

# Recent Progress in Parton Distributions and Implications for LHC Physics

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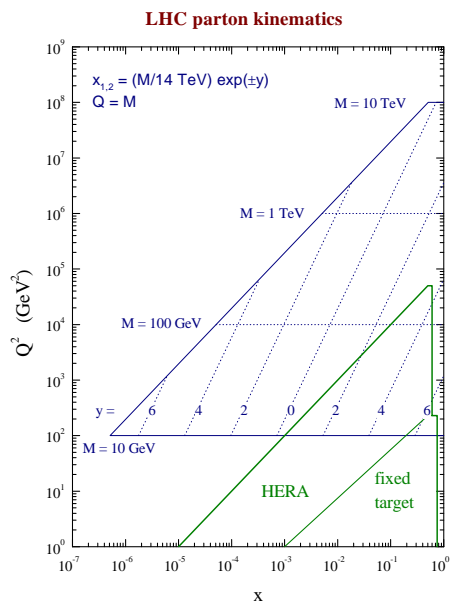
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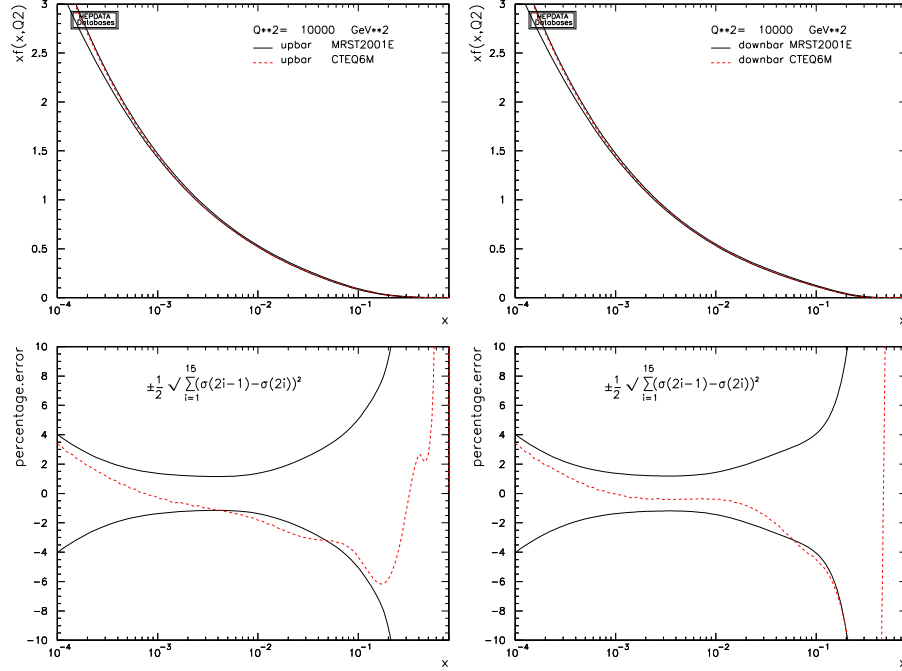
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**Abstract.** I outline some of the most recent developments in the global fit to parton distributions performed by the MRST collaboration.

At present there is a great deal of interest in the importance of parton distributions for studies at the LHC. This necessarily involves a certain kinematic range for the partons. The kinematic range for particle production at the LHC is shown in fig. 1. Parton distributions at  $x \sim 0.001 - 0.01$  are vital for understanding the standard production processes at the LHC. However, even smaller (and higher)  $x$  partons are required when one moves away from zero rapidity, e.g. when calculating the total production cross-section of the heavy boson. As well as the central values one needs the uncertainties on the partons, and there has been a lot of work on this [1]–[9]. This uncertainty is shown for the  $\bar{u}$  and  $\bar{d}$  quarks in fig. 2. Central rapidity production of  $W, Z$  Higgs at the LHC probes  $x = 0.006$ , which is ideal for the MRST partons. The current best



