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AGGREGATE LIQUIDITY SHORTAGES, IDIOSYNCRACIC LIQUIDITY SMOOTHING AND BANKING REGULATION

Wolf Wagner

Comments and suggestions would be welcomed by the author.

e-mail: w.wagner@cerf.cam.ac.uk

Aggregate Liquidity Shortages, Idiosyncratic Liquidity Smoothing and Banking Regulation*

Wolf Wagner[†]

CERF, University of Cambridge and Tilburg University

Abstract

This paper develops a model of banking fragility driven by aggregate liquidity shortages. Inefficiencies arise because liquidity smoothing across banks breaks down when there is such a shortage, causing an unnecessary and value-reducing transfer of assets between banks. We find that a standard Lender of Last Resort policy is ineffective in restoring efficiency as it leads to offsetting changes in the banks' supply of liquidity. In contrast, subsidizing the purchase of assets from troubled banks increases welfare by improving the banks' liquidity holdings. The first best, however, is achieved by redistributing liquidity from healthy to troubled banks in a crisis.

JEL classification: G21, G28

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1 Introduction

Banking crises are frequent and their costs are substantial: around two thirds of the member states of the IMF have experienced problems with their banking system in the last 25 years and have spent on average more than 10% of their GNP on resolving them (Daniel, 1997, and Honohan and Klingebiel, 2000). This fact has traditionally been explained by the liquidity of banks' liabilities: because demand deposits can be withdrawn at any time, they can give rise to self-fulfilling runs, which can force the costly liquidation of assets even at solvent banks. However, recent experiences with banking failures, at least in developed countries, suggest that bank runs are of lesser importance, a fact that can be attributed to the existence of deposit insurance systems in most countries.

This paper develops a model where costs in the banking sector arise from situations in which banks can no longer finance ongoing projects, rather than from bank runs. In our model, banks have uncertain liquidity inflows from their investments in firms, for example due to non-performing loans. Banks have a demand for liquidity because ongoing projects at these firms require new funds. In normal times, any liquidity needs can be satisfied through borrowing at the interbank market. However, when there is an aggregate liquidity shortage, interbank lending breaks down. Besides the projects that have to be liquidated because there is an insufficient amount of liquidity at the aggregate level, some projects also have be transferred to banks that have no liquidity shortage. This is value-reducing as it leads to the breakup of existing lending relationships.[1] The banking sector is fragile in that a small reduction in aggregate liquidity can lead to the breakup of many relationships.

Although liquidity is a public good as it avoids aggregate liquidity shortages, banks have private incentives to invest in liquidity in the hope of being able to buy up assets cheaply in a crisis. However, a bank cannot fully reap the social benefits of providing liquidity because the purchase of assets goes along with the breakup of the original bank-firm relationship. Banks therefore underinvest in liquidity. The inefficiency that arises is thereby solely related to losses from the transfer of assets; the liquidation of assets in a liquidity crisis is a necessary consequence of investing in high return but illiquid assets.

We explore the role of regulation in improving efficiency. We first show that liquidity provision by the central bank (Lender of Last Resort policy) is ineffective. This is because banks, anticipating the central bank's action, reduce their holdings of liquidity correspondingly, which completely neutralizes the impact of the liquidity injection. Even if this moral hazard could be controlled (for example through lending at punitive rates or

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[1]Empirical evidence suggests that such losses can be substantial: James (1991) has estimated for the U.S. that losses on bank assets sold to other institutions are around 30%.

through appropriate capital requirements), liquidity provision by the central bank is still not an effective tool. This is because it can only avoid a costly asset transfer by eliminating liquidity crises altogether, which, however, would imply an inefficiently low investment in risky assets.

We show next that sponsoring the purchase of assets from troubled banks in a crisis, commonly used by regulators to incentivize healthy banks to buy up assets, is welfare-improving: it increases banks' return from holding liquidity and thus motivates them to invest more in liquidity. Finally, we examine intervention that redistributes liquidity from healthy to troubled banks in a crisis and find that such a redistribution may achieve the first best. This is because it allows as many projects as possible to be continued at the originating banks, thus avoiding asset transfers altogether. Moreover, if appropriately designed, a liquidity redistribution can also provide the correct incentives for banks to invest in liquidity.

The remainder of the paper is organized as follows. The next section reviews the literature. Section 3 describes the model. The inefficiency of banks' liquidity provision and the resulting role for regulation are addressed in Section 4. Section 5 studies the scope for reducing the transfer of assets in a crisis. The final section concludes.

2   Related Literature

While in the present model liquidity needs arise from firms and there are no bank runs, most of the literature on banking crises has emphasized that bank runs can undermine the efficient provision of liquidity for consumers. The seminal work of Diamond and Dybvig (1983) has shown that intermediaries can provide liquidity insurance by subsidizing consumers that are experiencing high liquidity needs. However, this exposes them to runs. Allen and Gale (1998) have pointed out the principal optimality of bank runs in terms of achieving optimal risk sharing for consumers under the condition that the liquidation of assets in a crisis is costless. When liquidation is costly, or when assets can be traded at a secondary market, inefficiencies arise due to a coordination problem among depositors. They show that, in the presence of nominal contracts, the central bank can then improve welfare by injecting liquidity in order to avoid runs.

