GIRLS AND SCHOOL MATHEMATICS IN CHILE: SOCIAL INFLUENCES IN DIFFERENTIAL ATTAINMENT AND MATHEMATICAL IDENTITIES

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Abstract

Girls' relationship with mathematics has been an extensive and contested area of investigation during the last 40 years, mainly in developed countries. This contrasts with the small amount of research from developing countries, where the topic has been largely neglected but may present different challenges. In Chile, such lack of empirical evidence is surprising, particularly because of several national reports describing attainment differences in the national assessment test (SIMCE), where girls are consistently outperformed by boys. Currently, there are no studies which systematically explore gender differences in attainment in Chile. In addition, only a small number of studies have tried to explain why these differences, as well as others in engagement, attitudes and enrolment in mathematics, arise in this country. The main goal of this thesis is to critically examine these issues by investigating how girls relate to mathematics during early adolescence in Chile, and how such relationships are influenced/mediated by certain social variables (e.g. social class, classroom cultures and peer group identities).

In order to do this, this thesis has adopted a mixed methods approach, thus linking analysis and results from studies that use both quantitative and qualitative methodologies. Firstly, I investigate the size and distribution of the gender attainment gap in Mathematics in Chile using a Multilevel approach to analyse data from the national census of educational quality (SIMCE). Here, I analyse the naturalization of gender differences based on results, and conclude that differences found in attainment between boys and girls are small and dependent on socioeconomic status.

I then explore how girls' subjective relationships with mathematics are constructed, and how different social influences mediate this process. Using the concept of Mathematical Identities [MIs] as a main tool I explore the influence of social variables on the construction of girls' MIs in Chilean classrooms and I also consider how teaching practices and peer social relations in the classroom mediate these identities. A key finding here is the positive relationship between students' perceptions of their teaching as student-centred and more positive MI, which is in fact the same for girls and boys. A second key finding is that both representational and enacted aspects of girls' MI are mediated by their relationship with peers and peer groups. This mediation process can be described as a negotiation of different forms of belonging to social groups, which involved also the negotiation of different MIs inside the classroom.

The main conclusion of this thesis is that in order to understand the role of gender in mediating girls' relationships with mathematics, we need to acknowledge the profoundly situated nature of this relationship in the cultural practices of the classroom, including mathematical practices, but also peer group practices. This argues against discourses that essentialise and naturalize 'gendered relationships with mathematics' which appear to be pre-dominant in the collation of national assessment data (like SIMCE) where categories such as gender, class, ethnicity etc. are viewed as causal or explanatory rather than produced 'in practice'.

Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Copyright Statement

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Chapter 1. Introduction: Gaps in the Literature and Purposes of this Thesis

1.1. Introduction

The relationship between gender and mathematics, and in particular, between girls and mathematics, is a very complex and contested area of research. Extensive investigation over the last 40 years has documented differences which favour males in relation to various outcome variables (e.g. attainment and participation in mathematical courses and careers). However, there is little agreement on this phenomenon. For example, there lies the question of which gender differences are actually important and consistent across cultures, stages of schooling, or across different levels of the distribution of a particular outcome, etc. Different areas of research have struggled to build a comprehensive model in order to account for this phenomenon. Such difficulty appears to be related not only to the intrinsic complexity of the problem, but also to contradictory data reported in different studies.

One argument that explains why the study of the relationship between gender and mathematics poses so many challenges is that the definition of this relationship depends largely on how mathematics and gender are conceptualised within a particular context (Atweh & Cooper, 1995). An illustrative example comes from the UK context, when a new mathematics qualification was introduced for 16 year olds in the 1980s (GCSE). While its predecessor (O-level) was narrowly focused on the recall of procedural information, the new GCSE in mathematics included non-traditional assessment techniques (e.g. coursework), and was interested in a wider set of skills and abilities (Elwood, 1999). It was believed that these changes would encourage the development (and evaluation) of important skills not easily tested by other forms of assessment (such as problem solving), thus helping the teaching and learning process. In other words, this change in assessment was not simply a modification of assessment contents, but more importantly, a change in what was considered important knowledge and skills in mathematics at that point in time. However, an unexpected outcome of this modification was its impact on gender patterns of attainment, where girls' grades in mathematics aligned with their male counterparts (Gorard, Rees & Salisbury, 2001). Although it has since been shown that these changes were not the sole reason for this trend, it has been realised that girls tend to do better in these non-traditional sections (particularly coursework) in comparison with boys (Elwood, 1999).

In addition, several studies from social psychology have also documented how notions of ‘what it means to be a girl’ also have an impact on the complex relationship between gender and mathematics (Nguyen, & Ryan, 2008; Schmader, 2002; Spencer, Steele & Quinn, 1999). If beliefs about women not being suited for mathematics operate in a particular social context, girls tend to underperform when this stereotype is made salient. This has been called a *stereotype thread* (Spencer, Steele & Quinn, 1999). For example, a study set in Israeli classrooms has shown how changing these beliefs can have an impact on girls’ dispositions, and presumably also their performance (Mittelberg, Rozner & Forgasz, 2011). The authors compared two classrooms (one Jewish and one Druze) and showed that, whilst the Jewish teacher held conventional gender stereotypes of female inferiority in mathematics, the Druze teacher believed that girls needed extra affirmative actions in order to overcome gendered cultural barriers. In consequence, being a girl held remarkably different meanings for these two teachers, thus influencing their students' relationships with mathematics in accordance with their cultural contexts.

In consideration of how different contexts influence definitions of gender and mathematics, a theoretical and methodological consequence shows that researchers must adapt their work to changing historical social conditions (e.g. school reforms, educational policy, etc.), as well as expanding to different contexts. Nevertheless, this has not been the case, as research on gender and mathematics has been predominantly implemented in more developed countries [US, UK and Australia]. Only recently, investigations carried out in growing economies and international comparative studies have provided evidence from more varied contexts. Such evidence has supported a situated and heterogeneous nature of the relationship between gender and mathematics, with several studies reporting a wide variation between countries in terms of gender differences and mathematics attainment and performance (Else-Quest, Hyde & Linn, 2010; Guiso, Monte, Sapienza & Zingales, 2008) and attitudes and affective dispositions towards the subject (Else-Quest, Hyde & Linn, 2010).

In Chile, (the country which has been the focus of this study), although female under-attainment has been consistently reported, empirical evidence is almost non-existent. This situation contrasts with the significance attached to the issue presented, both by the media, and by policymakers, who have implemented national policies in order to address this problem. The combination of high salience media and a lack of empirical and context-based knowledge can be problematic for several reasons: (1) it can perpetuate the stereotype that girls are not suitable for mathematics, a stereotype already present in many Chilean teachers and students (del Río & Strasser, 2013; Mizala, Martínez, & Martínez, 2015); and (2) it can lead policy makers to take decisions not necessarily based on empirical evidence with consequent limited impact. For these reasons, research on this topic is required urgently. Following this, this thesis attempts to address this lack of knowledge by systematically exploring the relationship between girls and mathematics in Chile.

1.2. The Gender 'Gap' in Mathematics

Research on gender differences, and comparisons drawn between male and female students in terms of different mathematical outcomes, has been a key focus for the literature on gender and mathematics. Atweh and Cooper (1995) note that this approach has mainly conceptualised gender in two ways: as an independent variable (the effect of gender on any dependent variable) or as a mediator variable (how gender interacts with other variables in order to construct differences in any dependent variable). Following this approach, research on gender differences can be divided into two main areas of study depending on the main outcome under investigation: (1) Cognitive/Performance variables, such as performance, attainment and abilities; and (2) Affective variables, such as attitudes, emotions, dispositions and identities.

Studies which have focused on cognitive/performance variables have tended to report girls as being outperformed by boys (e.g. Hyde, Fennema & Lamon, 1990). However, there is also substantial evidence to suggest that these differences are not stable and, therefore, not easily described by simple mean differences. It has been noted, for example, that any gender difference in learning outcomes will depend on the context in which they are being measured. For example, some comparative studies show that, whilst in some countries gender differences in achievement are narrowing, in others differences are prevalent (Byrnes, Hong & Xing, 1997; Else-Quest, Hyde and Linn, 2010; Guiso, Monte, Sapienza & Zingales, 2008; Ma, 2008; Marks, 2008). Furthermore, it has also been noted by several studies that even inside the same country, gender differences in attainment may also vary depending on how they interact with other social categories, such as ethnicity and social class (Grant & Sleeter, 1986; Lubienski, 2008).

Evidence from studies exploring gender differences at different stages of schooling also points to a wide variability. One general tendency here is that gender differences in attainment tend to increase during compulsory education, reaching a peak during adolescence (Friedman, 1989; Hyde, Fennema, & Lamon, 1990). Such findings have led several researchers to suggest that while differences may be small or even non-existent during early education and primary school, social influences and school experiences that occur later on may have an effect on attainment and performance, thus increasing girls under-attainment.

Gender differences in attainment can also be explored at different levels of the outcome measure. Some authors have also suggested that gaps between male and female students are not stable at different points of the score distribution, with boys showing more variability than girls, and therefore, displaying increased higher and lower end scores. There is some evidence that this increased score variability leads to a greater number of males in the upper tail of the distribution (Feingold, 1992; Hedges & Nowell, 1995; Hyde, Lindberg, Linn, Ellis, & Williams, 2008), and hence, in a better position for accessing highly competitive mathematical and non-mathematical careers.

While international research has presented strong evidential support for variability in attainment differences between genders, there is sparse evidence of this in Chile. Until now no comprehensive study on the topic has been published, with most of the data used for policy making based on mean comparisons of the entire population. Following on from this gap in the Chilean literature, one of the goals of this thesis will be to culturally contextualise 'the problematic issue for girls' in school level mathematics by exploring the distribution of differences in attainment. These results are presented in Chapter 4, where gender differences in the National Assessment test (SIMCE) scores are explored in their interaction with other social variables (particularly socio-economic status), change at different levels of schooling (particularly in early and late primary education) and the variation displayed at different levels of the outcome variable (high and low attainment). In addition, this chapter also explores the relative contribution of schools, exploring the theory of differential effectiveness according to gender.

As previously noted, gender differences have not only been explored in terms of cognitive learning outcomes but also from an affective perspective (e.g. Eccles, 1994; Fennema & Sherman, 1977). In contrast with the 'cognitive' line of research, studies focusing on 'soft' mathematical outcomes have presented a much more consistent picture, with girls reporting a less positive attitude (Frenzel, Pekrun & Goetz, 2007; Frenzel, Goetz, Pekrun & Watt 2010), a lower disposition towards mathematics (Buschor, Berweger, Frei & Kappler, 2014; Nagy, Trautwein, Baumert, Köller & Garrett, 2006) and a lower self-concept or self-efficacy (Eccles, Wigfield, Harold & Blumenfeld, 1993). These effects appear to be independent of girls' attainment, thus suggesting that girls' subjective affective experience is not necessarily associated with an actual lack of mathematical skills.

Nevertheless, there are some studies which challenge the universality of this assumption, mainly by reporting that in some countries this disadvantage is not observed. For example, Kaldo and Hannula (2014) found that, at university level in Estonia, female students were more positive towards mathematics than their male counterparts. In this context women were observed as valuing mathematics more, feeling more competent and perceiving their teachers in a more positive light.

In relation to the study of gender differences in affective variables in Chile, the evidence is equally scarce. Data from PISA on attitudinal and affective results shows that female students score consistently lower on most affective measures (Agencia de Calidad de la Educacion, 2013a). According to these results, women are reported to be less open when solving problems and present less motivation towards studying mathematics. In addition, the same report also states that Chilean girls exhibit lower levels of self-efficacy and self-concept, higher levels of anxiety and a lower disposition to enable participation and continuation in mathematics.

1.3. Theoretical Accounts Used to Explain the 'Conflicted' Relationship Between Girls and Mathematics

By and large, two broad theoretical models have been used to account for the 'gender gaps'. A *psychological model* understands this gap to be the product of psychological variables, while a *sociocultural model* explains it as the product of social influences on gendered relationships with mathematics.

Studies based on a psychological model have attempted to determine the underlying psychological variables which explain girl's/women's under-attainment, as well as negative attitudes and emotions towards mathematics. These studies report that girls' lack of confidence (Ross & Bruce, 2012), lower levels of self-concept and self-efficacy (Eccles, Wigfield, Harold & Blumenfeld, 1993), high levels of anxiety and lower levels of enjoyment (Frenzel, Pekrun & Goetz, 2007) and self-attribution of failure, but not success (Stipek & Gralinsky, 1991), appear to be related to their lower attainment in mathematics and their choice to study mathematics (or not) beyond the compulsory level. For example, one psychological model which has been extensively used is the Control-Value model of academic choice (Eccles, 1994). According to this model individuals' academic choice and performance are affected by their expectations of success as well as the perceived value that they attribute to the task in question. Although this model acknowledges that the 'social milieu' influences students' expectations, the main focus lies in understanding how individual variables account for differential choices and performances. The emphasis is mainly focused on how self-concept, perceived difficulty, attribution of success and locus of control influence individuals' expectancy of success and the values attributed to the task (Boaler, 2002a).

A different line of research, which has also followed a psychological approach, has suggested that boys and girls, because of some psychological traits, approach the activity of doing mathematics differently. For example, some studies suggest that while girls are compliant and follow teacher instructions, boys tend to resist them (Fennema and Peterson, 1985; Fennema, Carpenter, Jacobs, Franke & Levi, 1998). According to these authors, this provides girls with better possibilities in lower ability problems, such as counting, but allows boys the chance to develop independent solutions for mathematics problems, which leads to further autonomous and flexible learning (Grieb & Easley, 1984, in Fennema and Peterson, 1985). Consequently, other authors suggest that girls perform better when solving familiar tasks (i.e. those which focus on contents already learnt), while boys are better at exploring unknown problems, again mentioning creativity and flexibility as male attributes (Kimball, 1989). These ideas are supported by the over-achievement of girls in school grade attainment (familiar tasks), and under-achievement in standardised tests (unfamiliar problems) (Kimball, 1989), and by girls' higher performance in standard computation and conventional problems (Hyde et al., 1990; Gallagher & De Lisi 1994; Gallagher et al., 2000). By contrast, boys are said to score higher in the assessment of applications with regards to extension problems (Hyde et al., 1990) and unconventional problems which are said to require flexible applications of familiar procedures or the generation of a new solution (Gallagher & De Lisi 1994) thus, reflecting an autonomous learning approach to mathematics.

Authors from a sociocultural model have criticized these psychological approaches, arguing that a focus on individual traits naturalises and essentialises differences without considering that they may be related to the particular social context. In other words, by naturalising gender differences, psychological approaches do not consider that these differences may be the product of differential social and cultural experiences of mathematics for boys and girls (Boaler, 2002a; Walkerdine, 1998).

This critique has led sociocultural researchers to develop an alternative model in order to explain gendered relationships in mathematics (Walkerdine, 1998). According to this model, gendered relations are embedded in any activity (including mathematics) and are the product of gendered (as well as classed and racialised etc.)[1] experiences which boys and girls encounter in those particular contexts where they grow up. In consequence, the emphasis of the sociocultural model is not on individual characteristics, but mainly on how gender relations with maths are constructed and produced within a social context, as well as the constraints and affordances offered by a particular environment or by certain discourses available in those contexts.

The interpretation of data in relation to different approaches to mathematics is a good example of how the sociocultural perspective contrasts with the psychological perspective. As explained above, from a psychological perspective it was suggested that some gender differences can be explained by boys' creativity and autonomous learning behaviour in contrast with girls' compliance when approaching mathematics. In contrast, Walkerdine and her colleagues (1998), came from a sociocultural perspective and suggested that this data could be interpreted

[1] I will consistently refer to gendered experiences, suggesting that experiences are influenced by social categories. How these experiences are racialised and classed will be assumed.

differently, since common beliefs about gender could be reproducing these differences. In other words, girls are not naturally compliant, but they become compliant through the social interactions they experience. After observing students, parents and teachers at different stages of schooling, Walkerdine concluded that girls were often encouraged by significant others (especially teachers) to be compliant and obedient. Boys, on the contrary, were encouraged to behave in a challenging and flexible manner. It was also suggested by these authors that girls and boys behaviours were valued differently, mainly because gender discourses lead teachers and parents to interpret boys' active and disruptive conduct as challenging (but still productive in terms of learning) while girls were perceived as passive -rule followers- in their learning. Since those early studies, there has been a growing interest in revealing how social (including gendered) inequalities are reproduced in the interactions between subject and society.

Although different concepts have been used to account for the relationship between subject and society in the reproduction of social inequalities, in the last 20 years the notion of identity has become increasingly popular, particularly in studies of gender relationships with mathematics (see Black et al., 2008). The concept of *identity* and its extension to *mathematical identity* are useful because they allow the exploration of processes underlying the relationship which students develop with the subject over time (Walls, 2009), as well as the affective aspects of this relationship (Hannula, 2012; Zan, Brown, Evans & Hannula, 2006). As an identity (or identities) is considered to be constructed in social practice, using the social products available in social relationships, the concept allows one to 'zoom out and zoom in' (Lerman, 2000) into the subjects' individual use of these products and how they are socially produced. As a consequence, some authors have suggested that identity provides the 'missing link' between the social and individual domains (e.g. Boaler, 1999a; Sfard & Prusak, 2005).

Even though identity has become a popular concept amongst researchers, its use has not been exempt from critics. Some authors have criticised the underdeveloped conceptualisation of identity itself (Brubaker & Cooper, 2000), and particularly the lack of theorisation on how identity links to social and individual domains (e.g. the use of *self* in Stetsenko & Arievitch, 2004). A possible explanation for this may well be that there is a wide variation in how the concept is used. Although all the sociocultural perspectives are equally interested in overcoming individualism (and the naturalisation/essentialisation of differences), they differ significantly in how the subject, its social context and the relationship between both is conceptualised (see Stetsenko & Arievitch, 2004), in particular with regards to the balance between the influences of social structure and individual agency. For example, some sociocultural approaches have produced a 'reductionism upwards' (Dunn, 1997, in Stetsenko & Arievitch, 2004), where the self is fused with its context or social practice, whilst others who are interested in local practices appear to ignore notions of power at play within these practices in relation to the self (Contu & Willmott, 2003; Hodge, 1998).

Despite these critiques, the concept of identity offers important advantages in order to explore gender differences in mathematics, at least in comparison to the available psychological models. Nevertheless, the limitations presented by several authors cannot be ignored. This thesis addresses this problem, by means of critically reviewing how the concept of identity has been used in Mathematics Education Research. Based on an examination of how identity has been defined and operationalised in the literature, a proposal of how conceptual coherence can be achieved is presented in Chapter 5.

1.4. Processes of Mathematical identity Construction During Adolescence

A key issue to consider when investigating the construction of mathematical identities is how this process interacts with developmental influences. Several authors have suggested that identity development begins with the first encounter between subject and society, but the course of this development changes according to the emergence of new abilities, as well as progressively more complex social requirements (e.g. Erikson, 1980; Leontiev, 2009). Throughout life, children experience many 'vital crises' (Erikson, 1968) and develop different 'motives', thus, progressively engaging in different activities that prepare them for their next stage of development (Leontiev, 2009). Authors from the sociocultural tradition have stressed the relevance of this process in order to carry out identity research, particularly in relation to how children encounter and relate to different activities during their lifetime (Hammack, 2008; Nasir & Saxe, 2003; Penuel & Wertch, 1995). As Nasir and Saxe have pointed out, “as individuals grow, the social meaning of participating in particular practices and the forms of participation may shift” (Nasir & Saxe, 2003, p.15).

Even though each developmental phase contributes to the construction of an individual’s identity, adolescence has been historically considered as a critical period. In fact, the classic work from Erikson proposed that during adolescence an identity crisis takes place as a consequence of the dramatic changes in physical, cognitive and emotional capacities, as well as the emergence of new social challenges. In the cognitive domain, for example, adolescence has been described as the period where formal operational thinking emerges (Inhelder & Piaget, 1958). This is not a minor developmental accomplishment for identity, since performing formal operations allows the individual to relate to the environment differently, via hypothetical thinking, abstraction and logical deduction (Inhelder & Piaget, 1958; Steinberg, 1989). In other words, the emergence of these cognitive abilities changes the way in which the individual relates to himself and others, becoming more self-aware, self-conscious and better able to use language to mediate own behaviour and will (Vygotsky, 1998). This increased reflection and self-awareness allows adolescents to develop conceptualisations and stories about themselves, which are directed at themselves and others (Nurmi, 2004), thus influencing their emerging ‘narrative identities’ (Sfard & Pusak, 2005). These narratives become particularly important, as they somehow produce, and at the same time guide, subjects consequent behaviour, development and choices (Holland Lachicotte, Skinner & Cain, 1998).

As noted above, during adolescence individuals are faced with new challenges and tasks that society expects them to engage with. In consequence, some authors have described this period as a preparing phase, where adolescents need to engage in exploring behaviours related to sexual relations and vocational choice (e.g. Erikson, 1968). In this sense, during adolescence, the emergence of new motives directs the subject’s attention to future-related activities where preferences and roles need to be explored and rehearsed (Elkonin, 1971).

Changes in capacities and motives are mirrored by changes in the activities that lead to development. Some authors have suggested that, while during childhood the leading activity is learning and systematic studies at school, during puberty and adolescence the leading activity is social peer relations (Elkonin, 1971; Karpov, 2003). This shift has been observed as an increased interest in social interaction (Brown, 2004), as well as more frequent conflicts with authority figures such as parents and teachers (Paikoff, & Brooks-Gunn, 1991; Steinberg, 1987). All these changes and subsequent 'identity crisis' challenge the adolescent to adjust their ideas of themselves in relation to the tasks that society asks of them.

Since adolescence is a critical period in the construction of individual's identities, it is likely that, due to developmental changes and shifts in leading activities, the construction of *mathematical identities* may also undergo important transformations during puberty and adolescence. In fact, several authors have reported a decline in the positive attitude-motivation, utility value, interests and positive emotions during this stage of development (Chouinard, 2008; Frenzel, Goetz, Pekrun & Watt, 2010; Ma & Cartwright, 2003). More importantly, this decline has been described to be more marked among female students (Brown, et.al., 2010; Fennema & Sherman, 1977; Stodolsky, 1985; Watt, 2004; Wigfield & Eccles, 1994). This data suggests that, in the developmental path in which mathematical identities are constructed, adolescence can also work as a critical period, where certain components of mathematical identity are crystallised in enduring dispositions that can influence a future relationship with mathematical activity [see chapter 6]. For all these reasons, this thesis has a strong focus on exploring the process of mathematical identity construction during adolescence. It focuses particularly on how patterns of differential attainment (which may be in turn related with students' identities) change in the transition from childhood to adolescence (between 9 and 14 years old), and in different forms of mediation of mathematical identities in a year 7 classroom (13-14 years old). The particular forms of mediation of mathematical identities in which this thesis is focused are described below.

1.5. Influences in the Construction of Mathematical Identities During Adolescence: Teaching Practice and Peer Relations

This thesis will focus on two processes of mediation in which girls' mathematical identities develop particularly during adolescence: the characteristics of the classroom and teaching practice and the relationship with peers inside the classroom.

In relation to classroom and teaching practices, it has been suggested that part of the decline in attitudes towards mathematics during adolescence can be attributed to a progressive shift during middle school to *teacher-centred teaching*, or what has been termed as *traditional pedagogies*. For example, it has been described that during middle school, when compared with primary school, there is a greater emphasis on teacher control and discipline (Brophy & Evertson, 1978; Moos, 1979, In Wigfield & Eccles, 1994), which translates into more time spent conducting whole class teaching (Rounds & Osaki, 1982, In Wigfield & Eccles, 1994). In addition, a shift from mastery goals (to develop and ability) to performance (to demonstrate ability or to avoid the demonstration of a lack of ability) has also been noted (Anderman & Midgley, 1997; in Urdan & Midgley, 2003), a trend that mirrors the more frequent use of public evaluation (Harter et.al., 1992, In Wigfield & Eccles, 1994). According to Wigfield and Eccles (1994) a potential limitation of these traditional practices is that they run against students' developmental needs and motives, since they encourage individual work, competition/comparison, and limit autonomy and active engagement. Furthermore, these practices appear to disrupt social networking when arguably during adolescence students become more interested in socialising with peers.

In addition with being counterproductive to students' developmental needs, it has also been noted that traditional teacher-centred pedagogies can be particularly detrimental to girls' needs. Becker (1995), for example, has proposed that girls learn better when mathematics is treated as a collaborative process which occurs between students and teachers, and not when mathematics is a product delivered by the teacher, which needs to be passively assimilated by the student. Since these theoretical hypotheses have not been empirically tested yet, Chapter 6 will explore the relationship between students' perceptions of their teaching practice and different forms of identification with mathematics, in terms of the level of positive and negative emotions attached to the activity, and the development of a mathematical self-concept and the future disposition to the activity. The main question addressed in this chapter, is whether increasing levels of student-centred teaching are associated with more positive identifications for students in general, and whether this effect is greater for girls.

A second interesting influence in the construction of girls' mathematical identities during adolescence is peer relations. As noted previously, social relations with peers are perhaps the most important leading activity during adolescence, thus contributing to the shaping of academic and mathematical identities. This claim has received indirect empirical support from different areas of research. Studies examining how the composition of the classroom affects individuals, have consistently reported that students use social comparison [with other peers] in the construction of their identities. This phenomenon has been called the 'big fish little pond effect' (Marsh, 1987; Marsh, Trautwein, Lüdtke & Köller, 2008), since students in classrooms with higher attainment tend to report lower levels of self-concept and self-efficacy. In addition, evidence from studies on gender patterns in the development of identities and subjectivities have reported that students often need to negotiate their academic identities in order to fit in with their peer culture (Francis, Skelton & Read, 2010). This can be observed in student behaviour as disengagement, or downplaying of achievements, in order to balance high attainment with popularity within the peer group. Other studies interested in mathematics education have also followed this line of enquiry and have showed how popularity can be used to negotiate access to mathematical resources in the classroom (Gholson & Martin, 2014).

If the construction of a mathematical identity is influenced by peer relationships, it can be expected that during the negotiation of belonging to different peer groups, girls will also negotiate different mathematical identities in the classroom. Unfortunately, the existing evidence does not offer any insights into how this process of negotiation takes place, including how such processes influence students' behaviours and relationships with mathematical activity. In Chapter 7 this gap was addressed by exploring the influence of peers on the construction of mathematical identities, using a case series approach in a group of high attainment girls. Furthermore, this Chapter explores in more detail how patterns of attainment (Chapter 4) influence a particular gendered relationship with the mathematics practice (Chapter 6), with diversity between girls being a main focus of exploration. An approach focused on diversity between girls is particularly useful for challenging essentialists' views of gendered relationships with mathematics, as it questions whether girls in groups share something simply because of their gender.

1.6. Summary of the Purposes of this Thesis

The main goal of this thesis is to understand how gender is implicated in the sociocultural processes which mediate students' identification with mathematics. In order to explore this complex issue, 4 specific research questions will be considered:

1. How can gendered patterns in academic attainment during primary education in Chile be described?

As mentioned previously, no comprehensive studies on this topic have been published yet in Chile, with most of the existing knowledge used for policy building based rather crudely on mean comparisons of the entire population. In Chapter 4 gender differences in mathematics will be explored, as well as how such differences interact with other variables [socioeconomic status of students' families and schools] and if they change during primary (between age 10 – 14) and at different levels of the attainment outcome.

2. How do girls relate to the mathematics they experience in their classrooms and how do these experiences mediate different mathematical identities?
3. How are different 'positive relationships' with mathematics constructed in the classroom and what is the influence of peers in such a process?

In relation to question 2 and 3, and as reviewed before, different social processes have been described as central to the mediation of mathematical identities. In particular, in this thesis I have focused on two which are particularly relevant during adolescence: students' relationships with the mathematical practice itself and with their peers inside the classroom. Regarding the first process, it has been proposed that the type of teaching (as an important part of how the mathematical practice is experienced) is relevant in mediating students' identifications with mathematics, mediation which may also be influenced by the gender of the student. In Chapter 6 the question of the relationship between perception of teaching and different forms of identifications with mathematics will be explored, including its mediation by gender.

Following the same line of enquiry, in chapter 7 the relationship between different girls with the mathematical practice in their classrooms will be explored. In particular, in this paper I will explore in detail how this relationship is also mediated by peers, who influence girls' behaviour, values and attitudes towards mathematics.

This last topic, although highly relevant during adolescence, has been surprisingly neglected in the mathematics education literature.

Finally, as this thesis makes use of the concept of mathematical identities, it also addresses the need to delineate a theoretical model in order to investigate students' mathematical identities from a sociocultural point of view. In doing so, this thesis attempts to answer a rather conceptual problem:

4. How identity has been conceptualised and operationalised in the Mathematics Education research community?

In order to address this specific research question, this thesis will explore different conceptualisations in the mathematics education research literature [Chapter 5] and will attempt to apply them in two empirical papers [Chapters 6 and 7].

In summary, this thesis consists of four research chapters [4, 5, 6, 7] written in the form of scientific papers, each of them tackling one or more of the aforementioned research questions. In addition to these research chapters, where results are presented, four additional chapters are also included. After the introduction (Chapter 1), Chapter 2 will offer contextual information about Chile, the country where this study took place. Of particular interest to the problem of gendered relationships with mathematics, this chapter will present a general overview of the Chilean educational system as well as some ideas regarding how the Chilean culture understands and positions women in education and society. Chapter 3 will extend the methodological approach followed by each of the result papers, thus offering a more detailed account of the methods that were used to treat the data, and the rationale behind such choices. Finally, Chapter 8 will present an overarching discussion of the main findings and contributions of this thesis, thus attempting to offer some insights into the 'problem' of girls' relationships with mathematics during adolescence in Chile. This section will also address the theoretical and technical limitations of this thesis and its implications for future research, policy making and mathematical practice.

1.7. Rational for an Alternative Format Thesis

The four result chapters that are included in this thesis, which specifically address each of the research questions, are written as scientific papers, thus using a format that is suitable for publication in peer-reviewed journals. This means that each of these chapters is a self-contained unit that includes a review of the relevant literature, methodology, results and discussion. I have decided to follow this alternative format for my thesis because such a structure has helped me to define and address with greater accuracy the key contribution to knowledge made by this thesis.

The alternative format also facilitates quick dissemination of results, both in the Chilean (and Latin-American community) and the mathematics educational research community. Furthermore, at the moment of submission two chapters of this thesis had been submitted for publication and were under review.

I could have not written this thesis without the support of my supervisors and colleagues from Manchester University and elsewhere. This is why various papers were co-authored by different collaborators according to their level of contribution. In relation to Chapter 4, I collected and analysed data, as well as generated the final draft. This is why I am the sole author of this article. Nevertheless, Prof. Julian Williams contributed by reviewing final versions of the drafts and suggesting modifications. In relation to Chapter 7 I also defined research questions, collected and analysed data and wrote the respective drafts. Nevertheless, both of my supervisors [Prof. Julian Williams and Dr Laura Black] contributed more extensively in the final product. Their involvement was mainly in defining theoretical gaps and helping in structuring the presentation and discussion of the data. This is the reason why they are both co-authors in this article. I also received help by Dr Christian Salas, who contributed with editing and structuring initial versions of this paper. This initial version was presented and accepted for publication in the proceedings of the Congress of European Research in Mathematics Education (CERME 9) (Radovic, Black, Salas & Williams, in press).

Chapter 2. Chilean Context: Its Educational System and the Position of Women in Society

2.1. Chile: a Growing Economy With High Levels of Social Inequity

Before presenting the methodology and research chapters, I would like to offer the reader some information about the specific context where these studies were carried out. Chile is a relatively small country located in Latin-America, which has been considered as one of the most stable and prosperous nations of the region (The World Bank, 2015). As an example, in 2010 Chile became the first South American country to join the OECD, with the World Bank considering a high-income economy.

Chile's population reaches about 16 million, with an approximate ratio of 0.97 males to females and only 11.11% of the total population claiming to belong to an indigenous minority (INE, 2012). Despite reports which describe Chile as being economically successful, the country exhibits high levels of inequity (The World Bank, 2015). For example, in 2011, the OECD ranked Chile as the country with the largest differences in terms of income. Whilst in other countries which belong to the OECD, the average income of the richest 10% is about nine times that of the poorest (9:1 ratio), in Chile this ratio is 27:1 (OECD, 2015). According to national reports, the number of people living below the poverty line in 2013 was 14.4%. However, this percentage rose by up to 20% when a broader definition of poverty was applied[2] (CASEN, 2013a).

2.2 The Chilean Educational System

Education in Chile is managed via a combined public and private system, where the government has only a conducting role. Both private and public schools teach under a national school curriculum framework, which sets foundational objectives and minimum contents for each subject per grade. However, within this framework, schools can design their own plans and use their own methods. The government maintains normative, and supervisory functions, as well as technical and financial support. The Ministry of Education is in charge of the national assessment for education quality (SIMCE[3]), a compulsory evaluation of teachers from public

[2] In addition to the definition of poverty and in relation to income, this broader definition was defined by other multiple dimensions (education, health, vocation, social security and housing).
[3] http://www.simce.cl

schools (DocenteMas[4]), and all international assessments in which Chile participates (TIMMS, PISA, ICILS, PIRLS, TERCE, ICCS[5]).

The educational system includes 4 levels of schooling: pre-school, primary education, secondary education and higher education. In Chile, both primary and secondary education are compulsory, comprising 12 years of mandatory schooling. Primary education (from 6 to 13 years) has been compulsory since 1965 and today approximately 100% of the population is enrolled (CASEN, 2013b). It is divided into 8 years (grades) and two sub-levels. In the first sub-level, students tend to have one general teacher for all subjects. In the second sub-level, there are different teachers for different subjects.

Secondary education (14 to 18 years) has only been compulsory since 2003, reaching a 98% enrolment total in 2009 (CASEN, 2013b). It consists of four grades and offers students a choice of two types of diploma. In the Scientific-humanities diploma students can choose a subject in either science (math, physics, chemistry or biology) or humanities (literature, history, philosophy), thus receiving a higher frequency of lessons in the area of their choice. In the Technical-Professional diploma students receive extra education in technical areas, such as electricity, mechanics, metal-work, etc. A total of approximately 63% of secondary education students attend Scientific-humanities schools (Mineduc, 2010). It is important to point out that students in Technical-Professional diplomas tend to come from lower income families, have lower access to higher education and obtain lower scores in the admission test for higher education (Brunner, Elaqua, Tillett, Bonnefoy, Gonzalez, Pacheco & Salazar, 2005).

After secondary education students can continue their studies in three types of institutions: Universities, Professional Institutes or Centres of Technical Training. The entire higher education system has a total enrolment of 51% (CASEN, 2013b). There are 25 traditional universities (which integrate the *Consejo de Rectores CRUCH*) and over 35 private institutions. Most of the traditional universities are better positioned in academic rankings and many of the private universities are considered to be poor quality. There is a single admission system to all the traditional institutions, called PSU (University Admission Test for its Spanish

[4] http://www.docentemas.cl
[5] As reported in http://www.agenciaeducacion.cl/estudios-e-investigaciones/estudios-internacionales/

Prueba de Seleccion Universitaria). The system includes knowledge tests on subjects like mathematics, languages, science and social sciences. In 2010, almost 100% of the students who finished their secondary education took the test[6].

In relation to funding, primary and secondary schools depend on public or private administration and funding. Public schools are administered by the local government and receive funds according to a per capita subsidy. The amount of money they receive depends directly on enrolment and does not depend on student characteristics. Students don't pay any fees and 44% of Chilean students attend public schools (Mineduc, 2010).

There are two types of private schools: non-subsidised and subsidised. Private non-subsidised schools finance themselves 100% from tuition payments and they account for nearly 7% of the students (Mineduc, 2010). Private-subsidised schools receive the same funding as public schools, but are administered by private institutions. They can also ask for small tuition payments from the family (the maximum amount is set by law). These schools account for 48% of the students in the country (Mineduc, 2010).

The financing mechanism used to subsidise schools (private and public) has been in operation since 1980. It works as a voucher system, where funding sits with the student, allowing families to select the school of their choice albeit public or private. There were 4 institutional features in the voucher system between its instauration and the year 2008[7] (Mizala & Torche, 2012). Firstly, the voucher provides a flat rate per-student subsidy, without adjustment according to socioeconomic status. Secondly, while public schools have to accept all applicants, private-subsidised schools can use selection (such as entry exams and parental interviews) and expulsion mechanisms. Thirdly, in public schools teachers are governed by special legislation (the Teacher Statute), with wages based on uniform, pay-scales independent of merit, and with restrictions for dismissal under-performance reasons. In contrast, in private-subsidised schools there are flexible criteria for personnel recruitment, dismissal and promotion. Finally, privately-subsidised and public

[6] CRUCH reports that 206.807 students who finished their education in 2009 took the test. Mineduc reports 207.948 as the total enrolment figure in the last year of secondary education that same year.

[7] The Chilean voucher system is undergoing a substantial transformation. A recent 2008 law introduces a means-tested voucher and forbids subsidised schools from selecting primary school students based on entry exams and parental interviews. Its implementation has started gradually in 2015.

schools differ in their ability to collect additional funding: whilst the former can charge parental fees, the latter is not entitled to (Mizala & Torche, 2012).

As shown in Table 2.1, there is an enormous socioeconomic stratification in the Chilean educational system. Private non-subsidised schools serve the upper class, with 90% of their population coming from the two richer quintiles. There is also a significant economic overlap between the public and the privately-subsidised schools. However, public schools tend to capture the lower and lower-middle groups, whilst private-subsidised schools attend middle and upper-middle groups.

Table 2.1:

Enrolment in school sector by family income quintile

Income quintile	Public	Private subsidized	Private non-subsidized
1	36.1	23.0	2.3
2	28.4	24.1	2.3
3	19.9	22.1	5.3
4	10.9	19.9	19.1
5	4.6	10.9	71.0
Total	100	100	100
Total Enrolment	44	48	7

Note. Percentage distribution and total enrolment in each type of school (percentage of the total distribution). Own elaboration based on CASEN 2009 survey

In a study which examined the socioeconomic distribution of achievement within and between schools, Misala and Torche (2012) found that, as a sector, privately-subsidised schools receive a broad population in terms of their socioeconomic status [SES], but each school shows high homogeneity in relation to the SES of their own students. The authors suggested that the characteristics of the Chilean voucher system, as well as the possibility for privately-subsidised schools to be able to select students, promotes the specialisation of privately-subsidised schools in different market niches (Mizala & Torche, 2012).

