

Two-Vehicle Dynamics of the Car-Following Models on Realistic Driving Condition

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Abstract: The paper discusses the traffic dynamics in microscopic level and analyzes the dynamics characteristics of the traditional Gazis-Herman-Rothery model, the optimal velocity model with delay, and the intelligent driver model. An essential feature differentiating those models is that the traditional Gazis-Herman-Rothery model only governs the vehicle dynamics in the car-following state, but the other two models encompass larger interaction state including the free-flow state and the acceleration from the vehicle initial state. From this study, it can be concluded: (i) the optimal velocity model and intelligent driver model are more complete than the traditional model; (ii) the existing optimal velocity model may produce an unrealistic vehicle interaction; (iii) the optimal velocity model with a realistic delay can produce a stable interaction, and (iv) the intelligent driver model still needs further development particularly to take into account the driver delay which is an important aspect in the traffic dynamics on the microscopic level, and finally, (v) those three models may produce similar dynamics characteristics.

Key Words: intelligent transportation; optimal velocity model; intelligent driver model; micro-simulation; car-following model

1 Introduction

The continuous-in-time car-following models have been studied for almost 60 years since the early publication of Pipes^[1]. This development is well-captured by Backstone and McDonald^[2]. At some point, the development converged to the Gazis-Herman-Rothery (GHR) model or the General Motor nonlinear model^[3–6]. Despite of this fact, we still find many new developments in this area to address limitations existed in the General Motor nonlinear model. For example, the model becomes more complex when we take into account realistic driving behaviors such as existence of a non-symmetric relationship between vehicle acceleration and its deceleration. To accommodate this fact, Aron^[7] and Ozaki^[8] suggested differentiating the model parameters for the vehicle acceleration from those for the deceleration. However, the approach increases complexity in implementing the governing equation in a micro-simulation framework because the status of each vehicle, accelerating or decelerating, must be checked on every time-step.

Beside the General Motor nonlinear model, the intelligent driver model (IDM)^[6,9,10] and the optimal velocity (OV) model

with delay^[6,11–14] also receive major attention currently. These two developments seem to be able to address a number of drawbacks of the general motor nonlinear model with a reasonable expense. In the case of IDM, the model has increased its number of model parameters to eight. The general motor nonlinear model only has four parameters including a delay parameter. From those eight parameters, some are used to define the vehicle acceleration, and some for the vehicle deceleration. Unlike the general motor nonlinear model, the user does not require to evaluate the vehicle status because the vehicle acceleration will automatically be governed by relevant model parameters depending on the vehicle state. The model unifies the acceleration and deceleration terms into a single formula, and the acceleration term will dominate the deceleration term when the vehicle accelerates, and vice versa. The optimal velocity method is rather similar to the IDM model. However, the user should supply a complete velocity function, not only model parameters.

In this paper, we review those models mentioned above. Particularly, we look into their features, compare one to others, conclude similarities and differences, and demonstrate that

those models can produce similar vehicle dynamics by properly tuning their parameters. In addition, we also raise an objection on the optimal velocity function proposed by Koshi et al.^[15], which has been considered as realistic by some researchers. In addition, we also scrutinize the stability of the delayed optimal velocity model of Davis^[16].

We limit our discussions to those three major models in the microsimulation. Both the GHR model and IDM model were developed on the basis of the stimulus-response model. The IDM is the most recent development in this group. Meanwhile, the OV model is mainly on the basis of the driver desired velocity. Another major model, not discussed in this paper, was developed on the basis of a safe braking distance or a collision-free driving^[17-21]. In this group, no differential equation needs to be solved. Hence, the computation cost is quite low. Another group that receives attention these days is those on the basis of the cellular automata^[6,22,23]. This particular approach is interesting because of its efficiency and fast performance.

2 Two-vehicle dynamic problem

On the microscopic level of the traffic simulation, a greater attention has been paid to the interaction of vehicles in a lane, particularly between a vehicle and its leader. The two-vehicle dynamic problem is the simplest model on the level that can be used to fully understand the nature of the interaction. Although the problem is extremely simple, but it reveals all necessary features of the car-following interaction and yet, the problem could accurately produce the characteristics of the traffic on the macroscopic level^[9]. Therefore, we selected and utilized the two-vehicle problem for the purpose of the present study.

