СДІТ

ПРОБЛЕМИ ПРИЙНЯТТЯ РІШЕНЬ
І УПРАВЛІННЯ В ЕКОНОМІЧНИХ, ТЕХНІЧНИХ,
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CURVATURE COORDINATES TO DESCRIBE THE EXPLOSION OF CHERNOBYL'S REACTOR CORE IN APRIL 1986, USING THE TENSOR EQUATIONS

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Abstracts. This research analyzes the process of the explosion of the reactor core of Chernobyl nuclear plant in April 1986, using the tensor equations. These tensor equations show a movement of a vector in the three dimensional curvature coordinates of time, water flow, and void. The equations shows that this vector moves along the geodesic in the curvature coordinates, which is described by fundamental tensor ($g_{\mu\nu}$), Christoffel symbol ($\Gamma^{\alpha}_{\mu\nu\sigma}$) and Ricci tensor ($R_{\mu\nu}$), where μ, ν, σ, α are suffixes that indicate the coordinates. The solution of the tensor equations shows that the geodesic of the vector has a singular point, which describes a turning point of the reactor core from the normal operation to the explosion, which we reported in our previous articles [1, 2].

Keywords: nuclear power plant, Chornobyl disaster, critical operation mode, regression analysis, void and water environment.

INTRODUCTION

In the previous research [1, 2], we analyzed the physical parameters [3] of nuclear reactor core, which were observed during the time period of five seconds before the explosion of Chernobyl nuclear plant, which happened in April 1986. With the first-order empirical analysis [1], we found that two parameters, water flow and void of the nuclear core, were strongly related to the sudden increase of the reactor power that led to the explosion. And, then, we found that the exponential model with these two parameters can explain the process of the immense power increase [2]. We also found that there was a turning point in the process between the normal operation mode and the explosion.

In this research we analyzed the explosion conditions, using the tensor equations, which show evolution of the three dimensional curvature coordinates of time, water flow, and void.

METHODOLOGY

At first, we assume that there is a scalar field that manages the water flow and void in the reactor core; and then, the derivatives of the scalar field produce a tensor field that is to describe the coordinates evolution in the three dimensional space.

Let S be a scalar field. It can be considered as a function of the three coordinates x^{μ} ($\mu = 0,1,2$). Here $\mu = 0$ corresponds to time (*t*), and $\mu = 1,2$ for coordinates of two parameters (water flow and void). Then

$$S_{\mu} = S_{\lambda} x^{\lambda}_{\mu}.$$

Here $S_{\mu} = \frac{d}{dx^{\mu}} S$, $S_{\lambda} = \frac{d}{dx^{\lambda}} S$, and $x^{\lambda}_{\mu} = \frac{d}{dx^{\mu}} x^{\lambda}$; and, the derivative of a scalar field (S_{μ}) is a covariant vector field by its definition [4].

And then, the movement of the vector in the curvature coordinates of time, water flow and void is described by tensor equations:

$$A_{\mu:\nu} = A_{\mu,\nu} - \Gamma^{\alpha}_{\mu\nu} A_{\alpha}. \tag{1}$$

Here A_{μ} is a vector field. And $\Gamma^{\alpha}_{\mu\nu}$ is the Christoffel Symbol of the second kind, while:

$$\Gamma^{\alpha}_{\mu\nu} = g^{\alpha\lambda} \Gamma_{\lambda\mu\nu},$$

$$A_{\mu,\nu} = \frac{d}{dx^{\nu}} A_{\mu},$$

$$\Gamma_{\lambda\mu\nu} + \Gamma_{\mu\lambda\nu} = g_{\lambda\mu,\nu},$$

$g_{\lambda\mu,\nu} = \frac{d}{dx^{\nu}} g_{\lambda\mu}$: and, $g_{\lambda\mu}$ are[1] fundamental tensors, which determine coordinate system and curvature of the space. The $g_{\lambda\mu}$ also lower and raise suffixes of vectors (contra-variant vector, A^{μ}, and covariant vector, A_{μ}), and which vary from a point to a point in a curvature space [4]. Also, it is noted that ":" denotes the covariant differentiation. The covariant derivative of A_{μ} is $A_{\mu:\nu}$, and its relation to $A_{\mu,\nu}$ is shown in the equation (1).

If we assume that there is no scalar field in the three dimensional space of time, water flow and void, the variables evolve straight in the space. In such a case, the geodesic of the variables is a straight line. On the other hand, if there is a scalar field, the variables move along curvature coordinates. So, in this case, the «curvature» gives the information about the evolvement of the variables.

