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ON THE VIABILITY OF ENERGY COMMUNITIES

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Following the development of decentralized production technologies, energy communities have become a topic of increased interest. While the potential benefits have been described, we use the framework of cooperative game theory to test the ability of such communities to adequately share the gains. Indeed, despite the potential value created by such coalitions, there is no guarantee that they will be viable: a subset of participants may find it profitable to exit the community and create another one of their own. We take the case of a neighborhood, having access to a limited resource e.g. a shared roof or piece of land which they can exploit if they invest in some renewable production capacity. By joining the community, participants also enjoy aggregation gains in the form of reduced network fees. We find conditions depending on the structure of renewable installation costs, on the magnitude of the aggregation effect and coordination costs and, most importantly, on the chosen sharing rule, under which the whole energy community is stable. Efficiency could require the intervention of a social planner or a change in network tariff structures.

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On the viability of energy communities*

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October the 9th, 2017

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Following the development of decentralized production technologies, energy communities have become a topic of increased interest. While the potential benefits have been described, we use the framework of cooperative game theory to test the ability of such communities to adequately share the gains. Indeed, despite the potential value created by such coalitions, there is no guarantee that they will be viable: a subset of participants may find it profitable to exit the community and create another one of their own. We take the case of a neighborhood, having access to a limited resource – e.g. a shared roof or piece of land – which they can exploit if they invest in some renewable production capacity. By joining the community, participants also enjoy aggregation gains in the form of reduced network fees. We find conditions depending on the structure of renewable installation costs, on the magnitude of the aggregation effect and coordination costs and, most importantly, on the chosen sharing rule, under which the whole energy community is stable. Efficiency could require the intervention of a social planner or a change in network tariff structures.

Keywords- Energy communities, Cooperative game theory, Decentralized power production, Consumer participation, Micro-grids

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1 Introduction

The notion of energy communities has received increased interest over the past few years, fostered by better information and communication technologies and an increase in environmental awareness. Even if the term itself has never been properly defined, energy communities are seen as crucial for facilitating the decentralization of power production and enhancing the management of energy resources at a local level. Small energy communities are thus blossoming in many countries (see [24] for a review of existing initiatives). This trend is often bolstered by substantial support from policymakers (see e.g., [30], [35]), for example, in the form of feed-in tariffs. However, despite the potential profits made by such communities, there is no guarantee that they will be viable. A subset of participants may indeed find it profitable to exit the community and create another one of their own if not properly remunerated.

For the purpose of this paper, we consider a narrow definition of an energy community: households of a common building or close geographical area may decide to combine their effort and jointly build solar panels on their roofs (or windmills in a nearby field). Instead of individual meters, they can then decide to install only one and use it for the whole community. There is therefore one source of costs (the costs of installation of the renewable resource), and two sources of gains: aggregation gains, in the form of decreased network fees, and energy gains, as the renewable energy can be consumed at zero marginal costs or re-injected in the network and given a feed-in tariff. The main goal of this paper is to analyze the viability and stability of such a community.

Our definition of an energy community is inspired by the recent German Mieterstromgesetz review that aims at the development of PV panels on the roofs of collective buildings or buildings that are physically close by[1]. The Mieterstromgesetz law, that was passed in June 2017, allows the owners or tenants of apartments in a collective building to self-consume locally produced electricity without using the public network. Self-consumed electricity allows the consumer not to pay fees that are usually collected with the network charges. Various other fees, like the renewable energy surcharge, have to be paid but can be partially compensated by a subsidy. Since tenants in collective houses can be very heterogeneous, the savings have to be allocated in a way that is considered acceptable enough by the community. The regulation also contains restrictions on gains sharing within the community that may further complicate the task of finding a feasible stable allocation.[2]

We treat this problem within the framework of cooperative game theory. An array of results are found, depending on the cost structure of renewables' installation costs. We show that the most basic sharing rules (per-capita, pro-rata of consumption or peak demand) usually fail to provide adequate remuneration to all players. In that case, some households may decide to opt out from the community. They may then try to create another smaller community with other unsatisfied households or may remain on their own. We find that diversified households with different generations, family size, occupation status, under the same roof create more value, and are therefore more likely to stick together as a community. More elaborate sharing rules, such as the Shapley value or the minimum variance allocation, though slightly more complex, have desirable properties and are more likely to enable communities to share their gains thereby enabling them to be viable. When the community cannot be stable, the intervention of a social planner or a change in network tariffs may

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[1]The German government estimates around 3.8 million households are eligible for the scheme. Further information on the Mieterstromgesetz can be found at https://www.bundesregierung.de/Content/DE/Artikel/2017/04/2017-04-26-mieterstrom.html

[2]See http://lexetius.com/EnWG/42a for a description of such provisions.

be required to restore efficiency. If such an intervention is not desired, we propose a way to optimally split the whole energy community into smaller stable groups of consumers, so that the lost value when splitting is minimized.

It is worth noting at this stage that we will restrain ourselves to assessing the game-theoretical implications of communities, inasmuch as they are motivated by financial incentives. Non-economic motivations, as well as potential externalities will not be explicitly modeled. This aims at making the paper concise while keeping the main insights unaltered. A key assumption is that households only have access to a limited amount of renewables (e.g., they only have access a pre-determined share of the roof surface in their apartment block). The framework is one of exchangeable utility and full public information.

The present paper is related to at least two strands of the literature.

First, it pertains to the literature on cooperative games. Clear expositions of how cooperative game theory can be applied to costs and surplus sharing in many sectors can be found in [46], [31] or [32]. These elements have in particular been applied to a wide variety of topics in the energy sector: [28] analyzes the surplus sharing among Liquefied Natural Gas exporters and casts doubts on the credibility of a logistic cooperation that would be exempt from market power. [17] finds that the Shapley value could be used to allocate generators' contribution to reliability in capacity adequacy problems. Cooperative games have also been successfully applied to the allocation of CO2 emission rights ([21],[34]), the allocation of network costs among customers ([42], [9], [20], [19]) or optimal system planning [43]. However, gainsharing within energy communities have, to the best of our knowledge, not yet been covered.

Second, this paper is closely related to the literature on decentralized energy systems. Substantial applied research has been done on decentralized generation, from an engineering or optimization perspective: [16], [1] and [22] provide insights into the optimal dispatch of decentralized generation. Likewise, operational research on energy communities and micro-grids has also been very active recently, showing an increased interest in these business models ([33], [3], [41], [29]), of which the benefits have been widely stressed, both theoretically ([25], [7], [6]) and empirically ([24] , [8]). All of the previously mentioned papers envisage rather sophisticated energy communities endowed with technologies such as storage or demand-response. In contrast, we do not explicitly model these aspects so as to focus on the issue of gain sharing within the community. Indeed, the literature has so far restricted the analysis to the technically achievable benefits yielded by such communities, while very little research has been made to date on the actual viability of the community seen as a coalition. [26], [44], [45], [30] and [10] discuss how energy communities or micro-grids may be integrated in the existing system but, likewise, do not address whether these coalitions hold in practice. Close in spirit to our paper, [27] exposes how cooperative game theory may shed light on the desirability of micro-grids. Echoing our results, they find that the misalignment between private and social objectives can lead to inefficient deployment. Similarly, [23] discusses how the gains of decentralized trading can be shared between end-users and suppliers. However, [27] and [23] mainly focus on the allocation of gains between energy communities and other players of the energy system, rather than between agents acting *within* the energy community. We believe the present paper is the first to apply cooperative game theory within energy communities as such, which is our main contribution. The other important contribution of this work is to propose an optimal stable partitioning as a way to treat the instability of the energy community. Each sub-group might then create a smaller viable community on its own.

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The rest of the paper is organized as follows: Section 2 exhibits a simple analytical model, that illustrates the main insights in a stylized situation. Section 3 describes the favorable situation when a community benefits from strong economies of scale. Conditions of non-emptiness of the core of the game, and of the stability of various allocations are also presented. Section 4 addresses the more complex case when coordination itself induces a cost. Both sections 3 and 4 are treated theoretically, and numerical applications on realistic situations are also proposed and analyzed. Finally, section 5 concludes the paper.