Other contributions have pointed out different inefficiencies, such as from contagion or the failure of the interbank market. Contagion has been shown to arise among banks when bank runs signal information about other banks (Chari and Jagannathan, 1988), banks have deposits at each other (Allen and Gale, 2000) or runs can change fundamentals by creating excess demand for liquidity (Diamond and Rajan, 2005). Bhattacharya and Gale

(1987) show in a variant of the Diamond-Dybvig model that, in the absence of aggregate uncertainty, an interbank market cannot implement the socially optimal allocation due to a coordination problem. They argue that central banks might be better equipped to carry out interbank risk sharing. Flannery (1996) develops a model where the interbank market does not function properly as it becomes cautious in times of crisis. The present paper develops another channel of an interbank market failure: in an aggregate liquidity shortage interest rates are very high, which would imply large repayments for borrowing banks and thus gives them an incentive to divert funds rather than to invest them in projects (this is basically the debt overhang problem of Myers (1977) where equity holders forego profitable investments when there is debt outstanding).

Similar to the present paper, Holmström and Tirole (1998) have developed a model where liquidity needs arise because firms require funds at an intermediate date. In their paper there is entrepreneurial moral hazard, which restricts firms' external financing and gives rise to a demand for liquidity. They identify a scope for government intervention as the private sector cannot self-insure against aggregate shocks. By contrast, in the present paper the government cannot remedy aggregate liquidity shortages and inefficiencies are due to idiosyncratic liquidity risks in the presence of aggregate uncertainty (idiosyncratic risk is not an issue in Holmström and Tirole as intermediaries can fully pool risk). The role of intervention arises then because in an aggregate crisis the interbank market stops functioning and can no longer smooth idiosyncratic liquidity needs.

An important difference between the present paper and most of the above contributions is that in our model there is no scope for liquidity provision by the central bank. Rather, a regulator should redistribute the *existing* liquidity in the banking system. Gorton and Huang (2004) have developed a model in which financing is restricted by agency problems at the level of the firm, as in Holmström and Tirole (1998). As in the present paper, liquidity is supplied by banks in order to buy up assets cheaply in a crisis (however, their paper views the private provision of liquidity as inefficient because it implies foregone returns from investing while in the present paper private liquidity is beneficial but not sufficiently provided). They show that subsidizing banks with bad projects, i.e., through bail-outs, can be optimal in a crisis. Such bail-outs, which can be understood as a liquidity redistribution in a crisis, are not optimal in the present paper as they harm banks' ex-ante incentives to avoid crises. Rather, the liquidity redistribution has to occur at high costs.

3 Setup

There is a continuum of banks of unit mass. Banks are owned by risk neutral households and invest on their behalf in firms. Households do not directly invest in firms because banks can make use of economies of scale, for example because of fixed monitoring costs or because firms' projects are indivisible. For concreteness, we assume that each bank collects one unit of funds from households.

There are three dates. At date 0, a bank can invest in a storage technology (liquidity) and in a risky asset. The storage technology simply transfers one unit of funds from the current period to the next. By contrast, the return on the risky asset materializes over two periods. At the intermediate date 1, each unit of the risky asset (a 'project') gives an uncertain intermediate return η_i, where $i \in [0, 1]$ is the bank index. The intermediate return consists of an aggregate component ε but also of a bank-specific component ε_i, arising because banks are specialized (e.g., regionally or into industries). The aggregate shock ε is uniformly distributed on $[-1/2, 1, 2]$ with density $\phi(\varepsilon) = 1$. The bank shock ε_i takes with equal probability the values s and $-s$ with $0 < s < 1$ and is assumed to be independently distributed across banks. Given these definitions, the expected value of the intermediate return, $\eta_i = \varepsilon + \varepsilon_i$, is zero (negative values can be interpreted as unexpected liquidity needs from the project).

In order to be continued, a project requires a liquidity injection of $l > 0$ at date 1 (as in Holmström and Tirole, 1998). If this injection is not provided, the project becomes worthless, i.e., returns zero at date 2. If the injection is provided, and the project is continued at the originating bank, it yields R at date 2 and returns the liquidity injection l. On the other hand, if the project is continued at another bank, the project returns only γR $(0 \leq \gamma < 1)$ plus the liquidity injection. $1 - \gamma$ is hence the value loss from transferring a project to another bank, which we interpret as arising from the loss of the bank-firm relationship.

At date 1, after the intermediate returns have materialized, banks can smooth their liquidity needs by trading liquidity at an interbank market. A bank's holding of liquidity L_i (before borrowing and lending) consists of investment in liquidity at date 0 plus the intermediate return on the risky asset

$$L_i = 1 - X_i + \eta_i X_i \tag{1}$$

where $X_i \in [0, 1]$ denotes the date 0 investment in the risky asset. By contrast, the bank's demand for liquidity L_i^D is given by the liquidity needed to continue its projects: $L_i^D = X_i l$. Thus the bank's liquidity needs $L_i^D - L_i$ have an idiosyncratic component.

From (1) we can derive the total (aggregate) amount of liquidity $L := \int_0^1 L_i di =$

$1 - X + \varepsilon X$, where $X := \int_0^1 X_i di$ is the aggregate investment in the risky asset (the bank specific shocks ε_i cancel out in L by the law of large numbers). The aggregate demand for liquidity is given by $L^D = \int_i L_i^D di = Xl$.