There are also large differences in the availability of resources across different types of schools. The amount of money spent per student in non-subsidised schools, for instance, is more than three times the amount of money spent in a public school, and

2.6 times the money spent in privately subsidised schools (Marcel & Tokman, 2005). The number of students per teacher is also significantly lower in private non-subsidised schools compared to subsidised schools. Whilst in private schools a teacher has on average 16 students, in a public and a privately subsidised school there are 25 and 30 students per teacher respectively (Mineduc, 2010). It has also been reported that schools in socially advantaged areas are more likely to have their own mathematics curriculum and better prepared teachers who have emphasised advanced mathematical content (Ramirez, 2006). All of the afore-mentioned characteristics of the Chilean educational system suggest that it is in part responsible for the large economic inequity in existence in the country.

In terms of access to higher education, large differences have been found between students of different socioeconomic status. In 2013, the estimated enrolment for the richest quintile reached almost 90%, with only 35% for the poorest (CASEN 2013b). Although these differences have decreased in the last 15 years (see figure 2.1), most students from lower socioeconomic backgrounds are currently attending private institutions, which have been described as less selective and of poor quality (Brunner et al., 2005). Traditional Universities with more rigorous selection processes, better quality of teaching and research, consistently appeal to students with higher social and economic capital (Brunner et al.,. 2005). Considering this scenario, Brunner and colleagues (2005) have argued that the characteristics of the higher education system have not contributed to social mobility. Low public funding, very high private funding and public subsidies in the form of loans (not scholarships), have resulted in a system which contributes towards maintaining social hierarchies and an accumulation of human capital.

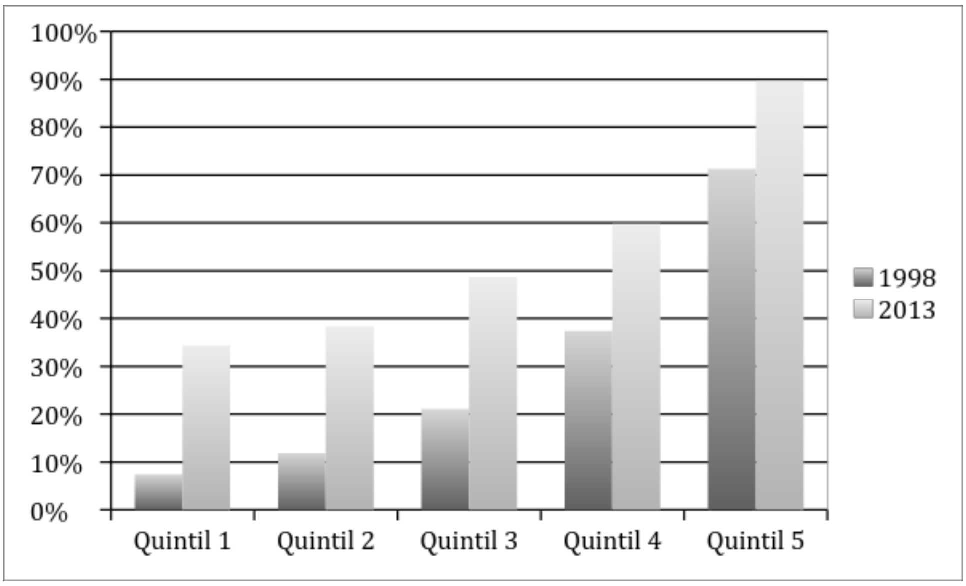


Figure 2.1. Students' enrolment in higher education by income quintile. 1998 – 2013. Own elaboration based on CASEN surveys, information available at CASEN 2013b.

2.3. Women and Girls in Chile

The role of women in Chile has changed dramatically since the 1960s. In the entire Latin-American region, the massive incorporation of women to the workforce, and increased access to formal education system, has contributed to reduce gender differences in educational outcomes and participation in the labour market (Esteve & Lopez-Ruiz, 2010). The participation of women in the economy has shown a sharp increase over the last 20 years, increasing from 28% in 1992 to 42% in 2012 (INE, 1992, 2012). However, the increased levels of participation are not comparable yet with that of men, which have remained constant over the last 20 years at around 70% (INE, 1992, 2012).

In addition to the differences in the participation of the workforce, inequalities in wages are still present in Chile. While Contreras and Puentes (2001) documented that differences have decreased since 1958, wage discrimination towards women is still estimated at about 30% (Fuentes, Palma & Montero, 2005), and it increases in higher levels of education and experience (Montenegro, 2001). This problem appears to be related to a differential distribution of participation in higher-level jobs. The participation of women has been observed as being concentrated in areas of service and trade (74%), occupying less competitive jobs with lower salaries (Voz de Mujer, 2013). This can be exemplified by the fact that only 21.7% of public and private management positions are occupied by women (Cardenas, Correa & Prado, 2012), as well as the low level participation of women in politics and economics. According to PNUD, Chile is the 75th country (out of 109) in terms of gender equity in leadership positions, both in politics and economics. Finally, women account for only 15.8% of parliament, a percentage that falls below the regional average of 25.5% (United Nations, 2015).

In Chile, differences in wages and participation are accompanied by social expectations that require women to conform to a feminine role, a role that has remained constant since colonial times (Escobar, 2001; Avalos, 2003). In Chile, it is commonly believed by people that the most important role for women in society is to take care of their families and for men to go out and work (Encuesta Voz de Mujer, 2012). For example, in a recent national survey, implemented between 2006 and 2012, this role was largely confirmed (Encuesta Nacional Bicentenario[8]). In this survey most women declared themselves as being prepared to leave the labour market after having children, or if their partners earned enough money to sustain their families. A total of 51% of the women reported as working full time before having their first child, with only 31% working full time afterwards. Furthermore, the socioeconomic background of the mother affected these differences: whilst 25% of women from a higher socioeconomic status remained at work full time after their first child, only 19% from poorer backgrounds did the same (Stuven, 2013). These differentiated roles are commonly accompanied by stereotyped beliefs about men and women, or what is expected from them: women should be caring, creative and communicative while men should be critic, able to build and research as well as to understand (see figure 2.2).

[8] UC-Adimark, http://encuestabicentenario.uc.cl/

Figure 2.2. Gender stereotypes present in the Chilean culture. The picture shows a publicity campaign used by the University of Chile in 2015 to recruit students. The University of Chile is the largest public educational institution in the country, and is widely known for its progressive views of Chilean society. The banner in the left side of the picture, depicting a male student, has the following attributes written on it –from top to bottom-: to question, to investigate, to understand, to build, to educate, to criticize, to judge, to transform. The right side banner, associated with a female student, reads from top to bottom: to imagine, to create, to communicate, to represent, to comprehend, to repair, to care.

In addition with different roles and attributes expected from men and women, Chile still presents important levels of male chauvinism and feelings of discrimination amongst women. In a recent survey on masculinity and gender equality, although several positive changes were reported, persistent stereotypical attitudes amongst men were observed (Aguayo, Correa & Cristi, 2011). Consequently, most women think that Chile is still a sexist country, and nearly 80% feel that women experience discrimination (Corporación Humanas, 2009).

In relation to gender equality in formal schooling, recent reports have shown that in Chile women have reached parity in participation at primary, secondary and higher education, with similar levels of pass rates and retention as males (Avalos, 2003).

However, participation after compulsory education is highly differentiated by gender. For example, when opting for technical specialisation in the last two years of secondary education, men tend to choose subjects associated with technology, electricity and building, while women select courses related to services, tailoring, social programs and tourism (Voz de Mujer, 2013). In higher education these differences are also replicated, with women choosing careers related to education and health while men prefer areas related to technology and science (see table 2.2).

Table 2.2:
Enrolment in tertiary education by field and gender.

OECD Areas	Male	Female	Female represent ation
Agriculture	13245	13012	50%
Science	53242	15018	22%
Social sciences, business and law	137588	176416	56%
Education	35407	99557	74%
Humanities and Arts	23666	27058	53%
Engineering, manufacturing and construction	188369	43380	19%
Health and welfare	64596	198779	75%
Services	66972	58555	47%

Note. Table own elaboration, source SIES Chilean Ministry of Education, 2014.

When gender differences in attainment are considered, most of the academic assessment tests have shown a higher performance of women in language and reading, while men tend to outperform girls in mathematics and science (MINEDUC, 2005). Data from PISA has shown that men's advantage in mathematics is the biggest, and women's advantage in language is the smallest in the region (Manzi, Strasser, San Martin & Contreras, 2008). In addition, women are outnumbered by men in achieving the higher score in the University Admission Test, both in Languages and Mathematics, which are crucial for admission into the high-demand universities (Avalos, 2003). Particularly in relation to mathematics, perpetual advantages have been reported for males in the post primary school phase (Manzi et al*.*, 2008; Mineduc, 2005) as well as in the national university admission test (Avalos, 2003). In relation to these differences, every year the SIMCE (System of Evaluation of Educational Quality) reports results which are aggregated at the national level, with differences between boys and girls remaining a stable topic (see figure 2.3). When comparing the gender gap during the educational pathway, it has been found that the largest differences occur in the 8th grade (MINEDUC, 2005). In the 10th grade, differences in both Mathematics and Language still exist, but they are smaller (MINEDUC, 2005).

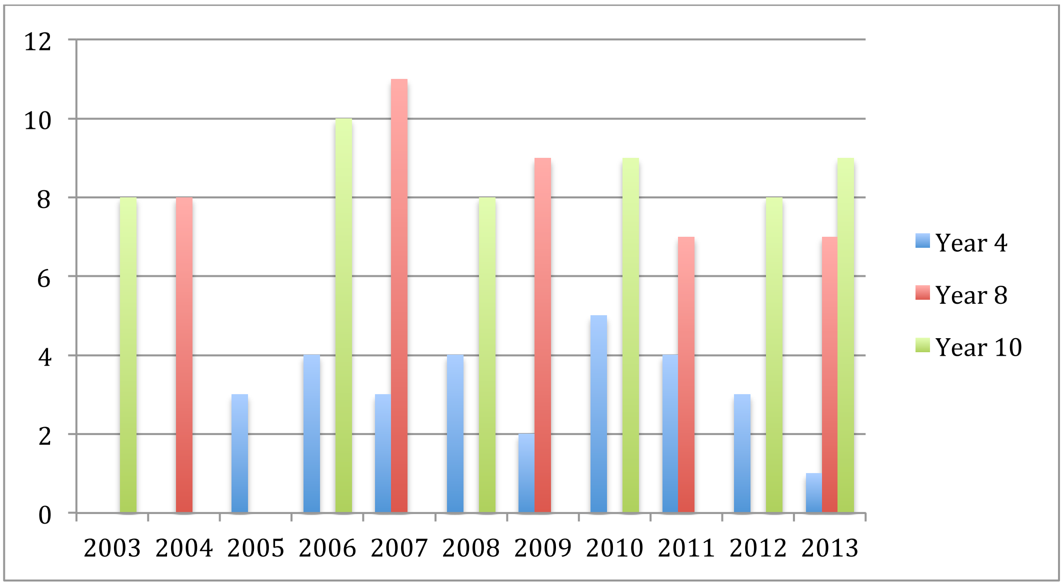


Figure 2.3: Descriptive male advantage in Mathematics SIMCE as reported in National Reports (Ministerio de educacion Chile, 2001 - 2010). SIMCE uses a scale with a mean of 250 and 50 standard deviation. y axis represents raw difference between male and female students. Own elaboration

In addition to these general differences in mathematics attainment, recent studies have also shown that students and teachers hold stereotypical beliefs on gender and mathematics. It has been reported, for example, that pre-service teachers have lower expectations of girls' mathematics attainment (Mizala, Martinez & Martinez, 2015). Similarly, a recent study with pre-school students (age 5) has shown that boys and girls already hold stereotypical expectations about each other's potential for academic attainment (del Rio & Strasser, 2013), for example, that girls will find mathematics more difficult and less enjoyable.

In summary, Chile can be considered a highly stratified country in terms of the distribution of goods and social opportunities, with the educational system playing an important role in reproducing such stratification. Gender differences are still quite marked in Chilean society, with working and social roles being highly differentiated. In relation to women's relationships with mathematics, there is a marked underrepresentation of women in mathematical careers. During school life, girls are consistently seen as under-performing and as having more negative attitudes towards mathematics. Taken together, these socio-cultural factors appear to have supported the widely held stereotype that women are less suitable for mathematics.

Chapter 3. Methodological Rationale and Methodology

3.1. Introduction

This research study adopts a mixed method design (Teddlie & Tashakkori, 2003), where different phases are combined (Gorard & Taylor, 2004) or integrated (Cresswell, 2003) in view of overcoming the polarisation of quantitative and qualitative methods (Teddlie & Tashakkori, 2003). Although some authors have criticised mixed method approaches because of the irreconcilable epistemological differences between quantitative and qualitative paradigms (Denzin & Lincoln, 1994; Guba, 1987), a growing number of researchers have argued that a mixed type of design provides the ability to overcome some weakness related to single method designs. For example, some have suggested that research claims are stronger, have greater impact and allow exploration at different levels of the same phenomenon when based on the triangulation of a variety of methods (Gorard & Taylor, 2004; Todd, Nerlich & McKeown, 2004). It has also been suggested that mixed method designs surpass difficulties associated with attempting to *describe and explain* at the same time. This provides the possibility to be able to explore (Harre & Crystal, 2004) the same phenomenon extensively and later intensively (Gorard & Taylor, 2004).

One of the most commonly employed philosophical supports for the use of mixed methods research has come from Pragmatism (Howe, 1988; Maxcy, 2003; Patton, 2002). Basically, this paradigm suggests that researchers need to be focused on their research question and use whatever philosophical and methodological approach that works best for the problem under study (Teddlie & Tashakkori, 2009). Tashakkori and Teddlie (2003) argue that pragmatism (1) rejects the incompatibility thesis, supporting the use of quantitative and qualitative methods in same and multistage research programs; (2) emphasises the relevance of the research questions over paradigms or methods required for answering them; and (3) presents a practical and applied research philosophy.

This study follows a pragmatist perspective to be able to engage with a wide and open research question. It acknowledges that the study of girls' relationships with mathematics can be carried out at multiple levels and in relation to different areas where this relationship develops (in this study, mainly attainment and mathematical identities). In particular, this study uses a sequential mixed methods design with three consecutive empirical phases. In this chapter I will discuss how a mixed methods design was used in this study, describing how the different methodological approaches connect and formulate the problem in different but complementary ways. I will then provide more detail on each phase of this research study, extending the information given in each paper's methodology section. At the end I will discuss ethical issues and how they were dealt with in this project.

3.2. Research Aims and Research Design

The main aim of this study was to explore girls' relationships with mathematics in the Chilean context. In particular it focussed on exploring how gender is implicated in the formation of mathematical identities for girls and its mediation of differences in attainment. Although this topic has attracted lots of public attention (always highlighting how girls are failing in maths), research on this topic in Chile is relatively weak (see Chapter 1 and 2 for the introduction and the Chilean context). In addressing this rather general aim, this study attempts to explore three empirical research questions (research question number four is of a more conceptual nature, so it has been omitted from the methodology chapter):

1. How can gendered patterns in academic attainment during primary education in Chile be described?

This question is best addressed through modelling attainment and gender from a national database that has gender and attainment of Chilean students. Such methodologies use national databases to model the relationship of gender and other background variables, as well as school effects using multi-level modelling (empirical phase 1 study, chapter 4).

2. How do girls relate to the mathematics they experience in their classrooms and how do these experiences mediate different mathematical identities?
3. How are different 'positive relationships' with mathematics constructed in the classroom and what is the influence of peers in such a process?

In order to answer questions 2 and 3, this study employs two different methodological approaches. Firstly, in the empirical phase number 2 (chapter 6), a quantitative survey is used in order to look at students' self-reports about their identification with mathematics and about the teaching they have experienced in their classrooms (answering mainly research question number 2). Then, in a sequential phase (empirical phase number 3, chapter 7), a classroom case study methodology is used in order to address research questions two and three. Using mainly qualitative, but also some quantitative methodologies, classroom interactions, classroom practices and forms of students participation and narratives are explored in order to best view in-depth processes where mathematical identities are negotiated. In particular the mediation of the mathematical practice (research question number 2) and relationships with peers (research question number 3) are explored in this empirical phase.

3.3. Methodological Rationale

As indicated above, this study followed a design which brings together three empirical studies. These three empirical studies link together with an over-arching mixed methods design which can be conceptualised as three sequential phases (Johnson & Onwuegbuzie, 2004). These sequential phases can be conceptualised differently according to the main methods employed, and according to their purpose in relation to the main aim of this thesis.

In relation to the methods employed, this study corresponds to a *sequential mixed methods* and *mixed model* design (Johnson & Onwuegbuzie, 2004). According to Johnson and Onwuegnuzie (2004) studies can be differentiated according to how each different phase makes use of qualitative and quantitative tools to explore the research questions. *Sequential Mixed Method* designs in their typology correspond to studies that link different phases with either a quantitative or qualitative emphasis. In addition, *Mixed Models* correspond to studies that make use of qualitative and quantitative tools within the same phase (Johnson & Onwuegbuzie, 2004). Following Johnson and Onwuegbuzie's (2004) typology, this study can be illustrated as shown in figure 3.1.

Phase 1		Phase 2		Phase 3
QUANTITATIVE	□	QUANTITATIVE	□	QUALITATIVE + quantitative

Figure 3.1. Diagram of sequential mixed method and mixed model design of this thesis. + stands for concurrent; □ stands for sequential; Capital letters denote high priority and lower case lower priority or weight (following Johnson & Onwuegbuzie, 2004).

In accordance with the relationship between each phase's purpose and the overall aim of this research study, the methodology can be conceptualised as expanding from a descriptive to an explanatory focus (Gorard & Taylor, 2004). Gorard and Taylor (2004) suggest that an explanation of a certain phenomenon requires an in-depth analysis approach which is necessarily focused on a small number of cases, and therefore harder to generalise. Following this, an in-depth analysis can follow a descriptive phase which outlines the general findings, and in these circumstances can help to elucidate how these general findings have come about (Gorard & Taylor, 2004).

In particular, this study started with a description of the available evidence in relation to girls' relationships with school mathematics in Chile. As discussed previously in Chapter 2, different sources of evidence were found which provided some support for the hypothesis of girls having a more conflicted/negative relationship with the subject in Chile. However, this evidence was restricted in several ways (see chapter 4).

Following the relatively easy access to the large scale dataset provided by the National Census of Education Quality (SIMCE), I decided to 'preface' the study with a routine analysis of the population in terms of Mathematical Attainment before moving into the in-depth data. This 'preface' also allowed me to start exploring the issue before collecting data. Following this, this research follows a design which resembles the 'New Political Arithmetic' (Gorard & Taylor, 2004):

> In its simplest form it involves a two-stage research design. In the first stage, a problem (trend, pattern, or situation) is defined by a relatively large-scale analysis of relevant numeric data. In the second stage, this problem (trend, pattern, or situation) is examined in more depth using recognised 'qualitative' techniques with a subset of cases selected from the first stage (Gorard & Taylor, 2004, p. 59).

Specifically in this study, the first stage (phase one) explored the general trends and patterns in girls' attainment in mathematics over the entire Chilean school population of year 4 and year 8 from one cohort. This descriptive analysis of attainment allowed the questioning of assumptions and constructions about girls' performance in this country. Although this first stage did not explore possible explanations of the differences described, by exploring their sizes and distributions in different groups of the population it provided evidence which contradicted essential explanations, such as ‘girls are not good at mathematics’ (see more detail in chapter 4).

By way of contrast, the more explicative phases (phases 2 and 3) in this thesis were concerned with exploring the processes through which girls' relationships with mathematics were built. In this sense, research questions two and three were interested in studying how possible differences between boys and girls and between different girls were constructed. Again the link between phases 2 and 3 goes from more ‘extensive’ to more ‘intensive’ explorations. By using quantitative and traditional statistical techniques, phase two explored the ‘average effect’ of the mathematical practice of different students’ mathematical identifications (see chapter 6). The process of mediation on mathematical practice was explored in more depth in phase three. Specifically looking at girls’ mathematical identities I explored different ways in which this ‘average effect’ came about, and how it was mediated by girls’ relationships with their peers. For this more ‘intensive exploration’ a different methodological approach was needed and taken. In contrast with an emphasis on ‘averages’, the focus of this phase was on ‘cases’: this involved three different girls who belonged to three different groups in a single classroom. This more detailed exploration allowed the exploration of the relationship between identities and practices and how these are linked in a two-way relationship (and not as one ‘causing’ the other) and how diversity was found between girls (complementing the ‘average effect’) (see chapter 7).

Another methodological contrast between phase two and three comes from the different conceptualisations of identity used in both studies. In addition to concerns regarding the implications of gender in the relationships girls have with mathematics; this study is also particularly concerned with exploring the use of mathematical identities to better understand this relationship. Following this, a conceptual question that permeated this entire thesis asked how the concept of a mathematics identity can be conceptualised and operationalized, particularly in the Mathematics Education research community. To address this question, I performed a literature review of research on mathematics identity which enabled me to construct a basic model of the defining features of this concept (chapter 5). In addition, in empirical phases two and three I explored two different methodological approaches in order to identify with two contrasting operationalisations.

In phase two the emphasis was placed on the self-reported, subjective relationship with mathematics in different forms of identifications with mathematics, collected using surveys. In this sense, I understood the activity of answering a set of closed questions (surveys) as students' identifications with the given statements. In phase three I complemented subjective self-reports through interviews and observational data, with all data also analysed in relation to situated meanings. In this sense, phase three corresponds with a more integrated model of identity (as defined by the literature review in chapter 5), and therefore its analysis allowed a more detailed and complex notion of how those identities were negotiated. While the survey approach of chapter 6 conceptualised identification as an individual activity, in chapter 7 these individual identifications were interpreted in context and were therefore conceptualised as built using social resources (i.e. they were both individual and social products). Having this comprehensive-integrative model of identity in mind when interpreting results from a more 'restricted' notion of identity (phase 2, chapter 6) allowed a discussion of results from this survey study, providing a more comprehensive and critical account of this data as well.

3.4. Details on each Empirical Phase Methodology

3.4.1. Phase 1: Gender Gaps in Mathematics Attainment in Chile

Results of this phase are reported in Chapter 4. In this sub-chapter more detail on methodological considerations and data handling are provided.

3.4.1.0. Introduction and research questions

As suggested previously, gender differences in academic attainment have been an area of interest in Chile. But despite this recognised relevance, the review of the literature suggests several weaknesses in terms of exploring differences in attainment in Mathematics. Firstly, although the SIMCE data included progress data for the last 10 years (since 2004), my literature research found no publications exploring how female students progress at the individual level during primary education. Secondly, despite the large amount of research reporting general attainment effects at school level and differential effects of different types of schools in the country (what has been called the evaluation of the 'Chilean voucher system'), to the best of my knowledge there has been no exploration of the school effect on gender differences. This is important for two reasons. Firstly, an exploration of systematic differences between schools of different types and their gender effects can help to explore which groups have access to educational opportunities. Secondly, an estimation of school effect on gendered patterns of attainment is a recent area of research and has been suggested as a relevant area for assessing policy on schools in Chile (see more detail in Chapter 4). Evidence with regards to this topic can assist the discussion on this national policy. Following this, this study focuses on research question one [How can gendered patterns in academic attainment during primary education in Chile be described?], examining 4 sub-questions:

1. How big are the differences in mathematics attainment between boys and girls and how do they change during primary school?

2. Is there a relationship between these differences and the socioeconomic status of students? How can this relationship be described?

3. Are gender differences distributed heterogeneously across schools from different SES and different administrations? How can this distribution be described?

4. Do schools vary in their relative effectiveness for boys and girls? How can this variation be described?

3.4.1.1. Data

The analysis presented in Chapter 4 was based on data from the National Assessment of Educational Quality (SIMCE), which is provided by the Chilean Ministry of Education. SIMCE consists of a series of standardised tests which have been applied since 1988. Since the year 2000 scores are comparable between years as tests started to be created using an IRT methodology with bridge (anchor) items between tests. Using this methodology, students are given an individual score, which in the first year of application has a mean of 250 with 50 points as a standard deviation. While initially students were expected to sit one exam during their educational career (either in year 4, year 8 or year 10), since 2004 students started sitting at least two SIMCE applications during their time in school (year 4 and year 8 or year 10), which has allowed the use of this instrument for exploring progress at the individual level. In addition with the attainment data, SIMCE collects background information of the students using a student and parents survey and some data at the classroom level (survey of teachers) and school level (information from the Ministry of Education regarding the administration of school and school composition).

3.4.1.2. Description of variables

Type of School

Type of school is a category which considers the funding and administration of primary and secondary schools. Schools are categorised into three main groups: Schools with private funding and administration (Private), schools with public funding and private administration (Subsidised) and schools with public funding and administration (Public). [See further detail in chapter 2 and 4].

Socioeconomic Status of school

The Ministry of Education is concerned with providing a fair comparison of schools with regards to their SIMCE data, and reports on a comparison of every school each year with schools which are similar in their socioeconomic status (SES). Consequently, every year a classification of schools is split into 5 different groups (see SIMCE, 2010b). School categories are created according to the socioeconomic status of their population using parents' questionnaires and information from the schools using a cluster analysis technique. This technique creates a number of predetermined groups (in this case, 5 groups) which maximise similarities within and differences between groups (SIMCE 2010b).

Rurality

Schools are also identified with regards to their location, with two levels used for this variable: schools which are located in urban locations and schools located in rural locations.

Previous attainment at school level

For the analysis of progress, schools were also categorised in terms of their level of previous attainment. In assigning this category, a raw mean of Year 4 attainment for each school was computed and then categorised into three levels: schools with low previous attainment (25% of schools with lower mean year 4 scores), schools with middle attainment (50% of schools in the middle), and schools with high previous attainment (25% of schools with a higher year 4 mean).

Student level socioeconomic status (Family SES)

When constructing the indicator of socioeconomic status or social class, concerns about the non-inclusion of only economic aspects were considered. For example, in the UK, Evans (2000) discussed how the inclusion of the parents' education allows the consideration of cultural facets pertaining to social positions beyond income. Following this suggestion, this study makes use of a composite measure of SES which includes both family income and parents' education.

Raw variables were obtained from the parents' questionnaire which is collected within the SIMCE application. Three questions from this survey were considered (see table 3.1).

Table 3.1:

Variables included in the composite measure of SES

Question	Levels of response	Treatment of responses
What was the highest level of education reached by the mother?	20 levels, from no formal schooling to PhD	Responses were transformed to years of education using SIMCE protocol (see table 9.1 in appendix 1). Then variable was standardized to have a mean of 0 and a SD of a unity.[1]
What was the highest level of education reached by the father?	20 levels, from no formal schooling to PhD	Responses were transformed to years of education using SIMCE protocol (see table 9.1 in appendix 1). Then variable was standardized to have a mean of 0 and a SD of a unity. [2]
In a normal month, in which of the following ranges represents better the total sum of incomes of the family?	15 levels of different ranges of income	Responses were transformed into the median of the range using SIMCE protocol (see table 9.2 in appendix 1). Then variable was standardized to have a mean of 0 and a SD of a unity.

Notes. [1] Missing data was imputated with level of education of father when available. [2] Missing data was imputated with level of education of mother when available.

In order to build a composite measure of SES a principal component analysis of the items/scale was performed. This kind of analysis is used to explain the correlations among variables in terms of more fundamental entities called factors (Cudeck, 2000). In this case it was used to assess the possibility of treating the SES scale as a uni-dimensional scale and this was assessed first at a descriptive level, by exploring the bivariate correlation among items [see tables 9.3 and 9.4 in appendix 1], and then by exploring the principal component solutions, following the suggestion of considering factors with eigenvalues higher than 1 (Kaiser, 1960, in Cudeck, 2000). After extracting factors, a single measurement was constructed using the regression method. This method constructs a single variable whilst considering the relative contribution of each factor to the scale. In addition to the factor analysis, an analysis of internal consistency was performed [see table 9.5 in appendix 1].

3.4.1.3. Sample

This study considered a cohort of students who were assessed both in year 4 (2005) and year 8 (2009), thus allowing the capture of gendered attainment patterns in both middle (year 4, 9-10 years old) and at the end of primary school (year 8, 13 – 14 years old). Every year around 250.000 students sit their examination, clustered in at least three levels (students, clustered in classrooms, clustered in schools).

3.4.1.4. Missing data analysis and treatment

SIMCE is a census of education attainment, in which the entire population of certain school years is assessed over a few days at the end of the academic year (around November). In the two examinations that were included in this study, SIMCE reported that 96% of the total enrolment of students in year 4 had participated (representing 90% of the total for schools, SIMCE, 2006) and 92.5% of the total enrolment of students in year 8 (representing 98.2% of the total for schools, SIMCE, 2010a). Following this high level of attendance, the population can be regarded as all of the students who sat their SIMCE in year 4, year 8, and in both for progress. However, this study dealt with some missing data problems particularly in relation to some covariates collected by parents' surveys. Several analyses were carried out in order to investigate the source of this missing data and to explore possible decisions with which to deal with these issues. Different approaches were taken regarding the outcome variables (mathematics attainment at different stages of schooling and on progress) and covariates and contextual variables (especially regarding SES at individual and school level).

For the outcome variable it was decided that the total population corresponded with students who sat their mathematics tests, and for progress, students that sat in both their year 4 and year 8 tests. Missing data was then defined as students who missed a response in one or more covariates. A few strategies were decided for minimising this missing data.

Regarding the use of contextual and covariate variables, this study's approach was to maximize the sample at the expense of losing control variables upon which this study was not focused. One example of this was the use of ethnicity as a control variable. Although initially ethnicity was considered a relevant control variable this was not used in the final analysis to avoid missing data problems. This was considered to be satisfactory as race/ethnicity is not the main focus of this study and Chile has been described as one of the countries in Latin America with a low number of people who identify as belonging to an indigenous ethnic group (about 11% INE, 2012). In addition, in this particular dataset a high percentage of missing data was found [around 20%, see table 9.6 in appendix 2], with no data on ethnicity existing for year 4 attainment data.

The effect of missing data on socioeconomic status of the family variables was also minimised by using information from the different variables at the individual level to input data. Firstly, mother and father's highest level of education was considered as two different variables but, when data was missing in one of these, then the value from the other variable was inputted. This allowed to decrease the level of missing data because of missing information in family socioeconomic variables to around 15% [see table 9.7 in appendix 2]. No missing data was found in all the school level variables (type of school, school SES and school location).

Analysis of the distribution of the sample and the population in relation to the main groups included in this study allowed us to accept that the sample was similar to the population, and therefore missing data could be regarded as Missing at Random (Graham, 2009) [See table 3.2].

Table 3.2:

Distribution of sample and population regarding the main variables of the study

		Sample	Population
Year 4 Maths Scores	Male	110810 (50.4%)	127376 (50.9%)
	Female	108872 (49.6%)	123116 (49.1%)
	Public	108567 (49.4%)	120737 (49.5%)
	Private Subv	97758 (44.5%)	108012 (44.2%)
	Private	13357 (6.1%)	15397 (6.3%)
	Urban	194848 (88.7%)	215530 (88.3%)
	Rural	24834 (11.3%)	28616 (11.7%)
	SES_1	23607 (10.7%)	27559 (11.3%)
	SES_2	75411 (34.3%)	83701 (34.3%)
	SES_3	76202 (34.7%)	3479 (34.2%)
	SES_4	31044 (14.1%)	33906 (13.9%)
	SES_5	13418 (6.1%)	15501 (6.3%)
Year 8 Maths Scores	Male	93883 (49.5%)	113583 (50.0%)
	Female	95952 (50.5%)	113396 (50.0%)
	Public	85.328 (44.9%)	102200 (45%)
	Private Subv	91536 (48.2%)	108159 (47.7%)
	Private	12971 (6.8%)	16620 (7.3%)
	Urban	170428 (89.8%)	204120 (89.9%)
	Rural	19407 (10.2%)	22859 (10.1%)
	SES_1	20089 (10.6%)	24609 (10.8%)
	SES_2	58809 (31.0%)	70088 (30.9%)
	SES_3	62401 (32.9%)	73363 (32.3%)
	SES_4	34756 (18.3%)	41212 (18.2%)
	SES_5	13780 (7.3%)	17707 (7.8%)
Progress	Male	76208 (48.0%)	91143 (48.5%)
	Female	82645 (52.0%)	96857 (51.5%)
	Public	69539 (43.8%)	82287 (43.8%)
	Private Subv	77855 (49.0%)	91094 (48.5%)
	Private	11459 (7.2%)	14619 (7.8%)
	Urban	143138 (90.1%)	169817 (90.3%)
	Rural	15715 (9.9%)	18183 (9.7%)
	SES_1	15353 (9.7%)	18371 (9.8%)
	SES_2	47067 (29.6%)	55420 (29.5%)
	SES_3	53179 (33.5%)	61957 (33.0%)
	SES_4	30951 (19.5%)	36498 (19.4%)
	SES_5	12303 (7.7%)	15754 (8.4%)

3.4.1.5. Data analysis

The analysis of this first phase used a *multilevel modelling* as the main framework. Basically, this model uses the shared variation of observations within structures to make estimations about an outcome variable, accounting for its nested structure [i.e. individual, classroom and school levels]. It does so by modelling interdependency within levels under the theoretical assumption that students inside the same context (classrooms/schools) share something that students from other contexts do not (for recent reviews on this method see Hox, 2002; Raudenbush & Bryk, 2002; Snijder & Bosker, 1999).

As this study is interested in making estimations at the student level (as gender is a variable at the student level), not considering the clustered organisation of data would incur two problems. Firstly, ignoring the hierarchical structure of data would imply a violation of the assumption of independency of observation, which is an assumption in multiple linear regression models. This methodological problem has been extensively discussed by researchers who have demonstrated that the violation of independency generates the underestimation of standard errors (Goldstein, 2003; Snijder & Bosker, 1999). This problem becomes larger when increasing intraclass correlation and sample size. In these situations the underestimation of the standard errors could often lead to the rejection of the null hypothesis and, consequently, overstatements about significant differences. Statistically, this happens because hypothesis testing is based on too many degrees of freedom (at the individual level) which are not truly independent, thus, inflating the type 1 error rate (Tabachnick & Fidell, 2007, p. 782).

Secondly, ignoring the hierarchical structure of data would have made it impossible to model higher level influences on individual variables. This is particularly important in this study, as one of the research questions is about the distribution of gender differences in the population of schools in the country. Without considering this hierarchical structure of data, this study would have had to rely on either aggregating gender differences at the school level or completely ignore the schools' contribution. The multilevel approach, in contrast, allows modelling of the independent contribution of each level by modelling variability independently. In other words, it allows the modelling of the relative contribution of individual, classroom and school variables to the outcome variable (e.g. Snijder & Bosker, 1999). As discussed within the paper which reports on this dataset (see chapter 4), this has been suggested as being particularly important in the Chilean context, where a large amount of students' attainment is explained at the school level (Manzi, Strasser, San Martin & Contreras, 2008; Mizala, Romaguera & Urquiola, 2007), with other levels also being observed as significant (Troncoso, Pampaka & Olsen, 2014).

Model building

As reported previously and as explained in chapter 4, several models were compared to assess the contribution of different variables from different levels of the outcome variables (attainment at year 4, attainment at year 8 and progress between these two years). Models were built on an equation that considered the variation of outcomes at the classroom and school levels, by the inclusion of a random coefficient for both. Different authors have suggested the 'stepwise' approach or the comparison of models of increasing complexity as a suitable analytical strategy (e.g. Raudenbush & Bryk, 2002, Tabachnik & Fidel, 2007).

1. *Empty models and models of gender* were used to assess the independent contribution of gender against the different outcome variables (year 4, year 8 and progress). By comparing the gender models with the empty model it was possible to assess the amount of variance explained only by gender. In the Progress model, gender was also assessed with and without the interaction effect of gender and previous attainment.

2. *The family socioeconomic status model* was used to assess if the effect of gender remained significant after controlling for SES. The interaction effect of gender and family socioeconomic status allowed the assessment of whether the effect of gender changed at different levels of family SES (in the individual level). For the progress model, this interaction was assessed independently and in interaction with previous attainment.

3. *School systematic effect models* were used to assess if the effect of gender was different in different types of schools, schools with different SES populations, schools with different levels of previous attainment (as measured by school averages of year 4 scores) and in rural schools (in comparison with urban). For comparing types of schools, the category

Private subsidised was considered the reference category, because there are private subsidised schools which compare with private and with public schools (in terms of population and results) (e.g. Mizala & Torche, 2012). For the socioeconomic status of the school, the middle category of the quintiles (defined as middle SES by SIMCE) was used as a reference category. Considering the strong relationship between school socioeconomic status and family status (Manzi et al., 2008), and the highly stratified population that public, private subsidised and private schools cater for (Mella, 2002; Torche, 2005), models were tested in two steps. Firstly, the independent contribution of each school level variable was tested independently (type of school, socioeconomic status and previous attainment of the school were tested independently, and with and without control of family socioeconomic status). Then, the joint contribution of these school variables was assessed, again with and without the control of family socioeconomic status. The increasing complexity of models was compared, with cross level interaction with gender added independently for the three school variables (two-way cross level interaction) and models where the dependent contribution of gender, school SES, school type and previous school attainment was considered (three-way cross level interaction).

4. *Schools' gender random effect* was assessed only in the progress models. School differential effectiveness was estimated by controlling the model using individual level variables (in this case, the family socioeconomic status of the student and previous attainment) and by allowing the intercepts of the school to vary in terms of gender. This means that two random variables and their correlation were estimated: the general intercept of the school and the difference between boys and girls in the school (the female coefficient) (Snijder & Bosker, 1999). Following these estimations, schools can roughly be classified into 9 different types of schools according to their general effectiveness (if students made less than expected progress, roughly what is expected and above expectation progress) and their differential effectiveness according to gender (boys' progress exceeds that of girls, no differences and girls' progress exceeds that of boys) (Gray, Peng, Steward & Thomas, 2004). This classification was made following Gray and colleagues' suggestion of

using a standard and more relaxed assumption of confidence intervals (2 and 1 standard deviation from general distribution respectively) (Gray et.al., 2004).[9]

Different models were compared using the likelihood test and the percentage of variance explained by models was defined as the proportional reduction in mean square prediction error at level 1 in comparison with a corresponding model[10]. The likelihood test allows testing if the inclusion of new parameters (both in the random and in the fixed part) provides a significant increase in the amount of variance explained by the (nested) models.