For a scalar field, S, its derivative is described with the covariant differentiation. And now we consider two differentiations in succession:

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$$S_{\mu:\nu} = S_{\mu,\nu} - \Gamma^{\alpha}_{\mu\nu} A_{:\sigma} = S_{\mu,\nu} - \Gamma^{\alpha}_{\mu\nu} A_{\alpha}.$$

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[1] Here "are" is appropriate to use, because $g_{\lambda\nu}$ are plural because λ and ν vary for time and spatial coordinates.

To deal with the second covariant derivative of a tensor, we use the Riemann-Christoffel tensor (or, covariant tensor): $R^{\beta}_{\nu\rho\sigma} = \Gamma^{\beta}_{\nu\rho,\sigma} - \Gamma^{\beta}_{\nu\rho,\sigma} + \Gamma^{\alpha}_{\nu\sigma}\Gamma^{\beta}_{\alpha\rho} - -\Gamma^{\alpha}_{\nu\rho}\Gamma^{\beta}_{\alpha\sigma}$. With the symmetries of the first and the last suffixes, we get $R^{\mu}_{\nu\rho\mu} = R_{\nu\rho}$, which is called as Ricci tensor:

$$R_{\nu\rho} = \Gamma^{\mu}_{\nu\mu,\rho} - \Gamma^{\mu}_{\mu\rho,\mu} - \Gamma^{\mu}_{\nu\rho}\Gamma^{\sigma}_{\mu\sigma} + \Gamma^{\mu}_{\nu\sigma}\Gamma^{\sigma}_{\rho\mu} . \tag{2}$$

These $\Gamma^{\alpha}_{\mu\nu}$ and $R_{\nu\rho}$ are indicators of the curvature of the physical coordinates.

In the curved space (Riemann space), we can also calculate the distance ds between a point x^{μ} and a neighboring point $x^{\mu} + dx^{\mu}$, by:

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} .$$

This gives the geodesic of the movement of a vector along the curvature coordinates.

RESULTS

Tensor equations of three dimensional curvature coordinates

According to the theory of the curvature tensors [4], if there is only one scalar field with no other physical field, the curvature space is empty; and, it is described by:

$$R_{\mu\nu} = 0 . \tag{3}$$

Then we will calculate the geodesic of the movement of the vector by:

$$ds^2 = g_{00}(dx^0)^2 - g_{11}(dx^1)^2 - g_{22}(dx^2)^2 . \tag{4}$$

Here time (t) and two coordinates (x and y) form a space, i.e., $t = x^0$, $x = x^1$, $y = x^2$, where x^0, x^1 and x^2 are contra-variant vectors, and g_{00}, g_{11}, g_{22} are fundamental tensors that lower the suffixes of the contra-variant vectors, to convert them to covariant vectors[2].

And then, we assume a static coordinate system, where $g_{\mu\nu}$ are constant in time. Therefore, $g_{\mu\nu,0} = 0$, where μ and ν are 0, 1, 2 (while, 0 is for time, and 1 and 2 are coordinates for water flow and void).

Where space is not flat, a vector of the variables moves along the curvature coordinates. This curvature coordinate system is nonlinear; so it is difficult to find solutions. In order to overcome this difficulty, we use spherically symmetric field to get solutions fairly easily. It is Schwarzchild' special solution [4]:

$$ds^2 = U dt^2 - V dr^2 - W r^2 d\theta^2 . \tag{5}$$

[2] In order to describe a physical space, we need to use covariant vectors.

Here U, V, W are functions of r only. For replacing the equation (5), we use the following equation:

$$ds^2 = e^{2\nu}dt^2 - e^{2\lambda}dr^2 - r^2 d\theta^2 . \qquad (6)$$

From the equation (5), we determine the values of $g_{\mu\nu}$ as follows:

$$g_{00} = e^{2\nu},\ g_{11} = -e^{2\lambda},\ g_{22} = -r^2, \text{ and } g_{\mu\nu} = 0 \text{ for } \mu \neq \nu .$$

And, then, $g^{00} = e^{-2\nu}$, $g^{11} = -e^{-2\lambda}$, $g^{22} = -r^{-2}$, and $g^{\mu\nu} = 0$ for $\mu \neq \nu$.