When $L \geq L^D$, there is sufficient aggregate liquidity to finance all projects. Competition ensures then that the interest rate on lending in the interbank market is zero, since this is the return of the storage technology. Thus, banks can fully insure their liquidity needs at zero cost. By contrast, when $L < L^D$, there is insufficient liquidity to finance all projects in the economy. Banks with liquidity needs compete then for the scarce liquidity. Given that the value of the project is zero in the absence of a liquidity injection, this would require the interbank interest rate to rise to a level that makes the return from financing assets (net of borrowing costs) equal to zero (if this were not the case, banks that are rationed would offer to pay a slightly higher interest rate, which would strictly increase their profits). We assume in the following that such financing of assets at a zero net return does not take place. Appendix A formalizes this as being due to borrowing banks having an incentive to appropriate funds rather than investing them because of the high debt burden implied by high interest rates.[2]

We refer to a situation of $L < L^D$ as a liquidity crisis. Such a crisis occurs when $L = 1 - X + \varepsilon X < L^D = Xl$ or, from rearranging, if the aggregate shock ε is lower than $\widehat{\varepsilon}$ defined by

$$\widehat{\varepsilon} := 1 + l - 1/X \tag{2}$$

Given that ε is uniformly distributed on $[-1/2, 1/2]$, the probability of a liquidity crisis $\pi := \Pr(\varepsilon < \widehat{\varepsilon})$ can be expressed as

$$\pi = \widehat{\varepsilon} + 1/2 = 3/2 + l - 1/X \tag{3}$$

The date 2 returns are as follows. When there is no liquidity crisis at date 1 (i.e., $\varepsilon \geq \widehat{\varepsilon}$), banks with liquidity needs (if they exist) can borrow their required amounts from the interbank market. Thus, all projects are financed at date 1 and continued at the originating bank. Since any excess liquidity $L_i - L_i^D$ (which may be positive or negative) is transferred at zero interest, a bank's return at date 2, $W_i^0(\eta_i)$, is simply the return on its risky asset plus liquidity holdings at date 1:

$$W_i^0(\eta_i) = RX_i + L_i(\eta_i) \tag{4}$$

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[2]Alternatively, imperfections that lead to a breakdown of interbank lending in crises could also arise from the inability of the interbank market to observe the financial health of banks (Berger et. al., 2000, provide empirical evidence for a general informational advantage of regulators in that respect), the interbank market becoming cautious in times of a crisis (Flannery, 1996) or because of coordination problems (e.g., Freixas, Parigi, Rochet, 2000).

When there is a liquidity crisis at date 1 (i.e., $\varepsilon < \widehat{\varepsilon}$), we presume that each bank holds an amount of liquidity such that when it experiences a negative idiosyncratic shock on its assets, it does not itself have sufficient liquidity to finance all its assets, whereas if it experiences a positive shock, it does ('partial insurance'). A bank that experiences a negative shock ('unlucky bank') will then use all its liquidity to finance as many of its own projects as possible. The remaining projects then become worthless for the bank, either because the bank cannot inject liquidity or because it sells projects to other banks. The latter is because, since there is an aggregate shortage of liquidity and the value of a project without liquidity injection is zero, the price of assets that are sold is driven to zero. The date 2 return, $W_i^1(\eta_i)$, of such a bank consists then of the return on the maximum amount of assets that can be financed with its own liquidity (L_i/l), plus liquidity holdings at date 1 :

$$W_i^1(\eta_i) = RL_i(\eta_i)/l + L_i(\eta_i) \tag{5}$$

In contrast, a bank that receives a positive idiosyncratic shock ('lucky bank') first uses its liquidity to finance its own projects and then uses the remaining liquidity to acquire assets from banks that have experienced a negative shock. Given remaining liquidity $L_i - X_i l$, it can purchase (at a zero price) and finance $(L_i - X_i l)/l = L_i/l - X_i$ assets, giving a return of γR. The bank's return is then

$$W_i^2(\eta_i) = RX_i + L_i(\eta_i) + \gamma R(L_i(\eta_i)/l - X_i) \tag{6}$$

The bank's total expected return, W_i, is given by

$$W_i = \int_{\widehat{\varepsilon}}^{1/2} \frac{W_i^0(\varepsilon - s) + W_i^0(\varepsilon + s)}{2} \phi(\varepsilon)d\varepsilon + \int_{-1/2}^{\widehat{\varepsilon}} (\frac{1}{2}W_i^1(\varepsilon - s) + \frac{1}{2}W_i^1(\varepsilon + s))\phi(\varepsilon)d\varepsilon$$

Using equations (4)-(6) this simplifies to

$$W_i = RX_i + (1 - X_i) - \frac{1}{2}\int_{-1/2}^{\widehat{\varepsilon}} R[X_i - L_i(\varepsilon - s)/l]d\varepsilon + \frac{1}{2}\int_{-1/2}^{\widehat{\varepsilon}} \gamma R[L_i(\varepsilon + s)/l - X_i]d\varepsilon \tag{7}$$

Denoting with $\overline{\varepsilon} = E[\varepsilon \mid \varepsilon < \widehat{\varepsilon}]$ the expected level of the aggregate shock in a crisis and using $\pi = \widehat{\varepsilon} + 1/2$ we can write (7) as

$$W_i = RX_i + (1 - X_i) - \frac{\pi}{2}R[X_i - L_i(\overline{\varepsilon}(\pi) - s)/l] + \frac{\pi}{2}\gamma R[L_i(\overline{\varepsilon}(\pi) + s)/l - X_i] \tag{8}$$

where

$$\overline{\varepsilon}(\pi) = (\pi - 1)/2 \tag{9}$$

(9 follows from $\overline{\varepsilon}(\pi) = \int_{-1/2}^{\widehat{\varepsilon}} \varepsilon\phi(\varepsilon)d\varepsilon / \int_{-1/2}^{\widehat{\varepsilon}} \phi(\varepsilon)d\varepsilon$). From (8) we have that W_i consists of the total expected return when there is no liquidity crisis, $RX_i + (1 - X_i)$, minus the

expected foregone returns from having to sell or liquidate assets when an unlucky bank in a liquidity crisis, $\frac{\pi}{2}R[X_i - L_i(\overline{\varepsilon} - s)/l]$, plus the expected gains from buying up assets when a lucky bank in a crisis, $\frac{\pi}{2}\gamma R[L_i(\overline{\varepsilon} + s)/l - X_i]$.

