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RESEARCH PAPER

Design and Implementation of Discrete-event Simulation Framework for Modeling Bus Rapid Transit System

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Abstract: It is clear that bus rapid transit is a strong contender for the solution of massive traffic congestion faced by many cities across the globe. However, the success or failure of this system will depend on many variables such as service planning, infrastructure, station design, passenger information systems, and integration and access. In this work, we established a computational framework on the basis of the discrete-event system for modeling the bus rapid transit system. This particular development allowed us to cost-effectively evaluate the effects of some of those variables on BRT performance. The results were a few sub-systems that could directly be used to model a typical BRT system. Some limited numerical trials revealed that the developed sub-systems could reasonably reproduce phenomena commonly observed in an actual BRT system.

Key words: urban traffic; discrete-event simulation; numerical model; traffic congestion; bus rapid transit

1 Introduction

This paper elaborates development of a computational framework for modeling bus rapid transit (BRT) on the basis of the discrete-event simulation (DES). The BRT system has some unique characteristics in comparison to a traditional bus system[1]. The developed computational framework is a set of sub-systems suitable for modeling the BRT system. Those sub-systems are built on top of the basic tools existed in a common DES system. We will explain the detailed development of each sub-system including their design decisions and functionalities.

A number of existing facts and earlier studies provide evidences of the necessity and importance of the present work. Campo[2] indicated the fast deployment of the BRT-based transportation system for public transport around the globe. It is clear, as shown in Fig. 1, that the number of BRT-based public transportation systems has been rising rapidly. But, the system, since its inception in the city of Curitiba, Brazil in 1974[3], initially received rather low acceptance. However, during the last decade, we witnessed a high rate of deployment of the system. Some large BRTs are TransMilenio started operating in Dec. 2000, TransJakarta BRT in Jan. 2004, and Guangzhou BRT in Feb. 2010[4].

Therefore, establishing a computational model for a BRT system may have many benefits and potential applications. One can use the model to study the service level of the system, and to evaluate the effectiveness of certain measures, or perhaps, to utilize the model to study the impact of the BRT to its environment. For an example, one can extend the structural equation model of Nugroho et al.[5] to the spatial dimension to further understand the spread of the secondary pollutants along a BRT corridor.

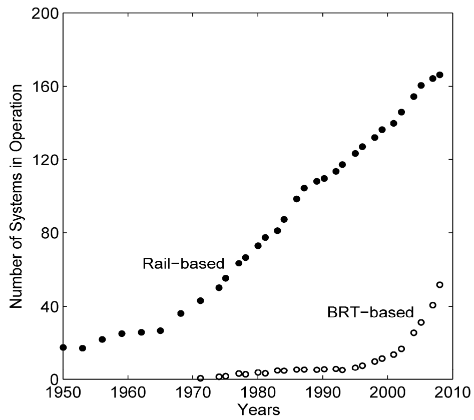

Fig. 1 The number of BRT-based systems and rail-based systems in operation[2]

In general, the importance of the computational model for transportation system had been realized since long ago, and many previous publications had addressed the issue from

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various aspects[6–13]. For an example, Valiguran et al.[13] focused on modeling a rail-based transportation system. Visser et al.[9] utilized a discrete-event simulation to evaluate a framework of the intelligent transportation systems. Dos Santos Silva et al.[12] employed discrete-event simulation and multi-criteria decision analysis (MCDA) to optimize a fleet of closed-loop maritime transportation of a steel manufacturing company. Moreover, Alves et al.[8] utilized a discrete-event simulation in conjunction with virtual reality for modeling a logistics system. Finally, Li et al.[7] employed discrete-time simulation approach to study factors influencing public bus travel efficiency in urban traffic in China.

We compose this paper in the following order. Section 2 describes the design of the basic sub-systems required to model a BRT system using DES approach. Each sub-system will be explained in detail including its Matlab SimEvents™ implementation. Then, Section 3 presents demonstrations of the use of the developed computational framework. Finally, Section 4 summerizes a few interesting findings related to this research.

