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An operational, nonlinear input-output system*

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Abstract: We develop a scale-dependent nonlinear input-output model which is a practical alternative to the conventional linear counterpart. The model contemplates the possibility of different assumptions on returns to scale and is calibrated in a simple manner that closely resembles the usual technical coefficient calibration procedure. Multiplier calculations under this nonlinear version offer appropriate interval estimates that provide information on the effectiveness and variability of demand-driven induced changes in equilibrium magnitudes. In addition, and unlike linear multipliers, the nonlinear model allows us to distinguish between physical and cost effects, the reason being that the traditional dichotomy between the price and quantity equations of linear models no longer holds. We perform an empirical implementation of the nonlinear model using recent interindustry data for Brazil, China and United States. When evaluating the robustness of the derived marginal output multipliers and the induced costs effects under the nonlinear approach, the results indicate that marginal indicators in physical terms can be perfectly used to infer average impacts; this is not the case, however, for the derived costs effects where average measures are seen to be more adequate. At the computational level, the analysis proves the operational applicability of the nonlinear system while at the methodological level shows that scale effects are relevant in determining sectoral multipliers.

Keywords: nonlinear input-output, scale-dependent equilibrium, general multipliers.

JEL: C67, D57, 022, R1

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1.INTRODUCTION

There is a glaring contrast between the theoretical advances in nonlinear input-output (NIO) theory and the surprisingly scarce list of applications in the empirical literature. This divorce cannot be attributed to the computational requirements for solving nonlinear models. With today's specialized software computation should not be a decisive issue. The question probably lies on the informational requirements needed for the implementation of NIO models, particularly on sectoral response elasticities. As Lahiri (1983) acutely points out, empirical estimation of NIO models is nearly impossible—too many parameters to estimate given the available data observations. The same type of problematic data requirement situation is also common for the specification of computable general equilibrium (CGE) models but this has not stopped practitioners at all (see Dervis *et al*, 1982, Mansur and Whalley, 1984). CGE modeling and research has become a very important area for policy analysis and evaluation and this has been possible, in part, thanks to the adoption of operational assumptions on agents' behavior and the use of calibration techniques. We believe practical implementation of NIO models is equally possible once we accept (i) some specific behavioral rules in the definition of production activities and (ii) are able to use observed empirical data for the parameterization of production processes.

The theory of NIO models has been concerned with establishing theorems that prove existence and uniqueness of solutions for a nonlinear version of the Leontief input-output (I-O) quantity equations. Under quite general conditions, but all of them sharing a modified system productivity assumption, existence and uniqueness can be proved. Sandberg (1973), Chander (1983), Fujimoto (1986), Szidarovsky (1989), Herrero and Silva (1991), among others, provide the necessary theoretical background for NIO logical consistency. In a NIO model technical coefficients are not taken as fixed. Their variability can be attributed to many different factors (technical innovation, input substitution, productivity changes, non-homogeneity, etc.) as Rose (1983) very clearly explains in his review and assessment paper. Theorists need not concern themselves with these possibilities but applied economists should at least explore them and consider how to sensibly incorporate them. The nonlinearities we consider in this paper are of the scale-dependent type, i.e. changes in total output need not be proportional to changes in total inputs but still a unique production mix is all that is available to firms. In other words, isoquants are L-shaped but the isoquant map is not necessarily homothetic. Price-induced nonlinearities in coefficients due to smooth input substitution, as dealt with in Tokutsu (1989) or Sancho (2010), are not considered here where we focus on the role of scale effects. West and Gamage (2001), in turn, is one of the few empirical examples of using a nonlinear assumption although restricted to the households' income account, where average coefficients are substituted by marginal ones. Zhao *et al* (2006) introduce a Cobb-Douglas production function for defining the interindustry technical coefficients but since their model does not contemplate any price behavior whatsoever, the selection of the input mix is very much based on some *ad-hoc* assumptions—such as maintaining total output constant when substitution takes place in some sector. This way of proceeding has little if any economic justification.

The paper follows this organization. Section 2 discusses the general characteristics of the proposed NIO model with scale effects. In Section 3 we undertake an empirical exercise with the proposed NIO model using 2011 interindustry data for Brazil, China and United States. Data is taken from the World I-O Database (WIOD). A summary and conclusion section completes the paper.

2. NONLINEAR INPUT-OUTPUT.

2.1 *Review of the conventional linear model.*

Interindustry data provide a detailed multisectoral depiction of the revenue-expenditure-output macroeconomic identities. Consider an economy composed of *n* distinct productive sectors indexed as *i*, *j*=1, 2,..., *n*. In the period when data is assembled, identified here by super index 0, the following identities representing the circular flow of income hold true for all *j*=1, 2,..., *n*:

$$\sum_{i=1}^{n} p_i^0 \cdot x_{ij}^0 + p_v^0 \cdot v_j^0 = \sum_{i=1}^{n} p_j^0 \cdot x_{ji}^0 + p_j^0 \cdot f_j^0 = p_j^0 \cdot x_j^0 \qquad (1)$$

In expression (1) the left-hand side collects total expenditure in intermediate purchases and value-added acquisition incurred by sector *j* to carry out the production of its output x_j^0, the middle part is total revenue accruing to sector *j* from the sale of its output x_j^0 to other sectors –as intermediate demand– and to final demanders. Finally, the right-hand side of the expression is the value of total production x_j^0 obtained in sector *j*. Expression (1) can therefore be seen as a sort of sectoral budget constraint in terms of volume. Interindustry data, however, is expressed in value and the distinction between physical magnitudes $(x_{ij}^0, x_j^0, v_j^0, f_j^0)$ and prices –for goods and value-added– (p_j^0, p_v^0) is not usually available. We can take observed transaction values as if they were physical magnitudes and in doing so we redefine units in such a way that every one of the new units have a worth of 1 currency unit. In other words, we use new prices p_j=1 for goods and p_v=1 for value added so that $p_i \cdot x_{ij} = p_i^0 \cdot x_{ij}^0$, $p_v \cdot v_j = p_v^0 \cdot v_j^0$, and $p_j \cdot f_j = p_j^0 \cdot f_j^0$.

With this implicit normalization it is customary in interindustry analysis to omit the presence of prices in the balance identities in (1) for the base year. Contrary to what has been common practise, for the time being we will keep them explicit for reasons that will become clear shortly. Consequently, from the expenditure perspective, identity (1) becomes:

$$\sum_{i=1}^{n} p_i \cdot x_{ij} + p_v \cdot v_j = p_j \cdot x_j \qquad (1a)$$

while, from the revenue perspective, takes the following form:

$$\sum_{i=1}^{n} p_j \cdot x_{ji} + p_j \cdot f_j = p_j \cdot x_j \qquad (1b)$$

Notice that since only the price p_j is involved in (1b), it can be eliminated altogether from it if so desired. However written, expressions (1a-1b) are nothing but the representation of observed data. The standard I-O model adopts the assumption that input-output ratios and value-added ratios are constant; in other words, it takes output as proportional to inputs by way of assuming nonnegative technical coefficients defined by $a_{ij} = x_{ij} / x_j$ and $\upsilon_j = v_j / x_j$. In production theory terms, this technological relationship takes the form:

$$x_j = Min\left\{\frac{x_{1j}}{a_{1j}}, \ldots, \frac{x_{nj}}{a_{nj}}, \frac{v_j}{\upsilon_j}\right\} \qquad (3)$$

