Modelling thin tissue compartiments using the immersed FEM (continuous Galerkin)

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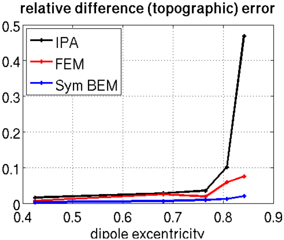
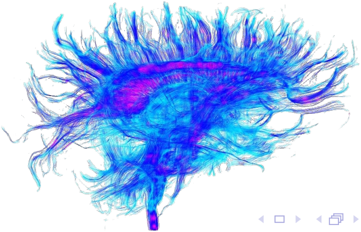
Motivations for Immersed FEM

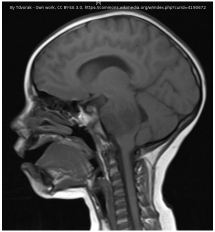
Long term goal: Democratize EEG/MEG/sEEG FEM models.
Reasons: Competitive, handles anisotropy, volumic, . . .

Challenges:

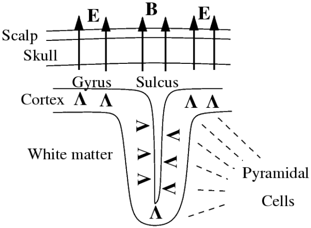
- Make software widely available [e.g. SimBio, NeuroFEM since 2013].
- Interface it with standard toolboxes [see Johannes talk].
- Environment and tools to easily build FEM head models.
  - Head tissue segmentation [FSL, Free Surfer, BrainSuite, SPM, Slicer, BESA, . . .]
  - **Build the FEM model from segmentations.**
  - Explore the solution.

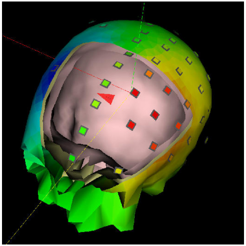






The EEG Forward Problem
Quasi-static electromagnetism



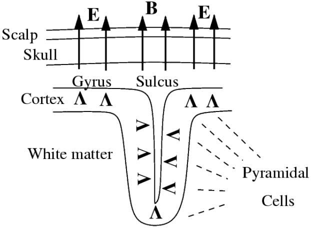


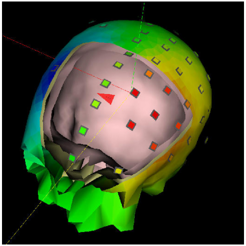
The potential V is related to the source currents $\mathbf{J}^p$ by:

$$\begin{cases} \nabla \cdot (\sigma \nabla V) = \nabla \cdot \mathbf{J}^p & \text{in} \quad \Omega \\ \sigma \nabla V \cdot \mathbf{n} = g & \text{on} \quad \partial\Omega. \end{cases}$$

- Ω: head volume.
- σ: local conductivity.
- $[V] = [\sigma \nabla V] = 0$ across interfaces.

The EEG Forward Problem
Quasi-static electromagnetism





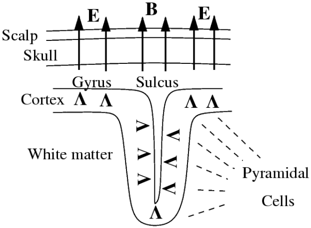
Weak formulation:

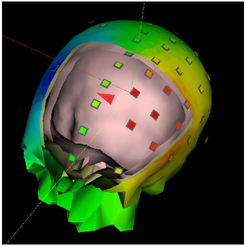
$$C(V) = \frac{1}{2}\int_{\Omega} \sigma(\mathbf{r})\|\nabla V(\mathbf{r})\|^2 d\mathbf{r} + \int_{\Omega} \nabla \cdot \mathbf{J}^{\mathrm{p}}(\mathbf{r}) V(\mathbf{r}) d\mathbf{r} - \int_{\partial\Omega} gVds \; .$$