2 Model developments

2.1 Canonical model of bus rapid transit system

To develop BRT sub-systems, we firstly reduce the size of a regular BRT system to a simple model but having all necessary sub-systems of the actual BRT system. This simplest model, which we called the canonical model, is depicted in Fig. 2.

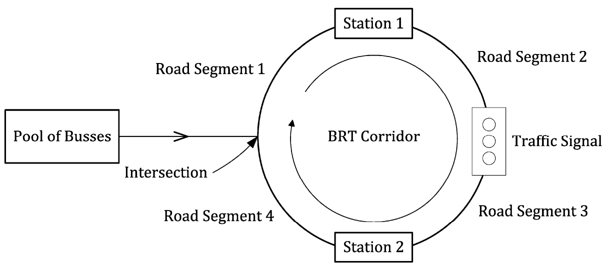


Fig. 2 A canonical model of a BRT corridor

The model in Fig. 2 is a simplified model of a BRT corridor where each bus serveing the corridor departs from the pool at the scheduled time. The schedule should relatively be high in bus frequency particularly during rush-hours (see Table 1). Then, the bus goes to Station 1 via the road-segment 1. For the case of TransJakarta BRT, the road-segment length varies from 0.3km to 1 km[14]. The bus stays at Station 1 for some amount of time to drop and pick up the passengers, and then, goes to Station 2. Finally, the bus will circulate in the corridor until it meets the end of the operation time for the day. Modern BRT is also required to operate until late at night[1]. In an actual BRT system, the bus will serve larger number of stations. TransJakarta BRT, for an example, has about 15–26 stations per corridor[14].

Table 1 Modern requirement for bus frequency during peak period[1]

Service Frequency (minute)	Points
< 3	4
3–5	3
5–7	2
7–10	1

It is clear that the canonical model in Fig. 2 has some necessary sub-systems to model a corridor of BRT system. Those sub-systems are the station, the road-segment, the pool of buses, the intersection, and the traffic signal. On the following, we will discuss the development of each BRT sub-system on the basis of Matlab SimEvents blocks.

2.2 The station sub-system

BRT station is very critical because transit activity mainly occurs. The BRT buses have to stop at the station for the passengers to board and alight, and the station platform should be at same level of the bus platform to reduce the passenger transfer-time.

The BRT system is designed so that the boarding and alighting activities can be performed within a short time. In comparison to the traditional bus system, the time required by the BRT buses is significantly shorter. This is achieved by three important design considerations of BRT system: alignment of the station platform and the bus floor, off-vehicle fare collection, and buses having wide doors[1].

An important feature of BRT station is that the system should be able to accurately capture the dynamics of passenger arrivals at the station. Fortunately, the issue has been of interest of many researchers, for examples: O'Flaherty and Mangan[15], Salek and Machemehl[16], Fan and Machemehl[17], Luethi et al.[18], Islam and Vandebona[19], and Gunawan et al.[20]. Some of those literatures had established the dynamics mathematically.

In general, the existing literatures identified the passenger dynamics and established the following conceptions. The times of the passenger arrivals inclined to follow, roughly speaking, two probability distribution functions. They are the uniform and log-normal distribution functions, see Fig. 3, depending on the bus headway. For a short headway, i.e., less than 5 min, the times of arrivals are inclined to follow the uniform distribution function, which means that the passengers arrived randomly. As for a long headway, i.e., longer than 5 min, the times of arrivals are inclined to the log-normal distribution function. In the latter case, the passengers mostly arrived in a few minutes before the bus scheduled arrival. Majority of the arrivals occurred about 4 min before the bus scheduled arrival. Many have identified that the 5-min-headway time as the transition of the arrival patterns. We should note that Fan and Machemehl[17] identified the headway of 10 minutes as the transition between the two distributions.

