TWO-LOOP AND N-LOOP EIKONAL VERTEX CORRECTIONS *

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I present calculations of two-loop vertex corrections with massive and massless partons in the eikonal approximation. I show that the n-loop result for the UV poles can be given in terms of the one-loop calculation.

1 Introduction

The eikonal approximation is valid for emission of soft gluons. The approximation simplifies the usual Feynman rules for the quark propagator and quark-gluon vertex as follows:

$$\bar{u}(p)(-i\gamma^\mu)\frac{i(\not p + \not k + m)}{(p+k)^2 - m^2 + i\epsilon} \rightarrow \bar{u}(p)\gamma^\mu \frac{\not p + m}{2p\cdot k + i\epsilon} = \bar{u}(p)\frac{v^\mu}{v\cdot k + i\epsilon} \tag{1}$$

with $k \rightarrow 0$ the gluon momentum, p the quark momentum after emission of the gluon, v a dimensionless vector $v \propto p$, and I have omitted overall factors of $g_s T_F^c$ with $g_s^2 = 4\pi\alpha_s$ and T_F^c the generators of SU(3) in the fundamental representation.

The eikonal approximation has numerous phenomenological applications in QCD, including threshold resummations for a variety of QCD processes [1, 2, 3, 4, 5, 6]. In these applications we are mainly interested in the ultraviolet (UV) pole structure (in dimensional regularization) of one-loop, two-loop, and higher-loop eikonal vertex corrections. In this talk, I discuss explicit calculations of one-loop and two-loop eikonal vertex corrections for diagrams with massive and massless partons and show that the n-loop UV poles are given simply in terms of the one-loop result [7].

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2 One-loop calculations

Let us denote by $\omega_{ij}^{(n)}$ the kinematics, color-independent, part of the n-loop correction to the eikonal vertex with lines i and j. At one loop, the expression

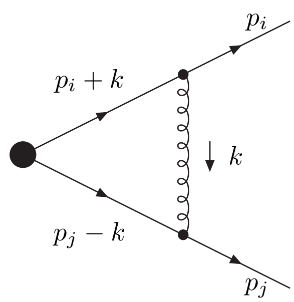

Figure 1: One-loop eikonal vertex correction diagram

for $\omega_{ij}^{(1)}$ (see fig. 1) in axial gauge is

$$\omega_{ij}^{(1)}(v_i, v_j) \equiv g_s^2 \int \frac{d^D k}{(2\pi)^D} \frac{(-i)}{k^2 + i\epsilon} N^{\mu\nu}(k) \frac{\Delta_i \, v_i^\mu}{\delta_i v_i \cdot k + i\epsilon} \frac{\Delta_j \, v_j^\nu}{\delta_j v_j \cdot k + i\epsilon} \tag{2}$$

with $\delta = +1(-1)$ when k flows in the same (opposite) direction as v, and

$$N^{\mu\nu}(k) = g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k} + n^2 \frac{k^\mu k^\nu}{(n \cdot k)^2} \,, \tag{3}$$

where n is the axial gauge vector. $\Delta = +1(-1)$ for a quark (antiquark) eikonal line, while for a gluon eikonal line $\Delta = +i(-i)$ for a gluon located above (below) the eikonal line.

Now let $I_l^{(1)}$ denote the contribution to $\omega_{ij}^{(1)}$ from the l-th term in the gluon propagator $N^{\mu\nu}(k)$ (i.e. $I_1^{(1)}$ is the contribution from the $g^{\mu\nu}$ term, $I_2^{(1)}$ from the $n^\mu k^\nu$ terms, and $I_3^{(1)}$ from the $k^\mu k^\nu$ term). In dimensional regularization with $\varepsilon = 4 - D$, the UV poles in $\omega_{ij}^{(1)}$ for the case of massive quarks, with mass m, are given by [1]

$$I_1^{(1)\,\mathrm{UV}} = \frac{\alpha_s}{\pi} \frac{1}{\varepsilon} L_\beta \,, \quad I_2^{(1)\,\mathrm{UV}} = \frac{\alpha_s}{\pi} \frac{1}{\varepsilon} (L_i + L_j) \,, \quad I_3^{(1)\,\mathrm{UV}} = -\frac{\alpha_s}{\pi} \frac{1}{\varepsilon} \,, \tag{4}$$