Note that the banking system displays fragility in the sense that a small reduction in liquidity (from $L = L^D$ to $L < L^D$) can lead to a breakdown of lending and the forced sale of assets at all banks with liquidity needs, even though the aggregate liquidity shortage is only infinitesimal.

4 Inefficiency of Banks' Supply of Liquidity and Regulation

In an unregulated economy, each bank will choose X_i in order to maximize its expected return W_i in (8), subject to its liquidity holdings given by equation (1) and taking as given the amount of aggregate investment X (and thus π and $\overline{\varepsilon}$). From (8) the bank's FOC is given by

$$\partial W_i/\partial X_i = R - 1 - \frac{\pi}{2}R[1 - (-1 + \overline{\varepsilon} - s)/l] + \frac{\pi}{2}\gamma R[(-1 + \overline{\varepsilon} + s)/l - 1] = 0 \qquad (10)$$

Note that the FOC does not specify a level of X_i but rather a level of aggregate investment X (because $\pi = \pi(X)$ and $\overline{\varepsilon} = \overline{\varepsilon}(\pi)$) for which an individual bank is indifferent between investing in the risky asset and holding liquidity.[3] In contrast, the socially efficient amount of investment maximizes total welfare in the economy, which consists of the sum of the returns of all banks

$$W = \int_i W_i di = RX + (1 - X) - \frac{\pi}{2}R[X - L(\overline{\varepsilon} - s)/l] + \frac{\pi}{2}\gamma R[L(\overline{\varepsilon} + s)/l - X] \qquad (11)$$

where $L(\varepsilon \pm s) = 1 - X + \varepsilon X \pm sX$. Proposition 1 shows that banks invest too much in the risky asset, i.e., they provide less than the socially efficient amount of liquidity.

Proposition 1 *Banks' provision of liquidity is inefficiently low, i.e., $dW/dX_i < \partial W_i/\partial X_i$. In particular, we have*

$$dW/dX_i = \partial W_i/\partial X_i - \frac{1}{2}(1 - \gamma)R\frac{sX}{l}\pi'(X_i)$$

Proof. *See Appendix C* ∎

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[3]Appendix B provides the conditions under which the X implied by (10) is indeed consistent with our setup: Appendix B.1 shows that the equilibrium X is consistent with partial insurance, Appendix B.2 verifies that a bank has no incentive to deviate to either full or no insurance.

The inefficiency of banks' liquidity holdings arises from an externality: when a bank holds less liquidity (i.e., invests more in the risky asset), it reduces the net aggregate liquidity $L - L^D$. This increases the probability of a liquidity crisis (in which assets have to be transferred) by increasing the domain of asset shocks for which a liquidity crisis occurs ($\widehat{\varepsilon}$ increases).

As the proposition shows, the efficiency loss due to this is $\frac{1}{2}(1 - \gamma)R\frac{sX}{l}$, consisting of the mass of lucky banks in a crisis $1/2$, the value loss from transferring a project $(1 - \gamma)R$ and the amount of projects that are bought by lucky banks sX/l. The latter is because for the new states where a liquidity crisis occurs we have $\varepsilon = \widehat{\varepsilon}$ and hence $L^D = L$. Thus, lucky banks have an excess liquidity of exactly $sX_i = sX$. Note that the externality is solely related to the value loss from transferring assets $1 - \gamma$: from Proposition 1 it can be seen that for $\gamma = 1$ banks' liquidity provision is efficient.

The presence of an externality creates a potential role for intervention and we shall now explore the role of regulatory measures to improve banks' liquidity holdings. Before doing this, let us clarify the sources of inefficiencies that arise. As shown above, there is an inefficient provision of liquidity in an unregulated economy because a single bank does not internalize the value losses from asset transfers due to the increased probability of an aggregate crisis. However, the costly asset transfer in itself constitutes a further inefficiency: *ceteris paribus*, welfare would be higher if assets were financed at the originating bank rather than sold.[4] The first best in our economy thus requires that banks provide an efficient amount of liquidity and that no assets are transferred. We shall now study the role of regulation in improving upon the first type of inefficiency. In the next section, we address the scope for avoiding the asset transfer altogether. Note that the occurrence of liquidity crises in itself is not inefficient here, i.e., liquidity crises are a necessary outcome of investing in high return but illiquid assets.

We consider first the provision of liquidity by a central bank (i.e., a lender of last resort, LOLR, policy). Assume therefore that the central bank commits to provide an amount of liquidity L^{CB} to the market in a crisis at market interest rates. Since liquidity needs are real in our setup (i.e., they are used to continue physical assets rather than to settle financial claims) a central bank would need to obtain this liquidity in a closed model through storage of L^{CB} at date 0, which in turn would need to be financed through

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[4]Banks could themselves avoid this inefficiency from the start by diversifying their assets across banks. However, the possibilities for banks to do so remain limited, despite the recent advent of credit derivatives (the liquid market for credit risk is currently limited to only around 500 names worldwide). Neither does granting each other liquidity lines remedy this inefficiency, as a bank would always have an incentive to draw upon its liquidity line in a crisis, regardless of whether it had liquidity needs or not (this is because liquidity can be used to buy up assets cheaply).

payments from the banks. This, obviously, is not a sensible intervention since it will simply transfer liquidity from banks to the central bank. Rather, we shall examine a situation where the central bank possesses its own liquid funds of L^{CB} that it can provide in a crisis. Since this amounts to injecting additional resources into the economy, studying the welfare implications is therefore not appropriate. However, it is informative to analyze the impact on the probability of a liquidity crisis.