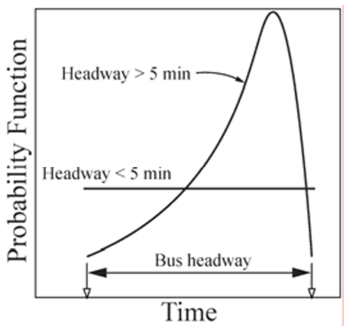


Fig. 3 Probability of passenger arrivals as a function of the bus headway

Those literatures, some are traditional, established the passengers waiting time (w) as a function of the bus headway (h). Most literatures agreed that $w = h/2$ for a short headway[17]. This particular model was established on top of three assumptions: (i) the passengers arriving randomly, (ii) the bus arriving regularly, and (iii) the passengers getting on their first bus. In reality, the BRT bus headways may vary considerably[21]. To take this aspect into account, Osuna and Newell[22], Holroyd and Scraggs[23] and Welding[24] advised:

$$w = \frac{\mu}{2}\left(1 + \frac{s^2}{\mu^2}\right) \tag{1}$$

where w is the expected passengers waiting time; μ is the mean headways between the buses; and s^2 is the variances of headways between the buses. In addition, various empirical formulas have been proposed, for example:

$$w = 1.79 + 0.14\mu \quad \text{O'Flaherty and Mangan}^{[15]} \tag{2}$$

$$w = 2.34 + 0.26\mu \quad \text{Seddon and Day}^{[25]} \tag{3}$$

$$w = 2.00 + 0.30\mu \quad \text{Salek and Machemehl}^{[16]} \tag{4}$$

To achieve those described features, we designed the station sub-system as shown in Fig. 4. Essentially, the sub-block consists of two parts: the lower sub-block and the upper sub-block with two different entity-types. The entity in the lower sub-block represents a passenger; meanwhile, the entity in the upper sub-block represents a bus.

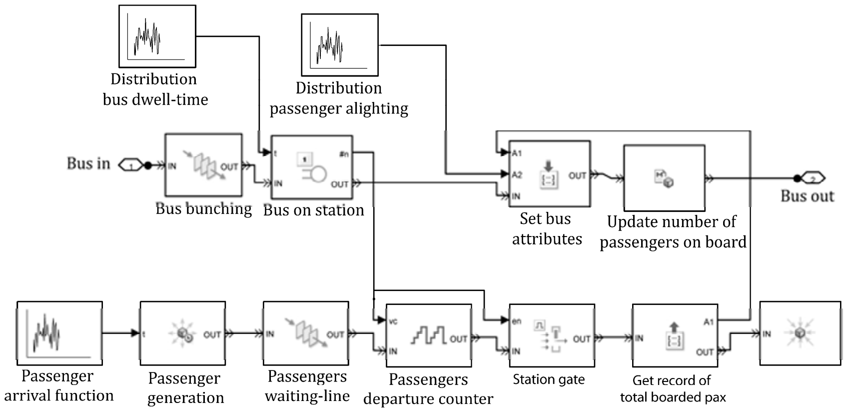


Fig. 4 SimEvents block components for the BRT bus station.

The lower sub-block starts with the passenger arrival function. Basically, the function is a statistical function that defines the nature of the passenger arrivals. The block signals the passenger generation block to initiate arrival of passengers. The generated passengers will then be transferred to a first-in first-out (FIFO) waiting-line where the passengers will wait until the opening of the station gate. The station gate will receive a signal from the upper sub-block. The signal will be initiated in the upper sub-block when a bus arrives and dwells at the station.

The upper sub-block starts with a port, which allows a bus to approach the station. The current design of the upper sub-block allows a bus to bunch at the station. The bus bunching is one of the biggest issues faced by the modern BRT system. When the station is empty, the bus will dwell at the station. The dwell-time will be determined by the distribution from bus dwell-time block. The bus entity will then receive data regarding the number of alighting passengers and the number of passengers waiting in line to board the bus. Finally, the bus will update those data, which are maintained as the bus attributes data, and departure data from the station to the next station.

2.3 The road-segment sub-system

This sub-system has to be able to simulate the traveling of a BRT bus along its corridor in a segment of road connecting two adjacent stations. This sub-system is characterized by the bus travel time and the number of buses accommodated by the road segment. Therefore, this sub-system is also modeled with a simple queue-server model having the FIFO queuing policy. This approach is appropriate considering the fact that majority of the busway is single lane; hence, a bus has no possibility to overpass other buses.

