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Essays on Financial Stability

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Abstract

This thesis consists of two essays concerning how banking regulations may promote financial stability. The first chapter investigates the competition-concentration-stability nexus from a novel perspective, by considering how concentration and, *inter alia* competition, affect the likelihood of an individual bank failing, and the likelihood of the bank failure spreading contagiously to the rest of the banking system. Competition is shown to reduce individual bank and systemic stability by reducing banks' profit buffers to absorb liquidity shocks. The impact of concentration on stability is more nuanced however, as increased concentration increases banks' profit buffers but also increases the concentration risk in the interbank market, widening the channel of contagion by which a liquidity shock can spread throughout the network. The second chapter concerns optimal ex-ante prudential regulation and ex-post resolution policy of globally systemically important banks. It characterises the conditions under which weakly capitalised, limitedly liable banks have incentives to 'gamble for resurrection' by investing in risky asset portfolios, in the knowledge that the downside risk is shifted onto the deposit insurance fund. In this context it is shown that a bank resolution by 'bailing in' unsecured debt holders can restore the incentive for banks to act prudently, and that the bail-in should occur above the point of insolvency to ensure the bank has sufficient skin in the game. The interplay of three ex-ante prudential regulatory instruments is analysed: the minimum capital and total loss absorbing capacity *requirements* and the minimum capital *buffer*. The minimum capital and TLAC requirements are set to ensure that the bank has sufficient skin in the game to invest prudently and tradeoff the ex-post costs of bailing in unsecured debt holders, the cost of bailing out depositors and the cost of equity issuance, and minimum equity buffer is set to ensure an appropriate trigger for resolution.

Contents

Chapter 1

Competition, Concentration and Contagion

1.1 Introduction

The "competition-concentration-stability nexus" which is the relationship between competition, concentration and stability in the banking industry has received renewed research interest in the wake of the financial crisis in 2008. This relationship has become particularly important as competition rules were relaxed during the financial crisis to stabilise banks by facilitating mergers, and in the EU an exemption in state aid rules was applied, which allows EU member states to use state resources to assist failing banks to prevent a serious disturbance in the economy. A number of observers have questioned whether developments in the state of competition in the banking industry such as the financial liberalisation in the 1970s or the recent consolidation in the banking sector have made the financial system more or less fragile. For example the UK Secretary of State approved the merger of HBOS and Lloyds TSB on advice from the UK prudential regulatory authorities that it was necessary to maintain financial stability, despite the UK competition authority (the OFT) objecting to the merger on the grounds of a material reduction in competition. Vickers (2010) reviewed this case and argued that "it would appear to have been a mistake to waive normal merger law to address the HBOS problem once it was clear, as it was by early October 2008, that a systemic solvency problem existed. Relaxation of competition law was not a good way to help financial stability in this case, and as the subsequent problems of LBG have shown, it may have worsened it."

A review of the history of banking regulation highlights that regulators' views on the relationship between competition, concentration and stability have evolved over time. Following the great depression in the 1930s, there was a view held in a number of advanced economies that there is a tradeoff between the benefits of greater competition promoting greater allocative, productive and dynamic efficiency and ultimately social welfare, versus the costs associated with an increased likelihood of bank failure. This led competition authorities to protect a highly concentrated banking industry with lower intensity of competition. For example, in the US the banking industry was exempt from the application of antitrust policy until the 1960s, and the European Commission did not apply competition policy in the banking sec-

tor until the early 1980s. The regulations in place in the period following the great depression until the period of liberalisation in the 1970s and 1980s suggest that regulators at the time held the 'competition-fragility' and 'concentration-stability' views. Since the 1970s it appears that these views have become less prevalent, as a period of deregulation reduced bank industry concentration and increased competition from domestic and foreign banks and other non-bank financial institutions. Following the entry and expansion of new banks in the 1980s and 1990s, there has been an increase in concentration resulting from a number of domestic and cross-border mergers in Europe and the US. There have been further increases in banking industry concentration since the onset of the financial crisis beginning in 2008 as, in order to allow banks to weather the crisis, a number of countries and the European Commission have relaxed competition policy and state aid rules by bailing out banks and permitting, and in some case forcing, the takeover of weak banks by larger banks with stronger balance sheets. Thus it appears that the 'concentration-stability' and 'competition-fragility' views have returned to the favour of regulators.

There is no consensus in theoretical and empirical academic literature on the effect of competition or concentration on financial stability. There are two opposing views on the nature of the relationship: there is the 'competition-fragility' view that more intense competition makes banks more fragile, and the 'competition stability' view that greater competition promotes greater stability of the banking system. Similarly there are 'concentration stability' and 'concentration fragility' views that increased concentration increases and reduces stability in the banking industry, respectively. In order to design an appropriate ex-ante competition framework for the banking industry an understanding of the relationship between competition, concentration and stability is key. If there is a tradeoff between the benefits of greater competition or reduced concentration in terms of promoting efficiency versus the costs of reducing stability then there is a need for competition authorities and prudential regulators to coordinate, for example to fully understand the ramifications of approving a proposed merger between two banks.

Post-crisis regulatory reform has aimed at increasing the resilience of individual banks to adverse shocks, but also has a greater focus on promoting *systemic* stability. A systemic banking crisis according to the 'narrow' definition adopted by De Bandt and Hartmann (2000), is a situation in which the failure of one financial institution "leads in a sequential fashion to considerable adverse effects on one or several other financial institutions or markets, e.g. their failure or crash". This definition includes crises in which financial distress in one bank spreads contagiously to others, but excludes situations in which banks are hit by a common shock such as poor macroeconomic fundamentals affecting highly correlated asset portfolio returns. Systemic banking crises are of particular interest to policymakers as there is a need to ensure that the continuity of the special role performed by banks in providing credit to the real economy, whereas the failure of a single, isolated bank should not cause major disruption to the economy as the failed bank's customers may switch to use one of the surviving banks. To understand how competition affects the incidence of banking crises, and in particular *systemic* crises the effect of changes in competition on the likelihood of contagion needs to be considered. Based on these observations the questions we seek to answer in this paper are the following:

1. *What is the effect of reduced concentration on systemic stability in the banking*

industry?

2. *What is the effect of increased competition on systemic stability in the banking industry?*

We distinguish between concentration and competition as the "structure conduct performance" paradigm which predicts a strong negative relationship between concentration and intensity of competition has been rejected in empirical literature. As Vives (2016) notes analysis of competition in the banking industry is complicated by the existence of a number of significant market failures including imperfect information, market power and externalities. Entry barriers into the traditional banking industry are high in part due to the high costs of establishing a branch network, building up a reputation for solvency and establishing a customer base, but also due to regulatory barriers such as the requirements to have in place appropriate risk management systems and meet prudential capital requirements. Nevertheless, concentration measures such as the Hirschmann-Herfindahl index (HHI)[1] cannot explain competitive outcomes alone. Banking markets are contestable to a certain degree as regulatory liberalisation has lead to banks facing competition, particularly in recent years from Fintech competitors which have disrupted the status quo in a number of banking markets.

In line with the above discussion we distinguish between the effect of competition and *inter alia* concentration on stability.[2] 'Concentration' is measured by the number of equally sized banks N, so a concentrated banking industry is one with a small number of banks. 'Competition' is defined as the intensity of competition between banks *for a given level of concentration*, and is determined by a parameter d which measures how much depositors value the specific banking services and other non-price product characteristics including physical locations of branches. The more depositors value these non-price product characteristics, the less competitive the market is i.e. the more banks will be able to exploit market power by offering lower deposit rates to depositors.

In answering the research questions posed above we aim to establish the magnitude, and more importantly the *direction* of any causal relationship from increased concentration or reduced intensity of competition on the incidence of systemic banking crises. To answer these questions we first review the relevant literature. The review highlights that theoretical banking crisis models typically assume perfect competition or monopoly in the banking sector, and thus do not provide an answer to the research questions above. The theoretical and empirical literature which focuses specifically on the relationship between competition and stability provides a rather mixed picture, and few of these papers considered the effect of competition on systemic, rather than individual bank stability. To examine the relationship in theory we introduce imperfect competition in the Allen and Gale (2000) model of contagion

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[1]HHI is equal the sum of the squares of all banks market shares. It is bounded between 0 (indicating perfect competition) and 1 (indicating monopoly).

[2]Note that the potential effect of competition on concentration and vice-versa are outside of the scope of the paper. Whilst it could be argued that as banks face increased competition from Fintech firms the endogenous response of the banking industry could be to consolidate to preserve profitability. The focus of this paper is rather to understand from a static perspective how competition and concentration affect financial stability, without analysing how bank industry structure may evolve.

through interbank markets. The modified model suggests that whereas an increase in industry concentration provides banks with a profit buffer which they can use to serve their depositors in the event of an increase of liquidity demand increasing the likelihood of contagion, it also broadens the channel of contagion through the interbank market meaning that there are two opposing effects of increased concentration on stability.

1.2 Literature Review

This paper bridges literature on financial contagion, and the literature on the relationship between competition, concentration and stability.

Literature on financial contagion shows how an idiosyncratic shock which causes the failure of one bank can be propagated to other banks, threatening the stability of the entire banking system. Transmissions channels put forward in the literature include idiosyncratic bank failures triggering a firesale of assets an environment of limited liquidity resulting in the asset price falling below its fundamental value (Diamond and Rajan (2011)), the failure of bank acting as a 'wakeup call' to other investors by revealing information on the vulnerability of banks with correlated asset portfolios (Chen (1999)) and connections through the interbank deposits (Allen and Gale (2000)). This literature typically assumes perfect competition so that the equilibrium deposit contract maximises depositor utility subject to the bank's resource constraints, and thus does not address the question of how competition and concentration affect the likelihood of financial contagion.

A number of papers have considered the effect of competition in the interbank market on stability. Allen and Gale (2004) show firms with market power in the interbank market have an incentive to provide liquidity to distressed banks in order to avoid contagion. On the other hand Acharya et al (2012) show that banks may strategically refuse to lend to distressed banks in order to induce the distressed bank to sell assets at fire sale prices and to increase market share. These papers focus on whether there are strategic gains from supplying or refusing to supply liquidity to rival banks in the event of a crisis. In contrast to these papers, we assume that the interbank lending rates are fixed ex-ante as in Allen and Gale (2000), and focus rather on how changes in industry structure affect concentration risk and profit buffers as discussed in section 1.3 below.

A separate body of literature which focuses specifically on how competition affects stability. Broadly speaking, it can be divided into two opposing views. The "charter-value" view proposed by Keeley (1990) states that increased competition erodes a bank's charter value (i.e. the net present value of future profits from keeping its charter), which reduces the opportunity cost if the bank goes bankrupt (i.e. it lowers the bank's skin in the game) and thus incentivises bank managers to hold a more risky asset portfolio. The charter value view assumes that banks can choose the riskiness of their asset portfolios. By contrast the 'risk-shifting' view of Boyd and De Nicolo (2005) assumes that risk is chosen by the borrower, so that greater competition in the loan market results which reduces interest rates will increase profits for debtor firms, and in turn incentivise firms to reduce risk, reducing

defaults and banking crises, meaning that greater competition is associated with greater stability. Martinez-Miera and Repullo (2010) show that if loans defaults are imperfectly correlated then increased competition has two counteracting effects on financial stability: it decreases interest rates, decreasing the riskiness of loans, however it also increases the margin on loans which do not default giving the bank a greater buffer against losses. This yields a more subtle inversed U-shaped relationship between competition and stability, in which if competition is weak then the risk-shifting effect dominates so that increased competition undermines stability, whereas if competition is intense, increasing competition increases stability as the margin effect dominates.

There are two key contributions this paper makes to the literature on the competition-concentration-stability nexus. Firstly, we analyse the impact of bank industry structure on *systemic* stability whereas extant literature examines the effect on the likelihood of an individual bank failing, abstracting from potential contagion effects.[3] Secondly, to the best of my knowledge, my paper is the first to separately analyse the effect of concentration, and competition on financial stability. The majority of papers use the number of banks as a proxy of competition, or in the case of Keeley (1990) they use a measure of competition (Tobin's q) and do not consider the potential interaction between the two. In contrast, by utilising a model of banking competition which incorporates parameters for both the number of banks *and* the intensity of competition for a given number of banks, we are able to separately identify the effect of competition and *inter alia* concentration on financial stability.

1.3   Model

1.3.1   Outline

The starting point of the model outlined below is the seminal paper by Allen and Gale (2000), hereafter 'AG', on financial contagion through the inter-bank deposit market. This model provides a parsimonious framework in which to analyse the effects of competition on the likelihood of contagion and systemic crises. We extend the model in two ways. Firstly, we introduce imperfect competition in the market for depositors, which gives banks a profit buffer which they can use to serve early depositors in the event of a liquidity shortage. Secondly, we extend the model beyond the four banks example given in the original paper to highlight how greater competition may spread risk across banks, stabilising the banking system. These modifications introduce a potential tradeoff in the relationship between concentration and stability, as higher concentration (equivalently a lower number of banks) means that banks are less able to diversify risk in the interbank market, but enjoy

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[3]Freixas and Ma (2015) analyse how competition affects systemic risk, in addition to the effect on portfolio, liquidity and insolvency risk. They find that competition reduces banks' profit buffer against loses, but also reduces solvency risk in part of the parameter space. In contrast to the present paper which models financial contagion as an equilibrium outcome of profit maximising banks, Freixas and Ma (2015) simply analyse a 2 bank model and assume that the loss in asset value in a firesale is greater if both banks fail, without explicitly modelling equilibrium in the market for assets during a firesale.

larger profits which can be used as a buffer with which to serve customers in the event of a run.

1.3.2 Environment

There are three periods: $t = 0, 1, 2$. There is a single good and a continuum of identical depositors normalised to unity, which are endowed with a single unit of the good at date $t = 0$. Depositors learn at period 1 whether they are an 'early type' which value consumption in period 1 only, or a 'late type' which value consumption in period 2 only. There is no discounting.

As in AG banks compete by offering contracts which promises depositors a fixed amount c_1 at $t = 1$ if the depositor is an early type or c_2 at $t = 2$ if the depositor is a late type, in exchange for the depositor's unit endowment at t=0. Each depositor deposits her unit endowment at a single bank, and the probability of being an early type is known by all agents in period 0.

We depart from the AG framework by assuming that the market for deposit contracts is imperfectly competitive.[4] Banks are assumed to be horizontally differentiated in the depositor market ala the Salop (1979) 'circular city' spatial model of competition.[5] In this model depositors tastes' are represented by their location on a circle of unitary circumference, and the continuum of depositors is uniformly distributed around the circle. Banks are assumed to be maximally horizontally differentiated, so that given N banks, each bank is located a distance $1/N$ from each of its two closest neighbours.[6]

The locations of banks and depositors on the unit circle have a natural interpretation of their location in physical space. However as in Salop (1979), locations may also be interpreted as the banks' and depositors' locations in characteristics space reflecting other non-price characteristics of the services offered by the bank, and preferred by the depositor respectively. Horizontal differentiation in the bank depositor market may take various forms. For example, banks may provide different combinations of

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[4]For tractability, we follow the assumptions employed by AG by assuming that the deposit contract offered to other banks in the interbank market is the same as the equilibrium $\{c_1, c_2\}$ offered to regular depositors. If we assume instead that there is a separate market for interbank deposits which is perfectly competitive then banks still optimally hold deposits in all other banks so the channel of contagion is unchanged and as the interbank deposit flows net to zero in expectation the expected profits of the bank are also unchanged meaning that the results of this paper are insensitive to this assumption .

[5]Note that this model has been used to model competition in the banking industry by a number of other authors including Friexas and Rochet (2008) and Cordella and Yayati (2002). Further note that Degryse and Ongena (2005) find some empirical support for spatial discrimination in loan pricing which is consistent with the view that as banks become closer in physical and other characteristics space market power falls.

[6]Note that banks do *not* decide on their location in this model, so their competition will take the form of a static game taking place in period 0 in which banks simultaneously decided on the optimal deposit contract and investment portfolio. Economides (1989) derives maximal differentiation as an equilibrium phenomenon by formulating a three stage game in which firms decide whether or not to enter, choose a location of the unit circle, and compete in prices, assuming quadratic transport costs.

ancillary services or have different brand identities which depositors have different tastes for.

The number of banks N is determined by the banking regulator which issues charters which permit banks to operate, so the number of banks is exogenously specified rather than specifying a fixed entry cost and determining the number of banks being determined endogenously by a condition that the number of the banks is equal to the number such that further entry is no longer profitable i.e. a zero net profit condition. This ensures that the model captures two key features of the banking industry: limited entry due to licensing and strictly positive profits. For example in the UK when Metrobank opened in 2010 it was the first new high-street bank to be issued a bank charter in over 150 years, however changes in industry concentration have also taken place more recently through merger and acquisition activity.