The two-vehicle problem essentially consists of a car approaching its leader. A rather large initial spacing is necessary to fully model the interaction spanning the acceleration state from a zero initial velocity, the free-flow state, and finally the car-following state. We need to note that the modern car-following models, such as the IDM and the OV model, govern those three states, unlike the traditional GHR model that only governs the vehicle dynamics in the car-following stage. This feature is a great advantage of the two models over the traditional model because it significantly simplifies the model implementation.

3 Theory

3.1 Traditional Gazis-Herman-Rothery Model

This traditional model^[3] has many names: Gazis-Herman-Rothery model, the General Motor nonlinear model, and the L&M model^[24]. The first name is clearly after the authors of the seminal paper^[3], the second is after the authors company, and the last name is because the model has

the constants m and l . The model can be written as:

$$a_n(t) = \alpha \frac{v_n^m(t) \cdot \Delta v_n(t - t_d)}{\Delta x_n^l(t - t_d)} \quad (1)$$

where $a_n(t)$ is the vehicle acceleration, $v_n(t)$ is the vehicle velocity, Δv_n and Δx , respectively, are the relative velocity and the relative position with respect to the leading vehicle. The model has three parameters α , m , and l , and has a delay of t_d .

Equation (1) was established on the basis of experimental data and an extensive use of the correlation analysis. Its historical development is well-documented by Brackstone and McDonald^[2], and also by Gazis^[24], which succinctly presented his personal account on the model development. Eq. (1) is nonlinear as Gazis *et al.*^[3] so strongly believed; they wrote, “nevertheless it has again been ascertained that a nonlinear model is necessary to account for observed flow versus concentration data. However, it is not clear, on the basis of presently available data, that a somewhat more complicated nonlinear model has any distinct advantages over the simple nonlinear model.”

A large number of research activities have been performed to address the issue of the model parameters m and l . As a result, many values for the parameters existed, and some that regarded as reliable values are reproduced in Table 1. In the table, the values for those parameters are differentiated mostly between vehicle acceleration and deceleration. Even some differentiate them between breaking and no breaking. These treatments on the model parameters make the GHR model harder to be implemented because the vehicle state, accelerating or decelerating, and breaking or no breaking, has to be checked before a proper model parameter is assigned.

3.2 Optimal velocity model

The optimal velocity method proposed by Newell^[29] can be expressed as^[11,13]

$$a_n(t) = \frac{1}{\tau} (V(\Delta x_n(t)) - v_n(t)) \quad (2)$$

with the velocity relaxation time τ , an optimal velocity function $V(\Delta x_n(t))$, and a distance to the leading vehicle Δx .

Furthermore, Bando *et al.*^[12] revised Eq. (2) by adding a delay t_d due to the driver reaction time, which is a significant feature in the traffic dynamics. They proposed a delayed-differential equation formula in form:

$$\tau \cdot a_n(t) + v_n(t) = V(\Delta x_n(t - t_d)) \quad (3)$$

Table 1 Most reliable estimates of parameters of GHR model^[2]

Source	m	l
Chandler <i>et al.</i> ^[25]	0	0
Herman and Potts ^[26]	0	1
Hoefs ^[27] (dcn no brk/dcn brk/acn)	1.5/0.2/0.6	0.9/0.9/3.2
Treiterer and Myers ^[28] (dcn/acn)	0.7/0.2	2.5/1.6
Ozaki ^[8] (dcn/acn)	0.9/-0.2	1.0/0.2

Note: dcn/acn: deceleration/acceleration; brk/no brk: deceleration with and without the use of brakes.

Table 2 Stopping distance at various speeds^[31]

Velocity (km/h)	Reaction distance (m)	Braking distance (m)
20	6	3
30	8	6
40	11	11
50	14	18
60	17	27
70	19	39
80	22	54
90	25	68
100	28	84

However, Davis^[16] concluded that Eq. (3) is incomplete in describing the traffic dynamics in the microscopic level because t_d must be set to an unrealistic small value, significantly smaller than the typical driver reaction time, in order to maintain a stable stream of a platoon of vehicles.