These coefficients are assumed to be unique and independent of the scale of production. Substituting these coefficients in (1a) and simplifying yields:

$$p_j = \sum_{i=1}^{n} p_i \cdot a_{ij} + p_v \cdot \upsilon_j \qquad (4a)$$

which, translated into vector-matrix notation, can be expressed and solved as:

$$\mathbf{p}' = \mathbf{p}' \cdot \mathbf{A} + p_v \cdot \boldsymbol{\upsilon}' = p_v \cdot \boldsymbol{\upsilon}' \cdot (\mathbf{I} - \mathbf{A})^{-1} = p_v \cdot \boldsymbol{\upsilon}' \cdot \mathbf{L} \tag{4b}$$

provided matrix $\mathbf{A}$, with $[\mathbf{A}]_{ij} = a_{ij}$, is productive and the value-added price p_v is taken as *numéraire*. Matrix $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ in equation (5a) is the so-called Leontief inverse. Similarly, substituting in expression (1b) and eliminating now the "irrelevant" price p_j gives:

$$\sum_{i=1}^{n} a_{ji} \cdot x_i + f_j = x_j \tag{5a}$$

In matrix terms we would obtain:

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{x} + \mathbf{f} = [\mathbf{I} - \mathbf{A}]^{-1} \cdot \mathbf{f} = \mathbf{L} \cdot \mathbf{f} \tag{5b}$$

The linear I-O model in expressions (4b) and (5b) is composed of two sets of equations, one for prices and one for quantities, which solve independently of each other. For a given technology matrix $\mathbf{A}$, cost covering prices $\mathbf{p}'$ depend only on the value-added technical coefficient vector $\boldsymbol{\upsilon}'$, while output levels $\mathbf{x}$ depend only final demand levels $\mathbf{f}$. This is the well-known dichotomy between prices and quantities in the conventional I-O model and it is a property that derives from the linearity assumption in the technology.

2.2 *A nonlinear input-output system.*

With the objective of describing the NIO model, the point of departure is a Leontief production function with no input substitution allowed. In the present formulation and in contrast with the standard case, however, output and inputs are no longer related through a linear relationship. Thus we posit:

$$x_j = Min \left\{ \frac{x_{1j}^{\beta_{1j}}}{a_{1j}}, \ldots, \frac{x_{nj}^{\beta_{nj}}}{a_{nj}}, \frac{v_j^{\beta_{vj}}}{\upsilon_j} \right\} \tag{6}$$

Under the production technology in (6), the efficient locus becomes:

$$x_j = \alpha_{ij} \cdot x_{ij}^{\beta_{ij}} = \eta_j \cdot v_j^{\beta_{vj}} \tag{7}$$

with the notational change $\alpha_{ij} = 1/a_{ij}$ and $\eta_j = 1/\upsilon_j$. Clearly for $\alpha_{ij}, \beta_{ij}, \beta_{vj} > 0$, expression (7) represents a monotonically increasing and continuous production function, i.e. more output can only be obtained when using more inputs. Parameters β_{ij} and β_{vj} can quickly be seen to be output-to-input, or scale, elasticities:

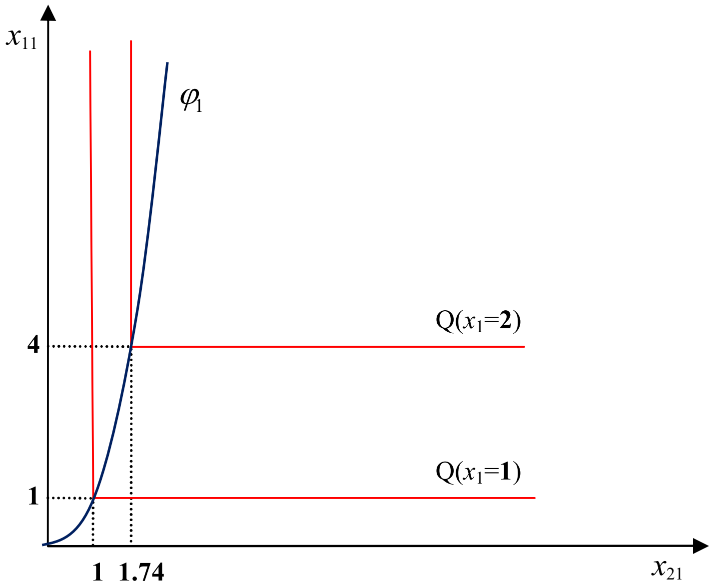
$$\left[\frac{\partial x_j}{\partial x_{ij}} \cdot \frac{x_{ij}}{x_j}\right] = \beta_{ij}$$

$$\left[\frac{\partial x_j}{\partial v_j} \cdot \frac{v_j}{x_j}\right] = \beta_{vj} \qquad (8)$$

Additionally, when $\beta_{ij} < 1$ (>1) the production coordinate linking output x_j with inputs x_{ij} can be seen to present decreasing (increasing) returns to scale denoted by DRS (IRS) hereafter. Notice that for the particular case $\beta_{ij} = \beta_{vj} = 1$, for all i and j, expressions (6) and (7) revert to the standard input-output linear assumption where input requirements vary proportionally with respect to output levels. In the linear input-output (LIO) model all scale elasticities are therefore implicitly assumed to be unitary. For these unitary scale elasticities and the observed benchmark data set (x_j, x_{ij}, v_j) all technical coefficients for goods a_{ij} and value-added υ_j are determined following the standard calibration procedure. Nothing, of course, would preclude using another, non unitary, set of values for the scale elasticities. In this case, the calibration of all required parameters should be adjusted to tackle the presence of these non-unitary elasticity parameters.

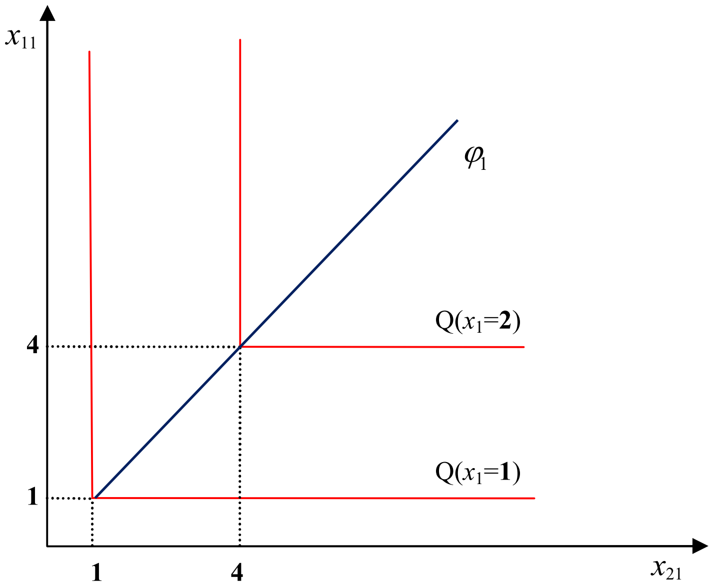
We illustrate these ideas in **Figure 1** where we can visualize the structure of a possible nonlinear production map for a fictitious two good economy, with production of good 1 following these hypothetical nonlinear technological relationships: $x_1 = \alpha_{11} \cdot x_{11}^{\beta_{11}}$ and $x_1 = \alpha_{21} \cdot x_{21}^{\beta_{21}}$ with $\beta_{11} = 0.5$, $\beta_{21} = 1.25$ and where, for the sake of simplicity, the parameters α_{11} and α_{21} are assumed to be unitary. We can see that the map is nonlinear on two counts. First, returns to scale are not constant as the non-unitary output elasticities show, and second the map of isoquant curves $\mathrm{Q}(x_j = \bar{x})$ is non-homothetic as the nonlinear efficiency path φ_1 indicates.

