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On Marilda Sotomayor´s Extraordinary Contribution to Matching Theory*

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Abstract

We report on Marilda Sotomayor´s extraordinay contribution to Matching Theory on the occasion of her 70th anniversary.

1 Introduction

Marilda Sotomayor's outstanding and innovative research in the field of matching models has made her renowned. In this short paper, we briefly report on her contributions on the occasion of her 70th anniversary.

Let us first mention that matching theory is not the first area Marilda was interested in. During her Ph.D. dissertation at IMPA, in Rio de Janeiro, from 1978 to 1981, she worked on the solution to growth models under the supervision of Jack Schechtman. In particular, she characterized the optimal dynamic behavior of consumers in several environments, including economies where the production function is random and not necessarily concave (part of her results during the thesis are included in Sotomayor, 1984).

Marilda wanted to continue developing her work on economic growth so in January 1983 she went to the University of California at Berkeley to learn from the famous mathematician David Gale, who had been Jack Schechtman's supervisor (that is, David Gale was the academic

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grandfather of Marilda). As reported by herself in "My encounters with David Gale" - published in Games and Economic Behavior in 2009 in an special issue to honor his memory - the first meeting between the two mathematicians ended with a sentence that was crucial in shaping Marilda's research career. David Gale stated: "I'm not interested in this area."

After this initial disappointment, Marilda decided to use her visit to learn from David Gale on the topic, or one of the topics, he was interested in at that time: matching theory. This theory deals with the endogenous formation of partnerships between, for instance, women and men, where each agent is supposed to have at most a partner (the "marriage problem"), or students and colleges, where each college may be matched with several students, although each student can only be admitted at one college (the "college admission problem").

Marilda, advised by David Gale, started reading the pioneer works on matching theory: the famous paper by David Gale and Lloyd Shapley (1962) on the college admissions problem, the short book by Donald Knuth (1976) on stable matchings, as well as the more recent papers (at that time) by Lester Dubins and David Freedman (1981) on the non-manipulability of the Gale-Shapley algorithm to allocate students to colleges. She also read the paper by David Gale and Gabrielle Demange (that was published in 1985) where the lattice property of the set of stable allocations and the non-manipulability result were extended to the assignment game (that is, to the matching model where money can be transferred among partners), a model that had been introduced by Lloyd Shapley and Martin Shubik in 1972.

2 Joint work with David Gale and Gabrielle Demange

Marilda visited Berkeley in 1983 and 1984, and she wrote with David Gale four very influential papers for the matching theory, two of them also coauthored with Gabrielle Demange. Three of these papers (Gale and Sotomayor, 1985a, 1985b; Demange, Gale and Sotomayor, 1987) published in mathematical journals, concerned marriage and college admission models and they proved properties that have since become classic. First, they proved the famous "Blocking Lemma" that for each non-stable and individually rational matching, identifies a particular blocking pair under which some agents of one side of the market are better off than under their optimal stable matching. The blocking lemma has been an instrumental result in proving key properties of stable matchings.[1]

They also proved, among other results, that the set of unmatched people is the same for

[1] In a lecture given by Marilda in 2011 dedicated to David Gale's memory, she reported that the proof of the blocking lemma was the first task that she received from him and that she solved it in one weekend (see Sotomayor, 2011).

all stable matchings; that the addition of, say, more positions in colleges cannot be bad for applicants and cannot be good for colleges; that the applicant-optimal stable matching is in fact Pareto optimal for the applicants among all the possible stable matchings; and that women have an incentive to manipulate the men-optimal stable matching.

In the fourth paper of this series (Demange, Gale and Sotomayor, 1986), the three authors analyzed dynamic multi-item auctions for the assignment game that achieve (either in an exact or in an approximate way) the best equilibrium from the bidders' point of view. In the International Workshop on Game Theory and Economic Applications of the Game Theory Society that celebrated in Sao Paulo the 70th anniversary of Marilda, we still heard new results on auctions of the type that Marilda, David, and Gabrielle proposed in their 1986 paper.

3 Joint work with Alvin Roth

Two months after she finished her visit to David Gale, that is, in March 1985, Marilda started the first of many visits that she would make during the following 13 years to the Department of Economics at the University of Pittsburgh. During that time she initiated a very fruitful collaboration with Al Roth. Marilda wrote two papers with Al, one on the college admission model and the other on the assignment model. Concerning the first of the models, they showed the surprising result that when preferences are strict, if we consider two different stable matchings, then each college prefers every student who is assigned to it in one of the matchings to every student who is assigned to it in the second matching but not in the first (Roth and Sotomayor, 1989). On the other hand, in Roth and Sotomayor (1988) they analyzed the structure of matching outcomes for the assignment games that are stable but are not optimal for one side of the market, extending and generalizing previous work by Sharon Rochfold (1984).

If her visit to and work with David Gale introduced Marilda to matching models, her collaboration with Al Roth was instrumental not only for the subsequent research that she has been developing but also for the advance of the theory of matching. Indeed, the book "Two-sided matching: A study in game-theoretic modeling and analysis" which they published in 1990 (Cambridge University Press) has been extremely influential and it has made the theory accessible to thousands of researchers and students. The elegance and rigor of the book makes for very pleasant and fruitful reading, as was recognized by the Lanchester Prize 1990, awarded by the Operations Research Society of America. These qualities also make the book appealing not only to mathematicians and economists, but also to many other researchers in social sciences and to public regulators. This was true in 1990 and not surprisingly is still true today.

In his premonitory praise of the book, Uriel Rothblum wrote: "Every once in a long while a book comes along with the potential to change the way in which an entire field of study is viewed. This book has that potential." It is clear to all of us that this potential, this prophecy, has really been fulfilled.