A parameter d measures the "transport cost" which is incurred by each depositor to move from her location on the unit circle to the bank. This parameter measures the intensity of competition for a given level of concentration. Market power is increasing in d, and in the limit in which the transport cost is zero banks have no market power and the contract offered is the first best contract offered in AG. The introduction of internet banking in the 2000s may have reduced this transport cost parameter d as depositors do not need to physically visit branches so often. More generally, the competition parameter d may also be interpreted as a proxy for competition for a given level of industry concentration. For example, the liberalisation of the banking industry in the 1980s and 1990s and the growth of shadow banks which do not have a banking license but offer deposit-like services, or Fintech firms, which compete with banks in other markets, may erode banks' profit margins even if the number of banks is unchanged.

Depositor i incurs 'transport costs' to travel from her location l_i to the bank j's location l_j in order to deposit its unit endowment at that bank. As discussed above the locations reflect the locations in characteristics space, and therefore the transport cost $d|l_i - l_j|$ reflects the depositors' disutility from depositing at a bank which offers a package of services (which may include physical bank locations) which are different from the depositors preferred package. In period 0 the expected utility of a depositor i of a deposit contract with bank j is given by

$$u_i^j = \lambda u(c_1^j) + (1 - \lambda)u(c_2^j) - d|l_i - l_j| \tag{1.1}$$

where λ is the probability of being an early type, the deposit contract of bank j offers (c_1^j, c_2^j), $u()$ is a twice differentiable, strictly increasing and strictly concave instantaneous utility function, d is the 'transport cost' and l_i and l_j are the locations of depositor i and bank j respectively.

Depositors choose the deposit contract which maximises their expected utility so a depositor i which is located at a distance $m \in (0, 1/N)$ away from bank j is indifferent between banks j and the other closest bank k if the following condition holds

$$u^j - dm = u^k - d(1/N - m) \tag{1.2}$$

where $m = |l_i - l_j|$ and $u^j = \lambda u(c_1^j) + (1-\lambda)u(c_2^j)$ and $u^k = \lambda u(c_1^k) + (1-\lambda)u(c_2^k)$. Re-arranging (1.2) and noting that as bank has two immediate neighbours and due to symmetry the demand is equal to $2m$ so the demand curve is given by:

$$D = 2m = \frac{u^j - u^k}{d} + \frac{1}{N} \tag{1.3}$$

At $t = 0$ banks can invest the unit of good they receive from each depositor in two assets: the short asset (y) which is a simple storage technology which has a gross return of one unit after one period, and the long asset (x) which yields a return $R > 1$ after two periods, and yields a return $r < 1$ if liquidated after one period. So each bank faces a per-depositor feasibility constraint of $x + y \leq 1$. Given that the assets both yield a positive return it is clear that this constraint is binding, so that the long asset holdings of the bank can therefore be written as $x = 1 - y$. Substituting this directly into the bank's objective function the bank's maximisation problem can be written as the following. The objective function (1.4) is the industry profit function multiplied the bank j's market share. Equations (1.5) and (1.6) are the resource constraints for serving early depositors at time $t = 1$ and late depositors at time $t = 2$ respectively. Note that at time $t = 1$ the bank serves its early depositors with the liquid asset y and at time $t = 2$ the bank serves its late depositors with the illiquid asset x which has return R. The final constraint (1.7) is an incentive compatibility constraint which ensures that late types are weakly better off by revealing their true type rather pretending they are early types and withdrawing early. As $u'(c_2) \geq u'(c_1)$ the incentive compatibility constraint is also satisfied.

$$\max_{\{c_1^j, c_2^j, y^j\}} \Pi = \left(y^j + R(1-y^j) - \lambda c_1^j - (1-\lambda)c_2^j\right)\left(\frac{u_j - u^k}{d} + \frac{1}{N}\right) \tag{1.4}$$

subject to

$$\lambda c_1^j \leq y^j \tag{1.5}$$

$$(1-\lambda)c_2^j \leq R(1-y^j) \tag{1.6}$$

$$c_1^j \leq c_2^j \tag{1.7}$$

The derivation of the optimal deposit contract and asset portfolio is shown in Appendix A.1. Note as the problem is symmetric the deposit contract and investment portfolio is the same for all bank so that $\{c_1^j, c_2^j, y^j\} = \{c_1, c_2, y\} \forall j = 1, 2...N$. We assume a logarithmic utility function $u(c) = log(c)$ for tractability and for ease of comparison of results with AG. This yields an optimal contract in which $c_1 = Rc_2$ so the incentive compatibility constraint (1.7) is not binding. This yields the following deposit contract $\{c_1^*, c_2^*\}$ and asset portfolio $\{x^*, y^*\}$:

$$(c_1^*, c_2^*, x^*, y^*) = \left(\frac{N}{N+d}, \frac{RN}{N+d}, \frac{N+Nd-N\lambda}{N+d}, \frac{\lambda N}{N+d}\right) \tag{1.8}$$

In the limit as the number of banks becomes large ($N \to \infty$), or as depositors care less about bank location ($d \to 0$) then the late consumptions approaches the first-best outcome in which profits are zero, and the special case of perfect competition assumed by AG is obtained i.e. $lim_{N \to \infty}\{c_1^*, c_2^*\} = lim_{d \to 0}\{c_1^*, c_2^*\} = \{1, R\}$. Due to symmetry each bank has an equal market share and has demand of $1/N$ so the total profit of each bank is given by:

$$\Pi^* = \frac{1}{N}[y + R(1 - y) - \lambda c_1 - (1 - \lambda)c_2] = \frac{Rd}{N^2 + Nd} \qquad (1.9)$$

1.3.3   Interbank deposits and network structure.

This subsection explains that in the model banks are subject to regional liquidity shocks such that they have excess demand for liquidity in one state, and excess supply of liquidity in another state. In order to provide the second best allocation in (1.8) above banks hold interbank deposit banks in interconnected banks. The interbank holdings of banks serve as a conduit through which crises can spread contagiously from one bank to other banks.

There are N banks. In non-crisis states 1 and 2 the liquidity demands of odd banks (i.e. banks 1,3,5,...) are negatively correlated with those of even banks, as summarised in Table 1.1. Liquidity shocks are bank, rather than depositor-specific which results in banks optimally holding their deposits in a greater number of banks, meaning that an increase in the number of banks dilutes the impact of potential losses due to an interconnected bank failing.

The total liquidity in the system is constant in both these states, so there are idiosyncratic liquidity shocks but no aggregate shocks. There is a third crisis state $\bar{S}$ which occurs with a zero probability, but given that the probability of the state occurring is zero banks do not factor this state into their interbank holdings decisions. That state is discussed in the following subsection, and the rest of this subsection considers the optimal interbank deposit holdings.

Table 1.1 – Banks' Liquidity Shocks

State	Bank 1	Other Odd Banks	Even banks
1	λ_H	λ_H	λ_L
2	λ_L	λ_L	λ_H
$\bar{S}$	$\lambda + \epsilon$	λ	λ

Banks can exchange interbank deposits at $t = 0$. The payoffs of these interbank deposits are the same as those for retail depositors, so 1 unit of interbank deposit returns c_1 if withdrawn at $t = 1$ and c_2 if withdrawn at $t = 2$. Arbitrage opportunities ensure that retail depositors and other banks are offered the same deposit contract.

AG consider a number of different networks structures for inter-bank borrowing. Firstly, they consider a complete network structure in which every bank is directly connected with every other bank. Secondly, they consider an incomplete network

structure in which each bank holds deposits in one other bank only, but all banks are directly or indirectly linked to each other. Finally they consider separated incomplete networks in which each bank can only borrow from one other bank in the interbank market the banks are not all (directly or indirectly) connected with each other.

The issue of endogenous network formation is outside of the focus of this paper. Instead we assume a complete network structure, as we wish to capture the possibility that in a less concentrated banking industry (i.e. with a greater number of equally sized banks) risk of contagion through interbank deposits is smaller as interbank deposits are spread over a larger number of connected banks, thus reducing the impact of the failure of a single rival bank on a bank's stability. The structure is also analagous to a 'hub and spoke' network structure in which a large number of atomistic banks only participate in the interbank deposit network through a larger regional 'hub', which is itself in a complete market structure with all other large hub banks, which are themselves each connected to a large number of atomistic banks, and all banks within the region are perfectly (positively) correlated within the region, and perfectly (negatively) correlated across odd and even regions.

In AG, given N banks in a complete network structure each bank holds interbank deposits in the other $N-1$ banks. This entails banks are holding interbank deposits in all other banks (which are equal in size by symmetry), so that odd banks hold deposits in other odd banks despite the fact that the benefit of doing so is zero. As the cost of holding interbank deposits is zero by assumption banks would be indifferent between holding deposits in other banks experiencing the same liquidity shock. For tractability we assume that there are no interbank deposits held by an odd bank in another odd bank. We note that if a complete network structure is assumed the qualitative nature of the results is unaffected. We further note that within the incomplete network structure each bank only holds deposits in one other bank so there is no risk sharing effect in the incomplete network structure.

Each bank holds enough interbank deposits to cover the excess demand for liquidity in the case of a high liquidity demand (λ_H). As there are $N/2$ banks with negatively correlated liquidity shocks each bank j holds $z^j = \frac{\lambda_H - \lambda}{(N/2}$ in each of the $N/2$ banks with the other liquidity shock.[7]

At t=1 banks with a high liquidity demand must pay c_1 to λ_H of its depositors and honour the claims of the $\frac{N}{2} - 1$ other high liquidity demand banks. In order to finance this they liquidate their short asset y and redeem their deposits from the $\frac{N}{2}$ even banks which have a low liquidity demand. The budget constraint is therefore given by

$$\lambda_H c_1 = y + \frac{N}{2} z c_1 \tag{1.10}$$
$$\implies \lambda_H c_1 = y + (\lambda_H - \lambda) c_1$$

This simplifies to the profit maximising condition $\lambda c_1 = y$. A bank with low liquidity

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[7]So an odd bank has interbank deposits in the $N/2$ even banks, and vice-versa. The results are not qualitatively affected if we assume the bank j holds deposits in all other banks $k \neq j$.

demand must pay c_1 to λ_L of its depositors and honour the claims of the $\frac{N}{2}$ high demand. In order to finance this they simply liquidate the short asset y. The budget constraint in this case is given by:

$$\left[\lambda_L + \frac{N}{2}\left(\frac{\lambda_H - \lambda}{N/2}\right)\right] c_1 = y \tag{1.11}$$

The above expression also reduces to the profit maximising condition $\lambda c_1 = y$. In period 2 banks liquidate all remaining assets, and *in contrast to AG, profits are retained.* Therefore the budget constraint for a bank which had a high liquidity demand at $t = 1$ can be written as

$$\left[(1 - \lambda_H) + \frac{N}{2}\left(\frac{\lambda_H - \lambda}{N/2}\right)\right] c_2 + \Pi = R(1 - y) \tag{1.12}$$

Substituting the profit maximising conditions (A.10), (A.13), (1.8) and (1.9) into this expression the condition is satisfied. The period 2 budget constraint for a bank which had a low liquidity demand at t=1 is given by, the following expression which is also satisfied.

$$(1 - \lambda_L)c_2 + \Pi = R(1 - y) + \frac{N}{2}\left(\frac{\lambda_H - \lambda}{N/2}\right) c_2 \tag{1.13}$$

Thus using the interbank deposit network banks are able to provide the second best deposit contract (c_1^*, c_2^*) in both states 1 and 2. The following subsection analyses the outcome in the third state $\bar{S}$ in which there is excess liquidity demand in the banking system leading to the possibility that one bank may fail and to that it may spread to other banks in the system.

1.3.4 Crisis state $\bar{S}$

In state $\bar{S}$ all banks other than bank 1 have a liquidity demand of λ and bank 1 has a liquidity demand equal to $\lambda + \epsilon$. The liquidity demands of each bank are summarised in table 1.1 As the state $\bar{S}$ occurs with zero probability it does not affect the allocation at $t = 0$. In state $\bar{S}$ early depositors always withdraw their deposits at $t = 1$, and late depositors now withdraw at $t = 1$ if $c_1 > c_2$, or withdraw at $t = 2$ otherwise. Banks are contractually required to pay c_1 to all who demand withdrawal at $t = 1$.

1.3.5 Collateralised borrowing at $t = 1$

As the deposit market is imperfectly competitive banks are able to earn strictly positive profits by obtaining returns on their investments which exceed their liabilities to depositors. The profits are obtained at $t = 2$ as the bank optimally invests as little in the short asset y as is required to serve its early depositors (which form a proportion λ of the banks total depositors, in expected terms). In the crisis state $\bar{S}$ we assume that banks are able to obtain a 1 period ahead loan from an external

lender of last resort at an interest rate l to be repaid at $t = 2$ using its profits earned in that period. Note there are zero profits at t=1 so if we were to relax this assumption and have the 1 period ahead loan also available in states 1 and 2 then the bank would be unable to borrow from the external lender at $t = 0$. Also note that in AG the profits are zero in both periods so the loan is not available to banks. As the liquidity shock has already been realised, and R is non-stochastic, the profits are determined in period 1 and known to the external lender so the rate l is an exogenously determined interest rate for risk-free lending.

1.3.6 Pecking order

As in AG we define a pecking order which defines the order in which banks liquidate their assets. The pecking order assumed is that banks first liquidate the short-asset, then borrow by sacrificing period 2 profits, then liquidate deposits, and finally liquidate their long asset. This requires the following assumption:

$$1 < l < \frac{c_2}{c_1} < \frac{R}{r} \tag{1.14}$$

where l is the gross interest rate on loans. By liquidating one unit of the short asset today the bank forgoes one unit of profit tomorrow, whereas obtaining one unit of loan today forgoes $l > 1$ unit of profit tomorrow. Note that the external lender is only willing to offer the loan if the bank will have sufficient profits to repay the loan i.e. $L \leq \Pi/l$ where L is the amount borrowed by the bank at t=1, so if part of the long asset is liquidated early the lender will be willing to lend less. From equation (A.9) we know that $c_2/c_1 = R > 1$, and finally if we assume that early liquidation of the long asset is costly i.e. $r < 1$ then the pecking order is established.

If a bank is bankrupt i.e. if it cannot meet the demands of it's depositors by either by using the profit-collateralised loan or by liquidating some of its long asset, it is required to immediately liquidate all its assets. There is no sequential service constraint so all depositors (including banks) receive an equal proportion of the liquidated assets. So the amount paid out at $t = 1$ is c_1 if the bank is not bankrupt, otherwise it is equal to a liquidation value. Thus the bank has limited liability, and depositors bear the risk of the bank not being able to deliver on their promised consumption c_1 or c_2. The equilibrium liquidation value q^j equates the liquidated banks' assets i.e. the short asset y, the loan from the external lender and the interbank claims on other banks, with the liabilities i.e. deposits held by consumers and other banks.

$$q^j = \frac{y + rx + \frac{N}{2}zq^k}{1 + N2z} \tag{1.15}$$

A bank which is unable to serve its early depositors with the short asset alone must forgo profits and/or liquidate other assets. In order to prevent a run the bank must provide late depositors with at least c_1 so if a fraction ω the bank's customers are early type, it must keep a buffer $b(\omega) = \max\left(\Pi/l, r[x - \frac{(1-\omega)c_1}{R}]\right)$. The maximum

operator here reflects the fact that a bank in crisis will first try to serve customers with the profit-collateralised loan, however if this is not possible the bank will resort to liquidating the long-asset, and liquidation of the long asset (x) means that the profits (Π) are zero.