Given this liquidity injection, a liquidity crisis will now occur only if $L + L^{CB} < L^D$.[5] Analogous to equation (3) one can derive the expression for the probability of a liquidity crisis

$$\pi = 3/2 + l - (1 + L^{CB})/X \qquad (12)$$

showing that for a given X, a liquidity injection reduces the probability of a crisis. However, a lower π has the effect of increasing banks' incentives to invest in the risky asset ($\partial W_i/\partial X_i$ is decreasing in π, as shown in Appendix B.1), which has an offsetting effect on the probability of a liquidity crisis.

Proposition 2 *A LOLR policy does not affect the probability of default.*

This neutrality result can be understood by noting that the introduction of L^{CB} does not directly enter the bank's FOC condition for X_i (equation 10); it enters only indirectly via π and $\overline{\varepsilon}$. Recalling that $\overline{\varepsilon} = (\pi - 1)/2$, the FOC is still fulfilled for the probability of default prior to the implementation of the LOLR policy, implying that the probability of a liquidity crisis is unchanged. Thus, banks increase their investment in the risky asset by an amount that exactly offsets the impact of L^{CB} on the probability of a crisis. The provision of liquidity by the central bank is hence neutral because it is anticipated by banks *ex-ante* and leads to a corresponding reduction in their liquidity holdings.[6] This moral hazard has long been recognized (e.g., Bagehot, 1873). What is remarkable here is that it completely neutralizes the impact of a LOLR policy on stability.

Another regulatory measure frequently implemented is sponsoring the purchase of assets from troubled banks. In the U.S., for example, institutions that acquire assets from

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[5] Note that when $L + L^{CB} \geq L^D$, the central bank lends at competitive, that is zero interest rates. By contrast, it has been argued since Bagehot (1873) that the lender of last resort should only operate at a penalty interest rate. However, the problem is here that high interest rates are precisely the reason why interbank lending breaks down as they increase the debt burden of borrowing banks, as shown in Appendix A (in Section 5 we discuss briefly the impact of an LOLR policy if lending at penalty interest rates were feasible). This may explain why in practice lending often occurs without a premium over the market interest rate (Goodhart and Schoenmaker, 1995).

[6] Similarly, neither should an (anticipated) private provision of liquidity from outside the banking sector (such as from institutional investors) reduce the occurrence of crises.

banks that have been closed down are sometimes granted tax deductions, or allowed to use goodwill on their books. We shall now analyze the effectiveness of such a policy. Assume that the regulator subsidizes the purchase of assets from unlucky banks in a crisis at rate δR, where $\delta > 0$. The subsidy is paid out at date 2 (which is consistent with goodwill and tax deductions not effective at the time of the crisis itself) and is financed through lump sum taxes T at date 2 from all banks.

This additional incentive to buy assets has no direct impact on asset purchases in a crisis since lucky banks already use all their liquidity to buy up assets. Given that there is not enough liquidity to finance all assets, the price at which assets are purchased is still zero. However, the subsidy has an effect on banks' ex-ante incentives to hold liquidity. The new expression for bank's equity is

$$W_i = RX_i + (1 - X_i) - \frac{\pi}{2}R[X_i - L_i(\overline{\varepsilon} - s)/l] + \frac{\pi}{2}(\gamma + \delta)R[L_i(\overline{\varepsilon} + s)/l - X_i] - \pi T \quad (13)$$

which differs from (8) because of the expected additional benefits from buying assets, $\frac{\pi}{2}\delta R[L_i(\overline{\varepsilon} + s)/l - X_i]$, and expected taxes πT.[7] The corresponding FOC writes

$$\partial W_i/\partial X_i = R - 1 - \frac{\pi}{2}R[1 - (-1 + \overline{\varepsilon} - s)/l] + \frac{\pi}{2}(\gamma + \delta)R[(-1 + \overline{\varepsilon} + s)/l - 1] = 0 \quad (14)$$

Proposition 3 *A subsidization of the purchase of assets from troubled banks improves welfare.*

Formally, this is because $\partial W_i/\partial X_i$ is decreasing with δ (because $s < 1$ and $\overline{\varepsilon} \leq 0$ we have that $(-1 + \overline{\varepsilon} + s)/l - 1 < 0$), hence an increase in δ reduces the bank's incentives to invest in the risky asset (or increases its incentives to hold liquidity). This increases welfare since, as Proposition 1 has shown, liquidity provision in the absence of regulation is too low.[8]

This result is noteworthy since the subsidization of asset purchases from troubled banks is usually justified on *ex-post* grounds, i.e., as a means to incentivize banks to buy up assets. Proposition 3 suggests that such subsidies also have a welcoming *ex-ante* effect by motivating banks to hold more liquidity in order to be able to buy more assets in a crisis.

Obviously, banks' liquidity holdings could also be improved through standard capital requirements: since capital is fixed in our model, capital requirements effectively restrict

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[7]Taxation of unlucky banks at date 2 creates similar problems of fund appropriation arising from borrowing at the interbank market (Appendix A). However, for sufficiently small δ, taxes become sufficiently small and thus also the potential incentives for appropriation (while the costs of appropriation, arising from lost projects, remain constant).