2 Mathematical formulation of the problem

2.1 Notation and assumptions

We assume that there is a set of households (i.e., consumers) $I = \{1, 2...n\}$, $n > 1$, who consider joining an energy community so as to share the cost of installing a photovoltaic (PV) panel to produce green energy. All households live close to each other (in the same house). If they form a coalition, they will subsequently also share the benefits of producing solar energy. Time is discretized into T periods representing a characteristic consumption year: $t \in \{1, 2...T\}$, where each time step represents a minute. The consumption of household i over time is denoted $f_i(t)$ and is expressed in kilowatt-hour (kWh). We assume that the electricity tariff has two components: one related to energy and one to capacity. Typically, a household with a profile $f(t)$ will pay $\alpha \mathrm{Max}_t f(t) + \delta$ for her capacity (α is expressed in euro per kilowatt €/kW and δ in €) and $\beta \sum_{t=1}^{T} f(t)$ for her energy (β is expressed in €/kWh). Component $\alpha \mathrm{Max}_t f(t)$ is the variable part of the grid tariff and δ is the fixed part that can correspond to the cost of installing a meter. This particular linear form of the electricity tariff is not too restrictive, as our setting can easily be generalized to other more elaborate tariff formulas. The installation cost of a PV panel is assumed to be a function of its capacity K. Once installed, we assume that the panel will deliver, on average, a yearly profile $Kg(t)$ (in kWh) with a peak production around noon (under clear sky conditions). The investment cost (expressed in €) of a PV panel with capacity K is denoted $c(K)$. Each individual household has access to an area in the premises to install a PV panel that is proportional to her living area. We assume her energy consumption is also proportional to her living area. Hence, if a group of households $S \subset I$ want to go green, we will make the assumption that they can install a PV capacity proportional to their total consumption. In other words, a set of households $S \subset I$ that install a PV panel will invest in a capacity that will allow them to cover a percentage $\mu(S) \in [0, 1]$ of their consumption $\sum_{i \in S} \sum_{t=1}^{T} f_i(t)$. The PV capacity to install is then $\mu(S) \dfrac{\sum_{i \in S} \sum_{t=1}^{T} f_i(t)}{\sum_{t=1}^{T} g(t)} \equiv \sum_{i \in S} k_i(\mu(S))$ (kW), where $k_i(\mu(S)) = \mu(S) \dfrac{\sum_{t=1}^{T} f_i(t)}{\sum_{t=1}^{T} g(t)}$ is the contribution of each household to capacity. This corresponds to a cost of PV installation $c\left(\sum_{i \in S} k_i(\mu(S))\right)$ (€). Parameter $\mu(S)$ is calculated such that the benefits (in terms of local consumption and injection in the grid) for the set of households is optimized (and therefore, $\mu(S)$ represents the PV investment decision for coalition S). We bound parameter $\mu(S)$ by an upper limit μ that reflects the fact that coalition S has access to a limited area on the roof and we set the same upper limit μ to all coalitions of the community, to model the fact that any coalition can have access on the roof to a surface that is proportional to its energy consumption. A more realistic description would optimize this investment when coupling it with the installation and operations of a battery, but this lies beyond the scope of this paper which focuses on stable benefit sharing among members of the community. We will see further on that our model already offers interesting insights regarding the viability of any energy community. The solar energy produced by the energy community can either be injected in the distribution network or locally consumed. When locally consumed, PV production reduces the

energy bill of the community at a marginal value equal to the electricity retail tariff β (expressed in €/kWh). When injected in the network, PV production is remunerated by the distribution system operator (DSO) at a marginal price of γ (expressed in €/kWh), that can represent the feed-in tariff or a market price with a premium attributed to the willingness to consume renewable energy. We assume in this paper that priority is given to local consumption, as we believe that this is the main objective of fostering energy communities from the point of view of policymakers (see [14]).

Households might benefit from economies of scales in the construction of PV panels by grouping into an energy community and sharing the benefits of green energy production. The question of fairness in sharing the benefit among the different players is crucial as it is a necessary condition for the viability of the project. The sharing should incentivize all players enough so that they have no interest in leaving and investing on a stand-alone basis. Cooperative game theory constitutes a nice framework to treat the subject inasmuch as it defines the notion of stability in the sharing and proposes (when possible) suitable allocation rules that make the energy community viable.

In this paper we only concentrate on the economic benefits of a PV installation that can be estimated. Non-economic motivations of the energy community like willingness to go green or to become energetically independent are neglected, even if they can represent an important part of the benefits.

The following table summarizes our notation:

I	set of households of the community. Indexed by i
T	time. Indexed by t
$\mathcal{P}(I)$	set of all coalitions of I, that we denote $S \subset I$
$f_i(t)$	consumption profile of household i (kWh)
$g(t)$	PV production profile (kWh per kW)
$\mu(S)$	factor of proportionality linking the invested PV capacity to consumed energy (no unit). This parameter is optimized for all coalitions $S \subset I$
μ	upper bound of all $\mu(S)$
$k_i(\mu(S))$	contribution of household i to PV capacity (kW): $k_i(\mu(S)) = \mu(S) \dfrac{\sum_{t=1}^{T} f_i(t)}{\sum_{t=1}^{T} g(t)}$
$c(.)$	PV investment cost as a function of capacity (€)
α	variable part of the grid tariff (€/kW)
δ	fixed part of the grid tariff (€)
β	electricity retail price (€/kWh)
γ	electricity wholesale price or feed-in tariff (€/kWh)

2.2 Modeling the game between households

2.2.1 Calculation of the value of coalitions

The interaction between the different households will be modeled by the characteristic function that gives the payoff (we will also refer to this payoff as the "value" throughout this paper) of a coalition of households that invest together in a PV panel. Given a coalition of players $S \subset I$, its value will be the difference between the cost it incurs by consuming electricity with and without the PV installation and the single meter.

- **Without the PV panel:** Each individual household of S has an individual profile $f_i(t)$. The total corresponding cost is then (s denotes the cardinal of the set S):

$$\text{cost}_1(S) = \alpha \sum_{i \in S} \text{Max}_t \left(f_i(t) \right) + \delta s + \beta \sum_{i \in S} \sum_{t=1}^{T} f_i(t) \tag{1}$$

- **With the PV panel:** the households join into a community and aggregate their profiles into: $\sum_{i \in S} f_i(t)$. Given the investment decision $\mu(S)$, the consumption peak can be reduced by the amount of PV that is locally consumed. Therefore, the peak consumption of the coalition will be $\text{Max}_t \left(\sum_{i \in S} f_i(t) - \sum_{i \in S} k_i(\mu(S))g(t) \right)^+$, where $(.)^+$ denotes the positive part of a real number. Households of the community invest in a panel, so as to produce a PV profile equal to $\sum_{i \in S} k_i(\mu(S))g(t)$ that is either locally conumed or injected in the network. Priority is given to local consumption: therefore, at times t when PV production $\sum_{i \in S} k_i(\mu(S))g(t)$ does not exceed electricity consumption $\sum_{i \in S} f_i(t)$, the cost of consuming electricity for the community is $\beta \left(\sum_{i \in S} f_i(t) - \sum_{i \in S} k_i(\mu(S))g(t) \right)$. On the contrary, at times t when PV production $\sum_{i \in S} k_i(\mu(S))g(t)$ does exceed electricity consumption $\sum_{i \in S} f_i(t)$, PV production – net from local consumption – will be injected in the grid. This provides an additional payment to the community: $-\gamma \left(\sum_{i \in S} k_i(\mu(S))g(t) - \sum_{i \in S} f_i(t) \right)$. To summarize, if the investment decision is $\mu(S)$ and we account for the PV investment costs, the total cost for a coalition S is expressed as follows:

$$
\begin{aligned}
\text{cost}_2(S) = \\
\alpha \text{Max}_t \left(\sum_{i \in S} (f_i(t) - k_i(\mu(S))g(t)) \right)^+ + \delta \\
+ \beta \sum_{t=1}^{T} \left(\sum_{i \in S} (f_i(t) - k_i(\mu(S))g(t)) \right)^+ - \gamma \sum_{t=1}^{T} \left(\sum_{i \in S} (k_i(\mu(S))g(t) - f_i(t)) \right)^+ \\
- c \left(\sum_{i \in S} k_i(\mu(S)) \right)
\end{aligned}
\tag{2}
$$

where the first line represents grid costs, the second line energy costs and revenues, and the last line the cost of installation of the PV panels.

- **The value of coalition S** is then the maximum possible benefit made by S when optimially deciding the PV investment $\mu(S)$. The PV benefit is estimated as the difference between $\text{cost}_1(S)$ and $\text{cost}_2(S)$:

$$
\begin{aligned}
v(S) = \quad &\text{Max} \quad (\text{cost}_1(S) - \text{cost}_2(S)) \\
&\text{s.t.} \quad 0 \le \mu(S) \le \mu
\end{aligned}
\tag{3}
$$

Using (1) and (2):

$$
\begin{aligned}
v(S) = \quad &\text{Max} \quad \alpha \left(\sum_{i \in S} \text{Max}_t (f_i(t)) - \text{Max}_t \left(\sum_{i \in S} (f_i(t) - k_i(\mu(S))g(t))^+ \right) \right) + \delta(s - 1) \\
&\qquad + \beta \sum_{t=1}^{T} \left(\sum_{i \in S} f_i(t) - \left(\sum_{i \in S} (f_i(t) - k_i(\mu(S))g(t))^+ \right) \right) \\
&\qquad + \gamma \sum_{t=1}^{T} \left(\sum_{i \in S} (k_i(\mu(S))g(t) - f_i(t)) \right)^+ \\
&\qquad - c \left(\sum_{i \in S} k_i(\mu(S)) \right) \\
&\text{s.t.} \quad 0 \le \mu(S) \le \mu
\end{aligned}
\tag{4}
$$

