SOLUTION OF THE BFKL EQUATION AT NEXT-TO-LEADING ORDER *

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We solve the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation in the next–to–leading logarithmic approximation for forward scattering with all conformal spins using an iterative method .

1 Introduction

The BFKL [1] formalism resums large logarithms appearing in the Regge limit, where the center of mass energy $\sqrt{s}$ is large and the momentum transfer $\sqrt{-t}$ fixed. The cross-section for the process $A + B \to A' + B'$ reads

$$\sigma(s) \quad = \quad \int \frac{d^2\mathbf{k}_a}{2\pi\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b}{2\pi\mathbf{k}_b^2} \; \Phi_A(\mathbf{k}_a) \; \Phi_B(\mathbf{k}_b) \; f\left(\mathbf{k}_a, \mathbf{k}_b, \Delta = \ln\frac{s}{s_0}\right), \qquad (1)$$

where $\Phi_{A,B}$ are the impact factors and $f\left(\mathbf{k}_a, \mathbf{k}_b, \Delta\right)$ is the gluon Green's function describing the interaction between two Reggeised gluons exchanged in the t–channel with transverse momenta $\mathbf{k}_{a,b}$. We use the Regge scale $s_0 = |\mathbf{k}_a|\,|\mathbf{k}_b|$. In the leading logarithmic approximation (LLA) terms of the form $(\alpha_s\Delta)^n$ are resummed. In the next–to–leading logarithmic approximation (NLLA) [2] contributions of the type $\alpha_s\,(\alpha_s\Delta)^n$ are also taken into account.

The Green's function is the solution of an integral equation where radiative corrections enter through its kernel. In this contribution we solve this equation using an iterative method considering the full kernel with scale invariant and running coupling terms. In this approach we keep all the angular information from the BFKL evolution, solving the equation for a general conformal spin without relying on any asymptotic expansion. For more details see Ref. [3].

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2 The BFKL equation in the NLLA

The BFKL equation is written in terms of a Mellin transform in Δ space, i.e.

$$f\left(\mathbf{k}_a,\mathbf{k}_b,\Delta\right) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} d\omega\; e^{\omega\Delta} f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right). \tag{2}$$

The BFKL equation in the NLLA in dimensional regularisation then reads

$$\omega f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right) = \delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right) + \int d^{2+2\epsilon}\mathbf{k}'\;\mathcal{K}\left(\mathbf{k}_a,\mathbf{k}'\right) f_\omega\left(\mathbf{k}',\mathbf{k}_b\right), \tag{3}$$

with the kernel $\mathcal{K}\left(\mathbf{k}_a,\mathbf{k}\right) = 2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)\delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}\right)+\mathcal{K}_r\left(\mathbf{k}_a,\mathbf{k}\right)$ depending on the gluon Regge trajectory and a real emission component [2].

We split the kernel $\mathcal{K}_r$ into a ϵ–dependent ($\mathcal{K}_r^{(\epsilon)}$) and a ϵ–independent ($\widetilde{\mathcal{K}}_r$) parts. To show the cancellation of the ϵ poles we split the integral over transverse phase space for $\mathcal{K}_r^{(\epsilon)}$ into two regions separated by a small cut–off λ. We then approximate $f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right)\simeq f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right)$ for $|\mathbf{k}|<\lambda$. This λ–dependence is negligible for large $|\mathbf{k}_a|$. In this way we express the BFKL equation as

$$\begin{aligned}\omega f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right) &= \delta^{(2+2\epsilon)}\left(\mathbf{k}_a-\mathbf{k}_b\right) \\ &+\left\{2\,\omega^{(\epsilon)}\left(\mathbf{k}_a^2\right)+\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)\theta\left(\lambda^2-\mathbf{k}^2\right)\right\} f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right) \\ &+\int d^{2+2\epsilon}\mathbf{k}\,\left\{\mathcal{K}_r^{(\epsilon)}\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)\theta\left(\mathbf{k}^2-\lambda^2\right)+\widetilde{\mathcal{K}}_r\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)\right\} f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right).\end{aligned} \tag{4}$$