Figure 1: Non-Unitary Heterogeneous Scale Elasticities. Mixture between Increasing and Decreasing Returns to Scale. Non-Homothetic Isoquant Map



The nonlinearity of the efficiency path φ_1 is due to the fact that returns to scale are different among intermediate inputs within the same sector, i.e. $\beta_{11} \neq \beta_{21}$. In **Figure 2**, in contrast, we consider the same scale elasticity for both inputs, i.e. $\beta_{11} = \beta_{21} = 0.5$. The isoquant map is now homothetic as the linear efficiency path shows while the production function itself presents overall DRS. From these examples we conclude that the efficient input ratios are no longer constant when the scale elasticities within the same industry are heterogeneous. In this case the input mix is variable, its value depending now on the scale of production. Notice that in the two numerical examples, the depicted L-shaped isoquants conform to the perfect complementarity property of input-output economics and thus the elasticity of substitution is equal to zero.

Figure 2: Non-unitary Homogenous Scale Elasticities. Decreasing Returns to Scale. Homothetic Isoquant map.



The derivation of the complete NIO system follows readily from the technology assumption outlined in (7). For the quantity equation we start from the balance expression (1b) which we simplify here as:

$$\sum_{i=1}^{n} x_{ji} + f_j = x_j \tag{9}$$

From (7) and recalling that $\alpha_{ij} = (a_{ij})^{-1}$ we find:

$$x_{ji} = \left(\frac{x_i}{\alpha_{ji}} \right)^{\frac{1}{\beta_{ji}}} = \left(a_{ji} \right)^{\frac{1}{\beta_{ji}}} \cdot \left(x_i \right)^{\frac{1}{\beta_{ji}}} = \left(a_{ji} \right)^{\frac{1}{\beta_{ji}}} \cdot \left(x_i \right)^{\frac{1-\beta_{ji}}{\beta_{ji}}} \cdot x_i \tag{10}$$

Substituting equation (10) in expression (9) we obtain:

$$\sum_{i=1}^{n} \left(a_{ji} \right)^{\frac{1}{\beta_{ji}}} \cdot \left(x_i \right)^{\frac{1-\beta_{ji}}{\beta_{ji}}} \cdot x_i + f_j = x_j \tag{11}$$

In matrix notation, expression (11) becomes:

$$\mathbf{x} = \mathbf{A}(\mathbf{x}) \cdot \mathbf{x} + \mathbf{f} \tag{12}$$

with the elements of matrix **A(x)** being:

$$\left[\mathbf{A}(\mathbf{x})\right]_{ji} = \left(a_{ji}\right)^{\frac{1}{\beta_{ji}}} \cdot \left(x_i\right)^{\frac{1-\beta_{ji}}{\beta_{ji}}} \tag{13}$$

Notice that unitary output-to-input elasticities everywhere, i.e. $\beta_{ji} = 1$, would yield back the standard technical coefficients matrix **A** of the standard input-output model. Under quite general conditions, as we mentioned above, nonlinear equations such as the one represented in expression (12), whereby industries' production functions present a mixture of returns to scale, have been proved to have a unique non-negative solution **x** for any possible non-negative vector **f** (Chander, 1983)**.** The requirements for solvability include the following set of assumptions: (**ass_1**) the vector function $\mathbf{A}(\mathbf{x}) \cdot \mathbf{x}$ is non-decreasing (i.e. more output **x** requires more intermediate inputs $\mathbf{A}(\mathbf{x}) \cdot \mathbf{x}$), (**ass_2**) continuity of $\mathbf{A}(\mathbf{x}) \cdot \mathbf{x}$**,** and (**ass_3**) a productivity condition guaranteeing that expression (12) holds true for some pair (**f**, **x**). The function **A(x)** in expression (13) can be seen to satisfy (**ass_1**) and (**ass_2**). Assumption (**ass_3**), on the other hand, is always satisfied in the case of an empirically implemented model by the benchmark or base year solution. Thus, equation (12) is in principle solvable, and for any other vector of final demand $\mathbf{f} \geq \mathbf{0}$ there will always be a unique production plan $\mathbf{x} \geq \mathbf{0}$ compatible with the nonlinear quantity equation (12).

The fact that the quantity equation is scale-dependent has another far-reaching implication, namely, that cost covering prices are no longer independent of quantities. With non-constant returns to scale, unitary costs are not constant either and their level depends on the actual production level. Despite the focus of the theoretical literature on the solvability of the quantity equation (12), the economic system as a whole has another component that needs to be factored in if overall balance, as described in expressions (1), is to be maintained after a change in final demand takes place. Plugging in expression (7) into expression (1a) and remembering that $\upsilon_j = (\eta_j)^{-1}$ we would obtain:

$$p_j \cdot x_j = \sum_{i=1}^{n} p_i \cdot \left(a_{ij}\right)^{\frac{1}{\beta_{ij}}} \cdot \left(x_j\right)^{\frac{1}{\beta_{ij}}} + p_v \cdot \left(\upsilon_j\right)^{\frac{1}{\beta_{vj}}} \cdot \left(x_j\right)^{\frac{1}{\beta_{ij}}} \tag{14}$$

Simplifying:

$$p_j = \sum_{i=1}^{n} p_i \cdot \left(a_{ij}\right)^{\frac{1}{\beta_{ij}}} \cdot \left(x_j\right)^{\frac{1-\beta_{ij}}{\beta_{ij}}} + p_v \cdot \left(\upsilon_j\right)^{\frac{1}{\beta_{vj}}} \cdot \left(x_j\right)^{\frac{1-\beta_{vj}}{\beta_{vj}}} \tag{15}$$

Recalling expression (13) and defining in a like manner the vector of value-added marginal coefficients as:

$$\left[\boldsymbol{\upsilon}'(\mathbf{x})\right]_j = \left(\upsilon_j\right)^{\frac{1}{\beta_{vj}}} \cdot \left(x_j\right)^{\frac{1-\beta_{vj}}{\beta_{vj}}} \tag{16}$$

we would obtain the scale-dependent system of prices:

$$\mathbf{p}' = \mathbf{p}' \cdot \mathbf{A}(\mathbf{x}) + \mathbf{p_v} \cdot \boldsymbol{\upsilon}'(\mathbf{x}) \tag{17}$$