- **The total value of the energy community** is then:

$$
\begin{aligned}
v(I) = \quad &\text{Max} \quad \alpha \left(\sum_{i \in I} \text{Max}_t (f_i(t)) - \text{Max}_t \left(\sum_{i \in I} (f_i(t) - k_i(\mu(S))g(t))^+ \right) \right) + \delta(n - 1) \\
&\qquad + \beta \sum_{t=1}^{T} \left(\sum_{i \in I} f_i(t) - \left(\sum_{i \in I} (f_i(t) - k_i(\mu(S))g(t))^+ \right) \right) \\
&\qquad + \gamma \sum_{t=1}^{T} \left(\sum_{i \in I} (k_i(\mu(S))g(t) - f_i(t)) \right)^+ \\
&\qquad - c \left(\sum_{i \in I} k_i(\mu(S)) \right) \\
&\text{s.t.} \quad 0 \le \mu(I) \le \mu
\end{aligned}
\tag{5}
$$

Given an investment decision $\mu(S)$, the benefit of a coalition can be split into several terms:

the first term $\alpha \left(\sum_{i \in S} \text{Max}_t \left(f_i(t) \right) - \text{Max}_t \left(\sum_{i \in S} \left(f_i(t) - k_i(\mu(S))g(t) \right)^+ \right) \right) + \delta(s - 1)$ is simply an aggregation benefit. It is not linked to the PV investment *per se*. Indeed, for simplicity of exposition, our theoretical developments assume that members of the community have consumption peaks outside the range of PV production (see assumption H1 in the next section). This assumption is relaxed in numerical applications. This term therefore simply accounts for the fact that households belonging to S have gathered into an energy community and have aggregated their consumption profiles accordingly. As a result, peak demand of the community is weakly smaller than the sum of individual peaks. The DSO will then charge less from the community. It is worth noticing that the aggregation benefit of a coalition may actually constitute free-riding. The electricity tariff part $\alpha \times$ capacity $+ \delta$ is usually imposed by the DSO to recover its grid cost. Therefore, the aggregation benefit of a coalition creates some loss that the operator can compensate only by increasing the tariff to the remaining consumers, which implies the existence of possible negative externalities between members and non-members of a community. For simplicity, we overlook these issues in this paper. The interested reader may find discussions on the issues of net metering in [4], [5], [15]. By construction, such externalities are indeed negligible if the energy community is small, as compared to the size of the distribution system. However, if the number of energy communities start to increase, one will have to take into consideration such effects, which we intend to do in future research.

The second term of the benefit of a coalition,

$\beta \sum_{t=1}^{T} \left(\sum_{i \in S} f_i(t) - \left(\sum_{i \in S} \left(f_i(t) - k_i(\mu(S))g(t) \right)^+ \right) \right)$, is the potential benefit to locally consume electricity.

The third term $\gamma \sum_{t=1}^{T} \left(\sum_{i \in S} \left(k_i(\mu(S))g(t) - f_i(t) \right) \right)^+$ is the value of the produced PV energy that is injected in the distribution system. The last term $c \left(\sum_{i \in S} k_i(\mu(S)) \right)$ is the cost of installation of $K = \sum_{i \in S} k_i(\mu(S))$ kW of capacity.

All optimization programs (4) are feasible and bounded and have continuous objectives. Therefore, they always have a solution which implies that the value function of our game is well-defined.

2.2.2 Two assumptions used in the theoretical developments

In our theoretical developments, we will have to make two important (and we believe necessary) assumptions to be able to conduct various calculations. This is due to the fact that the calculation of the value for a coalition is obtained in an implicit form by solving the first-order conditions of optimization programs (3) and hence, in principle these values do not have closed forms. Our assumptions will simplify the problem (in theory) allowing us to obtain closed forms and derive some interesting results regarding the stability of the community.

Assumption H1 assumes that consumers of the community have profiles with peaks occurring outside the range of PV production (which is the case of working people that have consumption peaks in the morning and in the evening). Therefore, the term $\text{Max}_t \left(\sum_{i \in S} \left(f_i(t) - k_i(\mu(S))g(t) \right) \right)^+$ simplifies into $\text{Max}_t \left(\sum_{i \in S} f_i(t) \right)$. We have observed that for a sufficiently small time granularity (a minute for instance), this assumption is quite realistic for the set of households we consider in our numerical applications.

Assumption H2 assumes that optimization programs (3) always have $\mu(S) = \mu$ as a solution. We justify this assumption by the fact that investment cost in PV is concave (it shows economies of scale, see section 3.2 for figures of this cost) and therefore a group of households will always have an incentive to invest up to the limit allowed by the area of the premises they have access to, provided the feed-in tariff is high enough. As a consequence, we will suppose the optimal variables $\mu(S)$ are always equal to μ. This assumption is no

more valid when the retail or wholesale prices are not high enough to always sustain the PV investment.

Using assumptions H1 and H2, the value of coalition S simplifies into:

$$v(S) = \begin{array}{l} \alpha \left[\sum_{i \in S} \text{Max}_t \left(f_i(t) \right) - \text{Max}_t \left(\sum_{i \in S} \left(f_i(t) \right) \right) \right] + \delta(s-1) \\ + \beta \sum_{t=1}^{T} \left(\sum_{i \in S} f_i(t) - \left(\sum_{i \in S} \left(f_i(t) - k_i(\mu)g(t) \right) \right)^+ \right) \\ + \gamma \sum_{t=1}^{T} \left(\sum_{i \in S} \left(k_i(\mu)g(t) - f_i(t) \right) \right)^+ \\ - c \left(\sum_{i \in S} k_i(\mu) \right) \end{array} \quad (6)$$

Both assumptions H1 and H2 are used only in our theoretical developments but are relaxed in all our numerical applications. These assumptions aim at simplifying the mathematical exposition. They are relaxed in all our numerical applications.

2.2.3 Definition of the game and allocation rules

We can now define the game of the energy community we are interested in as well as the notion of the sharing rule (also called the allocation rule):

Definition 1. *There is a set of players (households) I, consuming electricity. A coalition (community) S is a subset of the grand coalition I that generates value exposed in equation (3). Players can decide to join or not at most one coalition formed of some or all of the other households in I, according to the way the payment will be divided among coalition members, called the sharing rule.*

We can now define the core of the game between households. Intuitively, the core is constituted by all allocations of the total value of the game $v(I)$ such that all coalitions $S \subset I$ are incentivized to stay in the overall community. Formally, this gives the following definition:

Definition 2. *The core of the game $Ker(I)$ is the set of all allocations $x(v) = (x_1(v), x_2(v), ...x_n(v)) \in \mathbb{R}^n$ such that:*

$$\forall S \subset I, \quad \sum_{i \in S} x_i(v) \geq v(S) \quad (7)$$
$$\sum_{i=1}^{n} x_i(v) = v(I) \quad (8)$$

The notion of the core is an essential element of cooperative game theory. Relation (7) states that, if in the core, the sharing of the total benefit $v(I)$ should be done in a way that satisfies all coalitions: members of any coalition receive more than what they get when the coalition stands alone. Relation (8) states that any allocation belonging to the core is Pareto-optimal. In other words the core is the "set of payoff configurations that leave no coalition in a position to improve the payoffs to all of its members" ([38]), and satisfies individual and group rationality. We consider in this paper that an energy community is stable (or viable) if and only if the core is not empty.

The core of a cooperative game imposes conditions that make any allocation unquestionable by all potential coalitions. Unfortunately, the core suffers from two main drawbacks: first it is sometimes empty, and second, when not empty, it usually contains infinitely many allocations. One must then select in the core particular allocation rules that fullfil pre-specified properties. Many allocation rules have been defined that fulfill nice intuitive properties and belong to the core under some specific conditions. The Shapley value is certainly the most famous of them. In a seminal paper [37], Shapley defines three properties that an allocation rule should satisfy and draws the conclusion that there is a unique allocation rule that actually does it: the so-called Shapley value. These properties are:

- **Symmetry** An allocation $x(v) = (x_1(v), x_2(v), ...x_n(v)) \in \mathbb{R}^n$ of the value function v is symmetric if it fulfills the following ($\mathcal{S}_n$ is the set of permutations of n):

$$\forall \sigma \in \mathcal{S}_n, \ \forall i \in \{1, 2...n\} \ x_{\sigma(i)}(\sigma(v)) = x_i(v) \tag{9}$$

- **linearity** An allocation x is linear if:

$$\text{For all value functions } v_1 \text{ and } v_2, \ x(v_1 + v_2) = x(v_1) + x(v_2) \tag{10}$$

- **Pareto-optimality** An allocation x of the value function v is Pareto-optimal if it fulfills the following:

$$\sum_i x_i(v) = v(I) \tag{11}$$

The explicit formulation of the Shapley value is given below:

Definition 3. *The Shapley value $x^s(v)$ is the unique allocation rule satisfying symmetry, linearity and Pareto-optimality:*

$$\forall i \in \{1, 2..., n\}, \ x_i^s(v) = \sum_{i \in S \subset I} (v(S) - v(S/\{i\})) \frac{(n-s)!(s-1)!}{n!} \tag{12}$$

For a given coalition S containing player i, $(v(S) - v(S/\{i\}))$ represents the marginal contribution of player i to coalition S. The Shapley value averages these marginal contributions over all coalitions containing i. In [39], the author gives the link between the Shapley value and the core of game: if a game is convex then the core is not empty and the Shapley value belongs to the core. A game is convex if it satisfies the following:

$$\forall S \subset T, \forall j \notin S, \ v(S \cup \{j\}) - v(S) \leq v(T \cup \{j\}) - v(T) \tag{13}$$

Convexity is in fact a difficult condition to meet. In general, the energy community game is not convex. Therefore, it is not straightforward that the game has a non-empty core or that the Shapley value will be there.