and thus

$$\omega_{ij}^{(1)\,\mathrm{UV}} = \mathcal{S}_{ij}^{(1)} \left[I_1^{(1)\,\mathrm{UV}} + I_2^{(1)\,\mathrm{UV}} + I_3^{(1)\,\mathrm{UV}} \right] = \mathcal{S}_{ij}^{(1)} \, \frac{\alpha_s}{\pi\varepsilon} \left[L_\beta + L_i + L_j - 1 \right]$$

with $\mathcal{S}_{ij}^{(1)} = \Delta_i \Delta_j \delta_i \delta_j$ an overall sign and

$$L_\beta = \frac{1 - 2m^2/s}{\beta} \left[\ln\left(\frac{1-\beta}{1+\beta} \right) + \pi i \right] \tag{5}$$

with $\beta = \sqrt{1 - 4m^2/s}$. The functions L_i and L_j depend on the axial gauge vector n and are cancelled when we include the heavy-quark self energies.

When v_i refers to a massive quark and v_j to a massless quark we have [1]

$$\begin{aligned} I_1^{(1)\,\mathrm{UV}} &= \frac{\alpha_s}{2\pi} \left\{ \frac{2}{\varepsilon^2} - \frac{1}{\varepsilon} \left[\gamma_E + \ln\left(\frac{v_{ij}^2 s}{2m^2} \right) - \ln(4\pi) \right] \right\} , \\ I_2^{(1)\,\mathrm{UV}} &= \frac{\alpha_s}{2\pi} \left\{ -\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon} \left[2L_i + \gamma_E + \ln \nu_j - \ln(4\pi) \right] \right\} , \\ I_3^{(1)\,\mathrm{UV}} &= -\frac{\alpha_s}{\pi} \frac{1}{\varepsilon} , \end{aligned} \tag{6}$$

where $\nu_a = (v_a \cdot n)^2/|n|^2$, $v_{ij} = v_i \cdot v_j$, and γ_E is the Euler constant. Note that the double poles cancel in the sum over the $I^{(1)}$'s and we get

$$\omega_{ij}^{(1)\,\mathrm{UV}}(v_i, v_j) = \mathcal{S}_{ij}^{(1)} \, \frac{\alpha_s}{\pi\varepsilon} \left[-\frac{1}{2} \ln\left(\frac{v_{ij}^2 s}{2m^2} \right) + L_i + \frac{1}{2} \ln \nu_j - 1 \right] . \tag{7}$$

Finally, when both v_i and v_j refer to massless quarks we have [8, 1]

$$\begin{aligned} I_1^{(1)\,\mathrm{UV}} &= \frac{\alpha_s}{\pi} \left\{ \frac{2}{\varepsilon^2} - \frac{1}{\varepsilon} \left[\gamma_E + \ln\left(\delta_i \delta_j \, \frac{v_{ij}}{2} \right) - \ln(4\pi) \right] \right\} , \\ I_2^{(1)\,\mathrm{UV}} &= \frac{\alpha_s}{\pi} \left\{ -\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon} \left[\gamma_E + \frac{1}{2} \ln(\nu_i \nu_j) - \ln(4\pi) \right] \right\} , \\ I_3^{(1)\,\mathrm{UV}} &= -\frac{\alpha_s}{\pi} \frac{1}{\varepsilon} . \end{aligned} \tag{8}$$

Again, we note that the double poles cancel in the sum over the $I^{(1)}$'s and we get

$$\omega_{ij}^{(1)\,\mathrm{UV}}(v_i, v_j) = \mathcal{S}_{ij}^{(1)} \, \frac{\alpha_s}{\pi\varepsilon} \left[-\ln\left(\delta_i \, \delta_j \, \frac{v_{ij}}{2} \right) + \frac{1}{2} \ln(\nu_i \nu_j) - 1 \right] . \tag{9}$$