In state $\bar{S}$ bank 1 has an excess demand for liquidity of $(\lambda+\epsilon)c_1 - y = \epsilon c_1$. Assuming that all other banks have sufficient liquidity the value of the interbank deposits held by bank 1 is $q^k = c_1 \forall k \neq j$. In order to prevent a run in bank 1 the excess demand for liquidity must be less than the buffer so the following condition must hold:

$$\epsilon c_1 \leq b(\lambda+\epsilon) = \max\left(\Pi/l, r\left[x - \frac{(1-(\lambda+\epsilon))c_1}{R}\right]\right) \qquad (1.16)$$

Note that c_1, profits and the proceeds from sale of the long asset are all decreasing in N and increasing in d, however the terms on the right hand side of equation (1.16) decline more rapidly with N and increase more rapidly with d. Thus a crisis in bank 1 is triggered for a wider range of ϵ as concentration decreases (i.e. as N increases), and as competition increases (i.e. as d decreases) providing unequivocable support for the concentration-stability and competition-fragility views respectively *for individual banking crises*. However as this paper concerns *systemic* rather than individual bank crises, the key question is how competition and concentration affect the likelihood of the crisis spreading to other banks. Therefore we assume condition (1.16) is violated so that bank 1 fails. If bank 1 fails then the interbank claims on bank 1 are worth $q^1 < c_1$ and interconnected banks lose $(c_1 - q^1)z$. Note that the size of the shock ϵ affects whether or not bank 1 fails, but does not determine the liquidation value of bank 1's deposits, as all of bank 1's depositors withdraw in period 1 in the event of a run which does not depend on the proportion of early depositors. Since each bank holds just enough of the short asset y to satisfy its own early customers, all other banks will become bankrupt, and hence there is contagion, if the following condition is violated.

$$(c_1 - q^1)z \leq b(\lambda) = \max\left(\Pi/l, r[x - \frac{(1-\lambda)c_1}{R}]\right) \qquad (1.17)$$

The comparative statics of the above expression are contained in the Appendix. Increased competition increases the parameter space in which contagion occurs as increased competition reduces the profit buffer of each bank within the system, reducing the ability of the banking system to absorb the excess liquidity demand. Increased concentration has a more subtle effect as it increases the profit buffer with which losses can be absorbed, but also concentrates the interbank deposit network, increasing the breadth of the channel of contagion of the initial bank failure. Crises are more likely to spread contagiously if the interbank-deposit holding are large, which is the case if there are large variations in liquidity demand in non-crisis states 1 and 2 (i.e. the higher is $\rho = \lambda_H - \lambda$), and less likely to spread if the external is large (i.e. if R or d is large, and l is small).

As N increases the losses of interconnected banks (the left hand side of condition (A.2) falls as interbank deposits are held in a larger number of banks so the relative impact is lower, making systemic crises less likely. On the other hand, as N increases

the profits buffer falls and the proceeds from the early liquidation of the long asset falls, making systemic crises more likely. If profits are sufficiently high in a highly concentrated industry and the profit-collateralised loan is low, then the overall effect is that an increase in N is that the industry moves from a stable position to systemic crisis, supporting the concentration stability view. This is the case if the return on the long asset (R) is high and competition is weak (i.e. d is large)[8], the external loan interest rate (l) is low, and the losses from early liquidation of the long asset are small (i.e. $\frac{R-r}{R}$ is small). In the parameter space in which the buffer consists of the liquidated long asset (as the profit collateralised loan provides less funding), the losses from early liquidation are important as they determine the liquidation value of bank 1. If the losses from early liquidation is low, and competition is weak there is no systemic crisis regardless of the concentration in the industry. On the other hand if the losses from early liquidation are high (i.e. r/R is small), and competition is highly intense (i.e. d is small) a systemic crisis is unavoidable regardless of the concentration of the industry. If the liquidation of the long asset is low, banks buffers consist of the profit collateralised loan. In this case increased concentration may either reduce or increase the parameter space in which a banking crisis spreads contagiously as shown in the Appendix. Concentration promotes stability if the return on the long asset is high(R is high), competition is weak (d is high) and the difference in liquidity demand between states 1 and 2 (ρ) or the external loan interest rate l is small.

As d increases competition between banks becomes less intense and banks' enjoy a stronger degree of market power from greater horizontal differentiation. Both profits and the proceeds from early liquidation of the long asset are increasing in d so as the intensity of competition (for a given level of concentration) in the banking industry increases the buffer falls and crises become more likely. Note that unlike concentration, there is no risk-sharing effect from an increase in the intensity of competition so the effect is unambiguously to increase the likelihood of systemic crisis, thus supporting the competition-fragility view.

1.3.7   Numerical Results

The effect of greater intensity of competition, and greater concentration on financial stability can be examined by analysing how the number of banks N (equivalently a reduction in concentration), and the transport cost d respectively affect the incidence of systemic banking failure. The results summarised in Table 1.2 highlights how an increase in the number of banks affects the profit buffer, the liquidation buffer and the losses in interbank claims.

The results of the model depend crucially on the parameter values used. Parameter values were selected for comparability with AG and to preserve the liquidity pecking order. They are ($R = 1.5$, $r = 0.4$, $d = 0.2$, $\lambda_L = 0.4$, $\lambda_H = 0.6$, $\lambda = 0.5$, $\epsilon = 0.1$, $l = 1.3$). The critical value of ϵ required to induce a crisis in bank 1 is also reported to illustrate how concentration affects the individual stability of a bank to liquidity shocks. As the concentration of the industry falls the profit buffer of bank 1 falls and a crisis is triggered for a lower value of ϵ meaning that individual banking crises

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[8]See equation (1.9).

Table 1.2 – Impact of number of firms of buffers, losses and crises

Number of banks	2	4	6	8	10
Critical value of ϵ	0.067	0.033	0.022	0.017	0.013
Profit-collateralised loan	0.0524	0.0137	0.0062	0.0035	0.0023
Buffer	0.0524	0.0190	0.0129	0.0098	0.0078
Loss	0.0394	0.0205	0.0139	0.0105	0.0084
Buffer remaining	0.0131	-0.0015	-0.0010	-0.0007	-0.0006

Note: Negative values indicate a systemic banking crisis.

are less likely in more concentrated industries. As the research question concerns systemic crises the value of the following rows reported in Table (1.2) assume $\epsilon = 0.1$ in order to ensure that condition (23) is violated and there is a crisis in bank 1 for all values of N considered. The relationship between concentration and systemic stability is non-trivial. This partly follows from the fact that profits can only be used as a buffer if, by using the profit-collateralised loan, the bank avoid failure. This is the case for a highly concentrated duopolostic banking industry in which case the interconnected bank is able to absorb the loss in interbank deposits held in bank 1 with the profit buffer. However in a less concentrated industry (i.e. $N \geq 4$) the profits at $t = 2$ provide insufficient collateral with which to borrow and avoid a failure. In this case banks must liquidate their long asset early which provides greater liquidity, but this is also insufficient to prevent failure and all interconnected banks in industries with 4 or more banks fail. The results for the given parameter values therefore support the concentration-stability hypothesis. The buffer remaining however increases slightly as N increases reflecting the decreased concentration of interbank holdings in the failed bank 1, and the fact that profits are a convex function of N. Note that this result depends on the parameters used and whist concentration unambiguously makes an individual crisis less likely (in terms of increasing the size of the shock ϵ which is required to make a bank fail), the effect of contagion on the likelihood of contagion depends on the parameter values assumed.

In order to explore the relationship between concentration, competition and systemic crises we consider a range of values for the parameter d in Table 1.3. The table highlights that the stability of the system is highly sensitive to the intensity of competition. If $d = 0.1$, the profit buffer is low and all banks fail, regardless of the concentration. If $d = 0.2$ if one bank fails in a duopolistic banking industry the interconnected bank is able to use the profit buffer to avoid a crisis, whereas there would be a systemic crisis in a less concentrated industry. If $d = 1$ the system is stable regardless of the level of concentration, however the buffer remaining falls in less concentrated banking systems.

Table 1.3 – Impact of intensity of competition on buffer remaining

Number of banks	2	4	6	8	10
d=0.1	-0.0170	-0.0120	-0.0079	-0.0059	-0.0047
d=0.2	0.0131	-0.0015	-0.0010	-0.0007	-0.0006
d=1	0.1812	0.0677	0.0471	0.0362	0.0294

Note: Negative values indicate a systemic banking crisis.

These results generally support the concentration-stability views and competition-

fragility views. Note whilst concentration unambiguously supports individual bank stability, the contagion of the banking crisis depends on the values of parameters used as outlined in the Appendix. Note that crises can occur in both highly concentrated and highly diffuse banking industries, as even in highly concentrated market structures with low intensity of competition crises may occur if the loss from early liquidation is large enough (i.e. R/r is low enough), if the interbank-holdings large enough (i.e. if the variation in liquidity demands in states 1 and 2, $(\lambda_H - \lambda)$ is large enough, and if the interest rate on the profit-collateralised loan (l) is high enough.

1.4   Positive probability of crisis

The preceding analysis was predicated on the assumption that the probability of a state of excess liquidity demand ($\bar{S}$) occurring is zero. This assumption makes the maximisation problem of the banks considerably more tractable as it implies that banks' optimising behaviour does not take into account state $\bar{S}$. As AG note[9] when the probability of state $\bar{S}$ occurring is positive the banks' maximisation problem is considerably more complicated. AG argue that if the probability of state $\bar{S}$ is sufficiently low then banks would find it optimal to bear the risk of a crisis occurring rather than holding more of the liquid asset, as the opportunity cost of holding extra liquidity is too high. However AG do not characterise the conditions under which banks would seek to hold extra liquidity. This section analyses under what conditions banks find it optimal to hold extra liquidity, and which effects competition and concentration have on the likelihood of banks holding insufficient liquidity to avoid crises.

Table 1.4 presents the liquidity demands in crisis and non-crisis states, for the specific case of $N = 4$ crisis state. The number of crisis states is now equal to the number of banks, each occurring with equal probability. This ensures that the allocations will be symmetric and all banks will find it optimal to hold interbank deposits in all other banks.

Table 1.4 – Banks' Liquidity Shocks with proportional probability of a systemic event.

State	Probability	Bank A	Bank B	Bank C	Bank D	Average
1	$\frac{1-Np}{2}$	λ_H	λ_L	λ_H	λ_L	λ
2	$\frac{1-Np}{2}$	λ_L	λ_H	λ_L	λ_H	λ
$\bar{S}_A$	p	$\lambda + \epsilon$	λ	λ	λ	$\lambda + \epsilon/N$
$\bar{S}_B$	p	λ	$\lambda + \epsilon$	λ	λ	$\lambda + \epsilon/N$
$\bar{S}_C$	p	λ	λ	$\lambda + \epsilon$	λ	$\lambda + \epsilon/N$
$\bar{S}_D$	p	λ	λ	λ	$\lambda + \epsilon$	$\lambda + \epsilon/N$

For this section of the paper retail deposits are assumed to be fully insured, and thus retail depositors receive their promised repayment c_1 or c_2 in periods 1 and 2 respectively in all states so their expected utility of consuming a deposit contract

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[9]On page 29 of Allen and Gale (2000).

offering (c_1, c_2) is the same as in the above analysis.[10] Interbank deposits remain uninsured so interconnected banks remain subject to negative spillovers if the bank fails.

Each bank j faces excess liquidity demands in crisis state $\bar{S}_j$ with positive probability p, and each of the rival banks experiences excess liquidity demands in states $\bar{S}_{-j}$ with total probability $(N-1)p$. Given a sufficiently high probability p of crisis states $\bar{S}$ banks may wish to prevent runs by holding more of the short asset. The following section derives the contract (c_1, c_2, x, y, z) assuming that the bank wishes to be able to serve all early depositors in a crisis state without resorting to early liquidation of the long asset. The outcomes will then be compared with those in which the bank just holds enough liquidity to serve customers in states 1 and 2, and thus faces excess liquidity demand in all states $\bar{S}_j$. When the probability of a crisis state occurring (p) is low, it is better for the bank to just ignore the crisis states and bear the risk of failure. This result is partly due to limited liability which ensures that payoff for each bank is bounded from below at zero.

1.4.1 Safe asset portfolio

The following allocation ensured that the excess liquidity demand of the bank j in state S_j does not result in a crisis and that there is no need to liquidate the long asset. For the sake of tractability the profit-collateralised loan is assumed to be unavailable to banks, so that according to the pecking order banks first liquidate short asset holdings, then interbank deposits and finally long asset holdings.

In crisis states $\bar{S}_j$ there is an excess liquidity demand in the banking system as a whole of ϵc_1 relative to non-crisis states 1 and 2. Given that the liquidity demands are distributed symmetrically the excess liquidity demand can be met by each individual bank holding an equal amount of extra short asset relative to the allocation with zero-probability crisis state. Thus the amount of short asset each bank must hold in order to avoid a crisis in all states is given by

$$y = \frac{N\lambda}{N+d} + \frac{\epsilon c_1}{N} = \frac{N\lambda + \epsilon}{N+d} \tag{1.18}$$

The interbank deposits need to be large enough so that in both crisis and non-crisis states the interbank deposit channels are wide enough for the banks to be able to obtain the liquidity it needs. By assumption the excess demand for liquidity (ϵ) is greater than the variation in liquidity demand in non crisis-states (i.e. $2(\lambda_H - \lambda_L) > \epsilon$).

Therefore the interbank deposits can be expressed as:

$$z = max\left(\frac{\epsilon}{N}, \frac{\lambda_H - \lambda_L}{N/2}\right) = \frac{\epsilon}{N} \tag{1.19}$$

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[10]In the above analysis insurance against the excess liquidity demand in the crisis state $\bar{S}$ was not needed as the state occurs with zero probability.

From (1.19) it is evident that if the liquidity shock is larger than the variability of liquidity demands in non-crisis states 1 and 2 then the size of the liquidity shock determines the 'width' of the interbank deposit channels. The allocation is given by the following tuple.

$$(c_1, c_2, x, y, z) = \left(\frac{N}{N+d}, \frac{RN}{N+d}, \frac{N+d-N\lambda-\epsilon}{N+d}, \frac{N\lambda+\epsilon}{N+d}, \frac{\epsilon}{N} \right) \qquad (1.20)$$

Given that banks are symmetric, the increased liquidity demand can be accommodated through liquidation of the short asset, borrowing through the profit-collateralised loan and withdrawals of interbank deposits, so bank failure does not occur in any state and there is no contagion.

If the banks hold the safe asset portfolio the average profit earned in the non-crisis states S_1 and S_2 is lower as banks forgo the higher return on the long asset by holding liquidity in excess of the average liquidity demand, thus forgoing the higher return on the long asset. The average profit in these two states is given by:

$$\Pi_{1,2} = \frac{1}{N} (y + Rx - \lambda c_1 - (1-\lambda)c_2) = \frac{Rd - (R-1)\epsilon}{N^2 + Nd} \qquad (1.21)$$

Profits of bank j in state $\bar{S}_j$ are given by

$$\Pi_{S_j} = \frac{1}{N}(Rx - (1-\lambda-\epsilon)c_2 - (N-1)zc_2) = \frac{Rd}{N^2 + Nd} \qquad (1.22)$$

In states S_{-j} a single interconnected bank has its own liquidity demand shock and bank j provides it with liquidity zc_1 at $t=1$ and obtains zc_2 at $t=2$ in return. In state S_k all interbank deposit claims bank j holds in other banks other than bank k net out, and there is no loss in value of the interbank deposits held in bank k as bank failure is averted. The profits of bank j in state S_{-j} are given by

$$\Pi_{S_{-j}} = \frac{1}{N}(Rx + zc_2 - (1-\lambda)c_2) = \frac{Rd}{N^2 + Nd} \qquad (1.23)$$

The expected value of profits is given by the following expression:

$$\mathbb{E}\Pi^* = (1 - Np)\frac{Rd - (R-1)\epsilon}{N^2 + Nd} + Np\frac{Rd}{N^2 + Nd} = \frac{Rd - Np(R-1)\epsilon}{N^2 + Nd} \qquad (1.24)$$

1.4.2 When banks optimally hold extra liquidity buffers

If banks hold $y < \frac{N\lambda+\epsilon}{N+d}$ then in states S_j there is excess liquidity demand in the banking system. However with probability $1-Np$ there is no crisis and the profits of the bank are $\frac{Rd}{N^2+Nd}$. With probability p the bank faces a potential crisis stemming from excess liquidity demand from its own depositors, and with probability (N-1)p the bank faces a potential crisis stemming from contagion from other banks through interbank deposits. In these states the bank earns the profits it would receive in a

non-crisis state, less any losses due to the crisis. Banks have limited liability in the AG model so the minimum payoff in any state is zero.

