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Asset Prices and Portfolio Choice with Learning from Experience *

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Abstract

We study asset prices and portfolio choice with overlapping generations, where the young disregard history to learn from own experience. Disregarding history implies less precise estimates of output growth, which in equilibrium leads the young to increase their investment in risky assets after positive returns, that is, they act as trend chasers. In equilibrium, the risk premium decreases after a positive shock and, therefore, trend chasing young agents lose wealth relative to old agents who behave as contrarians. Consistent with findings from survey data, the average belief about the risk premium in the economy relates negatively to future excess returns and is smoother than the true risk premium.

Keywords: Learning from Experience Based Bias, Trend Chasing, Survey Based versus Objective Risk Premiums

JEL Classification: E2, G10, G11, G12

1 Introduction

Any risky investment decision, for example an investment in the stock market, requires the formation of expectations about fundamentals such as dividends and discount rates. Most models of financial decision making assume that investors form these expectations in an unbiased way. Yet, there are ongoing debates among academics and practitioners about the extent and nature of predictability in stock market returns and, more generally, about the level of the risk premium. Since academics and practitioners struggle to find common ground, it appears to be challenging to form the unbiased expectations required for optimal investment decisions.

Looking at survey data substantiates the concern that many economic agents fail to form unbiased expectations. According to the surveys, agents extrapolate stock returns, that is, when they see a high (low) return they expect to see more of it. However, this is contrary to what we see in the data and, therefore, forecasts of expected stock market returns are typically negatively correlated with actual future returns and ex ante measures of the risk premium.¹ This poses a serious challenge not just for models of financial decision making but for standard models of asset markets, in which agents perfectly understand the time-variation in stock market returns.

Given the complexity involved in producing economic forecasts, it seems that the question is not whether expectations or forecasts are biased but rather which are the decisive biases. Our focus is on how lifetime experiences affect expectations. For instance, stark experiences early on in life such as the Great Depression might drive the expectations of agents way beyond the end of the Great Depression. One would, however, expect that the bias declines over time as investors observe year after year that the Great Depression did not reoccur. Such behavior could be consistent with empirical evidence in support of the idea that experience matters for the formation of beliefs.² If this is the case, then personal experiences with stock

¹See Greenwood and Shleifer (2014).

²See Malmendier and Nagel (2016).

markets and the macroeconomy in general should manifest through savings and investment decisions and impact asset prices.

In this paper, we depart from standard models of asset markets that assume that every investor knows the underlying dynamics of fundamentals to instead allow investors to form expectations that are influenced by their own lifetime experiences. The learning from experience based bias generates a life-cycle of expectations about the risk premium that ranges from return extrapolation when young to contrarian when old. Contrary to the beliefs of the young, from the point of view of an econometrician with full information, the risk premium decreases after positive shocks. Hence, young agents increase their risky investment at times when the risk premium is low, leading to a slower wealth accumulation in the early years of life. Our model is sufficiently rich to accommodate a cross-section of beliefs that is consistent with findings from survey evidence such as the return extrapolation of the average investor and the negative correlation between the consensus forecast and future stock market returns.

Specifically, we consider an overlapping generations economy with incomplete information about expected growth in aggregate output. In the economy, agents learn about the true expected growth using Bayes' rule, but they only use the data observed during their own lifetime. Hence, agents overweight their own lifetime experiences relative to history, i.e., they exhibit a learning from experience based bias. Therefore, young agents with little experience on average make large mistakes and update their beliefs more aggressively in response to news than older agents with ample experience.

Since agents are learning from their own experience, beliefs about growth differ across all generations of agents. Consequently, the entire cross-section of beliefs in the economy determines asset prices, where the beliefs of wealthy agents have larger impact than that of poor agents. Thus, the "market view," which we define as the wealth weighted average expectation of growth, instead of the true expected growth rate, serves as the relevant statistic for asset pricing. As a result, the market view drives the interest rate and in times of an elevated market view, we see high interest rates due to the intertemporal smoothing

motive. In addition, from the perspective of an econometrician equipped with complete information about the true expected growth, the risk premium seems low in times of an elevated market view.

So how does the market view fluctuate over time? In our economy, all agents revise their expectations upwards in response to a positive shock. Consequently, the market view increases, which, in turn, pushes the interest rate up and the risk premium down. Importantly, changes in the market view in response to a shock are not only due to changes in the beliefs of individual agents, but also because the wealth distribution puts more weight on agents that have beliefs that happen to be more consistent with the direction of the shock. To understand this, consider again a positive shock, where all agents revise their expectations upwards, which increases the market view. Now, as the market view is the wealth weighted average belief and agents trade on their beliefs, a positive shock increases the wealth of the optimists relative to the pessimists and, therefore, it increases the market view beyond the change due to heightened expectations. Thus, there is an “overreaction” in the market view. We show that the effect coming from wealth reallocations is particularly large when disagreement is high and wealth is more evenly distributed among agents with differing beliefs. After large wealth reallocations, the market view is likely to revert back to the true mean at a faster rate than at other times because speculative trade is at a high, implying that agents with too optimistic or too pessimistic beliefs are likely to lose out to experienced traders.

Although the market view determines prices, all generations of agents perceive the risk premium differently. Expanding on this point, we show that while the true risk premium depends on the difference between the actual expected growth and the market view, the perceived risk premium depends on the difference between the individual agent’s belief about growth and the market view. Hence, agents who are relatively optimistic perceive a high risk premium on the stock market since from their point of view the stock appears to be “cheap,” or, put differently the discount rate seems to be high.

Next, we turn to the dynamics of the perceived risk premium by relating it to the true

risk premium. Specifically, the covariance between the true risk premium and the perceived risk premium depends on the variance of the market view minus the covariance between the market view and the individual agent’s belief about growth. Young agents with little experience update their beliefs much more aggressively in response to news than older more experienced agents and the covariance between the market view and their belief is higher than the volatility of the market view. Consequently, the belief about the risk premium of agents with little experience correlates negatively with the true risk premium. Instead of perceiving a low risk premium after positive shocks, the young perceive a high risk premium and respond by increasing their investment in the stock market. The behavior of the young mimics “return extrapolation.” Older agents with more experience act as “contrarians.” We see that the old counter-balance the demand of the young and, thereby, the market clears since in equilibrium there has to be contrarians to facilitate trade based on return extrapolation.

In the model, the average belief about the risk premium correlates positively with past returns and negatively with the true risk premium. Therefore, an econometrician studying a representative sample from our economy would conclude that the average investor is return extrapolating and has a belief that correlates negatively with the true risk premium. The reason for this is similar to why young agents have a negative correlation between the perceived risk premium and the true risk premium. In our economy, the average belief about the risk premium—a population survey—overweights young agents relative to older more experienced agents who are more important in determining prices. This effect is so strong that the covariance between the average or consensus belief about growth and the market view is higher than the variance of the market view and, consequently, there is a negative correlation between the true risk premium and the consensus risk premium.

Our paper relates to several strands of literature. First, there is an emerging but already influential empirical literature documenting learning from experience, which serves as the main assumption for our model. An early work in this field is Vissing-Jorgensen (2003),

which studies, among other things, whether investors extrapolate their own experience. A seminal paper in the finance literature is Malmendier and Nagel (2011); the paper shows that individuals who have experienced high stock market or bond market returns are more likely to take on further financial risks, i.e., are more likely to participate in the stock market or bond market and allocate a higher proportion of their liquid assets to stocks or bonds. Malmendier and Nagel (2011) also provide empirical evidence pointing directly to the importance of experience for beliefs.³ According to their view, experience effects could be the result of attempts to learn where all available historical data is used but not entirely trusted.⁴ Further, in a follow up paper, Malmendier and Nagel (2016) show that individuals adapt their inflation forecasts to new data but overweight inflation realized during their life-times, that young agents update more aggressively in response to news, and that learning from experience can explain the substantial disagreement in periods of high surprise inflation.⁵

Second, our paper speaks to the literature that studies the relation between risk premia extracted from survey data and statistical measures of risk premia. Greenwood and Shleifer (2014) show that survey based measures of expectations (i) correlate positively with past stock returns, but (ii) correlate negatively with future returns. The positive correlation in (i) suggests that survey respondents extrapolate returns, while the negative correlation in (ii) suggests that when survey respondents expect high returns, then future returns tend to be low. By analyzing global equities, currencies, and global fixed income, Koijen, Schmeling, and Vrugt (2015) highlight how pervasive the evidence of a negative relation between survey based expectations and future returns is. Further, Martin (2016) derives a measure for the equity premium from option prices and shows that it correlates negatively with average risk premia from survey data. Our model with learning from experience based bias proposes an

³Malmendier and Nagel (2011) point to the psychology literature that argues that personal, especially recent, experiences impact decisions to a greater extent than education and statistical summary information in books.

⁴Using age as measure of managers' investment experience, Greenwood and Nagel (2009) show that young managers trend-chase in their technology stock investments, while old managers do not.

⁵There is also a related literature that studies how experiences influence investment decision: Kaustia and Knüpfer (2008), Choi, Laibson, Madrian, and Metrick (2009), Chiang, Hishleifer, Qian, and Sherman (2011), Strahilevitz, Odean, and Barber (2011), and Knüpfer, Rantapuska, and Sarvimäki (2016).

equilibrium channel for this empirical regularity.

Third, our paper relates to the asset pricing literature with heterogeneous agents. We mention Gârleanu and Panageas (2015), who also study a continuous-time overlapping generations economy. Their focus is on the quantitative implications of heterogeneity in recursive preferences. Seminal works in the literature on asset pricing with disagreement include Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), and Basak (2000).⁶ Our model differs from this literature in that we employ a continuum of agents, where young agents endogenously chase trends in returns but over time endogenously become contrarians. Perhaps most importantly, we contribute to the literature by solving a continuous-time overlapping generations model with disagreement. This allows us to address a different set of economic questions than the above mentioned papers. Methodologically this is also quite different. The typical approach in the disagreement literature is to use a central planner with fixed Pareto weights. In contrast, the Pareto weights in our model depend on the state of nature at birth and are determined as a part of the equilibrium. Despite of this complication, our paper presents a closed-form solution. Moreover, it is standard in the disagreement literature to consider infinitely lived agents, which implies non-stationarity, since agents with more accurate beliefs accumulate wealth and eventually dominate the economy as shown in the market selection literature.⁷ In our model with overlapping generations, one cohort cannot dominate the economy because agents are continuously born and die. We close by relating to Barberis, Greenwood, Jin, and Shleifer (2015); they consider a model with exogenously defined return extrapolators and rational agents. In their economy, all return extrapolators perceive the same dynamics and never change type, while in our model there is an endogenous and smooth transition from appearing as a return extrapolator to

⁶Models with disagreement include, among many others, Basak (2005), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), Cvitanic and Malamud (2011), Cvitanic, Jouini, Malamud, and Napp (2012), Bhamra and Uppal (2014), and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2016).

⁷See, for instance, Blume and Easley (1992), Sandroni (2000), and Kogan, Ross, and Westerfield (2006). Although the market selection process can be slow as illustrated by Yan (2008) and Dumas, Kurshev, and Uppal (2009), it can be quite powerful if agents have access to many securities as in Fedyk, Heyerdahl-Larsen, and Walden (2013).

eventually become a contrarian.