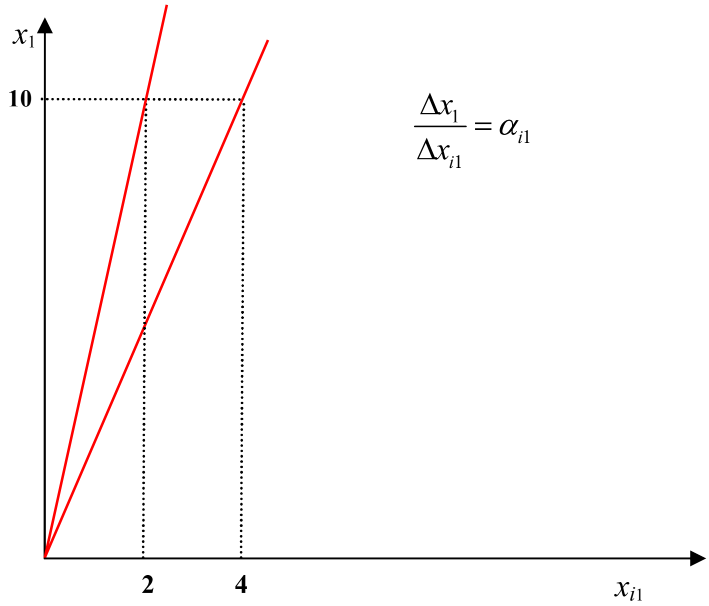
The system of prices depends now on quantities and the traditional—and very convenient in computing terms—separation of prices and quantities no longer holds. This is true even when the input mix is constant and consequently, the isoquant map is homothetic, i.e. scale elasticities are non-unitary but homogenous as shown in the case depicted in **Figure 2**. As expected, with unitary output elasticities everywhere, the price equation in (17) reverts to the standard price equation of the LIO model in (4b). The nonlinear, scale-dependent input-output system must therefore include both equations for quantities (12) and prices (17) for the system to be complete and all magnitudes to be in balance after external shocks are absorbed within the system. Prices in equation (17) can be interpreted as shadow prices, i.e. prices supporting the efficient production plans stemming from the quantity side of the economy (12), or as accounting prices, i.e. prices that guarantee the sectoral and economy-wide balance relationships between total costs and total resources. They should not be interpreted, however, as market prices since no demand behaviour is incorporated in this type of models.

2.3. Calibration issues of the nonlinear input-output system.

The numerical implementation of the NIO system in (12) and (17) requires that both the set of coefficients (a_{ij}, υ_j) and that of the output elastisticities $\left(\beta_{ij}, \beta_{vj}\right)$ introduced in the production function (6) be available prior to computation. In the LIO model things are particularly easy since $\beta_{ij} = \beta_{vj} = 1$ for all *i* and *j*. In this case, the a_{ij} technical coefficients needed for the computation of output multipliers can be calibrated from expression (7) by using the observations for output (x_j) and inputs (x_{ij}) read from the base year data:

$$a_{ij} = (\alpha_{ij})^{-1} = \frac{x_{ij}}{x_j} \tag{18}$$

Figure 3: Calibration to linear.



In the simple n=2 case, the inverse of these coefficients are the slopes of the linear rays going through the origin as we can see in the illustrative example in **Figure 3**. This is the consequence of taking all scale elasticities to be unitary. In the general case, when the relationship is not linear, we need to adjust the value of the a_{ij} coefficients to the non-unitary scale elasticities so that for the pair of parameters (a_{ij}, β_{ij}) and base year data (x_j, x_{ij}) expression (7) is upheld. Hence, in the NIO model we have:

$$a_{ij} = (\alpha_{ij})^{-1} = \frac{x_{ij}^{\beta_{ij}}}{x_j} \tag{19}$$

In the standard linear example with *n*=2 of **Figure 3** we find that the technical coefficients for *j*=1 are given by $a_{21} = x_{21} / x_1 = 4/10 = 0.4$ (or $\alpha_{21} = 2.5$) and $a_{11} = x_{11} / x_1 = 2/10 = 0.2$ (or $\alpha_{11} = 5$).

Figure 4: Calibration to non-linear.

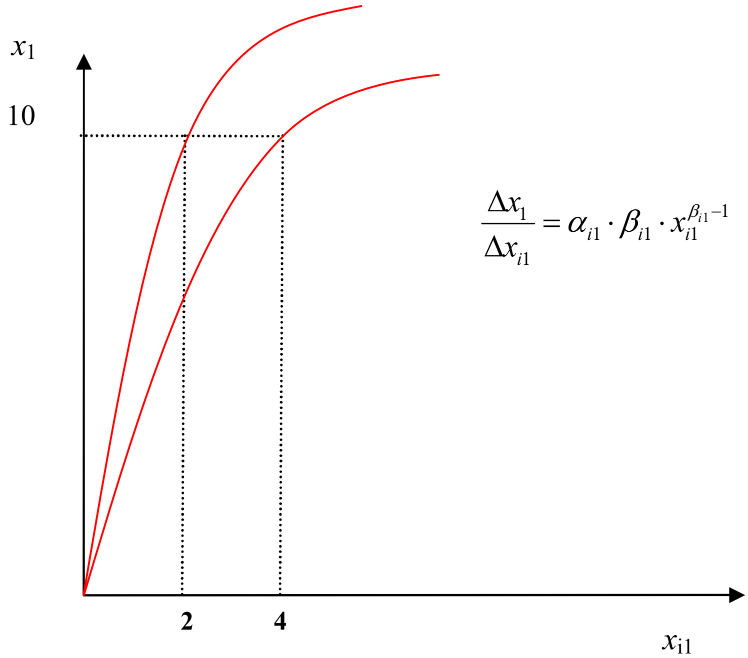


Figure 4 illustrates the nonlinear situation for the same base year data and, in assuming IRS with an elasticity value of $\beta_{11} = \beta_{21} = 1.2$, the calibrated coefficients turn out to be $a_{21} = x_{21}^{\beta_{21}} / x_1 = 4^{1.2} / 10 = 0.53$ (or $\alpha_{21} = 1.9$) and $a_{21} = x_{21}^{\beta_{21}} / x_1 = 2^{1.2} / 10 = 0.23$ (or $\alpha_{21} = 4.35$).

The calibration of the value-added coefficients υ_j for a given output-to-value added elasticity β_{vj} would follow the same procedure. Notice that this calibration procedure is exactly the same for the linear and the nonlinear cases, conditional in both cases to the elasticity values being adopted. In the linear case they are unitary everywhere in the production function but this set of values is just but one of the many possible ones in parameter space and is therefore as *ad-hoc* as any other selectable set. In probability terms the linear set is of course highly unlikely. The justification for linearity should, in any case, rest on economic grounds. The empirical evidence in favour of universal constant returns to scale (CRS), however, is not conclusive. Based on engineering evidence assembled for the manufacturing sectors of the United States economy, Jorgenson (1972) reported that CRS seem to be prevalent, at least from some minimal optimal size plant on. Dragonette (1983), in contrast, used time-series data on United States manufacturing too but concluded using concentration indicators that CRS seem to be more the exception than the rule, with a majority of DRS industries. Chirinko and Fazzari (1994), on their part, checked for market power using a time-series database for 11 manufacturing sectors of the same economy. Since market power is theoretically incompatible with CRS, this estimate provides indirect evidence for the presence of non constant returns to scale. He concludes that market power, hence IRS, are present in a majority of those industries. This contradictory evidence is for manufacturing industries alone, and no similar information seems to be available for non-manufactures, such as the services sectors.