The nucleolus is another allocation with desirable properties. This allocation rule, which has been defined in [36], always belongs to the core when it is non-empty. The idea behind the notion of the nucleolus is to minimize the maximal unhappiness of coalitions, the unhappiness being defined as the difference between the value of a coalition and what it receives from the allocation.

The calculation of the nucleolus is computationally heavy: it consists of a succession of solving of many optimization programs. For this reason, we rather focus in this paper on another new allocation rule, that we name MinVar, that is much simpler to calculate from a computational point of view because it requires solving only one optimization program. Intuitively, the Minvar allocation rule minimizes the inequality of treatment of the players. Formally, given an allocation $x_1, x_2, ..., x_n$, we measure the inequality in the treatment of the players by the variance of x_i:

$$Var(x) = \frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n} \right)^2 \tag{14}$$

and we require MinVar to look within the core for the allocation rule that minimizes the inequality of treatment:

$$\text{MinVar} := \text{Arg Min}_{(x_1, x_2, ...x_n) \in Ker(I)} \ Var(x) \tag{15}$$

By construction, like the nucleolus, the MinVar allocation is always in the core as long as it is not empty. Computationally, problem (15) is a quadratic optimization program with linear constraints, that always holds a unique solution when feasible (or when the core is non-empty). Therefore, like the nucleolus, MinVar will always define a unique allocation rule as long as the core is non-empty.

For simplicity of exposition, our theorems will focus on two specific types of households. We consider the cases when households are symmetric and anti-symmetric, defined as follows:

Definition 4. *Anti-symmetric players are households that have similar profiles but centered at different hours of the day in a way that their supports do not intersect: given a reference profile $f(t)$, each individual has a profile*

$$\forall i \in I, \ \forall t \in \{1, ...T\}, \ f_i(t) = f(t - t_i) \tag{16}$$

with $t_1, \ t_2...t_n$ are such that $\forall t \in \{1, 2...T\}, \ \forall i, j \in I, \ i \neq j \Rightarrow f_i(t).f_j(t) = 0$.

Definition 5. *Symmetric households are players that have similar load profiles: given a reference profile $f(t)$, each individual has a profile*

$$\forall i \in I, \ \forall t \in \{1, ...T\}, \ f_i(t) = f(t) \tag{17}$$

We now theoretically treat some examples.

3 The case of concave investment costs of PV

This section provides the framework for a concave cost function $c(.)$: there may be economies of scale in PV installation projects, which justifies the concavity of function $c(.)$.

3.1 Theoretical analysis

If investment costs are concave, the community benefits from returns to scale as it grows. One can show the following theorem:

Theorem 1. *When the investment cost is concave and players are either symmetric or anti-symmetric, the coalition game is convex.*

Proof. To prove convexity, from which it follows that the core is non-empty and that the Shapley value is there, we need to show that:

$$\forall S \subset T, \forall j \notin S, \ v(S \cup \{j\}) - v(S) \leq v(T \cup \{j\}) - v(T) \tag{18}$$

Whether households are symmetric or anti-symmetric, PV revenues are additive (they imply only volumes). Therefore, referring to relation (6), the expression of the PV revenues of a coalition of size s simplifies into the following expressions.

PV revenues from local consumption are equal to:

$$s\beta \sum_{t=1}^{T} \left(f(t) - (f(t) - k(\mu)g(t))^+ \right) \tag{19}$$

and for the grid injection, PV revenues are equal to:

$$s\gamma \sum_{t=1}^{T} (k(\mu)g(t) - f(t))^+ \tag{20}$$

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where $k(\mu)$ is the contribution of any individual household to the installed PV capacity:

$$k(\mu) = \mu \frac{\sum_{t=1}^{T} f(t)}{\sum_{t=1}^{T} g(t)} \tag{21}$$

Whether households are symmetric or anti-symmetric, the variable part of the tariff component $\alpha \left(\sum_{i \in S} \text{Max}_t \left(f_i(t) \right) - \text{Max}_t \left(\sum_{i \in S} f_i(t) \right) \right)$ is a constant that does not depend on coalition S. It is zero when households are symmetric and $\alpha \max_t f(t)$ when households are anti-symmetric. Hence, this term cancels out in the equality (18), together with the fixed component of the grid tariff δ and the PV revenues component. We should therefore focus on the cost term:

$$c \left(\sum_{i \in S} k_i(\mu) \right) = c \left(sk(\mu) \right)$$

A classical property[3] of a concave function is that:

$$\forall a < b \in \mathbb{R}, \forall x \geq 0, \quad c(a+x) - c(b+x) \geq c(a) - c(b) \tag{22}$$

With $a = \sum_{i \in S} k_i(\mu)$, $b = \sum_{i \in T} k_i(\mu)$, $x = k_j(\mu)$, we show that (18) holds.

$\square$

From theorem 1 it follows that the core of such a game is always non-empty and the Shapley sharing, rather comfortingly, is in the core. This means that such communities will indeed be stable, and there is no need for a central authority to intervene. The Shapley value is a stable allocation.

If players are neither symmetric nor anti-symmetric, the cost term remains unchanged and in addition, the coalition would benefit from gains of aggregation. Hence, we expect the coalitions to remain stable. A formal proof is, however, omitted due to its complexity. Instead, we propose numerical applications, based on realistic estimates of the costs and gains of energy communities.

3.2   A numerical example

In this section, we aim at providing a sense of the magnitudes at play in energy communities. While the theoretical developments modeled both the effects of grid tariffs savings and PV generation itself, the numerical application will only focus on the latter. Indeed, the issue of spillovers between consumers inside and outside a community has been extensively described. [4] or [5] show that current tariffs are inefficient and may result in the inadequate remuneration of owners of decentralized generation systems. For the sake of clarity in identifying the sources of gains and costs, we abstract away from grid tariff structures that vary from one jurisdiction to another and may be subject to changes in the future. In other words, we set these tariffs to zero and instead we focus on the fundamental costs and benefits of our energy communities, namely the joint installation of a PV system.

Communities will have to face many choices, including how they will share the gains amongst their members. We estimate the allocations resulting from rather simple and intuitive sharing rules (per capita, pro-rata of annual consumption, pro-rata of peak demand) to more complex ones (Shapley, MinVar). We show that simple allocations often fail to make the grand coalition stable (despite it being optimal, in the sense that it maximizes the gains of the grand coalition), while Shapley and MinVar are more likely to be stable.

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[3]See property 5 in https://www.irif.fr/ emiquey/MathsJPS/convexite.pdf

This also means that regulatory restrictions regarding the allocation of the gains within the community such as in the Mieterstromgesetz may further weaken the stability of a coalition. For illustration, we have built two composite communities, all composed of six households. They can create a community (within their apartment block or neighborhood) and jointly install PV panels. The costs of such installations are the same in both communities. However, these communities differ in their household composition: each building will have a different consumption pattern, not only overall, but also at the individual household level. This will affect the value of each coalition, through the channel of PV revenues and installation costs.

We consider typical buildings in north-west Germany. The timestamp granularity is one hour and we consider consumption and solar profiles over a whole year (we thus capture the seasonality inherent to power systems). Each household demand has been generated by a load profile generator that allows us to simulate detailed demand curves for various types of households ranging from their size, employment status, age, family status, etc. The PV production has been calibrated on ELIA data for solar generation, in year 2014. PV installation costs are calibrated on standard commercial PV panel prices. We assume a panel lifetime of 30 years and a discount factor of 5%. PV production is valued at the German retail price when it is consumed within the community. Excess production is injected into the grid at German wholesale prices. More detailed information on the data sources can be found in appendix A. It is worth mentioning that the PV installation cost is concave in theory. However, the concavity occurs starting from an installed capacity of the range of the megawatt and for the rather small capacities we are interested in in this paper, the cost can be considered as linear.

As previously explained, we generate two communities, which we believe are reasonable representations of buildings or neighborhoods one can find in developed countries, and yet provide rather contrasted occupation patterns. The first community (tables 1 and 2) is composed of retired people (i.e., rather symmetric households). The second one (tables 3 and 4) is completely mixed, with students, retired people, families with various occupations and, most importantly, a storekeeper. The latter has a particular consumption profile: it is broadly flat in the range between 9 am and 6 pm during all days of the year except weekends. Outside these periods, consumption is very low. All data or data sources are given in appendix A. All values are reported in euro/annum. To summarize, the first building is composed of rather symmetric households, while the second is more mixed, in terms of activity and size of the households.