In total, about 30 models were tested. Initially the contribution of school and classroom level variance in the model was assessed for each outcome variable. For this, a Null model was fitted and then compared with a model in which the variance of classrooms and schools were accounted for in the estimation of intercepts. Results of the analysis of the empty models confirm what has been found previously in the literature on Chile (e.g. Manzi et al., 2008; Troncoso, Pampaka & Olsen, 2015). Three model levels (students, nested in classrooms, nested in schools) displayed a significant improvement both for the null regression analysis, and for the level 2 model. This was confirmed in all of the different models (attainment at year 4, attainment at year 8 and progress). Following this, all the subsequent analyses were made considering this structure of the data.

The significance of individual variables was assessed using the Wald test. For categorical variables of more than one level (i.e. type of school, school socioeconomic status and school previous attainment) the Wald multivariate test was used and tested against a chi-square distribution (as suggested by Snijder & Bosker, 1999, p. 88). The significance of individual categories was only reported and interpreted, if this test was significant.

Following this methodology, the final models were expressed using the following equations:

[9] Only mixed gender schools were included in this analysis (schools with 5 or more boys and girls in each classroom)

[10] Proportional reduction of unexplained variance at level 1 – in comparison with previous models allows the estimation of the contribution of predictors in explaining variance at level 1 (estimated as suggested by Snijder and Boskert, 1999, p. 102)

Intercept models

$y_{(ijk)} = \beta_{0jk} + \beta_{1(ijk)} + \beta_{2(ijk)} + \beta_{3(ijk)} + \beta_{4(k)} + e_{(ijk)}$

$\beta_{0jk} = \beta_0 + v_{0k} + u_{0jk}$

where:

$v_{0k} = \sim \mathrm{N}(0, \sigma^2_{v0})$

$u_{0jk} = \sim \mathrm{N}(0, \sigma^2_{u0})$

$e_{0ijk} = \sim \mathrm{N}(0, \sigma^2_{u0})$

Here, $y_{(ijk)}$ corresponds to the standardised mathematics test scores obtained by year 4 and year 8 students 'i', in classroom 'j', and in school 'k'. β_{0ijk} corresponds to the national average (β_0) allowed to vary at the classroom and school level ('j' and 'k'). For the progress model, the expected increase in year 8 attainment ($y_{(ijk)}$) according to prior attainment ($\beta_{1(ijk)}$) was included. In addition gender ($\beta_{2(ijk)}$) and family SES ($\beta_{3(ijk)}$) were included at the individual level, and school SES, school type and rural school were included at the school level ($\beta_{4(k)}$). In both cases interactions with gender were omitted including what has been described above.

Random coefficient models

$y_{(ijk)} = \beta_{0jk} + \beta_{1(ijk)} + \beta_{2k} + \beta_{3(ijk)} + e_{(ijk)}$

$\beta_{0jk} = \beta_0 + v_{0k} + u_{0jk}$

$\beta_{2k} = \beta_2 + v_{2k}$

where:

$$\begin{bmatrix} v_{0k} \\ v_{2k} \end{bmatrix} \sim \mathrm{N}(0, \Omega_v) : \Omega_v = \begin{bmatrix} \sigma^2_{v0} & \\ \sigma_{v02} & \sigma^2_{v2} \end{bmatrix}$$

Here, $y_{(ijk)}$ corresponds to the standardised mathematics test scores obtained by a year 8 student '*i*', in classroom '*j*', and in school '*k*'. β_{0ijk} corresponds to the national average (β_0) allowed to vary at the classroom and school level ('*j*' and '*k*'). $\beta_{1(ijk)}$ corresponds to the variation expected according to previous attainment (year 4 scores). Control variables were included at the individual level (family SES, $\beta_{3(ijk)}$). For the random coefficient models, gender was allowed to vary at the school level (β_{2k}). Following this, the random coefficients of these models were expressed as v_{0k} (variation of schools around the national average) and v_{2k} (the random coefficient assigned to female students of school '*k*'). Following this, three variance components were estimated: variations of the schools around the national average (σ^2_{v0}), variation of the within school gender differences (σ^2_{v2}) and covariance between variation of the intercept and the gender coefficient (σ_{v02}).

3.4.1.6 Closing remarks on Phase 1 methodology

Using a quantitative methodology and multilevel techniques, the first phase of this project intended to provide a systematic description of gender differences in mathematics attainment in Chile, surpassing weaknesses observed in this literature in this context. In this methodological subsection, a detailed account of the different steps taken during this study was given, providing more information that was not included in the methodology section of the corresponding paper (see chapter 4).

3.4.2 Phase 2: Mathematical teaching and Mathematical Identifications

Results from this phase are reported in the paper presented in chapter 6. In this section, I will present more detail on the chosen methodology.

3.4.2.0 Introduction and research questions

As it will be presented in chapters 5 (literature review) 6 and 7 (exploring girls' developing mathematical identities), the relevance of students' subjective relationships with mathematics has been supported by several research studies and arguments. In particular, students' mathematical identities have been theorised as having an important place in learning and an influence on future engagement. While research has also been well documented on how girls tend to identify with mathematics less positively than boys, how these identities develop or are influenced by contextual or social variables is less clear. In particular, in this phase, the influence of students' perceptions of the mathematics teaching they experience in their classroom was explored as a possible influence on students' identifications with mathematics. While some theoretical arguments have suggested that 'student-centred pedagogies' could be beneficial for girls with some support from qualitative and intervention studies, there is no empirical support for this argument in the literature (see more detail in Chapter 6). Following this, this study will explore the relationship between students' perceptions of their teaching practice and four forms of identification with mathematics (positive and negative emotions, self-concept and mathematical dispositions). In addition this study will be particularly concerned with how this relationship may vary in accordance with gender. The particular research questions of this study were as follows: what is the relationship between students' perceptions of teaching practice and students' mathematical identifications? And does this relationship vary according to students' gender?

3.4.2.1 Sample and data collection

Data for this study was collected from a sample chosen by convenience. The only sample criteria were that it was representative of average schools in the Chilean population, with average attainment (as measured by SIMCE), medium socioeconomic status (as reported by SIMCE) and displaying the same curriculum and administration. These schools were all located in different neighbourhoods on the periphery of Santiago (the capital city of Chile) and catered for students from similar socioeconomic backgrounds. All schools participating in this study comprised of big institutions (about 1600 students), with full day programmes (between 8am and 4pm) and they all offered technical education for students after year 10 (gastronomy, administration and technology). Between about 60% and 70% of their students continued studies after compulsory schooling (either in professional courses or universities).

In total, 291 students from 8 different classrooms were surveyed. Data was collected during school hours by the main researcher of this project. Parents were informed of the purpose of the study by letter and were offered the opportunity to opt-out from the study. Students were informed on the application day of the purpose of the study and were also offered the possibility to opt out. Only one parent chose to exclude her child from the study.

3.4.2.2. Description of variables

As will be described in chapter 6, paper survey measured 4 different forms of identification with mathematics (negative and positive affects whilst performing mathematics, self-concept and disposition towards mathematics in the future) and perception of teaching practice in the classroom. Most of the instruments did not have a Spanish translation, so a native Spanish speaker conducted the translations from English. For the positive and negative affects scale, an existing translation validated in Chile was used (Dufey & Fernandez, 2012).

In table 3.3, the different instruments and their translations are presented.

Table 3.3:
Items from different scales (English and translations to Spanish)

Scale	English Items	Spanish translation
Student-Centred Teaching	We work together in group projects	Trabajamos en proyectos grupales.
	We talk with other about how to solve problems	Hablamos entre compañeros sobre cómo resolver problemas.
	We ask other to explain their ideas	Le preguntamos a otros alumnos que expliquen sus ideas.
	We do proyects that include other subjects	Hacemos proyectos o trabajos que incluyen otras asignaturas.
	We learn how maths has changed	Aprendemos cómo las matemáticas han cambiado en el tiempo.
	We learn that maths is about inventing rules	Aprendemos que matemáticas significa inventar reglas.
	We research topics on our own	Investigamos contenidos por nosotros mismos.
	We discuss ideas with the whole classroom	Discutimos ideas entre todo el curso.
	We explain our work to the whole class	Explicamos nuestro trabajo a todo el curso.
	We answer teacher's questions	Respondemos preguntas del profesor.
Positive Affects Scale	Interested	Interesado
	Excited	Entusiasmado
	Strong	Fuerte
	Enthusiastic	Optimista
	Proud	Orgulloso
	Alert	Alerta
	Inspired	Inspirado
	Determined	Decidido
	Attentive	Atento
	Active	Activo

Table 3.3 (continuation):

Items from different scales (English and translations to Spanish)

Scale	English Items	Spanish translation
Negative Affects Scale	Distressed	Molesto
	Upset	Enojado
	Guilty	Culpable
	Scared	Asustado
	Hostile	Hostil
	Irritable	Irritable
	Ashamed	Avergonzado
	Nervous	Nervioso
	Jittery	Intranquilo
	Afraid	Temeroso
Self-Concept	I can get good results in Maths	Puedo obtener buenos resultados en matemáticas.
	I can learn maths even if it's hard	Puedo aprender matemáticas incluso si es difícil.
	I have a mathematical mind	Tengo una mente matemática.
	Compare to my classmates, I'm good at maths	Comparado con mis compañeros, soy bueno en matemáticas.
Dispositions	Maths is one of the most interesting subjects in school	Matemáticas es una de las más interesantes asignaturas del colegio.
	Maths is important for my future	Las matemáticas son importantes para mi futuro
	I would prefer my future studies to include a lot of maths	Preferiría que mis estudios futuros incluyeran muchas matemáticas.
	I would look forward to studying more maths after school	Quiero seguir estudiando matemáticas después del colegio.
	I would like to be a mathematician	Me gustaría ser matemático.
	Maths is important for my future (after school)	Las matemáticas son importantes para mi futuro (después del colegio).

A Rasch model was used to check the validity of each scale and to construct an interval scale for each variable based on the ordinal measures used in each instrument. The Rasch model was developed during the 1960's and it is referred to as a one-parameter item-response model. Basically, the model performs a logarithmic transformation of the items and person data in order to transform ordinal into interval data (Bond & Fox, 2001). Items and persons are placed along a single line which represents the 'ideal model' against which the data is tested: this probabilistic model is unidimensional, conjoint and separable. This means that the Rasch model is based on the unidimensionality assumption: it involves the examination of only one variable/attribute at a time, on a hierarchy that represents a growing continuum. This assumption is tested by providing indicators of how well ability estimations (person data) and difficulty estimations (item data) fit the ideal model.

The logarithmic transformation of person and item data yields to one scale expressed in 'logits' (log odds units), for both ability (α) and difficulty (δ). As explained by Bond and Fox (2001), the logit scale is an interval scale with separability, i.e. differences between intervals, being consistent and having meaning for both person and items and probabilities dependent only on their difference. For instance, one can interpret that a person that has the same estimated ability as the estimated difficulty that a specific item will have a 50% chance of passing this item. "The probability of his success increases to 75% for an item that is 1 logit easier (perhaps item *O*) or decreases to 25% for an item that is 1 logit harder (perhaps item *T*)" (Bond & Fox, 2001, p. 29).

Because all items in each scale were asked using a Likert scale, in particular the application of the Rasch model for Rating Scales was used. This model is an extension of the simple Rasch model for dichotomous data that allows the estimation of probabilities of answering different levels for each question (Bond & Fox, 2001; Lamprianou & Athanasou, 2009). The model assumes that each category is higher than the previous category, but in an unspecific amount, therefore estimating differences between different levels of response based on empirical information (Bond & Fox, 2001, p.71). These different levels are called 'thresholds', and for the Rating Scale Model, one set of rating parameters fixed for all items are estimated. In other words, for the use of the Rating Scale Model all items need to share the same number of thresholds.

To check the validity of the scale, the Rasch model provides different indicators of unidimensionality which are usually assessed. Firstly, the Rasch analysis provides indicators of how well each item and person fit the model, with the *infit* and *outfit mean squares*. Both indicators are based on the residual between the expected response predicted by the Rasch model and their observed responses. Estimations close to 1 are evidence of unidimensionality. Some authors suggest that values need to be close to 1.0, or closer than 1.1 in samples over 500 (Smith, Schumacker & Busch, 1995) but others set a more relaxed interval between 0.5 and 1.5 for adequate fit (Linacre, 2002). Low fit statistics indicate items that are 'overfitting', or that are too predictable by the model and that therefore, do not provide much additional discrimination of ability (Lamprianou & Athanasou, 2009). In contrast, high fit statistics indicate items that are *misfitting* to the model and responses to these items do not fit expectations (Bond & Fox, 2001).

A second means of evidence testing for unidimensionality is the examination of a factor analysis performed on the residuals after the Rasch measurement has been extracted (Linacre, 1998). Linacre suggested that, if evidence of any substantial or interesting dimension was found, the researcher should consider creating a separate measurement for this dimension. Although there are no clear-cut criteria for deciding if there is evidence of further dimensions, it is usually considered that eigenvalues of the second contrast (first contrast after the Rasch measurement was extracted) over two give evidence of possible further dimensions (Raîche, 2005).

Finally, items and person reliability and separation index are indicators of whether there are enough items spread along the continuum of difficulty and whether there is enough ability spread amongst persons. Usually a separation index over 2 and person reliability higher than 0.8 are seen to be adequate.

3.4.2.3 Model building

In answering the research question (what is the relationship between students' perceptions of teaching practice and four forms of identification with mathematics? and does this relationship vary according to students' gender?) I used different linear regression models for the different identifications with mathematics which were collected. Regression analyses were created in order to predict one outcome variable (the dependent variable) from one or more predictor variables (independent variables) (Field, 2009). This basic model is created on the estimation of three different components: (1) an intercept, or the point where the line crosses the vertical axis (b_0), (2) a slope or gradient of the line (b_1), and (3) an error or residual term (ε) which represents the difference between the predicted score and the actual score. These components define a line (an intercept and a slope), which is estimated using an ordinary least square method.

As this study was interested in assessing the contribution of different predictors (at least gender and teaching) on the different outcome variables (the four forms of identification – positive affects, negative affects, self-concept and dispositions), four different models were applied (one for each of the outcome variables), entering predictors hierarchically. Hierarchical linear regression, or the blockwise entry method, uses theory or research questions to define how predictors are entered into the equation (Field, 2009). This method relies on good theoretical reasons for including the predictors, not purely based on mathematical criterion (such as those used in stepwise methods). In addition, with the theoretical strength of this method, it allows us to compare the contribution of the different predictors by comparing the goodness of fit of each consecutive model. Each consecutive model fit is tested using an R^2 statistic, and a change statistic is also provided to assess whether the change on R^2 between models is significant.

To answer the research questions of this study, models were tested in four consequent steps:

1. First, gender was included as a sole variable to assess differences in the outcome variables for boys and girls.

2. Attainment was included as a control variable to assess if the effect of gender on students' identifications was accounted for by differences in performance.

3. The simple effect of the perception of teaching was included in the third step to assess if, after controlling for gender and attainment, the perception of teaching had an effect on students' identifications.

4. The multiplicative effect (interaction effect) of gender and perception of teaching was included in the final step to assess if the effect of student-centred teaching was different for boys and for girls.

For assistance with the interpretation of the predictions and for decreasing problems of multicollinearity between variables and interaction terms (Aguinis, 1995), all predictors were centred and standardised.

3.4.2.4 Closing remarks on Phase 2 methodology

In this subsection I have provided more information on the methodology used in the second quantitative phase (phase 2, reported in chapter 6) of this research project. This phase intended to explore an explicative model of the more negative relationship of girls and mathematics by exploring the average 'effect' of different perceptions of teaching practice in students' mathematical identifications. In order to explore this, two different methodological steps were needed. Firstly, the validation of instruments for measuring outcome and predicting variables was performed using a Rasch analysis. Secondly, the modelling of effects in outcome variables was performed using hierarchical linear regressions of the different outcomes variables. This methodological approach was conceptualised as an 'explicative' phase after the descriptive phase detailed in the previous subsection. Both first phases were focused on gender differences and therefore, comparisons between boys and girls were explored. In the last empirical phase described in the following section, I change focus from differences between female and male students towards a more detailed account of differences between different girls. In this sense diversity between girls was a focus and different mediation mechanisms in the construction of mathematical identities were explored. Details of this methodology phase are described in the next section.

3.4.3. Phase 3: Diversity of Girls' Mathematical Identities

The paper submitted for publication in a peer-reviewed journal is presented in Chapter 7. In this section I present more detail on the methodological approach and methodological decisions.

3.4.3.0. Introduction and research questions.

Similarly with Phase 2, Phase 3 is also based on the relevance of students' subjective relationships with activities when understanding learning and engagement. In particular, this phase focuses on the development of positive mathematical identities (students that see themselves and are seen by others as successful and engaged in the mathematical practice), a topic that has been relatively neglected in the study of girls' relationship with mathematics during compulsory schooling. It focuses on the processes that foster development of these identities and the negotiations that are needed in order to be positioned by others and to position themselves as mathematicians in the classroom (see more detail on Chapter 7). In particular, this chapter explores how girls' relationships with their peers can be used to maintain or negotiate different positions of engagement and success. Consequently, three different 'successful' girls who belonged to different peer groups were chosen as case studies.

3.4.3.1. Methodological rational/framework

This study used a case study methodology (Yin, 2003). Case studies have been described as a suitable methodology for understanding complex social phenomenon in real life settings, relying on multiple sources of data for triangulation purposes (Yin, 2003). For the particularities of this study, the cases were constructed as nested case studies: students' cases nested in peer groups, nested in a classroom case. As will be explained in Chapter 7, the process of mathematics identification in the classroom is understood as being constructed at these three different levels. In addition, each student's identity was conceptualised as being produced in the intersection of two main elements: acts and narratives. While narratives involve self-reflection and crystallisation of statements that somehow define 'I am', 'I was' or 'I want' (Sfard & Pusak, 2005), acts in practice are the actual embodiment of these narratives in practice (Holland et.al., 1998). This model is summarised in figure 3.2.

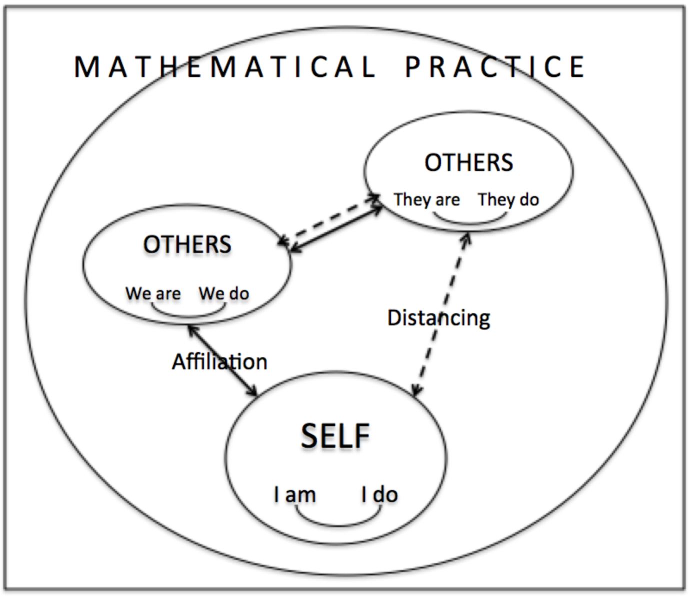


Figure 3.2: Process of Mathematical identification. It shows how one student's identities (both self-reflections/narratives and acts) are constructed in interactions with other peers and in a particular practice.

3.4.3.2. Participants

To put this model into practice, this study was focused on one year 7 case study mathematics classroom, with 3 girls and the peer clusters where they belonged also being used as case studies within the classroom. The school and classrooms were chosen purposefully to represent a 'typical' Chilean classroom. To find this typical classroom, I approached several different schools which catered for students with average socioeconomic status and had been consistently assessed as ‘average’ according to the national census of education, SIMCE. I visited schools from peripheral areas in Santiago, particularly Puente Alto, Maipu, San Bernardo and La Florida, and a few schools in the city center.

I decided to focus my study on the particular school and classroom where this study was carried out for two main reasons. Firstly, the school offered both vocational subjects and/or a scientific-humanist education as possibilities during secondary school (at year 10 students needed to choose to continue in either one of these possibilities). This characteristic opened different paths for the students to continue their educational trajectory and for this reason was considered interesting. Secondly, the school and the classroom teacher were very receptive to the idea of supporting this research. The Head of school and head of department offered support in organising initial activities and interviews with year 7 mathematics teachers, and one particular teacher was very positive and open to have someone visiting her classroom throughout an entire semester.

The study focused on one year 7 mathematics classroom case study, with 39 students. This group of students was one of the three year 7 groups in the school and was formed as a group only four months before the fieldwork of this study commenced. At the end of year 6 the three groups were mixed, forming three new groups. The school made this decision because of behavioural issues, using the criteria of forming three heterogenic groups in terms of attainment with no obvious relational issues. As usual, in the Chilean context, the group of students in the classroom remained the same in all the subjects they attended. This resulted in a highly stable peer group and a high variation in terms of mathematical attainment (non-ability grouping policy was in place, see Ramirez, 2007) within this classroom. In addition, because of the hold-back policy, the classroom was also diverse in terms of students’ age, displaying a range between 13 and 16 years.

The mathematics’ teacher [Ms P] had 2 and half years of experience in teaching mathematics. She came from a general teacher-training course (non-mathematics specific) in a non-traditional University, with relatively limited experience and specific preparation in mathematics. However, the Head of Department and her students characterised Ms. P. as a ‘very good teacher’.

Similar to many Chilean classrooms (e.g. Martinic & Vergara, 2007), Ms P's lessons followed a very consistent routine with three main parts: (1) whole class introduction of the day's topics; (2) students working on exercises; and (3) a closure. In this particular classroom, during the closure part 'reward points' were distributed amongst students according to whether they had completed the activities. As also described in previous Chilean studies (e.g. Radovic & Preiss, 2010), most of the talking was done by the teacher in the whole-class parts one and three, with students mainly focusing on answering teacher questions. Students had relative space for independence during exercises, where they could decide to work collaboratively.

The three girls that were chosen as individual case studies were selected to represent positive mathematical identities in the classroom (the three of them recognised themselves as good mathematicians and were part of the high attainment group in the classroom) that belonged to three different peer clusters (see below).

3.4.3.2. Procedure

Teachers, parents and students were informed about the study verbally and through information sheets. Signed consents were obtained from the teacher and parents of all students in the classroom, while students' assent was obtained verbally. All participation was voluntary.

I visited the case study classroom regularly during the second semester (between September and December). I observed the mathematics classroom at least once a week during this period, with low inference information (see the data collection section) which was collected during the last unit of the year. Interviews with students started 4 weeks after the beginning of fieldwork.

3.4.3.3 Social and subjective perspective

Following Cobb and colleagues' framework (2001), this study relied on several instruments designed to capture both 1) a social perspective and 2) a subjective perspective (individual student positioning in this culture) of identities. This contrasts with phase number 2, where identities were seen as mainly individual aspects of students' relationships with the mathematics (mainly a subjective perspective was considered in the previous phase). In this sense, this study takes on a more integrated perspective of identity, where the social (normative/taken as shared) and subjective perspective (diversity) are not considered as independent but as "two alternative ways of looking at, and making sense of what is going on in classrooms" (Cobb et.al., 2001, p. 122). This "two way relationship" is consistent with the theoretical framework which considers that " 'person' and 'society' are alike as sites, or moments, of the production and reproduction of social practices" (Holland et al., 1998, p.270).

Social perspective

The social aspect of identity could be seen as patterns of participation emerging in different observed lessons and the shared meanings of students and teachers around mathematics in the classroom. Two methodological considerations were required here: 1) a representative sampling of students [inside the classroom and groups] and lessons [inside a thematic unit]; 2) the use of structured coding to detect patterns and coincidences among different samples (i.e. students and lessons). Collecting and analysing data following a more quantitative perspective allowed searching for patterns and usual forms of participation and definitions of competence shared in the case study classroom. This included *spaces of participation* which students and teachers found available while engaging with the mathematical activities but also *frames of meanings* that allowed the interpretation of human action in this particular space.

Available spaces and patterns of participation

To enable the documentation of the social mathematical practice, the available spaces and patterns of participation, observation and analysis of didactic material were the main source of data. Regular classroom observations were performed during the second semester of the school year, with all the pedagogic material and examples of students' work being collected during those observations. Two different records were kept during these observations: notes on the general structure of the lesson and notes on different student's participation in the offered activities. A seating plan was used with the names of each student marked on it, with space for notes (see figure 3.3).

Initial observations were quite open: Any aspect of the students' participation that was deemed interesting was noted (with higher and lower levels of participation and disruptive behaviours presenting the more noticeable aspects). Comments were also made on the activities that were offered and how the available time was made use of.

These open observations provided a general sense of the norms within the classroom, the pedagogic approach of the teacher, and any student's participation which stood out from that of their peers. However, there was a concern that these observations did not allow the comparison all of the students' participation and only gave a general impression of the pedagogic approach of the teacher. For this reason, I decided to perform what I defined as a 'low-inference' collection of information during lessons in the last unit of the year. Four lessons distributed in an 8 lessons' unit were observed using this approach.

For these 'low inference' lessons, audio-recordings were introduced at different points of the classroom (one on the teacher's desk and 4 placed at different students' desks) and kept notes of every participation each student made during the whole classroom discourse. More detailed information was also collected on activities during the lesson (see figure 3.3).

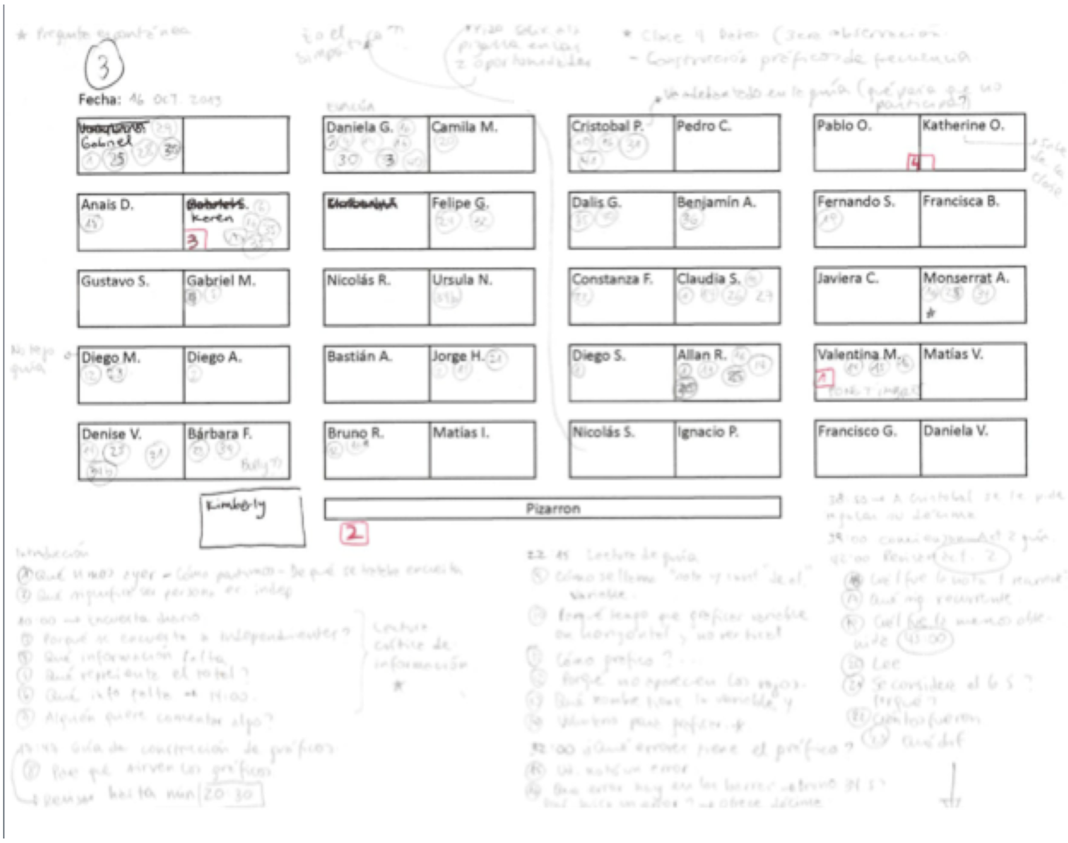


Figure 3.3. Example of seating plan after observation using 'low inference' collection of information. Bottom half records each question asked by the teacher with a number and upper half allows tracking which student answered each question (by recording questions' numbers in students' 'desks').

A written record was kept of every teacher's question and which students answered them by recording the question with a number (bottom half of figure 3.3) and by placing the same number on the student 'desk' (upper half of figure 3.3, seating plan). A record was also kept of every spontaneous question (by placing a mark on the recorded question) and signs of disruptive or clearly disengaged behaviour.

After the observations I contrasted my notes with the information audio recorded. This enabled three things: 1) the ability to make further notes on the different activities and tasks that were performed during the lesson (the general structure of the lesson); 2) the ability to complete any information that was missing regarding the questions that were noted during the lesson; and 3) the ability to complement this with information from answers and feedback which were provided after each question. During this checking process an individual table was created for each lesson detailing its activities and the format of these activities (structure of the lesson) and a database with all of the questions from each lesson, type of answers and the feedback which followed each question.

For the analysis of the lesson structure, I identified what different activities and tasks were used in order to achieve the main objectives of the lesson (and of the lesson in the unit). For this analysis, Wells (1993) application of Activity Theory concepts was used. This application states that an *activity* is a relatively self-contained, goal-oriented unit - such as carrying out an experiment or writing a story. A *task* is, according to the same author, a relatively well-defined component of an activity, which is recognised as such by the participants (Wells, 1993). Based on these concepts, the entire class was segmented into activities and tasks. This segmentation was made using linguistic markers (e.g. "Now…", "First…"), and explicit aims or instructions stated by the teacher (e.g. "We are going to…" "Each group start…"). After the class was segmented and its duration recorded, each activity was described and two codes were assigned to it, one in relation to how the activity was structured (whole class discussion, individual or peer group work and teacher guided) and one related to its purpose (introduction of new contents in the unit, formalisation, revision, reinforcement) (see example in table 3.4). This analysis enabled the identification of how the teacher organised time during a lesson and their approach at different points of a unit (see a summary of the results of this analysis in appendix 3).

Table 3.4:
Example table of structure of the lesson. Lesson 1.

Minute	LESSON 1	Description	Format	Duration
00:00	Management and organization of module	Counting points and intro to the module	Teacher guided	10:20
10:20	Intro new material	Focused on real life. Where have you seen surveys? Teacher elaborates by asking further on students' examples	Whole class	13:40
27:40	Intro concepts (frequency table, absolute frequency and cumulative frequency)	Based on analysis of an example	Whole class	10:30
38:10	Formalization	Group reading out loud	Whole class	4:50
43:00	Reinforcement concepts	Based on analysis of an example	Whole class	11:00
54:00	Intro new procedure	Based on an example (How to translate data into a frequency table)	Peer - individual	6:30
60:30	Revision	Students go to the whiteboard and write the table	Whole class	1:30
62:00	Reinforcement concepts	Based on analysis of an example	Peer - individual	5:40
67:40	Revision		Whole class	6:20

In relation to the patterns of participation, after creating the database of questions in each lesson (and maintaining the initial number that linked each question with the students who answered them), an analysis was performed on the 'type of questions'. I was interested in observing which were the most and the least common questions used by the teacher, but also if there were any patterns in the way students answered these different types of questions. Following categories developed by different authors (Hiebert, Gallimore, Garnier, Givvin, Hollingsworth, Jacobs, J. & Stigler, 2003; Preiss, 2009; Radovic & Preiss, 2010; Wells, 1999) I created a simplified coding structure which identified mainly *types of questions or demand*. Every question was analysed with consideration given to its function in the context of a sequence. This meant that each question was judged in terms of its content and the content of the utterances before and after it. In other words, every question was analysed according to what was said previously, the answer given, how the answer was/was not accepted and if a new question was used to elaborate the point in case. Detail on the coding structure is shown in table 3.5.

Table 3.5
Coding structure for teacher's Questions.

CODE	Description	Examples
CLOSE QUESTIONS		
ANALYZE	Student is asked to analyse information given (mainly extract information from table - including using concepts)	Which are the variables in this graph? How many people was surveyed?
CONCEPT	Student is asked to define or identify a concept	What is a frequency table? What does cumulative frequency mean? What is a sample?
PROCEDURE	Student is asked to describe a procedure	How do you calculate the total sample? How do you get to the cumulative frequency?
APPLICATION	Student is asked to filled a table - graph, according to the concept and info given	What number do you have to put on the relative frequency? Where would you put the percentage?
OPEN QUESTIONS		
OPINION-EVALUATION	Student is asked to give his/her opinion on some topic, with an emphasis on critiquing.	What do you think about the quality of information that we would get from this survey? What information is missing and what implications do you think this missed information will have on the conclusions?
JUSTIFICATION	Student is asked to justify a decision (about an application)	Why did you choose this variable on this axe? Why the cumulative relative frequency sometimes in not 100?
PERSONAL	Student is asked to talk about real life experiences regarding maths concepts	Where have you seen a survey? Why do you think people spends more electricity in winter time?

Table 3.5 (Continuation)
Coding structure for teacher's Questions.

CODE	Description	Examples
ACTS - PARTICIPATION		
BOARD	Go to work on the whiteboard	-----
READ	Student is asked to read something out-loud from the textbook or from a working sheet.	-----
REMEMBERS	Student is asked to remember previous topics (general question – does not fit with any other code)	What did we do on our previous lesson?

The analysis of the types of questions offered a general description of the spaces of participation provided by the teacher during whole class discussions. Every question was collated in accordance with its type (See Chapter 7 and a summary of these results in appendix 4).

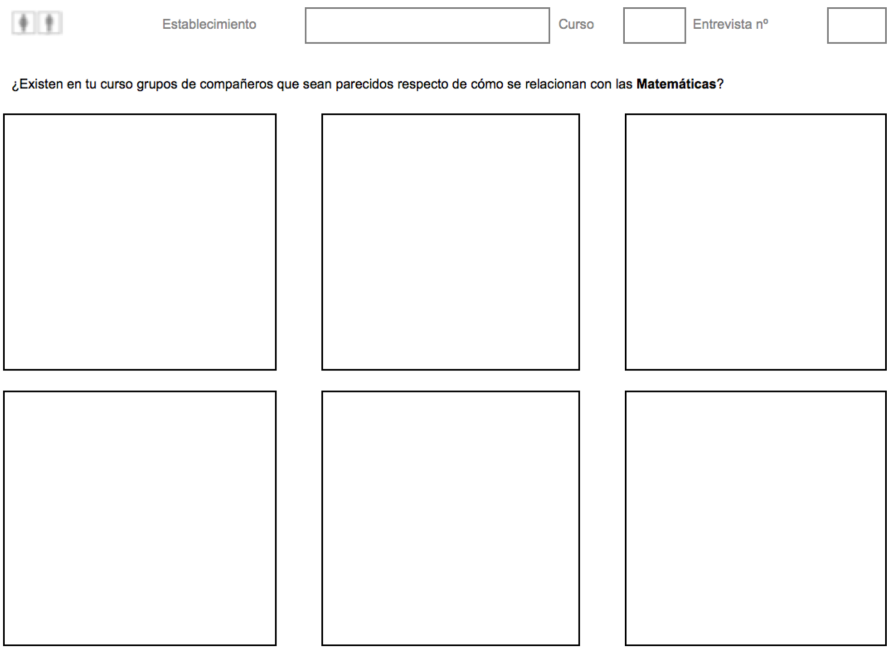
In addition, by analysing all of the questions which created the entire class discourse using this protocol, every utterance that included an open question was analysed in detail (opinion/evaluation, justification and personal experiences) including every question that was accepted by the teacher using elaboration or reformulation (going beyond the formalisation of language). In this analysis I was interested in which students were participating within this type of interaction and the quality of this participation. In order to do this all the utterances that included this kind of question and follow-up moves were transcribed and a preliminary analysis was conducted from the content of this interaction. This information was then used in detail when analysing each of the girls' 'mathematical identities' as forms of participation (see below).

Shared meanings

In order to collect and analyse shared meanings inside the classroom regarding the mathematical practice, interviews with students and teacher were used. Sixteen students were interviewed (7 boys and 9 girls), representing different levels of attainment and different peer social groups (which were identified as part of different social clusters, see below).

I was interested in collecting data on the different notions of competence in place in the classroom and how mathematical activity was perceived by students and the teacher. In order to collect this, each student and teacher was asked two open questions: 1) to describe a typical mathematics lesson; and 2) to group students by answering the following question: *are there groups of students who show a similar relationship with mathematics?* For the second question, students were given a sheet were they could create groups of different kinds of mathematics students (and thus, different peer group clusters, see below and see figure 3.4), and then they were asked to elaborate on the criteria they used for creating these groups. These short interviews (each student interview lasted between 5 to 10 minutes) were audio recorded and then transcribed verbatim.

Following the transcripts of each individual interview I analysed three separate aspects: 1) how mathematical activities were perceived; 2) how each mathematical group was formed; and 3) what criteria was used to create these groups. Perceptions of mathematical activities and criteria for grouping students were analysed thematically following an iterative method and constant comparison analysis as suggested by Strauss & Corbin (1998). This analysis was focused on identifying the main themes that students and teachers discussed with regards to each aspect, and which 'cultural models' (Holland, & Skinner, 1987) regarding mathematics and mathematics competence were in place for each of them. A cultural model was defined as a "stereotypical distillates, generalisations from past experience that people make" (Holland et.al, 1998, p.55), which become shared implicit knowledge on types and ways of discussing these types (Holland & Skinner, 1987). As I was interested in 'shared meanings', I also kept track of how many students mentioned each of the themes and cultural models identified.


Establecimiento Curso Entrevista nº

¿Existen en tu curso grupos de compañeros que sean parecidos respecto de cómo se relacionan con las **Matemáticas**?

Figure 3.4 Sheet used for collecting information about mathematics groups (Mathematics groups interviews) and peer social groups

In relation to the formation of each ‘kind of student’ group, the frequency with which each student was mentioned by their peers and teachers was analysed as part of a particular mathematical group. By contrasting the conformation of each group and definitions of these groups regarding the criteria used for grouping allowed the identification of ‘figures’ (Holland et.al, 1998). These represented the different types of students and how they were attached to particular cultural models (Holland & Skinner, 1987).

In summary, the analysis of the mathematical practice was based on information gathered from observations and interviews with students and teacher. Although the information was analysed separately, it was all brought back together to represent different spaces of participation and shared meanings with regards to mathematical practice. Building this ‘social’ perspective allowed the later contrast of each student’s mathematical identities, particularly when analysing the three girls who were the focus of this study.