2.4. Computing issues.

The complete input-output system includes the two nonlinear equations (12) and (17) and both need to be solved. The solution to equation (12) provides output levels **x** that satisfy final demand **f** and are compatible with the production mapping **A(x)**. In turn, the solution to equation (17) guarantees cost covering prices compatible with the dual cost functions. Considered together equations (12) and (17) yield the required economy-wide balance between uses and resources in expressions (1). For any possible level of final demand **f,** the solution **x(f)**, **p(x(f)**) is obtained using the GAMS (General Algebraic Modeling System) software[1]. The system of equations is transformed into a dummy nonlinear optimization program using a fake objective function with no relation to the endogenous variables. The nonlinear program includes expressions (12) and (17) as system constraints. Since GAMS looks first for a feasible solution to the constraints and only after finding it tries to increase the objective function while keeping feasibility, the process converges swiftly. The feasible solution to the nonlinear program exists and is unique, as implied by the theoretical I-O literature. The nonlinear solver is seeded with the benchmark data values and is run to obtain a first solution which of course coincides with the benchmark values. Once the first feasible solution is found, GAMS tries to find a better feasible one but, being the unique feasible solution, there is no room for improvement and the process quickly stops. This first run provides an initial basis for the subsequent, non-benchmark runs and facilitates convergence.

[1] See Brooke *et al* (1992).

These additional runs simulate sequential changes in final demand and are solved using the loop facility in GAMS. For instance, final demand for commodity 1 is assumed to change by one unit and a new NIO balance is found. Then final demand for 1 is reset to the benchmark level and next a unitary change in demand for commodity 2 is considered. The process is repeated for the rest of goods. This procedure allows the calculation of multiplier effects induced by changes in final demand for all goods both in quantities and in prices, a distinct and novel possibility not present in the standard linear model, where prices are known to be irresponsive to changes in quantities.

The fact that quantities and prices change at the same time has an interesting implication for the measurement of output multipliers. In the linear model, since prices do not change when final demand changes, the total multiplier value can be easily calculated by column summation, After all, units have been chosen to the effect that physical and value magnitudes coincide. With constant prices, aggregation is simple and adding up quantities is permissible. In the NIO system, however, prices are scale-dependent too and in the new economy-wide balance both quantities and prices will have changed. This requires the use of index numbers to isolate quantity effects from price effects.

3. EMPIRICAL EXERCISES WITH THE NONLINEAR I-O MODEL

3.1. Analysing the impact of demand-pull injections in a NIO model.

Given the absence of firm evidence on returns to scale, the substantial variety of possible sectoral situations, and the lack of data for economy-wide estimation of returns to scale, the second best option is to estimate economic effects using a set of distinct global scenarios which try to encompass some reasonable alternatives on output-to-input elasticities. For instance, a central scenario with CRS can be accompanied by two scenarios with DRS and IRS, providing interval estimates of effects quite richer than the usual point-estimate typical of the linear case. To this effect we use 2011 interindustry data for the Brazilian, Chinese and United States economies to calibrate all the needed I-O coefficients. The industry by industry I-O symmetric tables have been obtained from WIOD—the World I-O Database[2]. The industry break-down and its corresponding NACE[3] codes, Rev. 2, are summarised in the Annex.

The simulation strategy assumes three sets of values for the scale elasticities. The first one adopts the universal unitary elasticity value specific to the linear model (CRS), i.e. $\beta_{ij} = \beta_{vi} = 1$. We then modify those values downward (DRS) and upward (IRS) by 10 percent and proceed to recalibrate the needed two sets of a_{ij} and υ_j coefficients for matrix $\mathbf{A(x)}$ and vector $\boldsymbol{\upsilon}'\mathbf{(x)}$, respectively**.** The model is then repeatedly solved for "small" unitary sequential changes in each sectors' final demand. This allows us to obtain estimates of the quantity multiplier matrices for the three contemplated scenarios on returns to scale as well as estimates of the induced cost changes under the two non-constant returns to scale cases where prices are responsive to quantity changes.

The quantity effects on all output levels of these marginal changes[4] in the final demand for a specific commodity are captured by the matrix **M** of partial derivatives:

$$[\mathbf{M}]_{ij} = m_{ij} = \frac{\partial x_i}{\partial f_j} \tag{20}$$

In the terminology of key sector analysis, matrix **M** captures the so-called backward or demand-pull effects. Similarly, a matrix **C** of price changes can be calculated for each sector marginal change in final demand:

$$[\mathbf{C}]_{ij} = c_{ij} = \frac{\partial p_i}{\partial f_j} \tag{21}$$

From the information contained in matrices **M** and **C**, we derive both the quantity and price effects induced by each sector demand-pull stimulus under the two nonlinear scenarios. Quantity impacts are therefore computed following the classical procedure, i.e. aggregating the column elements of matrix **M**. The output multiplier for good j is calculated as $m_j = \sum_{i=1}^{n} m_{ij}$. However, and

[2] For more details about this database see Timmer et al.(2012).

[3] **N**omenclature statistique des **A**ctivités économiques dans la **C**ommunauté **E**uropéenne.

[4] In our approach, marginal changes in final demand refer to those exogenous changes for which the NIO quantity system started to be sensitive enough. This sensitiveness represents at least 0.00001 percent over the total domestic output benchmark equilibrium values as indicated by the Törnqvist quantity indexes reported in Table 1.

differently from the standard I-O model, under the non-linear system quantities affect prices. Consequently, as advanced in the previous section, it is necessary to use index numbers to compare the benchmark and the new equilibrium output levels, i.e. output before and after introducing the exogenous change in final demand must be measured in the same value terms. In fact, this is also contemplated but not made explicit in the LIO system. In this model, the initial and new equilibrium sectors' gross production levels are weighted by the unaltered unitary benchmark equilibrium prices. Under the NIO model, instead, the new equilibrium price vector is not longer the initial one.

There is a wide range of quantity index numbers that might be used to isolate the quantity effects obtained through the NIO system from the cost impacts. Behind each index number formula, there is a particular form of the production function. The Paasche and Laspeyres quantity indexes, for instance, imply an underlying fixed coefficient technology as that assumed by the LIO model. Nevertheless, the application of these quantity index numbers would be inadequate for the NIO model because, as can be asserted from **Figure 4**, input-output proportions are scale dependent. Taking these considerations into account, we have opted for using the Törnqvist quantity index; this indicator belongs to the family of "superlative" index numbers, a concept first introduced in the seminal work of Diewert (1976). Following Diewert's definition, an index is considered to be "superlative" if it approximates any type of production technology, including those that do not necessarily exhibit CRS. Hence, under the present approach, the Törnqvist quantity index TI^{Q} used to correct the evaluated total output backward stimulus of sector j takes the following form:

$$\mathrm{TI}_j^{\mathrm{Q}} = \prod_{i=1}^{n} \left(\frac{x_i + m_{ij}}{x_i} \right)^{\frac{1}{2} \cdot \left[\frac{x_i \cdot p_i}{\sum_{i=1}^{n} x_i \cdot p_i} + \frac{(x_i + m_{ij}) \cdot (p_i + c_{ij})}{\sum_{i=1}^{n} (x_i + m_{ij}) \cdot (p_i + c_{ij})} \right]} \qquad (22)$$

Using expression (22), the demand-pull output multiplier m_j under the general nonlinear specification would be calculated as:

$$m_j = (\mathrm{TI}_j^{\mathrm{Q}} - 1) \cdot \sum_{i=1}^{n} x_i \qquad (23)$$

The values reported in (23) measure the effect on total physical output in response to a marginal change in the final demand of sector *j*. Under the classical linear approach, the evaluated changes in output affect neither the cost structures nor the values of marginal multipliers. Expression (23) thus reverts to the familiar constant marginal or average demand-pull multiplier indicators obtained from the Leontief inverse **L**:

$$m_j = \sum_{i=1}^{n} m_{ij} = \sum_{i=1}^{n} [\mathbf{L}]_{ij} \tag{24}$$

The existence of nonlinearities in the quantity balance equation (12) makes marginal and average multipliers to no longer coincide because, under the NIO approach, output multipliers are scale dependent and consequently, they may vary with respect to the size of the exogenous change in final demand. Later in this empirical section, we will evaluate the potential multiplier bias when marginal effects in (23) are employed to infer minimum and maximum average impacts. This is also done as a way of testing the robustness of the NIO system.