The results will report on the maximum "strength of stability," that allows the core to be non-empty. This measure aims at providing a sense of the size of the core. First of all, as discussed in the theoretical developments, communities may extract value in the form of reduced grid payments (which we actually oversee in this numerical application). Furthermore, we acknowledge that many costs or benefits are not included in our analysis. Households can enjoy some non-economic benefits of joining a community or producing green energy (either because of genuine environmental concerns or a phenomenon known as "warm glow"[4].) Energy communities may also trigger innovation (see [11], [12]). Conversely, such communities may also induce costs we have not accounted for so far, such as costs of switching supplier, cognitive costs to make a decision, discounting due to risk-aversion etc. Appendix B further elaborates on these points by considering possible issues related to incentives to exert efforts in the community by its individual members. Hence, the "strength of stability" is defined as the maximum additional individual cost with which the

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[4]Warm glow (see [2]) is defined as an increase in utility resulting from the act of giving in addition to the utility generated by an increase in the total supply of the public good.

coalition is stable (equivalently, it is the minimum individual cost above which the grand coalition has an empty core). Formally this cost is defined by (s denotes the cardinal of S):

$$\overline{c} = \max c$$
$$s.t. \quad (n-1).c + \sum_{i \in I} x_i = v(I)$$
$$(s-1).c + \sum_{i \in S} x_i \geq v(S) \quad (\forall S \subset I, S \neq I)$$

This means a coalition generates an extra cost $s.c$, linearly increasing in the size of the coalition S.

We report our results for each building in two scenarios: the first scenario is the business as usual one that we have just described above. We will denote it BAU. The second scenario looks more into the future and assumes that the investment cost of PV will decrease by 30% due to technological advances. This assumption is in line with what one can find in the literature that forecasts the evolution of the cost of PV in Europe by 2025. We will denote this scenario TEC (which stands for TEChnology). We recall that realistic load curves have been simulated, meaning assumptions H1 and H2 are relaxed in this numerical application.

We report for each building the value of the game (value of the grand coalition I), the individual values of inhabitants as well as the outcome of some allocation rules: per-capita, pro-rata to energy, pro-rata to peak demand, the Shapley value and MinVar. We also indicate whether the core is empty or not. Furthermore, we show the size of the PV installation if agents install PV individually (no coalition, this is the case where the energy community is not formed) or if they all invest jointly (grand coalition).

From tables 1 to 4, it emerges that basic sharing rules (per-capita, pro-rata of volumes or capacity) fail to enable the community to be stable – meaning at least one subset of participants finds it profitable to exit the community. This happens despite the grand coalition creating the most value. However, the core is never empty, which means that there exists a feasible stable allocation – which is in line with theorem 1. In all cases but one (retired people, TEC scenario – table 2), the Shapley value is an adequate allocation rule. It is also worth noticing that the grand coalition always increases the installed PV capacity, as compared to the case with no coordination. Indeed, the grand coalition allows us to optimize the allocation of PV production, therefore making investment more profitable. A policymaker willing to encourage PV developments may therefore want to promote larger communities. The need to gather in larger communities is especially tangible when the PV technology is relatively immature and expensive (tables 1 and 3). Indeed, PV installations are close to break-even, which makes gains from aggregation a key requirement to secure profitability.

Considering the strength of stability of our communities, we observe that in each building, the condition for the grand coalition to be stable is that any extra cost (non accounted for *a prioiori*) does not exceed a threshold ranging from 0.1 to 5.3% of the value of the coalition. This means only a relatively small coordination cost may prevent the coalition from forming, even though the community may be value-maximizing overall.

Our results also suggest that simple allocation rules (per capita, pro-rata to volume or peak demand) are not in the core. They do not stabilize the community and should not be considered as sharing the benefit of the PV investment. On the contrary, the Shapley

Table 1: Building composed of retired people – BAU scenario

	Retired Man	Retired Woman	Retired Couple	Retired Couple	Retired Couple	Retired Couple	Total	In the core?
Annual demand (kWh)	1101	1016	2680	2088	1747	1747	10379	
Peak demand (kW)	5.4	5.1	8.2	7.3	7.0	7.0	40.0	
Individual value	21.5	20.1	50.7	40.5	32	32	196.8	n/a
per capita allocation	57.2	57.2	57.2	57.2	57.2	57.2	343.1	no
per volume allocation	36.4	33.6	88.6	69	57.8	57.8	343.1	no
per capacity allocation	50.3	32.1	68	74.3	59.2	59.2	343.1	no
Shapley	48.6	34.1	83.7	71.8	52.4	52.4	343.1	Yes
MinVar	58	38.4	68	68	55.4	55.4	343.1	Yes

Core is non-empty?	Yes	Installed PV (no coalition): 2.5 kW
Total value	343.1	Installed PV (grand coalition): 3.6 kW
Strength of stability	18.2	

Table 2: Building composed of retired people – TEC scenario

	Retired Man	Retired Woman	Retired Couple	Retired Couple	Retired Couple	Retired Couple	Total	In the core?
Annual demand (kWh)	1101	1016	2680	2088	1747	1747	10379	
Peak demand (kW)	5.4	5.1	8.2	7.3	7.0	7.0	40.0	
Individual value	42.7	33.2	97.3	78.6	68	68	387.8	n/a
per capita allocation	94.9	94.9	94.9	94.9	94.9	94.9	569.6	no
per volume allocation	83.4	53.2	113	123.3	98.4	98.4	569.6	no
per capacity allocation	83.4	53.2	113	123.3	98.4	98.4	569.6	no
Shapley	78.4	52	136.9	118	92.1	92.1	569.6	no
MinVar	87.9	56.7	118.7	118.7	93.8	93.8	569.6	Yes

Core is non-empty?	Yes	Installed PV (no coalition): 5.5 kW
Total value	569.6	Installed PV (grand coalition): 5.7 kW
Strength of stability	23.6	

Table 3: Building composed of various consumers – BAU scenario

	Couple Working	Family Working One child	Man Work from home	Student	Storekeeper	Retired Couple	Total	In the core?
Annual demand (kWh)	2623	2613	1601	1563	4003	1747	9930	
Peak demand (kW)	10.1	6.7	2.1	5.4	1.4	7.0	36.3	
Individual value	40.9	23.2	15.2	26.6	196.7	32	334.6	n/a
per capita allocation	82.5	82.5	82.5	82.5	82.5	82.5	495.3	no
per volume allocation	91.8	91.5	56	54.7	140.1	61.1	495.3	no
per capacity allocation	108.4	112.4	55.9	83.3	41.2	94.1	495.3	no
Shapley	63.2	58	33.5	38.7	231.8	70.1	495.3	Yes
MinVar	70	70.7	39.8	41.1	197.4	76.2	495.3	Yes

Core is non-empty?	Yes	Installed PV (no coalition): 3.7 kW
Total value	495.3	Installed PV (grand coalition): 4.7 kW
Strength of stability	0.5	

Table 4: Building composed of various consumers – TEC scenario

	Couple Working	Family Working One child	Man Work from home	Student	Storekeeper	Retired Couple	Total	In the core?
Annual demand (kWh)	2623	2613	1601	1563	4003	1747	9930	
Peak demand (kW)	10.1	6.7	2.1	5.4	1.4	7.0	36.3	
Individual value	74.2	65.6	35.1	45	302.8	68	590.7	n/a
per capita allocation	131.2	131.2	131.2	131.2	131.2	131.2	787.4	no
per volume allocation	145.9	145.4	89.1	87	222.8	97.2	787.4	no
per capacity allocation	172.3	178.7	88.8	132.4	65.5	149.7	787.4	no
Shapley	106	112.2	61.3	60	338.3	109.6	787.4	Yes
MinVar	113.3	121.4	69.4	61.5	302.9	118.8	787.4	Yes

Core is non-empty?	Yes	Installed PV (no coalition): 7.0 kW
Total value	787.4	Installed PV (grand coalition): 7.2 kW
Strength of stability	16.5	

value provides more stability and MinVar, by definition, is always stable (in the core).

It is worthwhile mentioning that for building 2 (composed of various consumers), the storekeeper receives the highest share of the total value of the community. This is explained by the fact that he has a consumption profile that is compatible with solar production and, therefore, his presence increases local consumption of PV by the community, that is remunerated at a higher value than when injected in the system. This is also reflected by the fact that his individual value is the highest among the inhabitants of the building.

Section 4 provides a more detailed analysis of the impact of coordination costs on the value and stability of coalitions.