Peer social practice

In order to map peer culture as mentioned in Cairns, Xie and Leung (1998) a methodology of natural groups was used. This method asks students to map their peer social clusters by asking them the question *are there people in this classroom* *who hang around together a lot?* (I used the same sheet for this instrument and for the mathematical group instrument, see figure 3.4). Because this study was concerned with accessing students' ideas with regards to these social clusters, this question was followed by an interview which explored the reasons why the interviewee thought the group had been formed or why they 'hung out a lot', what they shared and in what sense they thought they were different to the other groups in the classroom. The interviews commenced with students suggested by the teacher and was carried out by purposefully interviewing students from different peer clusters (as identified in initial/previous interviews). Although Cairns et al. (1998) suggest that half of the students should be surveyed, after the first 7 interviews, groups started to appear stable and stories from the groups in the follow-up interviews started to become repetitive. In total, 12 students [5 boys and 7 girls] were surveyed using this method.

The information collected by these interviews was analysed with two foci: 1) the conformation of each group; and 2) the contrasting definitions of each group. When analysing the conformation of each group, the Composite Social Maps software (SCM 4.0) was used. This software computes a co-occurrence matrix based on the information given by the students. After this co-ocurrence matrix is computed, it identifies groups in terms of similarities between co-occurrences of individuals using a correlation matrix (Cairns, Gariepy, Kinderman & Leung, 1996) (see a map of the peer cluster in the classroom in appendix 5).

After identifying the different social groups, a new matrix was created crossing each group with the definitions given by the students who had been interviewed from each of the groups (see table 9.8 in appendix 6). This data display allowed the identification of how each group was described by its own members, and also how it was perceived by others. The analysis of pronouns used also allowed the exploration of how the interviewed students perceived themselves as belonging or not to each peer cluster, i.e. supporting how the groups were formed by the software.

Subjective perspective

For the subjective perspective, the subjective positioning of each of the case study girls in the mathematical practice was analysed. As explained previously, their individual mathematical identities were conceptualised as being evident in two related processes: narrating and acting in the classroom. At the same time, both narratives and acts were only understood when contrasted against the social perspective of the classroom. In other words, narratives and forms of participation acquired meaning only when framed against those which were recognised as shared inside the classroom as described in the social perspective.

In order to analise mathematical identities of the three girls information was collated from the instruments previously described in the social perspective (all focus girls were interviewed in the 'mathematical and peer group interviews', and formed part of the observations which took place inside the classroom). They were also interviewed in detail about different aspects of their mathematical identities (using an 'identity map' and a 'maths story' instruments). Using an 'identity map' the students were asked to think about themselves in the maths classroom, describe the activities, how they behave in these activities, their feelings and thoughts and how they thought their peers and teachers perceived them when doing mathematics. The students made a few notes on what they said and at the end were asked to name their "maths character" (see figure 3.5).

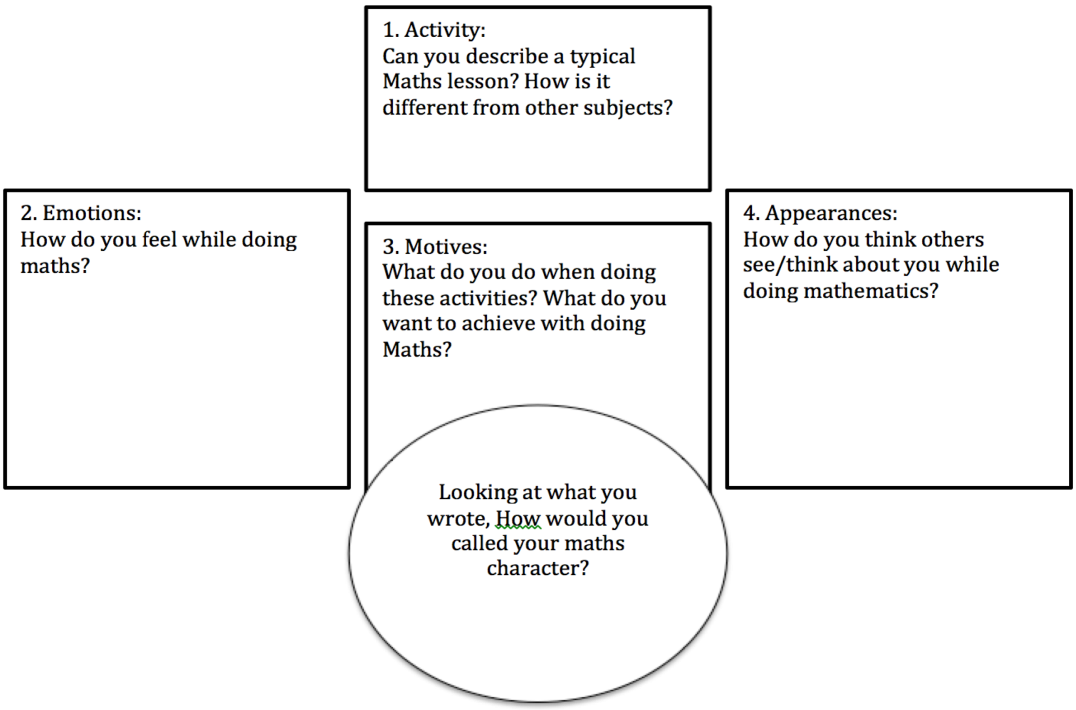


Figure 3.5: Mathematical Identity Mapping. Guiding questions inside the squares. Students were given instrument without promptings.

For their 'maths stories' they were asked to think and narrate on how they came to develop their current relationship with mathematics and how they projected this relationship into the future. They were given a map where they were asked to think on the different critical moments and to identify the higher (positive emotions) and lower (negative emotions) points in their mathematical lives (see figure 3.6).

All the information gathered by interviews and observations was collated together in a single document for each of the case study girls. This document included: (a) description of their individual participation in whole classroom discourse (lesson observations); (b) their own definitions of competence (mathematical group interviews); (c) their own positioning regarding these definitions (in which group they classified themselves); (d) their definitions of membership in their peer groups and how they distanced themselves from other groups (natural peer clusters interview); and e) their interviews regarding their mathematical stories and relationship with mathematics in the present (mathematical identities interviews). In addition, information from other students and the teacher [quotes where each girl was mentioned] were included in the document, as a way of assessing how they were positioned in classroom practices by others. All of this data was transcribed in its original language (Spanish).

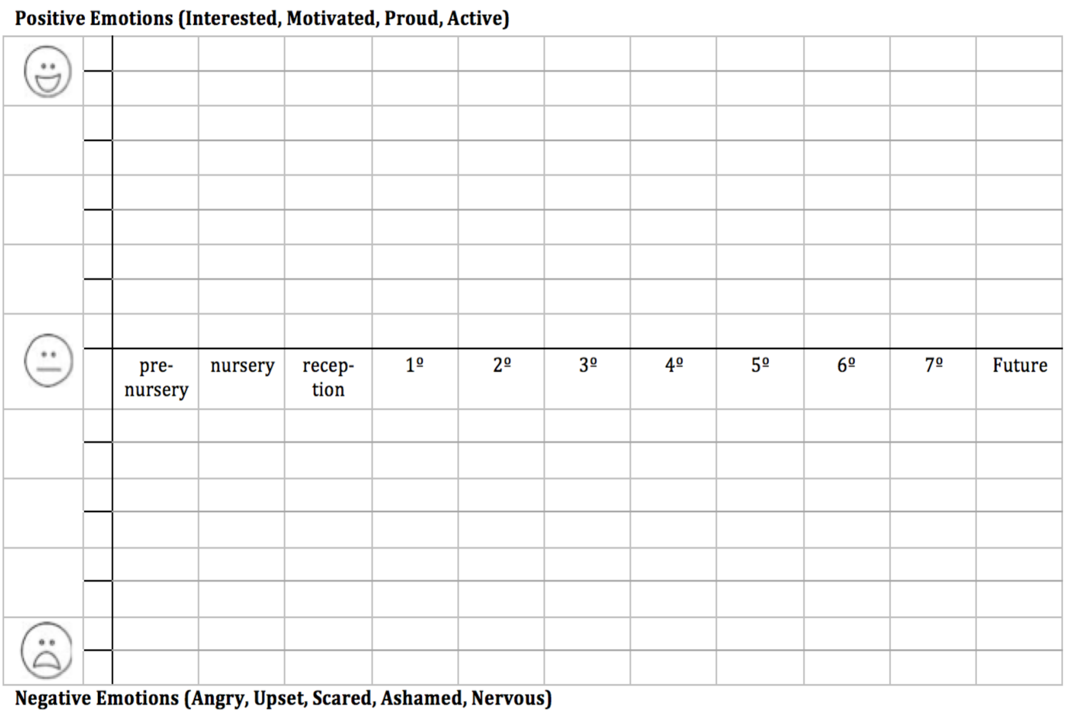


Figure 3.6: Mathematics life story. Students were asked to tell their mathematical stories and to mark highest and lowest point in this graph.

The analysis of each girls' mathematical identities was first carried out descriptively and then by looking for relationships between the different aspects of their mathematical identities in order to test possible explanations for their different positionings (both in comparison with the rest of the class, and in comparison with the other girls who were the focus of this study). With the descriptive analysis of the different aspects of these girls' mathematical identities, different forms of display were used to explore and compare the data. Firstly, each case was put back together, re-examining what they said and using their own metaphors in order to construct an 'identity narrative' (Clandinin & Conelly, 2000). Next, different tables with summaries on each aspect of each girls' identities were constructed in order to facilitate contrast and identification of similarities between the three girls and between the different sources of data. This final set of information was central in identifying potential areas of tension and conflict in girls' mathematical identities, which were central to understand their individual experiences.

3.4.3.4. Closing remarks on Phase 3 Methodology

In contrast to the methodological approach adopted in phases one and two, phase three was focused on exploring three different girls' mathematical identifications in the mathematical classroom in detail. Therefore, in contrast to the previous two phases, phase 3 was concerned with providing a complex and detailed account of these processes, rather than an extensive exploration of descriptive differences (phase 1) or 'average effects' (phase 2). In order to provide this detailed account, this study relied on a case study methodology with emphasis on qualitative approaches to data collection and analysis, but also triangulation with quantitative methods. This methodological approach allowed operationalising a more integrated notion of identity and the exploration of the complex social phenomenon of identity construction in real life settings (see chapter 7).

3.5. Ethical Considerations

With consideration for the characteristics of this study, four main ethical threads were identified: 1) Working with a vulnerable population (children under 16); 2) the inconvenience of participation; 3) the possible emergence of contentious issues during interviews; and 4) collection and use of personal data from secondary sources.

With regards to the necessity of working with a vulnerable population, the focus on students' relationships with mathematics during school made this compulsory. To limit the possible threats identified in relation to this issue, all data was collected inside the schools of the children and during school hours. In order to ensure that all children were fully aware and had agreed to participate in the study, the purposes and activities of data collection of all the different studies were explained verbally to them. After this explanation, the child's verbal assent was obtained in addition to an informed written consent from their legal guardian (parents) (see below).

Secondly, the inconvenience of participation in terms of time taken for interviews and the potential interference to learning due to processes of observation were kept to a minimum through the design of activities. Observations took place during scheduled lessons, without asking for any changes to the regular practice of the teachers and students. They were also conducted at a time as agreed with the teacher. Participation in interviews was always voluntary, and these interviews were also conducted at a time that was convenient and caused no negative consequence for the participants. For students, individual interviews were implemented during class meetings. Class meetings are a regular practice in Chile. During two pedagogical hours available a week for these meetings, the entire class discusses problems and plans group activities. As most interviews were short (between 5 to 30 minutes) each student did not miss much during these meetings (and did not lose time from any academic activity). A couple of group discussions were also organised with students, but these were planned to take place at the end of the academic year, when most academic activities had finished.

Several informal conversations with teachers before and after class were routine during the whole fieldwork. When more time was needed, a date for a longer interview was conjointly set which did not interrupt their usual activities.

Thirdly, all the activities of the study explicitly avoided contentious issues. Although no distress was expected, several ongoing checks were made to ensure that the interviewees were feeling comfortable during interviews. Participants were given the possibility to withdraw their participation in any research activity at any moment of time if they felt uncomfortable or didn't want to continue. This situation only happened once. Physical Education teacher gave permission to interview students during his lesson. During an interview with a boy, he said he wanted to apply for a football club, and added that physical education was his favourite subject. I asked him if he would prefer to be interviewed in some other time, and although at the beginning he said no, after just a little insistence, he said yes. We stopped the interview and continued it later.

Finally, information which may enable the identification of participants and/or institutions was altered to preserve anonymity. This included the use of secondary data (which was given using a mask of the national identification number) and data collected by surveys (study reported in Chapter 6) and case studies (study reported in Chapter 7).

Informed consent and assent was obtained from different sources in different stages of the study. Firstly, The Ministry of Education gave access to the datasets of the SIMCE data for the purpose of one of the studies of this thesis (reported in Chapter 4). This access was restricted for the use of this research and under the conditions detailed in the document "Condiciones de Uso de las Bases de Datos SIMCE", accessible on the following internet domain http://www.simce.cl/index.php?id=448.

For the survey data (study reported on Chapter 6), an opt-out form of consent from legal guardians was obtained. At least two weeks before starting the data collection, each school sent an information sheet and consent form (see appendices 7 and 8), asking parents to read and sign it if they did not wish to allow their pupils to participate. On the day in which the survey data was collected, students were informed of the purpose of the study and were offered the opportunity to opt-out if desired. Only students who wished to participate and whose parents had not sent an opt-out form were allowed to complete the surveys.

For the classroom study (case study reported in Chapter 7) an opt-in consent form from parents was obtained. Approximately two weeks before starting the data collection and visits to the case study classroom, I attended a parents meeting in school. In this school, parents meetings take place once a month, and I attended one organised near the beginning of the fieldwork. In this meeting I informed the parents verbally about the purpose of the study and answered questions. I gave them an information sheet and opt-in consent form that they were asked to return within two weeks if they agreed on their pupil's participation in the study (see appendices 9 and 10).

Finally, in both stages of this research project, teachers were invited to participate in the studies in a face-to-face meeting. In these meetings I discussed the purposes of the study and activities in which their students and themselves would be invited to participate. An information sheet and opt in consent form was also given and the teacher had at least two weeks to confirm his/her participation (see appendices 11 and 12).

Chapter 4. Gender differences in Mathematics Attainment in Chile: The Effects of Socioeconomic Status and Schools on Girls Underachievement

4.1. Abstract

This study explores the size and distribution of the gender gap in the national standardized test of mathematics attainment in Chile (SIMCE). Multilevel modelling was used to explore differences, the interactions between gender and socioeconomic status (SES) at the student and school level, and differential effects of schools for boys and girls. Results suggest that boys consistently and statistically significantly outperform girls in both early and late primary school, and that girls progress less between these years. In addition, a significant interaction with SES shows that this gap is larger for students with lower SES, with school SES effects being highly dependent on family SES effects. Small variability between schools gender gaps was found, with no school predicted to be better for girls than boys. Political relevance of these differences and further areas of studies are discussed.

Key Words: Gender and Mathematics; gender gap; gender and socioeconomic status; School Differential Effectiveness; Chile.

4.2. Introduction

The study of gender differences in academic attainment has been a major focus of international research for at least the last 40 years. Although in some first world countries recent reports and studies have documented a decrease in the general 'male advantage' in mathematics (e.g. Gorard, Rees & Salisbury, 2001; Hyde, Lindberg, Lynn, Ellis & Williams, 2008), this is not a generalised tendency: differences still persist in less developed countries, particularly in Latin-America (OCDE, 2013; Treviño et al., 2010). In Chile, for example, the national standardised test of mathematical attainment has reported persistent male overachievement over the last 10 years (MINEDUC, 2005). However, to the best of our knowledge, no study has systematically explored the nature of these differences with a multilevel modelling approach, where the socioeconomic context of schools and family are taken into account. The aim of this article is to address this gap by describing how gender differences are distributed in the population and the relative contribution of schools in maintaining or challenging these differences. This area of research has been highlighted by international research, but has been relatively neglected in the Chilean (and Latin American) context.

4.2. Background Literature

4.2.1. The State of the Gender ‘Problem’ in Attainment in Mathematics

As stated previously, the study of gender differences in academic and mathematical attainment has been the focus of intensive international research for at least the last 40 years. In the 1970s Maccoby and Jacklin reviewed over 1600 studies, concluding that boys achieved more than girls in mathematics, while girls outperformed boys in reading and writing (Maccoby & Jacklin, 1974). Since then, many studies have replicated these findings, by reporting that girls are better at literacy but not numeracy (e.g. Hyde, Fennema & Lamon, 1990; Willingham and Cole, 1997).

These findings have led some researchers to suggest that differences in mathematics could be of great importance beyond mathematics educational attainment. During the 1970s and 1980s the female disadvantage in education was viewed as one of the causes of gender inequalities in adulthood, especially in relation to the labour market (e.g. England & Browne, 1992). It was proposed that mathematics worked as a "critical filter", controlling access to many areas of advanced studies, which were linked to status and power in society (Sells, 1978). Recently, the same debate has re-emerged in the Latin-American context, with a study from the World Bank suggesting that lower scores in mathematics university admission tests could partially explain the gender gap in wages (Ñopo, 2012).

Even though there is robust evidence suggesting a decrease in gender differences in mathematics attainment in first world countries (Gorard, Rees & Salisbury, 2001; Hyde, Lindberg, Lynn, Ellis & Williams, 2008; Nowell & Hedges, 1998), a different picture can be observed in comparative international studies. Here, a large variation in the size of these gender differences between countries has been reported (Else-Quest, Hyde & Linn, 2010; Guiso et al., 2008). In Latin-America, the Latin American Laboratory for Assessment of the Quality of Education, coordinated by UNESCO (*LLECE*), has reported that boys outperform girls in mathematics in most countries (LLECE, 2000, 2008). The same conclusion has been reached by national studies in the region, with consistent male advantage shown at the end of primary and in secondary schools in Argentina (Cervini & Dari, 2009), Brazil (Gaviria, Martínez-Arias & Castro, 2004), Mexico (Gonzalez-Jimenez, 2003; Zorrilla & Muro, 2004), Colombia (Piñeros & Rodríguez, 1998) and Peru (Ministerio de Educacion Peru, 2001). Other studies have reported similar differences at primary school level in Brazil (Alves Macedo, 2004) Nicaragua (Navarrete, López & Laguna, 2008) and Mexico (Blanco et al., 2008). There are only a few countries where a male advantage in mathematics has not been found, all of them at the primary level: Bolivia (SIMECAL, 1998), Costa Rica (IIMEC, 1997), Paraguay (DOEE, 1998) and Peru (Ministerio de Educacion Peru, 2001).

In Chile, the national standardised test of mathematical attainment has also reported a consistent pattern of male advantage over the last 10 years (Agencia calidad de educacion, 2013a; 2013b; 2013c; MINEDUC, 2005). This situation has been confirmed by international studies in which Chile has participated, for example reporting a male advantage in mathematics in TIMSS (Mullis, Martin, Foy & Arora, 2012) and PISA (Manzi, Strasser, San Martın & Contreras, 2008; OECD, 2013). As shown in reports in the US and UK (e.g. Fryer & Levitt, 2010; Maccoby & Jacklin, 1974), in primary school these differences are rather small or even non-existent, but they start to increase as students progress in their school lives (MINEDUC, 2005).

A limitation of the Chilean and most Latin American studies is that general studies on associated variables with educational attainment have not considered how student progress is affected by their gender or the multilevel structure of the educational system (e.g. Cornejo & Redondo, 2007; see Cervini & Dari, 2009 for a progression analysis in Argentina). Modelling progress in a multilevel context is relevant for two main reasons. Firstly, it allows one to model individual trajectories during schooling and therefore, controls the estimation of gender differences by individual differences. And secondly, following the control of these individual variables, modelling progress allows one to also estimate the relative contribution of schools (and school variables) in this progress. The inclusion of a longitudinal design in the National Assessment of Educational Quality (SIMCE) since 2004, where the same student is evaluated during, and at the end of primary school, or the middle of secondary school, has recently made this possible: SIMCE now allows the modelling of individual progression between early and late primary school levels (between 9-10 years and 13-14 years) and between early primary and secondary school levels (between 9-10 years and 15-16 years).

4.2.2. Gender and Socioeconomic Status Influences in Mathematical Attainment

Although general differences between boys and girls have been widely studied, several authors have argued that more attention should be paid to the largest within-gender differences (Leder, 1992), particularly gender differences across ethnicity and social class groups (e.g. Archer, 1996; Boaler, Altendorff & Kent, 2011; Grant & Sleeter, 1986; Lubienski, 2008). This call for attention is also supported by several qualitative studies, which have shown how students' gendered identities, attitudes to school and mathematics, and corresponding attainment vary in relation to ethnicity (e.g. Mac an Ghaill, 1994; Martin, 2012) and social class identities (e.g. Black, 2004; Willis, 1977). For example, Willis (1977) highlighted the existence of a "laddish" culture among working class boys, where opposition and resistance to authority (and school) led these students to failure at school and in future working class jobs. More recently, some studies have suggested that this 'laddish' behaviour is not solely restricted to boys, but also to working class girls who support this resistant culture (Jackson, 2006). Following this logic, several studies have supported the notion that gender differences are ethnic- and culture- specific (which has also been supported by international comparison studies, e.g. Else-Quest, Hyde & Linn, 2010) and are strongly related to social categories (i.e. ethnicity and social class).

Although there is strong support for the study of *interaction effects* between gender and other social categories in the literature, surprisingly, this is still a relatively neglected area of research. For example, Grant and Sleeter (1986) reviewed 71 papers from four important journals between 1973 and 1983, and reported little integration between variables such as gender, social class and ethnicity. Twenty years later, Connolly (2006) also found a similar situation. By analysing GCSE data from England and Wales, he reported an independent effect of gender, smaller than the independent effect of class and ethnicity. Because no interaction was found between them, it was concluded that the gender effect is stable between different ethnic groups and social class (Connolly, 2006). Strand (2010) focused on ethnicity and explored its interaction effect with gender and poverty in a national cohort in England. He concluded that interaction analysis allowed the identification of black Caribbean boys, who were not entitled to free school meals as being the primary focus for the white-British black-Caribbean attainment gap (Strand, 2010).

In Chile, the available evidence with regards to the relationship between gender differences in mathematical attainment and socioeconomic status or ethnicity is even more limited. In terms of the ethnic differences in attainment, no studies that explored its interaction with gender were found. This can be explained by the restricted number of studies that focus on ethnicity itself, probably because of the small population in Chile which identify as ethnic minorities and because of the small differences in academic attainment that have been reported in the literature (mostly explained by socioeconomic status). For example, McEwan (2004) found that ethnic minorities (which represented between 4% and 6% of the school population) achieved between 0.3 and 0.4 standard deviations less than the non-ethnic minority population. He found that most of the difference was explained by family socioeconomic status or school and peer variables, with differences dropping to 0.05 when these effects were controlled for.

In contrast with the sparse literature on ethnicity, research on the effects of socioeconomic status in educational attainment have received much more attention in Chile. This country has been described as having high levels of inequity and low educational results (see for example Brunner & Elaqua, 2003; Cornejo & Redondo, 2005). This literature has reported large differences in educational attainment between students of different socioeconomic status (e.g. Mizala & Romaguera, 2000), and differences in these effects in relation to the administration of the school (e.g. Mizala & Torche, 2012; Torche, 2005). In relation to the interaction between gender and socioeconomic status, only studies which compared mean differences for boys and girls from different socioeconomic groups (i.e. different type of schools and schools from different socioeconomic backgrounds) were found. Some of these studies reported that differences in mathematics were stable in different socioeconomic groups (MINEDUC, 2005), while others suggested that male overachievement was greater in less privileged populations (Agencia de Calidad de la Educacion, 2013c). For example, a descriptive report from the Ministry of Education using TIMSS data found that mean differences between boy and girls were greater in subsidised schools, with no gender differences in private schools (which service 7% of the population in the country, including most of the richest) (Agencia de Calidad, 2013c).

There are no studies that have yet explored the relative contribution of schools and families' socioeconomic status in the distribution of the gender gap in educational attainment in Chile. Understanding this relationship is particularly relevant in view of the large educational gap in existence between students from different socioeconomic backgrounds (e.g. Torche, 2005), a socioeconomic status effect which has been described as being particularly strong at the school level (e.g. Mizala, Romaguera & Urquiola, 2007). This problem acquires even more relevance when considering the highly stratified Chilean educational system, where schools (particularly private schools and private schools with public subsidy) have become 'socioeconomic niches' for highly similar student populations in terms of their socioeconomic status (Mizala & Torche, 2012). As a consequence, educational differences could be systematically reproduced, and therefore a possible interaction between gender and socioeconomic status could imply a greater disadvantage for underperforming populations.

4.2.3. Schools Gendered Differential Effect in Mathematical Attainment

A final aspect that has received attention in the international literature regarding gender effects in educational attainment is the possible influence of schools on these differences. Following the general school effectiveness paradigm, which tries to link school characteristics with students' attainment (e.g. Scheerens, 1990), several authors have suggested that school effects may be differential for different sub-groups of pupils, i.e. for different ethnic, socioeconomic, previous attainment and gender groups (e.g. Palardy, 2008; Sammons, Nuttal & Cuttance, 1993; Strand, 2010).

The study of differential effectiveness across students' gender has been scarce and inconclusive. It has been suggested, for example, that some schools are actually narrowing gender gaps between students (Nuttall, Goldstein, Prosser & Rasbash, 1989), although this school effect has also been described as smaller than the effect of previous attainment and ethnicity on attainment (Thomas, Sammons, Mortimore & Smees, 1997). On the contrary, other studies have not found evidence of differential school effectiveness in terms of gender (Kyriakides, 2004; Strand, 2010).

Although in the Chilean educational system gender differences have been considered of great public concern and have been suggested as one of the criteria for assessing school effectiveness (Agencia de Calidad de la Educacion, 2014), there are no studies linking school characteristics -or school effects- to gender. In fact in the entire Latin American context, this area of research has been notably absent. One exception to this is Cervini and Dari's (2009) study using the National Census of Education at secondary school level in Argentina which found a significant variation in the gender effect between schools: in this context schools from less privileged backgrounds showed greater gender difference when compared with schools enrolling students from more privileged backgrounds.

In summary, the study of gender differences in mathematics attainment requires an integrated analysis that should include how these differences change during the school trajectory and how they differ between school contexts. There has been a strong requirement for studies to explore the interaction of gender with other social categories (such as social class) and with the estimation of school effect, with both areas being largely absent in the Latin American and Chilean context. This study will explore these aspects in detail for the first time in Chile. In order to accomplish this goal, a multilevel approach will be used in order to estimate differences in attainment where different factors will be controlled through the clustered organisation of data. More specifically, the questions that this study is attempting to answer are: 1) How great are the differences in mathematical attainment between boys and girls and how do they progress/change during primary school? 2) Is there a relationship between these differences and the socioeconomic status of the children? 3) Are gender differences distributed heterogeneously across schools of different socioeconomic status and different administrations? 4) Do schools vary in their relative effectiveness for boys and girls?

4.3. Methodology

4.3.1. Data and Variables

The following analysis was based on data from Mathematics SIMCE, which is provided by the Chilean Ministry of Education. SIMCE consists of a series of standardised tests, measuring attainment in different areas of the Chilean curriculum. This test has been applied since 1988, increasing its legitimacy within the Chilean context (Meckes & Carrasco, 2010), the stakes attached to its results (particularly at the school level) (Agencia de Calidad de la educacion, 2014), and the range of levels and subjects assessed. Since 2004, students started sitting at least two SIMCE applications during their school history (year 4 and year 8 or 10), which has allowed the use of this instrument in exploring progress at the individual level.

This analysis considered a cohort of students which was assessed in year 4 (2005) and year 8 (2009), thus allowing the assessment of gender differences in attainment in both the middle (year 4, 9-10 years) and at the end of primary school (year 8, 13 – 14 years). In addition it was also possible to explore individual progress between year 4 and year 8, looking at whether the gender gap decreased or expanded throughout primary school. This approach enables the measurement of schools' 'added' value, and therefore the ability to model a differential effect of gender at the school level.

Contextual variables were obtained from information provided by the Ministry and by parents through questionnaires also collected by the Ministry of Education. For a detailed description of each variable, its source and definition see Table 4.1.

Table 4.1:
Variable descriptions

Variable	Source	Description
Outcome	SIMCE data base	Year 4 / Year 8 Standardized Maths SIMCE scores (Mean= 0 SD=1)
Individual Level Variables		
Prior Attainment	SIMCE data base	Year 4 Maths standardized score used as control in Progression model (Mean= 0 SD=1)
Gender	SIMCE data set	Binary: 1 for female pupils
Family Socioeconomic status	Parents' questionnaire	Factor analysis (Principal component analysis) of mother's max education, father's max education and family income (as reported by parents), standardized to have a mean of zero and a standard deviation of unity

Table 4.1 (Continuation):
Variable descriptions

Variable	Source	Description
School Level variables		
Type of school	SIMCE data base	Categories according to administration and funds: Public (public funds – public administration) – Private subsidized (public funds – private administration) – Private (private funds and administration)
Urban / Rural	SIMCE data base	According to location
Socioeconomic status	SIMCE data base	Categories according to students' socioeconomic status. Provided by SIMCE, schools are classified in 5 different socioeconomic status categories, with 1 being the lowest and 5 being the highest category
School Previous Attainment	SIMCE data base	Categories according to mean average score in year 4 SIMCE. Three different categories built: schools with 25% lower mean scores, 50% schools of middle attainment and schools with 25% higher mean scores.

4.3.2. Analysis

4.3.2.1. Methodological rationale.

In order to answer the research questions above, a multilevel modelling analysis was used. This approach considers the nested structure of data [individual, classroom and school levels], by including the variation of higher hierarchies (for recent reviews on this method see Hox, 2002; Raudenbush & Bryk, 2002; Snijder & Bosker, 1999). This method allows us to model variability independently at the different levels and interactions between these levels. This implies that it is possible to assess the relative contribution of, for example, individual characteristics and school characteristics and, at the same time, how these different sources of variability interact in their effects on the outcome variable (e.g. Snijder & Bosker, 1999). In this particular case, previous literature with Chilean data has confirmed that a significant amount of variance in students' attainment is actually explained at the school level (Manzi, Strasser, San Martın & Contreras, 2008; Mizala, Romaguera & Urquiola, 2007). Based on these findings, it has also been suggested that by including intermediate (classroom) and higher (local authorities) levels this provides better parameter estimations (Troncoso, Pampaka & Olsen, 2015). As mentioned previously, most Chilean reports directly concerned with gender differences have not addressed these issues.

4.3.2.2. Analytical strategy.

A common analytical strategy often used in Multilevel Modelling is to compare models with increasing complexities (e.g. Snijder & Bosker, 1999; Raudenbush & Bryk, 2002). In this case, gender was assessed independently and in interaction with variables at the individual level [socioeconomic status of the family and previous attainment] and with variables at the school level [type of school, location of the school, socioeconomic status of the school and previous attainment at the school level] (see table 4.1). Three different 'outcome' variables were modelled independently: year 4 attainment, year 8 attainment and progress between year 4 and year 8 (year 8 score controlled by year 4 score). For each of these variables, several models were tested. Following this, data was analysed in four steps, thus, addressing each research question of this study:

1) *Empty models and Models of gender:* Models where the independent contribution of gender for different outcome variables (year 4, year 8 and progress) was assessed.
2) *Family Socioeconomic status models:* Models where the effect of gender was controlled by socioeconomic status and by the interaction effect of gender and socioeconomic status.
3) *School systematic effect models:* Models assessing the systematic differential effects of gender in schools of different administration, schools with different socioeconomic status, schools with different levels of previous attainment (as measured by school averages of year 4 scores) and in rural schools (in comparison with urban schools). Considering the strong relationship between schools' socioeconomic status and family status (Manzi, Strasser, San Martın & Contreras, 2008), and the highly stratified and homogeneous population which public, privately subsidised and private schools cater for (Torche, 2005), models were tested in two steps. Firstly, the independent contribution of each school level variable was tested independently. Then, the joint contribution of these school variables was assessed, again with and without the control of family

socioeconomic status. The increasing complexity of these models was compared, comprising cross level interaction with gender added independently for the three variables (two-way cross level interaction) and models where the dependent contribution of gender, school socioeconomic status, school type and school previous attainment was considered (three-way cross level interaction).

4) *School random effect of gender:* Models where the gender differential effectiveness of schools was estimated. In order to estimate this differential effectiveness, intercepts of schools were allowed to vary in terms of gender. These models were controlled by individual level variables (family socioeconomic status of the students' families and previous attainment). Two random variables and their correlation were estimated per model: the general intercept of the school and the difference between boys and girls in the school (the female coefficient) (Goldstein, 2003). Following these estimations, schools can roughly be classified into 9 different types of school according to general effectiveness (if students attained less than expected, roughly what was expected and above expectation progress) and to gender differential effectiveness (boys progress exceeds that of girls, no differences and girls progress exceeds that of boys) (Gray, Peng, Steward & Thomas, 2004). This classification was made following Gray and colleagues' suggestion of using a standard and more relaxed assumption of confidence intervals (2 and 1 standard deviation from general distribution respectively) (Gray et.al., 2004).[11]

Different models were compared using the likelihood test and the percentage of variance explained by models was defined as the proportional reduction in mean square prediction error at level 1 in comparison with a corresponding model[12]. The significance of individual variables was assessed using the Wald test. For categorical variables of more than one level (i.e. Type of School, School Socioeconomic Status and School Previous Attainment) the Wald multivariate test was used, tested against a chi-square distribution (as suggested by Snijder & Bosker, 1999, p. 88). The significance of individual categories was only reported (and interpreted) if this test was significant.

[11] Only mixed gender schools were included in this analysis (schools with 5 or more boys and girls in each classroom)
[12] Proportional reduction of unexplained variance at level 1 – in comparison with previous models allows one to estimate the contribution of predictors in explaining variance at level 1 (estimated as suggested by Snijder and Boskert, 1999, p. 102)

4.4. Results

4.4.1. Empty Models and Models of Gender

In relation to question 1 [How great are the differences in mathematical attainment between boys and girls and how do they change during primary school?] this study showed that girls were outperformed by boys in the three outcome variables, with an increase in this gender difference shown between early and late primary measures. Boys were also found to make more progress than girls during primary school (and mean differences were greater in year 8 than in year 4) (see table 4.2).

Table 4.2:
Gender effect in Mathematical attainment.

	Year 4 Maths Attainment Gender Coeff (S.E)	Year 8 Maths Attainment Gender Coeff (S.E)	Progress Gender Coeff (S.E)
Uncontrolled model (1 level)	-0.028 (0.007)	-0.174 (0.005)	-0.091 (0.003)
Uncontrolled model (3 levels)	-0.064 (0.006)	-0.206 (0.004)	-0.117 (0.003)
r^2 3 levels compared with Null	.10%	.42%	.22%
chi-square 3 levels model (compared with Null)	$x^2_{(1)}$= 468.06***	$x^2_{(1)}$= 2768.79***	$x^2_{(1)}$= 1331.88***

Note. Negative gender coefficients represent male advantage. P < .05 *; p < .01 **; p < .001***

By comparing all the empty models and models of gender differences it was confirmed that a main effect of gender remained significant for attainment in year 4 and year 8 and for progress, however changing the size of this difference. For the three outcome variables mean differences in gender were smaller when uncontrolled for the clustered structure of the data compared with that controlled by schools and fixed classroom effects. This difference suggests that part of the pattern indicating boys' over attainment could be explained at the school and classroom level.

In addition, by comparing the main effect of gender on the different outcome variables it was confirmed that differences increased while students advanced in their primary education. Differences in year 8 were nearly three times greater than differences in year 4, moving from 6% of a standard deviation to 21%. This was also depicted in the progress models, where girls were expected to increase their scores to around 12% of a standard deviation less than their male counterparts. Finally, overall gender explained only a small portion of the variance in attainment (between 0.10 and 0.42 percent).

4.4.2. Family Socioeconomic Status Models

In terms of the relationship between gender and socioeconomic status [research question 2: Is there a relationship between gender differences and socioeconomic status of the children?] this study showed different results in the different outcome variables. While the gender attainment gap was similar across low and high socioeconomic status in year 4, it tended to be smaller in year 8 for students with higher family socioeconomic status, particularly in the upper tail of the socioeconomic status distribution. In other words, in year 8 as family socioeconomic status increased, female disadvantage decreased. In progress, this relationship was mediated by the effect of previous attainment, with differences being greater for girls from lower socioeconomic status and lower previous attainment (see table 4.3).

Table 4.3:
Individual Level Models, Gender Controlled by Family Socioeconomic Status.

	Year 4 Maths Attainment	Year 8 Maths Attainment	Progress
Intercept	-0.007 (0.005)	0.002 (0.007)	0.095 (0.005)
Year 4 (Previous Attainment)			0.605 (0.002)***
Female	-0.076 (0.004)***	-0.197 (0.004)***	-0.119 (0.003)***
Family SES	0.292 (0.003)***	0.170 (0.003)***	0.073 (0.003)***
Female * Family SES	-0.002 (0.004)	0.013 (0.004)**	0.000 (0.003)
Female * Year 4			0.012 (0.003)***
Family SES * Year 4			0.015 (0.002)***
Female * Year 4 * Family SES			0.011 (0.003)***
r^2 (compared with Null)	16.1%	11.8%	4.7%
chi-square (compared with Null)	$x^2_{(3)}$= 10872.9***	$x^2_{(3)}$= 6894.3***	$x^2_{(6)}$= 2581.5***

Note. p < .05 *; p < .01 **; p < .001***

After the inclusion of the effect of the socioeconomic status of the family at the individual level there was a significant increase in the amount of explained variance, but across all models the gender coefficient remained significant and approximately the same size. The analysis of family socioeconomic status also confirmed a significant effect of this variable in all models, with this effect being greater than the effect of gender in year 4, but decreasing its size by year 8. In general, the effects of family socioeconomic status in students' attainment decreased by around 40% during primary education (between year 4 and year 8, from .29 to .17 of a standard deviation), but its joint effect with gender increased. Following this, while the effect of gender and socioeconomic status remained independent during year 4, in year 8 the effect of socioeconomic status was greater for girls than for boys (girls R^2 = .513; boys R^2= .481)[13]. This can be observed in figures 4.1a and 4.1b for mathematics attainment in year 8.