Fourth, we discuss several related papers that study the impact of an experience bias on asset prices. Perhaps the first paper in this strand of literature is Schraeder (2016). Her discrete-time and finite-horizon economy with up to eight overlapping generations connects trading volume to volatility, excess volatility, overreaction, and price reversals, among other results. Collin-Dufresne, Johannes, and Lochstoer (2016) consider two dynasties and recursive preferences with a focus on matching asset pricing moments. In their model, due to preference for early resolution of uncertainty, young agents behave as more risk averse than the old. Hence, experience has a dual role as inexperienced agents behave as more risk averse and have less accurate beliefs. Buss, Uppal, and Vilkov (2015) study a general equilibrium model with two risky assets where one of the risky assets is an alternative asset that is opaque and illiquid. They show that inexperienced agents initially tilt their portfolio away from the alternative asset, but eventually increase their position as they accumulate experience. Surprisingly, lower transaction costs for the alternative asset can amplify the initial portfolio tilt, as it is less costly to rebalance towards the alternative asset when the investor becomes more experienced. Recently, Malmendier, Pouzo, and Vanasco (2017) consider a discrete-time overlapping generations model with CARA utility and consumption from terminal wealth. They show that stock price volatility and autocorrelations are higher when more agents rely on recent observations. Moreover, when the disagreement across generations is high, then there is higher trading volume in the stock market. Our model differs in that it has a more general cohort and demographic structure, more general structure for priors, and still allows for closed-form solutions. Further, none of the above mentioned papers studies the negative relation between survey based measures of the risk premium and the true risk premium.

2 The Model

We consider a continuous-time overlapping generations economy in the tradition of Blanchard (1985). Currently living agents die at rate $\nu > 0$; dead agents are replaced by newborn agents at rate ν , so the total population size is constant and normalized to equal 1. The time- t size of the cohort born at time $s < t$ is, therefore, $\nu e^{-\nu(t-s)} ds$. At time t , all living agents receive an endowment of earnings $y_{s,t}$, where $y_{s,t} = \omega Y_t$ for $\omega \in (0, 1)$. In addition, there is a representative firm paying out $D_t = (1 - \omega)Y_t$ in dividends.⁸ Hence, aggregate output is $\int_{-\infty}^t \nu e^{-\nu(t-s)} \omega Y_t ds + D_t = Y_t$ and it follows the process

$$dY_t/Y_t = \mu_Y dt + \sigma_Y dz_t, \quad (1)$$

where z_t is a standard Brownian motion.

2.1 Information, Learning, and Disagreement

To introduce a role for experience, we make the following assumptions regarding information structure, the learning process, and disagreement across agents. Agents know ω and observe aggregate output, but do not know expected output growth μ_Y . An agent born at time s has a normally distributed prior about expected output growth with mean, $\hat{\mu}_{s,s}$, and variance $\hat{V} > 0$. Hence, different cohorts can have different initial beliefs about expected output growth, but share the same prior variance, \hat{V} .

Once born, agents use Bayes' rule to update their beliefs about expected aggregate output growth. By standard filtering theory, the dynamics of the expected output growth, $\hat{\mu}_{s,t}$, as perceived by an agent born at time s at time t , and its posterior variance, $\hat{V}_{s,t}$, are

$$d\hat{\mu}_{s,t} = \frac{\hat{V}_{s,t}}{\sigma_Y} dz_{s,t}, \quad \hat{V}_{s,t} = \frac{\sigma_Y^2 \hat{V}}{\sigma_Y^2 + \hat{V}(t-s)}, \quad (2)$$

⁸In the Internet Appendix, we consider a version of the model without a dividend paying stock, where agents instead trade in a security in zero net supply. This equilibrium corresponds to the limiting case $\omega \rightarrow 1$.

respectively, and where $z_{s,t}$ denotes a Brownian motion under the belief of an agent born at time s with associated probability P^s and information set (or sigma algebra) $\mathcal{F}_{s,t}^Y = \sigma(Y(u), s \leq u \leq t)$. The posterior variance, $\hat{V}_{s,t}$, decreases over time as agents learn about the true output growth.

Agents know σ_Y and since the perceived output dynamics of an agent born at time s is $dY_t/Y_t = \hat{\mu}_{s,t}dt + \sigma_Y dz_{s,t}$, it follows that perceived and true shocks are linked through

$$dz_{s,t} = dz_t - \Delta_{s,t}dt, \quad (3)$$

where $\Delta_{s,t} = \frac{\hat{\mu}_{s,t} - \mu_Y}{\sigma_Y}$ is the standardized estimation error of an agent born at time s . Hence, using Equation (3) the dynamics of the expected output growth of an agent born at time s , under the true probability measure, is

$$d\hat{\mu}_{s,t} = -\frac{\hat{V}_{s,t}}{\sigma_Y} \Delta_{s,t}dt + \frac{\hat{V}_{s,t}}{\sigma_Y} dz_t. \quad (4)$$

From the definition of the standardized estimation error and the solution of the stochastic differential equation in Equation (4), we obtain the following proposition.

Proposition 1. *The estimation error at time t of the cohort born at time s is*

$$\Delta_{s,t} = \frac{\sigma_Y^2}{\sigma_Y^2 + \hat{V}(t-s)} \Delta_{s,s} + \frac{\hat{V}}{\sigma_Y^2 + \hat{V}(t-s)} (z_t - z_s). \quad (5)$$

Moreover, we have that $\Delta_{s,s} = \frac{\hat{\mu}_{s,s} - \mu_Y}{\sigma_Y}$ and $\lim_{t \rightarrow \infty} \Delta_{s,t} = 0$ a.s.

2.2 Security Markets and Prices

Agents can trade in three securities: 1) an instantaneously risk-free asset, 2) units of a share in the representative firm, and 3) annuities to hedge mortality risk. The instantaneously

risk-free asset is in zero net supply and with dynamics given by

$$dB_t/B_t = r_t dt, \quad (6)$$

where r_t denotes the real short rate determined in equilibrium.

We normalize the supply of shares in the representative firm to one and denote its price by S_t . The corresponding return process, R_t , evolves according to

$$dR_t = (dS_t + D_t dt)/S_t = \mu_t^S dt + \sigma_t^S dz_t = \mu_{s,t}^S dt + \sigma_t^S dz_{s,t}, \quad (7)$$

where $\mu_{s,t}^S$ and σ_t^S are determined in equilibrium. Further, agents agree on current prices, but disagree about their probability distribution in the future. Using the relation between the perceived and actual shocks in Equation (3), we have that $\mu_{s,t}^S = \mu_t^S + \sigma_t^S \Delta_{s,t}$.

Annuity contracts, as in Yaari (1965), entitle to an income stream of $\nu W_{s,t}$ per unit of time, where $W_{s,t}$ is the financial wealth at time t of an agent born at time s . In return, the competitive insurance industry receives all financial wealth when the agent dies.

It is convenient to summarize the price system in terms of the stochastic discount factor. Since agents have different beliefs, they have individual stochastic discount factors that differ from the stochastic discount factor under the true probability measure. The stochastic discount factor as perceived by an agent born at time s , $\xi_{s,t}$, and the one under the true probability measure, ξ_t , follow the dynamics

$$d\xi_{s,t}/\xi_{s,t} = -r_t dt - \theta_{s,t} dz_{s,t}, \quad d\xi_t/\xi_t = -r_t dt - \theta_t dz_t. \quad (8)$$

We have that the relation between the market price of risk as perceived by the cohort born at time s , $\theta_{s,t}$, and the market price of risk under the objective probability measure, θ_t , is $\theta_{s,t} = \theta_t + \Delta_{s,t}$. Following Basak (2000), we define the disagreement process, $\eta_{s,t}$, through the relation between the stochastic discount factor under the objective measure and the belief

of an agent born at time s , i.e., $\xi_t = \eta_{s,t}\xi_{s,t}$. Formally, $\eta_{s,t}$ is the Radon Nikodym derivative that allows to move from the probability measure of an agent born at time s to the actual probability measure and vice versa. The dynamics of the disagreement process, $\eta_{s,t}$, is (see Appendix A for details)

$$d\eta_{s,t}/\eta_{s,t} = \Delta_{s,t}dz_t. \quad (9)$$

2.3 Preferences and Individual Optimization

Agents maximize lifetime utility given by

$$E_{s,s} \left[\int_s^\tau e^{-\rho(t-s)} \log(c_{s,t}) dt \right], \quad (10)$$

where τ is the stochastic time of death. In Equation (10), the first time subscript in the expectation operator denotes the probability measure of the expectation. We use the convention that expectation operators with one time subscript denotes the objective probability measure. Since the random time of death, τ , is independent of aggregate output and exponentially distributed, we integrate it out to write the expected lifetime utility as

$$E_{s,s} \left[\int_s^\infty e^{-(\rho+\nu)(t-s)} \log(c_{s,t}) dt \right]. \quad (11)$$

The dynamics of financial wealth, $W_{s,t}$, of an agent born at time s who is entitled to the earnings, ωY_t , follows

$$dW_{s,t} = (r_t W_{s,t} + \pi_{s,t} (\mu_{s,t}^S - r_t) + \nu W_{s,t} + \omega Y_t - c_{s,t}) dt + \pi_{s,t} \sigma_t^S dz_{s,t}, \quad W_{s,s} = 0, \quad (12)$$

where $\pi_{s,t}$ denotes the dollar amount held in the risky asset. Since agents are born without any financial wealth, we have that $W_{s,s} = 0$.

All agents maximize expected utility from lifetime consumption, Equation (11), subject to the wealth dynamics in Equation (12).

2.4 Equilibrium

We start by defining an equilibrium.

Definition 1. *Given preferences, endowments, and beliefs, an equilibrium is a collection of allocations $(c_{s,t}, \pi_{s,t})$ and a price system $(r_t, \mu_{s,t}^S, \sigma_t^S)$ such that the processes $(c_{s,t}, \pi_{s,t})$ maximize utility given in Equation (11) subject to the dynamic budget constraint given in Equation (12) and markets clear:*

$$\int_{-\infty}^t \nu e^{-\nu(t-s)} c_{s,t} ds = Y_t, \quad (13)$$

$$\int_{-\infty}^t \nu e^{-\nu(t-s)} \pi_{s,t} ds = S_t, \quad (14)$$

$$\int_{-\infty}^t \nu e^{-\nu(t-s)} (W_{s,t} - \pi_{s,t}) ds = 0. \quad (15)$$

After birth, the market is dynamically complete for each cohort.⁹ Thus, we solve the individual optimization problems by martingale methods as in Cox and Huang (1989). Consider an agent born at time s . The static optimization problem for this agent is

$$\max_{c_s} E_{s,s} \left[\int_s^\infty e^{-(\rho+\nu)(t-s)} \log(c_{s,t}) dt \right]$$

s.t.

$$E_{s,s} \left[\int_s^\infty e^{-\nu(t-s)} \xi_{s,t} c_{s,t} dt \right] = E_{s,s} \left[\int_s^\infty e^{-\nu(t-s)} \xi_{s,t} \omega Y_t dt \right]. \quad (16)$$

From the first order conditions (FOCs), we have

$$\frac{e^{-(\rho+\nu)(t-s)}}{c_{s,t}} = \kappa_s e^{-\nu(t-s)} \xi_{s,t}, \quad (17)$$

where κ_s denotes the Lagrange multiplier of the static budget constraint given in Equation

⁹We verify that the equilibrium risky security is spanning the output risk, i.e., $\sigma_t^S > 0$ for all times and states.