Changes in final demand also end up transferring to prices in the NIO model, as picked up in expression (21). We evaluate this price response using output weighted costs elasticities, so as to neutralize, in this case, the size effect. The induced demand-pull cost c_j^{ε} will be measured in elasticity terms by:

$$c_j^{\varepsilon} = \sum_{i=1}^{n} \left[\zeta_j^i \cdot \left((x_i + m_{ij}) \Big/ \sum_{i=1}^{n} (x_i + m_{ij}) \right) \right] \tag{25}$$

where ζ_j^i refers to price arc-elasticities (Allen, 1933), an appropriate approximation when dealing with unknown nonlinear functions. These elasticities are computed using the information provided by matrix **C**. Alternatively, we have also used the counterpart of Törnqvist's quantity index ($\mathrm{TI}_j^{\mathrm{P}}$) as a measure of the induced cost impacts.

Table 1: Output Multipliers for a Marginal increase in sectors' final demand. SIOTs 2011 for Brazil, China and United States.						
		Linear I-O System	**Nonlinear I-O System**			
		$\beta_{ij} = \beta_{vj} = 1$	$\beta_{ij} = \beta_{vj} = 1.2$		$\beta_{ij} = \beta_{vj} = 0.8$	
	Production units*.	m_j	$\mathrm{TI}_j^{\mathrm{Q}}$	m_j	$\mathrm{TI}_j^{\mathrm{Q}}$	m_j
Brazil	**S8_Coke, Refined Petroleum and Nuclear Fuel**	2.324832	1.000052	1.969570	1.000081	3.073935
	S3_Food, Beverages and Tobacco.	2.277771	1.000051	1.946911	1.000078	2.967103
	S15_Transport Equipment.	2.274310	1.000051	1.928298	1.000079	3.015420
	S9_Chemicals and Chemical Products.	2.077648	1.000047	1.792404	1.000070	2.677248
	S13_Machinery and Equipment.	2.071362	1.000047	1.786105	1.000070	2.673668
	S10_Rubber and Plastics	1.999289	1.000046	1.734222	1.000067	2.557452
	S12_Basic Metals and Fabricated Metal	1.998427	1.000046	1.733688	1.000067	2.555054
	S14_Electrical and Optical Equipment	1.995799	1.000046	1.733464	1.000067	2.546148
	Economy Average Effect	**1.778972**		**1.575239**		**2.204432**
China	**S14_Electrical and Optical Equipment**	3.660718	1.000013	2.635784	1.000037	7.703069
	S15_Transport Equipment	3.603732	1.000013	2.596690	1.000037	7.599560
	S10_Rubber and Plastics	3.524208	1.000012	2.571450	1.000035	7.166891
	S4_Textiles and Textile Products	3.426180	1.000012	2.515341	1.000033	6.907432
	S13_Machinery and Equipment	3.375347	1.000012	2.472013	1.000033	6.894549
	S5_Leather, Leather and Footwear	3.372044	1.000012	2.499715	1.000032	6.602942
	S12_Basic Metals and Fabricated Metal	3.331090	1.000012	2.463019	1.000032	6.647405
	S18_Construction	3.257593	1.000012	2.422504	1.000031	6.423375
	Economy Average Effect	**2.737364**		**2.103049**		**5.117849**
United States	**S3_Food, Beverages and Tobacco**	2.485166	1.000008	2.065708	1.000013	3.413082
	S15_Transport Equipment	2.433676	1.000008	2.039679	1.000013	3.283377
	S6_Wood and Products of Wood and Cork	2.224329	1.000007	1.890530	1.000012	2.943476
	S12_Basic Metals and Fabricated Metal	2.171321	1.000007	1.856162	1.000011	2.840914
	S9_Chemicals and Chemical Products	2.139690	1.000007	1.834402	1.000011	2.786367
	S1_Agriculture,Hunting, Forestry and Fishing	2.716089	1.000007	1.777687	1.000011	2.716089
	S4_Textiles and Textile Products	2.690939	1.000007	1.780708	1.000011	2.690939
	S7_Pulp,Paper, Paper , Printing and Publishing	2.616525	1.000007	1.775518	1.000010	2.616525
	Economy Average Effect	**1.804773**		**1.592368**		**2.250931**

** The prefix S is an abbreviation of Sectoral unit or Industry.*

Table 1 sums up some of the information on the output multiplier effects for the three selected economies and the three considered scale elasticity scenarios. For simplicity, we show in decreasing order the eight sectors with the strongest multiplier effects, along with their corresponding Törnqvist cost deflator[5]. Average quantity effects for each economy are also reported. From the results in **Table 1**, we can see that with the exception of the industry of *Transport equipment* (**Sector 15)** of the Brazilian economy under the IRS scenario, the presence of scale effects does not affect the hierarchy of sectors in terms of their demand-pull multipliers. This fact seems to be prevalent when elasticities are common for all sectors. As expected from the logic of economic theory relating inputs to output, output multiplier effects following a marginal change in final demand are lower (higher) under IRS (DRS) than under the traditional CRS linear case. This observation reinforces the consideration of the evaluated output multipliers of the nonlinear model as potential interval measures. Notice that, in comparing the output average effects across the three economies, the length of this interval is subjected not only to the structure of the industrial linkages of each economy but also to their intensity. In this regard, and not surprisingly, the largest emerging economy in the world, China, has the strongest output multiplier effects and presents the largest distance between multipliers under DRS and IRS, i.e. for the five sectors reported in **Table 1**, China's output multipliers under the IRS scenario represents less than half of those under DRS.

With the objective of approaching a more comprehensive understanding of the role played by sectoral interdependences within the NIO approach under the IRS assumption, we could make a parallelism between industrial linkages and market openness. Under IRS, firms tend to merge in order to favour, or even create, scale effects that improve their cost-efficiency levels and thus promote a competitive advantage. The DRS technology scenario, instead, can be compared to a situation of excess capacity where the presence of sectors connectedness affects economy wide efficiency levels in the opposite way.

[5] Detailed results for all sectors in the three economies are available from the authors upon request.