A number of studies and observations indicate that the travel time for the case can reasonably be approximated with the exponential distribution function with a single controlling parameter of the average travel speed of the bus or the average bus travel time (for an example, see Gunawan et al.[21]).

Fig. 5 shows Matlab SimEvents implementation of the design. The design allows buses to bunch on the road-segment, and the bus travel time will be determined by a block, which specifies the statistical distribution of the bus travel time.

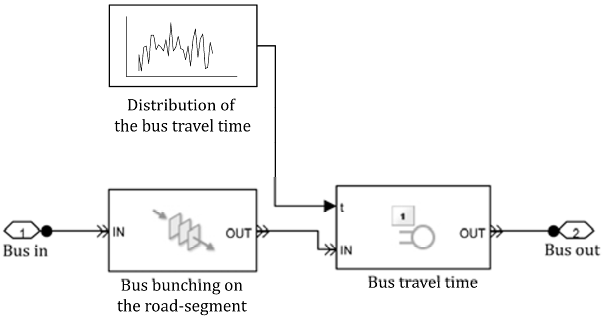


Fig. 5 SimEvents block components for the road-segment sub-system

2.4 The pool of buses sub-system

This sub-system has to be able to generate entities at a scheduled time where each entity represents a BRT bus. Our current SimEvents implementation is shown in Fig. 6. The sub-system is started with the Time Schedule for Bus Departure Block where the buses departure schedule is specified. The bus schedule specification is deterministic. When the bus has arrived, the Bus Departure Block will release a bus per unit time. Hence, the current design does not allow the bus pool sub-system to serve more than one bus per unit time. Subsequently, the sub-system will record the bus departure time, assign a bus number, and finally, create or attach each bus with some attributes. Those attributes are variables to hold the passenger data. At the current implementation, those attributes are the number of passengers on the bus, and the number of alighted and boarded passengers on the bus last station.

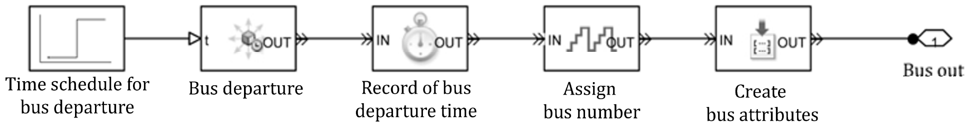


Fig. 6 SimEvents block components for the pool of buses sub-system

2.5 The traffic-signalsub-system

This sub-system has to be able to simulate traveling of the BRT buses across a manually controlled traffic-signal. Therefore, the system is characterized by the cycle time of the traffic-signal, t_c, and the duration of the green signal, t_g. Also, the sub-system has to allow vehicles queuing for the traffic-signal.

We designed a sub-system as shown in Fig. 7 to achieve the purpose stated above. Basically, the sub-system consists a FIFO queue block that facilitate the buses to wait for the green light. The FIFO block is then followed by a traffic signal block, which basically is a release gate block. The gate will open according to the time set by the traffic signal timing block. When the gate is open, the bus is allowed to move forward. The timing block is a time function regulated by two variables: the traffic-signal cycle-time, t_c, and the green-signal duration, t_g.

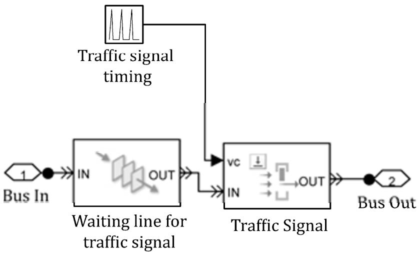


Fig. 7 SimEvents block components for the traffic signal sub-system.

Fig. 8 demonstrates the use of this sub-system. For this case, 30 vehicles were released from a pool, the cycle time was set to 20 s, and the duration of the green signal was set to 5 s. The figure indicates that the sub-system allowed vehicles to travel during the green traffic-signal duration, and fully blocked during the remaining time.