(16). For $s \leq u \leq t$ the FOCs imply

$$e^{-(\rho+\nu)(t-u)} \left(\frac{c_{s,u}}{c_{s,t}} \right) = e^{-\nu(t-u)} \frac{\xi_{s,t}}{\xi_{s,u}}. \quad (18)$$

Using the FOCs in Equation (18), we obtain the following proposition.

Proposition 2. *Optimal consumption at time t of agents born at time $s \leq t \leq \tau$ is*

$$c_{s,t} = c_{s,s} e^{-\rho(t-s)} \left(\frac{\eta_{s,t}}{\eta_{s,s}} \right) \left(\frac{\xi_s}{\xi_t} \right). \quad (19)$$

The optimal consumption in Proposition 2 contains the unknown initial consumption, $c_{s,s}$. What we see already is that the initial consumption relates inversely to the Lagrange multiplier, κ_s , which can be interpreted as a state dependent Pareto weight. The reason why it is stochastic is that agents cannot hedge against output fluctuations prior to birth.

Remark 1. *Interpreting the initial consumption as the inverse of the stochastic Pareto weight highlights a key difference in our model relative to the literature on disagreement discussed in the introduction, where the approach is often to specify the Pareto weights exogenously, then solve for the corresponding wealth allocations. This approach is not possible in our model as the economy imposes a specific relation between the stochastic aggregate output and the wealth of a newborn.*

The distribution of consumption or wealth is a state variable in models with heterogeneous agents and, therefore, we define the consumption and wealth share of each cohort as follows:

Definition 2.

1. *The consumption share of the cohort born at time $s < t$ is $f_{s,t}^c = \frac{\nu e^{-\nu(t-s)} c_{s,t}}{Y_t}$.*

2. *The wealth share of the cohort born at time $s < t$ is $f_{s,t}^W = \frac{\nu e^{-\nu(t-s)} \hat{W}_{s,t}}{\hat{W}_t}$,*

where \hat{W}_t denotes aggregate wealth in the economy.

Remark 2. *A property of models featuring infinitely lived heterogeneous agents is that there is a market selection mechanism through which one type of the agents dominates the economy in the long run. More specifically, with heterogeneous beliefs, the agents with beliefs closer to the correct estimate drive investors with less precise beliefs out of the markets. In our overlapping generations economy, all agents vanish in the long run since they die. Still, the market selection mechanism is at work because young agents with less precise beliefs lose out to older and more experienced agents on average.*

Due to log utility, the consumption and wealth shares are equal. To see this, consider the following. At time $u \geq s$, the value of the endowment of earnings of an agent born at time s is $H_{s,u} = \frac{1}{\xi_{s,u}} E_{s,u} [\int_u^\infty e^{-\nu(t-s)} \xi_{s,t} \omega Y_t dt]$. Total wealth is financial wealth, $W_{s,u}$, plus the present value of all future earnings, that is, $\hat{W}_{s,u} = H_{s,u} + W_{s,u}$. Using the static budget constraint, we obtain

$$c_{s,u} = (\rho + \nu) \hat{W}_{s,u}. \quad (20)$$

Equation (20) confirms that the well known result of a constant consumption-wealth ratio in a log utility setting also holds in our overlapping generations model with incomplete information and disagreement. Using the market clearing conditions and Equation (20), we have

$$Y_t = \int_{-\infty}^t \nu e^{-\nu(t-s)} c_{s,t} ds = \int_{-\infty}^t \nu e^{-\nu(t-s)} (\rho + \nu) \hat{W}_{s,t} ds = (\rho + \nu) \hat{W}_t, \quad (21)$$

and, consequently, aggregate wealth, \hat{W}_t , is given by

$$\hat{W}_t = \frac{Y_t}{\rho + \nu}. \quad (22)$$

It follows from Equation (20), Equation (22), and Definition 2 that the consumption and wealth shares equate and, thus, we use $f_{s,t} = f_{s,t}^W$ to denote the wealth share. Since the wealth share appears in several equilibrium quantities, we define two moments based on it.

Definition 3. *Define the infinite sequence $x_t = (x_{s,t})_{s < t}$ and the following operators:*

1. The wealth weighted average is $\mathcal{E}(x_t) = \int_{-\infty}^t f_{s,t} x_{s,t} ds$.

2. The wealth weighted variance is $\mathcal{V}(x_t) = \int_{-\infty}^t f_{s,t} (x_{s,t} - \mathcal{E}(x_t))^2 ds$,

where $x_{s,t}$ is any time t quantity associated with the cohort born at time s .

Using the market clearing in Equation (13) and the optimal consumption in Proposition 2, we solve for the stochastic discount factor as a function of the disagreement processes, $\eta_{s,t}$, aggregate output, Y_t , and initial consumption, $c_{s,s}$. While the disagreement processes are determined by the learning and aggregate output is exogenous, the initial consumption shares have to be computed as a part of the equilibrium. To do so, we conjecture that the initial consumption share of a newborn is constant across time and states, then we verify this by solving for the value of the aggregate endowment of earnings and the stock market. Following this approach, the next proposition characterizes the stochastic discount factor.

Proposition 3. *In equilibrium, the stochastic discount factor is*

$$\xi_t = \bar{\eta}_t \frac{e^{-(\rho+\nu(1-\beta))t}}{Y_t}, \quad (23)$$

where $\bar{\eta}_t$ solves the integral equation

$$\bar{\eta}_t = \int_{-\infty}^t \beta \nu e^{-\beta \nu(t-s)} \bar{\eta}_s \frac{\eta_{s,t}}{\eta_{s,s}} ds, \quad (24)$$

where $\beta = \frac{\rho+2\nu}{2\nu} - \frac{\sqrt{\rho^2+4(\rho+\nu)\nu(1-\omega)}}{2\nu}$ represents the fraction of total output consumed by a newborn agent, i.e., $c_{t,t} = \beta Y_t$. Moreover, $\bar{\eta}_t$ is a local martingale with dynamics

$$d\bar{\eta}_t = \bar{\Delta}_t \bar{\eta}_t dz_t, \quad (25)$$

where $\bar{\Delta}_t = \mathcal{E}(\Delta_t)$ denotes the wealth weighted average standardized estimation error in the economy and where

$$f_{s,t} = \beta \nu e^{-\beta \nu(t-s)} \frac{\eta_{s,t}/\eta_{s,s}}{\bar{\eta}_t/\bar{\eta}_s}, \quad (26)$$

is the equilibrium wealth share.

It is useful to decompose the stochastic discount factor:

$$\xi_t = \underbrace{\frac{e^{-\rho t}}{Y_t}}_{\text{Log utility discount factor}} \times \underbrace{e^{-\nu(1-\beta)t}}_{\text{OLG effect}} \times \underbrace{\bar{\eta}_t}_{\text{Experience effect}}. \quad (27)$$

Equation (27) shows that the stochastic discount factor has three parts: 1) a discount factor that prevails in an economy with an infinitely lived representative agent with log utility and complete information, 2) an effect coming from the overlapping generations structure, and 3) an effect from disagreement about expected output growth across cohorts. While the first part is straightforward, it is worth discussing the second and the third parts.

The overlapping generations structure affects the stochastic discount factor through a generational replacement effect. Agents are born without financial wealth and, hence, they have a lower consumption than the population average. The agents invest to buy units of the share of the representative firm and by doing so they have an expected consumption growth that is higher than the expected growth in aggregate output. This effect holds even in a setting with complete information.

The third part, $\bar{\eta}_t$, is due to learning from experience. As pointed out in Proposition 3, $\bar{\eta}_t$ is a local martingale and has the properties of a Radon Nikodym derivative. We can interpret the stochastic discount factor as the discount factor of a hypothetical representative agent with a belief given by the wealth weighted average belief, $\bar{\mu}_t = \mathcal{E}(\hat{\mu}_t)$, which we call the market view. The market view captures the fact that agents with a larger wealth share are more important in determining the price and, therefore, their belief carries a larger weight in the belief of this “representative agent.” Using this belief, $\bar{\Delta}_t$ in Proposition 3 then measures the standardized estimation error of the market view, i.e., $\bar{\Delta}_t = \frac{\bar{\mu}_t - \mu_Y}{\sigma_Y}$.

Proposition 3 also characterizes the wealth shares of the different cohorts, $f_{s,t}$. The first part, $\beta\nu e^{-\beta\nu(t-s)}$, represents the wealth share in an overlapping generations economy without learning from experience. In such an economy, the wealth share declines at a rate of $\beta\nu$,

which reflects the mortality risk and that young agents are born without financial wealth and, thus, save to accumulate it over time. Given that $\beta < 1$, the rate of decay of the cohort wealth share is slower than that due to mortality. The second part, $\frac{\eta_{s,t}/\eta_{s,s}}{\bar{\eta}_t/\bar{\eta}_s}$, captures the likelihood of observing a particular value of output at time t as a realization under the belief of an agent born at time s relative to that of the market view. Consequently, the wealth share is increasing whenever the data is more supportive of the belief of an agent born at time s relative to the market view. Given the stochastic discount factor in Proposition 3, we can apply Ito's lemma and match the drift and diffusion coefficients with the dynamics in Equation (8) to solve for the real short rate and the market price of risk.

Proposition 4. *In equilibrium, the real short rate is*

$$r_t = \rho + \bar{\mu}_t + \nu(1 - \beta) - \sigma_Y^2, \quad (28)$$

and the market price of risk is

$$\theta_t = \sigma_Y - \frac{1}{\sigma_Y} (\bar{\mu}_t - \mu_Y). \quad (29)$$

The expression for the real short rate deviates from the one in an economy with an infinitely lived log utility agent by two terms. First, in a log utility economy, the intertemporal smoothing motive depends on the expected output growth. A higher growth implies a higher interest rate as the demand for borrowing increases. However, in Equation (28) the interest rate does not depend on the true expected output growth. Instead, it depends on the market view. The reason for this is that wealthier agents are more important in determining prices and, therefore, the real short rate reflects the view of the wealthier agents more than the view of the poorer agents. Second, due to the generational replacement effect the real short rate is higher in the overlapping generations economy than in a representative agent economy.

We see that in addition to the standard compensation for output risk, the market price of risk is taking into account that agents in the economy might have a belief about output

growth that differs from the true value. Specifically, we see that when the market is optimistic about expected growth, i.e., $\bar{\mu}_t > \mu_Y$, then the market price of risk is low. Indeed, under the true probability measure, the risky asset is expensive and, thus, the market price of risk must be low.

Remark 3. *An alternative way of expressing the real short rate and the market price of risk is to explicitly write them in terms of the market's estimation error, $\bar{\Delta}_t$,*

$$r_t = \underbrace{\rho + \mu_Y - \sigma_Y^2}_{\text{Log utility economy}} + \underbrace{\nu(1 - \beta)}_{\text{OLG}} + \underbrace{\sigma_Y \bar{\Delta}_t}_{\text{Experience effect}}, \quad (30)$$

$$\theta_t = \underbrace{\sigma_Y}_{\text{Log utility economy}} - \underbrace{\bar{\Delta}_t}_{\text{Experience effect}}. \quad (31)$$

From the point of view of an econometrician with the correct estimate of the expected output growth, the real short rate is distorted by the market's estimation error. Hence, when the market is optimistic about output growth, $\bar{\Delta}_t > 0$, then the econometrician perceives the real short rate as too high and the compensation for risk too low.