$\Longrightarrow$ after FEM discretization: $[\Delta_\sigma]\mathbf{V} = [\nabla]^T \mathbf{J}^{\mathrm{p}}$

m

The EEG Forward Problem
Quasi-static electromagnetism



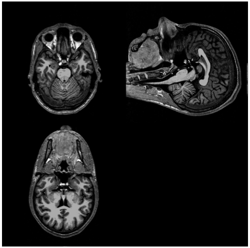


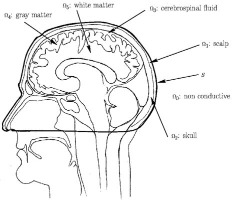
MEG can be put in the same fraemwork (adjoint method).

EEG Forward Model (FEM)

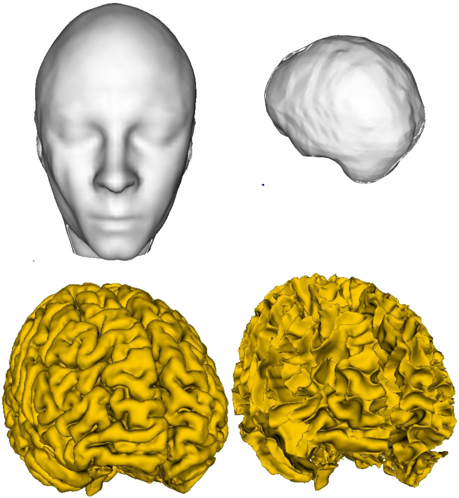
Head segmentation and mesh generation

MRIs, CT-scan, . . .



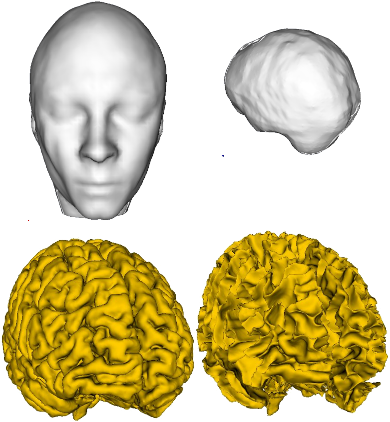


$\Longrightarrow$



EEG Forward Model (FEM)

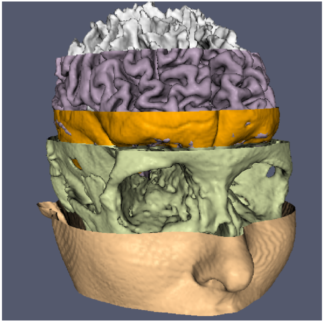
Head segmentation and mesh generation

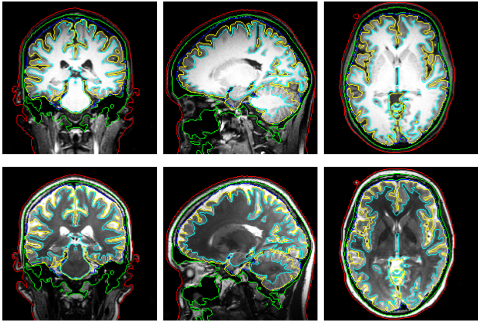


- Very thin structure:
  - Esp. for infants.
  - Very resolved meshes.
  - Derefinement.
- Topology changes.
  - Self intersections.
  - Holes in the model.
  - Complicate head modelling.
- Lots of work.
- Huge meshes.
- Computational quality.

EEG Forward Model (FEM)

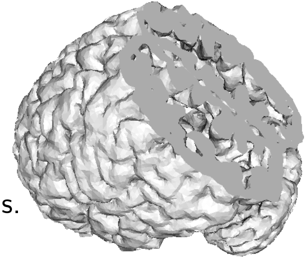
Potentially much more complex geometries. . .





- Complex skull geometry.
- Blood vessels.

From surfacic meshes to matrices $[\Delta_\sigma]$



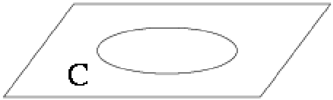
- Volumic meshing between surfaces.
- Merge volumic meshes.
- Matrix computation from base functions.

