial frequency of input DW pulse. We develop an analytic formalism describing the external acting-force perturbation. It is directly related to the peak power of the external DW pulse and the walk-off length because of a mismatch in the group-velocity of two injected pulses. Additionally, two second-order solitons with DWs trapped between them can create an effective higher-order solitonic well, which act as the temporal analog of planar dielectric multimode waveguides. When the external DW power is increased, it can be observed that the temporal analogy of multimode waveguide structure is destroyed, leading to the collision of the more energetic main solitons after the fission event occurs. These results elucidate the fundamental physical scaling and dynamics of soliton fission in nonlinear dispersive medium and can find applications in improved supercontinuum sources.

Funding

This work was supported in parts by the European Horizon 2020-MSCA-IF under Grant 701493, the National Natural Science Fund Foundation of China (NSFC) (Grants 61505124, 61505122, and 61606166), the Science and Technology Planning Project of Guangdong Province of China under Grant 2016KCXTD006, Shenzhen Government’s Plan of Science and Technology (Grant No. JCYJ20170302153731920), and Natural Science Foundation of SZU (Grant No. 2017022).