[8]A related effect has been emphasized in Perotti and Suarez (2002), where banks have an incentive to survive in a crisis in order to benefit from lessenend competition due to failing rivals.

the amount that can be invested in the risky asset. However, a subsidization of asset purchases seems to be a more appropriate tool for several reasons. First, administrative requirements should be lower for asset subsidization. This is because regulation does not have to be applied continuously as with capital requirements, only once a crisis occurs. Moreover, it is more straightforward to measure the amount of assets that are purchased by a bank than the riskiness of a bank's portfolio. Second, subsidization incentivizes banks to hold the right amount of liquidity instead of forcing them to do so, as capital requirements do. Third, and relatedly, subsidization does not give banks an incentive to engage in asset substitution or off-balance sheet transactions in order to increase their risk, which has been a problem with capital requirements under the old Basel regime. Fourth, it is directly related to the source of inefficiency, that is the inefficient provision of liquidity, while capital requirements are designed to restrict the amount of risk on banks' portfolios.

5   The Transfer of Assets and Regulation

We shall now examine whether regulation can also avoid the transfer of assets in a crisis. Obviously, this requires the regulator to have some advantage over the interbank market. The literature has emphasized that such an advantage may arise, for example, because the regulator has superior information about banks (Berger et. al., 2000) or because of a coordinating role (e.g., Freixas, Parigi, Rochet, 2000). In our context, it arises because a regulator can, due to its oversight role, observe the amount of borrowing that takes place in the interbank market, while an individual bank cannot. This allows control over a bank's total borrowings and thus mitigates the appropriation problem. Appendix D provides a condition under which the regulator's ability to restrict borrowing can indeed make full liquidity smoothing feasible in a crisis.

We show in the following that a policy of transferring liquidity from lucky to unlucky banks in a crisis can then achieve the first best. We assume thereby that the government cannot observe banks' asset shocks (i.e., whether banks have been lucky or unlucky), so it has to base its transfers on banks' actual liquidity holdings. Suppose that the transfer scheme is such that in a crisis a bank receives an amount of liquidity of $\triangle L_i^1 := L - L_i$ (if $\triangle L_i^1 < 0$ the bank has to give up liquidity). It follows that banks' post-transfer liquidity holdings are $L_i + \triangle L_i^1 = L$, i.e., all banks hold the average pre-transfer amount of liquidity, implying that the redistribution is feasible in that it does not create or destroy liquidity. Furthermore, assume that liquidity is distributed back at date 2 with an interest of R/l, i.e., we have $\triangle L_i^2 = -(R/l) \triangle L_i^1$. Since this is tantamount to borrowing and lending that smooths idiosyncratic liquidity needs at interest $r = R/l$, these transfers are also feasible

in that they do not incentivize banks to appropriate funds at date 1 (Appendix D).

Consider now a bank's return under this scheme. Since in a liquidity crisis all banks have identical liquidity after the transfer, there is no scope for transferring assets. Moreover, both lucky and unlucky banks are then unable to provide liquidity for all their assets, i.e., they lose an amount of $X_i - L/l$ of their assets (we have $X_i - L/l > 0$ since in a liquidity crisis $X - L/l > 0$ and since $X_i = X$ in equilibrium because of symmetry). The equation for the bank return thus writes

$$W_i = RX_i + (1 - X_i) - \pi R[X_i - L/l] + \frac{\pi}{2} \triangle L_i^2(\overline{\varepsilon} - s) + \frac{\pi}{2} \triangle L_i^2(\overline{\varepsilon} + s)$$

Using the definition of $\triangle L_i^2$, this can be rearranged to

$$W_i = RX_i + (1 - X_i) - \frac{\pi}{2} R[X_i - L_i(\overline{\varepsilon} - s)/l] - \frac{\pi}{2} R[X_i - L_i(\overline{\varepsilon} + s)/l] \qquad (15)$$

Observing that this is identical to equity in the unregulated economy for $\gamma = 1$ (equation 8) and recalling that for $\gamma = 1$ there are no externalities from liquidity provision (Proposition 1), it follows that banks choose a socially efficient level of X_i. Since there are also no costly asset transfers, we obtain the following result:

Proposition 4 *An appropriate redistribution of liquidity in times of crisis can achieve the first best.*

Note that our redistribution of liquidity is not a bail-out, since the latter would subsidize banks: liquidity is here provided at effectively punitive interest rates, which do not distort *ex-ante* incentives.[9]

One may be tempted to conclude from this analysis that an LOLR, if it could also operate at punitive rates, is also an effective intervention. This is not the case because a first best under an LOLR policy would require the aggregate supply of liquidity plus the central bank's liquidity injection L^{CB} never to fall short of the demand for liquidity (otherwise there would be a liquidity crisis with costly asset transfer). Thus, *ex-post* efficiency requires the probability of an aggregate liquidity crisis to be zero, which is generally not efficient.[10] The intuitive reason for the superiority of liquidity transfers over an LOLR policy is that the former directly tackles the inefficiency arising from asset transfer by reducing the heterogeneity of bank's liquidity holdings in a crisis. An LOLR can only avoid the asset

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[9]Technically, the reason why regulation can improve upon the market allocation here is that by equalizing liquidity holdings, it limits a bank's borrowing in a crisis and thus avoids the overborrowing that causes the breakdown of the interbank market in a crisis.

[10]This can be easily verified from the corresponding FOC for equation 15, which by Proposition 4 leads to efficient investment.

transfer by ensuring that there is never a liquidity crisis, which is not efficient since the holdings of liquidity required to achieve this imply foregone returns from investing in the risky asset.