4 Recent contributions

Following her collaboration with David Gale, Gabrielle Demange, and Al Roth, Marilda's contributions to matching theory have been numerous. She has extended the existing models in various dimensions, provided new properties, proposed simple mechanisms to implement stable allocations in several models, and she has also given simple, short, and elegant proofs of classic results.

We will certainly not review all the results that have emerged from Marilda's papers. However, we would like to very briefly illustrate some of her contributions. We will do this brief account of her published papers by topic, not by publication date.

First, Marilda has extensively worked on the college admission problem. In one of her papers, for instance, she provided a very short and simple proof of the existence of stable matchings in marriage markets. The proof in Sotomayor (1996a) is non-constructive and hence quite different from the original proof in Gale and Shapley (1962), which relies on the famous algorithm. She has also developed several analyses on the differences between the various possible definitions of stability in college admission problems. In Sotomayor (1999a), in a paper included in the issue that Mathematical Social Sciences dedicated to David Gale on his 75th birthday, she discussed the differences between the core and the set of pair-wise stable matchings, two sets that she proved can be non-empty but disjoint. She also proved that pair-wise stable matchings always exist in college admission models if preferences are substitutable. Very recently, she has come back to discussing the right definition of stability in the marriage market, including additional possible blockings by the grand coalition.

Also for the college admission model, Marilda has analyzed agents' non-cooperative behavior in several natural mechanisms (Sotomayor, 2003a, 2008). The typical type of games that she has analyzed has the following sequential structure: in a first period, the players from one side of the market (say men or firms) choose a set of players from the other side; in a second period, the players from the other side of the market (women or workers) select a partner (or partners) out of those who chose them in the first place. Marilda has shown that the outcomes of this type of sequential mechanism are often nice, in the sense that they are stable. And the nice properties of equilibria hold for the marriage (one-to-one) model as well as for the many-to-many model, although to a lesser extent. Finally, she has also analyzed agents' behavior when stable rules are applied. That is, what happens if a "naïve" designer organizes an institution that leads to a good (stable) outcome if the agents are honest about their preferences. Do they really have incentives to report their true preferences?

The second group of Marilda's contributions refers to matching problems where monetary transfers are feasible and are crucial part of the deals. The assignment game (Shapley and Shubik, 1972) constitutes the simplest class of problems as it concerns situations where agents from each side of the market can deal, at most, with one agent from the other side. In this assignment game, in addition to her initial contributions with Gabrielle Demange, David Gale, and Al Roth, Marilda worked on the characteristics of the core, in particular when there is only one optimal matching, in which case the core necessarily has infinitely many payoffs (Sotomayor, 2003b).

She also proposed, in a joint paper with David Pérez-Castrillo which they initiated while she was visiting the Universidad Autónoma of Barcelona in 1996, a simple selling and buying procedure that, in equilibrium, leads to the maximum equilibrium price vector together with an optimal matching. That is, this paper contributes to the message that matching models are not only nice because they are useful and have very elegant and beautiful properties concerning stability and competitive equilibria; they are also nice because we can design reasonable non-cooperative games whose equilibrium outcomes lead to stable (or competitive) allocations. Pérez-Castrillo and Sotomayor (2002) received the Prize Haralambos Simeonidis awarded "to the best paper by Brazilian Economists."

Already in 1992, in a contribution to the book "Dynamics and Equilibrium: Essays in Honor to David Gale," Marilda proposed an extension of the assignment game to what she named the "multiple partners assignment game," which is a model where each participant can form more than one partnership. In a series of papers (Sotomayor 1999b, 2007a), she has proven that many of the properties of the assignment game also hold for the multiple partners assignment game. In particular, the set of stable payoffs is non-empty and they form a complete lattice, with a unique optimal stable payoff for each side of the market. She also compared, for this class of games, the concepts of cooperative and competitive equilibria. Additionally, she has proposed mechanisms that lead to competitive equilibrium payoffs and she has analyzed other variants of markets where each agent can form several partnerships (Sotomayor, 2004, 2009b).

Marilda has also contributed to models outside the classic marriage and assignment models. She has studied the stable set of "mixed economies," and "hybrid markets," in which some firms are flexible in the sense that they can choose salaries whereas other firms have no flexibility (Sotomayor, 2000, 2007b). That is, these are models where some agents behave as in the assignment game and other agents behave as in the marriage market (their preferences are ordinal because they cannot choose salaries). The set of stable matching is also non-empty in this extension and the lattice property of the core is preserved. Properties concerning the entrance of firms are not preserved.

Finally, let us mention that, motivated by the problems detected in the admission of candidates to graduate economic schools in Brazil, Marilda published in 1996 a paper that led the Association of Graduates Centers in Economics in Brazil (ANPEC) to change their decentralized allocation procedure to the Gale and Shapley algorithm one year later (see Bardella and Sotomayor, 2014, and Sotomayor, 1996b).

5 Final Remarks

We are sorry we have to leave you in suspense but we cannot tell you the end of the story of Marilda's contributions to matching theory. We cannot because it is an ongoing history. For example, in the International Workshop on Game Theory and Economic Applications of the Game Theory Society in Sao Paulo, Marilda presented a paper on "cooperative equilibria" (Sotomayor, 2013). Similarly, in the Workshop on Game Theory in Rio de Janeiro organized by Danilo Coelho and Humberto Moreira, David Pérez-Castrillo presented a recent working paper on the manipulability of competitive equilibrium rules in many-to-many buyer-seller markets (Pérez-Castrillo and Sotomayor, 2014). And she has other working papers and projects in the pipeline.

Therefore, we can only say that we look forward to the next celebration, where her friends and peers can gather again and can continue to report on Marilda's impressive, innovative, and influential contribution to matching theory.

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