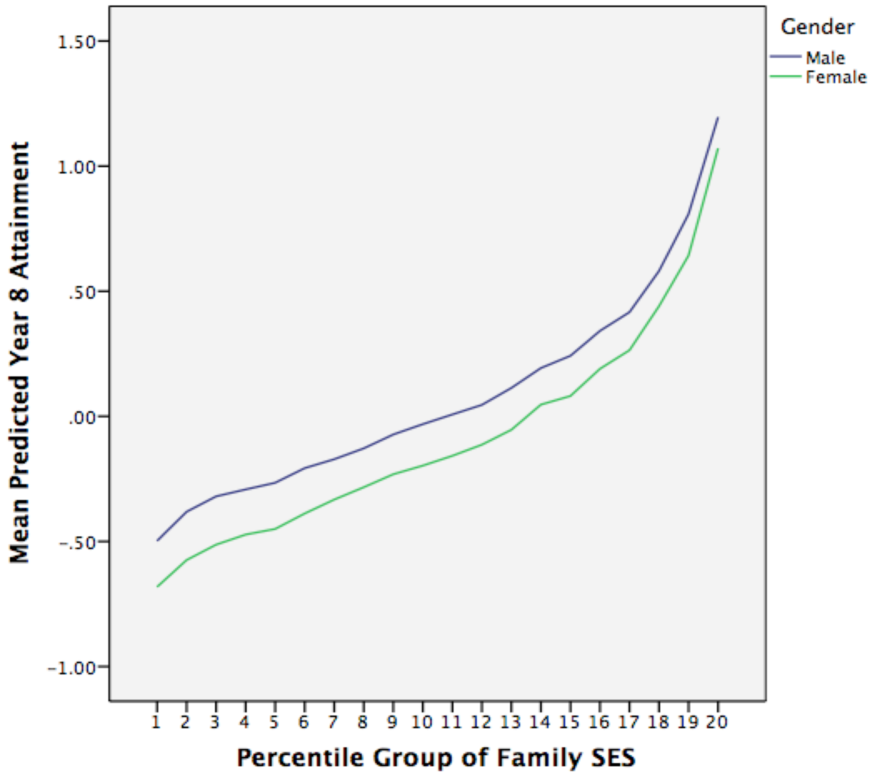


Figure 4.1a. Year 8 Predicted Mathematical scores by Family Socioeconomic Status (Total Mean)

[13] These are the r^2 of the variables family SES on the predicted scores. In the raw scores these values are .166 - .184 and .185 - .161 respectively.

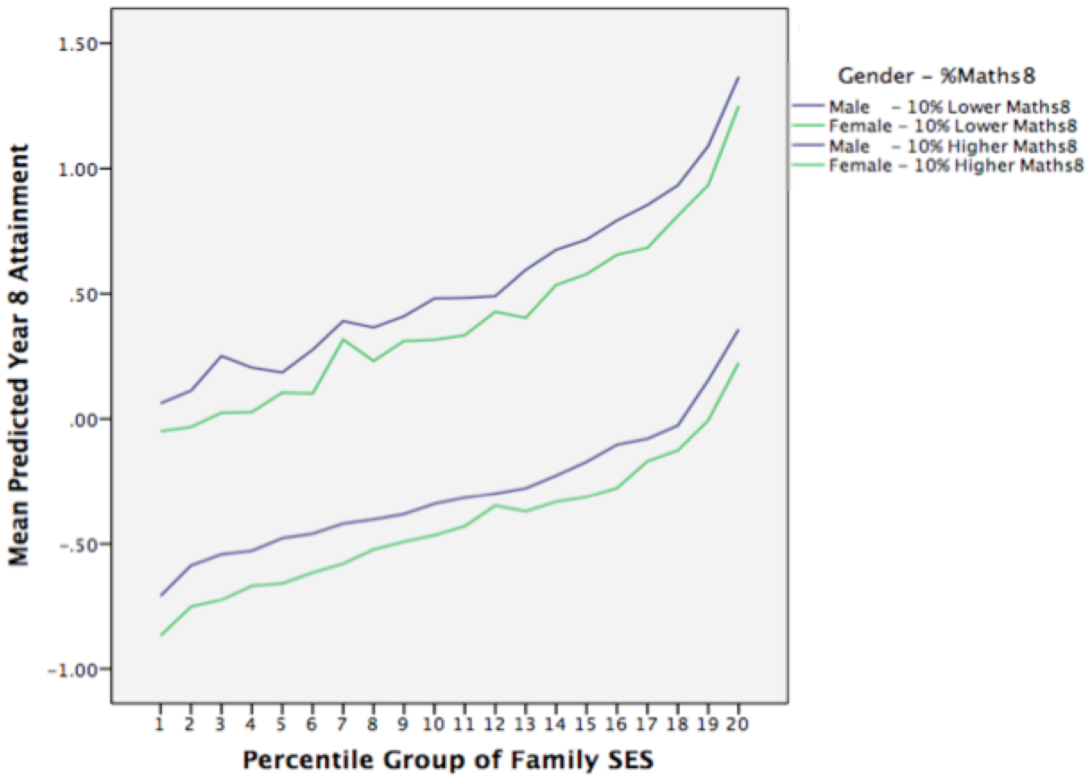


Figure 4.1b. Year 8 Predicted Mathematical scores by Family Socioeconomic Status (Mean for 10% higher and 10% lower scores of Attainment.

The graphs suggest that the differential effect of socioeconomic status for boys and girls in year 8 attainment was stronger for the total population than for the highest and lower scores (namely the 10th percentile highest and lowest attainment in terms of year 8 scores). For the population, gender differences were relatively consistent at different points of the family socioeconomic status distribution, but started to decrease at the upper tail of the distribution. This means that girls from the 10% higher socioeconomic status were performing slightly lower than boys, while girls in the rest of the socioeconomic status distribution were predicted to have much lower scores than boys from the same socioeconomic status.

The somehow “simple interaction effect” of gender and socioeconomic status became more complex when adding previous attainment (and therefore, when modelling progress). Firstly, the main effect of previous attainment was highly significant, with an increase of 1 SD in previous attainment adding more than half SD in year 8 attainment. Secondly, the effect of previous attainment was not independent of family socioeconomic status or gender. As indicated by the positive coefficient of the interaction between year 4 attainment and socioeconomic status, the negative effect of socioeconomic background became greater as previous attainment increased. Finally, the effect of previous attainment was greater for girls. Putting all of this together a summative effect of previous attainment and family socioeconomic status was observed (see figures 4.2a and 4.2b).

These graphs show how the effect of socioeconomic status was stronger for students with a higher previous attainment (namely the 10% highest scores in year 4), for boys and girls, and how gender differences were greater for students with lower previous attainment. This latter gender effect was slightly lower for girls of higher family socioeconomic status. In other words, girls who came from less privileged backgrounds and who were performing at a lower level during early primary school level (year 4) were predicted to make the least progress (relative to boys) when reaching year 8.

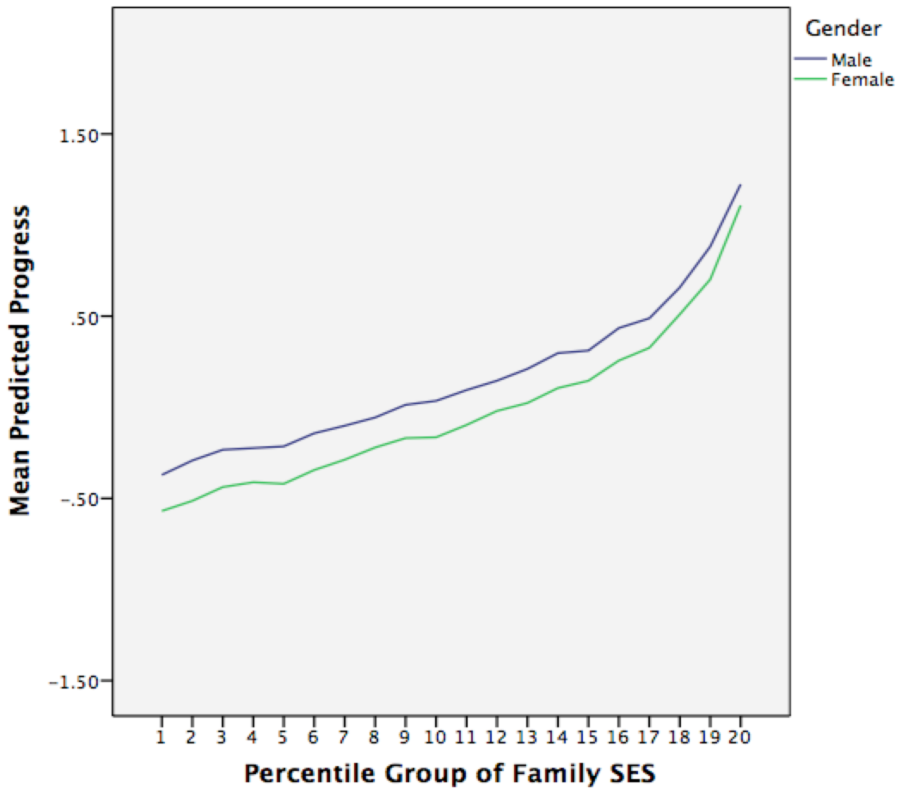


Figure 4.2a. Predicted Progress in Mathematics scores by Family Socioeconomic Status (Total Mean)

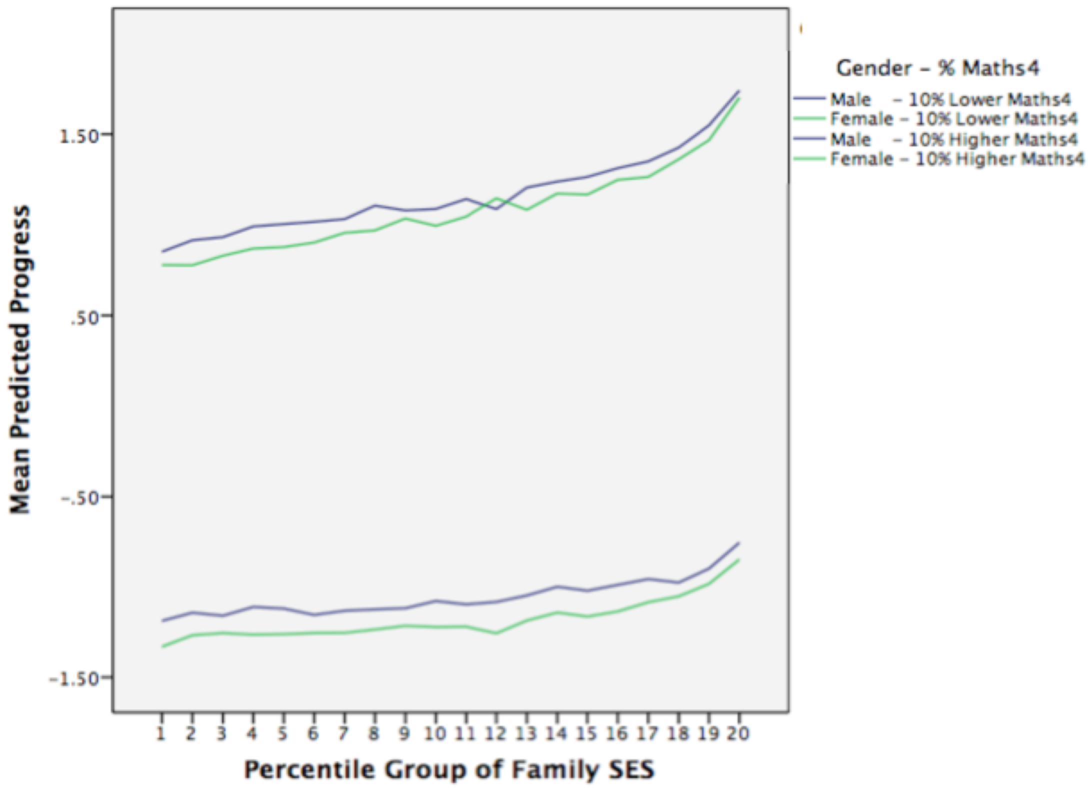


Figure 4.2b. Predicted Progress in Mathematics scores by Family Socioeconomic Status (Mean for 10% higher and 10% lower scores of Attainment.

4.4.3. School Level Variables

Regarding research question number three [Are gender differences distributed heterogeneously across schools from different socioeconomic statuses and different administrations?] this study showed differences in the different outcome variables. Whilst in Year 4 most school variables did not have an effect on differences between boys and girls, schools from different types, different socioeconomic statuses and different levels of previous attainment seemed to have a different effect in Year 8 and on progress. The most consistent results across outcome variables showed that private schools seemed to narrow the gap for girls only when no other socioeconomic variable was accounted for. If family socioeconomic status and/or school socioeconomic status was considered, public schools seemed to be more beneficial for girls than privately subsidised schools (see tables 4.4, 4.5 and 4.6 at the end of this paper).

In terms of the systematic effects of schools' variables, it was confirmed a main effect of the type of school, rurality and socioeconomic status of the school. For attainment in year 4 and year 8, schools from rural areas showed lower mean scores than schools from urban areas. However these differences disappeared when socioeconomic variables at individual and school level (i.e. family socioeconomic status and/or socioeconomic status of the school) were controlled for. Furthermore, when modelling progress this difference was even inverted, with rural schools improving on average more than urban schools. In terms of the effects of type of school, privately subsidised schools showed higher scores in year 4, year 8 and made more progress than public schools. Private schools showed higher attainment and progress than privately subsidised schools only when they were not controlled by school socioeconomic status. When comparing a middle socioeconomic status school, private and privately subsidised schools showed similar results. Finally, even after controlling for family socioeconomic status, the socioeconomic status of the school is significant in explaining students' attainment: schools from a lower socioeconomic status achieve less than an average school and schools from a higher socioeconomic status achieve better. The size of these effects is greater than the effect of family background (see tables 4.4, 4.5 and 4.6).

In terms of systematic school effects on gender differences, in year 4 only a small effect of the Type of school was observed, with private schools narrowing the gap for girls (table 4.4, model 1). However this effect seems to be determined mostly by family socioeconomic status, as after including these effects in the equation, the Type of school effect disappears (table 4.4, model 2). No differential effect of school socioeconomic status was observed (see table 4.4).

In Year 8, the systematic effect of type of school on gender (interaction between gender and type of school) was significant in most models, but as in year 4 it was also influenced by the effects of socioeconomic status both at the individual level and at the school level. What this analysis shows is that without any other socioeconomic status control, girls tend to do better in private schools than in privately subsidised schools (see table 4.5, model 1). However this effect changes when controlling for family or school socioeconomic status: After these controls were included, public schools seemed to be more beneficial for girls than privately subsidised schools, and private schools were performing at the same level as privately subsidised schools (see table 4.5, models 2, 5 and 6). Similarly, this was observed in the effects of schools' socioeconomic status: when the effect of schools' socioeconomic status was included without any other socioeconomic status control in middle and lower-middle socioeconomic status schools, girls appeared to be attaining less than boys (see table 4.5, model 4). However, these differences tended to disappear with the inclusion of family socioeconomic status controls. In this case it seemed that schools from a lower socioeconomic status were more beneficial for girls than schools of a middle socioeconomic status (see table 4.5, models 4 and 6).

Progress models produced similar findings to what was observed in the Year 8 Attainment Models i.e. the effect of Type of school changed when including family socioeconomic status and school socioeconomic status controls: Public schools and private schools were more beneficial for girls when no other socioeconomic variables were included (table 4.6, model 1). If the socioeconomic status of the school or the family were accounted for, then differences were observed only between public and privately subsidised schools (with public schools reducing the effect for girls in middle socioeconomic status schools) (table 4.6, models 2, 7 and 8). The socioeconomic status of the school did not appear to change female attainment when controlled for by other school and family socioeconomic variables (table 4.6, models 4 and 8). Female disadvantage was observed to be greater in schools with a higher year 4 attainment (table 4.6, models 5 and 6).

4.4.4. Gendered differential school effectiveness

Finally, in relation to research question 4 [do schools vary in their relative effectiveness for boys and girls?] this study showed that variation between schools regarding their gender gaps is small, with no schools predicted to show more progress for girls than for boys (see table 4.7).

The exploration of school variation in terms of their general effectiveness (Intercept variation) and differential effectiveness for girls and boys (Gender coefficient variation) suggested that schools varied significantly. This variation was greater in terms of general effectiveness than in differential gender effectiveness (almost 10 times greater). As table 4.7 shows, there was some variability shown between schools in terms of gender-gaps or some schools displayed less differences between boys and girls than others, but this variability was fairly low. This variability was correlated with the general achievement of the school: whilst average attainment increased, gender differences (favouring boys) also tended to increase. The inclusion of school level variables (schools' socioeconomic status, school type and schools' previous attainment) produced a change in the variation between schools in their general effectiveness (intercepts) but not on the variation of schools in terms of their differential effectiveness for boys or girls (gender coefficient).

Table 4.7:

Random part of progress models

Progress Models	Controlled by Family SES		Controlled by Family SES and School Variables	
Random Part	Parameter	SE	Parameter	SE
Average Gender effect	-0.117	0.004	-0.125	0.007
Intercept Variation (General Effectiveness)	0.084	0.003	0.053	0.002
Co-Variance Coefficient*Intercept	-0.011	0.001	-0.008	0.001
Gender Coefficient Variation (Differential Effectiveness)	0.005	0.001	0.005	0.001
Correlation Coefficient*Intercept	-0.531		-0.526	
n schools	5415		5415	
n students	158853		158853	

When plotting school residuals it is possible to observe that with a 2 standard deviation confidence interval most of the schools are performing similarly to an average school in terms of general effectiveness and differential effectiveness in terms of gender (see figure 4.3). In addition, the graph of predicted gender differences shows that there are some schools which show greater progress for boys (schools with predicted gender difference below zero), but apparently no schools are showing greater progress for girls.

To analyse this further, schools were classified according to Gray and colleagues' (2004) typology in relation to their predicted gender difference[14], with 1 and 2 standard deviations. In accordance with what was described, no schools were identified as being particularly beneficial for girls (see table 4.8).

[14] Predicted differences were computed by adding the school gender residual to the overall gender effect.

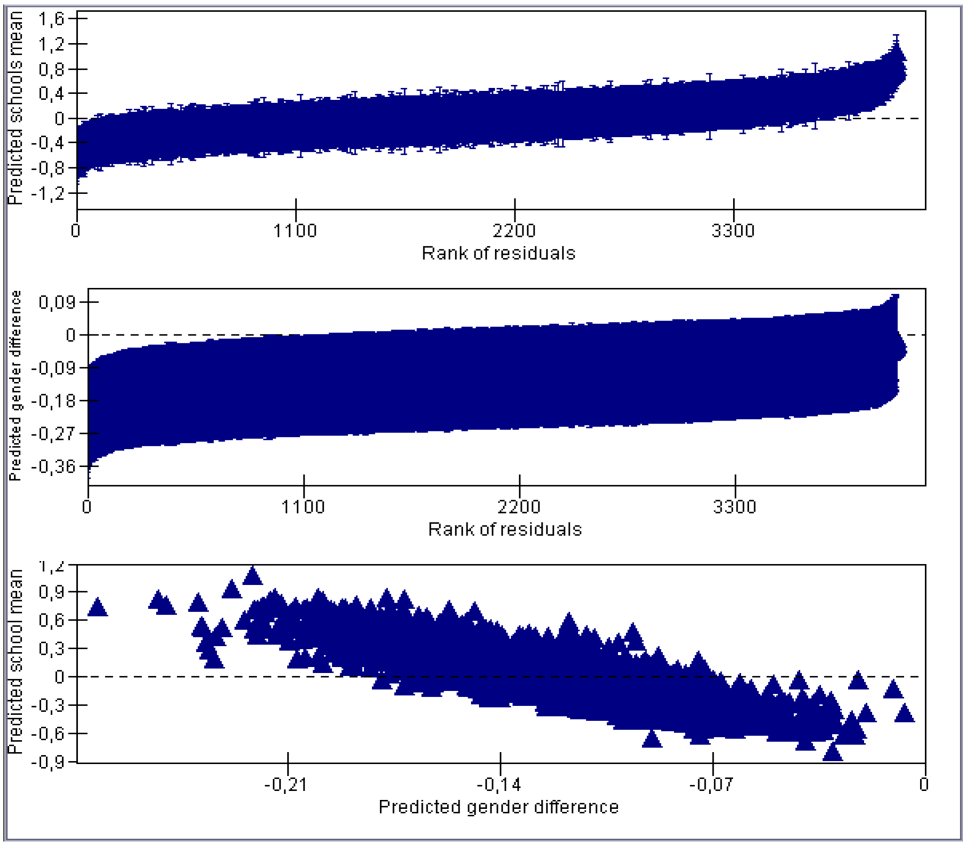


Figure 4.3: Schools residuals for Intercept and Gender Coefficient and relationship between them. Model controlled by Individual level variable (Family SES).

Table 4.8:

Classification of schools in relation to overall effectiveness and differential effectiveness by gender (as measured by school level residuals).

	2SD in comparison with Mean			1SD in comparison with Mean		
	Boys' progress exceeds that of girls	Boys and girls are predicted similar progress	Girls' progress exceeds that of boys	Boys' progress exceeds that of girls	Boys and girls are predicted similar progress	Girls' progress exceeds that of boys
Progress below expectation	1 (0%)	282 (7%)	-	717 (17%)	190 (5%)	-
Progress around expectation	755 (18%)	2520 (61%)	-	2057 (50%)	11 (0%)	-
Progress above expectation	520 (13%)	44 (1%)	-	1147 (28%)	-	-

Note. Model controlled by Individual level variable (Previous attainment and family SES).

4.4.5. Conclusion (Summary of Results)

In summary, the analysis revealed a consistent gender effect in mathematics attainment in year 4 and year 8, with boys making more progress than girls. Regarding the size of these differences, these results suggested that although gender is one of the strong coefficients in all the models explored, it accounts for only 1% of the variance in attainment. In addition, the differences between boys and girls reached a maximum of 20% of a standard deviation. This gap was smaller when modelling progress, where girls progressed one tenth of a standard deviation less than boys. This effect was about a quarter of the difference between students from more privileged and less privileged backgrounds (family socioeconomic status) in year 8, and nowhere near as comparable with the effect of previous attainment, where students with lower year 4 scores can fall more than 2 standard deviations behind students with higher initial scores.

In relation to the effect of family socioeconomic status on students' attainment, this study showed that this effect tended to decrease during primary education (between year 4 and year 8), but increased in its differential effect for girls and boys. Following this, while gender and family socioeconomic status effects were independent during year 4, there was a significant interaction in year 8. At this point differences between boys and girls lessened when family socioeconomic status increased, with fewer differences in the higher levels of socioeconomic status (in some cases differences disappeared altogether). The interaction between gender and family socioeconomic status in progress showed that girls from less privileged backgrounds and those who performed at a lower level during year 4 were predicted to make the least progress when reaching year 8.

The analysis of the systematic effect of different groups of schools suggested that, while the effect of family socioeconomic status decreased during primary school, the effect of school variables (i.e. type of school, school socioeconomic status and school previous attainment, all linked with socioeconomic status as well) increased. The size of the interaction effects between these variables and gender was always minimal, but allowed the identification of private schools as achieving more for girls only when family socioeconomic status was not accounted for, and that privately subsidised schools for average family socioeconomic status students were the ones which increased female disadvantage (for year 8 and for progress scores).

In terms of the differential effectiveness of schools in terms of gender, the main result of this study showed that in all schools, girls were doing similarly or making less progress than boys.

4.5. Discussion

This paper explored in detail the state of gender differences in mathematics attainment in Chile. Several reports in this country have raised concerns about the pervasive nature of male advantage, but no previous studies had explored this in detail and with suitable methodological tools. As stated by Lubienski (2008): "detailed analyses of gaps can help researchers and practitioners more effectively target their efforts towards equity, illuminating which groups to target and what aspects of instruction to address" (p.353).

Overall this study supports the persistence of gender differences in the academic attainment of Chilean students in mathematics. Importantly, it also shows that such differences were minimal compared with the effect of socioeconomic status and previous attainment, becoming even smaller for students from more privileged backgrounds. This interaction (between gender and social class) is in opposition to what other studies have reported (e.g. Grant & Sleeter, 1986), particularly in the UK and England (e.g. Machin & McNally, 2005; Connolly, 2006). However, in the US and Australia some researchers have described similar patterns of interaction (Lamb, 1996; Teese et al., 1995), suggesting that the higher socioeconomic status of middle class girls – and their corresponding educational opportunities - offsets the negative impact of gender.

In the Chilean context, the scarce previously available evidence on gender had suggested that girls' underachievement was stable and an important matter of concern (e.g. Agencia de calidad de la educacion 2013abc). In fact, gender differences are now being considered as one of the relevant criteria for assessing the quality of education according to the new Chilean educational policy. This policy provides a system for assessing and ranking schools, and includes non-academic criteria (what is termed 'other evidence of quality') as well as attainment. Evidence of gender equity in terms of attainment is one of the variables that has been proposed (Agencia de Calidad de la Educacion, 2014). Although differences in Language (which are commonly known to favour girls in Chile) are also considered in this new policy, most of the preliminary studies that the agency in charge of this process has published are related to girls' disadvantages in mathematics (as shown on their webpage). This policy issue suggests that girls' failure in Mathematics is a powerful discourse in the Chilean educational system.

A question raised by the data presented here is whether the actual size of gender differences in attainment justifies a public discourse which positions girls as failing (for a discussion of this social positioning see for example Hodgetts, 2008). In other words, does such a discourse reinforce existing cultural stereotypes, instead of reflecting the real scope of the problem? Recent research has shown that stereotypes of mathematics being a male domain are present even in the early stages of the Chilean school system (del Rio, 2012), when differences in attainment are non-existent -or as this study showed, much smaller. Following a similar logic, another study has shown that Chilean teachers expect (and evaluate) lower results from girls (Mizala, Martinez & Martinez, 2014). The obvious question here is whether the existing discourses on differences in attainment are contributing further to the reproduction of these realities by influencing individual expectations and beliefs. As Gee (2000) notes, public debates made available in the media and in everyday social interactions can form what has been called big D-Discourses (Gee, 2005), that can impact on D-Identities (Gee, 2000). Therefore, one could argue that perpetuating the notion that there are differences in mathematical attainment and that girls' relationships with this subject are problematic is complicated since it reproduces a large D-Discourse on girls and mathematics which becomes 'a given' rather than questioned. This paper attempts to questions this by exploring the complexity of gender differences in mathematical attainment. In doing so, a discussion is facilitated throughout this paper which considers the size of this gender difference, and suggesting that, although it is fairly stable, it really does not appear to be that big (particularly in comparison with the effect of socioeconomic status).

Finally, this study's main limitation is its sole focus on hard learning outcomes i.e. academic attainment: in terms of the relationship between gender and mathematics it is also necessary to consider how gender may play a role in girls' affective relationships with this subject. Existing evidence has shown that girls in Chile are reporting more negative attitudes than boys in this subject (see Agencia de calidad, de la educacion 2013a for PISA evidence), an aspect that has been linked to more developed countries with lower levels of participation in mathematically related careers (Stevens, Wang, Olivárez & Hamman, 2007). As reported by the Chilean Ministry of Education (MINEDUC, 2011) and as discussed in the series Comunidad de Mujer (Comunidad de mujer, 2014), there is a highly stereotypically marked selection of careers in Chile. Women tend to choose careers related with caring roles (e.g. teaching, nursing) and service, while men tend to choose problem solving and 'hands on' careers (e.g. engineering, technology, industry). These differences in career choice have been related by some authors to an inequality in wages which is still observed in the Latin-American context (Nopo, 2012). Even though differences in attainment could also relate to these issues it is clear that they are not the main factor. For instance, this study shows that differences are small and there are also girls that do very well in mathematics. Therefore, it can be argued that further studies which explore attitudinal aspects that relate with career choice, and the relation between these attitudinal aspects with cultural discourses (e.g. by using statistics to create discourses), as well as studies centred on high attainment girls, are urgently needed in the Chilean context.

Table 4.4:

Contribution of School level variables in Year 4 Mathematics Attainment models

Fixed Effects/Model	MODEL 1: Type School	MODEL 2: Type School - Family SES	MODEL 3: SES School	MODEL 4: SES School - Family SES	MODEL 5: School Level Model	MODEL 6: School Level Model - Family SES
Rural	-0.25 (0.015)***	-0.079 (0.014)***	-0.005 (0.015)	0.061 (0.014)***	-0.01 (0.015)	0.055 (0.014)***
Female * Rural	0.082 (0.012)***	0.085 (0.012)***	0.083 (0.013)***	0.086 (0.013)***	0.084 (0.013)***	0.087 (0.013)***
Public	-0.216 (0.014)***	-0.1 (0.012)***			0.049 (0.013)***	0.064 (0.012)***
Private	0.784 (0.027)***	0.277 (0.024)***			0.006 (0.055)	-0.115 (0.053)*
Female * Type (joint chi-square)	$x^2_{(2)}$= 6.15*	$x^2_{(2)}$= 1.27			$x^2_{(2)}$= 6.41*	$x^2_{(2)}$= 4.74
Female * Public	-0.008 (0.008)	-0.001 (0.008)			-0.009 (0.01)	-0.004 (0.009)
Female * Private	0.036 (0.018)*	0.022 (0.019)			0.109 (0.047)*	0.098 (0.046)
SES 1			-0.543 (0.018)***	-0.337 (0.017)***	-0.561 (0.019)***	-0.36 (0.018)***
SES 2			-0.377 (0.014)***	-0.239 (0.013)***	-0.398 (0.015)***	-0.266 (0.014)***
SES 4			0.433 (0.018)***	0.225 (0.017)***	0.445 (0.018)***	0.246 (0.018)***
SES 5			0.808 (0.023)***	0.308 (0.024)***	0.818 (0.057)***	0.436 (0.055)***
Female * SES (joint chi-square)			$x^2_{(4)}$= 4.94	$x^2_{(4)}$= 1.04	$x^2_{(4)}$= 4.66	$x^2_{(4)}$= 4.24
Female * SES 1			-0.001 (0.014)	0.001 (0.015)	0.004 (0.015)	0.002 (0.015)
Female * SES 2			0.005 (0.009)	0.008 (0.01)	0.01 (0.01)	0.01 (0.011)
Female * SES 4			0.018 (0.013)	0.003 (0.013)	0.013 (0.013)	0 (0.013)
Female * SES 5			0.032 (0.018)	0.009 (0.021)	-0.072 (0.047)	-0.082 (0.047)
r^2	8.8%	1.1%	15.5%	3.5%	15.4%	3.5%
chi-square	$x^2_{(6)}$= 1986.624	$x^2_{(6)}$= 404.317	$x^2_{(10)}$= 4214.931	$x^2_{(10)}$= 1353.225	$x^2_{(14)}$= 4235.84	$x^2_{(14)}$= 1388.584

Note. Non-controlled and controlled by Family Socioeconomic variables. R^2 and chi-square are in comparison with Individual Models. P < .05 *; p < .01 **; p < .001***. N schools= 6740; N classrooms= 9865; N students= 219682.

Table 4.5:

Contribution of School level variables in Year 8 Mathematics Attainment models

Fixed Effects/Model	MODEL 1: Type School	MODEL 2: Type School - Family SES	MODEL 3: SES School	MODEL 4: SES School - Family SES	MODEL 5: School Level Model	MODEL 6: School Level Model - Family SES
Rural	-0.114 (0.018)***	-0.044 (0.017)**	0.105 (0.017)***	0.118 (0.017)***	0.112 (0.017)***	0.125 (0.017)***
Female * Rural	0.051 (0.013)***	0.061 (0.013)***	0.047 (0.014)***	0.052 (0.014)***	0.046 (0.014)**	0.051 (0.014)***
Public	-0.377 (0.015)***	-0.308 (0.014)***			-0.077 (0.014)***	-0.071 (0.014)***
Private	0.875 (0.028)***	0.616 (0.026)***			-0.065 (0.053)	-0.122 (0.053)*
Female * Type (joint chi-square)	$x^2_{(2)}$= 16.77***	$x^2_{(2)}$= 9.55**			$x^2_{(2)}$= 4.50	$x^2_{(2)}$= 6.21*
Female * Public	0.011 (0.008)	0.023 (0.009)*			0.02 (0.01)*	0.024 (0.01)*
Female * Private	0.067 (0.017)***	0.031 (0.018)			0.023 (0.038)	0.014 (0.038)
SES 1			-0.543 (0.02)***	-0.43 (0.02)***	-0.511 (0.021)***	-0.4 (0.021)***
SES 2			-0.392 (0.015)***	-0.326 (0.015)***	-0.36 (0.016)***	-0.296 (0.016)***
SES 4			0.471 (0.019)***	0.376 (0.019)***	0.454 (0.019)***	0.363 (0.019)***
SES 5			1.013 (0.024)***	0.762 (0.025)***	1.048 (0.054)***	0.851 (0.054)***
Female * SES (joint chi-square)			$x^2_{(4)}$= 26.77***	$x^2_{(4)}$= 12.06*	$x^2_{(4)}$= 14.03***	$x^2_{(4)}$= 8.59
Female * SES 1			0.033 (0.015)*	0.045 (0.015)**	0.024 (0.015)	0.034 (0.016)*
Female * SES 2			0.006 (0.01)	0.015 (0.01)	-0.002 (0.011)	0.005 (0.011)
Female * SES 4			0.032 (0.012)**	0.018 (0.012)	0.036 (0.012)**	0.023 (0.013)
Female * SES 5			0.073 (0.016)***	0.034 (0.019)	0.059 (0.038)	0.028 (0.038)
r^2	13.0%	6.8%	20.5%	11.8%	20.6%	11.9%
chi-square	$x^2_{(6)}$= 2239.029	$x^2_{(6)}$= 1434.831	$x^2_{(10)}$= 4220.703	$x^2_{(10)}$= 2760.349	$x^2_{(14)}$= 4250.273	$x^2_{(14)}$= 2791.362

Note. Non-controlled and controlled by Family Socioeconomic variables. R2 and chi-square are in comparison with Individual Models. P < .05 *; p < .01 **; p < .001***. N schools= 5467; N classrooms= 8188; N students= 189835.

Table 4.6:

Contribution of School level variables in Progress models

Fixed Effects/Model	MODEL 1 Type School	MODEL 2 Type School - Family SES	MODEL 3 SES School	MODEL 4 SES School - Family SES	MODEL 5 School Year 4	MODEL 6 School Year 4 - Family SES	MODEL 7 School Level Model	MODEL 8 School Level Model - Family SES
Rural	0.054 (0.013)***	0.075 (0.012)***	0.105 (0.013)***	0.108 (0.013)***	0.029 (0.012)*	0.056 (0.012)***	0.104 (0.013)***	0.107 (0.013)***
Female * Rural	0.002 (0.011)	0.003 (0.011)	-0.001 (0.012)	0.001 (0.012)	0.003 (0.011)	0.005 (0.011)	-0.001 (0.012)	0.001 (0.012)
Public	-0.22 (0.01)***	-0.194 (0.01)***					-0.1 (0.011)***	-0.097 (0.011)***
Private	0.459 (0.018)***	0.35 (0.019)***					0.037 (0.04)	0.014 (0.04)
Female * Type (joint chi-square)	x2= 20.80***	x2= 18.21***					x2= 10.86**	x2= 12.01**
Female * Public	0.029 (0.007)***	0.03 (0.007)***					0.026 (0.008)**	0.027 (0.008)***
Female * Private	0.034 (0.013)**	0.014 (0.016)					-0.011 (0.031)	-0.017 (0.031)
SES 1			-0.171 (0.015)***	-0.138 (0.016)***			-0.074 (0.017)***	-0.041 (0.017)*
SES 2			-0.167 (0.012)***	-0.146 (0.012)***			-0.078 (0.013)***	-0.059 (0.013)***
SES 4			0.234 (0.014)***	0.198 (0.014)***			0.114 (0.015)***	0.083 (0.015)***
SES 5			0.556 (0.018)***	0.451 (0.019)***			0.359 (0.041)***	0.281 (0.041)***
Female * SES (joint chi-square)			x2= 12.74*	x2= 6.67			x2= 10.91**	x2= 7.11
Female * SES 1			0.03 (0.013)*	0.025 (0.013)			0.001 (0.014)	-0.002 (0.014)
Female * SES 2			0.017 (0.008)*	0.015 (0.008)			-0.006 (0.009)	-0.008 (0.009)
Female * SES 4			0.001 (0.01)	-0.002 (0.01)			0.027 (0.011)*	0.023 (0.011)

Female * SES 5			0.034 (0.013)**	0.015 (0.016)			0.075 (0.031)*	0.06 (0.032)
School Year 4 Low					-0.068 (0.012)***	-0.06 (0.012)***	-0.032 (0.012)**	-0.036 (0.012)
School Year 4 High					0.394 (0.011)***	0.335 (0.011)***	0.192 (0.013)***	0.187 (0.013)
Female * Year 4 (joint chi-square)					x2= 17.84***	x2= 21.18***	x2= 10.08**	x2= 19.36**
Female * Year 4 Low					0.024 (0.009)**	0.019 (0.01)	0.021 (0.01)*	0.018 (0.01)
Female * Year 4 High					-0.022 (0.008)**	-0.032 (0.008)***	-0.037 (0.01)***	-0.038 (0.01)***
r^2	7.13%	4.07%	9.07%	5.20%	6.91%	4.52%	10.15%	6.33%
chi-square	$x^2_{(6)}$= 1476.583***	$x^2_{(6)}$= 971.734***	$x^2_{(10)}$= 2015.53***	$x^2_{(10)}$= 1316.66***	$x^2_{(6)}$= 1444.432***	$x^2_{(6)}$= 1083.835***	$x^2_{(18)}$= 2337.51***	$x^2_{(18)}$= 1628.24***

Note. Non-controlled and controlled by Family Socioeconomic variables. R2 and chi-square are in comparison with Individual Models. P < .05 *; p < .01 **; p < .001***. N schools= 5415; N classrooms= 8130; N students= 158853.

Chapter 5. Towards Conceptual Coherence in the Research on Mathematics Learner Identity: a Systematic Review of the Literature

5.1. Abstract:

This paper presents the results of a systematic review of the empirical literature on Mathematics Learner Identity, reported in research journals. In mathematics education credible research arguments have been made that the conceptualisation of 'mathematics identity' is inconsistent and this makes the literature as a whole incoherent. This study aims to review the range of conceptualisations used in such empirical studies, and how these conceptualisations can be accounted for with research epistemologies and questions in the field of mathematics education. A grounded analysis of the selected 68 papers revealed 3 themes as defining features of the concept of identity [ontological; forms of expression; and process of identity formation] and 5 main categories of conceptualisation [identity as individual attributes; identity as narratives; identity as a relationship with specific practices; identity as ways of acting; and constraints and affordances of local practices]. An emphasis on representational aspects of identities during post-compulsory education (particularly narratives) and enacted and practice-related identities during compulsory education research was found. In conclusion, a discussion reveals how the field would be clarified if studies made their choices of conceptualisation clear in terms of these (or other) dimensions, and if research as a whole becomes more aware of restrictions in identity research at different points of the educational trajectory of students.