6 Summary

This paper has developed a model where costs in the banking sector, rather than from bank runs, arise from situations in which banks can no longer finance ongoing projects. The banking sector is fragile because a small reduction in aggregate liquidity can lead to the collapse of interbank lending and force the breakup of lending relationships at banks with liquidity needs, even though there is sufficient aggregate liquidity to avoid this.

Banks underinvest in liquidity as they cannot fully reap the social benefits from providing liquidity when buying up assets. This can be corrected by subsidizing the purchase of assets from troubled banks. Liquidity provision by the central bank is ineffective as it undermines banks' incentives to invest in liquidity and also does not stop the unnecessary breakup of lending relationships in a crisis. However, there is a role for the regulator in redistributing *existing* liquidity, which can in principal achieve the first best.

Appendix A (Interbank Market Breakdown when $L < L^D$)

We shall now model the breakdown of the interbank market when there is an aggregate liquidity shortage. The breakdown stems partly from the possibility for borrowing banks to appropriate funds but also from an imperfection in the interbank market itself. The latter arises because market participants cannot fully observe the level of a bank's total borrowing. In particular, we assume in the following that a bank can appropriate at date 1 all its liquid funds (after borrowing), instead of using them to inject liquidity into projects. Moreover, banks can borrow up to an amount of $\overline{B}$ without other banks observing its total level of borrowing.

Suppose that the interbank market works when $L < L^D$. The net return from financing an additional project through borrowing at the market interest r is $(R+l) - (1+r)l = R - rl$. An equilibrium would thus require $r = R/l$ in order to make banks indifferent between financing projects and not injecting liquidity (which would make the projects worthless). The date 2 return of a bank with liquidity needs that borrows B_i and invests all liquid funds in projects is thus (see also equation 6)

$$RL_i/l + L_i + (R+l)B_i/l - (1+r)B_i = R \cdot L_i/l + L_i$$

On the other hand, if the bank appropriates all funds, its date 1 return is simply $L_i + B_i$, since there is no return at date 2 as all assets are then worthless. Noting that the gains from appropriation are increasing in B_i and assuming zero discounting between periods (consistent with a return of 1 on the storage technology), a bank hence finds it optimal to appropriate funds iff

$$R \cdot L_i/l < \overline{B}$$

In such a situation, lending to smooth banks' idiosyncratic liquidity needs cannot take place and the interbank market breaks down (note that the complete breakdown of lending is really due to market participants' inability to control a bank's borrowing level: for sufficiently small $\overline{B}$ the above condition would always be violated)

By contrast, the return from borrowing at the interbank market and investing when there is no aggregate liquidity shortage is $RX_i + L_i$ (from equation 4), since all projects can then be continued at zero borrowing costs. Since the return from appropriating funds is unchanged, interbank lending takes place in the absence of a liquidity shortage iff

$$RX_i > \overline{B}$$

Since $L_i/l < X_i$ for a bank that has financing needs, the gains from borrowing and investing are higher when there is no aggregate liquidity shortage. Thus there exist $\overline{B}$ that fulfill the

above conditions, for which we have then that the interbank market operates in normal times but breaks down in a crisis.

Appendix B (Existence of Equilibrium)

Appendix B.1 (Partial Insurance in an Equilibrium)

We show that there are parameters for which an X that fulfills the FOC (10) implies partial insurance: i.e., in a crisis a lucky bank always has excess liquidity, while an unlucky bank has liquidity needs. The latter follows immediately from the fact that there is an aggregate liquidity crisis, since an unlucky bank will have less liquidity than the average liquidity per bank. For the former, since liquidity increases with the aggregate shock, the condition that a lucky bank always has sufficient liquidity to finance its own assets is fulfilled whenever there is sufficient liquidity at the minimum asset shock $\varepsilon = -1/2$. This condition writes

$$L_i(\varepsilon = -1/2) - L_i^D = 1 - X_i + (-1/2 + s)X_i - X_i l \geq 0$$

Dividing by X_i, using $X_i = X$ (because of symmetry in equilibrium) and substituting X using (3) this condition simplifies to $\pi \leq s$.

We show next that there are parameter values that jointly fulfill this inequality and the FOC. To make use of the inequality, we first show that $\partial W_i / \partial X_i$ is decreasing in π. From comparing W_i and $\partial W_i / \partial X_i$ (equation 8 and 10) we find that

$$W_i = (\partial W_i / \partial X_i)X_i + 1 + (1 + \gamma)\frac{\pi}{2}R/l$$

Partially differentiating wrt. π and using $\partial W_i / \partial \pi < 0$ (analogous to Proposition 1 one can show that $\partial W_i / \partial \pi = -(1 - \gamma)s\frac{R}{2l}X_i$) we get

$$\frac{\partial W_i}{\partial \pi} = \frac{\partial(\partial W_i / \partial X_i)}{\partial \pi}X_i + (1 + \gamma)\frac{1}{2}R/l < 0$$

from which it follows that $\partial W_i / \partial X_i$ is decreasing in π.

Using $\overline{\varepsilon} = (\pi - 1)/2$ to write the FOC (10) as a function of π and substituting s for π in the FOC hence yields $\partial W_i(\pi, s)/\partial X_i = 0 \geq \partial W_i(s, s)/\partial X_i$, which gives the following condition :

$$
\begin{aligned}
\partial W_i(s, s)/\partial X_i &= R - 1 - \frac{s}{2}R[1 - (-1 + \frac{s-1}{2} - s)/l] + \frac{s}{2}\gamma R[(-1 + \frac{s-1}{2} + s)/l - 1] \\
&= R - 1 + \frac{s}{2}R(1 - \gamma) - \frac{s}{2}R/l[3/2(1 + \gamma) + s(1/2 - 3/2\gamma)] \leq 0
\end{aligned}
$$

This condition can be fulfilled for a variety of parameter values: for example for $\gamma < 1/3$, the expression in the squared brackets is positive; by letting l become small, the last term can then be made an arbitrarily large negative number.