3 Numerical trials

3.1 Passenger arrival models

In this section, we present the results, and discussion, of a numerical study that performed to evaluate the passenger arrivals on a BRT station and their relation to the bus arrivals. The relation is rather simple that more passengers are expected to be in queue when the bus headway is longer. In addition, the passengers waiting time, obtained from the present computational model, should also agree, up to some extent, with those predicted by the existing passenger waiting models (see Eq. 1–Eq. 4).

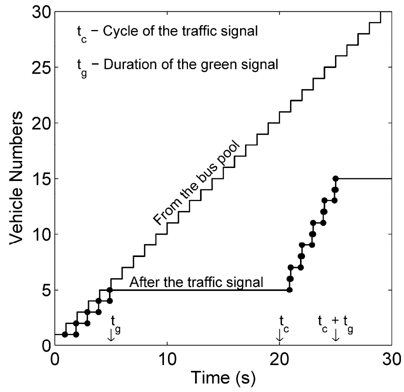


Fig. 8 A test case of the traffic signal sub-system

Before the discussion, we should restate a few notes regarding the present BRT station model. Basically, the station sub-system consists of two sub-blocks: the upper and lower sub-blocks. The upper sub-block consists of queue-server components that allow a BRT bus to bunch and settle at the station, and the lower sub-block only models the passenger arrivals. The latter sub-block generates entities to represent passengers, and holds the passengers in queue until the bus arrives on the upper sub-block. Once the bus settles at the station on the upper sub-block, the block will send a signal to the lower block that the bus has arrived. The lower sub-block will open a gate to board passengers, will count the number of passengers boarding the bus, and will send the data to the upper sub-block. The received data of the number of passengers will be maintained on the upper sub-block in the bus attribute data.

To evaluate the model of a BRT station, we established a numerical model with the following input data. The passengers would arrive randomly at the station at an average rate of 2 passengers/min following the exponential distribution. This data was a norm in many TransJakarta BRT station as observed earlier by Gunawan et al.[20]. The buses would arrive at the station randomly according to the exponential distribution after traveling on a road segment. The mean of the bus headways was 5 min. This assumption is based on observation of Gunawan et al.[21], who measured the travel times of TransJakarta BRT (see Fig. 9).

The arrived bus at the station would wait for the passengers to board and to alight. The waiting time was set random according to a triangular distribution with the minimum, maximum, and mode times of 1 min, 3 min, and 2 min, respectively.

The simulation was executed for a final simulation time of 5000 min involving about 1 000 buses. However, only the results of the first 50 min simulation time are reproduced in Fig. 10.

Some notes outlined from the results are the following. Fig.10(a) shows the arrival and departure data of the first-nine buses. The data are reproduced in Table 2, and then used to determine the actual dwell time and headway. The data indicate that the third bus departed when the fourth bus arrived. The same thing also occurred for the fourth and fifth buses, the sixth and seventh buses, and the seventh and eighth buses. Therefore, we should not expect any queuing building up before the arrival of the fourth, fifth, seventh, and eighth buses. Finally, we expect the first bus to gather more passengers than the other buses due to long bus-headway.

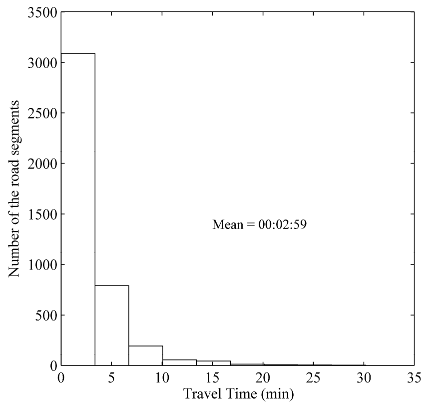


Fig. 9 Distribution of the travel times between two-adjacent stations of TransJakarta BRT[21]

Table 2 The arrival, departure, dwell, and headway times of the first-nine buses on the simulation.