The next proposition characterizes the equilibrium excess return and the volatility of the stock market.

Proposition 5. *In equilibrium, the expected excess return on the stock market is*

$$\mu_t^S - r_t = \sigma_t^S \left(\sigma_Y - \frac{1}{\sigma_Y} (\bar{\mu}_t - \mu_Y) \right), \quad (32)$$

and the volatility of the stock market is

$$\sigma_t^S = \sigma_Y. \quad (33)$$

From Equation (33), we see that the volatility of the stock market is not affected by learning from experience and is identical to that of an economy with complete information, which is due to log preferences.

The next proposition characterizes the optimal portfolio policy of an agent born at time s .

Proposition 6. *The optimal dollar amount invested in the risky asset, $\pi_{s,t}$, for an agent born at time s is*

$$\pi_{s,t} = W_{s,t} + \frac{\hat{\mu}_{s,t} - \bar{\mu}_t}{\sigma_Y^2} \hat{W}_{s,t}. \quad (34)$$

To understand the optimal portfolio choice in Proposition 6, first consider the case with homogeneous beliefs which corresponds to $\hat{\mu}_{s,t} = \bar{\mu}_t$ for all $s \leq t$. In this case, the second term in Equation (6) vanishes and the optimal choice is simply to invest the entire financial wealth in the risky asset. This is intuitive as there is no form of heterogeneity in beliefs or preferences and, therefore, in equilibrium the optimal choice must be to hold the market portfolio. Now consider the case in which $\hat{\mu}_{s,t} > \bar{\mu}_t$, i.e., the agent is more optimistic about expected output growth than the market view. The optimal choice is to deviate from the market portfolio by investing more in the risky asset. The amount is determined by the “excess risk premium,” $\hat{\mu}_{s,t} - \bar{\mu}_t$, perceived by the agent born at time s . Given log preferences, the optimal choice is to increase total exposure of wealth by the excess risk premium scaled by the variance of the market.

3 Dynamic Properties of the Model

In this section, we examine the dynamics of the equilibrium in Section 2. First, we study the properties of the equilibrium stock market risk premium. Second, we show how agents perceive the risk premium on the stock market and how this translates into differing optimal portfolios. Third, given the heterogeneity in beliefs about the risk premium, we examine how the average belief (instead of the market view), frequently used in the empirical literature, relates to the true risk premium.

3.1 Parameters

The model has seven parameters $(\rho, \nu, \omega, \mu_Y, \sigma_Y, \hat{\mu}_{s,s}, \hat{V})$. We follow Gârleanu and Panageas (2015) and set the time discount factor, ρ , at 0.1%, the birth and death intensity, ν , at 2%, and the share of earnings in output, ω , at 0.92 to match the fraction of capital income in national income. The drift, μ_Y , and volatility, σ_Y , of aggregate output are set to 2% and 3.3%, respectively, which is similar to Gârleanu and Panageas (2015) and to the long sample in Campbell and Cochrane (1999). The parameters determining the learning from experience based bias are the prior belief, $\hat{\mu}_{s,s}$, and the prior variance, \hat{V} . To put discipline on how we set these two parameters, we assume for the prior belief, $\hat{\mu}_{s,s}$, and the prior variance, \hat{V} , of an agent “born” at time s that the agent observes the output during the first part of the life before starting to trade at age of 20. Further, the initial prior at time $s - 20$, i.e., at the actual birth of the agent, is diffuse.¹⁰ Consequently, the prior variance at the point when the agent enters the market is $\hat{V} = \frac{\sigma_Y^2}{20} = \frac{0.033^2}{20}$. According to Equation (2), a newborn updates her belief about expected growth by $\frac{\hat{V}}{\sigma_Y} = \frac{0.033}{20} = 0.165\%$ of the shock, which corresponds to 5% of the volatility of aggregate output. For comparison, according to the estimate in Malmendier and Nagel (2016) the response to inflation shocks of a 20 year old’s belief about expected inflation is 4% of the volatility of inflation. The prior belief about expected output growth depends on the realizations of the shocks to output over the first 20 years and is given by $\hat{\mu}_{s,s} = \mu_Y + \sigma_Y \frac{z_s - z_{s-20}}{20}$. Hence, on average agents start with the correct belief and the 95% confidence interval on the initial belief is (0.0055, 0.0345). With this specification of the prior beliefs, the cross-sectional standard deviation of beliefs (disagreement) in the economy is 26 basis points. In comparison, the cross-sectional standard deviation about real GDP growth using the Survey of Professional Forecasters over the period Q1 1992 to Q4

¹⁰One alternative to using 20 years as a pre-trading period to learn is to use $\hat{\mu}_{s,s}$ and \hat{V} as free parameters. A natural choice for $\hat{\mu}_{s,s}$ is the correct value, i.e., assuming that the agent is born with an unbiased prior. One can think of this as the agent being told what the correct value is, but she does not fully trust it. The drawback of it is that at birth all newborn agents are more correct than someone who has been trading for a while. Another alternative is to use a distribution for the priors. The Internet Appendix contains one example of both alternatives.

2016 is 48 basis points. The value of the birth and death intensity, ν , implies an average life of 50 years and including the pre-period gives an effective average age of 70 years. Although we set the initial belief by using 20 years of data, below we still refer to an agent as newborn when entering the market to trade and all references to age are from the time when the agent starts trading.¹¹

3.2 The Dynamics of the Risk Premium and the Real Short Rate

Proposition 4 shows that the real short rate and the market price of risk depend on the market view, $\bar{\mu}_t$. Hence, to understand their dynamics it is important to examine the properties of the market view.

Proposition 7. *The dynamics of the market view, $\bar{\mu}_t$, is*

$$d\bar{\mu}_t = \beta\nu (\hat{\mu}_{t,t} - \bar{\mu}_t) dt - \frac{\bar{V}_t}{\sigma_Y} \bar{\Delta}_t dt + \frac{\bar{V}_t}{\sigma_Y} dz_t. \quad (35)$$

where $\bar{V}_t = \mathcal{E}(\hat{V}_t) + \mathcal{V}(\hat{\mu}_t) > 0$,

Proposition 7 shows that the diffusion depends on both the wealth weighted average posterior variance $\mathcal{E}(\hat{V}_t)$ and the wealth weighted variance of the beliefs about output growth $\mathcal{V}(\hat{\mu}_t)$. Comparing the first term, $\frac{\mathcal{E}(\hat{V}_t)}{\sigma_Y}$, to the diffusion of the individual agents' beliefs in Equation (4), we see that this captures the wealth weighted average diffusion coefficient in the individual agents' belief.

The second term, $\frac{\mathcal{V}(\hat{\mu}_t)}{\sigma_Y}$, is the cross-sectional variance of beliefs about the output growth in the economy scaled by the output volatility, σ_Y . For the same reason as for the first term, the variance is calculated using the wealth distribution. Both $\mathcal{E}(\hat{V}_t)$ and $\mathcal{V}(\hat{\mu}_t)$ are positive. Hence, in response to an output shock the market view increases. This is intuitive as all agents in the economy revise their expectations upwards after a positive shock and

¹¹Given the parameters, the unconditional values of the real short rate and the risk premium are 2.52% and 0.11%, respectively. These values correspond to the one of an equivalent economy without learning from experience. Hence, the model cannot speak to the equity premium and the interest rate puzzles.

the market view is simply the wealth weighted average belief in the economy. However, as $\mathcal{V}(\hat{\mu}_t)$ is positive, the market view reacts more than just the wealth weighted average update by the agents in the economy, i.e., there is an overreaction. To understand the overreaction, consider a positive shock to output. In this case, all agents revise their expectation upwards and, therefore, the market view becomes relatively more optimistic, which is the first effect captured by $\mathcal{E}(\hat{V}_t)$. In addition, after a positive shock to output, relative more optimistic agents accumulate wealth, which in turn increases the weight on their belief in the market view. The second effect is due to the trading among the agents based on their beliefs, and the trading is more aggressive when the disagreement is high. Consequently, when disagreement is high in the economy and wealth is relatively evenly distributed, then the market view reacts stronger to shocks to output than when the disagreement is low and wealth is concentrated among few agents with relatively similar beliefs.

Now turning to the drift of the market view, we see that it also contains two terms. The first term, $\beta\nu(\hat{\mu}_{t,t} - \bar{\mu}_t)$, is due to the overlapping generations structure. The aggregate wealth share of the newborn is $\beta\nu$ and the term captures that newborn agents, in general, have an initial belief that differs from the market view. The second term, $-\frac{\bar{V}_t}{\sigma_Y}\bar{\Delta}_t$, mean-reverts with speed of mean reversion given by $\frac{\bar{V}_t}{\sigma_Y}$. Hence, the market view has the same ratio between the speed of mean reversion and the diffusion coefficient as the belief of the individual agents. As expected, the speed of mean reversion of the market view depends on the wealth weighted average posterior variance. However, it also depends on the wealth weighted variance of the beliefs, $\mathcal{V}(\hat{\mu}_t)$, since the market also “learns through market selection.” Specifically, consider a shock that increases the wealth weighted variance of the beliefs. In this case, relative to before the shock, the economy has more room for speculative trade.¹² Hence, the trading based on beliefs is more aggressive and individual agents have large exposure to output shocks. Importantly, as agents with particular high or low belief about output growth are expected to lose on average and the market selection is stronger

¹²The wealth weighted variance of beliefs can increase because the disagreement of individual agents is higher than before the shock or because wealth is less concentrated among agents with similar beliefs.

the higher the exposure, the market view is pushed towards the true expected growth at a higher rate.

As the discussion above shows, the updating of the market view has similarities with how individual agents update their beliefs, but it also has differences. One way to think about the market view is that it is the belief of a “representative agent” that prices the market. However, the dynamics of the belief of this representative agent is different from that of any of the individual agents in the economy. Specifically, from this hypothetical representative agent’s point of view, the wealth weighted cross-sectional distribution of beliefs acts as uncertainty about expected output growth, because shocks to output not only move individual agents’ beliefs, but also the wealth distribution and, therefore, the “representative agent” puts more weight on agents with beliefs that are more consistent with the direction of the shock. Hence, in the eyes of the “representative agent” the changes to the wealth distribution act as preference shocks correlated with output shocks.

So how much is the market view reacting to news? Using the above parameter values, the average value of the diffusion coefficient, $\frac{\mathcal{E}(\hat{V}_t)}{\sigma_Y} + \frac{\mathcal{V}(\hat{\mu}_t)}{\sigma_Y}$, is 7.5 basis points, which is 45% of that of a newborn, which is 16.5 basis points. The unconditional standard deviation is 35 basis points. Further, $\frac{\mathcal{E}(\hat{V}_t)}{\sigma_Y}$ and $\frac{\mathcal{V}(\hat{\mu}_t)}{\sigma_Y}$ in the diffusion coefficient of the market view are both important as they account for 74% and 26%, respectively. Therefore, more than a quarter of the response of the market view to a shock is due to wealth reallocations.