Appendix B.2 (Global Optimum)

We show that an X with partial insurance that fulfills the FOC (10) does indeed constitute a global equilibrium. This requires that it neither pays off for a single bank to deviate to a situation where it always has sufficient liquidity in a liquidity crisis (full insurance) nor to a situation where it never has sufficient liquidity in a crisis (no insurance). In the following we therefore presume that (10) holds, i.e., X is such that $\partial W_i/\partial X_i = 0$ and show that a bank has no incentive to deviate from partial insurance.

No deviation to full insurance: a fully insured bank never has to liquidate assets in a crisis and can always buy up assets from other banks. Its return is then analogous to (8)

$$W_i^{FI} = RX_i + (1 - X_i) + \frac{\pi}{2}\gamma R[L_i(\bar{\varepsilon} - s)/l - X_i] + \frac{\pi}{2}\gamma R[L_i(\bar{\varepsilon} + s)/l - X_i]$$

Taking derivative wrt. X_i and comparing to the FOC condition under partial insurance (equation 10) we get

$$\partial W_i^{FI}/\partial X_i = \partial W_i/\partial X_i + \frac{\pi}{2}(1 - \gamma)R[1 - (-1 + \bar{\varepsilon} - s)/l] > 0$$

where the inequality follows from $\partial W_i/\partial X_i = 0$ and $1 - (-1 + \bar{\varepsilon} - s)/l > 0$. Thus, under full insurance, the bank's return is strictly increasing in its risk taking. A bank therefore has an incentive to increase its investment in the risky asset, which pushes the bank into partial insurance. Thus, full insurance cannot be a worthwhile deviation from partial insurance.

No deviation to no insurance: under no insurance, a bank always has to liquidate assets in a crisis and can never buy up assets from other banks. The expression for its return is thus

$$W_i^{NI} = RX_i + (1 - X_i) - \frac{\pi}{2}R[X_i - L_i(\bar{\varepsilon} - s)/l] - \frac{\pi}{2}R[X_i - L_i(\bar{\varepsilon} + s)/l]$$

Taking derivative wrt. X_i and making use of equation (10) we find that

$$\partial W_i^{NI}/\partial X_i = \partial W_i/\partial X_i - \frac{\pi}{2}(1 - \gamma)R[1 - (-1 + (\bar{\varepsilon} + s))/l] < 0$$

Hence, the bank has an incentive to reduce its risk, which would push the bank eventually into partial insurance. Thus, no insurance can also not be a worthwhile deviation.

Appendix C (Proof of Proposition 1)

From (11) we have the efficient level of investment fulfills $dW/dX_i = \partial W_i/\partial X_i + \partial W/\partial \pi \cdot \pi'(X_i)$. Using W_i from (7) and $\pi = \hat{\varepsilon} + 1/2$ (equation 3) we can write $W = \int_i W_i di$ as

$$W = RX + (1 - X) - \frac{1}{2}\int_{-1/2}^{\pi - 1/2} R[X - L(\varepsilon - s)/l]d\varepsilon + \frac{1}{2}\int_{-1/2}^{\pi - 1/2} \gamma R[L(\varepsilon + s)/l - X]d\varepsilon$$

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Differentiating wrt. π and simplifying gives $\partial W/\partial \pi = -(1-\gamma)s\frac{R}{2l}X < 0$. From $\pi'(X_i) = 1/X^2$ (since $X_i = X$ in equilibrium) we have that $\pi'(X_i) > 0$. It follows that $dW/dX_i < \partial W_i/\partial X_i$, hence banks invest more than the socially optimal amount in the risky asset.

Appendix D (Feasibility of Lending when Borrowing Can Be Restricted)

We first derive a condition that the amount of lending (or transfers) that is required in a crisis (i.e., that completely smooths banks' idiosyncratic liquidity needs) is feasible when banks' total borrowings can be sufficiently restricted. Suppose that banks can only borrow up to an amount of $\overline{B}_R = -\varepsilon_i X_i$ (lend if negative). This just permits banks to fully smooth their liquidity holdings. The feasibility condition is then, analogous to the second condition in Appendix A,

$$R \cdot L_i/l \geq \overline{B}_R = sX_i$$

i.e., a borrowing bank (a bank with $\varepsilon_i = -s$) does not find it optimal to appropriate liquid funds (while interbank lending still breaks down for borrowing restrictions $\overline{B}$, $\overline{B} > \overline{B}_R$, that fulfill the conditions in Appendix A).

Next we show that this condition can be fulfilled in an equilibrium consistent with (10) for appropriate parameter values. We have that L_i becomes minimal for $\varepsilon = -1/2$, thus $L_i \geq 1 - X_i - 1/2X_i - sX_i$. Using equation (3) and $X_i = X$ (because of symmetry in equilibrium) this can be transformed to $L_i \geq (l-s-\pi)X_i$. Using the equilibrium condition for partial insurance $\pi \leq s$ (from Appendix B.1), we have then that $L_i \geq (l-s-\pi)X_i \geq (l-2s)X_i$. Hence, the condition for the feasibility of idiosyncratic smoothing is fulfilled when $R(l-2s)X_i/l \geq sX_i$. Rearranging for s gives

$$s \leq Rl/(l+2R)$$

which is fulfilled, for example, for sufficiently small s.

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