Bus Number	1	2	3	4	5	6	7	8	9
Arrival Time (min)	10.4	19.1	24.6	27.1	29.5	35.3	37.5	39.5	45.8
Departure Time (min)	13.0	21.2	27.1	29.5	31.1	37.5	39.5	41.9	48.0
Dwell Time (min)	2.6	2.1	2.5	2.5	1.6	2.2	2.0	2.4	2.2
Bus Headway (min)	10.4*	6.0	3.4	3.4	0.0	4.2	0.0	0.0	3.9

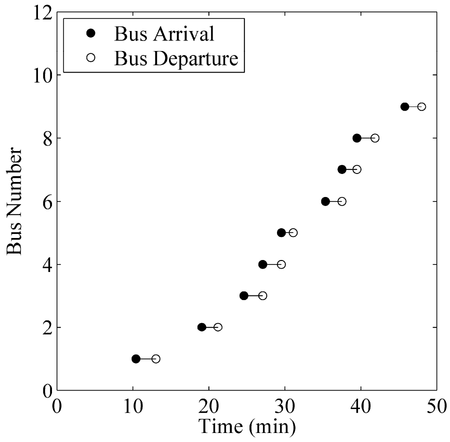
*Calculated from the starting of the simulation

Fig. 10(b) shows the arrival of the passengers at the station that are presented as a counting function $N(t)$, which is only defined for $t \geq 0$. It is clear that the passenger arrivals were a stationary Poisson process. Fig. 10(c) shows the passengers boarding the bus. In this case, the first-twenty-eight passengers boarded the first bus including 23 passengers that had to wait for the bus, and 5 passengers arrived in time with the bus. In this figure, the passenger waiting time is marked with a long vertical line. The data also support the previous assessment that the passengers did not need to queue for the fourth, fifth, seventh, and eighth buses. Fig. 10(d) shows the number of passengers boarding on the first-nine buses. The first bus picked up 28 passengers, the most among these buses. This phenomenon agrees well with our early observation on the basis of the bus headway. Fig. 10(e) shows the passenger waiting time distribution and fig. 10(f) shows the distribution in form of a boxplot. The boxplot is also overlaid with the empirical estimations of the waiting times given by Eq. 1–Eq.4, and the both seem to have a strong agreement. In fact, the estimations of three out of the four models were above the median and below the upper quartile of the simulated results.

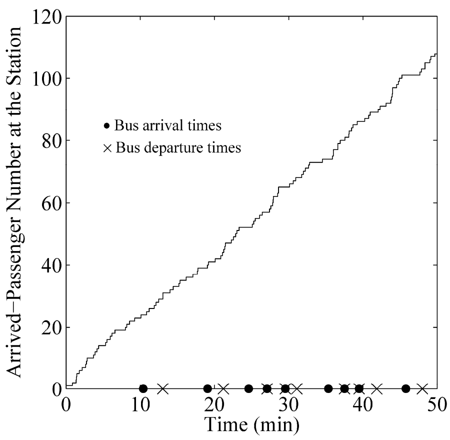
Those evidences lead us to a conclusion that the present numerical passenger arrival model is suitable to model the arrival of passengers on a BRT station.

3.2 Simple corridor

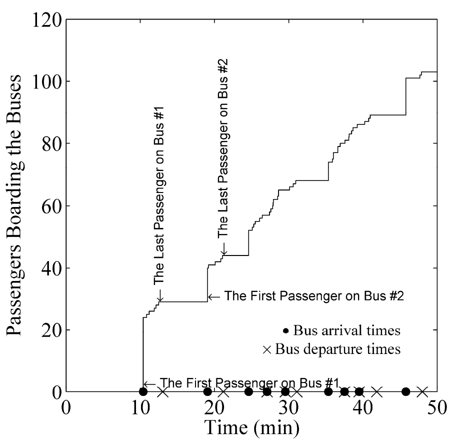
We established a small-symmetric BRT network for this case of numerical trials. The network consists of a bus pool and two BRT stations that are connected by two road-segments. In the simulation, two buses were dispatched from the bus pool and entered the network. Once the buses were in the network, they would circulate from Station 1 to Station 2, then, from Station 2 to Station 1, and so on. The network is shown in Fig. 11 as a SimEvents model.