**FIGURE 1.** The kinematic range for particle production at the LHC



**FIGURE 2.** Uncertainty on MRST  $\bar{u}$  and  $\bar{d}$  distributions, along with CTEQ6

estimate for the uncertainty due to experimental errors is  $\delta\sigma_{W,Z}^{\text{NLO}}(\text{expt. pdf}) = \pm 2\%$ , but we note that there is a theoretical uncertainty, which is potentially large due to possible problems at small  $x$ . This is because the large rapidity  $W$  and  $Z$  cross-sections sample very small  $x$ . However, the ratio  $\sigma(W^+)/\sigma(W^-)$  is a *gold-plated* prediction, where  $R_{\pm} = \frac{\sigma(W^+)}{\sigma(W^-)} \simeq \frac{u(x_1)\bar{d}(x_2)}{d(x_1)\bar{u}(x_2)} \simeq \frac{u(x_1)}{d(x_1)}$  and using the MRST2001E partons  $\delta R_{\pm}(\text{expt. pdf}) = \pm 1.4\%$ . Assuming all other uncertainties cancel, this is probably the most accurate SM cross-section test at LHC.

This suggests that  $\sigma(W)$  or  $\sigma(Z)$  could be used to calibrate other cross-sections, e.g.  $\sigma(WH)$ ,  $\sigma(Z')$ . As an example we consider  $W$  plus Higgs production.  $\sigma(WH)$  is more precisely predicted than  $\sigma(W)$  because it samples quark pdfs at higher  $x$  and scale. However, the ratio shows no improvement in uncertainty, and can be worse, see fig. 3. This is because partons in different regions of  $x$  are often anti-correlated rather than correlated, partially due to sum rules. Similarly, there is no obvious advantage in using  $\sigma(t\bar{t})$  as a calibration SM cross-section, except maybe for very particular, and rather large,  $M_H$ , where the gluon is probed in the same region for both. However, a light (SM or MSSM) Higgs is dominantly produced via  $gg \rightarrow H$ , and the cross-section has a small pdf uncertainty because  $g(x)$  at small  $x$  is well constrained by HERA DIS data. The current best MRST estimate, for  $M_H = 120\text{GeV}$ , is  $\delta\sigma_H^{\text{NLO}}(\text{expt pdf}) = \pm 2 - 3\%$  with less sensitivity to small  $x$  than  $\sigma(W)$ . This is a much smaller uncertainty than that from higher-order corrections, for example [10],  $\delta\sigma_H^{\text{NNLL}}(\text{scale variation}) = \pm 8\%$ . In contrast, the error on predictions for very high- $E_T$  jets at the LHC is dominated by the parton uncertainties, because it is sensitive to the relatively poorly known high- $x$  gluon.

Different approaches to fits generally lead to similar uncertainties for measured quantities, but can lead to different central values [9]. For the true uncertainty one must consider the effect of assumptions made during the fit and the correctness of fixed order QCD. The failings of NLO QCD are indicated by some areas where the fit quality could be improved. There is a good fit to HERA data, but there are some problems at the high-

pdf uncertainties on W, WH  
cross sections at LHC (MRST2001E)