It is most interesting to consider the situation in which systemic crises would occur if banks hold the risky asset portfolio. This requires some technical condition to be satisfied which are detailed in the Appendix. In this case bank holding the risky asset portfolio obtain profit given by the expression in (1.9) in non-crisis states which occur with probability 1-Np, and zero profit in crisis states which occur with probability Np, so the expected payoff is given by:

$$\mathbb{E}\Pi_{RISKY} = (1 - Np)\frac{Rd}{N^2 + Nd} \tag{1.25}$$

Given the safe asset portfolio the profits in normal times are

$$\frac{Rd - (R-1)\epsilon}{N^2 + Nd} \tag{1.26}$$

The profit in the state in which the bank has excess liquidity demand from its retail depositors is

$$\frac{Rd - RN\epsilon}{N^2 + Nd} \tag{1.27}$$

As each bank avoids failure there are no contagion to other banks. Thus the profits earned in the states in which other banks have excess liquidity demand is given by (1.26). The expected profits of holding the safe asset portfolio are thus given by:

$$\mathbb{E}\Pi_{SAFE} = (1 - p)\frac{Rd - (R-1)\epsilon}{N^2 + Nd} + p\frac{Rd - RN\epsilon}{N^2 + Nd} = \frac{Rd - (1-p)(R-1)\epsilon - pRN\epsilon}{N^2 + Nd} \tag{1.28}$$

The bank earns higher profits ex-ante if it simply bares the risk of crisis occurring by holding the risky asset portfolio if the following condition holds

$$\mathbb{E}\Pi_{RISKY} = (1 - Np)\frac{Rd}{N^2 + Nd} > \mathbb{E}\Pi_{SAFE} = \frac{Rd - (1-p)(R-1)\epsilon - pRN\epsilon}{N^2 + Nd} \tag{1.29}$$

which simplifies to

$$p < p^* = \frac{(R-1)\epsilon}{(R-1)\epsilon + NR(d-\epsilon)} \tag{1.30}$$

The critical probability p^* is strictly decreasing in N as $\frac{\delta p^*}{\delta N} = -\frac{R(d-\epsilon)}{(R-1)\epsilon + NR(d-\epsilon)^2} < 0$, given that $d > \epsilon$.[11] Thus as the concentration decreases banks prefer to hold the safe asset portfolio as the number of banks proportionately increases the probability

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[11]Recall that $\epsilon < 1 - \lambda$ and d is unbounded.

of a crisis state occurring. As the intensity of competition increases i.e. as the transport cost d falls, the marginal effect of an increase in N on p^* is reduced. The critical probability p^* is also strictly decreasing in the 'transport cost' d as $\frac{\delta p^1}{\delta d} = -\frac{NR}{(R-1)\epsilon + NR(d-\epsilon)^2} < 0$ implying that the intensity of competition increases the banks prefer to hold the risky asset portfolio.

The above comparison neglects the possibility that banks may hold more liquidity than in the safe asset portfolio but less liquidity than in the risky asset portfolio. If the bank A hold liquidity $y \in (\lambda c_1, (\lambda + \frac{\epsilon}{N})c_1)$ then in the state $\bar{S}_A$ the bank will need to use some buffer to meet the excess liquidity demand.

Without loss of generality the portfolio choice of the bank can be represented as

$$(x, y, z) = \left(1 - y, (\lambda + \frac{\alpha}{N})c_1, \frac{\epsilon}{N}\right) \tag{1.31}$$

where $0 < \alpha < \epsilon$ and $\frac{\epsilon}{N} > \frac{\lambda_H - \lambda_L}{N/2}$. In the state $\bar{S}_A$ bank A has liquidity demand $(\lambda + \epsilon)c_1$ but only has available liquidity $(\lambda + \alpha)c_1$. If the bank is able to accommodate this excess demand using its buffer the cost is $\frac{R}{r}(\epsilon - \alpha)c_1$. It can be shown that unless the bank j is able to prevent early liquidation of the long asset in its own crisis state $\bar{S}_j$ it would not hold extra liquidity as to do so would be of no benefit.

1.5 Conclusions

This paper investigated the relationship between concentration, competition and stability in the banking industry by introducing imperfect competition ala Salop (1979) in the Allen and Gale (2000) model of contagion. The relationship between concentration and stability primarily depends on the intensity of competition between banks as measured by the intensity of competition and the quantity of inter-bank holdings, which depend on variations in liquidity demands in non-crisis states. The main result is that increased competition and reduced concentration tends to make the bank system more fragile as in more concentrated systems the profit buffer exceeds the increased concentration of interbank holdings. However systemic crises may still arise in a range of market structures.

If the probability of crisis is small but strictly positive banks find it optimal to hold the risky asset portfolio if the cost of holding the extra liquidity (i.e. forgone return on the long asset) is greater in expected value terms than the lost profit from a crisis occurring. As concentration increases and competition decreases crises may be absorbed more easily meaning that crises are triggered for a smaller range of parameter values. As concentration increases the critical probability increases and as competition decreases the critical probability falls. This supports the concentration-stability and competition-fragility views.

Chapter 2

Debt, Equity and Moral Hazard: The Optimal Structure of Banks' Loss Absorbing Capacity

Joint with Misa Tanaka, Bank of England[1]

2.1 Introduction

A key aim of the post-crisis global regulatory reform effort has been to end the problem of large globally, systemically important banks (G-SIBs) being 'too big to fail'. A bank is 'too big to fail' if it is so large that its failure could destabilise the entire financial system so that public authorities are compelled to bail it out by providing financial assistance to prevent a systemic failure. The expectation of such a bailout in turn encourages its shareholders and executives to take socially excessive risks, thus increasing the expected liabilities for taxpayers.

The post-crisis regulatory reforms included a number of new requirements for G-SIBs, aimed at ending 'too big to fail'. First, G-SIBs are required to maintain a higher capital ratio during normal times relative to the smaller, less systemic banks. Under the new Basel III capital regulation, banks are subject to minimum capital *requirements* and minimum capital *buffers*: whereas the minimum capital requirements must be met at all times by banks, banks can 'use' minimum capital *buffers* to absorb unexpected losses on their asset portfolios without entering resolution.[2]

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[1]The views expressed in this paper are those of the authors and do not necessarily reflect those of the Bank of England, the MPC or the FPC.

[2]It is important to make this distinction as prior to Basel III there was no minimum capital buffer specified in the Accords. However the debate around minimum capital *requirements* often focuses on the role of capital as a buffer against losses, even though the minimum capital requirements should be met at all times. Goodhart (2008) made the following metaphor for prudential requirements: "the weary traveller who arrives at the railway station late at night, and, to his delight, sees a taxi there who could take him to his distant destination. He hails the taxi, but the taxi driver replies that he cannot take him, since local bylaws require that there must always be one taxi standing ready at the station... required minimum capital [is not] fully usable capital from the point of view of a bank.

In addition to these requirements under the Basel III, G-SIBs are also required to maintain an additional capital buffer in normal times, reflecting their greater impact on systemic stability. Second, the Financial Stability Board also introduced a new total loss absorbing capacity (TLAC) requirement applicable to G-SIBs from 1 January 2019. TLAC may consist of capital instruments which count towards the minimum capital requirements, and other eligible unsecured debt instruments with residual maturity of over one year. The primary aim of the TLAC requirement is to ensure that there is sufficient loss-absorbing and recapitalisation capacity available in resolution to implement an orderly resolution of bank failures, reducing potential systemic instability and recourse to taxpayer funds.[3] A breach or likely breach of the TLAC requirement will be treated as severely as a breach or a likely breach of minimum capital requirements.

In this paper, we develop a simple framework in order to examine the optimal setting of the triplet of regulatory requirements: a minimum capital requirement and a minimum TLAC requirement, which both need to be met at all times, and a capital *buffer* requirement, which banks can deplete to absorb losses in stressed conditions without entering resolution. In our model, banks are funded by equity, long-term unsecured (bail-inable) debt, and insured deposits. Equity is privately costly relative to the unsecured debt and insured deposits, which is passed through into tighter credit conditions reducing economic activity. However, equity has the advantage of absorbing losses without creating wider social costs whilst the bank remains a going concern i.e. whilst the amount of equity is non-negative, however a "bail-in" of unsecured debt holders in which some or all of the interest and principle is written down will require a modification of the contract which is likely to generate social deadweight costs, including legal and administrative costs incurred by the bank and resolution authority, and could lead to contagion by triggering a generalised freeze of unsecured debt market for other systemic banks.[4] Finally, 'bail-out' of either insured depositors or other creditors is funded by distortionary taxation which creates a deadweight cost.

Our model also incorporates two moral hazard problems, which are dependent on the bank's funding structure: i) ex-ante, banks that maximise shareholder returns have the incentive to shirk on costly project monitoring efforts in the presence of flat-rate deposit insurance, thus increasing the risk of a failure, and ii) ex-post, poorly capitalised, limitedly liable banks have the incentive to engage in asset substitution by taking excessive risks at the expense of the deposit insurance fund (or taxpayers) and the long-term unsecured debt holders. Thus, the regulator optimally sets the trio of regulatory requirements to eliminate these two types of moral hazard, while trading-off the ex-ante and ex-post costs and benefits of equity against debt and deposits as outlined above.

Our analysis yields the following key results. First, we demonstrate that the *ex-ante* moral hazard problem can be addressed by ensuring that in the event of a bank failure, private investors (as opposed to taxpayers) will absorb sufficient amount of losses. That means that the regulator needs to ensure that the TLAC *plus* equity capital buffer is sufficiently high ex-ante but it also means that the *composition*

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[3]FSB Principles on Loss-absorbing and Recapitalisation capacity.

[4]As suggested by Goodhart and Avgouleas (2014).

of this private loss absorbing capacity is irrelevant for ex-ante moral hazard when unsecured debt market can price risks and penalise risk-taking through a higher interest rate. Second, we show that, in order to prevent the ex-post moral hazard, the resolution authority needs to intervene and 'bail in' unsecured debt as soon as the minimum capital requirement is breached. Hence, the minimum capital requirement should be set at a level below which shareholders' 'skin in the game' is too low such that banks have the incentive to engage in socially suboptimal asset substitution. Third, the optimal level of capital buffer is determined by a trade-off between the social cost of equity issuance and the benefit of reducing the probability and the cost of a bank failure.

Our simple framework also enables an analysis of how changes in economic conditions might alter the optimal trio of regulatory requirements. Our analysis suggests that a higher bail-in cost calls for a higher capital buffer and a lower TLAC requirement, whereas a higher bail-out cost calls for both the capital buffer and the TLAC requirement to be raised. Furthermore, our analysis suggests that, in response to an elevated risk of a system-wide shock, the regulator may want to increase the capital buffer while reducing the TLAC requirement. This is because an increased risk of a system-wide shock increases the expected cost of bank insolvency, and an additional capital buffer helps to reduce the risk of bank insolvency.

2.2 Literature Review

Our paper is related to a number of existing papers that have examined the impact of capital requirements on banks' risk-taking incentives. The traditional view is that increased capital requirements promote social welfare by curtailing banks' incentives to shift risks onto the taxpayers when banks are limitedly liable and benefit from implicit (e.g. the expectation that creditors will be bailed out as the bank is TBTF) or explicit (e.g. flat-rate deposit insurance) government guarantees (e.g. Keeley(1990), Rochet (1992)). This view has been challenged by more recent studies which suggest a more nuanced effect. For example, Hellman et al (2000) show that capital requirements have two counteracting effects on the incentive for a bank to take on a more risky investment project. On the one hand, an increase in capital requirements increases shareholders' 'skin in the game' by putting more of the their equity capital at risk: this has the effect of reducing the incentive to take on risk. But on the other hand, an increase in capital requirements also reduces the bank's profit margin, thus eroding its franchise value and incentivising it to take on greater risks. The authors conclude that the overall effect of capital requirements on banks' risk-taking is ambiguous. Allen et al (2011) show how optimal capital regulation depends on the presence of deposit insurance, and competition in the deposit and credit markets. With perfect competition for uninsured deposits, there would be no need for capital regulation as market discipline induces banks to maintain some capital to ensure they have skin in the game, resulting in a lower deposit interest rates. If deposits are insured, however, a moral hazard problem arises as limitedly liable banks may shift risk onto the deposit insurance (given flat-rate deposit insurance). In our model, the presence of limitedly liable banks which are partly financed by insured deposits yields the same incentive to shift risk onto the deposit insurance

fund.

In addition to the impact on banks' *ex-ante* incentives to take risk, bank capital also has a role in acting as a buffer between the asset portfolio realisation and liabilities, reducing the probability and cost of failure (Dewatripont and Tirole, 1994). Our model captures both the skin-in-the-game and buffer views of capital regulation, as the bank's liability structure determines its incentives to invest in riskier projects, as well as the probability of a risk-shifting problem arising ex-post.

Furthermore, our paper considers the optimal minimum capital requirement, TLAC requirement, and capital buffer in a single framework, reflecting the new regulatory environment of G-SIBs. Given the new nature of these requirements, there has been little research in this area to date, although a number of existing papers have examined the issue of how contingent-convertible bonds (Cocos) and bail-inable debt might affect banks' incentives. For example, Martynova and Perotti (2015) analyse how contingent-convertible bonds affect banks' risk-taking behaviour, and argue that Cocos that trigger when banks are solvent will reduce their risk-shifting incentives more than bail-in bonds, which they assume will convert into equity or be written down only when the bank becomes insolvent. While we do not explicitly examine the role of Cocos, we also argue that, for bail-in bonds to reduce banks' (ex-post) risk-shifting incentives, they need to convert while equity value is still positive. We also examine how a bank's ex-ante risk-taking incentives are influenced by the funding mix and show that equity and bail-in debt have the same disciplining effect on banks.

Finally, our paper is also related to the literature on bank failure resolution. In our model, the rationale for resolution arises due to the need to intervene promptly to prevent undercapitalised banks from 'gambling for resurrection' by investing in excessively risky but high return assets, as in Tanaka and Hoggarth (2006) and De Niccolo et al (2014). The existing literature has also examined the possibility of time-inconsistent resolution authority (e.g. Mailath and Mester (1994)). In our analysis, the resolution authority can credibly intervene as soon as the bank breaches the minimum capital requirement whilst the bank is still solvent because allowing undercapitalised banks to continue operating will encourage them to gamble for resurrection and magnify the losses for unsecured debt holders and the deposit insurance fund and expected social cost of resolution, so resolution is time-consistent in our model.

The rest of the paper is organised as follows. Section 2.3 develops the baseline model, in which the probability of a bad state, or a system-wide shock – which reduces asset return for all banks – is assumed to be exogenous: thus, we focus on the impact of *ex-post* moral hazard in the baseline analysis. Section 2.4 solves for the model equilibrium and clarifies the determinants of the optimal regulatory requirements. Section 2.5 endogenises the probability of the bad state, which is now determined by banks' decision about whether to monitor the project or not. This extended model allows us to analyse policies to mitigate *ex-ante* moral hazard. Section 2.6 concludes.

2.3 The model

This section outlines our baseline, three-period model. We assume that the regulator initially imposes regulatory requirements consisting of a minimum capital requirement, a TLAC requirement, and a capital buffer on ex-ante identical banks. There are two macroeconomic states of the world, H ('good') and L ('bad'), which determine the interim return that banks receive. Banks are ex-post heterogeneous, because in the bad state, they receive different returns, which are unknown ex- ante. In the baseline model, banks are only subject to ex-post moral hazard: banks with little skin in the game i.e. returns net of liabilities have the incentives to engage in asset substitution – or gamble for resurrection – in order to maximise shareholder returns at the expense of the unsecured debt holders and the deposit insurance fund (DIF). The presence of this ex-post moral hazard creates a case for the resolution authority to intervene and resolve undercapitalised banks in order to prevent them from failing with larger losses – which also imply larger social costs – at a later date.

2.3.1 Ex-ante regulatory requirements and investment (t=0)

There are three periods: $t = 0, 1, 2$. All agents are risk-neutral, and the banking sector consists of a continuum of ex-ante identical banks of mass 1. At *the start* $t = 0$, ex-ante identical banks each have an investment opportunity which requires a unit of funds, and the regulator sets rules on how it should be funded. The bank can fund its investment using three different types of instruments to maintain the following balance sheet identity:

$$E_0 + G + D = 1$$

where E_0 is initial ($t = 0$) equity investment; G is uninsured, unsecured debt; and D is the insured deposits. We define the 'private loss absorbing capacity' as $\theta \equiv E_0 + G$, and the share of equity within this as e_0. Hence, the items on the liability side of the balance sheet can be rewritten as: $E_0 = \theta e_0$, $G = \theta(1 - e_0)$, and $D = 1 - \theta$. Debt and deposits mature at $t = 2$, and their holders are repaid the full principal and interest if the bank is solvent, whereas equity holders receive the residual value. But if it is insolvent, then equity holders are wiped out, whereas the uninsured debt holders receive the residual return. As explained in Section 2.3.2, the regulator can also 'resolve' the bank if it is found to be in breach of the minimum capital requirement at $t = 1$. Some banks that are resolved at $t = 1$ are balance sheet solvent, even though they are in breach of the minimum capital requirement: in this case, none of the claimholders will suffer any losses. If, however, a bank is found to be insolvent at $t = 1$, equity holders receive zero, whereas the uninsured debt holders receive the residual return after depositors are repaid. The interest rate i on the unsecured debt G factors in the state of the world in which the unsecured creditors are bailed in so that $i \geq 1$, depending on the return R_L in the bad state of the world as shown in section 2.4.3 below.