Then, we calculate the Christoffel symbols as follows:

$$\Gamma^1_{00} = g^{11}(\Gamma_{001} + \Gamma_{001}) = 2g^{11}g_{00,1} = 2e^{-2\lambda}(e^{2\nu})_{,r} = \nu' e^{2\nu - 2\lambda};$$

$$\Gamma^0_{10} = g^{00}(\Gamma_{001} + \Gamma_{001}) = 2g^{00}g_{00,1} = 2e^{-2\nu}(e^{2\nu})_{,r} = \nu' e^{2\nu - 2\nu} = \nu';$$

$$\Gamma^0_{11} = g^{11}(\Gamma_{111} + \Gamma_{111}) = 2g^{11}g_{11,1} = -2e^{-2\lambda}(-e^{2\lambda})_{,r} = \lambda' e^{2\lambda - 2\lambda} = \lambda';$$

$$\Gamma^2_{12} = g^{22}(\Gamma_{221} + \Gamma_{221}) = 2g^{22}g_{22,1} = 2(-r^{-2})(-r^2)_{,r} = r^{-1};$$

$$\Gamma^1_{22} = g^{11}(\Gamma_{221} + \Gamma_{221}) = 2g^{11}g_{22,1} = 2e^{-2\lambda}(-r^2)_{,r} = -re^{-2\lambda} .$$

With these Christoffel symbols rewrite the equation (2) as follows:

$$R_{00} = \left(-\nu'' + \lambda'\nu' - \nu'^2 - \frac{2\nu'}{r}\right)e^{2\nu - 2\lambda}; \qquad (7)$$

$$R_{11} = \nu'' - \lambda'\nu' + \nu'^2 - \frac{2\lambda'}{r}; \qquad (8)$$

$$R_{22} = (1 + r\nu' - r\lambda')e^{-2\lambda} - 1 . \qquad (9)$$

Here $\nu' = d\nu/dr$, $\nu'' = d^2\nu/dr^2$, and $\lambda' = d\lambda/dr$.

From the equation (3),

$$R_{00} = R_{11} = R_{22} = 0 . \qquad (10)$$

From the equations (7), (8) and (10):

$$\lambda' + \nu' = 0 .$$

For large values of r, the space must become approximately flat; so, when $r \to \infty$, $\lambda \to 0$ and $\nu \to 0$.

From the equations (9) and (10): $(1 + 2r\nu')e^{2\nu} = 1$, and then:

$$(re^{2\nu})' = 1 . \qquad (11)$$

By integrating the equation (11), we get:

$$re^{2\nu} = r - 2m .$$

Here m is a constant of integration.

Because $g_{00} = e^{2\nu}$, we get:

$$g_{00} = 1 - \frac{2m}{r}. \tag{12}$$

From the equations (4), (6), and (12), we get the equation to calculate the geodesic of the movement of a vector along the curvature coordinates:

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\theta^2. \tag{13}$$

The equation (13) becomes singular at $r = 2m$, because then $g_{00} = 0$ and $g_{11} = -\infty$.

Estimation of the magnitude of the scalar field

Thus, $r = 2m$ is the critical radius of the geodesic equation (13). From our previous research [2], we assume that the value of $r = 40$. Now, we neglect the angular component, $x^2 = \theta$, therefore the critical length of r occurs when the void (%) becomes the maximum, while the water flow (m^3/sec) becomes minimum. The critical value of the void is 40 (%).[3]

From our previous research [2], the water flow and void influenced the reactor power. With the total output energy (Mega Joules) as the dependent variable, the relations are shown in the following equation:

$$TotalEnergy = 1903.8 + \exp(11.809 - 3.2934 MCPflowrate + 0.7552 void). \tag{14}$$

As far as we neglected the angular component, the critical length of *r* takes place when the void acquires its critical (maximum) value, and the water flow approaches its minimum level. With keeping this in mind, equation (14) allows to compute the total energy that is to be released in critical cases using the values of MCP flow rate and of the void. From the equation developed in [2], it was shown that the 40 percent of void is leading to approximately $5.0 \cdot 10^4$ Mega Joules of the total output kinetic energy of the reactor.

CONCLUSIONS AND RECOMMENDATIONS

Thus, it was shown that the curvature coordinates, fundamental tensor ($g_{\mu\nu}$), Christoffel symbol ($\Gamma^{\alpha}{}_{\mu\nu\sigma}$) and Ricci tensor ($R_{\mu\nu}$) appear as indicators of development the processes in time, which describe the curvature of the coordinates of time, water flow and void, along which a vector of the variables considered evolves. The track of the movement of a hypothetical particle was described by the tensor equation (4), $ds^2 = g_{00}(dx^0)^2 - g_{11}(dx^1)^2 - g_{22}(dx^2)^2$, given ds is a distance of the vector's movement, x^0 is the coordinate of time, x^1 and x^2 are coordinates for water flow and void. And then, the coordinates are converted into the symmetrical two dimensional curvature coordinates; after that the fundamental tensors were calculated.

[3] Here, x^2 does not mean "squared x". It is the suffix for the third coordinate, θ.

The tensor equations considered showed qualitatively and quantitatively that there is a minimum distance to describe evolution of the process of possible explosion, as in the minimum radius of three-dimensional curvature coordinates. The order of magnitude of the reactor power is approximately in the order of 10^{-8} (Mega Joules^{-1}), at the singular point in the curvature space.

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