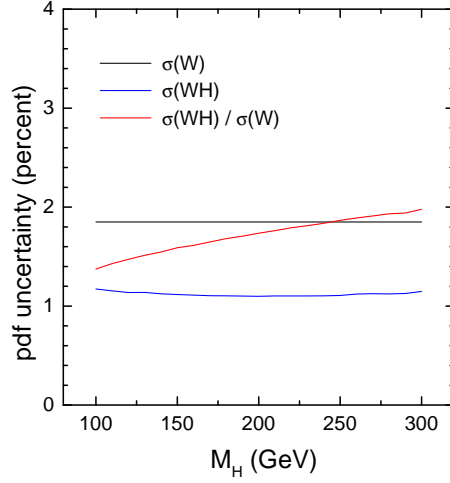


FIGURE 3. The uncertainty on W and Higgs production

est  $Q^2$  at moderate  $x$ , i.e. in  $dF_2/d\ln Q^2$ . Also the data require the gluon to be small or negative at low  $Q^2$  and  $x$ , and this is needed by all data (e.g. Tevatron jets), not just low- $Q^2$ , low- $x$  data. Other groups find similar problems with the gluon at low  $x$ . CTEQ have a valence-like input gluon at  $Q_0^2 = 1.69\text{GeV}^2$  which would marginally prefer to be negative [11]. There is also instability in the physical, gluon-dominated quantity  $F_L(x, Q^2)$  going from LO to NLO to NNLO, seen in fig. 4. The exact NNLO coefficient function [12] has a very large effect, a possible sign of  $\ln(1/x)$  corrections being required.

As an example of the effect assumptions can make to the fit, MRST found only a

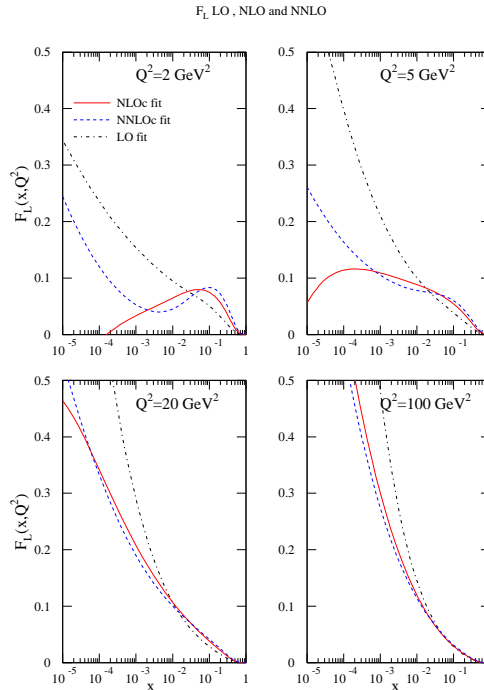
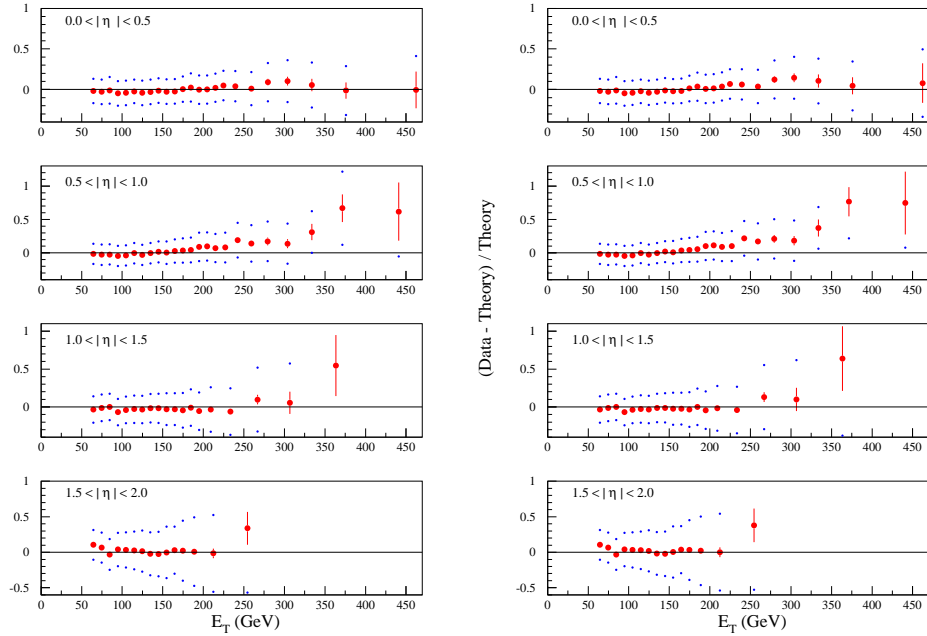