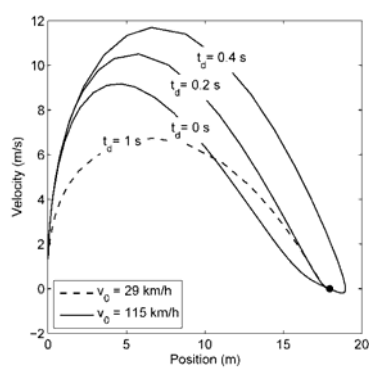
As for the driver reaction time, many existing data indicate that 1 s is a reasonable value. The data in Table 2 of Japanese regular passenger give us the reaction time of 1.00 ± 0.03 s for the vehicle velocity in range of 20 km/h up to 100 km/h. In the table, the reaction time is associated with the reaction distance; the sum of the reaction distance and the braking distance gives us the stopping distance. Others data, obtained from the study of Mehmood and Easa^[30], show the reaction time in range of 1.32 s and 1.39 s on the normal condition, but it may drop to a range of 0.70 s and 0.74 s in a stationary condition.

As for the optimal velocity function, it has to satisfy two properties. The function should monotonically increase and should have an upper bound^[11]. Those behaviors clearly exist on a sigmoidal function such as $\tanh(\Delta x - 2) + \tanh 2$ ^[11]; hence, many previous studies relied on such a function^[11,12,30,32]. However, Newell's early proposal was $V(\Delta x_n) = v_0(1 - \exp(-(\Delta x_n - s_0)/v_0 T))$, where s_0 is the jam distant and T is the headway time. For a Japanese highway, Koshi, Iwakai and Ohkura^[15], as cited by Bando *et al.*^[12] and Davis^[16] proposed:

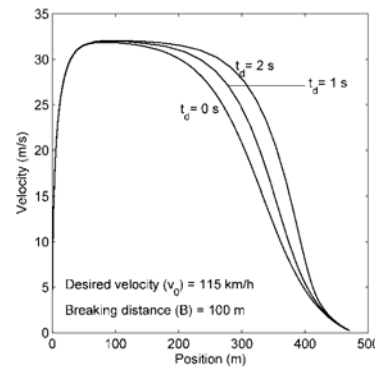
$$V(\Delta x) = 1.68(\tanh[0.086(\Delta x - 25)] + 0.913) \quad (4)$$

Finally, Davis^[16] generalized the optimal velocity function in form of

$$V(\Delta x) = v_0 \left[\tanh\left(\frac{\Delta x - D}{b} - C_1\right) + C_2 \right] \quad (5)$$



(a) A close distance



(b) A reasonable distance

Fig. 1 Effect of the delay on the two-vehicle dynam

where v_0 is the desired velocity, D is the effective vehicle length, B is the braking distance, the length constant C_1 , and the dimensionless constant C_2 .

The vehicle dynamics regulated by Eq. (5) is as follow: (i) the vehicle will start braking at a distance of $D + (C_1 \times b)$ behind the leading vehicle; (ii) the vehicle free-flow velocity will actually reach $v_0(1 + C_2)$. Therefore, to preserve the physical meaning of the parameter v_0 , D and b , it might be convenient to set C_1 and C_2 to zero.

As for the relaxation time t , a value of 0.5 s or 1.0 s has been widely used^[16]. Treiber *et al.*^[9] mentioned that collision can be avoided only if $t < 0.9$ s for the realistic optimal velocity function in Bando *et al.*^[11]. Despite of the fact, a large relaxation time will lead to a slow development of the velocity that is the velocity will slowly increase or slowly decrease. Hence, a large relaxation time may lead to vehicle collision.

Following, we briefly review the work of Davis^[16], who suggested that the optimal velocity method is incomplete on the basis of his numerical studies. His studies were performed on a platoon of 100 vehicles with the initial headway of 25 m, the relaxation time of 0.5 s, and the initial velocity of 15.34 m/s, except the leader that running at a velocity of 14 m/s. Utilizing Eq. (4), he finally found that the delay must be smaller than 0.22 s to avoid collision within the platoon, and this magnitude is unrealistic because the existing data suggest the value should be about 1 s.

We believe his results were based on unrealistic combination of data of the desired velocity, the initial headway and the braking distance. The desired velocity was 32 m/s (115 km/h), the initial headway was 25 m, and the braking distance was 11.5 m. According to Table 2, which provides data for a realistic driving condition, a vehicle will require a minimum braking distance of 100 m at 115 km/h velocity to avoid collision. Or, for the 25 m headway, the vehicle velocity should not exceed 8.3 m/s (30 km/h).