5.2. Introduction

Over the last two decades numerous studies in mathematics education have used the concept of identity, or 'mathematical identity/ies', in order to understand learning and development. Proof of the increased popularity of this concept is shown in the publication of books (e.g. Black, Mendick & Solomon, 2008; Solomon, 2008; Walls, 2009) and a journal special issue (Mathematics Education research Journal, 2015, volume 27, issue 1), specifically targeting the topic. There are several reasons that appear to explain the growing interest of researchers in this concept of mathematical identity(ies). It has been noted, for example, that exploring how students identify with mathematics can offer valuable information with regards to how a personal relationship with the subject develops over time (e.g. Walls, 2009) as well as offering insight into the affective aspects of this relationship (Hannula, 2012). In addition, other authors have also claimed that the concept of identity(ies) can be a useful tool in exploring students' difficulties engaging with mathematical activity (e.g. Solomon, 2007), or in understanding how identifying with some social categories [e.g. gender, race] can negatively influence the construction of a mathematical identity (e.g. Nasir & Cobb, 2007).

Historically speaking, the increased popularity of the concept of identity in Mathematics Education appears to be related to two theoretical 'turns'. Starting in the 70's, a so-called 'affective turn' established a relationship between *affective* variables -beliefs, emotions and attitudes- and attainment (Fennema & Sherman, 1977). These affective variables became relevant due to a large number of studies which focused on the development of dispositions to choose maths after compulsory education (Johnston, 1994). A predominant psychosocial model at that time proposed that *affective* variables, such as expectations of success, incentives, motivations and personal values, influenced choices made by mathematics students (e.g. Eccles, 1994). Inevitably, focusing on these affective variables led several authors to also pay attention to students' self-schemas, self-concepts and self-efficacy beliefs (see also Eccles, 1994 model) since a focus on emotions led them to position the self at the centre of the learning process. Several years later these ideas still persist. For example, some authors have suggested recently that identity can be considered as one affective variable, or as an umbrella concept for affective variables (Hannula, 2012).

Years later, during the 80's, a *social turn* took place as a response to limitations of a predominant individual perspective, which explained attainment, engagement and disposition as processes that occurred 'inside' the subject. This change of paradigm proposed 'the emergence into the mathematics education research community of theories that see meaning, thinking, and reasoning as products of social activity' (Lerman, 2000, p.23). A movement towards social theories of learning changed the focus from the individual to its social reality -activities, contexts and relationships-, thus emphasising the influence of social inequalities and practices of exclusion in mathematics education rather than individual agency (Lerman, 2000). One of the main challenges of this paradigm shift was to bridge social and individual domains.

In other words: “to develop accounts that bring together agency, individual trajectories (Apple, 1991), and the cultural, historical, and social origins of the ways people think, behave, reason, and understand the world. Any such analysis must not ignore either: it should not reduce individual functioning to social and cultural determinism nor place the source of meaning making in the individual” (Lerman, 2000, p. 36). Several authors from this tradition have proposed that the concept of identity can help overcome this ‘divide’ between social and individual, thus, constituting a ‘missing link’ (e.g. Boaler, 1999a; Sfard & Prusak, 2005).

Even though both *affective* and *social* turns have stressed the relevance of the concept of mathematical identity(ies), and used it in their formulations, several authors have pointed to the need for better definitions and more refined methodological approaches (Brubaker & Cooper, 2000; Cobb, Gresalfi & Hodge, 2009; Sfard & Prusak, 2005). So far, researchers have used a wide range of conceptualisations of identity which, coming from diverse epistemologies, have resulted in a relatively small field [Mathematics Educational Research] overcrowded by numerous approaches. In addition, some of these definitions have lacked a clear operationalisation. By trying to fight essentialism and stable definitions of identity -a prevailing trend in the psychological tradition using the concept of personality-researchers after the social turn have ‘softened’ the concept, transforming it into something vague and thus limiting its analytical power (Brubaker & Cooper, 2000). In summary, the development of an adequate concept of MI has been made difficult by two main factors: a) a lack of conceptual coherence between studies because of co-existing multiple definitions; b) a resistance from the scientific community to defining the concept due to the danger of essentialising it in the process.

A lack of conceptual coherence has important implications to the development of the field. It may not only compromise the communication and debate between researchers, but also obstruct the testing of theoretical ideas and replication of findings. Furthermore, if we consider that identity is, from any theoretical perspective, an extremely complex phenomenon, a lack of conceptual coherence could seriously threaten the process by which such richness is captured.

Recently, researchers concerned about these issues have begun to develop more suitable definitions of MI, as well as more explicit analytical frameworks (Cobb, Gresalfi & Hodge, 2009; Sfard & Prusak, 2005; Varela, Martin & Kane, 2012).

These attempts, although valuable, still present some limitations. For example, in order to 'solve' the problem of conceptual coherence, some authors have simply decided to adhere to one definition of identity and discard alternative or contesting points of view (e.g. Sfard & Pusak and their definition of identity as narrative and discursive). Others, have developed more comprehensive theoretical frameworks, mainly based on their own approaches to the problem (Cobb, Gresalfi & Hodge, 2009; Varela, Martin & Kane, 2012). A limitation of these frameworks is that they are not based on the systematic study of how the concept of identity has been used in the literature. By not considering all the available evidence in the field, the proposed frameworks are often influenced by researchers' theoretical allegiances and their respective private 'languages'.

The main goal of this article is to address the lack of conceptual coherence when defining MIs. In order to accomplish such a task, this paper will use a systematic review methodology, where existent conceptualisations of identity in mathematics education will be gathered and analysed. The study of how identity is both *conceptualised* and *operationalised* by researchers can provide valuable information for critically situating the concept which researchers opt to use, without necessarily 'killing' the concept, as suggested by Brubaker and Cooper (2000). Despite repeated calls to bring conceptual clarity and coherence to the concept of identity in mathematics education, this is the first systematic review on identity published. Three main research questions guided this review: 1) What are the defining features in the conceptualisation of identity/identities? 2) Which features of the concept of identity/identities are emphasised by different studies in their theoretical and methodological approaches? 3) Are these emphases related to the study of MIs at different stages of schooling (primary, secondary, transition to post-compulsory, and post-compulsory)? While research question number one was concerned with identifying what were the main features that were considered in all the conceptualisations of identities, research questions number two and three were concerned with identifying how these features were differently emphasised in different study groups.

5.3. Methodology

5.3.1. Sample

As a first step, and in order to define the sample of papers/studies to include in this literature review, a systematic search of the concepts of 'identity' (identification/identities/identity) and 'mathematics' (maths/mathematical) was carried out using several search engines [Web of Science, ERIC, BEI, and PsychInfo] and journals specialising in Mathematics Education Research [Educational Studies in Mathematics, For the Learning of Mathematics, International Journal of Science and Mathematics Education, Journal for Research in Mathematics Education, Journal of Mathematical Behavior, Research in Mathematics Education, Mathematical Thinking and Learning]. Only peer reviewed articles which included both key concepts in their title or abstract were considered as part of the initial sample. This review was updated up to studies published in June 2015.

As a second step, the initial set of papers (over 600) was reviewed and articles were excluded if they did not fit the topic of 'mathematics learner identity/ies'. The exclusion of papers was based on the following criteria: 1) use of the term identity as a mathematical concept (eg. trigonometric identities, series identities); 2) use of identity to signal a structural category (e.g. gender and race/ethnic identity) with no relation to mathematical identities; 3) focus on another discipline (although studies on STEM were retained since they could be related to mathematical identity); 4) focus on teachers or researchers' mathematical identities and not students' identities; 5) literature reviews and theoretical discussions where no data was presented. This last criterion was included in order to consider only articles where identity conceptualisations were operationalised, thus, anchoring theoretical definitions in concrete methodological approaches.

5.3.2. Analysis

Based on the inclusion and exclusion criteria described above, 68 articles were selected for analysis. Each paper was exported to an excel file, with basic information such as title, source, keywords, date and abstract. Following a "Thematical Synthesis" model (Thomas & Harden, 2008) definitions of identity used by each study [and associated *conceptualisations and operationalisation*] were coded. This model follows a similar approach to grounded theory, but uses studies as objects of analysis (rather than qualitative data as in Strauss & Corbin, 1998). The analysis is performed in different stages. Initially, there is a close focus on the data itself only later to build up to more abstract conceptualisations. In other words, it moves from line-by-line coding towards descriptive and analytical coding (Thomas & Harden, 2008).

More specifically, the first step of the procedure involved the analysis of definitions and operationalisations of identity. This was accomplished by determining the 'elements' that were referred to by studies as central to their conceptualisation of identity (Research Question Number 1). Information regarding the use of mathematical identity was gathered in the form of literal quotes from theoretical and operational definitions of identity, description of research purpose or research questions and methodology. As a second step, literal quotes related to definitions of identity were used as raw data for coding. Initial coding remained closely attached to data, and similar codes were constantly compared, in order to advance the analysis towards the generation of more general categories (Strauss & Corbin, 1998). Each time the raw data was insufficient for attaching a code - and later in the process to assign a category - the entire paper was again revisited. As a main result of this analysis several 'defining' features of what constituted an identity were highlighted.

When a level of 'saturation' was reached in terms of the emergence of defining features of what constitutes an identity, different combinations of how each paper emphasised these defining features led to a categorisation of different conceptualisation categories (research question number 2). A methodological challenge of this coding procedure was that some papers emphasised certain features of an identity as central to their conceptualisations, while others were less specific in their approaches. In these cases information from the respective methods sections was used to determine conceptualisations of identity based on the operationalisation of the construct. An example of the different steps of the coding process can be observed in Table 5.1.

Table 5.1:

Working example of analysis procedure

Step 1: **Quote Conceptualisation of Identity**	**Step 2:** **Analysis defining features**	**Step 3:** **Emphasis**
Study A: I define identity as a **dynamic view** of self, **negotiated** in a specific **social context** and informed by past history, events, personal **narratives**, **experiences, routines, and ways of participating**. An identity is who one is in a given community and, as such, is both **individually and collectively defined**. Although an identity is related to a role or ways of participating (e.g., the role of a problem poser in a mathematics classroom), I define identity in a broader sense. An identity also encompasses ways of being and talking; narratives; and affective components such as feelings, attitudes, and beliefs (Bruner, 1994; Gee, 2005; Wenger, 1998)—aspects not necessarily included in the term role. Drawing heavily from Sfard's discursive approach and Martin's 2006 definition, I use the term mathematics identity to mean the ideas, often tacit, one has about who he or she is with respect to the subject of mathematics and its corresponding activities. Note that this definition includes a **person's ways of talking, acting, and being and the ways in which others position one with respect to mathematics**. Moreover, a mathematics identity is dependent on what it means do mathematics in a given community, classroom, or small group. As such, identity is **situated**; learned; stable and predictable, yet malleable; and is both individual and collective.	- Representational (ideas / meanings) AND Enacted (ways of acting, talking, participating) - Social (who one is in a particular community) AND Subjective (view of self - ideas of who one is). - Process of Identity construction: Identity as learned / negotiated in context.	**Enacted in Micro-Identities:** Purpose 'Show how identities are enacted at the microlevel' Method patterns of social interaction, microanalysis of discourse.
Study B: Mathematics identity encompasses the **dispositions** and deeply held **beliefs** that individuals develop about their **ability** to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives. A mathematics identity encompasses a person's **self understandings** as well as how they are **constructed by others** in the context of **doing** mathematics. Therefore, a mathematics identity is expressed in **narrative** form as a negotiated self, a **negotiation** between our own assertions and the external ascriptions of others.	- Representational (beliefs / self-understandings) - Enacted as source of information (representations about ability to participate = identities). - Social as context and as collectively defined (build in the context of doing mathematics and constructed by others) AND Subjective (self-understanding). - Process of identity construction: As negotiation between self and others 'ascriptions'	**Identities as Narratives:** Emphasis on representation. Purpose and methods based on 'life-stories' 'narrative interviews'

Note. Text in bold letters refers to the first step of analysis, where main concepts were highlighted. The studies used in this working example were: Study A (Bishop, 2012) and Study B (McGee & Martin, 2011).

As a next step, and in order to explore the distribution of the different emphases in research on different stages of schooling (research question 3), studies were categorised according to the age of their participants: primary (compulsory); middle-school (compulsory); secondary/high school (compulsory); transition between compulsory and post-compulsory (transition); higher education (college, higher education, and postgraduate education). Longitudinal studies during the transition between compulsory and post-compulsory, and studies that focused either on compulsory or post-compulsory (but which studied anticipation, expectation or decision making about continuing in maths), were all categorised as part of transition. As a final step, the frequency of the use of different conceptualisations of identity across stages of schooling was explored (Research Question 3).

5.4. Results

5.4.1. Descriptive results

Overall, the above analysis indicates that empirical research published on Mathematical Learner Identities has increased steadily over the last two decades, and especially over the last 10 years (see Figure 5.1).

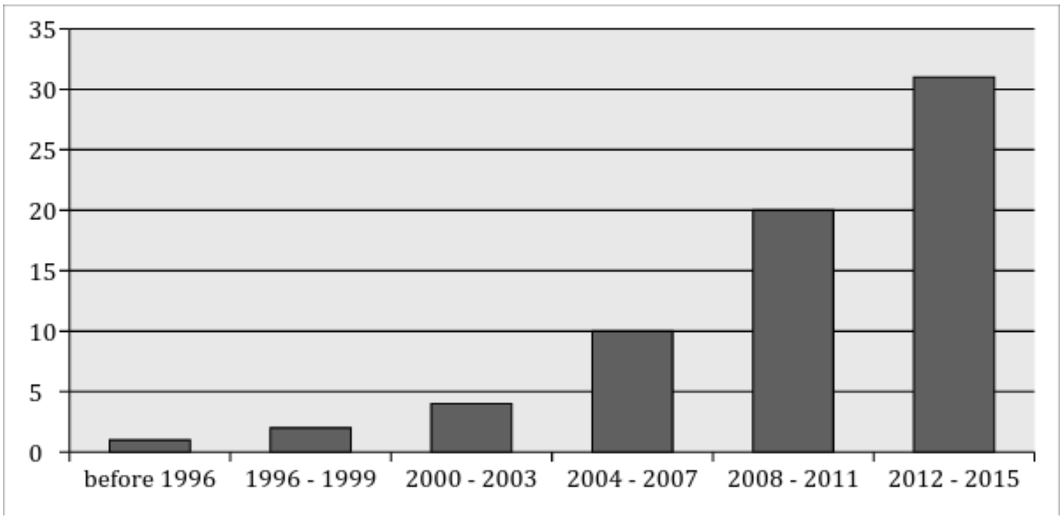


Figure 5.1: Evolution of research in students' mathematics identities in the last 20 years

According to this review, the first paper which specifically explored students' mathematical identity as an object of study dates from 1995. Furthermore, the analysis highlights how studies have investigated mathematical identities at different stages of schooling. As can be observed in Figure 5.2 half of the studies were focused on compulsory schooling and the other half on transition and post-compulsory [compulsory= 51% (n=34); transition= 20% (n=13); post-compulsory= 29% (n=19)]. Notably, there were significantly less studies addressing the issue of mathematical identities in primary and middle school [less than 25%].

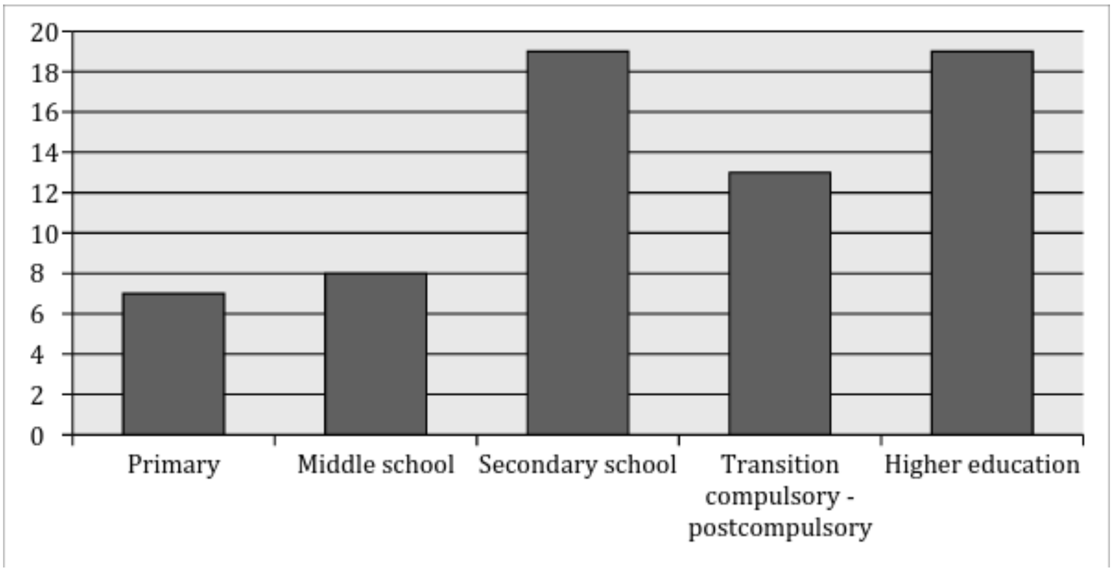


Figure 5.2: Distribution of studies regarding stage of schooling of participants (primary includes 1 study in early years – two studies did not report age or stage of schooling).

It is interesting that over 70% (n=48) of the studies explored identity in relation to issues of equity, either by using equity as a main topic of their research question, or by targeting a specific population where the topic of equity was relevant. Amongst these studies, problems such as unequal access and underrepresentation were often referred to as a justification to explore students' developing mathematical identities. Importantly, gender was the most frequently addressed equity issue [53%, n = 36] particularly in relation to differential access and underachievement. Whilst most of these gender studies addressed problems related to girls'/women's difficulties constructing a mathematical identity, or under-representation in mathematics, a growing number of articles were also concerned with difficulties experienced by males from ethnic minorities.

Ethnicity was also a frequently explored dimension in relation to equity and developing mathematical identities. Nearly 40% of the studies included in this review had ethnicity as central to their inquiries (n= 27), particularly in relation to contexts with a high percentage of ethnic minorities (predominantly the black population in the US) or contexts where there was an intersection of different ethnic groups. While most of these studies considered 'class' or 'socioeconomic status' (SES) as an intersecting dimension with ethnicity, only one explored the effect of 'class' by itself. Interestingly, very few [n = 3] articles considered the intersection of all three social categories -gender, social class and ethnicity- as a central focus of their investigations.

5.4.2. What are the Defining Features of Identity?

In relation to research question number 1 [What are the defining features in the conceptualisation of identity/identities], three main themes were identified as 'defining features' of identity: its ontological nature (what an identity is), its forms of expressions (how it is expressed) and the process of identity formation (how identity develops). All ontological definitions were observed as being located somewhere between the tension of *subjective/individual* and *social*, and all forms of expression placed somewhere between *embodied/enacted* and *representational* forms of expression.

In relation to the ontology of identity, when identity was seen as a subjective notion of oneself, authors described it as a subjective 'sense'. Identity in this subjective aspect was defined as a 'sense of continuity', a 'sense of a place in the world', a 'sense of being', a 'sense of connection or belonging', a 'self-view', or even as 'self-descriptions' which created a private experience of who one is. When identities were seen as mainly social, studies described it as a 'social product', something constituted by 'social discourses', 'a space in which discourses work and are worked'.

The relationship between what can be described as the two poles of the ontological tension between *subjective/social* was conceptualised differently in different studies. Some papers described the social aspect as being constitutional of the subjective. Here the 'sense of who one is' was constituted by the subjective perception of a place in the social world, such as a 'sense of belonging' and a 'sense of connection to others'. Others conceptualised the social aspect as a product from which the subjective sense was constituted. Here, social discourses or cultural tools were resources from which subjective senses were built. Finally, others conceptualised individual identities as influenced by external, separate, social aspects (such as social constructions of gender, and gender and mathematics).

The second 'tension' that was observed in the different definitions of identities was more related to how different studies conceptualised the expression of identities in the social world. When identities were seen as expressed in representational forms, the 'sense of who one is' (subjective) or the 'discourses/social resources' that were used for building an identity (social), they were seen as mediated by discourse or language and expressed in the use of these languages. These representational forms were described as 'self-concept', 'self-reflection', 'identity discourses' and 'narratives about one-self'. Researchers that viewed identity as an embodied/enacted phenomenon, on the other hand, emphasised the idea of identity expressed and performed in action, with no necessary mediation of language representations. In these studies, the idea of 'engagement in action' was central, with concepts such as 'forms of participation', 'forms of engagement' or 'roles performed during activities' used to stress how different ways of being can express different identities.

Furthermore, in terms of the relationship between the representational and enacted expressions of identities, different conceptualisations were found. Most studies (as will be described later) relatively neglected the enacted aspects of identities by focusing mainly on its representational forms. However, some studies considered enacted aspects as information used for constructing representations and others saw identities as expressed mainly in enacted forms. Finally, a few studies conceptualised 'representational narratives' as performances or enactments in a particular context (the interview context).

Another relevant finding of the coding process was that, independently of which ontological and forms of expression were emphasised by each study, no definition described it as a stable or fixed trait. In contrast, most of the papers considered identity as being constructed in a process, and in consequence, learnt and open to change. This learnt quality was explicitly discussed by some studies, which emphasised identity as something 'fluid', 'in process', 'malleable' or 'dynamic'.

One implication of considering identity as being constructed in a process was the consideration of a temporal aspect in the development of mathematical identities. This temporal aspect was expressed in three different ways. Firstly, some studies particularly focused on the longitudinal analysis of identities and directly analysed how the identification of students with mathematics changed over time. In particular, these studies were concerned with exploring the choice of mathematics after compulsory school/education, and how expectations changed when approaching and straight after a decision point (e.g. Black et.al., 2010; Buschor et.al., 2014a; 2014b; Holmegaard et.al., 2014). A second group of studies which explored identities as narratives, were particularly concerned with the articulation of these narratives in a past, present and future tense (e.g. Braathe & Solomon, 2015; Solomon, 2012). Finally, another group of studies also considered a temporal dimension by exploring the development of enacted forms of identities in time. By also performing a longitudinal analysis of forms of participation (enacted forms), they explored how these performances become crystallised in more restricted forms of participation or even in representational forms (e.g. Bishop, 2012; Langer-Osuna, 2015).

5.4.3. Different Conceptual and Methodological Emphases in Approaches to Identity

In relation to the second research question [which defining features of the concept of identity/identities are emphasised by different studies in their theoretical and methodological approaches?], the analysis suggested that the studies included in this review could be categorised into five groups: (1) identity as individual attributes; (2) identity as narratives; (3) identity as a relationship with specific practices; (4) identity as ways of acting; and 5) constraints and affordances of local practices (see table 5.2 with all papers included in this review with their final categorization). These groups were different in their emphasis, but also in how they conceptualised the relationship between the different defining features as previously described. As mentioned before, most studies emphasised *representational* forms of expressing identities, with differences in the relationship between *social* and *subjective* features being the main aspect which differed between studies (see figure 5.3).

Table 5.2:

Distribution of studies according to their conceptualizations of identity.

Conceptualizations of Identity	**Studies included in the review**
Identity as individual attributes (n=10)	Andersen & Ward (2014); Axelsson (2009); Boe (2012); Buschor, Berweger, Frei & Kappler (2014); Buschor, Kappler, Keck Frei & Berweger (2014); Hernandez, Schultz, Estrada, Woodcock & Chance (2013); Kizzie Rouland, Johnson Rowley & Kurtz-Costes (2013); Lee (2002); Lesko, Corpus (2006); Pronin, Steele & Ross (2004)
Identity as narratives (n=24)	Archer, Dewitt, Osborne, Dillon, Willis & Wong (2012); Bartholomew, Darragh, Ell & Saunders (2011); Berry (2008); Black, Williams (2013); Black, Williams, Hernandez-Martinez, Davis, Pampaka & Wake (2010); Braathe, Solomon (2015); Craig (2013); Epstein, Mendick & Moreau (2010); Hernandez-Martinez, Williams, Black, Davis, Pampaka & Wake (2011); Holmegaard, Madsen & Ulriksen (2014); Krogh, Andersen (2013); Lim (2008a); McClain (2014); McGee, Martin (2011); McGee, Pearman (2014); McGee, Pearman (2014); Mendick (2005a); Oppland-Cordell (2013); Smith (2010); Solomon (2012); Solomon, Lawson & Croft (2011); Stinson (2008); Walshaw (2005); Ward-Penny, Johnston-Wilder & Lee (2011)
Identity as a relationship with a specific practice (n=9)	Ahlqvist, London & Rosenthal (2013); Campbell, Lee, Kwon & Kyungsuk (2012); Darragh (2013); Darragh (2015); Lee (1998); Lim (2008b); Oppland-Cordell, Martin (2015); Solomon (2007); Tate, Linn (2005)
Identity as ways of acting (n=11)	Bishop (2012); Black (2004); Empson (2003); Forster (2000); Gholson & Martin (2014); Heyd-Metzuyanim (2013); Heyd-Metzuyanim & Sfard (2012); Landers (2013); Langer-Osuna (2015); Turner, Dominguez, Maldonado & Empson (2013); Wood (2013)
Constraints and affordances of local practices (n=14)	Anderson & Gold (2006); Boaler (1999b); Boaler, Staples (2008); Buxton (2005); Chronaki (2005); Cobb, Gresalfi & Hodge (2009); de Abreu (1995); Hand (2010); Hodge (2008); Horn (2008); Hughes, Nzekwe & Molyneaux (2013); Nasir (2002); Nasir & Hand (2008); Solomon, Croft & Lawson (2010).

Note. N=68. Complete references of each paper included in this review are provided in Appendix 13.

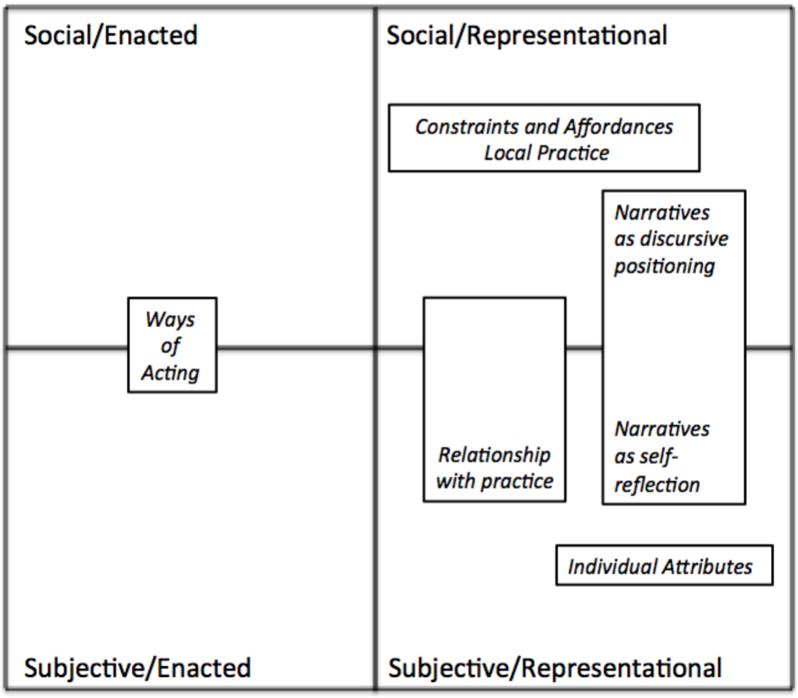


Figure 5.3. Identity conceptualizations. Each quadrant represents how social/subjective and representational/enacted features are emphasized. Each rectangle represents different conceptualizations. Rectangles (and its text) are positioned according to their emphasis and to how the relationship between social and subjective features is conceptualized.

5.4.3.1. Identities as individual attributes

The first group of studies was characterised by a definition of identity which emphasised its subjective aspect, a notion of oneself and attributes of the self (n=10, 15%). These notions had a representational nature and, in consequence, were predominantly investigated via self-reports (mainly surveys). Even though the social world was considered an influence in the construction of MIs in this category, individuals were considered as being separate from their social context. Amongst these studies, identities were generally defined as 'self-views', 'self-descriptions', 'self-concepts', 'self-competence' or 'self-esteem'. Mathematics (or STEM) and the related process of self-identification was viewed as a process of trying to fit the 'individual identity' with ideas of what mathematics was. In this sense, a 'mathematical identity' was seen as something relatively independent from the subject, which was expressed mainly as 'social stereotypes' regarding mathematics which influenced students' subjective identities.

Three main features characterise how this group of studies treated the concept of identity. Firstly, identity was predominantly explored from an individual point of view -via the use of surveys- and no major attention was placed on how the identity related to a particular, situated mathematical activity (or STEM activity). This can be exemplified in typical self-reports. By employing questions such as 'I am good at mathematics' or 'I see myself as a mathematician', it is implicitly assumed that there is only one way of doing mathematics (in which I am good or bad) or that there is only one type of mathematician (with which I identify/or not) independent of context.

Secondly, studies which belong to this category tended to see individuals and their social contexts as two separate entities, with social factors being considered as an influence in students' personal variables (i.e. identity attributes). From this perspective, the process of identity construction was explained as the individual response to a social stereotype by 'identity bifurcation' (i.e. separating oneself from the stereotyped activity), by 'identity correspondence' (i.e. seeing the 'individual identity' as compatible with the stereotyped activity) (e.g. Lesko & Corpus, 2006; Pronin, Steel & Ross, 2004) or by developing 'consistency of identities' (e.g. Andersen & Ward, 2014; Boe, 2012; Hernandez, Schultz, Estrada, Woodcock & Chance, 2013).

Finally, it is interesting to note that there was a notion of fluidity in how the process of identification with mathematics was described in this group of studies. As the salience of a stereotype could be manipulated, a subject's ability to perform or identify with maths could be facilitated or limited according to changes in context (such as making a social identity more salient). However, the process by which 'individual identity' or its attributes -self-view or self-concept - can change according to variations in contexts was largely unexplained, and even a matter of debate. While some authors explicitly defined 'mathematical identities' -or 'domain identity' according to their model- as a relatively 'stable and enduring' 'personal factor' (Hernandez, et.al., 2013), others stated that the conceptualisation of identity as a stable phenomenon may need to be revised (Axelsson, 2009).

5.4.3.2. Identities as narratives

Data analysis revealed a second group of studies (n=24, 35%) which emphasised the representational aspect of identity in the form of narratives. In other words, the stories students tell about themselves and their relation to the subject constitutes their MIs. A narrative view of identity radically distances this approach from those which conceive identity as a personal attribute, since cultural and social resources were seen as being used in shaping these narratives and therefore conceptualised as intrinsic to the construction of MIs. In other words, researchers from this group recognised the subjective nature of identity but, at the same time, acknowledged that individuals are not isolated from their social and cultural milieus. It is interesting to note that, in these studies, narratives were not only the object of study but also the method of choice for collecting data. This has important theoretical and methodological implications; narratives were seen as being re-constructed during the process of recollection, thus preserving their dynamic and fluid quality.

Even though all studies included in this group emphasised representational forms of identities [in the form of narratives], and acknowledged that the subjective experience of identities was intrinsically related to social influences, variations in the emphasis of social and subjective dimensions were observed. While some studies defined narratives as personal stories which allowed self-understanding and self-reflection (therefore with an emphasis on subjective experience), others viewed narratives as self-positioning, or the way in which individuals positioned themselves in discursive spaces (therefore with an emphasis on social aspects). More importantly, whilst studies which conceptualised narratives as self-reflective processes emphasised the subjects' agency in the construction of their personal histories, those who defined narratives as self-positioning placed more emphasis on the structural restrictions and power of social dominant discourses within this process.

Studies which conceptualised identities as *self-reflected* stories emphasised the individual self-production of these stories. Although they all acknowledged how 'storying' occurs in response to culture, and thereby employs cultural objects, identity was seen as the 'conscious' self-reflection regarding these cultural objects and stories which embedded them. Interestingly, authors that adhered to this perspective suggested that the act of self-reflection, which is inherent in the construction of a self-narrative, could have different functions. To some authors the generation of narratives allowed students to manage stereotypes in relation to what constituted a mathematician (McGee & Martin, 2011; McGee & Pearman, 2014a, 2014b). Others emphasised that the construction of stories could be used as a tool for self-regulation, thus influencing motives and dispositions to study mathematics

(Black et al., 2010; 2013). Finally, some researchers also emphasised the use of narratives as a reflective process which allowed the orchestration of multiple voices, facilitating the construction of MIs (Solomon, 2011, 2012; Braathe & Solomon, 2015).

Studies which conceptualised narratives as *positioning* in discursive spaces saw narratives as the production of subjects' stories through socially dominant discourses. Here, narratives were considered as a form of *identity work* used by students to manage a view of themselves utilising dominant discourses. A major theoretical consequence of considering narratives as an act of positioning the self is that a particular emphasis was placed on understanding and exposing how dominant discourses influence or restrict, what the individual sees as 'possible and/or desirable' positions (Acher, et al., 2013; Holmegaard, Madsen & Ulriksen, 2014), with some authors even exploring the origin of such discourses (e.g. Epstein, Mendick & Moreau, 2010; Mendick, 2005a).

There are two further issues that deserve to be mentioned regarding the different conceptualisations of identities as narratives. Firstly, as mathematical identity was seen as the narrative of a relationship between individuals and a mathematical activity, mathematics itself became a constructed phenomenon, which was built through discourses and cultural models. Such a de-naturalisation of mathematics implied that there were as many mathematics as narratives and that students could become active agents -either by contributing or resisting- in the construction of these mathematics.

A second issue that deserves to be mentioned in relation to MIs as narratives is the relevance of time as a structuring element. It has been noted by several authors that identities unfold in time (Sfard & Prusak, 2005). Identities can extend towards the future when future aspirations or motives are narrated (Acher, et.al., 2012, Black & Williams, 2013; Craig, 2013), or become the product of temporal tensions when elements of the past, present and future are articulated in one narrative (Solomon, 2012).

5.4.3.3. Identity as a relationship with a specific practice

A third group of studies emphasised how identities were defined by the relationship that individuals establish with a particular mathematical practice (n=8; 12%), a relationship that was described mainly as subjective and in representational forms. It was in this group of studies that identities were defined predominately as ‘a sense of belonging to’ or ‘forms of membership of’ these particular local communities of practices. In order to explore this, most researchers used self-report tools again for data collection, including interviews and surveys, with some of them triangulating data with observations of individuals in practice.

This group of studies can be placed in between the two previous groups with regards to how the relationship between subjective and social features of identities was conceptualised. In contrast with the conceptualisation of identity as an individual attribute, which saw individuals as somehow separated from their social environment, in this group subjective and social features were seen as unified in individuals’ identities. This means that the ‘subjective sense of oneself’ was constructed by an individual ‘sense of belonging’ (e.g. Ahlqvist, London, & Rosenthal, 2013; Solomon, 2007) or by continuous participation in these practices (e.g. Darragh, 2013; Lee, 2002), and was impossible to be understood independently of the collective, shared practice.

Secondly, as their emphasis was placed on representations regarding local practices, these studies emphasised the situated nature of the process by which meaning is constructed, as well as identity development as a process where meanings are negotiated. This perspective is similar to the one adopted by narrative approaches to identity, but it differs in that there is a greater emphasis on how shared meanings are negotiated in a particular practice, with less interest in how general discourses are negotiated. Although in this group of studies discourses or institutional general practices are considered, they are often addressed through the study of their concrete manifestations in specific practices (e.g. schools with/without ability grouping; segregated/diverse classrooms, etc.) (e.g. Solomon, 2007), or by the use of these general discourses in the specific practice (e.g. Darragh, 2013). Following this difference, in these studies the relationship to the particular practice becomes the centre of the narrative. Here, narratives are not the essence of identity, but simply a means to investigate the relationship of individuals with a practice.

A final contrast between these studies and those which emphasise narratives as identities is in their emphasis on temporal dimensions of identities. Studies which explored sense of self in relation to the practice explored identities in different 'dimensions', such as self-perceived competence (in comparison with definitions of competence in place in the specific context), or interests (in relation to the resources available in the specific practice), and not as changing trajectories in an individual's life (as per those focused on narratives).

5.4.3.4. Identities as 'ways of acting':

The fourth category suggested by the data included studies (n=11; 16%) which addressed MI '*ways of acting*', shifting the focus from representational to enacted aspects of identities. This perspective understands *participation* as a constantly changing process where identities are in constant negotiation in *moment-by-mo*ment interactions, which have led to some authors calling them 'micro-identities' (Bishop, 2012). This approach differs from those who see identity as a personal attribute, as a narrative or as a relationship with practice, in that it does not place a major emphasis on the representational aspect of identity. Researchers who adhere to this perspective commonly use case studies and ethnographies as main methodologies in order to provide an in-depth account of students' enacted identities. Here, observed interactions between subjects was the main unit of analysis used for capturing identities.

Studies that considered micro-identities as *ways of acting* focused on students' 'forms of subjectivisation' and 'forms of participation'. 'Forms of subjectivisation' were operationalised as the way in which individuals positioned themselves in relation to others during conversations (e.g. Bishop, 2012; Heyd-Metzuyanim, 2013), while 'forms of participation' were defined as a way of showing that one was able, or unable to act in certain ways (Bishop, 2012; Turner et al., 2013). In both operationalisations there is a clear social aspect in students positioning, emphasising (as in narrative and relation with practice conceptualisations) the unified relationship between subjective and social features of identity.

Two main elements can be extracted from this conceptualisation of identities. First, the in-depth analysis of students' micro-identities has offered interesting insights into how identities are constructed *while* interacting with others. These observations have led many authors to propose a definition of identity as something fluid and changeable according to contextual influences, but also as an emergent process which can become crystallised. In this regard, longitudinal studies have shown evidence that repetitive patterns of interaction can restrict students possible 'ways of acting' in relation to teachers (Black, 2004; Empson, 2003; Heyd-Metzuyanim, 2013) and peers (Turner, Dominguez, Maldonado & Empson, 2013) thus crystallising these positionings.