4 Introducing coordination costs

From the empirical data we have, it appears that investment costs for PV are largely linear in the range of capacity we are considering for a standard building or village. In the rest of this paper, we will therefore make the assumption that the PV installation cost is linear. We now take into consideration that forming a coalition induces a cost of coordination. Indeed, one imagines that if a coalition appears in the game, its members will have to meet in order to agree on the allocation rule and effectively share the benefit of installing the PV panel. For the purpose of this illustration, we will assume that the cost of coordination of a coalition is a function of its size. A typical example would consider that this cost will depend on the number of handshakes between its members, $\frac{s(s-1)}{2}$, which will make it strictly convex with respect to the size of the coalition. More generally, in mathematical forms, the coordination cost of a coalition S of size s is will be denoted:

$$c_{coordination}(S) = c'(s) \tag{23}$$

Where $c'(.)$ is assumed to be a strictly convex and smooth function. Appendix B elaborates more on another interpretation of these costs incurred by the community.

4.1 Theoretical analysis

This section details the theoretical treatment of the stability of the energy community when the coordination cost is considered. We recall that whether households are symmetric or anti-symmetric, we have already seen in section 3 that PV revenues of a coalition of size s can be expressed as:

$$s \left(\beta \sum_{t=1}^{T} \left(f(t) - (f(t) - k(\mu)g(t))^+ \right) + \gamma \sum_{t=1}^{T} (k(\mu)g(t) - f(t))^+ \right) \tag{24}$$

where $k(\mu)$ is expressed by:

$$k(\mu) = \mu \frac{\sum_{t=1}^{T} f(t)}{\sum_{t=1}^{T} g(t)} \tag{25}$$

To ease the notation, we will call ζ the term (recall that μ is an exogenous parameter):

$$\zeta = \beta \sum_{t=1}^{T} \left(f(t) - (f(t) - k(\mu)g(t))^+ \right) + \gamma \sum_{t=1}^{T} (k(\mu)g(t) - f(t))^+ \tag{26}$$

Therefore, using expression (6), the (new) value of any coalition S of size s is then given by (the PV investment cost is written in the linear form $c(K) = cK$):

$$v'(S) = \alpha \left(\sum_{i \in S} \text{Max}_t \left(f_i(t) \right) - \text{Max}_t \left(\sum_{i \in S} f_i(t) \right) \right) + \delta(s-1) + s(\zeta - ck(\mu)) - c'(s) \tag{27}$$

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4.1.1 The case of anti-symmetric households

In this part, we treat the case of anti-symmetric households (see definition 4).

Theorem 2. *When the coordination cost is taken into account and players are anti-symmetric, the following two propositions are equivalent:*

1. *The core of the game is not empty and the Shapley value is in the core*

2.

$$\alpha Max_t f(t) + \delta \geq (n-1)c'(n) - nc'(n-1) \tag{28}$$

Proof. In this proof, the individual subscript i will be dropped if there is no possible confusion.

$1) \Rightarrow 2)$

Let us assume that the core is not empty. When players are anti-symmetric, the value of a coalition S of size s is calculated from (27):

$$\forall S \subset I, \ v'(S) = (\alpha Max_t f(t) + \delta)(s-1) + s(\zeta - ck(\mu)) - c'(s) \tag{29}$$

Let us assume that the core is not empty: there exists a stable allocation $x = (x_1, ... x_n) \in \mathbb{R}^n$ such that:

$$\forall S \subset I, \quad \sum_{i \in S} x_i \geq v'(S) \tag{30}$$

$$\sum_{i=1}^{n} x_i = v'(I) \tag{31}$$

Considering equation (30) for all coalitions of size $s < n$, one gets ($||$ denotes the cardinal of a set):

$$\forall s \in \{1, 2...n-1\}, \forall S \subset I \text{ such that } |S| = s$$

$$\sum_{i \in S} x_i \geq (\alpha Max_t f(t) + \delta)(s-1) + s(\zeta - ck(\mu)) - c'(s) \tag{32}$$

Summing over all coalitions of size s, one gets the following: on the left-hand side, each term x_i will appear exactly $\binom{n-1}{s-1}$ because by fixing player i, one is left with the choice of $s-1$ members among $n-1$ players. On the right-hand side, each term depends only on the size s, which means that each term should be counted $\binom{n}{s}$ times. Therefore, one will get:

$$\binom{n-1}{s-1} \sum_{i=1}^{n} x_i \geq \binom{n}{s} \left((\alpha Max_t f(t) + \delta)(s-1) + s(\zeta - ck(\mu)) - c'(s) \right) \tag{33}$$

and using (31) (the Pareto-optimality condition of the core) with some simplifications:

$$\forall s \in \{1, 2, ..., n\}, \ \alpha Max_t f(t) + \delta \geq \frac{ns}{(n-s)} \left(\frac{c'(n)}{n} - \frac{c'(s)}{s} \right) \tag{34}$$

Let us now consider function $h : x \longrightarrow h(x) = \frac{xn}{n-x}\left(\frac{c'(n)}{n} - \frac{c'(x)}{x} \right)$ defined over $[0, n)$. Function h is differentiable over $[0, n)$ and:

$$\forall x \in [0, n), \ \frac{\mathrm{d}h}{\mathrm{d}x}(x) = \frac{n}{n-x}\left(\frac{c'(n) - c'(x)}{n-x} - \frac{dc'}{dx}(x) \right) \tag{35}$$

Since c' is strictly convex, one can state that: $\forall x \in [0, n)$, $\frac{c'(n) - c'(x)}{n-x} - \frac{\mathrm{d}c'}{\mathrm{d}x}(x) \geq 0$ and therefore h is an increasing function over $[0, n)$: $h(1) \leq h(2) \leq ... \leq h(n-1)$. Hence, relation (34) is equivalent to:

$$\alpha Max_t f(t) + \delta \geq h(n-1) = n(n-1)\left(\frac{c'(n)}{n} - \frac{c'(n-1)}{n-1} \right) \tag{36}$$

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Or

$$\alpha \mathrm{Max}_t f(t) + \delta \geq (n-1)c'(n) - nc'(n-1) \tag{37}$$

2) $\Rightarrow$ 1)

Suppose relation (28) holds. Using the fact that function h is increasing, one obtains:

$$\forall s \in \{1, 2, ...n-1\}, \ \alpha \mathrm{Max}_t f(t) + \delta \geq \frac{ns}{(n-s)} \left(\frac{c'(n)}{n} - \frac{c'(s)}{s} \right) \tag{38}$$

When we calculate the Shapley value $x^s(v')$ of the game using relation (12), we get:

$$\forall i \in \{1, 2...n\}, \ x_i^s(v') = (\alpha \mathrm{Max}_t f(t) + \delta) \frac{n-1}{n} + \zeta - ck(\mu) - \frac{c'(n)}{n} \tag{39}$$

By construction, the Shapley allocation is Pareto-optimal, which means that:

$$\sum_{i=1}^{n} x_i^s(v') = v'(I) \tag{40}$$

Consider now a coalition of players $S \subset I$ and let us prove that the Shapley allocation gives more to S than what it earns by standing alone. In other words, we want to prove that $\sum_{i \in S} x_i^s(v') - v'(S) \geq 0$.

$$
\begin{aligned}
\sum_{i \in S} x_i^s(v') - v'(S) = & \left(s(\alpha \mathrm{Max}_t f(t) + \delta)\tfrac{n-1}{n} + s(\zeta - ck(\mu)) - s\frac{c'(n)}{n} \right) \\
& - ((s-1)(\alpha \mathrm{Max}_t f(t) + \delta) + s(\zeta - ck(\mu)) - c'(s)) \\
= & \quad \tfrac{(n-s)}{n}(\alpha \mathrm{Max}_t f(t) + \delta) - s\left(\tfrac{c'(n)}{n} - \tfrac{c'(s)}{s} \right)
\end{aligned} \tag{41}
$$

and using relation (38), one can conclude that the Shapley value is in the core.

$\square$

Theorem 2 stipulates that when coordination costs are taken into account and when the PV investment cost is linear, the benefit from aggregation of the energy community has to be sufficient to compensate the increase of the marginal cost of the community due to coordination costs, to ensure stability.

4.1.2 The case of symmetric households

In this part, we treat the case of players with similar profiles (see definition 5):

Theorem 3. *When the coordination cost is taken into account and players are symmetric, the following two propositions are equivalent:*

1. *The core of the game is not empty and the Shapley value is in the core*

2.
$$\delta \geq (n-1)c'(n) - nc'(n-1) \tag{42}$$

Proof. When players are symmetric, the value of a coalition S of size s is calculated from (27):

$$\forall S \subset I, \ v'(S) = \delta(s-1) + s(\zeta - ck(\mu)) - c'(s) \tag{43}$$

When players are symmetric, the aggregation term reduces to the fixed component of the grid tariff δ.

The expression of the value of a coalition is similar to that of the non-symmetric case (29) when one replaces the aggregation term $\alpha \mathrm{Max}_t f(t) + \delta$ by δ. The demonstration of theorem 2 can therefore be directly applied here to obtain our result.