Combining the dynamics of the market view with the expressions for the real short rate and market price of risk, we have the following proposition.

Proposition 8. *After a positive shock to aggregate output ($dz_t > 0$), the risk-free rate increases, i.e., $\frac{\partial r_t}{\partial z} > 0$, and the market price of risk and the risk premium on the stock market decrease, i.e., $\frac{\partial \theta_t}{\partial z} < 0$ and $\frac{\partial(\mu_t^S - r_t)}{\partial z} < 0$.*

The intuition for Proposition 8 is the following. Since the market view increases after a positive shock, the real short rate increases as the relevant expectation of the aggregate output growth is that of the market view, and a higher market view implies a higher in-

tertemporal smoothing motive. Also, from the point of view of an econometrician with perfect knowledge of the true parameter the stock price looks “too high,” or put differently the risk premium has decreased relative to before the shock.

Remark 4. *Consumption based asset pricing models in which the risk premium declines in response to a positive shock are sometimes interpreted to be consistent with the empirical evidence in Fama and French (1989) that the risk premium on the stock market is counter-cyclical, although there is no cycle in these models in the conventional sense as shocks are permanent. According to Proposition 8, our model produces a joint decline in the market price of risk and the risk premium on the stock relative to a positive output shock and an increase in the real short rate relative to a positive output shock. The decrease in the risk premium after a positive shock is qualitatively comparable, for example, to Campbell and Cochrane (1999).*

3.3 Perceived Risk Premium

The objective probability measure will, in general, be different from the probability measure of individual agents. Specifically, from Proposition 5 and the relation between the perceived and the true shock, the risk premium on the stock market as perceived by an agent born at time $s < t$ is

$$\mu_{s,t}^S - r_t = \overbrace{\sigma_Y^2 - \bar{\mu}_t + \mu_Y}^{\text{Risk premium under the true measure}} + \overbrace{\hat{\mu}_{s,t} - \mu_Y}^{\text{Experience based bias}}. \quad (36)$$

Simplifying Equation (36), the perceived risk premium can be written as $\mu_{s,t}^S - r_t = \sigma_Y^2 + \hat{\mu}_{s,t} - \bar{\mu}_t$ and, consequently, it is higher than the true risk premium whenever $\hat{\mu}_{s,t} - \bar{\mu}_t > 0$.

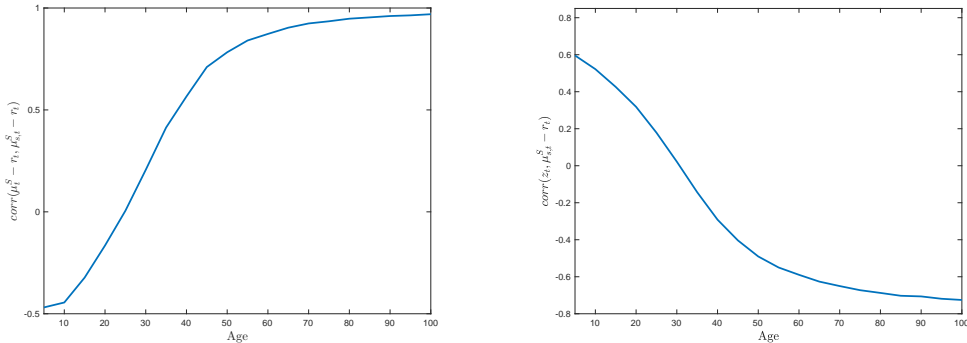
We know from Proposition 8 that the true risk premium decreases after a positive shock. But how does the belief about the risk premium of an agent born at time s react to a shock?

To examine this, consider the covariance between the true risk premium and the perceived risk premium

$$\text{cov}(\mu_{s,t}^S - r_t, \mu_t^S - r_t) = \text{var}(\bar{\mu}_t) - \text{cov}(\bar{\mu}_t, \hat{\mu}_{s,t}). \quad (37)$$

From Equation (37), we see that the variance of the market view always pushes the covariance between the true risk premium and the perceived risk premium towards the positive region. However, as the perceived output growth, $\hat{\mu}_{s,t}$, and the market view both increase after a positive shock, the covariance between the belief about the output growth of the agent born at time s and the market view is positive. Hence, it contributes towards pushing the covariance of the true and the perceived risk premium towards the negative region. If the covariance between the two is sufficiently high, then this outweighs the variance term and, consequently, the correlation between the perceived and the true risk premium can be negative.

Figure 1: *True and Perceived Risk Premium.* The figure shows the correlation between the risk premium under the true measure and the perceived risk premium (left plot) and the correlation between the perceived risk premium and stock market shocks (right plot) by cohort lifespan. Each observation is calculated using a window of 60 non-overlapping observations (5 years). The figure is averaged from 10,000 simulations with 1200 periods or 100 years per simulation.



The left plot in Figure 1 shows that the average correlation between the true and the perceived risk premium as a function of age is strongly negative with a value of -0.45 over the first 5 years of trading, from where it increases monotonically in age. The reason for this is that young agents update very aggressively and, therefore, the variance of their belief about output growth is high and, consequently, the covariance term in Equation (37) outweighs the variance term. As an agent gains more experience, the covariance term becomes less important and the correlation between the true and the perceived risk premium increases, reaching one in the limit. We see that the correlation in the left plot in Figure 1 is not

symmetric around zero, which is because of the positive contribution from the variance of the market view.

The right plot in Figure 1 shows that the beliefs of young agents are positively correlated with shocks to the stock market. As they gain experience, the correlation declines and becomes strongly negative when old. Therefore, young agents form expectations that mimics return extrapolation. Specifically, consider a positive shock to output which also corresponds to a positive shock to the stock market. In this case, young agents revise their beliefs about expected output growth upwards by more than the market does and, hence, from their point of view the stock is now relatively “cheap.” In other words, the risk premium must be high to justify the stock price from the point of view of young agents. Hence, the young keep raising their beliefs about future returns when experiencing positive shocks to the stock market. To formally link our model with return extrapolation, we express the belief about output growth of an agent born at time s as a function of past stock returns.

Proposition 9. *The belief about output growth at time t of an agent born at time s is*

$$\hat{\mu}_{s,t} = -(\rho + \nu(1 - \beta)) + RE_{s,t}, \quad (38)$$

where $RE_{s,t} = \frac{1}{20+t-s} \int_{s-20}^t dR_u$ is the average return experienced over the period $s - 20$ to t .

Therefore, the belief about output growth depends on the observed history of stock returns through $RE_{s,t}$, which we refer to as the return extrapolation term. From the extrapolation term, we see that more experienced agents observe a longer history of stock returns and, thus, the average return over their lifetime is closer to the true population value. Given Equation (38), at time t the perceived risk premium of an agent born at time s is

$$\mu_{s,t}^S - r_t = \overbrace{\sigma_Y^2 - \mathcal{E}(RE_t)}^{\text{True risk premium}} + RE_{s,t}. \quad (39)$$

Hence, an agent who has experienced a better history of market returns than the wealth

weighted population average perceives a higher risk premium. Moreover, agents for whom $RE_{s,t}$ increases more than the wealth weighted average, $\mathcal{E}(RE_t)$, after positive returns appear as if they extrapolate from past returns.

Remark 5. *Barberis, Greenwood, Jin, and Shleifer (2015) consider a model with two types of agents, namely return extrapolators and rational agents. Extrapolators believe that expected stock price changes are linearly increasing in an index given by $I_t = b \int_{-\infty}^t e^{-b(t-u)} dS_{u-dt}$ for $b > 0$, where dS_t is the instantaneous change in the stock price. They show that in equilibrium the true expected stock price change is negatively related to the index. From Equation (39) in our model, we see that the true expected risk premium relates negatively to the extrapolative term $RE_{s,t}$ and that young agents for whom $RE_{s,t}$ reacts strongly to past returns appear as if they extrapolate returns. However, in our economy a fraction of agents behaves as return extrapolators endogenously and there is a smooth transition from appearing as a return extrapolator when young to eventually become a contrarian when old, while in Barberis, Greenwood, Jin, and Shleifer (2015) all return extrapolators perceive the same dynamics and never change type.*

3.4 Consumption and Portfolio Choice

In this subsection, we study the consumption and portfolio choice of individual agents. Within our model, the only reason to trade is differences in beliefs generated by learning from experiences. The simplicity allows for a transparent analysis of the portfolio choice within our model, with the caveat that we do not model features like incomplete markets and life-cycle profiles of earnings, which are important determinants of consumers' portfolio choice.¹³

Our results regarding the perceived risk premium provide a direct view at optimal portfolio allocations. A positive shock increases young agents' expectations about the future stock

¹³We solve a model with a life-cycle profile of earnings as in Gârleanu and Panageas (2015) in the Internet Appendix.

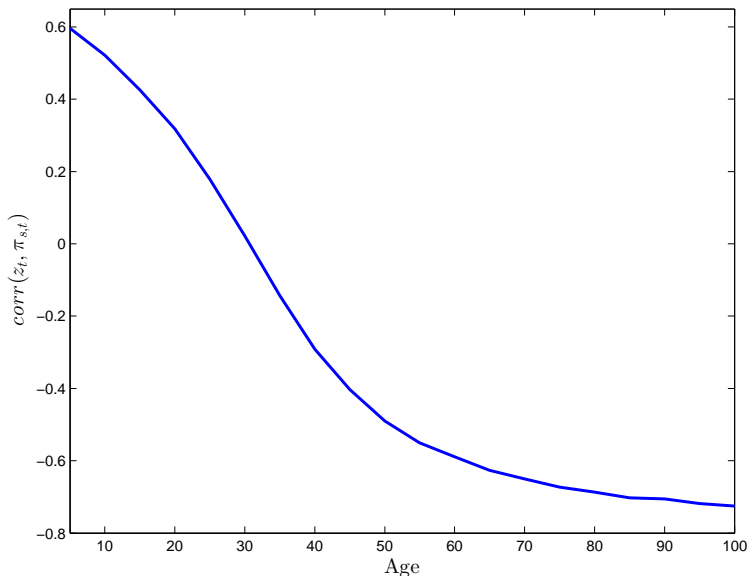
market return. In turn, their demand for the risky asset increases. Old agents reduce their expectations about the future excess returns relative to the young and, therefore, they reduce their portfolio holdings in the risky asset. Figure 2, which shows the correlation between portfolio allocations and shocks by cohort age, confirms this intuition. From the figure, we see that young agents increase their position in the stock market after a positive shock, but that the correlation declines monotonically over time, reaching a strong negative correlation in ripe old age. The decline in the correlation as an agent ages can be understood from the general equilibrium properties of the model. For the market to clear, old agents counter-balance the portfolio allocations of the young. From Proposition 6, we see that the optimal portfolio allocation is driven by the difference between agents' belief about output growth and the market view, $\hat{\mu}_{s,t} - \bar{\mu}_t$. Following a positive shock, $dz_t > 0$, the young revise their expectations about aggregate output growth more than the revision in the market view, since $Var_t(d\hat{\mu}_{s,t}) \geq Var_t(d\bar{\mu}_t)$. Hence, the young increase their allocation in the risky asset. Old agents revise their expectations less than the market view, since $Var_t(d\hat{\mu}_{s,t}) \leq Var_t(d\bar{\mu}_t)$. Therefore, they counter-balance the behavior of the young by reducing their demand for the risky asset, thereby the market clears.