3 Two-loop and n-loop calculations

For the two-loop diagram in fig. 2 we have

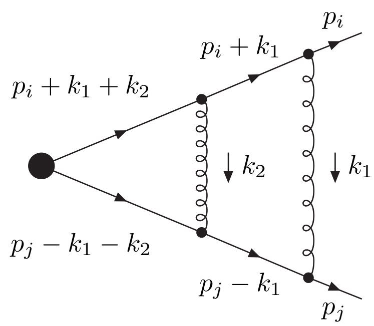

Figure 2: Two-loop eikonal vertex correction diagram

$$\omega_{ij}^{(2)}(v_i, v_j) = g_s^4 \int \frac{d^D k_1}{(2\pi)^D} \frac{(-i)}{k_1^2 + i\epsilon} N^{\mu\nu}(k_1) \frac{\Delta_{1i}\, v_i^\mu}{\delta_{1i} v_i \cdot k_1 + i\epsilon} \frac{\Delta_{1j}\, v_j^\nu}{\delta_{1j} v_j \cdot k_1 + i\epsilon}$$
$$\times \int \frac{d^D k_2}{(2\pi)^D} \frac{(-i)}{k_2^2 + i\epsilon} N^{\rho\sigma}(k_2) \frac{\Delta_{2i}\, v_i^\rho}{\delta_{2i} v_i \cdot (k_1 + k_2) + i\epsilon} \frac{\Delta_{2j}\, v_j^\sigma}{\delta_{2j} v_j \cdot (k_1 + k_2) + i\epsilon}. \tag{10}$$

Let $I_{kl}^{(2)}$ denote the contribution to $\omega_{ij}^{(2)}$ from the product of the k-th term in the axial-gauge gluon propagator $N^{\mu\nu}(k_1)$ with the l-th term in $N^{\rho\sigma}(k_2)$. Then we can rewrite

$$\omega_{ij}^{(2)}(v_i, v_j) = \mathcal{S}_{ij}^{(2)} \sum_{k,l=1,2,3} I_{kl}^{(2)}(v_i, v_j)\,. \tag{11}$$

When both partons are massive, an explicit calculation of the two-loop diagram gives the following results for the leading UV $(1/\varepsilon^2)$ poles at two loops:

$$I_{11}^{(2),\,\mathrm{UV}} = \frac{\alpha_s^2}{\pi^2}\frac{1}{\varepsilon^2} L_\beta^2\,, \quad I_{12}^{(2),\,\mathrm{UV}} = \frac{\alpha_s^2}{\pi^2}\frac{1}{\varepsilon^2} L_\beta\,(L_i + L_j) = I_{21}^{(2),\,\mathrm{UV}}$$
$$I_{22}^{(2),\,\mathrm{UV}} = \frac{\alpha_s^2}{\pi^2}\frac{1}{\varepsilon^2}\,(L_i + L_j)^2\,, \quad I_{13}^{(2),\,\mathrm{UV}} = -\frac{\alpha_s^2}{\pi^2}\frac{1}{\varepsilon^2} L_\beta = I_{31}^{(2),\,\mathrm{UV}}$$
$$I_{23}^{(2),\,\mathrm{UV}} = -\frac{\alpha_s^2}{\pi^2}\frac{1}{\varepsilon^2}\,(L_i + L_j) = I_{32}^{(2),\,\mathrm{UV}}\,, \quad I_{33}^{(2),\,\mathrm{UV}} = \frac{\alpha_s^2}{\pi^2}\frac{1}{\varepsilon^2}\,. \tag{12}$$

Then

$$\omega_{ij}^{(2)\,\mathrm{UV}}(v_i,v_j)=\mathcal{S}_{ij}^{(2)}\,\frac{\alpha_s^2}{\pi^2}\frac{1}{\varepsilon^2}\left[L_\beta+L_i+L_j-1\right]^2+\mathcal{O}\left(\frac{1}{\varepsilon}\right)\,, \qquad (13)$$

where $\mathcal{S}_{ij}^{(2)}=\Delta_{1i}\Delta_{1j}\Delta_{2i}\Delta_{2j}\delta_{1i}\delta_{1j}\delta_{2i}\delta_{2j}$. The calculation of the $1/\varepsilon$ terms is given in [7].

We now note that the leading two-loop UV poles are simply the square of the one-loop result since $I_{mn}^{(2),\,\mathrm{UV}}=I_m^{(1),\,\mathrm{UV}}I_n^{(1),\,\mathrm{UV}}$. Similar results hold for the cases when one or both partons are massless. For example, the leading poles in $I_{23}^{(2),\,\mathrm{UV}}$ when one of the partons is massless and the other is massive are $(\alpha_s^2/\pi^2)(1/\varepsilon^3)$.

We noted that the coefficient of the leading UV pole in $\omega_{ij}^{(2)}$ is simply the square of the coefficient of the leading UV pole in $\omega_{ij}^{(1)}$. We can show by induction that this generalizes to n loops [7]. Thus the leading UV pole in the n-loop corrections for massive partons, $\omega_{ij}^{(n)}$, is $(\alpha_s/\pi)^n(1/\varepsilon^n)\left[L_\beta+L_i+L_j-1\right]^n$. Furthermore a similar structure holds for non-leading UV poles [7].

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