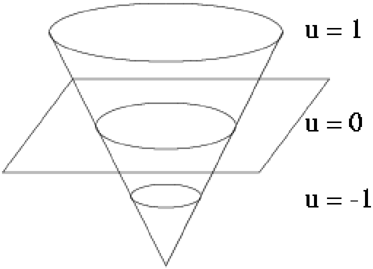
- For each subject.
- Size/accuracy tradeoff.
- Image data ignored during simplication.

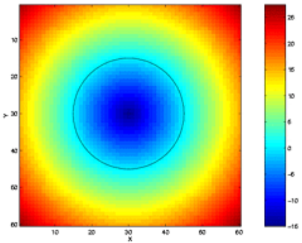
Solution: **Compute matrices directly from segmentations.**

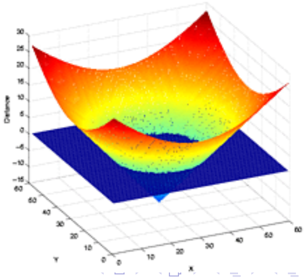
Direct Use of Segmentations

Level sets representation of surfaces





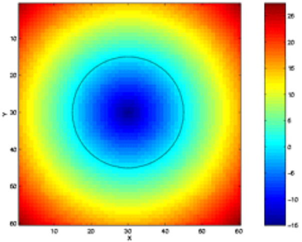


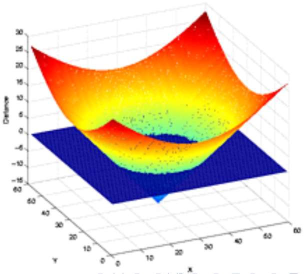


Direct Use of Segmentations
Level sets representation of surfaces

Most curve (2D) or surface (3D) manipulations can be expressed as levelset manipulations.

Segmentation.



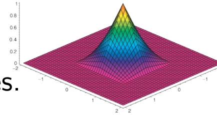


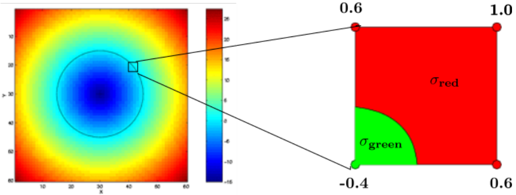
Direct Use of Segmentations

Direct computation of $[\Delta_\sigma]$ from levelsets

The MRI image space is the computation reference space.

- Use the "image mesh". Trilinear interpolation method (Q1).
  $\Longrightarrow$ The solution is an image of potential.
- Interfaces defined by implicit functions (levelsets).
  $\Longrightarrow$ Computation of partial integrals wrt interfaces.





Implicit boundary:

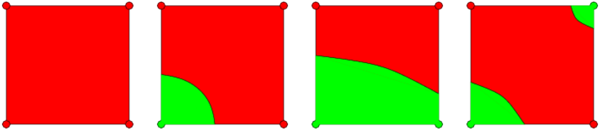
$f(x, y) = c_{11}xy + c_{10}x + c_{01}y + c_{00} = 0$.

Sum green and red contributions

$\Longrightarrow$ Closed form formula in 2D (integral of a rational fraction).

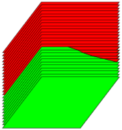
Direct Use of Segmentations
2D/3D algorithms

2D:
- Apply transform (rotations, symmetries, sign inversion) to reduce to:



- Compute the explicit 2D formulae.
- Correct for the transform action.

3D: Numerical integration of 2D values:



2D topological changes.

Direct Use of Segmentations
2D/3D algorithms

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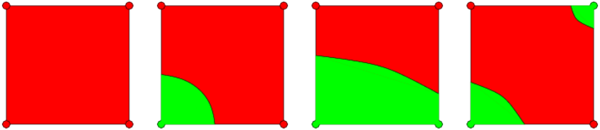

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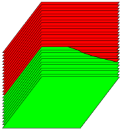
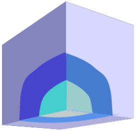


2D topological changes.