The discussion above implies that in equilibrium there is an entire cross-section of extrapolators and contrarians and that there is an endogenous and smooth transition from appearing as a return extrapolator to eventually become a contrarian. As the true risk premium is decreasing after positive returns, young agents are on average buying at the “wrong” time. So, what are the financial consequences of buying at the wrong time? To answer this question, we characterize in Proposition 10 the dynamics of agents' log consumption.

Proposition 10. *The dynamics of the log consumption, $\log(c_{s,t})$, for an agent born at time s with $t < \tau$ is*

$$d\log(c_{s,t}) = \left(\mu_Y + \nu(1 - \beta) - \frac{1}{2}\sigma_Y^2 + \frac{1}{2}(\bar{\Delta}_t^2 - \Delta_{s,t}^2) \right) dt + (\sigma_Y + \Delta_{s,t} - \bar{\Delta}_t) dz_t. \quad (40)$$

Figure 2: *Portfolios and Shocks*. The figure plots the correlation between portfolio allocations and stock market shocks by cohort lifespan. Each observation is calculated using a window of 60 non-overlapping observations (5 years). The figure is averaged from 10,000 simulations with 1200 periods or 100 years per simulation.

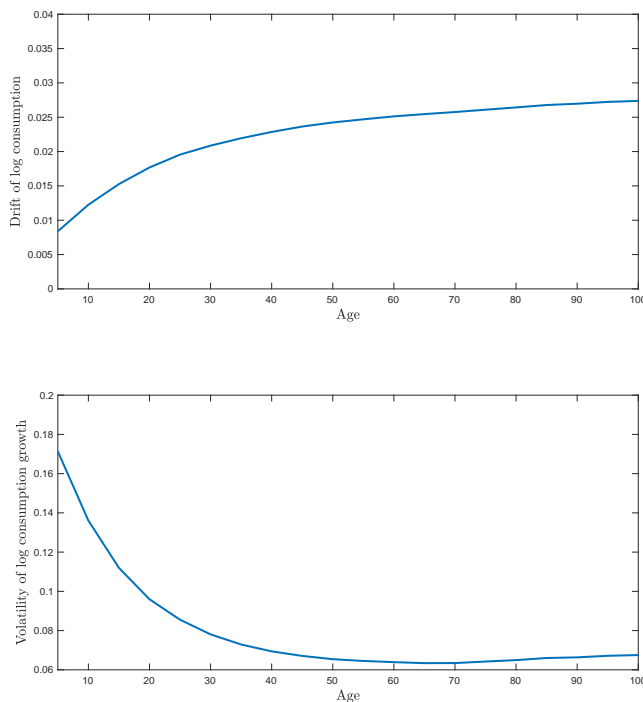


The bottom plot in Figure 3 shows the volatility of log consumption growth. Here we see that the volatility of individual agents' consumption is substantially higher than the aggregate consumption volatility, but more so for the very young agents. The reason for this is that the agents trade on their beliefs and this amplifies the volatility of consumption of each individual relative to the aggregate output volatility due to movements in wealth shares. Further, the volatility does not decrease monotonically. This is because at a certain age, an agent's belief looks very much like the wealth weighted average, which is the belief that determines prices. Hence, for this age, the speculative component is low. However, for both younger and older agents for which the market view dynamics differ substantially from their own belief, the speculative trade is large.

The top plot in Figure 3 shows the expected value of the drift of log consumption as a function of age. One can see that young individuals have much lower expected log consumption growth, because they are making larger mistakes than older agents with more experience. Proposition 10 captures this effect by the difference between the squared estimation error of the market view and the squared estimation error of the individual agent. As

long as the expected value of the squared estimation error is larger than that of the market, the agent is expected to lose out and, hence, has a lower expected log consumption growth than in a corresponding economy without the learning from experience bias.

Figure 3: *Cohort Specific Log Consumption Growth and Volatility.* The figure plots the drift term of log consumption under the objective measure and the volatility of consumption growth by cohort lifespan. The figure is averaged from 10,000 simulations with 1200 periods or 100 years per simulation.



Remark 6. *Agarwal, Driscoll, Gabaix, and Laibson (2009) use a proprietary database to provide evidence for the hypothesis that older adults make fewer financial mistakes than younger adults, that is, they transition from inexperienced to experienced. More specifically, Agarwal, Driscoll, Gabaix, and Laibson (2009) show that older and experienced investors have greater investment knowledge. In addition, the survey based analysis in Arrondel, Calvo-Pardo, and Tas (2014) suggests that investors’ measure of information “increases with past experience.”*

3.5 The Average Belief About the Risk Premium and the True Risk Premium

In this subsection, we study the average belief about the risk premium. The average belief is important as it is frequently used in the empirical literature to study if agents have biased beliefs. Using survey data, Greenwood and Shleifer (2014) show that the average belief about expected returns is highly positively correlated with past stock returns. Moreover, they show that the average belief is negatively correlated with future returns. Hence, when the average belief about future returns is high, on average realized returns tend to be low.

The findings that the average belief about expected excess returns is positively correlated with past stock returns and negatively correlated with measures of ex ante risk premia and future excess returns pose challenges to standard rational expectations models. On the onset, it is not clear if the model with learning from experience can replicate such a relation between returns and beliefs about risk premium. For instance, one could imagine that individual mistakes about the risk premium wash out in aggregate and, therefore, the average belief about the risk premium is unbiased. Moreover, one can imagine that the average belief about the risk premium is positively correlated with the actual risk premium as agents are using Bayes' rule to learn about expected output growth. However, as we show below, within our economy we see a similar relation between 1) past stock returns and the current average belief about the risk premium and 2) the average belief about the risk premium and the true risk premium as in Greenwood and Shleifer (2014).

Definition 4. *The average belief about the risk premium is*

$$\hat{\mu}_t^S - r_t = \int_{-\infty}^t \nu e^{-\nu(t-s)} \mu_{s,t}^S ds - r_t. \quad (41)$$

The average belief about the risk premium is simply a population survey. Substituting

Equation (36) into the definition of the average belief about the risk premium, we obtain

$$\hat{\mu}_t^S - r_t = \sigma_Y^2 + \overbrace{\int_{-\infty}^t \nu e^{-\nu(t-s)} \hat{\mu}_{s,t} ds}^{\text{Average belief about output growth}} - \bar{\mu}_t. \quad (42)$$

From Equation (42), we see that the average belief about the risk premium depends on the difference between the average belief about output growth and the market view. Importantly, the average risk premium is positively related to the average belief about output growth, but negatively related to the market view. Hence, if the average belief reacts more aggressively than the market view in response to a shock, then the average belief about the risk premium increases after positive output shocks. Since positive output shocks also correspond to positive shocks to the stock market, the average appears as if there is return extrapolation. To test this, we follow Greenwood and Shleifer (2014) and regress the average belief about the risk premium in Equation (42) onto last year's realized return.

Table 1: *Returns, Risk Premium and Beliefs* The table shows coefficient estimate from a regression of (1) the average belief onto the return over the past 12 month: $\hat{\mu}_t^S - r_t = a + bR_{t-12,t} + e_t$, (2) the true risk premium onto the return over the past 12 month: $\mu_t^S - r_t = a + bR_{t-12,t} + e_t$, and (3) the true risk premium onto the average belief: $\mu_t^S - r_t = a + b(\hat{\mu}_t^S - r_t) + e_t$. $R_{t-12,t}$ is the cumulative return over the previous 12 months. The regression uses data from 10,000 simulations with 6000 periods (monthly observations) or 500 years per simulation, where coefficients, t-statistics, and R^2 are averaged across the 10,000 sample paths.

	b	t-stat	R^2
(1)	0.001	5.665	0.077
(2)	-0.056	-90.077	0.708
(3)	-1.410	-12.963	0.168

From regression (1) in Table 1, we see that the average belief about the risk premium is positively correlated with past stock returns. Hence, from the point of view of an econometrician, the average investor in our economy looks like a return extrapolator. In comparison, Table 3 in Greenwood and Shleifer (2014) shows the result from regressing six different sur-

vey measures onto the stock market returns over the past 12 months.¹⁴ All the six survey measures are positively related to past stock market returns and the R^2 range from 0.002 to 0.611 with an average value of 0.210.¹⁵ If we instead regress the true risk premium onto past stock market returns, we have a strong negative correlation as illustrated in regression (2) in Table 1. Hence, after positive shocks, the risk premium declines consistent with Proposition 8.

Given the opposite sign in Table 1 for the average belief about the risk premium and the true risk premium, we expect a negative relation between the average belief and the true risk premium. Indeed, this is what we find. In our model, the correlation between the average belief about the risk premium and the true risk premium is -0.193. For comparison, Table 5 in Greenwood and Shleifer (2014) shows that the correlations between four different measures of expected returns and average beliefs from six different surveys range from -0.807 to 0.366, with only 4 out of the 24 correlations being positive.¹⁶ The average value is -0.298, which is slightly more negative than the correlation in our model. This is also consistent with the evidence in Martin (2016). He derives a bound on the risk premium based on option prices and shows that this measure of the risk premium not only predicts future returns with the expected sign, but is negatively correlated with four different survey based measures of the risk premium with correlation coefficients ranging from -0.37 to -0.53.¹⁷

In addition to being negatively correlated with the true risk premium, the average belief about the risk premium is much less volatile. Specifically, the volatility of the average belief about the risk premium in the model is only 15% of the volatility of the true risk premium. This finding is consistent with Piazzesi, Salomao, and Schneider (2015), who find

¹⁴Specifically, Greenwood and Shleifer (2014) consider the following surveys: 1) Gallup, 2) Graham and Harvey, 3) American Association of Individual Investors survey (AAII), 4) Investors' Intelligence Newsletter Expectations, 5) Shiller, and 6) Michigan Surveys of Consumer.

¹⁵As the survey measures of expectations in Greenwood and Shleifer (2014) all have different units, the slope coefficients are not comparable.

¹⁶Greenwood and Shleifer (2014) consider the log dividend price ratio, the surplus consumption ratio, cay, and the predicted risk premium using the log price dividend ratio, Treasury-bill rate, the default spread, and the term spread jointly as measures of expected returns.

¹⁷Specifically, the estimates of the four correlation coefficients are -0.37 using AAII's survey, -0.46 using Shiller's survey, -0.50 using Graham-Harvey's survey, and -0.53 using Gallup's survey.

that statistical measures of the bond risk premium are more volatile than the bond risk premium measured from survey data. They show that the ratio of the standard deviation of survey based expectations of returns to statistical measures of the expected return ranges from 0.53 to 0.65, depending on the statistical measure of expected returns. For the stock market, Martin (2016) shows that the option based measure of expected return on the S&P 500 is highly volatile with standard deviations of 4.60% for 1 month horizon decreasing to 2.43% for the 1 year horizon using data from January 4, 1996 to January 31, 2012. For comparison, the Graham-Harvey survey of the risk premium has a standard deviation of 1.28% over the period 2000 to 2011. Further, from regression (3) in Table 1 we see that regressing the true risk premium onto the average belief about the risk premium yields a slope coefficient of -1.41 . This reflects the negative correlation between the belief about the risk premium and the true risk premium and the fact that the true risk premium is more volatile than the belief about the risk premium.