Insured deposits have a unit gross return in all states of the world, because the deposit insurance fund (DIF) will cover any shortfall if the bank has insufficient

resources to pay depositors. The DIF is funded by flat-rate contributions which for paucity we normalise to zero.[5]

We assume that, when setting regulatory requirements, the regulator knows that the bank can invest in a project that yields a 'high' return equal to R_H in the 'good' macroeconomic state, which occurs with probability q, and a 'low' return $\tilde{R}_L$ in the 'bad' state which occurs with probability $1 - q$ at $t = 1$: $1 - q$ could be interpreted as the probability of a system-wide stress which reduces return on assets across the banking sector. In this section we assume that q is exogenous, but in Section 2.5 we endogenise the choice of q.

We assume that, in the 'good' macro state when $R = R_H$, banks will be solvent with certainty: $R_H > (1-\theta)+i\theta(1-e_0)$, where i is the interest rate on unsecured debt. *At the start of* $t = 0$, however, the low return is stochastic, with $\tilde{R}_L \sim Unif[0, R^{max}]$, where $R^{\max} \leq R_H$, and we assume that banks and the regulator only know the distribution of the low return. Thus, the solvency of each bank in the 'bad' macro state depends on the realisation of R_L relative to it's liabilities $D + G$. We interpret R_L as bank 'type' which is ex-ante uncertain, and the regulator sets all requirements without observing the realised R_L for individual banks.

The regulator sets the following requirements, which are expressed as a ratio of the bank's (unweighted) assets.

1. the minimum capital requirement E^* which the bank has to maintain both at $t = 0$ and $t = 1$ in order to remain in business until $t = 2$

2. the capital buffer, denoted E^b, which the bank can use to absorb losses at $t = 1$ without facing resolution when the return on assets turns out to be low

3. the minimum TLAC requirement τ^*.

Note that, throughout our analysis, the risk weights are normalised to one, such that capital ratios are defined as equity (which is equal to the value of assets minus the value of liabilities) divided by the value of assets; and as the value of assets is equal to one at $t = 0$, E_0 can be interpreted as both the level of capital and the capital ratio. At $t = 0$, the TLAC requirement can be satisfied using regulatory capital instruments that are not used to meet regulatory capital buffers, $E_0 - E^b$, and uninsured debt, G. As the capital buffer can be depleted to absorb losses at $t = 1$, the TLAC requirement is satisfied as long as the sum of equity at $t = 1$, denoted as E_1, and the uninsured debt G exceeds the minimum TLAC ratio τ^*multiplied by the value of the bank's assets at $t = 1$. Formally the minimum capital and TLAC requirements and capital buffers are given by (2.1),(2.2) and (2.3) respectively, where B_t is the bank's balance sheet size at time t, which is equal to 1 at time $t = 0$ and either R_H or R_L at time $t = 1$, and (2.3) exploits the fact that $B_0 = 1$.

$$E_t \geq E^* B_t \forall t \in \{0, 1\} \tag{2.1}$$

$$E_t + G \geq \tau^* B_t \forall t \in \{0, 1\} \tag{2.2}$$

$$E_0 - E^* \geq E^b \text{ at } t = 0 \tag{2.3}$$

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[5]Insured deposits can be also interpreted as any debt liabilities that are expected to be bailed out by the government in the event of a bank failure.

At $t = 0$ the bank has to satisfy all of the above requirements in order to start operating, whereas at $t = 1$ the bank needs to satisfy both (2.1) and (2.2) in order to avoid resolution. As we detail in the subsequent sections, the regulator chooses these requirements in order to maximise the social welfare, taking into account the following considerations:

1. The minimum capital ratio requirement E^* needs to be set at a level below which it can intervene to prevent undercapitalised banks from engaging in asset substitution ('gambling for resurrection').

2. The capital buffer E^b needs to be set at levels that enable banks to absorb some losses without facing resolution, while taking into account the social cost of equity funding relative to debt and deposits.

3. The TLAC requirement τ^* needs to be set at levels that represents the best trade-off between the cost of bail in (of uninsured creditors) and the cost of bail out (of depositors).

At the end of period $t = 0$, R_L is realised for each bank and becomes publicly observable: this makes banks heterogeneous ex-post. Each bank then chooses the amount of deposits, uninsured debt and equity to issue, subject to regulatory constraints (2.1), (2.2), and (2.3). We assume that all of the depositors, unsecured creditors and equity holders are risk neutral and have access to a safe asset which yields a certain return normalised to equal 1.

The assumptions of a risk-insensitive deposit insurance premium creates a departure from the Modigliani and Miller (1958) result that, in a frictionless world, a firm's total cost of funding is independent of its funding structure. In our model, insured deposits are the least costly form of funding for banks, as they benefit from the implicit subsidy provided by the deposit insurance fund. The unsecured debt is the second most expensive form of funding, as the unsecured debt holders demand a premium over the safe interest rate (normalised to 1) in order to be compensated for the credit risk (see Section 2.4.3 below). In addition, following the existing corporate finance literature, we assume that equity capital is most costly, and a bank incurs a *private* cost $\delta \equiv 1 + \delta_s$ in issuing equity, where 1 is the equity investors' opportunity cost of funding (given by the safe rate of return) and δ_s captures the social deadweight costs associated with equity issuance. The social cost could, for example, reflect the cost of overcoming the asymmetric information problem between the bank's potential equity investors and its executives, e.g. through cumbersome disclosure. As Myers and Majluf (1984) have shown, the cost of equity issuance is higher than the cost of debt issuance as the executives who have inside information about the bank's assets and investment opportunities have an adverse incentive to issue equity when equity is overvalued. Thus, in our model, δ_s captures the *social cost* of requiring banks to issue more capital, as we discuss more in detail in Section 2.4.4. As the issue of whether requiring banks to fund themselves with more capital is *socially* costly or not has been a controversial one, we will also consider explicitly in Section 2.4.4 the regulatory implications of having no social cost of equity issuance.

Thus, as we demonstrate formally in Section 2.4.3, banks' capital and TLAC regulations are binding in equilibrium: banks only issue as much equity and unsecured debt as required by the regulator, in order to maximise the benefit derived from the

implicit subsidy provided by the deposit guarantee.

We have chosen to model the sequence of events at $t = 0$ as above for two main reasons. First, we want to allow for the possibility of bank failures, by capturing the fact that the regulator typically sets requirements without full knowledge of how each bank would perform under stressed conditions. If the regulator can set all requirements separately for individual banks based on perfect knowledge of the performance of their assets under all states of the world (i.e. R_L for each bank is known at the start of $t = 0$), then bank failures can be entirely eliminated by setting the capital requirements sufficiently high to ensure that $R_L > D + G$, and hence we will not have an interesting problem to analyze. Second, we also wanted to ensure that unsecured debt fully prices in the credit risk, in order to examine the role it plays in providing market discipline and mitigating ex-ante moral hazard, which we will analyze in Section 2.5.

2.3.2   Resolution and bail in in the interim period (t=1)

At time $t = 1$, the macroeconomic state is realised. Each bank receives the interim return, which is either R_H or R_L depending on the macro state, and this will determine its capital ratio at $t = 1$:

$$E_1 = R - D - iG \text{ where } R \in \{R_L, R_H\} \tag{2.4}$$

If the interim return is R_L and R_L is sufficiently low, such that a bank does not meet the pre-specified minimum regulatory capital *ratio* E^*, the regulator (or the resolution authority) triggers 'resolution', and converts unsecured debt into equity in order to recapitalise the bank, or writes down the value of the debt ('bail in').[6] Specifically, we assume that the regulator triggers resolution if the following condition holds:

$$E_1 = R_L - (1 - \theta) - i\theta(1 - e_0) < E^* R_L \tag{2.5}$$

The consequence of resolution and 'bail in' is explained more fully in Section 2.4. We assume that, in the absence of resolution at $t = 1$, a bank can choose between reinvesting the interim return either in a risky or a safe asset. The risky asset yields a gross return $\gamma > 1$ at $t = 2$ with probability p and 0 with probability $1 - p$, whereas the safe asset is a simple storage technology which yields a unit gross return at $t = 2$. We assume that the risky asset has a negative net present value, such that $p\gamma < 1$. Thus, the socially optimal choice is for the bank to reinvest its asset return at $t = 1$ in the safe asset. However, the presence of long-term debt, which only needs to be repaid at $t = 2$, and flat-rate (and therefore non actuarially fairly priced) deposit insurance both create the incentive for the bank's shareholders to 'risk shift', i.e. to take excessive risks at the expense of debt holders so as to maximise shareholder returns. As we demonstrate in Section 2.4, undercapitalised

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[6]As the intervention of the resolution authority is assumed to be costless, there is no loss in economic value.

banks have the incentive to 'gamble for resurrection' by investing in risky assets at $t = 1$, thus magnifying the expected losses for creditors and the DIF.[7] Thus, the minimum capital requirement E^* needs to be set in such a way to allow the regulator to intervene to prevent this privately optimal but socially sub-optimal gambling behaviour, once this threshold is breached.

2.3.3 Debt repayment in the final period (t=2)

At $t = 2$, all debt is repaid if the bank is solvent, and the equity holders receive the residual return. If the bank is insolvent, insured depositors are paid in full by the DIF but the unsecured debt holders receive zero. If the bank is insolvent but the value of its assets exceed the liability to insured depositors, unsecured debt holders receive the residual return after the payment to the depositors have been made; and equity holders receive zero return.

We assume that, whilst equity is a costly means of financing banks, equity holders can absorb losses without imposing costs on the rest of the society. By contrast, we allow for the possibility that losses imposed on unsecured debt holders via a 'bail in' at $t = 1$, or at insolvency at $t = 2$, could give rise to a social cost which depends on the size of the losses that are imposed on unsecured debt holders. We allow for this possibility for two reasons. First, losses can be imposed on equity holders without modifying the contract, whereas the imposition of losses on debt holders will require a modification of the original debt contract. The process of modifying the debt contract is likely to give rise to social deadweight costs, in the form of legal costs and administrative costs for the resolution authority, which are likely to increase with the amount of losses that need to be imposed and the number of creditors that are affected as a result. Second, unlikely equity, debt will need to be rolled over. For example, Avgouleas and Goodhart (2014) have raised the possibility that the imposition of losses on unsecured debt holder in one bank could cause a dry-up of the market for such unsecured bank debt, especially when the financial system is already under stress. Thus, the possibility that a large-scale 'bail-in' of creditors of a systemic bank might impose negative externalities on other banks in the form of contagion cannot be ruled out. Furthermore, we also allow for the possibility that the 'bail out' of depositors might give rise to a social cost which depends on the size of losses imposed on the DIF: this captures the possibility that any DIF deficit will need to be funded by distortionary taxes which create a deadweight loss. The details of the model specification are described in Section 2.4.4, which examines how the regulator should set the various requirements in order to maximise the social benefits. The timing of the model is illustrated in Figure 2.1.

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[7]For evidence of 'gambling for resurrection' during the 1990s Japanese banking crisis, see Peek and Rosengren (2005) and Nelson and Tanaka (2014).

Figure 2.1 – Timing of the model
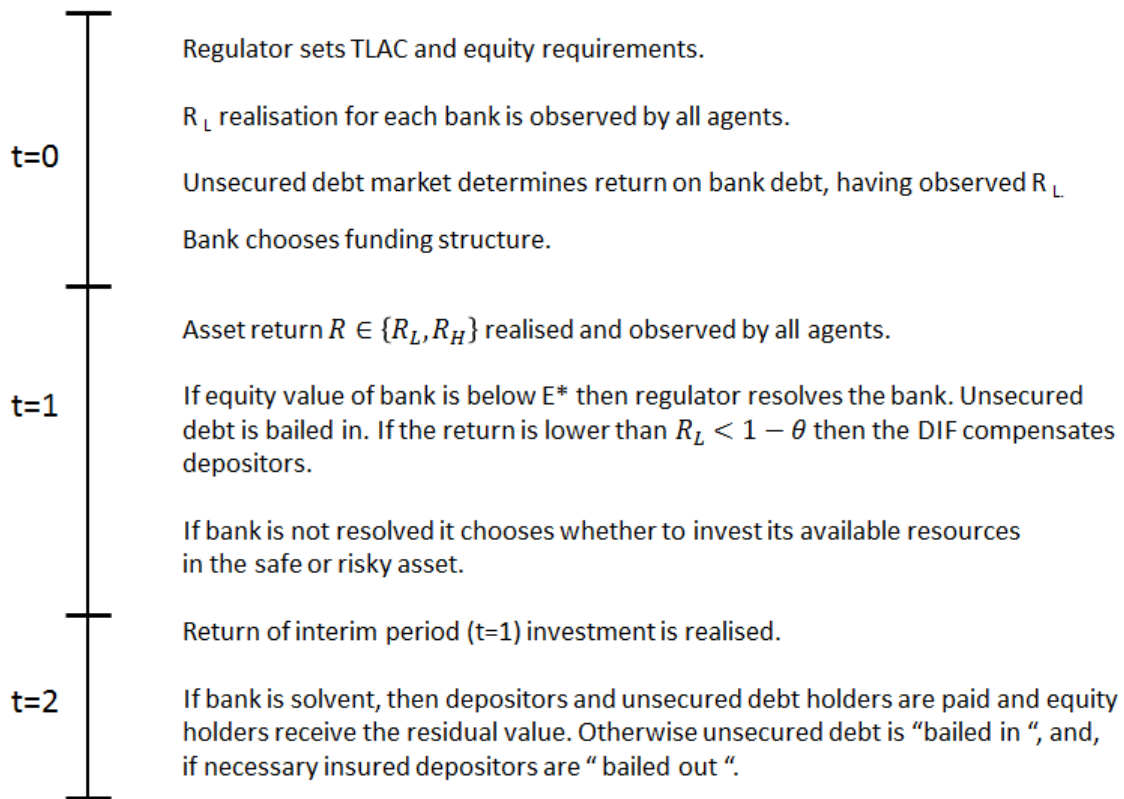
2.4 Socially optimal regulatory requirements and resolution

We now solve this model backwards in order to illustrate the determinants of the socially optimal minimum capital requirement, regulatory capital buffer and TLAC requirement. We demonstrate that ex-ante regulatory requirements determine the share of insolvent banks in the bad state, as well as the size of the losses imposed on different stakeholders of the bank – the shareholders, the unsecured debt holders and the deposit insurance fund (DIF). Hence, the regulatory requirements will need to be set by taking into account the possible social costs associated with imposing losses on different stakeholders.

2.4.1 Optimal minimum capital requirement

At $t = 1$, the regulator optimally intervenes in the bank if its continuation without intervention would result in asset substitution that could magnify the eventual losses for creditors and the DIF, which also give rise to social costs. Thus, the optimal minimum capital requirement is given by the capital ratio below which the bank's shareholders have an incentive to 'gamble for resurrection'.

Since $p\gamma < 1$, a bank will always choose to reinvest in the safe asset if the macro state is 'good' and the interim return is high:

$$p\left[\gamma R_H - i\theta(1 - e_0) - (1 - \theta)\right] < R_H - i\theta(1 - e_0) - (1 - \theta) \qquad (2.6)$$

Hence, there is no need for a regulatory intervention in a 'good' macro state when banks receive a high return.

When the realised macro state is 'bad', some banks receiving very low returns may have the incentive to invest in the risky asset if their interim returns are low, in order to 'gamble for resurrection'. This *ex post* moral hazard arises because shareholders are limitedly liable, and interest rate on long-term debt cannot adjust once debt has been issued: thus, a bank which has received a low interim asset return can increase expected returns for shareholders at the expense of debt holders by investing in risky, negative NPV assets. More specifically, a bank has the incentive to 'gamble for resurrection' if the expected return from investing the interim return in risky asset is higher than that of investing in the safe asset:

$$p[\gamma R_L - i\theta(1 - e_0) - (1 - \theta)] < \max\left\{R_L - i\theta(1 - e_0) - (1 - \theta), 0\right\} \qquad (2.7)$$

Rearranging the above, we obtain that banks will gamble for resurrection in the absence of any regulatory intervention if $R_L < R^T$, where R^T is defined as follows:

$$R^T \equiv \frac{1 - p}{1 - p\gamma}[(1 - \theta) + i\theta(1 - e_0)] \qquad (2.8)$$

The investment in risky assets by a weakly capitalised bank ultimately magnifies expected losses for its creditors and the DIF (and the associated social costs), thus creating a rationale for an early intervention by the authorities to prevent asset substitution when $R_L < R^T$. It can be shown that some of these banks are still solvent. By substituting $R_L = R^T$ from (2.8) into (2.5), we obtain the minimum equity ratio below which the authorities need to intervene at $t = 1$ to prevent asset substitution by the bank (see Annex):

$$E^* = \frac{p(\gamma - 1)}{1 - p} \qquad (2.9)$$

We interpret E^* as the optimal minimum regulatory capital requirement, or equivalently in our model, the optimal trigger for resolution. If a bank's capital ratio falls below this level, then it has the incentive to engage in socially sub-optimal asset substitution, and this creates a case for an early intervention by the authorities before the point of insolvency is reached.