A second common element among these studies is the strong emphasis on the on-going process of identifying in practice. Several authors have noted that observing these enactments allows one to explore the processes that facilitate or discourage certain types of identifications. For example, there is evidence suggesting that students can identify with positions of increased or decreased competence and confidence (Bishop, 2012; Empson, 2003; Turner, Dominguez, Maldonado & Empson, 2013) as well as positions which offer increased or decreased access to participation and resources (Bishop, 2012; Black, 2004).

5.4.3.5. Constraints and affordances of local practices

A final group of studies which were identified in the analysis comprised of studies which talked about identities, but which focussed on the particular '*Constraints and Affordances of Local Practices'* in terms of the spaces offered for students to develop, narrate or act in particular identities (n= 14, 21%). In contrast with all of the other groups, this group of studies did not emphasise the subjective aspect of identities, but rather how particular contexts provided resources which made some identities more possible. Most of these studies rely on case studies of particular shared practices, where different sources of data were used in order to develop an understanding of how these practices worked and which identities were offered. Following this, mixed methods and ethnographies were the preferred methodological approaches.

At a practice level, studies often compared activities in relation to their social status (usually classed), and also explored whether students could identify or not with practices. More specifically, researchers from this tradition were interested in investigating the *normative identity(ies)* of a particular practice, its particular *constraints and affordances* (Boaler, 1999b; Cobb, Gresalfi & Hodge, 2009), how competence was defined by each practice (e.g. Hodge, 2008; Horn, 2008), and how different forms of engagement and identities were allowed within each practice (e.g. Nasir & Hand, 2008).

The main result from these studies showed how different contexts provided different available identities, in which students needed to position themselves accordingly. Different pedagogical practices (e.g. Boaler & Staples, 2008; Cobb, Gresalfi & Hodge, 2009; Hodge, 2008), different cultural contexts such as classroom culture and culture at home (Anderson & Gold, 2006; Chronaki, 2005; de Abreu, 1995) and different social activities such as school mathematics and sports (Nasir, 2002; Nasir & Hand, 2008) were some of the contexts that were explored within these studies.

5.4.4. Intersection of Definitions of Identities and the Context of Education

The final research question of this review asked whether different definitions of MIs were related to different school stages (primary, secondary, transition to post-compulsory, and post-compulsory). Data from this review appears to support this idea, since some conceptualisations of identity were more frequent in research on certain school stages.

In relation to primary education, there is a notable absence of studies recognising representational expressions of identities, with all studies being mainly focused on enacted identities (ways of Acting identities) or on the identities offered by the particular practice of the classroom (practices' constraints and affordances). This tendency is observed at middle and secondary school levels, where these two conceptualisations account for more than 60% of the studies. In contrast, studies operationalising identity as a subjective and representational aspect of oneself (Identities as Narratives, Relationships with Practices and Individual Attributes) altogether account for the other 40% in middle and secondary schools (see figure 5.4).

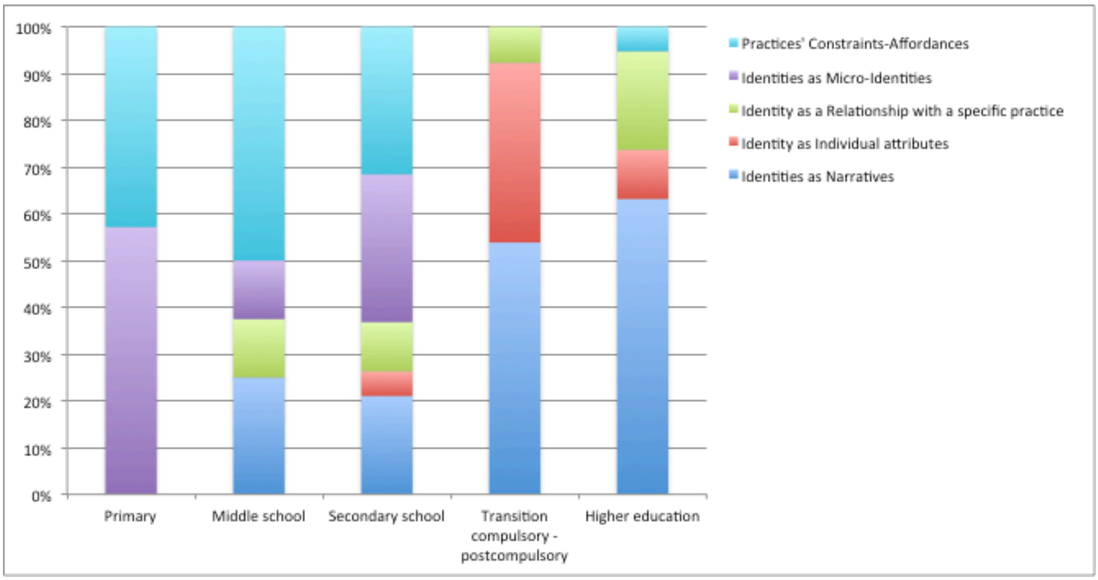


Figure 5.4: Representation of studies from each category of identities in each stage of schooling.

In contrast to the predominance of studies exploring enacted and practice related identities in compulsory education, most of the studies in post compulsory education tended to focus on 'representational' and subjective definitions of identity. More specifically, over half of the studies focused on identities in the form of narratives (transition 54%; post-compulsory 63%), with an emphasis on the increase of shared practices in post-compulsory education (transition 8%; post-compulsory 26%). This contrast raises important questions with regards to the reasons and implications for focusing on different 'defining features' of identity during the first and second half of the mathematical developmental path (see figure 5.4).

5.4. Conclusions and Discussion

The main goal of this review was to address the lack of conceptual coherence in the use of identity when applied to mathematics. This responds to the necessity for the definition and operationalisation of identity (e.g. Cobb, Gresalfi & Hodge, 2009; Sfard & Prusak, 2005), with some authors even suggesting that perhaps the concept should be replaced with something more specific (Brubaker & Cooper, 2000). By reviewing the literature on MIs, this article attempts to summarise how the concept of identity has been employed by different empirical research in Mathematics Education. Such an analysis provides a shared language which can be used to bridge epistemological, theoretical and methodological disparities between traditions, thus advancing the development of a more coherent conceptual framework of identity.

The main outcome of this review considers that conceptual coherence is possible, despite the fact that the literature on identity and mathematics was populated by multiple definitions, which referred to diverse theoretical backgrounds. Even though differences between theoretical approaches which emphasize one or another feature of identity may seem insuperable, data from this review suggests that there are also important commonalities. These communalities include a widely shared notion of identity as learned or constructed, as well as the idea that it is formed by subjective and social aspects, and mediated by mainly representational but also enacted forms.

Variations of how identity was defined appeared to be a consequence of how differences in ‘ontological’ and ‘expression’ dimensions were emphasised, as well as how the relationship between each of the two ‘poles’ of each dimension (social-subjective and representational-enacted) was conceptualised. For example, in relation to the social/subjective tension, some studies considered social and subjective features as separate but interacting elements while others viewed them as an indissoluble unity. Additionally, some studies acknowledged both elements in their definitions of identity, but ‘zoomed’ in on one of them as an object of research. This idea is consistent with several authors who have proposed that research on identity has an adjustable focus (Cobb, Stephan, McClain & Gravemeijer, 2001; Lerman, 2001), thus suggesting that the various definitions of identity may be more related to research ‘choices’ or ‘interests’ than to irreconcilable theoretical differences (Penuel & Wertch, 1995). These different foci can include local meanings and practices (e.g. the meaning of mathematics in a particular didactic or pedagogic context/practice), but also wider socio-political contexts in which these practices and subjects develop (e.g. the meaning of mathematics in a context of performativity or in a context with a tracking policy in place). Following this logic, studies can be differentiated by the defining features of identity, thus becoming the main focus of study, and also by the specific frames of references used to scrutinise these objects (Lerman, 2001; Martin 2012).

As the ‘poles’ of these two dimensions (social/subjective and enacted/representational) are not in opposition, but rather in dialectical unity (Stetsenko & Arievich, 2004), it is the proposal of this article that an ideal conceptualisation of identity should take all of them into account. By ignoring the complexity of identity as a phenomenon, which has a subjective and social quality, as well as an enacted and representational expression, this may seriously limit the validity of efforts to understand the process by which MIs are constructed or developed. For example, investigating MIs exclusively from an individual level, thus only exploring its personal and subjective aspect, can lead to essentialist and un-contextualised notions of identities as attributes, without accounting for the influence of social elements. This danger has been extensively discussed by sociocultural authors (e.g. Atweh & Cooper, 1995; Boaler, 2002a), and as this study/review has shown, has been used-up by most of the research which is currently shaping the definition of mathematical identities in the empirical literature. Ignoring the subjective aspect is also fraught with danger, creating the impossibility of exploring how particular selves are produced and how individual agency is implicated in this process (Stetsenko & Arievich, 2004).

A similar idea is applicable if we consider the representational/enacted dimension. Here, a clear example is the overemphasis of representational aspects based on discursively mediated meanings of identity over enacted ones. A similar tendency has been observed outside the field of mathematics, where some authors have raised a concern regarding the increased number of 'quotation driven studies' (Jerolmack & Khan, 2014). It has been noted that such an overemphasis can lead to an attitudinal fallacy, or the error of inferring situated behaviour from verbal accounts (Vaisey, 2009).

This movement towards the integration of different defining features of identity can be observed in a growing number of studies that have started to explore the relationship between enacted and representational elements in the constitution of MIs. The work of Bishop (2012) is remarkable in this respect, since she has presented evidence to support the idea that representational aspects of identity (e.g. "She's always been the smart one. I've always been the dumb one") are intimately related to enacted and performed aspects of identity which unfold during practice (e.g. how these positions were reproduced in interactions between the 'smart one' and the 'dumb one'). Furthermore, several authors have suggested that consistent and pervasive positionings in classroom interactions can become reified in specific forms of 'representational mathematical identities' (e.g. I am the dumb one) (Bishop, 2012; Black, 2004). These reifications and routinized forms of participation can not only limit further opportunities of enactment but also limit the ability to represent oneself as a different 'kind' of mathematical student (e.g. I will never be the smart one) such as the 'smart one'.

Even though approaching the problem of MIs from an integrated approach is ideal, the difficulties and challenges implied in implementing such an approach are evident. First of all, it requires the collection of multiple sources of data, which are often nested at different levels. At the subjective/individual level, behavioural observations of enacted behaviours as well as self-reports and narratives would be required. In addition, both subjective meanings and actions would need to be taken into consideration in relation to sources of social influence as well as the particular practice under scrutiny. This is also the case at different levels of aggregation. For example the mediation of shared meanings with peer clusters (i.e. groups of friends), classroom practices (e.g. definitions of competence, forms of value in the classroom context), school institutional practices (e.g. ability grouping) and dominant social discourses (e.g. value of mathematics in the labour market) are some of the different 'social' levels which can become the focus of identity research.

Converging data from all these sources requires not only suitable methodological tools, but also adequate theoretical integration of these tools. The challenges implied in designing studies with such complex approaches are evident. This is where researchers need to be explicit with how their research designs include some concepts/elements of identity and ignore others. Having explicit conceptual operationalisations with an emphasis placed on research would allow a more correct interpretation of the data, thus avoiding reductionism. In addition, it would help to suggest gaps in the research to be addressed in future studies and also theorise the implications of the results for areas that were not explored in a particular study.

Finally, this study highlights the large differences between emphases in identity approaches in compulsory, transitionary and post-compulsory education. In particular, it showed that during compulsory education emphasis was given to practice linked identities and enacted aspects. During post-compulsory education most studies focused on representational and subjective aspects of identity. This contrast could be explained by a notion of identity as something which is learnt and constructed throughout an individual's experience with the subject. Following this, developmental factors (changing capacities and changing social requirements during individuals' lives) play a crucial role in the construction of MIs as well as in how this process can be investigated. For example, the development of cognitive abilities and the emergence of operational thinking during adolescence (Inhelder & Piaget, 1958) enables students to engage in reflective thinking, thus allowing them to become more self-aware and self-conscious. In addition, the social requirement of 'committing' with an educational and labour trajectory at the same point of time (Erikson, 1968), may also engage students in the generation of narratives on their lives that can give meaning to present and future choices (Smith, 2010). Due to these cognitive and social changes, which begin to take shape in puberty and continue developing until early adulthood, it is no surprise that narratives become the methodological approach of choice in order to capture the construction of identity, and in particular MIs. Younger children, in contrast, are not yet cognitively mature enough to engage in this narrative activity, and society does not expect the generation of mental representations of who they are, but rather expects that they become 'industrious' by learning and adjusting to social norms (Erikson, 1968). As a consequence, it makes sense that the study of MIs at this age focuses on enacted aspects of their identities or practice identities, since representational aspects have not yet fully developed. However, being aware of the developmental influences of the study of MIs has important theoretical and methodological consequences. First of all, it suggests that the observation of enacted narratives in early schooling years and narrative identities during post-compulsory education has a strong methodological bias, and that the evidence generated by these studies must be critically assessed. By exclusively relying on methodological approaches which explore 'identity as a practice' during primary and middle school, the possibility of understanding the early development of representational aspects of MIs is lost. Similarly, over-reliance on narratives as the main tool with which to explore identity during post-compulsory education neglects the fact that even in adulthood, human beings position themselves, and are positioned, when engaging in social-activities. As a consequence, important aspects of MIs which have an enacted nature, or that are not directly accessible to self-reflection, are neglected. Of course, addressing both narrative identities during early schooling and positioned identity during post compulsory education poses important methodological challenges to researchers, and new methods will be required in order to capture such processes. There is however a small area of studies heading in this direction. Some researchers have used, for example, pictures (Ali-Khan & Siri, 2014; Noland, 2006) or identity maps (Radovic, Black, Salas & Williams, 2015) in order to facilitate the generation of narratives during early childhood and puberty.

Following the growing popularity of the use of the concept of Mathematical Identity in the research literature, this review argues that studies need to place their conceptual framework carefully in relation to a number of existing conceptual frames. It also argues that these conceptual difficulties cause important limitations in the interpretation of the empirical evidence currently available, and suggests an inclusive framework with which to situate conceptualisations of the concept. This framework can allow researchers from different methodological and theoretical traditions to enter into a dialogue. This dialogue is necessary for the research community to move towards a conceptual coherence in mathematics identity research.

Chapter 6. Is there a Female-Friendly Mathematics? The Relationship between Students' Perceptions of Mathematical Teaching and Mathematical Identifications

6.1. Abstract

This paper explores the relationship between students' perceptions of the type of teaching they experience in mathematics [more or less student centred] and how they identify with mathematics [their reported emotions, self-concepts and dispositions]. It also addresses the question of whether there are differences in this relationship for boys and girls. It uses survey data from nearly 300 Chilean students from YEAR 7, clustered over 8 different classrooms. A correlational analysis suggests that there is a positive significant association between how student centred the teaching is perceived to be and students' more positive mathematical identifications. Nevertheless, this effect was independent of gender, suggesting that more student-centred pedagogies (as perceived by the students) do not necessarily offer an advantage for girls, but are positive for both boys and girls.

6.2. Introduction

For over 40 years, girls' relationships with mathematics has been a main focus in mathematics educational research. Several studies have consistently shown that girls do worse than boys in mathematics education and in mathematics related careers, both in attainment (e.g. Hyde, Fennema & Lamon, 1990) and participation after compulsory education (e.g. Solomon, 2007a; WISE, 2012). Research has also documented that part of this lower performance and participation can be attributed to a negative relationship that girls develop with mathematics, both as a subject and with its associated careers. Girls report lower levels of self-concept, self-efficacy and attribute less value to this subject throughout their lives (e.g. Jacobs et al., 2002; Nagy, et al., 2006; Watt, 2004). They also report lower levels of enjoyment when engaged in mathematical activities (Frenzel, Pekrun & Goetz, 2007) and a lower disposition to continue studying mathematics in the future (Buschor, et al., 2014). Some have suggested that this negative relationship with mathematics is part of an overall negative identification with the subject, which can result in the development of negative mathematical identities or identities of marginalisation (e.g. Solomon, 2008).

In addressing these concerns, many studies have focused on what is wrong with girls and with their socio-cultural environment in order to identify the explanatory factors involved in this uneasy positioning in relation to mathematics (e.g. Eccles, 1994; Walkerdine, 1998). However, since the end of the 1980s' some researchers have shifted attention, suggesting there may be something wrong with the way in which mathematics is taught in schools. Following the idea that traditional mathematical teaching does not suit female 'ways of knowing' (Belenky *et al*, 1986; Becker, 1995) or learning styles (Philbin, Meier, Huffman & Boverie, 1995), several forms of teaching have been suggested as more 'female-friendly pedagogies'. In order to become more 'female-friendly', authors have suggested that mathematics should be treated as a process (rather than product) (Becker, 1995), with students learning in a conditional rather than an absolute way (Anglin, 2008; Brew, 2001; Buek, 1985). In addition it has been suggested that teachers and students should engage in problems solving together and share their thinking in a cooperative, rather than a competitive, environment (Becker, 1995; Lavasani & Khandan, 2011; Peterson & Fennema, 1985).

Building on this argument, a question emerges as to whether some forms of teaching which are conceptualised as more 'female-friendly' in the literature mediate how students identify with mathematics and whether this mediation process is different for boys and girls. This study seeks to address such questions by exploring the link between students' perceptions of the teaching in their classrooms and four proxy measures for successful (and unsuccessful) identification with mathematics (positive and negative emotions whilst doing mathematics, self-concept and dispositions towards mathematics in the future). In particular, it will look to explore if these relationships differ for girls and boys, and will then discuss the implications for the 'female-friendly' teaching theory.

6.3. Background Literature

6.3.1. Mathematical Identifications

Despite the global focus on increasing student performance and raising standards, over the last 15 years there has been an increased concern around developing students' positive relationships with mathematics and their consequential positive mathematical identifications (e.g. Nardi & Steward, 2003; Solomon, 2008). Several authors have suggested that the ability to see oneself as a particular kind of person (Gee, 2000) in relation to mathematics is a relevant aspect to be developed further in schools (e.g. Boaler & Greeno, 2000). Positive forms of identifications have been linked to students' persistence in mathematics practices (Black, *et al*, 2010; Boaler, William & Zevenberger, 2000), and has been recognised as an important dimension that can explain differential participation and even performance according to different social categories (Martin, 2012; Lubienski, 2000; Black, 2004). Girls or women have been one of the social groups that have been described as finding it more difficult to see themselves as "legitimate" mathematicians, usually reporting feeling more marginalised from mathematics courses and careers (Solomon, 2007a).

Even though the process of identification with mathematics has sustained increased research and has been suggested as an important dimension for understanding students' engagement with this subject, how this process can be conceptualised and operationalised is still a contested issue (Brubaker & Cooper, 2000; Sfard & Prusak, 2005). This study focuses on 4 variables which relate to different processes of identification with mathematics. Firstly, following Sfard and Pusak (2005) students identify with mathematics in two different temporal dimensions: the present and future. These two processes result in what they call actual and designated identities, i.e. identities which relate to a current state of affairs and identities which relate to an expectation of who one may become in the future.

> Actual identities are usually told in the present tense and formulated as factual assertions. Statements such as 'I am a good driver', 'I have an average IQ', and 'I am an army officer' are representative examples. Designated identities are stories believed to have the potential to become a part of one's actual identity. They can be recognised by their use of the future tense or of words that express wish, commitment, obligation, or necessity, such as *should*, *ought*, *have to*, *must*, *want*, *can*, *cannot*, and so forth (Sfard & Pusak, 2005, pp. 18).

In relation to the development of designated identities this study explores how students expect their current relationship with mathematics to continue in the future. These *mathematical dispositions* have been recently explored showing that positive dispositions tend to decrease during secondary school (Pampaka, Williams, Hutcheson, Wake, Black, Davis & Hernandez-Martinez, 2011a) and are strongly associated with future enrolment in mathematics or STEM (Science, Technology, Engineering and Mathematics) and persistence in mathematical careers (Buschor, et al., 2014). In addition, previous research has also linked these dispositions to pedagogy, giving support to the mediation of pedagogy in students future identifications with the subject (Pampaka et al., 2011a).

In relation to actual identities, three variables that represent aspects of current identification of the students with the subject were considered. First, and following several authors (Bartholomew, Darragh, Ell & Saunders, 2011; Heyd-Metzuyanim & Sfard, 2012; Op' T Eynde, de Corte & Verschaffel, 2006), it is considered that the identification with the subject involves an emotional and affective relationship. How the students feel when doing mathematics is an important aspect of the relationship they are building and, therefore, the mathematical identities they are developing (e.g. Hannula, 2012). Emotions and affects in mathematics education have also been an emerging area of research in the last decade, with several studies reporting the relationship of how students feel when doing mathematics and their performance and persistence in the area (Daniels, et al., 2009; Goetz, 2008; Roth & Radford, 2011; Wigfield, et al., 2002). It can be argued that, although emotions can be seen as changeable aspects of students' relationships with the subject (Evans, 2000), consistent *positive* and/or *negative emotions/affects* can become routinized forms of identification resulting in particular 'mathematical identities'.

A final form of identification considered in this study is students' perceived *self-concept* in relation to mathematics. The reification of seeing one-self as a 'good' or 'bad' mathematician is key to the crystallization of a mathematical identities (e.g. Darragh, 2015; Mendick, 2005a). Research on this topic using different individuals' self-perceptions of competence have documented a strong relationship between these self-beliefs and how learners perform and engage in mathematics (e.g. Abu-Hilal et al., 2014; Bandura, 1986; Pajares, 1996; Pajares & Graham, 1999). In particular, self-concept, or normative judgments that people make about their abilities in comparison with other people and with oneself in different domains (Parker, Marsh, Ciarrochi, Marshall & Abduljabbar, 2013), have received strong support as connected with academic attainment and engagement (March, 1990; Marsh, 2007; Marsh et al., 2008; Valentine et al., 2004).

These four forms of identification with mathematics are of special interest for the study of girls' relationships with the subject. Several different studies have reported that girls usually feel less positive, have decreased dispositions towards mathematics in the future and have a lower sense of self-concept. For example, in relation to emotions/affects girls usually report less enjoyment and pride and more anxiety and shame when doing mathematics than boys (Frenzel, Pekrun & Goetz, 2007). They also display less interest in mathematics (Frenzel, et al., 2010), low aspirations for mathematics (Buschor, et al., 2014; Nagy, et al., 2006), and consistently low self-concept and self-efficacy (Eccles, et al., 1993; Fredricks & Eccles, 2002). Most of these studies have shown how these more negative attributes are often independent of girls' actual attainment, suggesting that the subjective experience of girls is not necessarily associated with an actual deficit in mathematical skills.

In contrast with the large number of studies reporting on the relationship between identification and performance, engagement and participation in mathematics in accordance with gender, there is very little research on how contextual aspects can influence this relationship. Most of the research which has explored these issues has focused on how parents' and teachers' expectations influence students, and particularly the development of girls' mathematical identifications (e.g. Eccles, Jacobs & Harold, 1999; Gunderson, et al., 2012; Jacobs, et al., 2005). One area that has only recently been explored is the influence of the pedagogic style experienced by students. This contextual variable may be particularly important given that forms of teaching have been linked with women's particular ways of knowing (see below).

6.3.2. Female-friendly, Connectionist/Student-centred and Traditional Pedagogy

Ever since the middle of the eighties there has been a growing movement in the introduction of feminist ideas into education and mathematics education research (Kaiser & Rogers, 1995). The work of Belenky and colleagues (1986) explored women's epistemological development and suggested that women might have different ways of knowing when compared to men. They suggested that women tend to develop knowledge particularly when connecting with others, requiring a method of 'connected-teaching' in order to help them to attain better understanding. In their descriptions of this particular way of teaching they suggested that (1) knowledge should be treated as a process, not as a finished product; (2) students should actively engage in producing their own ideas, without being seen as recipients for depositing knowledge; and (3) the process of actively constructing knowledge should be done via speech, in a public dialogue where teachers and students collaborate to construct a new interpretation. The authors described the 'connected classroom' as a classroom which:

> … recognises the core of truth in the subjectivist view that each of us has a unique perspective that is in some sense irrefutably 'right' by virtue of its existence. But the connected class transforms these private truths into 'objects', publicly available to the members of the class who, through 'stretching and sharing', add themselves as knowers by absorbing in their own fashion their classmates' ideas (Belenky, et al., 1986, pp. 223).

Some mathematics educators (e.g. Becker, 1995; Buerk, 1985; Morrow & Morrow, 1995) have followed Belenky's (1986) theory proposing a feminist perspective on mathematics teaching, "one that considers the experiences of women as central to their mathematics development, and in which emotion and reason play balanced roles" (Kaiser & Rogers, 1995, p. 8). Becker (1995), for example, analysed the different stages of development described by Belenky and suggested that the way in which women experience the stage of procedural knowing conflicts strongly with traditional mathematics teaching. Becker states that knowledge through induction (connected knowledge – preferred by women) should have the same value as deductive knowledge (separated knowledge – preferred by men). When applying 'connected teaching' to mathematics, she describes mathematics as a process, whereby teachers and students should engage in problem solving together, alternating methods of solutions, and shared thinking via a public dialogue (Becker, 1995).

From a different viewpoint, Boaler (Boaler, 2002b, Boaler & Greeno, 2000) compared the effect of different pedagogic practice on students' engagement and performance in her studies and came to a similar conclusion. Using concepts such as traditional versus reform teaching (Boaler, 2002b), or didactic versus collaborative/discussion-based classrooms (Boaler & Greeno, 2000), she described how girls tended to become more disengaged and disinterested in the traditional/didactic classroom. While boys tended to reposition their goals by pursuing competition and relative success, girls did not engage in this repositioning

(Boaler, 2002b). In their study on two different advanced placement algebra classrooms, Boaler and Greeno explored the link between these different methods of teaching and the different ways of knowing that were required from the students, using Belenky's typology (Boaler and Greeno, 2000). In the first (individual/didactic) classroom, mathematics was narrowly defined as closed, rule-bound and thoughtless where students were forced to become passive receivers of knowledge in order to become successful and competent. In the second (discussion based collaborative) classroom, where students were offered more open activities which included discussion with consideration given to different plausible solutions, they described themselves as active learners whose roles went beyond memorisation. They argued that these different ways of knowing and identification with pedagogic practices are crucial aspects of mathematical academic success and positive dispositions, with the second classroom being more accessible/engaging for girls (Boaler, 2002b; Boaler and Greeno, 2000) and more equitable between different ethnic and cultural groups (Boaler & Staples, 2008; Boaler, 2008).

Following Boaler's lead it is possible to link what was called a 'connected/female-friendly' mathematical pedagogy with the more general concepts of 'student-centred' and 'connectionist' pedagogy (e.g. Askew, Brown, Rhodes, Johnson & Wiliam, 1997; Cuban, 1983; Pampaka et al., 2011a; Schuh, 2004) in order to make comparisons with 'traditional' forms of teaching. Traditional pedagogy has been described as mostly based on transmission (teacher-centred), memorisation, rote recitation, and is thought to emphasise abstract and 'out-of-context' learning (see, e.g. Schuh, 2004; Boaler, 2002b; Cuban, 1983). In contrast, a non-traditional, student-centred pedagogy gives more relevance to the active construction of knowledge from the student's perspective, and the role of the teacher as learning facilitator (Kember & Gow, 1994). This pedagogic style has been linked with problem-solving approaches, where collaboration and discussion between students and teachers are major aspects of how students are expected to learn (Swan, 2006a). Connections inside and outside the same area of mathematics, between different methods, and between students and teachers via the discussion of mathematical concepts are also aspects which are considered central (Askew et al., 1997). From this framework (connectionist teaching and student-centred teaching) some researchers have built teacher and student surveys for measuring adherence to these types of pedagogies (e.g. Swan, 2006b; Pampaka, et al., 2011a). This study links one of these measurement surveys with the conceptual framework of the connected/female friendly pedagogy.

6.3.3. Relationship between Pedagogic Practices and Educational Outcomes

Most of the literature which has tried to link different pedagogies with educational outcomes has worked with different measures of attainment, trying to link pedagogy with performance (e.g. Desimone & Long, 2010; Freeman, Eddy, McDonough, Smith, Okoroafor, Jordt & Wenderoth, 2014; Hamilton, McCaffrey, Stecher, Klein, Robyn & Bugliari, 2003; Le, Lockwood, Stecher, Hamilton & Martinez, 2009; Palardy & Rumberger, 2008). The relationship between pedagogy and students' subjective relationship with mathematics has undergone less research. The few studies that have focused on these subjective variables have consistently reported a positive relationship between more student-centred pedagogies and more positive affects and dispositions (e.g. Cooper, 2014). For example, Pampaka and colleagues (2011a) found no relationship between pedagogy and grades or drop-out rates, but a small significant relationship between transmissionist teaching (as being the opposite of a more connectionist and student-centred pedagogy) and a decline in dispositions for the continuation of mathematics study in future years. Similarly, other studies have reported that this kind of pedagogy relates with students' perceptions of maths utility and self-efficacy (Gilbert, Musu-Gillette, Woolley, Karabenick, Strutchens & Martin, 2014), students' favourite subjects, general enjoyment of school (Irelson & Hallam, 2005; Noyes, 2012), and comfort and motivation (Timmermans, Van Lieshout & Verhoeven, 2007).

However, despite this evidence, the differential effect of different pedagogies for girls and boys has not been appropriately tested. The support for a female friendly pedagogy comes from qualitative studies (e.g. Boaler, 2002b) or small intervention studies which include the comparison of different classroom/interventions (e.g. guided v/s direct instruction in Timmermans, Van Lieshout & Verhoeven, 2007). This means that further studies exploring the association between forms of teaching and identification variables are needed. This study will fill this gap by exploring the relationship between students' perceptions of their teaching practice and the four forms of identification mentioned above (positive and negative emotions, self-concept and mathematical dispositions) and will consider how this relationship may vary in accordance with gender. Following the literature we expect to find: (1) A more negative mathematical identification for girls over boys (lower levels of positive affects, self-concept and dispositions, and higher levels of negative affects); (2) A general positive effect of the experience of higher levels of student-centred pedagogy on positive forms of identification (i.e. positive affects, self-concept and dispositions) and a negative effect on negative forms (i.e. negative affects); and (3) A strong interaction between student-centred pedagogy and gender. Following the aforementioned 'female-friendly theory', it can be expected that the positive effect of experiencing more student-centred teaching will be higher for girls than for boys.

6.3. Methodology

6.4.1. Procedure, Data and Sample.

This study was situated in eight year 7 mathematics classrooms (13 to 14 years old) classrooms from different schools in Santiago de Chile. These schools were chosen by convenience sampling in order to be representative of a ‘normal’ population in this country: medium socioeconomic status, average attainment in the national evaluation (SIMCE) and a privately subsidised administration. All of these schools were administered by the same non-profit, catholic foundation, scheme of administration that has been found to be one of the most efficient schemes for low-income families in Chile (McEwan, 2001). All mathematics teachers followed the same curriculum and were encouraged to implement the same method of teaching in mathematics lessons; a method which emphasised students’ learning as a process with a central social participation.

The classrooms were visited during school hours and students were surveyed in their own groups. Parents were given an information sheet two weeks prior to evaluation, which allowed them to opt their children out of the study if so desired (see appendices 7 and 8). Only one parent opted out. During evaluation, students were informed of the purpose of the study and were given the possibility to opt out as well. The surveys lasted approximately 40 minutes. 291 students (137 males and 154 females) returned completed surveys.

6.4.2. Instruments

Each student survey included 4 different measurement instruments and also general information about the student. Students were asked to report on their own relationship with mathematics in relation to the four forms of identification previously mentioned (negative and positive affects when doing mathematics, perception of self-concept and dispositions towards mathematics in the future). In addition, students' perceptions of mathematical activities in the classroom, of their attainment in comparison with their classmates and their favourite subjects in schools were included as independent questions within the survey (see appendix 14 for survey in Spanish – translation to English are presented in methodology page 64).

In relation to the *emotional experience* of students when doing mathematics, two broad, general components, typically labelled '*positive emotions'* and *'negative emotions'*, were surveyed. These factors have reliably emerged as the dominant dimensions of emotional experience and conform to the most basic model in order to understand approach and avoidance tendencies. To measure these factors, Watson, Clark, and Tellegen (1988) developed the Positive and Negative Affect Schedule (PANAS), which consists of two 10-item scales for positive and negative affects. Participants are asked to rate each of the 20 emotional words (e.g., interested, proud, irritable, hostile, etc.) indicating the extent to which they experience each emotion while doing mathematics according to a 5-point scale (*very slightly or not at all, a little, moderately, quite a bit,* and *extremely*). These factors account for most of the variance in self-rated affect (see Crawford & Henry, 2004), and emerge consistently across cultures, response formats and languages, including Spanish (Dufey & Fernandez, 2012; Moriondo, Palma, Medrano & Murillo, 2012; Robles & Paez, 2003).

Students' self-concepts and dispositions were measured using items from the Pampaka and Wo survey (2014) in order to create two new composite measures. These items were based on previous surveys of mathematical attitudes (e.g. Fennema & Sherman, 1977) and a particular survey developed by Pampaka and colleagues in order to measure dispositions for studying mathematics (Pampaka et al., 2013) and self-efficacy (Pampaka, Kleanthous, Hutcheson & Wake, 2011b). In particular the Self-Concept scale included 4 items which measured students' attributions of their own ability in doing mathematics (e.g. I can get good results in maths). The Mathematical Dispositions scale considered 6 items asking students to report how they expected their relationship with mathematics to continue in the future (e.g. maths is important for my future). In both sub-scales students were asked to select their level of agreement in a 5 point scale (from *strongly disagree* to *strongly agree*).

Finally in order to measure teaching practice, a modified scale of teaching-centeredness based on Pampaka and colleagues scale was used (Pampaka, et al., 2011a). This survey measures students' perceptions of the teaching in their classroom, and was constructed based on previous surveys built to measure similar constructs (e.g. Swan, 2006b). Different studies have given support to the use of students' perceptions of teaching as a measure of pedagogy. For example, Ellis and colleagues (2007) found a moderate correlation between students and outside observers in perceptions of classroom environment (pedagogy, content, tasks and interactions). Desimone (2010) found that students and teachers reported similar practices. Finally, several studies have found that students' reports of standards-based teaching practices are more predictive of outcomes than teacher reports (Kahle, Meece, & Scantlebury, 2000; McCombs & Quiat, 2002). Following this, students' perceptions were considered a measure of the teaching in the classroom, but also a more specific measure of each student's experience of this teaching. This is important, as several qualitative studies have documented how students can experience highly differentiated forms of teaching inside the same classroom (e.g. Black, 2004). The student-centred teaching instrument used here included 10 items which described different activities related to a connectionist/student-centred pedagogy (e.g. we work together on group projects; we discuss ideas with the whole group). Students were asked to answer how frequently each of these activities happened in their classrooms in a 4 point scale (*never, rarely, frequently,* and *always or almost always*) (see methodology for a complete list of items).

Finally, for measuring attainment a final year assessment test was used. This test assessed students' knowledge of the curriculum, linking the assessment with the national standardised test (SIMCE).

6.4.3. Analytical Approach

The first step was to test the model of different forms of identification used in this study. Three steps were taken in testing this model. Firstly, the property of each dimension as measured by its instrument was analysed, testing to see if there was enough evidence of uni-dimensionality. For assessing this a Rasch methodology was used in its application for rating scales. This methodology allows the construction of interval measurements from raw data (in this case, ordinal data), at the same time testing the plausibility of using it as a one-dimension scale. This analysis models item difficulty in relation to person 'ability' (e.g. 'ability' to respond with higher levels of emotional experience), and assesses the validity of the scale by providing an item and person fit to the overall construct (Bond and Fox, 2001). It also provides support to the reliability of the scale by providing information as to whether there are enough items spread along the continuum and enough spread of ability among persons (Bond and Fox, 2001). All the analyses were performed using Winsteps 3.72.3 (Linacre, 2011). For assessing uni-dimensionality three different indicators were observed. Firstly, the infit and outfit indicators assessed the item fit to the model. Authors suggest estimations close to 1 are evidence of uni-dimensionality, with some suggesting that values need to be close to 1.1 in samples over 500 (Smith, Schumacker & Busch, 1995) although others set a more relaxed interval between 0.5 and 1.5 for adequate fit (Linacre, 2002). Secondly, following Linacre (1998) a factor analysis of the residuals after the scale had been built was performed to check if any further dimensions might be identified. It was considered that Eigenvalues of the second contrast (first contrast after the Rasch measurement was extracted) over two were evidence of possible further dimensions (Raîche, 2005). Finally, items and person reliability were observed, on the basis that a separation index over 2 and a person reliability over 0.8 was thought to be adequate. These same analyses were performed for the student-centred teaching practice scale.

After analysing each scale, evidence of relationship between forms of identification was tested. As the model suggests, the different measurements are proxy of different forms of identifications with mathematics which are theoretically related. Furthermore, it was expected to find medium positive correlations between the positive identifications [positive affects, self-concept and dispositions] and a negative relationship with negative forms of identification [negative affects], giving evidence to the theoretical relationship but confirming the relative independency of each dimension.

Finally, although all components of identity were measured as a current state of affairs (students' current perceptions of their affects, self-concept and dispositions), it was hypothesised that current aspects of identity would predict designated aspects (as expectations of the future). Consequently, it was expected that dispositions towards mathematics in the future would be predicted by a positive relationship of positive affects and self-concept, and a negative relationship of negative affects. This model was tested by running a multiple linear regression, with dispositions set as a dependent variable and the three other components of identity set as predictors. It was also assessed if the relationships were independent of attainment and gender, by the inclusion of these variables as controls and by testing each variable's interaction with these control variables.

In order to address the association between teaching practices (as perceived by the students) and the different proxies of identification, a linear regression analysis was followed. Multiple linear regressions were used to regress the scores of the perceived teaching practice on each of the forms of identification. A hierarchical approach was taken, entering gender and perceived teaching first, and later the interaction between teaching and gender. For assistance with the interpretation of the predictions and for decreasing problems of multicolinearity between variables and interaction terms, all predictors were centred and standardised (Aguinis, 1995). In addition, as interaction between variables is more difficult to detect in field study designs and as the sample of this study was limited (particularly when dividing the sample, see below), interaction effects were tested against a p value of 0.1 (as suggested by Aguinis, 1995; McClellan & Judd, 1993). Attainment was entered as a control variable.