So what is the reason for the low volatility of the average belief and the negative correlation between the average belief about the risk premium and the true risk premium in our model? To understand this, recall that both the average belief about output growth and the market view increase after positive shocks to output, implying that they are highly correlated with an unconditional correlation coefficient of 0.989. Importantly, the average belief about the risk premium depends positively on the average belief about output growth, but negatively on the market view. The covariance between the average belief about the risk premium and the true risk premium is $cov(\hat{\mu}_t^S - r_t, \mu_t^S - r_t) = var(\bar{\mu}_t) - cov\left(\int_{-\infty}^t \nu e^{-\nu(t-s)} \hat{\mu}_{s,t} ds, \bar{\mu}_t\right)$ and, consequently, if the covariance between the average belief about output growth and the market view is sufficiently high, the correlation between the average belief about the risk premium and the true risk premium is negative. The unconditional standard deviation of the average belief about output growth is 36 basis points while it is 35 basis points for the market view, and given the high correlation between the two, the covariance is sufficiently high to outweigh the variance of the market view. Further, the average belief about the

risk premium is increasing in the average belief about output growth and decreasing in the market view. Given that the two are almost perfectly correlated and have similar volatility, the volatility of the average belief about the risk premium is low.

From the discussion above, we see that the key reason for the negative correlation between the average belief about the risk premium and the true risk premium is that the average belief about the output growth is highly correlated with the market view and more volatile. The intuition for this result is that the actual risk premium is driven by the market view and not the average belief about output growth. From the “representative agent’s” point of view (using the market view), the risk premium is in fact constant. In contrast, the average belief puts too much weight on the young and inexperienced agents with low wealth. Because young agents perceive a high risk premium after a series of positive shocks to the stock market, the survey forecast reflects their view more heavily than what the market view does and, therefore, correlates negatively with the true risk premium which is decreasing.

Remark 7. *One might conjecture a negative relation between the true risk premium and the average belief in a similar model with a representative agent with CRRA utility who learns about consumption dynamics. This turns out to be difficult. For this case, the perceived risk premium from the point of view of the representative agent is $\gamma\sigma_Y$, where γ is the coefficient of relative risk aversion. This is the standard constant risk premium in a full information model with CRRA utility. Hence, there is no variation in the risk premium from the point of view of the representative agent. Therefore, even though under the true probability measure it looks like as if there is predictability, one cannot generate a negative correlation between the true and the perceived risk premium. The negative correlation between the true and the average belief about the risk premium in our model is due to the large cross-sectional variation in the aggressiveness of the updating of the beliefs and a market selection process that favors the more experienced agents who update less aggressively.*

4 Conclusions

Empirical evidence suggests that experience matters for the formation of beliefs. If this is the case, then we should ask how personal experiences with the stock market and the macroeconomy in general manifest through savings and investment decisions and impact asset prices more broadly.

In response, in this paper we develop a general equilibrium model with overlapping generations in which cohort specific experience drives beliefs about output growth and through that affects equilibrium outcomes. We use the model to analyze how an experience based bias impacts beliefs, consumption, portfolio choice, and ultimately drives asset prices.

We show that in equilibrium the wealth weighted average belief, which we call the “market view,” is the relevant statistic for asset prices as the beliefs of wealthy agents influence prices more than the beliefs of poor agents. This has important implications for how the risk premium responds to shocks. After a positive shock, agents revise their belief about output growth upwards. However, there is also an additional effect due to wealth reallocations between agents with different beliefs as more optimistic agents gain from the shock and hence their belief impact prices more. This second channel works as an amplification mechanism and the risk premium becomes more volatile.

Our analysis offers a theoretical foundation for the empirical regularity that young individuals update expectations more strongly in the direction of recent surprises than old. In equilibrium, the risk premium decreases after positive shocks. Yet, young agents increase their risky investment at times when the risk premium is low, leading to a slower wealth accumulation in the early years of life. Our model is sufficiently rich to accommodate an endogenous life-cycle of expectations about the risk premium ranging from return extrapolation when young to contrarian when old and a cross-section of beliefs that is consistent with findings from survey evidence such as the return extrapolation of the average investor and the negative correlation between the consensus forecast and future stock market returns.

Why are the consensus forecast and future stock market returns negatively correlated?

After a positive shock, the stock market is priced as if the expected dividend growth is higher than prior to the shock. From the point of view of an econometrician with full information the stock looks “expensive,” i.e., the risk premium is lower than before the shock. In contrast, the average belief puts too much weight on young agents who update aggressively in the direction of the shock relative to the market view. Therefore, from the standpoint of the average investor, the risk premium goes up rather than down, creating a negative correlation between the true risk premium and the average belief about the risk premium. The mechanism above highlights the importance of considering the whole cross-section of beliefs to understand apparent puzzling features of the average beliefs, such as the negative correlation with statistical measures of risk premia.

A The Dynamics of the Disagreement Process, $\eta_{s,t}$

Following Basak (2000), we derive the disagreement process, $\eta_{s,t}$, from the stochastic discount factor under the objective probability measure over the stochastic discount factor under the subjective probability measure, $\eta_{s,t} = \frac{\xi_t}{\xi_{s,t}}$. Formally, this is the dynamics of the Radon-Nikodym derivative. Solving the two stochastic differential equations in Equation (8) yields

$$\xi_t = \xi_s e^{-\int_s^t (r_u + \frac{1}{2}\theta_u^2) du - \int_s^t \theta_u dz_u}, \quad \xi_{s,t} = \xi_{s,s} e^{-\int_s^t (r_u + \frac{1}{2}\theta_{s,u}^2) du - \int_s^t \theta_{s,u} dz_{s,u}}. \quad (43)$$

Their ratio is

$$\frac{\eta_{s,t}}{\eta_{s,s}} = \frac{\xi_t/\xi_s}{\xi_{s,t}/\xi_{s,s}} = e^{-\frac{1}{2} \int_s^t (\theta_u^2 - \theta_{s,u}^2) du - \int_s^t \theta_u dz_u + \int_s^t \theta_{s,u} dz_{s,u}}. \quad (44)$$

Using the equalities $dz_{s,t} = dz_t - \Delta_{s,t} dt$ and $\theta_{s,t} = \theta_t + \Delta_{s,t}$ and rearranging terms leads to

$$\frac{\eta_{s,t}}{\eta_{s,s}} = e^{-\frac{1}{2} \int_s^t \Delta_{s,u}^2 du + \int_s^t \Delta_{s,u} dz_u}. \quad (45)$$

Lastly, an application of Ito’s lemma yields the dynamics of the disagreement process

$$d\eta_{s,t}/\eta_{s,t} = \Delta_{s,t} dz_t, \quad (46)$$

where the disagreement process is a local martingale.

B Proofs of Propositions

B.1 Proof of Proposition 1

Following standard filtering theory, i.e., Liptser and Shiryaev (1974a,b), the dynamics of the expected output growth as perceived by an agent born at time s is given by Equation (2). Defining the standardized estimation error at time t for an agent born at time s as

$$\Delta_{s,t} = \frac{\hat{\mu}_{s,t} - \mu_Y}{\sigma_Y}, \quad (47)$$

and applying Ito's lemma yields

$$d\Delta_{s,t} = -\frac{\hat{V}}{\sigma_Y^2 + \hat{V}(t-s)}\Delta_{s,t}dt + \frac{\hat{V}}{\sigma_Y^2 + \hat{V}(t-s)}dz_t. \quad (48)$$

The solution to this stochastic differential equation yields the desired result:

$$\Delta_{s,t} = \frac{\sigma_Y^2}{\sigma_Y^2 + \hat{V}(t-s)}\Delta_{s,s} + \frac{\hat{V}}{\sigma_Y^2 + \hat{V}(t-s)}(z_t - z_s). \quad (49)$$

By the strong law of large numbers, we have $\lim_{t-s \rightarrow \infty} \frac{z_t - z_s}{t-s} = 0$ and, hence, $\lim_{t-s \rightarrow \infty} \Delta_{s,t} = 0$ a.s.

B.2 Proof of Proposition 2

An agent born at time s solves the static optimization problem in Equation (16). Rearranging Equation (18) leads to the optimal consumption at time t under the probability measure of an agent born at time s

$$c_{s,t} = c_{s,s}e^{-\rho(t-s)} \begin{pmatrix} \xi_{s,s} \\ \xi_{s,t} \end{pmatrix}. \quad (50)$$

Inserting the relation between the perceived and the true stochastic discount factor in Equation (45) yields the result.

B.3 Proof of Proposition 3

Proposition 3 presents the equilibrium expression for the stochastic discount factor. Let β_t define the fraction of aggregate output consumed by newborn agents:

$$\beta_t = \frac{c_{t,t}}{Y_t}. \quad (51)$$

We conjecture, and verify later, that β_t is constant, i.e., $\beta_t = \beta$. Plugging in the optimal consumption at time t of an agent born at time s , Equation (19), into the market clearing condition for the goods market, Equation (13), we have the following:

$$\begin{aligned}
Y_t &= \int_{-\infty}^t \nu e^{-(\rho+\nu)(t-s)} c_{s,s} \frac{\xi_s}{\xi_t} \frac{\eta_{s,t}}{\eta_{s,s}} ds \\
Y_t \xi_t &= \int_{-\infty}^t \nu e^{-(\rho+\nu)(t-s)} \frac{c_{s,s}}{Y_s} Y_s \xi_s \frac{\eta_{s,t}}{\eta_{s,s}} ds \\
e^{(\rho+\nu(1-\beta))t} Y_t \xi_t &= \int_{-\infty}^t \beta \nu e^{-\beta\nu(t-s)} e^{(\rho+\nu(1-\beta))s} Y_s \xi_s \frac{\eta_{s,t}}{\eta_{s,s}} ds \\
\bar{\eta}_t &= \int_{-\infty}^t \beta \nu e^{-\beta\nu(t-s)} \bar{\eta}_s \frac{\eta_{s,t}}{\eta_{s,s}} ds,
\end{aligned} \tag{52}$$

where $\bar{\eta}_t = e^{(\rho+\nu(1-\beta))t} Y_t \xi_t$. Applying Ito's lemma to $\bar{\eta}_t$, we obtain

$$\begin{aligned}
d\bar{\eta}_t &= \left(-\beta\nu \int_{-\infty}^t \beta \nu e^{-\beta\nu(t-s)} \bar{\eta}_s \frac{\eta_{s,t}}{\eta_{s,s}} ds + \beta\nu \bar{\eta}_t \right) dt + \bar{\Delta}_t \bar{\eta}_t dz_t \\
&= \bar{\Delta}_t \bar{\eta}_t dz_t
\end{aligned} \tag{53}$$

where

$$\bar{\Delta}_t = \int_{-\infty}^t f_{s,t} \Delta_{s,t} ds = \mathcal{E}(\Delta_t), \tag{54}$$

and where

$$f_{s,t} = \beta \nu e^{-(\rho+\nu)(t-s)} \left(\frac{\bar{\eta}_s}{\bar{\eta}_t} \right) \left(\frac{\eta_{s,t}}{\eta_{s,s}} \right) = \nu e^{-\nu(t-s)} \frac{c_{s,t}}{Y_t}, \tag{55}$$

which represents the share of aggregate output at time t that accrues to agents born at time s as in Equation (26).