$\square$

Theorems 2 and 3 are intuitive: with a linear PV investment cost and if the aggregation benefit is small, any energy community facing convex coordination costs creates a positive value that unfortunately cannot always be shared in a stable way: there might always remain a coalition that will want to play apart to marginally decrease the coordination cost. Furthermore, comparing condition (28) with (42) indicates that the energy community has more chance of remaining stable when its members have anti-symmetric profiles. Indeed, any anti-symmetry of agents creates a potential additional aggregation benefit $\alpha \mathrm{Max}_t f(t)$ that helps compensate the coordination cost.

We now propose a way to stabilize the energy community, even if the core of the game is empty.

4.1.3 Stabilizing the energy community in the case of an empty core

When the core of the game between members of an energy community is empty, there is no stable way to share the benefits since at least one coalition cannot be incentivized enough to remain in the community. In that case, one can look for a judicious partition of the set of players I into different sub-communities such that each sub-community becomes stable and the total value generated by the partition is maximal. Let us first define a partition of the community I and its corresponding value:

Definition 6. *A partition $P = \{S_1, S_2, ..., S_p\}$ of size p of the community I is a collection of subsets of I satisfying:*

- $\forall i \in \{1, 2, ..., p\}, \ S_i \subset I$
- $\forall i \in \{1, 2, ..., p\}, \ S_i$ *is not empty:* $S_i \neq \Phi$
- $\forall i, j \in \{1, 2, ..., p\}, \ i \neq j \implies S_i \cap S_j = \Phi$
- $\cup_{i=1}^{p} S_i = I$

The value of such a partition is defined as follows:

$$VP(P) := \sum_{S \in P} v(S) \tag{44}$$

A partition is simply a subdivision P of the whole set I into smaller (non-empty) coalitions, or subsets that never intersect. We will sometimes refer to these subsets as clusters. The value of P is the sum of the values of all coalitions belonging to P. As an example, the simplest possible partition of the community contains only singletons:

$$P_0 = \{\{1\}, \{2\}, ..., \{n\}\} \tag{45}$$

and its value is the sum of the individual values of all members of the energy community:

$$VP(P_0) = \sum_{i=1}^{n} v(\{i\}). \tag{46}$$

Let us consider a subset S of a partition of the community. If members of S join together in a small energy community, one can wonder whether S generates enough value that can be shared in a stable way among its members. This leads to the definition of a stable partition:

Definition 7. *P is a stable partition of I if:*

- *P is a partition of I*

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- $\forall S \in P$, the game constituted by members of $S = \{s_1, s_2, ..., s_k\}$ if they gather in an energy community has a non-empty core:

$$\exists x_1, x_2, ...x_k \geq 0 \text{ such as}$$

$$\forall \{j_1, j_2, j_m\} \subset \{1, 2, ...k\}, \sum_{q=1}^{m} x_{j_q} \geq v(\{s_{j_1}, s_{j_2}, ...s_{j_m}\})$$

$$\sum_{q=1}^{k} x_q = v(S)$$

The set of stable partitions will be denoted $\Pi(I)$.

A stable partition is a subdivision of I into smaller stable energy communities. As an example, the obvious partition P_0 defined in equation (45) is stable. However, it might not ensure the highest value. We then define the notion of efficient partition:

Definition 8. *P is an efficient partition of I if it provides the highest value:*

$$\forall P' \text{ partition of } I, \ VP(P') \leq VP(P) \tag{47}$$

A simple and intuitive way to find the "best" partition of a community that is not stable (leading to an empty core) is then to look for the partition that gives the highest value, among all stable partitions. This leads to the definition of an optimal partition:

Definition 9. *P is an optimal partition of I if it is stable and provides the highest value among stable partitions:*

- *P is stable*
- $\forall P'$ *stable partition of* I, $VP(P') \leq VP(P)$

In other words, an optimal partition P of the community I solves the following:

$$P := \text{Arg Max}_{Q \in \Pi(I)} \ VP(Q) \tag{48}$$

The previous optimization program always has a solution: it is feasible since the obvious partition P_0 is stable and it is bounded because there is a finite number of possible partitions. However, it does not always lead to a unique solution.

It is worthwhile mentioning that an optimal partition is not always efficient because any efficient partition is not always stable. Besides, intuitively, an optimal partition will split the community among smaller subgroups with members having sufficiently different consumption profiles, in order to reduce the coordination cost while creating enough aggregation benefits to ensure stability.

4.2 A numerical example

Again, we omit grid payments and focus on PV costs and benefits. Tables 5 and 6 expose the simulations for the building composed of varied households (building 2) in the BAU and TEC scenarios, respectively. We keep the same specifications as in section 3.2, but now consider that the coordination cost of a coalition of size s depends on the number of handshakes in the coalition: $c'(s) = c'\frac{s(s-1)}{2}$. We assume a cost per handshake of $c' = €5.5$. All data or data sources are given in appendix A. All values are given in euro/annum and, again, we relax assumptions H1 and H2 in this numerical application.

Coordination costs are a lump-sum cost that differ from one coalition to the other, but do not vary with the installed PV within the coalition. Hence, it does not modify the

Table 5: Building composed of various households – BAU with coordination costs

	Couple Both working	Family Working One child	Man working at home	Student	Storekeeper	Retired Couple	Total	In the core?
Annual demand (kWh)	2623	2613	1601	1563	4003	1747	9930	
Peak demand (kW)	10.1	6.7	2.1	5.4	1.4	7.0	36.3	
Individual value	40.9	23.2	15.2	26.6	196.7	32	334.6	n/a
per capita allocation	68.8	68.8	68.8	68.8	68.8	68.8	412.8	no
per volume allocation	76.5	76.2	46.7	45.6	116.8	51	412.8	no
per capacity allocation	90.3	93.7	46.6	69.4	34.3	78.5	412.8	no
Shapley	49.5	44.3	19.7	25	218	56.4	412.8	no
MinVar	n/a	n/a	n/a	n/a	n/a	n/a	n/a	no

Core is non-empty?	no	Installed PV (no coalition): 3.7 kW
Total value	412.8	Installed PV (grand coalition): 4.7 kW
Strength of stability	-23.5	
Optimal partition	{Man working at home and student} and	
	{Couple, family, storekeeper, retired couple}	
Value of the optimal partition	426.6	

Table 6: Building composed of various households – TEC with coordination costs

	Couple Both working	Family Working One child	Man working at home	Student	Storekeeper	Retired Couple	Total	In the core?
Annual demand (kWh)	2623	2613	1601	1563	4003	1747	9930	
Peak demand (kW)	10.1	6.7	2.1	5.4	1.4	7.0	36.3	
Individual value	74.2	65.6	35.1	45	302.8	68	590.7	n/a
per capita allocation	117.5	117.5	117.5	117.5	117.5	117.5	704.9	no
per volume allocation	130.7	130.2	79.7	77.9	199.4	87	704.9	no
per capacity allocation	154.2	160	79.5	118.6	58.6	134	704.9	no
Shapley	92.3	98.4	47.5	46.2	324.5	95.9	704.9	no
MinVar	n/a	n/a	n/a	n/a	n/a	n/a	n/a	no

Core is non-empty?	no	Installed PV (no coalition): 7.0 kW
Total value	704.9	Installed PV (grand coalition): 7.2 kW
Strength of stability	-26.4	
Optimal partition	{Student} and	
	{Couple, family, Man working at home, storekeeper, retired couple}	
Value of the optimal partition	715.8	

optimal installed capacity for a given coalition, but will potentially affect their stability. It comes as no surprise that in both scenarios treated in tables 5 and 6, the value of the game is positive but the introduction of a coordination cost makes it impossible to share it among the inhabitants in a stable way (the core is empty). This is reflected by the fact that the "strength of stability" is negative in both scenarios: members of the community should receive a minimum amount of money to accept any allocation rule. Given this outcome, we then go further by looking for the optimal partition of the buildings that makes the PV installation feasible. In both buildings it appears that the community should be divided in two: this has the advantage of reducing the coordination costs and creating clusters with quite heterogeneous members, to exploit the aggregation benefit. Surprisingly, dividing the community into two groups does not destroy value in either scenario as a comparison between the value of the game and the value of the optimal partition reveals. Conversely, there is an increase in value, meaning such a partitioning is actually desirable, as it allows a saving to be made on coordination costs. Once the division of the community has been performed, it will then be possible to allocate the value of each coalition in the optimal partition in a stable way, using an allocation rule, like MinVar, that is in the core, which by construction is possible because the optimal partition is always stable.

5   Conclusion

The European Commission recently released its winter package "clean energy for all Europeans" (see [14]). The document makes it clear that consumers will be put at the center of energy systems. In particular, more engagement in clean energies and decentralized production and consumption is promoted. Consumer participation and local energy communities will be encouraged. The German Mieterstromsgesetz is one of the first actual implementations of this vision of the local sharing of energy and inspires our definition of an energy community. These communities may create a wealth of benefits, both at the consumer or system level. However, it is not clear how these gains can best be shared within communities. This paper shows that an inadequate allocation of gains may jeopardize the stability of these communities. Despite positive gains of cooperation in energy communities, the usual intuitive rules for sharing these gains may not satisfy all participants. In this case, members may leave the community simply because the gains are not adequately shared. Restrictions on gains sharing such as provisioned in the Mieterstromgetz may also impede the stability of coalition. This paper finds conditions under which communities may -or may not- be stable, and the allocation rules that appear to be most suitable for them to be viable and equitable.