In dimensional regularisation the gluon Regge trajectory [2] reads [1]

$$\begin{aligned}2\,\omega^{(\epsilon)}\left(\mathbf{q}^2\right) &= -\bar{\alpha}_s\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\left(\frac{1}{\epsilon}+\ln\frac{q^2}{\mu^2}\right)-\frac{\bar{\alpha}_s^2}{8}\frac{\Gamma^2(1-\epsilon)}{(4\pi)^{2\epsilon}}\left\{\frac{\beta_0}{N_c}\left(\frac{1}{\epsilon^2}+\ln^2\frac{q^2}{\mu^2}\right)\right. \\ &+\left.\left(\frac{4}{3}-\frac{\pi^2}{3}+\frac{5}{3}\frac{\beta_0}{N_c}\right)\left(\frac{1}{\epsilon}+2\ln\frac{q^2}{\mu^2}\right)-\frac{32}{9}+2\zeta(3)-\frac{28}{9}\frac{\beta_0}{N_c}\right\}.\end{aligned} \tag{5}$$

The ϵ-dependent part of the real emission kernel [2] is

$$\begin{aligned}\mathcal{K}_r^{(\epsilon)}\left(\mathbf{q},\mathbf{q}+\mathbf{k}\right) &= \frac{\bar{\alpha}_s\mu^{-2\epsilon}}{\pi^{1+\epsilon}(4\pi)^\epsilon}\frac{1}{\mathbf{k}^2}\left\{1+\frac{\bar{\alpha}_s}{4}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\left[\frac{\beta_0}{N_c}\frac{1}{\epsilon}\left(1-\left(\frac{\mathbf{k}^2}{\mu^2}\right)^\epsilon\right.\right.\right. \\ &\times\left.\left(1-\epsilon^2\frac{\pi^2}{6}\right)\right)+\left(\frac{\mathbf{k}^2}{\mu^2}\right)^\epsilon\left(\frac{4}{3}-\frac{\pi^2}{3}+\frac{5}{3}\frac{\beta_0}{N_c}+\epsilon\left(-\frac{32}{9}+14\zeta(3)-\frac{28}{9}\frac{\beta_0}{N_c}\right)\right)\Bigg]\Bigg\}.\end{aligned} \tag{6}$$

[1] $\beta_0\equiv\frac{11}{3}N_c-\frac{2}{3}n_f$, $\bar{\alpha}_s\equiv\frac{\alpha_s(\mu)N_c}{\pi}$, $\alpha_s(\mu)=\frac{g_\mu^2}{4\pi}$, μ is the $\overline{\mathrm{MS}}$ renormalisation scale.

When we combine the trajectory with the integration of $\mathcal{K}_r^{(\epsilon)}$ over the phase space limited by the cut–off, the poles in ϵ cancel and we obtain

$$\omega_0\left(\mathbf{q}^2,\lambda^2\right) \equiv \lim_{\epsilon\to 0}\left\{2\,\omega^{(\epsilon)}\left(\mathbf{q}^2\right)+\int d^{2+2\epsilon}\mathbf{k}\,\mathcal{K}_r^{(\epsilon)}\left(\mathbf{q},\mathbf{q}+\mathbf{k}\right)\theta\left(\lambda^2-\mathbf{k}^2\right)\right\} \tag{7}$$
$$= -\bar{\alpha}_s\left\{\ln\frac{\mathbf{q}^2}{\lambda^2}+\frac{\bar{\alpha}_s}{4}\left[\frac{\beta_0}{2N_c}\ln\frac{\mathbf{q}^2}{\lambda^2}\ln\frac{\mu^4}{\mathbf{q}^2\lambda^2}+\left(\frac{4}{3}-\frac{\pi^2}{3}+\frac{5}{3}\frac{\beta_0}{N_c}\right)\ln\frac{\mathbf{q}^2}{\lambda^2}-6\zeta(3)\right]\right\}.$$

Using $\omega_0\left(\mathbf{q}^2,\lambda^2\right) \equiv -\xi\left(|\mathbf{q}|\,\lambda\right)\ln\frac{\mathbf{q}^2}{\lambda^2}+\eta$, $\eta \equiv \bar{\alpha}_s^2\frac{3}{2}\zeta(3)$ and $\xi\left(\mathrm{X}\right) \equiv \bar{\alpha}_s + \frac{\bar{\alpha}_s^2}{4}\left[\frac{4}{3}-\frac{\pi^2}{3}+\frac{5}{3}\frac{\beta_0}{N_c}-\frac{\beta_0}{N_c}\ln\frac{\mathrm{X}}{\mu^2}\right]$ we can write the equation in the simple form