We have performed two numerical analysis for the optimal velocity function with delay using the two-vehicle problem to further study the instability observed by Davis.

In the first study, the vehicle approached its leader at 25 m headway, 1.34 m/s initial velocity, and 32 m/s desired velocity. These data were obtained from Davis^[16]. As a result, we obtained a vehicle dynamic shown in Fig. 1(a) for various driver reaction times or delay. The result supports Davis observation that only at an unrealistically small delay, e.g., $t_d=0$ and $t_d=0.2$ s in the figure, the vehicle could avoid collision. At $t_d=0.4$ s, although this value is smaller than the realistic delay of 1 s, the vehicle collided with its leader. However, the collision can be avoided if the desired velocity is adjusted to a reasonable value for the given headway, which is less than 8 m/s (29 km/h). For this case, the vehicle could avoid the collision with the realistic delay.

In the second study, the above desired speed was maintained, but the braking distance was set to 100 m, and the vehicle was initially located 500 m behind its leader. The result, reproduced in Fig. 1(b), shows that no collision happens even at a significantly high delay time. Hence, we conclude the local stability can be maintained by the optimal velocity method with a realistic delay.

3.3 Intelligent driver model

For a vehicle moving in a lane, the IDM postulates that the vehicle will accelerate according to^[9,10]:

$$a_n(t) = a_{\max} \left[1 - \left(\frac{v_n(t)}{v_0} \right)^\delta - \left(\frac{s^*(t)}{s_n(t)} \right)^2 \right] \quad (6)$$

where s_n is the gap of the n -vehicle. The gap is the headway space, or mathematically, is expressed as

$$s_n(t) = x_{(n-1)}(t) - x_n(t) - L_n$$

where L_n is the actual vehicle length. The s^* denotes the desired minimum gap, and is defined as:

$$s^*(t) = s_0 + s_1 \sqrt{\frac{v_n(t)}{v_0}} + T v_n(t) + \frac{v_n(t) \Delta v_n(t)}{2 \sqrt{a_{\max} a_{\min}}} \quad (7)$$

In total, the IDM acceleration model has eight parameters, which can be categorized into three groups. A parameter in the first group determines the steady state motion of the vehicle; three parameters in the second group determine the vehicle acceleration/deceleration; finally, four parameters in the last group determine a non-collision gap with respect to the front vehicle. The member of the first group is the vehicle desired velocity v_0 . Meanwhile, the members of the second group are the vehicle maximum acceleration a_{\max} , the minimum deceleration a_{\min} , and the acceleration exponent δ . As for the last group, the members are the safe time-headway T , the jam distances s_0 and s_1 , and the vehicle length D . Non-zero s_1 is only necessary on special conditions such as when equilibrium flow-density requiring an inflection point^[9].

4 Dynamic characteristics of GHR, IDM, and OV models using two-vehicle problem

In this section, we study the dynamic characteristics of the GHR, IDM and OV models by means of the two-vehicle problem. For the purpose, the leading vehicle on the problem

is halted at 500 m ahead, and the other vehicle approaches the leader at a certain free-flow velocity.

4.1 Dynamic characteristics of two-vehicle model based on Japanese high-way condition

The vehicle dynamic characteristics on a Japanese highway have been quantified by Koshi *et al.*^[15], and expressed in an optimal velocity function of Eq. (4). By associating the equation with Eq. (5), we obtain $v_0=17$ m/s (61 km/h), $D=5$ m, $b=11.6$ m, $C_2=1.72$ m and $C_2=0.913$. We also obtain the final headway Δx of 5 m. These data imply that the vehicle has an actual free-flow velocity of 32.1 m/s (115.7 km/h) and it will start to apply the brake at a distance of 25 m behind the leading vehicle.

The vehicle dynamic characteristics are clearer when we study using the two-vehicle problem. In this problem, the vehicle was placed 500 m behind its leader and had a zero initial velocity.