We show that most commonly used sharing rules (per capita, per capacity, per energy) fail to stabilize the community. When PV installation costs are concave, the Shapley or MinVar sharing rules are, in general, stable enough to adequately share the economies of scale among the members of the community. When coordination costs are introduced, the community is stable only if aggregation benefits can compensate them, which happens if consumption profiles are distinct enough and the community is diversified. In that case the Shapley or MinVar allocations ensure stability. However, when aggregation costs are not sufficient to provide stability, we propose an optimal way to split the community into different stable subcommunities so that the total value is maximized. As a result, we observe that our optimal clustering leads to heterogeneous subcommunities mixing employed and unemployed households of different sizes. More generally the paper shows that communities with the ability to measure individual households consumption, and able to produce personalized billing are more likely to be stable. Hence, the success or failure of such communities relies not only on the engineering of renewable generation, but also on the ability of such communities to adopt recent technologies such as smart metering devices

and engage in dynamic pricing.

Future work can extend our setting in different ways: first, one can endogenize the PV investment decision when coupling it with an installation and operations of a battery. Second, one can capture non-economic motivations of the energy community: willingness to go green and become energetically independent, etc. All these effects tend to further stabilize the community. Finally, one could also take into consideration negative externalities that a coalition may impose on the rest of the community, when it wants to play apart, in the calculation of its value. As an illustration, the grid tariff structure is such that the aggregation term reduces the payment for grid services, but not its costs. Hence, the system operator may have to recoup its cost by increasing the tariff of the remaining consumers. We envisage treating this aspect in another paper.

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APPENDIX

A   Data sources

The energy sector being particularly complex, there exists a wide diversity of candidate parameters on which to base our analysis, reflecting the diversity of existing grid tariffs, PV costs, profiles of demand and production, etc. In addition, PV and wind installations are characterized by a relatively long duration, and many economic or regulatory parameters are likely to evolve throughout the lifetime of a community. Even though we are aware of these difficulties we believe the present sources and estimates give a reasonable account of the information available to households when they take their decision to form a community, as of today. All the sources of our numerical applications are public.

On the demand side, we simulated composite household load curves using a publicly-available load generator LoadProfileGenerator (www.loadprofilegenerator.de). The timestamp is per minute, reduced to hourly averages for ease of computation. On the supply side, the shape of the solar generation curve is calibrated on 2014 solar production, as reported by ELIA (in Belgium). The data is per 15-minutes granularity, converted into hourly granularity by averaging. We assumed each household or community could cover a maximum of 80% of their annual demand with PV production ($\mu = 0.8$). This value has been calibrated so that it corresponds to up to 35 square meters of available surface to install PV panels, for a typical building composed of representative households.

Costs of PV have been calibrated on the observed current prices of commercial PV panels. We assume a 30-years panel lifetime with a 5% discount rate. These values are in line with most of the recent estimates for PV panels – see e.g., [18], [13] or [40].

Two sources of gains are taken into consideration in the numerical applications. First, the PV production can be injected in the grid. In that case we assume communities receive the average 2014 spot price in Germany. [5] Second, PV can be locally consumed. This marginally decreases the community bill by the retail price of electricity in Germany (€280/MWh).

Regarding coordination costs, we assume a cost of 5.5 euro per handshake, which results in a total coordination cost of $c'(6) = 5.5 \times \frac{5*6}{2} = 82.5$€per year for the community as a whole (our buildings are composed of six households). While coordination costs may be much higher in practice, in some cases such a low value suffices to undermine the stability of our simple communities.

Results are in 2017 euro.

B   Impact of individual incentives on collective stability

This section takes things a little further and analyzes the impact of individual incentives on the collective stability of the community. We recall that section 4 showed that introducing convexities in the costs of a coalition could cause the coalition to become unstable. For simplicity of exposition, we defined it as a general "coordination cost" coalitions may incur. However, other difficulties related to the behaviour of households may arise, further justifying the ideas of coordination costs. This section proposes an alternative micro-foundation for this cost, namely diminished incentives to exert efforts individually, which harms the

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[5]These prices can be found on the EPEX website.

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community as a whole.

Energy communities are often motivated by environmental concerns. Two main ways to mitigate a households' environmental footprint are to produce green energy, or simply strive to consume less. The first point is the main driver of the present paper. The second one should, however, not be neglected, when taken within the paradigm of coalitions. Indeed, the value derived from efforts can again be divided into two parts. First, households decrease their energy demand, thereby reducing the energy cost. Second, and more concerning, energy reductions may reduce the capacity component of the community's electricity bill. Individual consumption and peak demand may, however, no longer be observed if the community is formed. The value created depends on the composition of the community and the fruits of any individual efforts will be shared with all participants. We show below that this may cause the coalition to lose its stability, if the community cannot contract on efforts.

For illustration, assume households may reduce their peak demand by $\theta \geq 0$, which comes at convex cost of effort $m\theta^2$, incurred by the household. When households are isolated, an effort of magnitude θ translates into a reward $\alpha\theta$, which corresponds to the reduction in network fees. A rational household individual maximizes her surplus:

$$\max_{\theta} \alpha\theta - m\theta^2 \Rightarrow \theta^* = \frac{\alpha}{2m}$$

In the case of perfectly symmetric or asymmetric households, the optimal level of efforts is the same as the privately optimal one if the households were isolated. However, the sharing rule implemented in the community may aggregate some of these benefits, hampering a households' incentive to make such efforts, since some of the benefits are not appropriated but are shared with the community. More precisely, assume now that households are part of a coalition of n players. If metering is no longer individual but collective, the network fee can only be allocated according to some observable parameter (household surface, number of residents, etc.). The only equilibrium is one where all households shirk and produce efforts so as to maximize their individual surplus:

$$\max_{\theta} \frac{1}{n}\alpha\theta - m\theta^2 \Rightarrow \theta_n = \frac{\alpha}{2nm} \tag{49}$$

Hence, lower individual efforts are observed when households aggregate behind a single meter, as consumers cannot internalize the full benefits of their actions. This is a classical case of moral hazard. This is all the more concerning when there is a large number n of participants in the coalition. Anticipating that members are more prone to exert efforts when coalitions are small, the larger coalition may no longer be stable, since the direct aggregation gains are mitigated by lessened efforts to mitigate peak demand.

Theorem 4. *If efforts are not observable, members of a coalition may find it individually optimal to exert (coalition-wise) suboptimal efforts. This decreases the value of coalitions and may make them unstable.*

Proof. Assume efforts and individual peak demand are not observable and are hence not contractible. The network fee to the coalition can then only be shared pro-rata of individuals' surface. Assume for simplicity the n households are symmetric: $\forall i \in I, \forall t \in T, f_i(t) = f(t)$. Households will exert efforts θ_n (see (49)).

All households being symmetric, we write again that $\forall S \subset I \sum_{i \in S} k_i(\mu) = sk(\mu)$. Let us assume that the core is not empty: there exists a stable allocation $x = (x_1, ...x_n) \in \mathbb{R}^n$ such that:

$$\forall S \subset I, \quad \sum_{i \in S} x_i \geq v(S) \tag{50}$$
$$\sum_{i=1}^{n} x_i = v(I) \tag{51}$$

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Considering equation (50) for all single-player coalitions, one gets:

$$\forall i \in I, \ x_i \geq \gamma k \sum_{t=1}^{T} g(t) - ck - \alpha(\max_t f(t) - \theta^*) - m(\theta^*)^2 \tag{52}$$

and summing over all players, one gets:

$$\sum_{i=1}^{n} x_i \geq n\gamma k \sum_{t=1}^{T} g(t) - nck - n\alpha(\max_t f(t) - \theta^*) - nm(\theta^*)^2 \tag{53}$$

using (51) ,we must have that:

$$- c(nk) + \alpha\theta_n - nc(\theta_n)^2 \geq -nck + n\alpha\theta^* - nc(\theta^*)^2$$
$$\Leftrightarrow nck - c(nk) \geq \underbrace{n\alpha(\theta^* - \theta_n) - nm\left((\theta^*)^2 - (\theta_n)^2\right)}_{\text{incentive component}}$$

It is easy to show that the incentive component is positive and increases in n. A first observation is that in the case when the capacity cost function is linear ($c(nk) = cnk$) this condition is never met and the coalition is necessarily unstable. When capacity costs show insufficient economies of scale, the inequality is less likely to be met as n grows. As the coalition grows, incentives considerations tend to decrease its value. If m is small enough, and hence the incentive effect is stronger, the equality may not be verified, even though costs are concave and the relation would hence hold, absent considerations on incentives. $\quad\square$

Thus, the incentives to exert efforts, and the lack of commitment to do so, may induce a form of diseconomies of scale, which in turn may cause the grand coalition to be unstable.