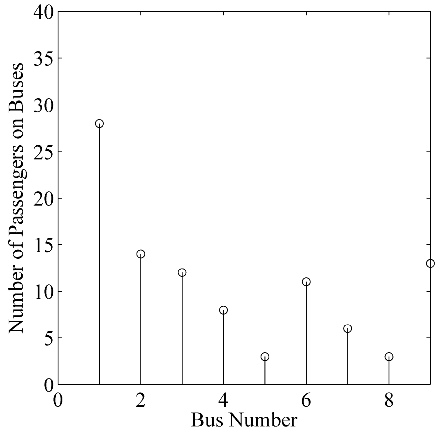
(a) Bus number arrived and left the station. Only the first 50-minute simulation time is shown



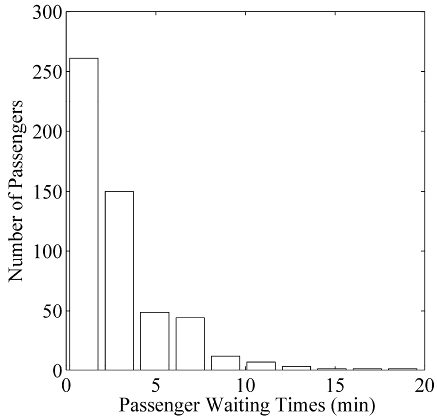
(b) The passengers arrived at the station



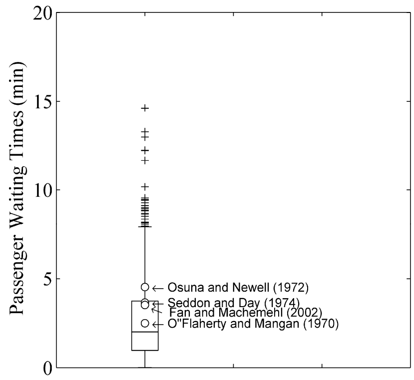
(c) The passengers boarding the buses



(d) The passenger on the buses



(e) The distribution of the passenger waiting times



(f) The boxplot of the passenger waiting times and its comparison to some mathematical model

Fig. 10 Simulation results

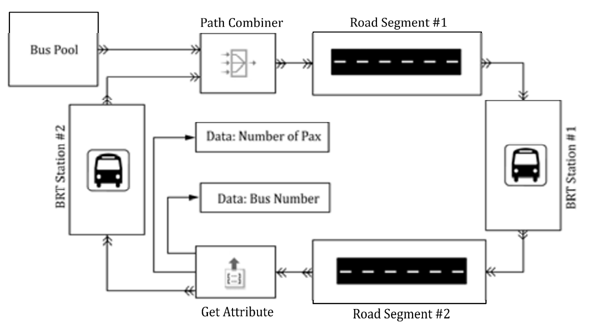


Fig. 11 The SimEvents-numerical model of the two-station corridor

Besides the sub-systems of the road-segments, the bus pool and BRT stations, the SimEvents model above also has other sub-systems: Path Combiner and Get Attribute. The earlier sub-system allows the bus, either from the bus pool or from the station #2, to travel to the road segment #1. The latter sub-system allows us to extract the bus internal data or the buses attribute data; in this case, the data were the bus number and the number of passengers on board.

A complete list of the model parameters, associated with the SimEvents-numerical model, is given in Table 3. Those parameters were for the BRT stations, the road-segments, and the bus pool. The two BRT station were assumed to be identical, and so do the two road-segments. As for the BRT station, the passengers were assumed to arrive according to a Poisson process with an arrival rate of 1 person/min, slightly lower than that observed by Gunawan et al.[20]. The number of alighted passengers was assumed to be discretely and uniformly distributed between 0 and 25 persons. Meanwhile, the bus service time was short with a mode of 2 min. As for the road-segment, the bus was assumed to travel with a duration following the exponential distribution function with a mean of 5 min. The exponential distribution for the bus traveling time was also observed by Gunawan et al.[21]. Finally, the bus inter-departure time was set to exactly be 5 min that was based on the BRT standard for the bus frequency during the rush-hours[1].