The results of the study are reproduced in Fig. 2 with the solid line labeled ‘OV Model’. One striking feature shown by Fig. 2(a) is that the vehicle velocity quickly rises from zero velocity to its actual-desired velocity, and then, quickly decreases to zero velocity. The numerical computation indicated that the vehicle requires a maximum acceleration of 64.3 m/s² and a maximum deceleration of 25.6 m/s². These results are in contrast with following actual data: (i) the maximum deceleration of an actual vehicle is usually higher than the maximum acceleration; (ii) the maximum acceleration and deceleration of a modern passenger vehicle is significantly lower; The BMW 523i, for an example, only has a maximum acceleration of 3.5 m/s². In addition, Treiber *et al.*^[9] noted that the acceleration and deceleration for normal traffic should be within 1 m/s² to 2 m/s². Therefore, we conclude that the obtained optimal velocity function for the Japanese highway may produce an unrealistic acceleration and deceleration, and the braking distance is too short for the velocity.

Following, we numerically demonstrate that the above vehicle dynamics can also be achieved using the GHR and IDM models. From the above data and analysis, we obtain following parameters for the IDM model: $v_0=115.7$ km/h, $L=5$ m, $T=0.6$ s, $a_{\max}=64.3$ m/s², $a_{\min}=25.6$ m/s², and $s_0=s_1=0$. It is difficult to obtain the IDM acceleration exponent directly; therefore, we perform the analysis using $\delta=1$ and 2, which are common values for the parameter. Finally, we obtain the GHR data using the least-square method: $m=0.3$ and $l=0.5$, and the sensitivity coefficient τ can be related to the velocity relaxation time τ by $\alpha=1/\tau$. For the present study, $\tau=0.5$ s.

The result that the headway time is 0.6 s gives us an additional reason to question Koshi *et al.*^[15] proposal of the Japanese highway optimal velocity function. In fact, the realistic headway time should be within 0.8 s and 2.0 s as recommended by many German driver schools.

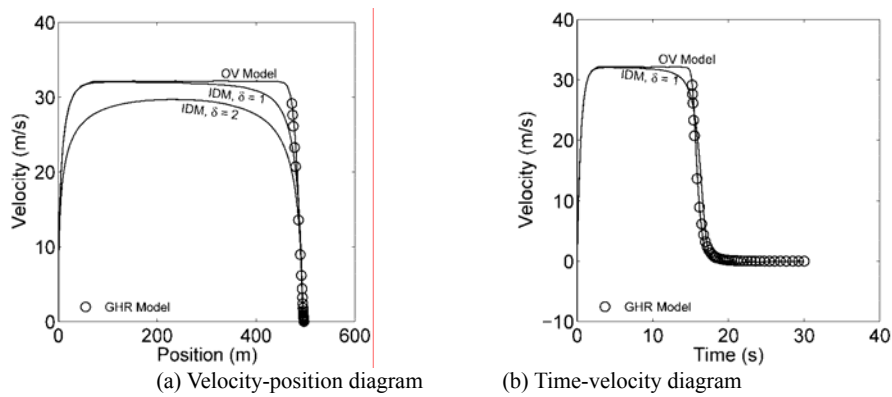


Fig. 2 Vehicle dynamic characteristics using Japanese highway model of Koshi *et al.*^[15]

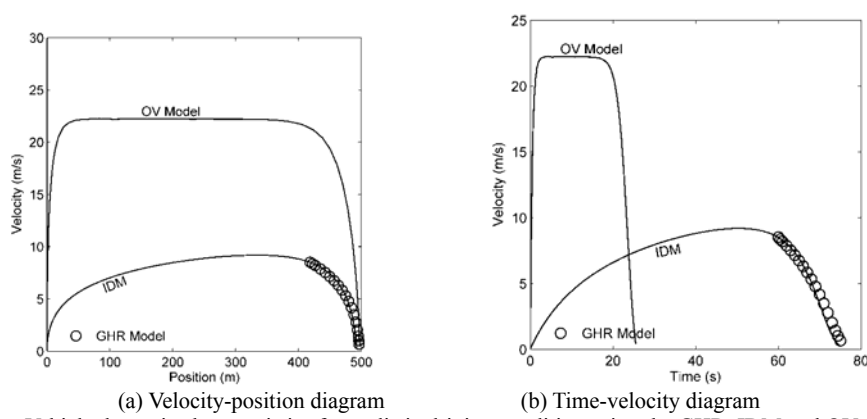


Fig. 3 Vehicle dynamic characteristics for realistic driving condition using the GHR, IDM and OV models

When we compare the dynamic characteristics obtained by GHR and IDM models, as shown in Fig. 2, to those of the OV model, we find some interesting points of following:

Some equivalence exists between the IDM model and the OV model: the free-flow velocity of the OV model, $v_0(1+C_2)$, is equivalent with the desired velocity of the IDM model, the vehicle effective length D is equivalent with $L+S_0+S_1$ in the IDM model, and finally, the braking distance b is inversely proportional with T .