2.4.2 Bail in and resolution

We assume that, if the bank is found to be in breach of the minimum capital requirement (2.9) at $t = 1$, the regulator (or the resolution authority) will use the following decision rule:

Resolution Rule:

1. The resolution authority intervenes at $t = 1$ whenever the capital requirement is breached, i.e. whenever (2.5) holds, where E^* is defined by (2.9).

2. If there is sufficient amount of unsecured debt G that can be written down to restore the bank's capital ratio to E^* or above, then write down unsecured debt and let the bank continue operating as normal ('bail in').

3. If there is insufficient amount of unsecured debt that can be written down to restore the bank's capital ratio to E^* or above, then wind down the bank. This could, for example, include transferring all, or part of failed bank's assets and liabilities to another entity. In this case, the claimholders are compensated in accordance with the claimholder hierarchy, i.e. the DIF is compensated first, followed by the unsecured debt holders, and finally the equity holders.

Importantly, we assume that resolution does not destroy the value of the bank, regardless of whether the bank is successfully recapitalised through bail in, or wound down. Rather, resolution only prevents the asset substitution (or gambling for resurrection) by an undercapitalised bank, and thus benefits the uninsured creditors and the DIF relative to the counterfactual of no intervention. Thus, resolution is value preserving, in the sense that it maximises the sum of the expected returns to shareholders, unsecured debt holders, depositors and the DIF, although shareholders are prevented from engaging in a profitable risk-shifting opportunity and hence are made worse off. This simplifying assumption is made in order to capture the 'no creditor worse off than liquidation' principle for resolution, as outlined in the FSB's (2015) *Principles*. Thus, it is time-consistent for the regulator to follow the Resolution Rule, as it minimises expected losses for unsecured debt holders and the deposit insurance fund, and this is consistent with minimising the social cost of bank failure. This point is explained more fully in Section 2.4.4.

This assumption implies that the unsecured debt holders will suffer losses only if the bank is *insolvent*, and not just in breach of the minimum capital requirement E^*, regardless of whether the bank is recapitalised or wound down, i.e.:

$$R_L < (1 - \theta) + i\theta(1 - e_0) \equiv R^S \tag{2.10}$$

Thus, if the bank is solvent at the point of resolution (i.e. $R_L \geq R^S$), then the unsecured debt holders receive their promised return $i\theta(1 - e_0)$ in full. If the bank is insolvent, i.e. $R_L < R^S$, the unsecured debt holders will only receive the residual claim at wind down, given by $\max\{R_L - (1 - \theta), 0\}$. The unsecured debt holders will receive nothing, although the depositors are compensated in full by the deposit insurance full when $R_L < R^D$ where:

$$R^D \equiv 1 - \theta \tag{2.11}$$

The stakeholder payoffs and the allocation of the asset return R_L in the bad state are summarised in Table 2.1:

<div align="center">Table 2.1 – Payoffs of the stakeholders in the bad state</div>

	$R_L \in (0, R^D)$	$R_L \in (R^D, R^S)$	$R_L \in (R^S, R^T)$	$R_L \in (R^T, R^{max})$
RA decision	Resolve	Resolve	Resolve	Don't resolve
		Stakeholder Payoffs		
Shareholders	0	0	$R_L - i\theta(1 - e_0) - (1 - \theta)$	$R_L - i\theta(1 - e_0) - (1 - \theta)$
Unsecured debt holders	0	$R_L - (1 - \theta)$	$i\theta(1 - e_0)$	$i\theta(1 - e_0)$
Depositors	$(1 - \theta)$	$(1 - \theta)$	$(1 - \theta)$	$(1 - \theta)$
DIF	$R_L - (1 - \theta)$	0	0	0

2.4.3 The pricing of unsecured debt

The pricing of unsecured debt depends on the realisation of R_L *at the end* of the period $t = 0$, given the Resolution Rule as described above. Thus, the market assumes that the resolution authority will intervene at $t = 1$ whenever the minimum capital requirement is breached, and hence do not expect gambling for resurrection to occur in equilibrium. We assume that the bail-in of unsecured debt is fully credible, such that the credit risk is fully priced in the interest rate. Note that there are three ex-post types of banks depending on the realised return R_L:

1. $0 \leq R_L \leq R^D$ (Type 1: insolvent in the low state, with zero recovery value for unsecured debt holders)

2. $R^D \leq R_L \leq R^S$ (Type 2: insolvent in the low state, with positive recovery value for unsecured debt holders)

3. $R^S \leq R_L \leq R^{max}$ (Type 3: solvent in the low state)

Type 1 banks will be insolvent in the low state, and the return R_L will be insufficient to cover the liability to depositors. Thus, the unsecured debt holders of Type 1 banks are paid their promised return i if $R = R_H$ which occurs with probability q and receive 0 with probability $1 - q$. The unsecured debt holders need to earn the expected return which is at least equal to the interest rate on the outside option of storage. Thus, the equilibrium interest rate for Type 1 banks' unsecured debt, denoted as i_1, is determined by the following equilibrium condition:

$$qi_1\theta(1 - e_0) + (1 - q)0 = \theta(1 - e_0) \tag{2.12}$$

This yields the following interest rate which includes a risk premium to reflect the fact that unsecured debt holders only get paid their promised $i_1\theta(1 - e_0)$ with probability q.

$$i_1 = \frac{1}{q} > 1 \tag{2.13}$$

Thus, the expected profit of Type 1 banks at the end of period $t = 0$ (net of equity holders' opportunity cost of investing in the bank) is given by:

$$
\begin{aligned}
\Pi_1 &= q(R_H - (1-\theta) - \frac{1}{q}\theta(1-e_0)) - \delta\theta e_0 \\
&= q\left[R_H - (1-\theta)\right] - \theta(1-e_0) - \delta\theta e_0
\end{aligned}
\tag{2.14}
$$

The private opportunity cost of equity funding enters negatively in the profit function, as equity investors only care about returns in excess of this opportunity cost. Note that the profits of Type 1 banks are distorted by the implicit subsidy from the DIF, as losses will be borne by the DIF in the bad state. It can be shown that $\frac{\partial \Pi_1}{\partial e_0} < 0$ and $\frac{\partial \Pi_1}{\partial \theta} < 0$ as long as $\delta > 1$, so Type 1 bank will issue no more equity and unsecured debt than required by the regulator at $t = 1$. As the shareholders of Type 1 banks capture positive profits in the good state while they at the most lose all their initial investments in the bad state due to limited liability, they benefit from the implicit subsidy from the deposit insurance fund.

Type 2 banks will also be insolvent in the low state, but the low return will be sufficient to cover the liability to depositors, and the unsecured debt holders will receive the residual claim in the low state. Thus, the equilibrium unsecured debt interest rate for Type 2 banks, denoted as i_2, is determined by the following condition:

$$
qi_2\theta(1-e_0) + (1-q)(R_L - (1-\theta)) = \theta(1-e_0)
\tag{2.15}
$$

Thus, the interest rate on unsecured debt for Type 2 banks is given by:

$$
i_2 = \frac{1}{q} - \frac{1-q}{q}\frac{R_L - (1-\theta)}{\theta(1-e_0)}
\tag{2.16}
$$

The expected profit of Type 2 banks is given by:

$$
\begin{aligned}
\Pi_2 &= q\left(R_H - (1-\theta) - \left(\frac{1}{q} - \frac{1-q}{q}\frac{R_L - (1-\theta)}{\theta(1-e_0)}\right)\theta(1-e_0)\right) - \delta\theta e_0 \\
&= qR_H + (1-q)R_L - (1-\theta e_0) - \delta\theta e_0
\end{aligned}
\tag{2.17}
$$

Note that, because all losses in the low state will be borne by equity and unsecured debt holders who can fully price all risks, profits of Type 2 banks are not distorted by the presence of an implicit subsidy from the DIF. Type 2 banks will also issue as much equity and unsecured debt as required by the regulator, as $\frac{\partial \Pi_2}{\partial e_0} < 0$ and $\frac{\partial \Pi_2}{\partial \theta} < 0$.

Type 3 banks will be solvent in the low state, such that the low asset return R_L will be sufficiently high to pay both depositors D and the unsecured debt holders their initial investment G in full, if the unsecured debt carries a risk free rate. This implies that the unsecured debt of Type 3 banks carries a risk free rate, such that:

$$
i_3 = 1
\tag{2.18}
$$

Thus, the expected profit of Type 3 banks is given by:

$$
\begin{aligned}
\Pi_3 &= q(R_H - (1-\theta) - \theta(1-e_0)) + (1-q)(R_L - (1-\theta) - \theta(1-e_0)) - \delta\theta e_0 \\
&= qR_H + (1-q)R_L - (1-\theta e_0) - \delta\theta e_0 \quad\quad\quad\quad\quad\quad (2.19)
\end{aligned}
$$

As with type 2 banks, type 3 banks' profits are not distorted by the implicit subsidy from the DIF, as the losses are fully borne by equity holders. The expected profits are simply the expected value of the asset minus the liabilities. Type 3 banks will also issue as much equity and unsecured debt as required by the regulator, as $\frac{\partial \Pi_3}{\partial e_0} < 0$ and $\frac{\partial \Pi_3}{\partial \theta} < 0$.

Thus, for all banks, the capital buffer requirement (2.3) and TLAC requirement (2.2) are binding, such that:

$$
\begin{aligned}
E_0 - E^* &= \theta e_0 - E^* = E^b \quad\quad\quad\quad\quad\quad (2.20) \\
E^* + G &= E^* + \theta(1-e_0) = \tau^* \quad\quad\quad\quad\quad\quad (2.21)
\end{aligned}
$$

The above results also imply that, if the minimum capital requirement (2.1) is violated at $t = 1$, then the TLAC requirement (2.2) is also violated. Note that, the sum of capital buffer and TLAC is equal to the private loss absorbing capacity, θ:

$$
E^b + \tau^* = \theta
$$

The above implies that the expected profit of a bank *at the start* of period $t = 0$ is given by:

$$
\begin{aligned}
E\Pi &= \int_0^{1-\theta} \left\{ q\left[R_H - (1-\theta)\right] - \theta(1-e_0) \right\} f(R_L) dR_L \\
&+ \int_{1-\theta}^{R^{max}} \left\{ qR_H + (1-q)R_L - (1-\theta e_0) \right\} f(R_L) dR_L - \delta\theta e_0
\end{aligned}
$$

2.4.4 Ex-ante social welfare

The capital buffer and TLAC requirements, E^b and τ^*, will determine the share of each type in the banking sector when the bad macro state – or a system-wide stress – materialises ex-post. Thus, we now consider how these two regulatory requirements are optimally set, when the optimal minimum capital requirement E^* is determined by (2.9). In order to pin down interior solutions for E^b and τ^*, we make the following assumptions in writing the social welfare function:

Assumption 1 (loss-absorbing equity): Equity can absorb losses ex-post without imposing a social cost.

Assumption 2 (ex-post costly bail in): The imposition of losses on the holders of unsecured debt is associated with a social deadweight loss $\psi(L_{G,T})$ that are increasing and convex in the loss given default $L_{G,T}$ imposed on unsecured debt holders of Type T bank in the bad state (where $T \in \{1,2\}$), so that $\psi(0) = 0$, $\psi(L_{G,T}) \geq 0 \; \forall L_{G,T} > 0$, $\psi'(L_{G,T}) > 0$, $\psi''(L_{G,T}) > 0$.

Assumption 3 (ex-post costly bail out): The imposition of losses on the deposit insurance fund (DIF) are associated with a social deadweight loss $\chi(L_{D,1})$ that is increasing and convex with respect to the loss given default $L_{D,1}$ imposed on the DIF at resolution (for Type 1 banks), where $\chi(0) = 0$, $\chi(L_{D,1}) \geq 0 \; \forall L_{D,1} > 0$, $\chi'(L_{D,1}) > 0$, and $\chi''(L_{D,1}) > 0$.

Assumption 4 (value-preserving resolution): The resolution of a failing bank will not destroy the recovery value of its assets, R_L, which will be distributed amongst the claimholders according to hierarchy. The resolution is triggered whenever $E < E^*$.

Assumption 4 implies that all of the bank's asset returns are captured by a combination of its shareholders, the unsecured debt holders, and the DIF at resolution. This assumption is made purely for expositional simplicity. We expect that dropping this assumption would simply increase the optimal capital buffer by increasing the cost of bail in and bail out. Assumptions 1-3 imply that the imposition of a given loss on unsecured debt holders or the DIF is socially more costly than the imposition similarly sized loss on equity holders, and that the social cost of a bank failure is captured by the externalities caused by the bail in and bail out. As discussed in Section 2.3, the externalities from bail-in (Assumption 2) could arise from the fact that, while imposition of losses on equity holders can be done without modification of the contract, imposition of losses on debt holders will require modification of the original debt contract, which will be subject to legal and administrative costs. Moreover, debt needs to be rolled over. Thus, there is a risk that the imposition of losses on unsecured debt holders of one bank gives rise to 'contagion externalities' by raising concerns about the solvency of other banks: this can create funding difficulties for other banks that need to roll over their maturing unsecured debt. This possibility is of particular concern when the market is already under stress and there is a generalised concern about the stability of the banking system as a whole. We assume that the social deadweight loss from bail-in, $\psi(L_{G,T})$, is convex, thus capturing the possibility that, while a small scale bail in is done relatively easily without causing contagion, large scale bail-in is more likely to create investor uncertainty across the banking sector.

Assumption 3 implies that the imposition of losses on the DIF is also associated with a social cost. We interpret this as a deadweight loss associated with funding any deficit of the DIF via distortionary taxes. For completeness, however, we do consider in Section 2.4.5 the cases in which Assumptions 2 and 3 are dropped.

Since capital buffer and TLAC requirements are binding, as in (2.20) and (2.21), setting E^b and τ^* amounts to selecting the socially optimal θ and e_0, subject to

resolution occurring whenever the equity ratio falls below E^* (given by (2.9)) at $t = 1$. Thus, the regulator sets these parameters to maximise the social welfare, denoted as W, which is given by the expected return on investment of ex-ante identical banks – which is equal to the sum of the expected payoffs of equity investors, uninsured creditors, depositors and the deposit insurance fund – net of the social opportunity cost of equity funding and the expected social costs of bail in and bail out (see Annex for derivation):

$$
\begin{aligned}
W \equiv{} & \bar{R} - \delta_s \theta e_0 \\
& - (1 - q) \left(\int_{R^D}^{R^S(i=i_3)} \psi(L_{G,2}) f(R_L) dR_L + \int_0^{R^D} \left[\psi(L_{G,1}) + \chi(L_{D,1}) \right] f(R_L) dR_L \right)
\end{aligned} \tag{2.22}
$$

where

$$
\begin{aligned}
\bar{R} &\equiv qR_H + (1-q) \int_0^{R^{max}} R_L f(R_L) dR_L \\
\delta_s &\equiv \delta - 1 \\
L_{G,1} &\equiv i_1 \theta (1 - e_0) & (2.23) \\
L_{G,2} &\equiv i_2 \theta (1 - e_0) - [R_L - (1 - \theta)] & (2.24) \\
L_{D,1} &\equiv (1 - \theta) - R_L & (2.25)
\end{aligned}
$$

and, using (2.10), (2.18), the point of insolvency can be expressed as:

$$
R^S(i = i_3) = (1 - \theta) + \theta(1 - e_0) = 1 - \theta e_0 \tag{2.26}
$$

As explained in Section 2.3.1, δ_s in (2.22) reflects the *social* opportunity cost of equity funding, which captures the higher (deadweight) transaction. The derivation of the social welfare function (2.22) is in the Annex.