Table 3 Parameters of the simple corridor model

Model Parameters	Characteristics	Value
BRT station		
Passenger Inter-arrival Time	Stochastic; Exponential Distribution	Mean = 1 person/min
Number of Alighted Pax	Stochastic; Uniform Distribution	Min. = 0, Max. = 25 persons
Bus Service Time	Stochastic; Triangle Distribution	Min. = 1 min, Max = 3 min, Mode = 2 min
Road segment		
Travel Time	Stochastic; Exponential Distribution	Mean = 5 min
Bus pool		
Bus Inter-departure Time	Deterministic	5 min

The time frame for the simulation was 24 hours although BRT usually operates about 18 hours per day, and the analysis was replicated for 200 days. The simulation results are reproduced in Fig. 12–Fig.14.

The results in Fig. 12 show the distribution of passenger numbers on the time-averaged sense on both the buses when the analysis has reached its steady-state condition. The figure reveals an interesting phenomenon that the distribution of the time-averaged number of passengers was rather similar for both buses. This fact seems acceptable by considering the model design and its input parameters that were designed to be symmetric. The actual numbers of passengers on the two buses were 14 passengers on average with a deviation of 18 passengers. When the buses left the BRT Station #2, they were boarded with about 8 time-averaged number of the passengers for 85% of the cases. On a few cases, the second bus was boarded by about 60 passengers. Variation of the number of passengers was much wider on the second bus, about twice wider than that on the first bus.

The time history data of passenger numbers are reproduced in Fig. 13(a) for the first bus and in Fig. 13(b) for the second bus. On the right side of each figure, a boxplot is provided to show the final state of the time-averaged number of passengers for 200 replications. In general, the passenger numbers was highly fluctuating in the range of 0 to about 70 passengers. For the historical data depicted in the figure, the passengers were on board for 62% of the time for the bus 1 and 55% of the time for the bus 2.

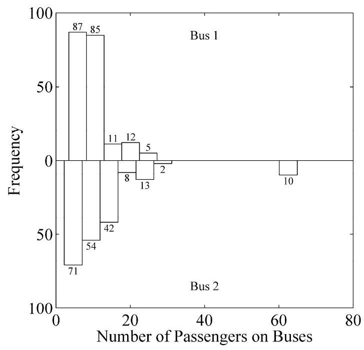
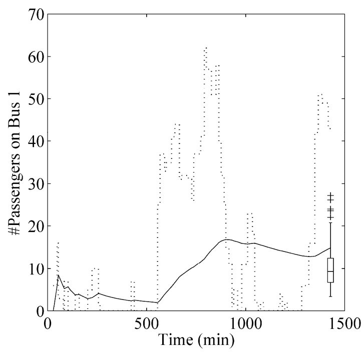
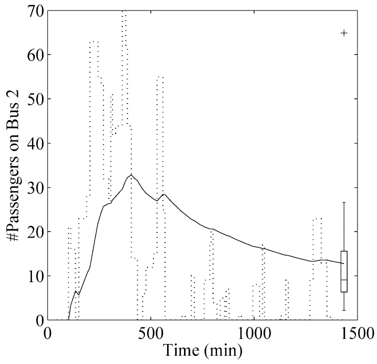


Fig. 12 Bi-histogram of the distribution of the time-averaged number of passengers on both the buses in steady-state condition for 200 replications



(a)



(b)

Fig. 13 The history of the number of passengers on Buses 1 and 2 (broken line), time-averaged (solid line), and a boxplot of the time-averaged number of passengers at the end of the analysis

4 Conclusions

This paper has discussed a potential use of the simulation framework to model the dynamics of a bus-rapid transit system. The framework was developed on the basis of the standard features existed in a common discrete-event simulation system. For this particular application, significant complexity of modeling was found in modeling a BRT station. A few assumptions have to be made to allow development of the sub-system. The established model, in limited numerical trials, systematically produced well observed phenomena of the actual BRT system.

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