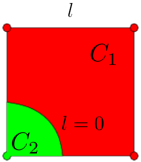
Multiple interfaces per voxel can be handled.

Non-differentiable Elements

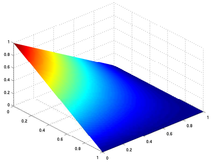
V and $\sigma\nabla V$ continuous over Ω . . .
. . . but σ is discontinuous across interface boundaries
$\Longrightarrow$ ∇V discontinuous: **not representable with Q1 elements.**
$\Longrightarrow$ **Use non-differential continuous elements (piecewise Q1):**

$$\begin{cases} \phi(x,y,z) = p_1(x,y,z) & in \quad C_1, \\ \phi(x,y,z) = p_2(x,y,z) & in \quad C_2, \end{cases} \quad p_1, p_2 \in \mathrm{Q1}$$

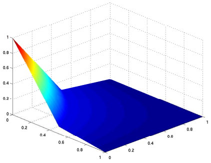
$$p1 - p2 = \lambda l, \quad \lambda \in \mathbb{R} \qquad \int_{l=0} \sigma_1 \nabla p1 \cdot \mathbf{n} - \sigma_2 \nabla p2 \cdot \mathbf{n} = 0$$



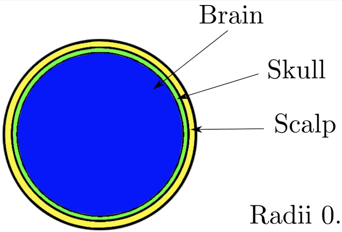
$\sigma_1 = 10$
$\sigma_2 = 1$



$\rightsquigarrow$

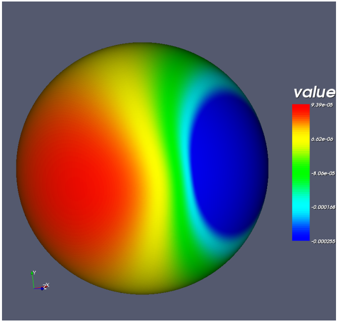


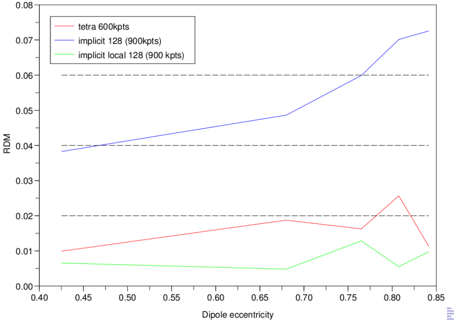
Results: Spherical Model



$$RDM = \left\| \frac{V_{num}}{\|V_{num}\|} - \frac{V_{anal}}{\|V_{anal}\|} \right\|$$

Radii $0.87, 0.92, 1.$
Conductivities $0.33, 0.0042, 0.33\ \ S.m^{-1}$.





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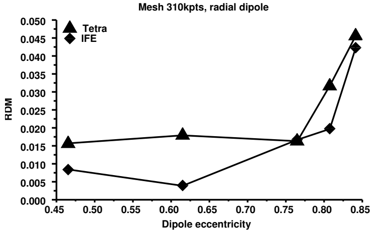
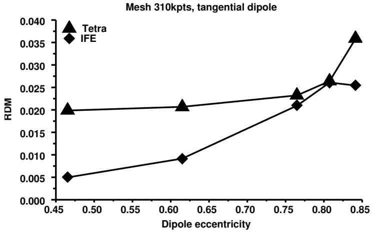
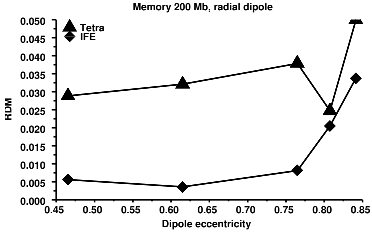
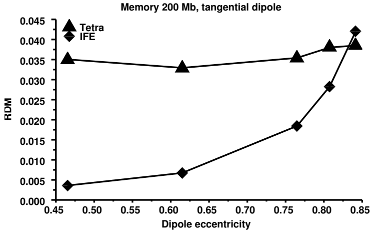