It is worth clarifying at this point what the social costs and benefits of increasing the capital buffer are in our model. In our model, increasing the capital buffer reduces both the probability of a bank failure (by lowering the point of insolvency, (2.26)) and the cost of a bank failure (by lowering the losses imposed on unsecured debt holders, (2.23) and (2.24)). But this comes at the cost of reducing output by 'crowding out' investments in other sectors that banks' equity investors could have funded instead. Note that, in the absence of social deadweight costs associated with bank failures ($\psi(.) = 0$ and $\chi(.) = 0$), the regulator simply maximises the expected total return of the bank – which will be shared amongst all claimholders – net of the social cost of equity funding.

2.4.5 Determinants of optimal regulatory requirements

We now examine the key determinants of the socially optimal capital buffer E^b and TLAC requirement τ^*, given the optimal minimum requirement E^* given by

(2.9), through numerical simulations of (2.9) and (2.22). To do this, we assume the following quadratic functional forms for the social costs of bail in and bail out:

$$
\begin{aligned}
\psi(L_{G,T}) &= \lambda_G L_{G,T} + \tilde{\lambda}_G L_{G,T}^2 \\
\chi(L_{D,T}) &= \lambda_D L_{D,T} + \tilde{\lambda}_D L_{D,T}^2
\end{aligned}
$$

The derivation of the social welfare function under these assumptions is provided in the Annex. Under the baseline calibration, we assume the following: $p = 0.3$, $\gamma = 1.14$, $R_H = 2$, $R^{\max} = 1.045$, $\delta_s = 0.0025$, $\lambda_G = 0.02$, $\tilde{\lambda}_G = 0.0256$, $\lambda_D = 0.023$, $\tilde{\lambda}_D = 0.0089$, $q = 0.9$. The baseline parameterisations are chosen purely to reproduce the minimum TLAC, minimum capital and capital buffer requirements as set out by the Financial Stability Board (FSB) and Basel Committee on Banking Supervision (BCBS). As these parameters are not chosen based on empirical estimates, we are primarily interested in the directional (rather than quantitative) change in the optimal trio of regulatory requirements caused by the changes in the assumptions about the key parameters. But we note that the baseline parameterisations reflect the view that the social cost of equity funding is small, and that the cost of bail-in is smaller than the cost of bail-out as long as the size of loss at insolvency is relatively small.

The tables below summarise the results from numerical simulation. The third column of Table 2.2 below shows the optimal regulatory requirements under the baseline calibration: in this scenario, we obtain an optimal minimum capital requirement of 6.0% (consistent with Basel III end-point Tier 1 capital ratio), capital buffer of 5.0% (consistent with Basel III end-point capital conservation buffer of 2.5% plus G-SIB buffer of 1-2.5%), and TLAC of 18.0% (consistent with the full implementation of the FSB (2015)'s *Principles* in 2022).

A key unknown parameter is the cost of bail-in, given that there is little historical precedents for orderly imposition of losses on private bond holders of large banks. In the fourth column of Table 2.2, we present the 'low bail in cost' scenario, in which $\tilde{\lambda}_G = 0.02$: i.e. the bail-in cost is both lower and less convex than in the baseline. The optimal equity buffer falls in this case to 4%, while the optimal TLAC rises to 25%. This is because in this scenario, bail-in is lower cost than bail-out, and unsecured debt is a close substitute to equity in its ability to absorb losses without creating large externalities. Conversely, a higher and more convex bail-in cost ($\tilde{\lambda}_G = 0.03$, shown in the fifth column of Table 2.2) would imply a lower optimal TLAC and a higher capital buffer than the baseline. Note that the minimum capital requirement is invariant to the cost of bail-in, as this is determined by the need to ensure that the bank has enough 'skin in the game' to incentivise it to invest in the safe asset.

Bailout costs also matter. In the 'low bailout cost' scenario presented in column 4 of Table 2.3 (in which $\tilde{\lambda}_D = 0.008$), both the capital buffer and TLAC are lower than the baseline (shown in column 3). Conversely, in the 'high bailout cost' scenario (shown in column 5) in which $\tilde{\lambda}_D = 0.0095$, so that bailout costs are both higher and more convex, both the capital buffer and TLAC can be considerably higher relative to the baseline.

Table 2.2 – Optimal regulation and sensitivity to bail-in costs

	Expression	Baseline	Low bailin cost	High bailin cost
Minimum Capital Ratio	E^*	6.0%	6.0%	6.0%
TLAC	$\tau*$	18.0%	21.0%	16.5%
Capital buffer	E^b	5.0%	4.0%	5.6%
Minimum TLAC + Capital Buffer	θ	23.0%	25.0%	22.0%

Table 2.3 – Optimal regulation and sensitivity to bail-out costs

	Expression	Baseline	Low bailout cost	High bailout cost
Minimum Capital Ratio	E^*	6.0%	6.0%	6.0%
TLAC	τ^*	18.0%	17.2%	18.6%
Capital buffer	E^b	5.0%	3.8%	5.9%
Minimum TLAC + Capital Buffer	θ	23.0%	21.0%	24.5%

There are also other important parameters that determine the optimal regulatory ratios, as shown in Table 2.4. For example, a higher social cost of equity ($\delta_s = 0.00255$) would imply that the regulator should require banks to hold a lower capital buffer than the baseline (column 3 in Table 2.4). Interestingly, stronger incentives for failing banks to engage in gambling, or asset substitution ($\gamma = 1.15$, shown in column 4 in Table 2.4) implies that the regulator should set a higher minimum capital requirement, but also a lower capital buffer and a higher TLAC requirement, so as to maintain the sum of TLAC and capital buffer (θ) the same as in the baseline: this is intuitive, as the cost of insolvency is unchanged from the baseline, while the need for early intervention has increased. Finally, if the probability of adverse macro shock becomes higher ($q = 0.895$, column 5 in Table 2.4), then the capital buffer should be increased while the TLAC requirement can be reduced. This reflects the higher expected social cost of bank failure, which calls for a higher capital and lower unsecured debt to reduce the probability of bank failure. This analysis raises an interesting possibility that, as the capital buffer is raised with increased risk of system wide distress, the TLAC requirement may actually need to be *reduced* to allow banks to fund themselves with more capital and less debt.

For completeness, we also examine the extreme case in which both bail-out and bail-in are costless ($\lambda_D = \lambda_G = 0$ and $\tilde{\lambda}_D = \tilde{\lambda}_G = 0$): thus, our Assumptions 2 and 3 are dropped, and both deposits and debt become perfect substitutes to equity as loss-absorbing instruments. In this case, while a minimum capital requirement E^* is still needed in order to prevent ex-post moral hazard, and there is no need for a capital buffer or TLAC requirement in this case. In the next section, we demonstrate that, when ex-ante moral hazard is present, then the regulator should

Table 2.4 – Optimal regulation under different scenarios

	Baseline	High equity cost	Strong gambling incentive	Higher macro risk
Minimum Capital Ratio	6.0%	6.0%	6.4%	6.0%
Subordinated debt	12.0%	12.1%	12.0%	11.3%
TLAC	18.0%	18.1%	18.4%	17.3%
Capital buffer	5.0%	3.4%	4.6%	8.7%
Minimum TLAC + Capital Buffer	23.0%	21.5%	23.0%	26.0%

still require that the bank has sufficient private loss absorbing capacity, θ.

We also study what happens if only Assumption 2 is dropped. Suppose that bail-in is costless ($\lambda_G = \tilde{\lambda}_G = 0$) but bail-out remains costly (as in the baseline). In this case, it is optimal to require banks to meet a TLAC requirement of 100% and a minimum capital requirement of E^* (=6% as in the baseline), but there will be no need for a capital buffer. This is intuitive: if bail-in is costless, then unsecured debt and equity are perfect substitutes ex-post as loss absorbing instruments, and hence unsecured debt is favoured over equity as the latter is socially more costly ex-ante. Deposits are socially less desirable than unsecured debt in this case. We note, however, that deposits may carry benefits that are not considered in this model, such as the convenience of on-demand withdrawal, such that a 100% TLAC requirement may not be desirable even when bail-in is costless. Finally, we note that, when $\delta_s = 0$, our model suggests that bank should be fully funded by equity.

2.5 Ex-ante moral hazard and the impact of regulatory policies

Thus far, we have assumed that the probability of a high return, q, is exogenous. In this section, we assume that q is determined endogenously by banks' decisions over the monitoring of their projects, which requires them to exert costly effort. We then examine how ex-ante regulation affects the incentive to monitor.

Suppose that, in the absence of any monitoring by banks at $t = 0$, the probability of the project yielding high return is q_L. Following Holmstrom and Tirole (1997), suppose that, if banks choose to exert a monitoring effort which has private cost C, they can increase the probability of project success from q_L to q_H, where $q_H > q_L$. We assume that the bank chooses the monitoring effort *at the end of $t = 0$*, after the realisation of R_L has been observed. At this point there is no information asymmetry between banks and potential unsecured debt holders, so the market can fully price in the risk depending on whether or not the bank monitors its investment at t=0.

2.5.1 Socially efficient monitoring

Monitoring is socially efficient for Type 1 banks with $R_L \in (0, R^D)$, if the following condition holds:

$$
\begin{aligned}
q_H R_H + (1 - q_H)\left[R_L - \psi(L_{G,1}) - \chi(L_{D,1})\right] - C &\geq \\
q_L R_H + (1 - q_L)\left[R_L - \psi(L_{G,1}) - \chi(L_{D,1})\right], \forall R_L &\in (0, R^D)
\end{aligned}
$$

In other words, the private monitoring cost C incurred by the banks must be sufficiently small relative to the social costs of bail in and bail out. This can be simplified to:

$$
C \leq C_1^* \equiv (q_H - q_L)(R_H - R_L) + (q_H - q_L)[\psi(L_{G,1}) + \chi(L_{D,1})], \forall R_L \in (0, R^D) \quad (2.27)
$$

The right hand side is the sum of the benefits from monitoring that accrue to banks' claimholders, $(q_H - q_L)(R_H - R_L)$, and to the society in the form of reduced externalities from bail ins and bail outs, $(q_H - q_L)[\psi(L_{G,1}) + \chi(L_{D,1})]$.

Similarly, monitoring is efficient for Type 2 banks with $R_L \in (R^D, R^S)$, as long as:

$$
q_H R_H + (1 - q_H)\left[R_L - \psi(L_{G,2})\right] - C \geq q_L R_H + (1 - q_L)\left[R_L - \psi(L_{G,2})\right], \forall R_L \in (R^D, R^S)
$$

This can be reorganised as:

$$
C \leq C_2^* \equiv (q_H - q_L)(R_H - R_L) + (q_H - q_L)\psi(L_{G,2}), \forall R_L \in (R^D, R^S) \quad (2.28)
$$

Finally, monitoring is efficient for Type 3 banks with $R_L \in (R^S, R^{\max})$, as long as:

$$
q_H R_H + (1 - q_H)R_L - C \geq q_L R_H + (1 - q_L)R_L, \forall R_L \in (R^S, R^{\max})
$$

This can be reorganised as:

$$
C \leq C_3^* \equiv (q_H - q_L)(R_H - R_L), \forall R_L \in (R^S, R^{\max}) \quad (2.29)
$$

In what follows, we assume that $C \leq C_1^*$, $C \leq C_2^*$, and $C \leq C_3^*$ for all Types 1, 2, and 3 banks, respectively. In other words, the regulator would always want to ensure that banks monitor their projects.

2.5.2 Private monitoring incentives

We now consider the monitoring incentives of a bank which seeks to maximise shareholder returns in a context in which monitoring is socially desirable. From previous analysis in Section 2.4.3, we know that profits of Type 1 banks are distorted by the presence of implicit subsidy from the DIF. Below, we demonstrate that only Type

1 banks are therefore prone to ex-ante moral hazard; and that having a sufficiently high private loss absorbing capacity, θ, which is the sum of TLAC plus equity buffer, can help reduce this.

For Type 1 banks with $R_L \in (0, R^D)$, unsecured debt has zero recovery value if $R = R_L$. From (2.13), we know that the interest rate on unsecured debt is $\frac{1}{q_H}$ if the bank chooses to monitor, and $\frac{1}{q_L}$ if it decides not to monitor, such that $\frac{1}{q_H} < \frac{1}{q_L}$. Type 1 banks will monitor as long as the expected profit from monitoring, net of monitoring costs, exceed the expected profit of not monitoring:

$$\Pi_1(q_H) - C > \Pi_1(q_L)$$

where $\Pi_1(.)$ is given by (2.14). Reorganising the above, we can show that Type 1 banks will monitor their projects as long as $\theta \geq \theta^*$ where:

$$\theta^* = \frac{C}{q_H - q_L} - R_H + 1 \tag{2.30}$$

This shows that, unless θ is sufficiently high, Type 1 banks will be subject to moral hazard and will not monitor their project. Thus, the regulator can ensure that Type 1 banks take actions to reduce risks by setting θ – TLAC *plus* equity buffer – above θ^*. We emphasise that the incentive to engage in moral hazard is influenced by the total amount of losses that can be absorbed by private claimholders as opposed to taxpayers. In particular, the monitoring incentives are unaffected by the split of θ between equity and unsecured debt, as unsecured debt prices in the risks that banks are taking. Thus, as long as θ is sufficiently high, distortions in monitoring effort arising from a limitedly liable bank being funded by insured deposits is eliminated.[8]

For instance, for baseline parameterisations $C = 0.45$, $q_H = 0.9$, $q_L = 0.5$ and $R_H = 2$, $\theta \geq \theta^* = 12.5\%$ is needed to induce monitoring by Type 1 banks. But if there is an exogenous shock that reduces the probability of a high return that banks can achieve through monitoring, θ will need to be raised in order to induce monitoring: for instance, for $q_H = 0.89$, $\theta \geq \theta^* = 15.4\%$ is needed to induce monitoring by Type 1 banks, consistent with our previous analysis. Note that the optimal θ – which maximises social welfare (2.22) – could be higher than the minimum required to induce monitoring by Type 1 banks.

For Type 2 banks with $R_L \in (R^D, R^S)$ and Type 3 banks $R_L \in (R^S, R^{\max})$, we have already seen that their profits are not distorted by the presence of the guarantee on deposits, because the DIF will not have to pay out anything even in the bad scenario. Type 2 banks will monitor as long as:

$$\Pi_2(q_H) - C > \Pi_2(q_L)$$

where $\Pi_2(.)$ is given by (2.17). Similarly, Type 3 banks with $R_L \in (R^S, R^{\max})$ will monitor as long as:

---

[8]Or equivalently the distortion created from risk-insensitive interest rates on insured deposits is sufficiently small.

$$\Pi_3(q_H) - C > \Pi_3(q_L)$$

where $\Pi_3(.)$ is given by (2.19). Reorganising the above, it can be shown that both Type 2 and Type 3 banks will choose to monitor as long as:

$$C < (q_H - q_L)(R_H - R_L)$$

The right hand side is the benefits of monitoring that accrue to the banks' claimholders. Comparing the above with (2.28) and (2.29), it is clear that Type 3 have the socially optimal incentive to monitor, regardless of the level of θ. This is because Type 3 banks will remain solvent even in the bad state, so that all costs and benefits are internalised. By contrast, Type 2 banks have a sub-optimal incentive to monitor, if $(q_H - q_L)(R_H - R_L) < C < C_2^*$: if so, Type 2 banks do not have the incentive to monitor, because the private cost of monitoring outweighs the benefit, even though it is socially optimal for them to monitor once the cost of bail in is taken into consideration. Note that Type 2 banks' incentive to monitor is independent of θ, because they will not impose losses on the DIF even in the bad state and hence are not subject to moral hazard driven by the implicit subsidy. This is why ex-ante regulatory requirements cannot induce them to monitor, if $(q_H - q_L)(R_H - R_L) < C < C_2^*$.

2.6 Conclusion

The interactions between the new regulatory requirements on banks, which were introduced after the global financial crisis of 2007-2008, is still a relatively unexplored area. In this context, our paper makes two key contributions. First, we clarify within a simple framework what factors determine the optimal settings of the trio of regulatory requirements: the minimum capital requirement, the TLAC requirement, and the capital buffer. Second, we show which of the trio of the regulatory requirements should be adjusted cyclically.