FIGURE 4. The MRST prediction for  $F_L(x, Q^2)$  at LO, NLO and NNLO

MRST 2004 NNLO DIS-type and D0 jet data,  $\alpha_s(M_Z)=0.1167$ ,  $\chi^2=64/82$  pts

MRST 2004 NNLO DIS type and D0 jet data,  $\alpha_s(M_Z)=0.1167$ ,  $\chi^2=75/82$  pts



**FIGURE 5.** Change in fit to D0 data with weak corrections

reasonable fit to jet data [13, 14], but needed to use the large systematic errors, while the result is better for CTEQ6 [6] due to different cuts on other data, and a different type of high- $x$  parameterization. However, for the CTEQ6.1M partons, which give a good fit to the jet data, the gluon is very hard as  $x \rightarrow 1$ . MRST have recently utilised the fact that, under a change of scheme from  $\overline{MS}$  to DIS schemes, the scheme transformation will dominate the high- $x$  gluon if valence quarks are naturally biggest at high  $x$  [15]. This allows a high- $x$  gluon in the  $\overline{MS}$  scheme which is determined from the quarks. At NLO the  $\chi^2$  for jets reduces from 154 to 116. This prescription works even better at NNLO –  $\chi^2$  for the jets goes from 164 to 117, and the total  $\Delta\chi^2 = -79$ .

Regarding high  $E_T$  jets, there has recently been a calculation of weak corrections [16], which implies that  $\sigma_{QCD} \rightarrow \sigma_{QCD} \left(1 - \frac{2}{3} C_F \frac{\alpha_W}{\pi} \log^2(E_T^2/M_W^2)\right)$ . This is a 12% correction at  $E_T = 450\text{GeV}$ , and the authors question the validity of recent partons due to this. We have studied the phenomenological impact, and the movement of both CDF and D0 data is relatively small, as shown in fig. 5. The total  $\chi^2$  changes by  $\sim 15$  without refitting, which is significant but not a disaster. The correction is more important at higher  $E_T$ , but there are positive real corrections to be added which depend on the jet definitions.

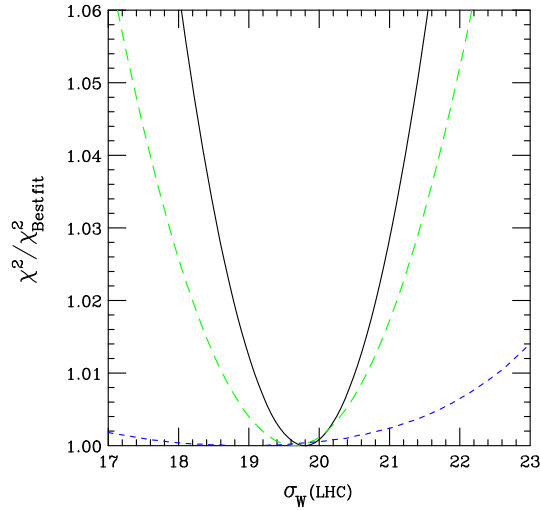
There has also been a study of the inclusion of QED effects by MRST [17]. The overall effect is small, but does lead to small isospin violation because  $u_V^p(x)$  quarks radiate more photons than  $d_V^n(x)$  quarks. This is in the correct direction to reduce the NuTeV  $\sin^2 \theta_W$  anomaly [18], and with current quark masses it is halved. Our approach is supported by data on wide-angle photon scattering, i.e.  $ep \rightarrow e\gamma X$  [19] where the final state electron and photon have equal and opposite large transverse momentum.