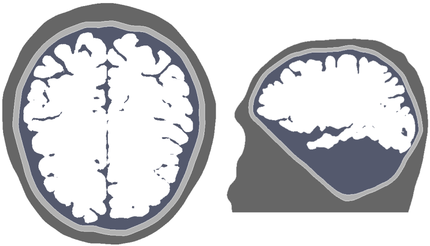


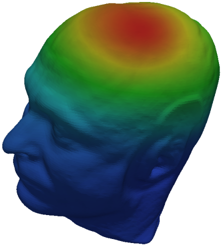




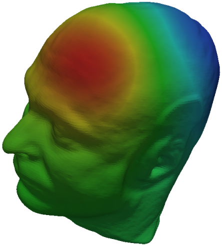
Results: Real Data

MRI image of size $256 \times 256 \times 160$
5 conductivity domains segmented using four levelsets.
Rescaled to $128 \times 128 \times 80$
$\Longrightarrow$ **many thin structures and double interface cells are present !**





Radial



Tangential

Conclusions
Take home messages

Immersed FEM:

- Use MRI as the computational space.
- Direct connexion Q1 Levelsets $\longleftrightarrow$ FEM matrices.
- **Simple, fast and unsurprising: a real use case advantage.**
- **Thin structure easy to deal with.**
- **Accuracy similar to standard FEM.**
- **Easy visualization and exploration of the solution.**
- Multiscale and parallelisation are easy.
- Control of boundary condition of solution more complex.

Acknowledgments:
Sylvain Vallaghé, Jérome Piovano, EADS foundation grant n°2118.

Details: *A trilinear immersed finite element method for solving the EEG forward problem,* Vallaghé, Papadopoulo.

Thank you !!

Direct Use of Segmentations (2)

The MRI image space is the computation reference space.

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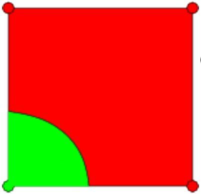
- Interfaces defined by implicit functions (levelsets).

$\Longrightarrow$ Computation of partial integrals wrt interfaces.



Implicit boundary: $f(x,y) = c_{11}xy + c_{10}x + c_{01}y + c_{00} = 0$.

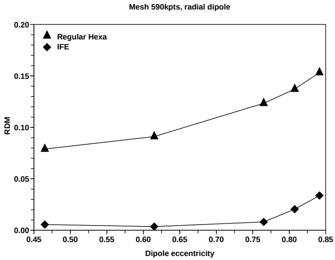
Green contribution:

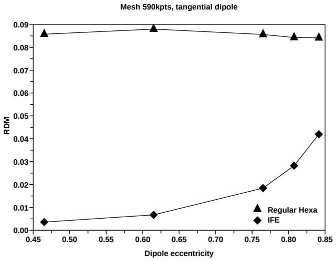
$$\int\int_{Green} m(x,y)dydx = \int_0^{x_0}\int_0^{-\frac{c_{10}x+c_{00}}{c_{11}x+c_{01}}} m(x,y)\, dydx = \int_0^{x_0} M\left(x, -\frac{c_{10}x+c_{00}}{c_{11}x+c_{01}}\right) dx \, .$$

$\Longrightarrow$ Analytic formula (integral of a rational fraction).

- Special cases (polynomial expressions): $c_{11}c_{00} - c_{10}c_{01} = 0$, $c_{11} \simeq 0$ (taylor expansion).

Comparison with FD





More Comparisons with P1 Implementation