Equation (53) corresponds to Equation (25) in Proposition 3. Using the fact that $\bar{\eta}_t = e^{(\rho+\nu(1-\beta))t} Y_t \xi_t$ and solving for ξ_t , we get the stochastic discount factor in Equation (23) of Proposition 3.

Lastly, the remainder of the proof verifies our conjecture about β . Using the result in Equation (20), we rewrite β as follows

$$\beta = \frac{c_{t,t}}{Y_t} = \frac{(\rho + \nu) \hat{W}_{t,t}}{Y_t} = \frac{(\rho + \nu) H_{t,t}}{Y_t}, \tag{56}$$

where the last equality follows from the fact that an agent is born without financial wealth.

Next, we solve for the value of the aggregate endowment of earnings

$$\begin{aligned} H_{t,t} &= \frac{1}{\xi_t} E_t \left[\int_t^\infty e^{-\nu(u-t)} \xi_u \omega Y_u du \right] = \omega Y_t E_t \left[\int_t^\infty e^{-(\rho+\nu+\nu(1-\beta))(u-t)} \frac{\bar{\eta}_u}{\bar{\eta}_t} du \right] \\ &= \frac{\omega Y_t}{\rho + \nu + \nu(1 - \beta)}, \end{aligned} \quad (57)$$

where, in the second line, we plug in the equilibrium stochastic discount factor, Equation (23), and in the third line, we use the martingale property of $\bar{\eta}_t$ and solve the integral. Combining Equation (56) and (57), we get

$$\beta = \frac{(\rho + \nu)\omega}{\rho + \nu + \nu(1 - \beta)}. \quad (58)$$

Solving the quadratic equation for β gives two solutions

$$\beta^+ = \frac{\rho + 2\nu}{2\nu} + \frac{\sqrt{\rho^2 + 4(\rho + \nu)\nu(1 - \omega)}}{2\nu}, \quad \beta^- = \frac{\rho + 2\nu}{2\nu} - \frac{\sqrt{\rho^2 + 4(\rho + \nu)\nu(1 - \omega)}}{2\nu}. \quad (59)$$

Next, we show that only the second solution, β^- , is feasible. As we demonstrate below, the expression for the equilibrium stock market price, S_t , is

$$S_t = \frac{1 - \omega}{\rho + \nu(1 - \beta)} Y_t, \quad (60)$$

for $\rho + \nu(1 - \beta) > 0$. This provides an upper bound on the values that β can take, specifically $\beta < \frac{\rho + \nu}{\nu}$. For $\rho > 0$, we have that $\beta^+ = \frac{\rho + 2\nu}{2\nu} + \frac{\sqrt{\rho^2 + 4(\rho + \nu)\nu(1 - \omega)}}{2\nu} > \frac{\rho + 2\nu}{2\nu} + \frac{\sqrt{\rho^2}}{2\nu} = \frac{\rho + \nu}{\nu}$ and hence β^+ is not a feasible solution. For β^- , we have that $\beta^- = \frac{\rho + 2\nu}{2\nu} - \frac{\sqrt{\rho^2 + 4(\rho + \nu)\nu(1 - \omega)}}{2\nu} < \frac{\rho + 2\nu}{2\nu} - \frac{\sqrt{\rho^2}}{2\nu} = 1$ and, therefore, $\rho + \nu(1 - \beta) > 0$ when $\rho > 0$.

Finally, using Equation (22), (57), and (60) we have that $H_t = \int_{-\infty}^t \nu e^{-\nu(t-s)} H_{s,t} ds = \beta \hat{W}_t$ and $(1 - \beta) \hat{W}_t$.

B.4 Proof of Proposition 4

Applying Ito's lemma to Equation (23) gives

$$\begin{aligned} d\xi_t &= d \left(\bar{\eta}_t \frac{e^{-(\rho+\nu(1-\beta))t}}{Y_t} \right) \\ &= \left(\bar{\eta}_t \frac{e^{-(\rho+\nu(1-\beta))t}}{Y_t} \right) [(-\rho - \nu(1 - \beta) - \mu_Y + \sigma_Y^2 - \sigma_Y \bar{\Delta}_t) dt - (\sigma_Y - \bar{\Delta}_t) dz_t]. \end{aligned} \quad (61)$$

Taking the dynamics of the stochastic discount factor, Equation (8), and the dynamics of Equation (61), matching the drift and diffusion terms leads to the following equilibrium expressions for the real short rate and market price of risk, respectively

$$r_t = \rho + \mu_Y + \nu(1 - \beta) - \sigma_Y^2 + \sigma_Y \bar{\Delta}_t \quad (62)$$

and

$$\theta_t = \sigma_Y - \bar{\Delta}_t. \quad (63)$$

To obtain the equilibrium real short rate and market price of risk as in Equations (28) and (29), it suffices to substitute the definition $\bar{\Delta}_t = \frac{\bar{\mu}_t - \mu_Y}{\sigma_Y}$ in the equations above.

B.5 Proof of Proposition 5

Discounting the dividends, D_t , with the stochastic discount factor yields

$$\begin{aligned} S_t &= \frac{1}{\xi_t} E_t \left[\int_t^\infty \xi_u D_u du \right] = \frac{1 - \omega}{\xi_t} E_t \left[\int_t^\infty e^{-(\rho + \nu(1 - \beta))u} \bar{\eta}_u du \right] \\ &= \frac{1 - \omega}{\xi_t} \bar{\eta}_t \int_t^\infty e^{-(\rho + \nu(1 - \beta))u} du = \frac{1 - \omega}{\rho + \nu(1 - \beta)} Y_t. \end{aligned} \quad (64)$$

Applying Ito's lemma to the equilibrium stock price leads to

$$\frac{dS_t}{S_t} = \frac{dY_t}{Y_t}. \quad (65)$$

Using Equation (7) and Equation (1) to match drift and diffusion terms yields the equilibrium expected return

$$\mu_t^S = \rho + \mu_Y + \nu(1 - \beta), \quad (66)$$

and the stock market volatility

$$\sigma_t^S = \sigma_Y. \quad (67)$$

Equation (67) is the stock market volatility as in Proposition 5. Subtracting the equilibrium real short rate from the expected return on the stock market, Equation (66), yields the risk premium on the stock market, Equation (32), as in Proposition 5.

B.6 Proof of Proposition 6

The total wealth at time t of an agent born at time s under the objective probability measure follows from the static budget constraint

$$\hat{W}_{s,t} = \frac{1}{\xi_t} E_t \left[\int_t^\infty e^{-\nu(u-t)} \xi_u c_{s,u} du \right] = \frac{c_{s,t}}{\rho + \nu}. \quad (68)$$

Substituting in optimal consumption, Equation (19), and rearranging terms, we have

$$\xi_t (W_{s,t} + H_{s,t}) = c_{s,s} e^{-\rho(t-s)} \frac{\eta_{s,t}}{\eta_{s,s}} \frac{\xi_s}{\rho + \nu}. \quad (69)$$

Applying Ito's lemma to both sides of Equation (69), using Equation (12) and (57) and equating the diffusion terms, we get

$$\xi_t \left(\pi_{s,t} \sigma_t^S + H_{s,t} \sigma_Y - \hat{W}_{s,t} \theta_t \right) = \Delta_{s,t} c_{s,s} e^{-\rho(t-s)} \frac{\eta_{s,t}}{\eta_{s,s}} \frac{\xi_s}{\rho + \nu}. \quad (70)$$

Using Equation (20) and simplifying yields

$$\pi_{s,t} \sigma^Y = \hat{W}_{s,t} (\sigma_Y - \bar{\Delta}_t + \Delta_{s,t}) - H_{s,t} \sigma_Y. \quad (71)$$

Solving the above equation for the optimal portfolio, $\pi_{s,t}$,

$$\pi_{s,t} = \frac{\Delta_{s,t} - \bar{\Delta}_t}{\sigma^Y} \hat{W}_{s,t} + W_{s,t}, \quad (72)$$

leads to Equation (6).

B.7 Proof of Proposition 7

Applying Ito's lemma to the market view, $\bar{\mu}_t = \int_{-\infty}^t f_{s,t} \hat{\mu}_{s,t} ds$, we get

$$d\bar{\mu}_t = f_{t,t} \hat{\mu}_{t,t} dt + \int_{-\infty}^t f_{s,t} d\hat{\mu}_{s,t} ds + \int_{-\infty}^t \hat{\mu}_{s,t} df_{s,t} ds + \int_{-\infty}^t df_{s,t} d\hat{\mu}_{s,t} ds. \quad (73)$$

Hence, we need the dynamics of the wealth shares and the individual agents' beliefs, $\hat{\mu}_{s,t}$. The dynamics of the beliefs are given in Equation (4). Applying Ito's lemma to the expression for the wealth share in Proposition (3), we find:

$$\frac{df_{s,t}}{f_{s,t}} = (-\beta\nu + \bar{\Delta}_t^2 - \bar{\Delta}_t \Delta_{s,t}) dt + (\Delta_{s,t} - \bar{\Delta}_t) dz_t. \quad (74)$$

Inserting Equation (4) and (74) into Equation (73) and after some algebra, we obtain the dynamics of the market view

$$d\bar{\mu}_t = \beta\nu(\hat{\mu}_{t,t} - \bar{\mu}_t)dt - \frac{\bar{V}_t}{\sigma_Y}\bar{\Delta}_t dt + \frac{\bar{V}_t}{\sigma_Y}dz_t, \quad (75)$$

where $\bar{V}_t = \mathcal{E}(\hat{V}_t) + \mathcal{V}(\hat{\mu}_t)$. Hence, the diffusion coefficient is guaranteed to be positive if both $\mathcal{E}(\hat{V}_t)$ and $\mathcal{V}(\hat{\mu}_t)$ are positive. This is indeed the case as $\hat{V}_{s,t}$ is positive for all agents and $\mathcal{V}(\hat{\mu}_t)$ is the variance (using the wealth distribution) of the beliefs which is positive.

B.8 Proof of Proposition 8

This follows directly from the expression for the real short rate, market price of risk, and risk premium combined with the fact that $\frac{\partial \bar{\mu}_t}{\partial z_t} > 0$.

B.9 Proof of Proposition 9

First, note that given the specification of the prior belief and the pre-trading period of 20 years the belief at time t of an agent born at time s is

$$\hat{\mu}_{s,t} = \mu_Y + \sigma_Y \frac{z_t - z_{s-20}}{t - (s - 20)}. \quad (76)$$

Next, we express the Brownian motion as a function of the return process and the expected return:

$$dz_t = \frac{dR_t - \mu_t^S dt}{\sigma_t^S}. \quad (77)$$

Using the expression for μ_t^S in Equation (66), the equilibrium stock market volatility in Equation (67), the representation of the beliefs in Equation (76), and the expression for the shocks in Equation (77) yield the result.

B.10 Proof of Proposition 10

Applying Ito's lemma to the log of the equilibrium consumption in, Equation (19), we get

$$d\log(c_{s,t}) = \left(\mu_Y + \nu(1 - \beta) - \frac{1}{2}\sigma_Y^2 + \frac{1}{2}(\bar{\Delta}_t^2 - \Delta_{s,t}^2) \right) dt + (\sigma_Y + \Delta_{s,t} - \bar{\Delta}_t) dz_t. \quad (78)$$

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