A much more important correction is NNLO QCD. The NNLO splitting functions are now complete [20], but very similar to the average of previous best estimates, so lead to no large change in our previous NNLO partons [21]. The NNLO corrections improve the quality of the fit slightly, and reduce  $\alpha_s$ . However, to perform an absolutely correct

NNLO fit we need not only exact NNLO splitting functions, but also require a rigorous treatment of heavy quark thresholds [22], NNLO Drell-Yan cross-sections [23], and a complete treatment of uncertainties. All this is in hand, and an essentially full NNLO determination of partons will appear very soon. Only the NNLO jet cross-section is missing. This is probably not too important – the NLO corrections themselves are not large, except at high rapidities, being  $\sim 10\%$  at central rapidities. There are also good NNLO estimates, i.e. the threshold correction logarithms, which are expected to be a major component of the total NNLO correction [24]. These give a flat 3 – 4% correction, i.e. smaller than the systematic errors on the data. Hence, the mistakes from ignoring jets in the fits are bigger than the mistakes made at NNLO by not knowing the exact hard cross-section. There is reasonable stability order by order for (quark-dominated)  $W$  and  $Z$  cross-sections, but this fairly good convergence is largely guaranteed because the quarks are fit directly to the data. This is much worse for gluon-dominated quantities, e.g.  $F_L(x, Q^2)$ , which is unstable at small  $x$  and  $Q^2$ , as seen in fig. 4.

Given this theoretical uncertainty, we devised an approach to look for the safe theoretical regions, i.e. change  $Q_{cut}^2$  and  $x_{cut}$ , re-fit and see if the quality of the fit to the remaining data improves and/or the input parameters change dramatically [25]. Raising  $Q_{cut}^2$  from  $2\text{GeV}^2$  in steps, there is a slow, continuous and significant improvement for higher  $Q^2$  up to  $> 10\text{GeV}^2$ . Raising  $x_{cut}$  from 0 to 0.005, there is a continuous improvement, and at each step the moderate  $x$  gluon becomes more positive. This led to the MRST2003 conservative partons, which should be the most reliable method of parton determination, but are *only* applicable for a restricted range of  $x$  and  $Q^2$ . We also have NNLO conservative partons, with similar cuts and improvement in fit quality, but the change in the partons is considerably less in this case because NNLO includes important theoretical corrections lacking at NLO. The variation in predictions with the cuts indicates the range of possible theoretical errors. There is a large change in  $\sigma_W$  at the LHC since this is sensitive to the low  $x$  region. The prediction is much more stable at NNLO, and LHC uncertainties are  $\sim 3 - 4\%$  including the theoretical uncertainty. Hence,  $\sigma_W$  is a good candidate for luminosity determination. CTEQ have repeated this type of analysis and see a similar type of behaviour with cuts [11], although much less dramatic. With conservative cuts on data their input gluon again marginally prefers to have a negative component, confirming that a negative/small gluon at low  $x$  and  $Q^2$  is not due to the data at low  $x$  and  $Q^2$ . They also find that the prediction for  $\sigma_W$  at the LHC moves down, but only a little, as more cuts are imposed. However, the loss of data with more cuts leads to larger errors, and the  $\chi^2$  profile is very flat indeed in the downwards direction, as seen in fig. 6. There is not really any inconsistency with MRST. If one is cautious about the accuracy of theory at low  $x$  and  $Q^2$ , the conclusion that the uncertainty is large on small  $x$ -sensitive quantities holds. CTEQ claim no reason to be cautious. This theoretical uncertainty is not so much of an issue at NNLO though, as discussed above.

In conclusion, we determine the parton distributions and predict cross-sections by performing global fits, and the fit quality using NLO or NNLO QCD is fairly good. There are various ways of looking at uncertainties due to errors on data, and they are 1 – 5% for most LHC quantities. Ratios often do not reduce uncertainties. QED corrections are small, but introduce important isospin asymmetry. The uncertainty from using different approaches is often comparable to or even larger than deriving from the



**FIGURE 6.** CTEQ  $\chi^2$  profile for  $\sigma_W$  [11], where the wide profile is for conservative cuts

errors on the data. For example, a model for the input form of the gluon can solve the apparent high- $E_T$  jet problem. Errors from higher orders/resummation are potentially large. Conservative cuts on  $x$  and  $Q^2$  allow an improved fit to the remaining data, and altered partons. CTEQ see some effects from this type of study, but these are much smaller. NNLO is much more stable than NLO, and more theoretically reliable. For MRST full NNLO fits are imminent, and should become the new standard.

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