Table 2: Sectoral Induced Costs Effects for a Marginal increase in sectors' final demand. SIOTs 2011 for Brazil, China and United States.					
		Nonlinear I-O System			
		$\beta_{ij} = \beta_{vj} = 1.2$		$\beta_{ij} = \beta_{vj} = 0.8$	
	Production units	TI_j^P	c_j^ε	TI_j^P	c_j^ε
Brazil	S8_Coke, Refined Petroleum and Nuclear Fuel.	0.999962	-0.001178	1.000084	0.002611
	S3_Food, Beverages and Tobacco.	0.999997	-0.000491	1.000008	0.001509
	S15_Transport Equipment.	0.999992	-0.000741	1.000021	0.001827
	S9_Chemicals and Chemical Products.	1.000034	0.001908	0.999955	-0.002487
	S13_Machinery and Equipment	0.999991	-0.000417	1.000022	0.001116
	S10_Rubber and Plastics	1.000002	0.000004	0.999998	-0.000003
	S12_Basic Metals and Fabricated Metal	0.999985	-0.000382	1.000034	0.000884
	S14_Electrical and Optical Equipment	0.999989	-0.000381	1.000024	0.000863
	Economy Average Effect		**-0.000092**		**0.001007**
China	S14_Electrical and Optical Equipment	0.999993	-0.004602	1.000032	0.022310
	S15_Transport Equipment	0.999994	-0.002107	1.000031	0.010426
	S10_Rubber and Plastics Products	0.999992	-0.000368	1.000033	0.001517
	S4_Textiles and Textile Products	0.999994	-0.002011	1.000027	0.009012
	S13_Machinery and Equipment	0.999994	-0.002513	1.000029	0.011574
	S5_Leather, Leather and Footwear	0.999995	-0.000504	1.000025	0.002308
	S12_Basic Metals and Fabricated Metal	0.999992	0.000555	1.000031	-0.002178
	S18_Construction	0.999995	-0.009694	1.000025	0.049592
	Economy Average Effect		**-0.001034**		**0.004653**
United States	S3_Food, Beverages and Tobacco	0.999997	-0.001187	1.000007	0.003074
	S15_Transport Equipment	0.999998	-0.000804	1.000006	0.002085
	S6_Wood and Cork Products.	0.999997	-0.000021	1.000007	0.000047
	S12_Basic Metals and Fabricated Metal	0.999997	-0.000132	1.000007	0.000291
	S9_Chemicals and Chemical Products	0.999997	-0.000633	1.000006	0.001447
	S1_Agriculture, Hunting ,Forestry and Fishing	0.999997	-0.000264	1.000006	0.000590
	S4_Textiles and Textile Products	0.999998	-0.000061	1.000005	0.000141
	S7_Pulp, Paper, Paper, Printing and Publishing	0.999998	-0.000377	1.000005	0.000831
	Economy Average Effect		**-0.000710**		**0.001543**

To qualify these parallelisms we now move to compare the sectoral quantity backward effects of the three economies with their induced costs impacts. These costs effects are presented in **Table 2**. In line with the two types of technological scenarios considered for the NIO approach, the signs of the two induced costs measures, namely, the Törnqvist price index and the weighted costs elasticities, clearly indicate that the existing industrial interdependencies intensify and spread the presence of returns to scale occurring at the sectoral level, generating a decline (increase) in domestic prices when IRS (DRS) are present. In terms of the output multipliers commented above, China followed by the United States economy shows the strongest economy average costs effects.

An interesting peculiarity of the NIO model is that the intensity of the cost impacts at the industry level significantly differs from that of the quantity effects that induced them. In comparing the results in **Table 1** and **Table 2**, some sectors with an above economy average output multiplier lose this "position" in terms of their costs effects. For example, in the context of the United States economy, this is the case of the *Wood and Products of Wood and Cork* industry (**Sector 6**) that, under any of the scenarios, see that its derived price-costs impacts are far from the economy's average. Furthermore, the induced price effect may present an unexpected sign such as the *Chemicals and Chemical Products* industry (**Sector 9**) and the *Rubber and Plastics Industry* (**Sector 10**) in Brazil, where a marginal increase in its final demand with IRS (DRS) generates an increase (decline) in economy-wide costs levels. The reason behind this surprising result is that, differently to the economies of United States and China, to obtain the benchmark equilibrium prices under NIO system net taxes on products had to be explicitly calibrated for the Brazilian economy. In the original symmetric I-O tables of the other two economies compiled by the WIOD project, these taxes were already integrated within the I-O flows. Then, if a sector presents strong interdependencies with net subsidised industries (industries with a negative tax margin) and the economy exhibits IRS (DRS), the economy-wide influence that this sector and its interactions exert over total output and thus, on all sectors' costs structures may decrease (increase) with the evaluated exogenous positive change in its final demand, i.e. less (more) output is needed to cover one unit of final demand. Consequently, the shrinking impact of subsidies may compensate the scale effects leading to a net increase (decrease) in economy-wide price levels. Summing up, the empirical exercise with the NIO model shows that the existence of nonlinearities within production chains leads to asymmetries between quantity multipliers and their corresponding costs impacts.

3.2. Evaluating the robustness of the NIO model.

The presence of scale effects, as captured in the NIO model, makes that marginal multipliers differ from average multipliers. Hence, in contrast with the standard I-O system, output multipliers and their induced cost impacts depend on the size of the exogenous change in final demand. At this stage of the analysis a question that arises is whether the intervals of marginal impacts shown in **Table 1** are informative enough for the appropriate design of policy actions. In order to shed some light over this question, we consider a scenario whereby the evaluated exogenous change in final demand is heterogeneous rather than the common unitary change in all sectors. The exogenous shock consists in a shared 10 percent increase over the benchmark equilibrium final demand levels. This change represents a plausible upper bound for potential positive demand shocks in the short term. The exercise will also serve to evaluate the robustness of the marginal demand-pull multipliers and their induced costs effects in the NIO system presented in **Table 1** and **Table 2**, respectively.