Our analysis illustrates that the optimal size of the capital buffer and TLAC, and the optimal composition of TLAC, depend on the social cost of a crisis (i.e. the cost of bail-in and bail-out), as well as the probability of a system-wide shock. This implies that the policymakers will need to take a view on how costly they expect bail-in – which is yet to be tested – to be. If they expect system-wide externalities from bail-in to be limited, then setting a low capital buffer and a high TLAC requirement would be optimal, when equity funding is socially expensive. By contrast, if policymakers fear that bail-in could potentially cause contagion, or would be subject to high legal costs, then a relatively low TLAC requirement and a high capital buffer would be desirable. We note that the cost of bail-in is fundamentally uncertain, given that it is untested and hence there is a lack of evidence to enable us to estimate the cost. Thus, a precautionary approach might suggest that a relatively low TLAC requirement combined with a high capital buffer might be desirable, especially if the social cost of equity is considered to be relatively small.

Our analysis also raises an interesting possibility that it may be desirable to make TLAC as well as capital buffer time-varying: in particular, as the risk of system-wide shock increases, the capital buffer should be raised and TLAC should be reduced in order to reduce the probability of bank insolvency. This result could alternatively interpreted as stating that forbearance on the minimum TLAC requirement could be justifiable if the bank in question is well capitalised, and that such a policy may not give rise to the same adverse incentives associated with forbearance on the minimum capital requirement.

Appendix A

Appendix Chapter 1

A.1 Equilibrium Deposit Contract

The Lagrangian is given by expression (A.1).

$$
\begin{aligned}
\mathcal{L} \; = \;\; & \left(y^j + R(1 - y^j) - \lambda c_1^j - (1 - \lambda)c_2^j\right)\left(\frac{u_j - u^k}{d} + \frac{1}{N}\right) && \text{(A.1)} \\
+ \;\; & \mu_1(y^j - \lambda c_1^j) + \mu_2(R(1 - y^j - (1 - \lambda)c_2^j) + \mu_3\left(c_2^j - c_1^j\right)
\end{aligned}
$$

The first order conditions (FOCs) with respect to holding of the short asset, consumption in periods 1 and 2 (y, c_1 and c_2, μ_1 and μ_2 respectively) are below

$$
\mu_1 - R\mu_2 = \frac{1}{N}(R - 1) \tag{A.2}
$$

$$
\mu_1 = \frac{1}{d}u'(c_1)\left[y + R(1 - y) - \lambda c_1 - (1 - \lambda)c_2\right] - \frac{1}{N} \tag{A.3}
$$

$$
\mu_2 = \frac{1}{d}u'(c_2)\left[y + R(1 - y) - \lambda c_1 - (1 - \lambda)c_2\right] - \frac{1}{N} \tag{A.4}
$$

$$
min\left\{y - \lambda c_1, \mu_1\right\} = 0 \tag{A.5}
$$

$$
min\{R(1 - y) - (1 - \lambda)c_2, \mu_2\} = 0 \tag{A.6}
$$

$$
min\{c_2^j - c_1^j, \mu_3\} = 0 \tag{A.7}
$$

Where the above simplified FOCs make use of the fact that in a symmetric equilibrium all banks give the same deposit contract yielding the same utility i.e. $u^j = u^k = u$. Substituting (A.3) and (A.4) in (A.2) the following expression is obtained:

$$[u'(c_1) - Ru'(c_2)] \left[y + R(1-y) - \lambda c_1 - (1-\lambda)c_2 \right] = 0 \tag{A.8}$$

This implies that at least one of the bracketed terms in (A.8) equals zero. The first bracketed term is an Euler equation and the latter bracketed term is the zero profit condition. Note that if the bank makes zero profits both Lagrange multipliers are strictly positive $\mu_1 > 0, \mu_2 > 0$ as the period 1 and 2 budget constraints are binding. Substituting the zero profit condition into (A.3) and (A.4) gives $\mu_1 = \mu_2 = -1/N$ which contradicts with (A.5) and (A.6). Thus the above FOC can simplified to the following expression.

$$u'(c_1) = Ru'(c_2) \tag{A.9}$$

As the return on the long asset is greater than the return on the short asset banks will only hold as much short asset as required to provide c_1 to early depositors. Thus (1.5) is binding, and (1.6) is non-binding, implying that $\mu_1 > 0, \mu_2 = 0$ so that the following expression holds.

$$c_1 = \frac{y}{\lambda} \tag{A.10}$$

Substituting these conditions in (A.4) we obtain the following expression:

$$u'(c_2)[R(1-y) - (1-\lambda)c_2] = \frac{d}{N} \tag{A.11}$$

Using the specific example of the logarithmic utility function $u(c) = ln(c)$ the expression becomes:

$$\frac{R(1-y)}{c_2} - (1-\lambda) = \frac{d}{N} \tag{A.12}$$

which yields the expression for c_2

$$c_2 = \frac{R(1-y)}{(1-\lambda) + \frac{d}{N}} \tag{A.13}$$

So as the number of banks becomes large ($N \to \infty$) or the banks become less horizontally differentiated ($d \to 0$) then the late consumption approaches the 1st best outcome, as in AG ($c_2 \to \frac{R(1-y)}{1-\lambda}$) implying that profits tend to zero also.

Using (A.10) and (A.13) in the Euler equation and re-arranging we obtain the solution to the above maximisation problem. Formally, these are the symmetric Nash equilibrium strategies of all banks within the system.

$$(c_1^*, c_2^*, x^*, y^*, \mu_1^*, \mu_2^*, \mu_3^*) = \left(\frac{N}{N+d}, \frac{RN}{N+d}, \frac{N+Nd-N\lambda}{N+d}, \frac{\lambda N}{N+d}, \frac{R-1}{N}, 0, 0 \right) \tag{A.14}$$

A.2   Comparative Statics

In order to understand how the degree of competition and concentration we rearrange the expression in condition to obtain the following contagion condition, which can be expressed as a function of the number of firms N and the intensity of competition d.

$$\Psi(N, d) = b - z(c_1 - q^1) \geq 0 \tag{A.15}$$

If the liquidation value of the long asset is small ($r \to 0$) then the buffer b is given by $b = \Pi/l$, meaning that expression (A.15) can be rewritten as the following, where $\rho = \lambda_H - \lambda$.

$$\Psi(N, d) = \frac{1}{N^2 + Nd} \left(\frac{Rd}{l} - \frac{2\rho(1-\lambda)N}{1+\rho} \right) \tag{A.16}$$

The marginal effect of reducing the intensity of competition, i.e. increasing the transport cost parameter d, is given by equation (A.17). Both bracketed terms in the parentheses are strictly positive so the effect of increasing competition (a reduction in d) reduces the buffer remaining, meaning that increased competition *ceteris paribus* increases the parameter space in which contagion occurs.

$$\frac{\partial \Psi}{\partial d} = \frac{1}{(N^2 + Nd)^2} \left(\frac{Rd}{l} + \frac{2\rho(1-\lambda)}{1+\rho} \right) \tag{A.17}$$

The marginal effect of reducing concentration, i.e. increasing the number of banks N, is given by equation (A.18). The sign of expression (A.18) is ambiguous. The first term in the parentheses is strictly positive and the second term is strictly negative. The first term is the concentration-risk effect, which results from an increase in the number of banks reducing the losses from a single counterpart in the interbank deposit network failing. The second term is the profit buffer effect, which results from an increase in the number of banks reducing the profit buffer which banks can use to absorb losses.

$$\frac{\partial \Psi}{\partial N} = \frac{1}{(N^2 + Nd)^2} \left(\frac{2\rho(1-\lambda)N^2}{1+\rho} - \frac{Rd}{l}(2N + d) \right) \tag{A.18}$$

Which of the two effects dominates depends on whether the first term in the parentheses is larger in magnitude than the second term in parentheses. Rearranging equation (dpsidN) we obtain the following condition which must be satisfied for there to be concentration stability.

$$\frac{Rd}{l}(2N + d) > \frac{2\rho(1-\lambda)}{1+\rho}N^2 \tag{A.19}$$

From the above expression it can be deduced that concentration promotes systemic stability (given an initial bank failure) if the return on the long asset R is high,

competition is weak (d is high), the share of early customers λ is high, the interest rate on the external loan l is low, and the difference in liquidity demands in states 1 and 2 (ρ) is low as this determine the breadth of the channel of contagion. A narrower channel of contagion means that the concentration risk reducing effect of increased number of banks N is reduced.

Appendix B

Appendix Chapter 2

B.1 Minimum Capital Requirement

The minimum capital requirement is obtained by substituting the gambling threshold $R_L = R^T$ from (2.8) into (2.5). Rearranging (2.5) we obtain $R_L = \frac{1-\theta+i\theta(1-e_0)}{1-E^*}$. Thus the minimum capital requirement E^* solves the following condition:

$$\frac{1-\theta+i\theta(1-e_0)}{1-E^*} = \frac{1-p}{1-p\gamma}[(1-\theta)+i\theta(1-e_0)] \tag{B.1}$$

which becomes

$$1-E^* = \frac{1-p\gamma}{1-p}$$

Rearranging the above, we obtain (2.9).

B.1.1 Derivation of the social welfare function

To derive (2.22), note that the social welfare consists of two components: i) the expected return from banks' investment (which are divided between their claimholders) net of the social opportunity cost of funding that investment, and ii) the expected social cost of bank failure. The first component in the social welfare function is given by the following expression.

$$
\begin{aligned}
\text{Expected return on investment net of funding cost} &= \bar{R} - (1-\theta) - \theta(1-e_0) - \delta\theta e_0 \\
&= \bar{R} - 1 - (\delta-1)\theta e_0 \\
&= \bar{R} - \delta_s\theta e_0 - 1 \tag{B.2}
\end{aligned}
$$

where $\bar{R} \equiv qR_H + (1-q)\int_0^{R^{max}} R_L f(R_L)dR_L$ is the expected return on banks' investment (which is the sum of the expected payoffs of the bank's shareholders, uninsured creditors, depositors and the deposit insurance fund), and $\delta_s \equiv \delta - 1$

is the social opportunity cost of funding the bank with equity instead of debt or deposits.

The second part of the social welfare function is given by the expected cost of bank failure, which arises in a bad state when the bank becomes insolvent:

$$\text{Expected social cost} = -(1-q) \quad \left(\int_{R^D}^{R^S(i=i_3)} \psi(L_{G,2}) f(R_L) dR_L \right.$$
$$\left. + \int_0^{R^D} \left[\psi(L_{G,1}) + \chi(L_{D,1}) \right] f(R_L) dR_L \right) \quad \text{(B.3)}$$

where losses imposed on uninsured debt holders and depositors of Type 1 and Type 2 banks are given by:

$$
\begin{aligned}
L_{G,1} &\equiv i_1 \theta (1 - e_0) \\
L_{G,2} &\equiv i_2 \theta (1 - e_0) - [R_L - (1 - \theta)] \\
L_{D,1} &\equiv (1 - \theta) - R_L
\end{aligned}
$$

and, using (2.10), (2.18), the point of insolvency can be expressed as:

$$R^S(i = i_3) = (1 - \theta) + \theta(1 - e_0) = 1 - \theta e_0$$

Summing up (B.2) and (B.3) after dr opping the constant -1, we obtain (2.22).

B.2   Social welfare function used for numerical simulation

 We now derive the specific functional form of the welfare function (2.22), under the following two assumptions.

$$
\begin{aligned}
\psi(L_{G,T}) &= \lambda_G L_{G,T} + \tilde{\lambda}_G L_{G,T}^2 \\
\chi(L_{D,1}) &= \lambda_D L_{D,T} + \tilde{\lambda}_D L_{D,T}^2
\end{aligned}
$$

In this case, (2.22) can be rewritten in the following form:

$$W = \bar{R} - (X + \tilde{X}) - (Y + \tilde{Y}) - \delta_s \theta e_0 \qquad \text{(B.4)}$$

where:

$$X = (1-q)\lambda_G \int_{1-\theta}^{1-\theta e_0} L_{G,2} f(R_L)\, dR_L$$

$$\tilde{X} = (1-q)\tilde{\lambda}_G \int_{1-\theta}^{1-\theta e_0} L_{G,2}^2 f(R_L)\, dR_L$$

$$Y = (1-q) \int_0^{1-\theta} (\lambda_G L_{G,1} + \lambda_D L_{D,1}) f(R_L)$$

$$\tilde{Y} = (1-q) \int_0^{1-\theta} \left(\tilde{\lambda}_G L_{G,1}^2\right) f(R_L)$$

Expanding the above:

$$\bar{R} = qR^H + (1-q)\int_0^{R^{\max}} R_L f(R_L) dR_L$$
$$= qR^H + (1-q)\frac{R^{\max}}{2}$$

The losses suffered by unsecured debt holders of Type 1 and 2 banks in the event of insolvency are given by:

$$L_{G,1} = i_1\theta(1-e_0) = \frac{\theta(1-e_0)}{q}$$

$$L_{G,2} = i_2\theta(1-e_0) - (R_L - (1-\theta))$$
$$= \left(\frac{1}{q} - \frac{1-q}{q}\frac{R_L - (1-\theta)}{\theta(1-e_0)}\right)\theta(1-e_0) - (R_L - (1-\theta))$$
$$= \frac{1}{q}(1-\theta e_0 - R_L)$$

Substituting the above:

$$X = \frac{(1-q)\lambda_G}{qR^{\max}} \int_{1-\theta}^{1-\theta e_0} [(1-\theta e_0 - R_L)]\, dR_L$$
$$= \frac{(1-q)\lambda_G}{qR^{\max}} \left[[(1-\theta e_0) R_L]_{1-\theta}^{1-\theta e_0} - \left[\frac{R_L^2}{2}\right]_{1-\theta}^{1-\theta e_0} \right]$$
$$= \frac{(1-q)\lambda_G}{qR^{\max}} \left[(1-\theta e_0)((1-\theta e_0) - (1-\theta)) - \frac{1}{2}\left[(1-\theta e_0)^2 - (1-\theta)^2\right] \right]$$
$$= \frac{(1-q)\lambda_G}{qR^{\max}} \left[\theta(1-\theta e)(1-e_0) - \frac{1}{2}\left[(1-\theta e_0)^2 - (1-\theta)^2\right] \right]$$

$$
\begin{aligned}
\tilde{X} &= \frac{(1-q)\tilde{\lambda}_G}{R^{\max}} \int_{1-\theta}^{1-\theta e_0} \left(\frac{1}{q}(1-\theta e_0 - R_L)\right)^2 dR_L \\
&= \frac{(1-q)\tilde{\lambda}_G}{q^2 R^{\max}} \int_{1-\theta}^{1-\theta e_0} \left[(1-\theta e_0)^2 - 2R_L(1-\theta e_0) + R_L^2\right] dR_L \\
&= \frac{(1-q)\tilde{\lambda}_G}{q^2 R^{\max}} \left[(1-\theta e_0)^2[(1-\theta e_0)-(1-\theta)] - (1-\theta e_0)[(1-\theta e_0)^2 - (1-\theta)^2] + \frac{[(1-\theta e_0)^3 - (1-\theta)^3]}{3}\right] \\
&= \frac{(1-q)\tilde{\lambda}_G}{q^2 R^{\max}} \left[\theta(1-\theta e_0)(1-\theta)(e_0-1) + \frac{[(1-\theta e_0)^3 - (1-\theta)^3]}{3}\right]
\end{aligned}
$$

$$
\begin{aligned}
Y &= (1-q)\int_0^{1-\theta} \left[\lambda_G \frac{\theta(1-e_0)}{q} + \lambda_D((1-\theta)-R_L)\right] f(R_L)\, dR_L \\
&= \frac{(1-q)}{R^{\max}} \left[\left(\lambda_G \frac{\theta(1-e_0)}{q} + \lambda_D(1-\theta)\right)R_L\right]_0^{1-\theta} - \frac{(1-q)\lambda_D}{R^{\max}}\left[\frac{R_L^2}{2}\right]_0^{1-\theta} \\
&= \frac{(1-q)}{R^{\max}} \left[\left(\lambda_G \frac{\theta(1-e_0)}{q} + \lambda_D(1-\theta)\right)(1-\theta) - \lambda_D\frac{(1-\theta)^2}{2}\right] \\
&= \frac{(1-q)}{R^{\max}} \left[\left(\lambda_G \frac{\theta(1-\theta)(1-e_0)}{q}\right) + \lambda_D\frac{(1-\theta)^2}{2}\right]
\end{aligned}
$$

$$
\begin{aligned}
\tilde{Y} &= \frac{(1-q)}{R^{\max}} \int_0^{1-\theta} \left[\tilde{\lambda}_G\left(\frac{\theta(1-e_0)}{q}\right)^2\right] dR_L \\
&= \frac{(1-q)\tilde{\lambda}_G}{R^{\max}}\left(\frac{\theta(1-e_0)}{q}\right)^2 (1-\theta)
\end{aligned}
$$

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