$$\left(\omega-\omega_0\left(\mathbf{k}_a^2,\lambda^2\right)\right)f_\omega\left(\mathbf{k}_a,\mathbf{k}_b\right) = \delta^{(2)}\left(\mathbf{k}_a-\mathbf{k}_b\right) \tag{8}$$
$$+\int d^2\mathbf{k}\left(\frac{1}{\pi\mathbf{k}^2}\xi\left(\mathbf{k}^2\right)\theta\left(\mathbf{k}^2-\lambda^2\right)+\widetilde{\mathcal{K}}_r\left(\mathbf{k}_a,\mathbf{k}_a+\mathbf{k}\right)\right)f_\omega\left(\mathbf{k}_a+\mathbf{k},\mathbf{k}_b\right),$$

where $\widetilde{\mathcal{K}}_r\left(\mathbf{q},\mathbf{q}'\right)$ contains the full angular information of the BFKL evolution and can be found in Ref. [3].

3 Iterative solution in the NLLA

We solve Eq. (8) using an iterative procedure in the ω plane similar to the one in [4] for the LLA. In the NLLA we take the renormalisation scale $\mu=\mu\left(\mathbf{k}_a^2,\mathbf{k}_b^2\right)$ depending on the large scales in the scattering process. Our result reads [2]

$$f(\mathbf{k}_a,\mathbf{k}_b,\Delta) = \exp\left(\omega_0\left(\mathbf{k}_a^2,\lambda^2,\mu\left(\mathbf{k}_a^2,\mathbf{k}_b^2\right)\right)\Delta\right)\left\{\delta^{(2)}(\mathbf{k}_a-\mathbf{k}_b)\right. \tag{9}$$
$$+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d^2\mathbf{k}_i\left[\frac{\theta\left(\mathbf{k}_i^2-\lambda^2\right)}{\pi\mathbf{k}_i^2}\xi\left(\mathbf{k}_i^2,\mu\left(\left(\mathbf{k}_a+\sum_{l=0}^{i-1}\mathbf{k}_l\right)^2,\mathbf{k}_b^2\right)\right)\right.$$
$$\left.+\widetilde{\mathcal{K}}_r\left(\mathbf{k}_a+\sum_{l=0}^{i-1}\mathbf{k}_l,\mathbf{k}_a+\sum_{l=1}^{i}\mathbf{k}_l,\mu\left(\left(\mathbf{k}_a+\sum_{l=0}^{i-1}\mathbf{k}_l\right)^2,\mathbf{k}_b^2\right)\right)\right]$$
$$\times\int_0^{y_{i-1}}dy_i\,\exp\left[\left(\omega_0\left(\left(\mathbf{k}_a+\sum_{l=1}^{i}\mathbf{k}_l\right)^2,\lambda^2,\mu\left(\left(\mathbf{k}_a+\sum_{l=0}^{i}\mathbf{k}_l\right)^2,\mathbf{k}_b^2\right)\right)\right.\right.$$
$$\left.\left.\left.-\omega_0\left(\left(\mathbf{k}_a+\sum_{l=1}^{i-1}\mathbf{k}_l\right)^2,\lambda^2,\mu\left(\left(\mathbf{k}_a+\sum_{l=0}^{i-1}\mathbf{k}_l\right)^2,\mathbf{k}_b^2\right)\right)\right)y_i\right]\delta^{(2)}\left(\sum_{l=1}^{n}\mathbf{k}_l+\mathbf{k}_a-\mathbf{k}_b\right)\right\},$$

[2] Using the notation $y_0\equiv\Delta$.

where we have made use of an inverse Mellin transform to write the final solution in energy space [3].

4 Conclusions

We have presented a method to solve the BFKL equation for forward scattering in the next–to–leading logarithmic approximation using the kernel in dimensional regularisation and introducing a cut–off in phase space. This allows us to write the solution in a compact form, Eq. (9), suitable for numerical studies, which will be presented in a future work. We keep the full angular information in our solution by solving the equation for any conformal spin. This will allow the study of spin–dependent observables in the NLLA.

In recent years there have been many studies of the behaviour of the gluon Green's function in the NLLA [5]. Work is in progress to understand the BFKL resummed gluon Green's function using this novel approach, and to quantify the effect of those terms related to the running of the coupling [6] compared to the scale invariant ones.

Our ultimate goal is the calculation of cross–sections in the NLLA. In principle, using this procedure it will be possible to disentangle the structure of the final state allowing the study of, e.g., multiplicities, extending the work of [7] to the NLLA.

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