The only parameter in IDM that could not be reduced from the OV model is the acceleration exponent δ . The effect of the parameter is shown in Fig. 2 where the results for $\delta=1$ and $\delta=2$ are provided. Regarding the acceleration exponent, we conclude that $\delta=1$ of the IDM provides a best fit for the above Japanese highway velocity function. However, this result contradicts with the following: a rather clear separation between the non-congested traffic and congested traffic existed on the fundamental diagram of a Japanese highway as shown in Fig. 1 of Sugiyama *et al.*^[33]. From the IDM perspective, such a relation can be produced by the model when δ is set to a large number, theoretically, $\delta=\infty$. The small value of δ , for an example $\delta=2$, is much more relevant with an urban motorway characteristics. Therefore, the good fit of $\delta=1$ for highway traffic is inconsistent with the general

characteristics of the IDM.

Regarding the GHR model, equivalence of its model parameters with the remainder models does not exist. Therefore, the model parameters were obtained by means of the least-square method. The obtained value of $m=0.3$ is close to that obtained by Hoefs^[27] for the case of deceleration with braking, and $l=0.5$ is close to that obtained by Ozaki^[8] for the same case. Finally, we note that the GHR model is only applicable when the two vehicles are in a close distance. As shown in Fig. 2, as soon as the vehicle reached the braking distance, its movement can accurately be described by the GHR model.

The above numerical trial clearly shows that the three models could produce almost similar vehicle dynamic characteristics. The IDM and OV model could model the dynamic characteristics spanning the initial acceleration stage, the free-flow stage and the car-following stage; however, the GHR model is only applicable for the last stage.

4.2 Dynamic characteristics of two-vehicle model based on realistic driving condition

The IDM has many more parameters in comparison to the rest two models; it allows us to control the model interaction not only in the free-flow velocity, and the safe time-headway but also the acceleration of the vehicle. Despite of this fact,

the present IDM model does not take into account the driver delay.

We study a vehicle dynamic for the IDM parameters of following: $\delta=4$, $a_{\max}=0.7 \text{ m/s}^2$, $a_{\min}=1.7 \text{ m/s}^2$, $S_0=2$, $S_1=0$, $L=5 \text{ m}$, $T=1.6 \text{ s}$, $v_0=80 \text{ km/h}$. Those data are equivalent with $D=7 \text{ m}$, and $B=4 \text{ m}$ for the OV model. In addition, we assume $t_d=1 \text{ s}$ and $\tau=0.5 \text{ s}$ ($\alpha=2 \text{ 1/s}$ for the GHR model). As for the GHR model, we obtain $m=0.2$ and $l=1$.

On this driving condition, the dynamics characteristics of the vehicle are shown in Fig. 3. We note following behavior: due to the limitation applied on the vehicle maximum acceleration, the vehicle in the IDM model is not able to reach the desired velocity within this short distance. The IDM disregards the desired velocity to maintain the vehicle acceleration within the prescribed limits. However, since the acceleration is not regulated in the OV model, the acceleration quickly arises such that the vehicle reaches its desired velocity.

5 Conclusions

We have scrutinized three continuous-in-time car-following models: the traditional Gazis-Herman-Rothery model, the optimal velocity model with delay and the intelligent driver model. An essential feature differentiating those models is that the traditional Gazis-Herman-Rothery model is only valid when a car involves in a car-following state; meanwhile, the other two models also govern vehicle dynamics in the free-flow state and in the accelerating state. Therefore, it is easier to implement the last two micro-simulation models into a microscopic simulation system. The intelligent driver model is the most complete among them; however, it still needs improvement particularly in respect with the driver delay, which is a significant feature existing in the traffic dynamics in the microscopic level. The existing optimal velocity model also needs improvement in the issue of finding a physically acceptable optimal velocity function. Finally, we note that the Gazis-Herman-Rothery model is only accurate in a limited domain in the car-following regime, and on common selection of parameters m and l , the model led to a stiff delay-differential equation that hard to converge upon the numerical integration.

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