Table 3: Average Output Multipliers and Induced Price Effects for a 10 % increase in sectors' final demand. SIOTs 2011 for Brazil, China and United States.							
		$\beta_{ij} = \beta_{vj} = 1.2$			$\beta_{ij} = \beta_{vj} = 0.8$		
	Production units	**Distance from LIO m_j in %**	m_j	c_j^{ε}	**Distance from LIO m_j in %**	m_j	c_j^{ε}
Brazil	**S8_Coke, Refined Petroleum and Nuclear Fuel**	18.478654	1.962237	-0.038113	-24.911146	3.096108	0.085276
	S3_Food, Beverages and Tobacco	17.886493	1.932173	-0.095858	-24.420479	3.013741	0.285738
	S15_Transport Equipment	18.989906	1.911347	-0.069354	-25.900742	3.069275	0.173245
	S9_Chemicals and Chemical Products	16.450690	1.784144	0.106253	-23.112723	2.702200	-0.139247
	S13_Machinery and Equipment	16.772303	1.773847	-0.022205	-23.463458	2.706370	0.059612
	S10_Rubber and Plastics	15.302242	1.733955	0.000006	-21.844248	2.558083	-0.000004
	S12_Basic Metals and Fabricated Metal	15.620738	1.728433	-0.010319	-22.226092	2.569534	0.024060
	S14_Electrical and Optical Equipment	15.828556	1.723063	-0.014484	-22.447494	2.573481	0.033114
	Economy Average Effect	**13.417207**	**1.568518**	**-0.051145**	**-19.587244**	**2.212298**	**0.109981**
China	**S14_Electrical and Optical Equipment**	39.981347	2.615147	-3.336766	-118.400789	7.995037	3.660718
	S15_Transport Equipment	39.866349	2.576554	-0.740591	-116.266637	7.793670	3.603732
	S10_Rubber and Plastics	37.263790	2.567471	-0.017913	-104.272307	7.198981	3.524208
	S4_Textiles and Textile Products	37.549586	2.490869	-0.688177	-108.431431	7.141236	3.426180
	S13_Machinery and Equipment.	37.346965	2.457533	-1.046487	-108.703194	7.044457	3.375347
	S5_Leather. Leather and Footwear	35.988927	2.479646	-0.048636	-99.7436845	6.724684	3.372044
	S12_Basic Metals and Fabricated Metal	35.115600	2.465363	-0.041470	-98.7436845	6.620331	3.331090
	S18_Construction	35.797829	2.398855	-19.889433	-108.680059	6.797947	3.257593
	Economy Average Effect	**30.6316164**	**2.095483**	**-0.92023803**	**-89.1168292**	**5.176816**	**4.686921**
United States	**S3_Food. Beverages and Tobacco**	21.338827	2.048121	-0.057912	-28.572693	3.479294	0.153080
	S15_Transport Equipment	20.343415	2.022276	-0.029217	-27.120131	3.339298	0.076983
	S6_Wood and Products of Wood and Cork	17.783717	1.888486	-0.000015	-24.572323	2.948956	0.000035
	S12_Basic Metals and Fabricated Metal	17.149348	1.853464	-0.000595	-23.792495	2.849222	0.001315
	S9_Chemicals and Chemical Products.	17.348226	1.823368	-0.016499	-24.130575	2.820227	0.038273
	S10_Rubber and Plastics	16.955581	1.809705	-0.000265	-23.712084	2.774425	0.000598
	S1_Agriculture. Hunting. Forestry and Fishing	15.846650	1.772025	-0.002725	-22.252725	2.732723	0.006142
	S4_Textiles and Textile Products	13.877919	1.772185	-0.000168	-20.506498	2.713841	0.000391
	Economy Average Effect	**14.110000**	**1.768500**	**-0.006037**	**-20.756992**	**2.635138**	**0.013427**

The results for this new simulation exercise are summarized in **Table 3.** When comparing the size of the average output multipliers for the across-the-board 10 percent increase in sectoral final demand to those presented in **Table 1**, we observe that the difference between them is indeed very small. Considering the three economies and the two nonlinear scenarios together, the relative distance between the marginal and average multiplier can be seen to be within the range of 0.3 and 1 percent. This clearly shows that the NIO output multipliers have little sensitivity to the size of the exogenous shocks.

The marginal multiplier intervals provided by the NIO model may therefore be used to infer average and total output multipliers since the bias of using these approximations is negligible. In contrast, the multiplier bias derived from assuming CRS when either IRS or DRS are present is remarkably large. To illustrate empirically this idea, we have added two columns in **Table 3** that inform about the relative distance between the LIO and NIO sectors' average multipliers with IRC and DRS for each of the three economies. As can be asserted from **Table 3**, the Chinese economy presents the highest bias. The size of the potential upward bias should the Chinese economy exhibit IRS approaches 31 percent over the NIO average effect. The possible downward bias when using the standard average multipliers, when DRS are present, is even larger, close to 90 percent. Notice that for most of the sectors with the highest backward effects in the Chinese economy, the output multiplier under this DRS scenario more than doubles the LIO average effects.

The fact that the NIO marginal multipliers appear to be quite robust to the intensity of the evaluated change in final demand does not translate to their induced cost impacts, as can be verified from the size of the weighted costs elasticities reported in **Table 3**. This indicates that the scale effects that are built into the NIO model affect to a larger relative extent the price system in equation (17) than the quantity system in equation (12). This observation suggests that when considering the two dimensions of a policy decision, i.e. the impact on prices and the impact on quantities, the best option for a more balanced design of policy actions should rest in the use of the intervals provided by the NIO average impacts (**Table 3**) rather than the marginal ones (**Tables 1** and **2**).

4. CONCLUDING REMARKS.

We have shown that NIO models of the scale dependent type can be quickly implemented using standard interindustry data and nonlinear capable software packages. The numerical implementation is possible thanks to the existence and uniqueness theorems provided by the theoretical literature on the solvability of nonlinear models. The determination of the nonlinear equilibrium requires the use of standard computing techniques. We use here a nonlinear programming algorithm as backbone solver. The algorithm computes both quantities and prices simultaneously since the classical dichotomy no longer holds and both quantities and prices are now scale dependent.

We have outlined the step-by-step procedures needed to put into practice a scale-dependent NIO model, i.e. parameter specification from benchmark data, adaptable calibration to returns-to-scale scenarios, and equilibrium computing issues. We have also shown its operational utility in applied work and pointed out some of its major implications too. To this effect, we have carried out an empirical exercise using WIOD data for three large but structurally different economies, namely, China, Brazil and United States. This exercise has consisted in perturbing the initial equilibrium to generate backward or demand-pull output multipliers and the induced costs effects in several returns-to-scale scenarios including that implicitly assumed by the LIO system, the standard CRS scenario.

The results indicate that, in the absence of accurate information about the production structure of an economy, the output multiplier intervals derived from the two non-constant returns to scale scenarios may provide a more comprehensive data set to base policy decisions than the point-estimates provided by the standard I-O model. The underlying bias resulting from computing output multipliers using the LIO system, should the economy exhibit either IRS or DRS, appears to be quite large for some economies, as it is the case of China. In addition, the comparison of the output multipliers of the NIO system between the three economies, allows us to shed some light about the role played by industrial linkages when nonlinearities are present. In this regard, the strength of interindustry connectedness seems to intensify the presence of scale effects at an economy-wide level.

Lastly, since by construction the NIO model presented here is scale dependent, we have also tested the robustness of the marginal output multipliers with respect to the size of the exogenous changes in final demand. Our findings clearly legitimate the use of the NIO output multiplier intervals because, even when considering a relatively large exogenous shock, average and marginal effects turned out to be remarkably close. This was not the case in terms of the average induced costs indicators, implying that scale-effects influence to a larger extent the price system than the quantity system. Therefore, if both prices and quantities are relevant for policy purposes, average measures would be preferred than marginal indicators. Summing up, this analysis proves that a scale-dependent NIO models can be made operational and that scale effects seem to matter in determining sensible I-O multipliers, an aspect which should be taken into consideration by researchers and policy makers.

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Annex: Sectoral Break-down of the National Symmetric Input-Output tables. World Input-Output Tables Database.

Description of the sectors.		**NACE codes Rev. 2 correspondence**
Sector 1	Agriculture, Hunting, Forestry and Fishing	A-B
Sector 2	Mining and Quarrying	C
Sector 3	Food, Beverages and Tobacco	15-16
Sector 4	Textiles and Textile Products	17-18
Sector 5	Leather, Leather and Footwear	19
Sector 6	Wood and Products of Wood and Cork	20
Sector 7	Pulp, Paper, Paper , Printing and Publishing	21-22
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