Finally, to further explore these results, the total sample was divided into three groups: those who identified strongly with mathematics (students who reported mathematics as being one of their favourite subjects), those identified least with mathematics (students who reported mathematics as being one of their least favourite subjects) and neutral (students who did not consider mathematics to be one of their favourite or least favourite subjects). Similar linear regressions were carried out separately for these three groups of students. In these analyses we tried to explore if relationships between variables were different for students who identified strongly and students who identified least with mathematics

6.5. Results

6.5.1. Testing the identification model and scale of Student-centred teaching practice

Using a Rasch analysis of students' perceptions of the activities in the classroom it is possible to build a scale of perception of frequency of student-centred mathematical activities: students can be differentiated in terms of their perception with an adequate person separation index and no misfitting items (see table 6.1 and appendix 14).

In terms of the different forms of identification, it was possible to measure a scale of general positive affects, dispositions towards mathematics in the future and a self-perceived concept (see table 6.1 and appendices 15, 17 and 18). No evidence was found regarding further dimensions on each scale. Although the negative affect scale showed a lower separation and reliability index, the scale was included in the analysis (see table 6.1 and appendix 16).

Table 6.1:
Summary Rasch analyses scales.

Scale	Items	Total Person Infit [1]	Total Person Outfit [2]	Person Separation index	Person Reliability	Variance explained by measurement	Eigenvalue first contrast [3]
Student-centred	10	1.02 (0.66)	1.05 (0.75)	1.68	0.74	43.70%	1.7
Positive Affects	10	1.01 (0.75)	1.02 (0.73)	2.20	0.83	48.90%	1.6
Negative Affects	10	1.05 (0.60)	0.97 (0.62)	1.37	0.65	39.90%	2
Self-Concept	4	0.97 (0.85)	0.98 (0.88)	2.03	0.80	67.70%	1.6
Dispositions	6	0.98 (0.72)	0.99 (0.82)	2.30	0.84	69.80%	1.7

Note. [1] Infit meansquare (SD); [2] Outfit meansquare (SD); [3] After extracting Rasch measurement, principal component of the residuals.

Regarding the relationship between different variables included in this study, it was observed that most of the variables used as proxies of identification showed a positive correlation between them. These correlations were of medium to large size suggesting that these different variables were highly correlated but relatively independent. As expected, the only dimension that showed a negative correlation with the other variables was *negative affects*, providing further support to the validity of the scales. The same direction of relationship was observed between the scales of teaching and attainment and identification: a negative relationship with the *negative affects scale* and a positive relationship with *positive affects*, *self-concept* and *dispositions* (see table 6.2), giving preliminary support to the hypothesis of a general relationship between student-centred teaching and students' positive identification with mathematics.

Table 6.2:
Bivariate correlations.

	Student centred TP	Positive Affects	Negative Affects	Self-concept	Disposition	Attainment
Student centred TP	1	.429***	-0.005	.276***	.331***	-0.038
Positive Affects		1	-0.078	.619***	.569***	.247***
Negative Affects			1	-.315***	-.168***	-.359***
Self-concept				1	.589**	.465***
Dispositions					1	.222***
Attainment						1

Note. * p< .05; ** p< .01; *** p< .001

Finally, when testing the predictive power of current forms of identification on dispositions for the use of mathematics in the future (a designated form of identification), a general support for this relationship was found. Higher identification with positive affect and with positive self-concept predicted more positive dispositions towards using/studying mathematics in the future, with the level of negative affects not affecting this variable (see table 6.3). These effects were independent of attainment and gender.

Table 6.3:

Model of current aspects forms of identification predicting dispositions towards mathematics in the future

	Beta	r2
Step 1		0.43****
Positive Affects	0.30****	
Negative Affects	0.00	
Self Concept	0.44****	
Step 2		0.43****
Positive Affects	0.30****	
Negative Affects	-0.02	
Self Concept	0.47****	
Gender	0.01	
Attainment	-0.07	
Step 3		0.45****
Positive Affects	0.23***	
Negative Affects	-0.11	
Self Concept	0.56****	
Gender	0.00	
Attainment	-0.06	
Gender_ZPositiveAffects	0.10	
Gender_ZNegativeAffects	0.16**	
Gender_ZSelfConcept	-0.11	
Attainment_ZPositiveAffects	0.08	
Attainment_ZNegativeAffects	-0.11**	
Attainment_ZSelfConcept	-0.12	

Note. * p< .10; ** p< .05; *** p< .01; **** p< .001. Gender coefficient represents effect of being female (male is the reference category)

6.5.2. Descriptive Statistics

In comparison to males, female students reported student-centred teaching practices to be less frequent, felt less positive, reported being less competent, and placed less value on doing mathematics in the future. All differences were significant at a level of .05, except attainment and identifications with negative affects. Using Cohen's D, the size of the differences for the significant effects were between small and medium (ranging from .24 to .39). Differences in attainment, forms of identification and perceived teaching practice by gender are presented in table 6.4.

Table 6.4:
Descriptive results.

	Gender	N	Mean	Std. Deviation	Std. Error Mean	Cohens D
Student-centred teaching practice	male	133	0.14	1.00	0.09	0.24*
	female	153	-0.09	0.96	0.08	
Positive Affects	male	128	0.25	1.13	0.10	0.39***
	female	148	-0.18	1.10	0.09	
Negative Affects	male	129	-0.95	0.84	0.07	0.05
	female	148	-0.99	0.87	0.07	
Self-concept	male	137	1.47	2.49	0.21	0.34**
	female	154	0.72	2.00	0.16	
Dispositions	male	137	0.40	1.96	0.17	0.25**
	female	154	-0.05	1.66	0.13	
Attainment	male	132	60.49	14.84	1.29	0.02
	female	151	60.25	15.20	1.24	

Note. * $p< .05$; ** $p< .01$; *** $p< .001$

The descriptive analysis shows that girls not only report less positive identifications with mathematics (as noticed on significantly lower scores in all positive variables), but they also report that they experience student-centred practices less often in their classroom. These differences are not necessarily related to differences in attainment, as this variable was the only aspect in which girls attained at a similar level to boys.

6.5.3. Model Building

Models showed that gender was a significant predictor of all of the identification variables, except identification with negative affects. Girls were predicted to have lower scores in positive affects, self-concept and mathematical dispositions, both before and after controlling for mathematical attainment (see table 6.5, steps 1 and 2). The perceived teaching practice (as a measurement of student-centredness) was significant in predicting all of the positive identifications after controlling for students' academic attainment. The perception of their teaching practice, as being more student centred, was associated with higher scores on the positive affects measure, higher dispositions towards mathematics in the future and higher levels of self-concept. This effect was not found in the measure of negative affects (see table 6.5, step 3).

Table 6.5:

Forms of Identification predicted by gender and student-centred practice. Total sample.

	Positive Affects (Beta)	Negative Affects (Beta)	Self-Concept (Beta)	Dispositions (Beta)
Step 1				
Gender	-0.20**	-0.01	-0.19**	-0.12*
Step 2				
Gender	-0.20**	-0.01	-0.20***	-0.12*
Attainment	0.25***	-0.36***	0.47***	0.23***
Step 3				
Gender	-0.14**	0.01	-0.16**	-0.08
Attainment	0.27**	-0.36***	0.49***	0.24***
Student centred teaching	0.42***	-0.02	0.29***	0.34***
Step 4				
Gender	-0.14**	-0.01	-0.16**	-0.08
Attainment	0.27***	-0.36***	0.48***	0.24***
Student centred teaching	0.43***	-0.05	0.33***	0.26***
Gender * Student centred teaching	-0.01	0.05	-0.05	0.11
r2 step 1	0.03**	-0.00	0.03**	0.01*
r2 step 2	0.09***	0.12***	0.26***	0.06***
r2 step 3	0.27***	0.12***	0.34***	0.17***
r2 step 4	0.27***	0.12***	0.34***	0.17***

Note. * p< .10; ** p< .05; *** p< .01; **** p< .001. Gender coefficient represents effect of being female (male is the reference category)

When observing the gender effect in positive forms of identification before (step 2) and after (step 3) controlling for teaching practice, a decrease in its size was observed. This meant that part of the gender differences in positive identifications with mathematics may be explained by different perceptions of the teaching practice in the classroom: As girls perceive the teaching in their classroom to be less student-centred they tend to report lower levels of positive affects, mathematical self-concept and dispositions towards mathematics in the future. However, this mediation effect is not complete, as some differences remain after the teaching practice variable is included (see table 6.5 step 3).

Finally, the inclusion of the interaction between gender and perceived teaching practice did not produce a significant increase in the explained variance in any of the identification variables (*positive affects model* ΔR^2= .00, $F_{(1, 260)}$= 0.021, p= .88; *negative* affects *model* ΔR^2= .00, $F_{(1, 261)}$= 0.42, p= .52; *self-concept model* ΔR^2= .00, $F_{(1, 270)}$= 0.57, p= .45; *dispositions model* ΔR^2= .01, $F_{(1, 270)}$= 1.79, p= .18). This suggests that the effect of student-centred teaching is equal for girls and boys, which does not support the female-teaching theory (see table 6.5, step 4).

To further explore these results the sample was divided into three different groups: strongly identified with mathematics (students who reported mathematics as being one of their favourite subjects), strongly dis-identified with mathematics (students who reported mathematics as being one of their least favourite subjects) and neutral (students who did not consider mathematics to be one of their favourite or least favourite subjects). After dividing the sample, the same analysis was conducted. The general tendencies observed for the overall sample were replicated in the divided sample, but instability was found in how the effects of gender and teaching practice affect different forms of identification for students with different levels of identification with the subject (see tables 6.6a and 6.6b).

Table 6.6a:

Forms of identification (with positive and negative affects) predicted by gender and student-centred practice. Sample divided by levels of identification with the subject.

	Positive Affects (Beta)			Negative Affects (Beta)		
	Dis-Identified	Neutral	Identified	Dis-Identified	Neutral	Identified
Step 1						
Gender	-0.21*	-0.13	-0.21*	0.14	-0.13	-0.05
Step 2						
Gender	-0.23*	-0.14	-0.19	0.19	-0.10	-0.07
Attainment	0.13	0.09	0.19	-0.28**	-0.33****	-0.36***
Step 3						
Gender	-0.16	-0.13	-0.10	0.13	-0.10	-0.05
Attainment	0.13	0.16*	0.23**	-0.28**	-0.30****	-0.35***
Student centred teaching	0.35***	0.31****	0.49****	-0.28**	0.16*	0.11
Step 4						
Gender	-0.12	-0.12	-0.06	0.19	-0.10	-0.03
Attainment	0.12	0.16*	0.21*	-0.29**	-0.30****	-0.36***
Student centred teaching	0.28*	0.38***	0.60****	-0.37**	0.07	0.17
Gender * Student centred	0.13	-0.09	-0.18	0.18	0.11	-0.10
r2 step 1	0.03*	0.01	0.03*	0.004	0.01	-0.01
r2 step 2	0.03	0.01	0.05*	0.07**	0.11****	0.10**
r2 step 3	0.14***	0.10***	0.28****	0.12***	0.13****	0.10**
r2 step 4	0.13**	0.10***	0.029****	0.13***	0.13****	0.09*

Note. * p< .10; ** p< .05; *** p< .01; **** p< .001. Gender coefficient represents effect of being female (male is the reference category).

Table 6.6b:

Forms of identification (with self-concept and dispositions) predicted by gender and student-centred practice. Sample divided by levels of identification with the subject.

	Self-Concept (Beta)			Dispositions (Beta)		
	Dis-Identified	Neutral	Identified	Dis-Identified	Neutral	Identified
Step 1						
Gender	-0.13	-0.20**	-0.16	-0.10	-0.05	-0.15
Step 2						
Gender	-0.15	-0.22***	-0.12	-0.12	-0.05	-0.14
Attainment	0.18	0.40****	0.49****	0.08	0.04	0.11
Step 3						
Gender	-0.09	-0.22***	-0.08	-0.08	-0.04	-0.09
Attainment	0.18	0.45***	0.51****	0.08	0.10	0.13
Student centred teaching	0.30**	0.23***	0.19*	0.18	0.30***	0.22*
Step 4						
Gender	-0.09	-0.21***	-0.10	-0.02	-0.04	-0.16
Attainment	0.18	0.44****	0.52****	0.08	0.10	0.16
Student centred teaching	0.32**	0.45****	0.14	0.07	0.40***	0.05
Gender * Student centred	-0.02	-0.28**	0.08	0.19	-0.13	0.30**
r2 step 1	0	0.04**	0.01	-0.01	-0.01	0.01
r2 step 2	0.02	0.20****	0.25****	-0.01	-0.01	0.003
r2 step 3	0.09**	0.25****	0.27****	0.002	0.07***	0.04
r2 step 4	0.08*	0.28****	0.26****	0.004	0.07***	0.08*

Note. * p< .10; ** p< .05; *** p< .01; **** p< .001. Gender coefficient represents effect of being female (male is the reference category).

These models showed that there was still a tendency for girls to identify less positively with mathematics, reporting lower self-concept and lower mathematical dispositions, but these effects were not stable for the different levels of identification. For the measures of positive affect and disposition to study mathematics in the future, the effect of gender was higher with students who either strongly identified with maths or strongly dis-identified with it (with the effects of dispositions not having a statistical significance in any group). For self-concept the effect was greater for students who report being neutral about mathematics. In other words, girls who strongly identified and dis-identified with mathematics tended to be less positive than their male counterparts, but did not necessarily report a lower self-concept. In this last variable, only neutral girls (girls who did not mention mathematics as favourite or least favourite subject) displayed lower levels of self-concept than boys. Again the main effect of gender was not significant for negative affects when doing mathematics (see tables 6.6a and 6.6b, step 1).

With regards to the effect of students' perception of their teaching practice, again the general tendency was for higher levels of student-centred teaching to be associated with higher levels of positive affect, self-concept and mathematical dispositions. Interestingly, when dividing the sample, this variable also had an effect on negative affect levels when doing mathematics, an effect that varied in the different subsamples. For students who dis-identify with maths (least favourite subject), there was a negative effect between the perceived high frequency in the use of student-centred teaching in their classrooms and their level of negative affect when doing mathematics. In other words, when these students perceived a high usage of student-centred practices in their classrooms, they tended to report feeling less negative when doing mathematics. In contrast, for students who reported a neutral relationship with mathematics, the perception of a high frequency of student-centred teaching activities in their classroom was associated with higher levels of negative affect when doing mathematics (see tables 6.6a and 6.6b, step 2).

Finally, in relation to the interaction between gender and perception of the frequency of student-centred teaching practices, again most models gave no support to the female-friendly teaching theory when the group was differentiated in accordance with how they identify with mathematics. However, two interactions appeared significant. Firstly, the positive effect of student-centred teaching in students' self-concepts appeared to be less strong for girls who reported being neutral about mathematics. In other words, and contrary to the prediction of the female-friendly teaching theory, neutral girls reported lower levels of self-concept than neutral boys when they perceived higher levels of student-centred teaching in their classroom. Secondly, for strongly identified girls the effect of student-centred teaching on their dispositions to study mathematics in the future was positive, i.e. there was a tendency in this group of girls to report higher dispositions towards mathematics in the future when experiencing higher levels of student-centred teaching (see tables 6.6a and 6.6b, step 3).

6.6. Conclusions and Discussion

The main purpose of this paper was to explore the relationship between students' experiences of mathematics teaching in their classroom and different variables related with their identifications with mathematics. In particular, the emphasis was placed on exploring if the perception of their teaching as more student-centred had a positive effect on students' positive emotions, positive self-concept and dispositions towards the subject in the future, and a negative effect on students' reports of experiencing negative affect when doing mathematics. It was also concerned with exploring if this relationship was similar for boys and girls, testing if any evidence of these effects was greater for girls or boys and if therefore, support could be provided to the claim that student-centred pedagogies are more 'female-friendly'.

Three main findings can be extracted from this study. Firstly, students who perceived teaching in their classroom to be more student-centred reported higher identification with positive self-concepts, positive dispositions and positive affects when doing mathematics. This was particularly important for the measures 'positive affect' and 'disposition to study mathematics in the future', where perceived teaching style had a greater effect on these measures than attainment. This confirms previous studies (e.g. Gilbert et al., 2014; Pampaka, et al., 2011a) and is in line with recent reforms in mathematics education which have encouraged teachers to follow this type of pedagogy in their classrooms (see for example in the US the NCTM standards). For example, in the US, what has been called '*reform oriented teaching*' or '*standards based instruction*' has been described as instruction which engages students as active participants in their own learning through communicating with others, working in cooperative learning groups, and making connections to real life situations (Hamilton et al., 2003; Le, Lockwood, Stecher, Hamilton, & Martinez, 2009). While most studies which have evaluated this reform approach have been focused on its impact on attainment (e.g. Desimone & Long, 2010; Hamilton et al., 2003), this study confirms that when students perceive a higher frequency of such teaching in their classroom they also tend to identify more positively with mathematics. This adds to the literature which has reported this type of effect predominately in attainment and performance, but has neglected the effects of teaching on affective/subjective variables.

A second result relates to the hypothesis that girls may experience more positively student-centred pedagogies. Following this hypothesis, a stronger effect of student-centred teaching was expected for girls when compared with boys. This was not confirmed by the data: the positive effect was of a similar magnitude for boys and girls in general. However this study still provides some support for the use of student-centred activities for enhancing girls' positive identifications with mathematics. It was found that girls reported the experience of their classroom activities as being less student-centred, which was consequently associated with their report of feeling less positive, competent and less positively disposed to mathematics. This highlights the need to further explore why girls report different perceptions of the teaching in their classroom when compared with boys. One hypothesis which needs to be contrasted is that girls may actually be experiencing a different mathematics in their classroom. International and national studies have tried to explore this issue by researching the distribution of opportunities of participation in the classroom between boys and girls. Some studies have documented how teachers direct more questions to boys (in Chile, Sernam, 2008) or provide them with more visual attention (French & French, 1984; Graddol & Swann, 1989), which in turn gives boys more opportunities for participation, even without the teacher noticing this differential treatment (Black, 2004). This may be one possible explanation for the differences in perceptions of pedagogy which requires further research.

A final third result that also provides some support for the female friendly teaching argument is the relationship between student-centred teaching and dispositions towards studying mathematics in the future for girls who identified mathematics as their favourite subject. According to this study, the effect of student-centred teaching on future dispositions was greater for girls than for boys who named mathematics as one of their favourite subjects. This is important as dispositions, aspirations and intentions towards mathematics in the future have been linked with future engagement in mathematically related careers (Buschor, et al., 2014). In other words, increasing girls' perceptions of this type of pedagogic practice in their classrooms can enhance girls' intentions to follow STEM careers, which have been linked with better career opportunities in the future (e.g. Ñopo, 2012; Sells, 1978).

However, two main questions remain unanswered here and may be useful for further research. Firstly, it is not clear if the effect of the perception of teaching on students' identifications with mathematics corresponds with a general effect of the teaching in the classroom or with individual experiences inside these classrooms. Although some variability within classroom was found, the small sample of classrooms did not allow comparing this variability with the variability between them. Future research might include a larger sample of classrooms to allow further explorations of classroom effects in this relationship. This is particularly relevant if this or future similar studies intend to inform prescriptions of different pedagogic practices for teachers. A second question asks whether the perceived experience of pedagogy reported by girls is related to what actually unfolds in classrooms. Linking these perceptions with observational data may provide further information for suggested interventions on teachers' activities.

Finally, while a number of writers (eg. Kaiser & Rogers, 1995; Becker, 1995) have advocated a 'female-friendly' mathematics, this analysis suggests that student centred pedagogy should be conceptualised as 'student-friendly' mathematics, as it was found to be beneficial for both boys and girls. Therefore, this critiques previous literature which has advocated for transforming mathematics for girls by highlighting the necessity of transforming mathematics for all. It can be concluded that, in line with others (e.g. Walshaw, 2001; Mendick, 2005b), linking different pedagogies for different genders may essentialise differences that are not necessarily attributes of boys and girls.

Chapter 7. Being a Girl Mathematician:

Analysis of the diversity of positive mathematical identities in a secondary classroom

7.1. Abstract

The construction of 'positive' mathematical identities [MIs] is a complex and central issue in School Mathematics, where girls are usually 'counted out' of the field. This study explores positive MIs (high attainment and positive relationship) in three girls. A theoretical and methodological framework with narrative and practiced aspects of identity in relation with peers and with the particular mathematical practice was employed. Results highlighted diversity in how these girls experienced mathematics. They valued different forms of doing mathematics [independent-collaborative; wider-complex; straightforward-procedure oriented], showed different forms of engagement [detachment; protagonist and challenging of practice; compliant and support seeker] and positioned themselves differently when narrating their MIs [effortless/efficient; different; responsible]. The role of mathematical practice(s) and social relations with peers in these different forms of identification was also explored and discussed.

Keywords: Mathematics Identities, Peer Relations/hips, Gender, Positive Identities, Classroom Interactions, Narratives

7.2. Introduction

Several studies over at least the last 40 years have documented how women tend to live conflicted mathematical lives, dealing with a relative underperformance as a group (and social discourses documenting these differences) and participating less than men in post compulsory mathematics and related careers (e.g. Hyde, Fennema, & Lamon, 1990; WISE, 2012). When attempting to explain these difficulties, studies have predominantly followed a psychological perspective, trying to identify what is 'wrong' with girls/women in their relationship with mathematics - an approach that has been called 'blaming the victims' (Kaiser & Rogers, 1995). Several authors have criticized such work, suggesting that by focusing on girls' individual-psychological characteristics, as inherent fixed traits, a 'naturalization' process may take place where girls are seen as ill-suited to mathematics (Boaler 2002a). In addition, this psychological approach neglects the fact that the process of learning mathematics does not occur inside an isolated individual, in a historical vacuum, but is embedded in a wider and particular social context (Kaiser & Rogers, 1995; Boaler, 2000). Consequently, some have suggested that, in order to understand students' relationships with mathematics and with any other social activity, the emphasis needs to be placed on the relationship between individual and social context (e.g. Wenger, 1998; Holland, Lachicotte, Skinner & Cain, 1998). It is here where the concept of identity has been suggested as a useful lens to explore this intersection (Sfard, 2005), with particular benefits for understanding underrepresented and underperforming groups (e.g. Varelas, Martin & Kane, 2012, Martin, 2012).

Recently, a growing number of studies in mathematics education have used concepts such as identity or 'mathematical identity' [MI] from a social perspective (Stentoft & Valero, 2009; Radovic, 2015). In particular, when used for understanding girls' relationship with mathematics, a strong contrast can be found between studies in compulsory and postcompulsory education (which is also the case in more general research on mathematical identities, see Radovic, 2015). Research on post-compulsory mathematics has mainly explored womens' MIs by capturing the 'singularity' of their experiences and their reflection on this experience through the use of a narrative approach (Herzig, 2004; Mendick, 2005a, 2005c; Rodd & Bartholomew, 2006; Solomon, 2012; Solomon, Radovic and Black, 2015). Several ideas have come out of this line of investigation. For example, it has been noted that women with a 'positive' identification with mathematics, those that decided to pursue math as a career path, need to negotiate a series of complexities and conflicts in order to engage with maths. The role of others in building a mathematical story (Solomon, Lawson & Croft, 2011; Solomon, 2012), the power of discourses that define mathematics as a male domain (Mendick, 2005a) and the positioning of women as an invisible minority in a male peer context (Rodd & Bortholomew, 2006), are some of the complexities described by this literature. As a consequence, a main proposition of this line of research is that women have to perform complex 'identity work' (Black & Williams, 2013; Holmegaard, Madsen, & Ulriksen, 2014; Mendick, 2005a; Solomon, 2012), suggesting that being a female mathematician does not come easily.

In contrast with the focus on a 'positive' (but complex) identification with mathematics, and the use of a narrative approach, studies in compulsory education have tended to focus on how girls interact with mathematical activity in schools, particularly paying attention to how they disengage from it. This line of research has suggested that predominant mathematical practices -where teachers have a central role, individual/competitive work is dominant and rote repetition and rule-bound learning is encouraged- have a central role in producing girls' difficulties, and disengagement from mathematics (Atweh & Cooper, 1995; Black, 2004; Boaler, 2002b; Boaler & Greeno, 2000; Solomon, 2007b). Although both boys and girls have been observed as experiencing this practice as disengaging (Nardi & Steward, 2003), boys have been described as more able to reposition their goals, or to resist the practice, by pursuing competition and relative success. Girls, in contrast, have been observed as finding this re-positioning more difficult, thus, tending towards disengagement and disidentification from mathematics (Boaler, 2002a, Boaler, 2002b). It is interesting to note here that some of these studies have anecdotally reported the existence of girls that have a 'positive' relationship with maths, who engage in mathematical activity even in 'traditional' mathematical classrooms (e.g. Solomon, 2007b). But accounts of how such 'against the odds' positioning within traditional school mathematics practices is made possible and how it is experienced by such girls is an under researched area.

One interesting area of investigation in terms of developing our understanding of how girls' positive mathematical identities are made possible in the classroom is the mediation of peer relationships, as peer social relationships are the main source of identity development during adolescence (El'konin, 1971). Peer relationships in the classroom have been regarded as a source of social comparison, relevant in the development of adolescents' self-concept and self-efficacy (e.g. Marsh, 1987; Marsh et.al.; 2008; Zeidner & Schleyer, 1999), and influential on the values and meanings that students give to education and engagement (Francis, Skelton, & Read, 2010; Jackson, 2006). In mathematics, a few recent studies have explored how belonging to different social-peer groups in the classroom can also play a role in the process of mathematical identity construction (e.g. Gholson & Martin, 2014; Lim, 2008a). For example, Gholson and Martin (2014) explored how two primary girls negotiated their positions in the mathematics classroom by using resources from their peer clusters as well. Belonging to the 'nuclear peer cluster' (the cluster that was regarded as popular in the classroom) increased students' access to mathematical resources (such as teacher attention), with other students being somehow positioned as more marginal. Esmonde and Langer-Osuna (2013) in their study with secondary students showed how, when students perceived themselves as less powerful in the mathematics practice, negotiated more powerful positions by using their social/peer capital. Although both of these studies provided support for the intersection between peer cultures and the mathematics cultures in the mathematics classrooom, and therefore between developing mathematical and peer-social identities, they did not explore how constructions of 'who we are' may have a particular impact on particular notions of 'who I am… in relation to mathematics'.

In summary, previous studies alert us to the conflicts and contradictions that girls and women face in being or becoming successful in mathematics. While studies on post-compulsory have suggested that these conflicts relate with their difficulty in finding a narrative of themselves as 'mathematically successful', studies in compulsory education have not consistently researched this issue. In addition, we hypothesize that these identity negotiations relate with girls peer relations, which are dominant in adolescence. Yet there is little research on the role of girls' peer groupings and their gendered dimensions within the classroom and how they mediate the 'becoming' of successful mathematical identities. Following this, this paper will systematically study how a group of successful girls construct mathematical identities in compulsory education, during early adolescence (13 – 14 years old). In particular, this study will explore how girls' relationship with their peers (and their 'peer cluster's identity') mediates these processes, by exploring how these relationships can be used to maintain or negotiate different positions of engagement and success.

7.3. Theoretical Model of Identity and Methodological Rationale

We follow a sociocultural perspective on identity(ies) which are defined as the dynamic process by which individuals recognize themselves, and are recognized by others, as a particular 'kind of person' (Gee, 2000). As a process, it is mediated by the use of symbols that are learnt in practice -and therefore are context specific- and which become internalized through social interaction (Holland & Lachicotte, 2007). In other words, students learn to identify themselves with different cultural models or types of persons, such as 'nerd', 'geek', 'freak', 'capable', or 'at risk' (Holland &

Skinner, 1987), which then become tools for further reflection on inner-speech (Holland & Lachicotte, 2007).

The role of others is central in this process, in that they become available 'figures', or 'kinds of person', which students can identify with. In addition, it is through social interaction with these others that identities are also negotiated inside the classroom, either by identifying or dis-identifying from them (Holland et.al., 1998). As a consequence of this negotiation process, students often develop a sense of belonging to different peer groups, which become an important resource for identity construction and also part of identity itself (Gholson & Martin, 2014; Lim, 2008a; Renold & Allan, 2006). For example, becoming a 'freak' or a 'geek' is an identity process that occurs not only in the use of symbols associated with these figures, but also in the process of participating with 'the freaks' and differentiating from 'the geeks' and vice versa.

In order to account for this complexity, this study uses a nested model of identity based on a case study approach (Yin, 2003). Such a model proposes that a case [i.e. a female mathematics student] can be nested in another case [i.e. a group of students inside a mathematical classroom], which can be also nested in a larger case [i.e. a mathematics class]. Through the use of a nested model of identity, this study will explore the process of identity construction of three particular girls that belonged to three different peer groups in one mathematical classroom.

This study conceptualizes identity as constructed and expressed by two main elements: narratives and classroom acts/actions in practices. While narratives emphasize the stories regarding 'who we are', 'how we came to be this way' and 'who we would like to become' (Gee, 2000; Sfard & Pusak, 2005; Hammack, 2008), acts/actions emphasize the actual embodiment of these narratives in practice (Holland et.al., 1998). Narratives are reflective self statements which crystallise notions of 'I am', 'I want to', 'I should' or 'I was' during mathematical practice (Sfard & Pusak, 2005), while acts point at behaviors, expressions that related with 'I do' or 'I engage in this way' as observed in practice. Both self-statements and acts emerge from a process of 'self-positioning' in relation to two main aspects of mathematical practice. Firstly -and based on the principle that an individual case is always nested in a peer cluster case- girls' sense of 'I am' and 'I do' will be understood in relation to notions of 'we are' and 'we do' [affiliation to a group] and 'they are' and 'they do' [distancing from other groups] (Lim, 2008a). Secondly, both narratives and acts will be considered always in relation to the mathematical practice in the classroom. In other words, this study attempts to capture how each of the girls, and their peer group, engages, complies with or resists the mathematical practice(s) in their classroom (e.g. Cobb, Gresalfi & Hodge, 2009; Solomon, 2008). It is our opinion that this model provides a dynamic and relational operationalization of identity, thus emphasizing a view of identity as multiple, flexible and not fixed. In order to see a more detailed description of this model of MI see figure 7.1.

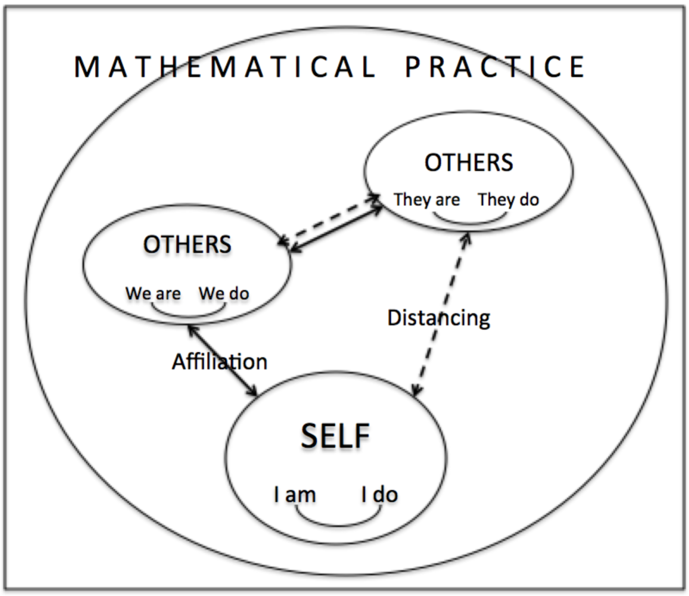


Figure 7.1: Process of Mathematical identification. It shows how one student's identities (both self-reflections/narratives and acts) are constructed in interactions with other peers and in a particular practice.

7.4. Methodology

7.4.1. Participants

This study was purposefully situated in a school with average attainment in the national evaluation (SIMCE), medium socioeconomic status (SES) and private subsidized administration, a school that could be regarded as a 'normal' school for a working class population in Santiago, Chile. The current Chilean policy allows this type of school to select their students and to charge a small tuition fee. As a consequence, these schools are highly homogeneous in terms of the socioeconomic status of their students (Mizala & Torche, 2012). In contrast, due to a policy of mixed ability teaching, these schools present at the same time a high variation in students' attainment inside the classroom (Ramirez, 2007). This school, in particular, was considered as a demanding school where, up to year 8, students were expected to reach a 70% mark in order to pass. Students that did not reach this score were held back a year.

This study is focused on a year 7 mathematics classroom with 39 students (16 girls). The teacher [Ms P] had two and half years of previous experience teaching maths. She came from a general teacher-training course (non-mathematics specific) in a non-traditional University, with relatively limited experience and specific preparation in mathematics. Despite her relative inexperience, Ms. P was considered by the Head of Department and her students to be a 'very good teacher'.

The three girls chosen for this study (pseudonyms Maria, Carla and Katia) were purposefully selected in order to represent girls that were developing a positive relationship with mathematics but belonged to different peer clusters inside the classroom (see Natural Peer Group Interview). All three girls were among the highest attainment students in the classroom and all of them reported that mathematics was their favourite subject in an initial survey. In addition, all of them were mentioned by their teacher and by some of their classmates as students with a good relationship with the subject (in the 'Mathematical peer groups interviews', see below).

7.4.2. Procedure

The case study classroom was visited regularly during the second semester of the academic year (between September and December). Observations were distributed during this term, with low inference information collected from four of the last Units of the year (see below). Interviews (Mathematics and Peer cluster Interviews) started 4 weeks after observations began. The teacher collaborated selecting students for initial interviews and information from these initial interviews was used to further select students for further interviews (until saturation). Finally, Identity interviews with each of the three girls, a formal interview with the teacher and group interviews about mathematical groupings with students were collected later.

7.4.3. Data Collection

Several instruments were designed, or adapted, in order to consider both social and psychological (subjective) perspectives in the analysis of students' identities (Cobb, Stephan, McClain & Gravemeijer, 2001). Four main instruments were used.

1. *Lesson Observations*: The main purpose of lesson observation was to describe the predominant mathematical practice in the classroom and also how students' participation was exercised and distributed. Field notes and audio recordings were collected during these observations. Low inference information (every public intervention by each student in whole class discussions) was also gathered from four of these lessons that focused on the topic of Data Management and Probabilities.

2. *Mathematical Group Interviews:* The main purpose of this instrument was to determine which groups, or 'kind of students', existed in the classroom, and to explore where and how each of the three girls was positioned in relation to these. All participants were asked to group students by answering the following question: *are there groups of students that show a similar relationship with mathematics?* They were asked to build as many groups as they could. It was explicitly stated to them that students in a group should be similar within the group and different from other groups. After this procedure, both all students and teacher were asked to elaborate on the criteria they used to group their classmates/students. The students that participated in this interview were sampled from the whole class to represent different levels of attainment. They were interviewed both individually (7 boys and 9 girls) and in groups (7 boys and 7 girls). In addition, the teacher was asked the same set of questions during an individual interview.

3. *Natural Peer Clusters Interview*: This interview was adapted from Cairns, Xie and Leung's (1998) in order to map the classroom's peer group structure in terms of naturally occurring -not academic- peer clusters. Students were purposely sampled in order to represent different peer groups. A total of 12 students were interviewed (5 boys and 7 girls) until data saturation was reached (i.e. no new groups were mentioned). Interviews lasted on average 10 minutes. Students were asked to group their classmates by answering the following question: *Are there people in this classroom who hang around together a lot?* Similarly to the previous interview, students' responses were further explored in order to capture meanings associated to each group.

4. *Mathematical Identity Interviews:* Individual interviews were employed to capture each girls' MIs. These interviews were an adaptation of two existing instruments, the Mathematical Life Story (Lewis, 2013) and the Identity Mapping

(Ylvisaker, Mcpherson, Kayes & Pellett, 2008). The Mathematical Life Story was used to engage each girl in building a narrative regarding her historical relationship with mathematics. They were asked to think about their maths story or how come they came to develop their current relationship with mathematics and how they projected this relationship to continue in the future. In order to develop this idea, a map was used to identify critical moments in which they saw their mathematical relationship changing. In addition, girls also were asked to identify in this map the highest (positive emotions) and lowest (negative emotions) points in their mathematical life [see figure 7.2]. The Identity Mapping was employed to describe the feelings and behaviors that each girl associated to herself when doing maths, as well as their perception of how others saw them while doing mathematics. At the end of this interview, they were asked to think of a name [a mathematical character or metaphor] that could portray their unique relationship with the mathematical activity [see figure 7.3]. Both interviews lasted about half an hour and during the entire interview the researcher encouraged girls to think of people that influenced their stories and current relationship. In other words, interviews had a focus on how others have shaped or were shaping these girls mathematical identities.

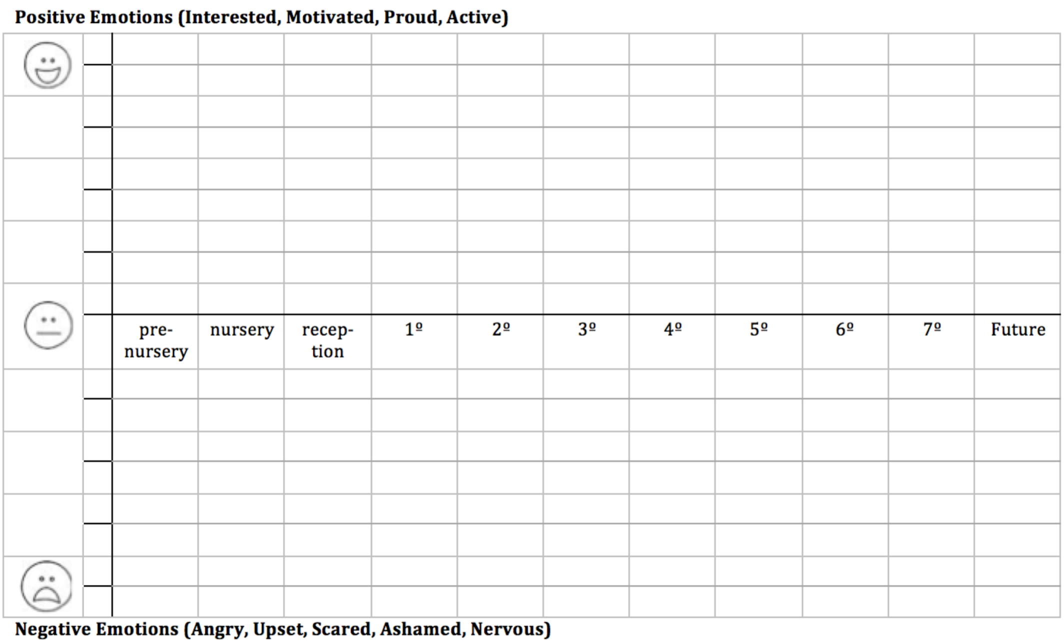


Figure 7.2: Mathematics life story. Students were asked to tell their mathematical stories and to mark highest and lowest point in this graph.

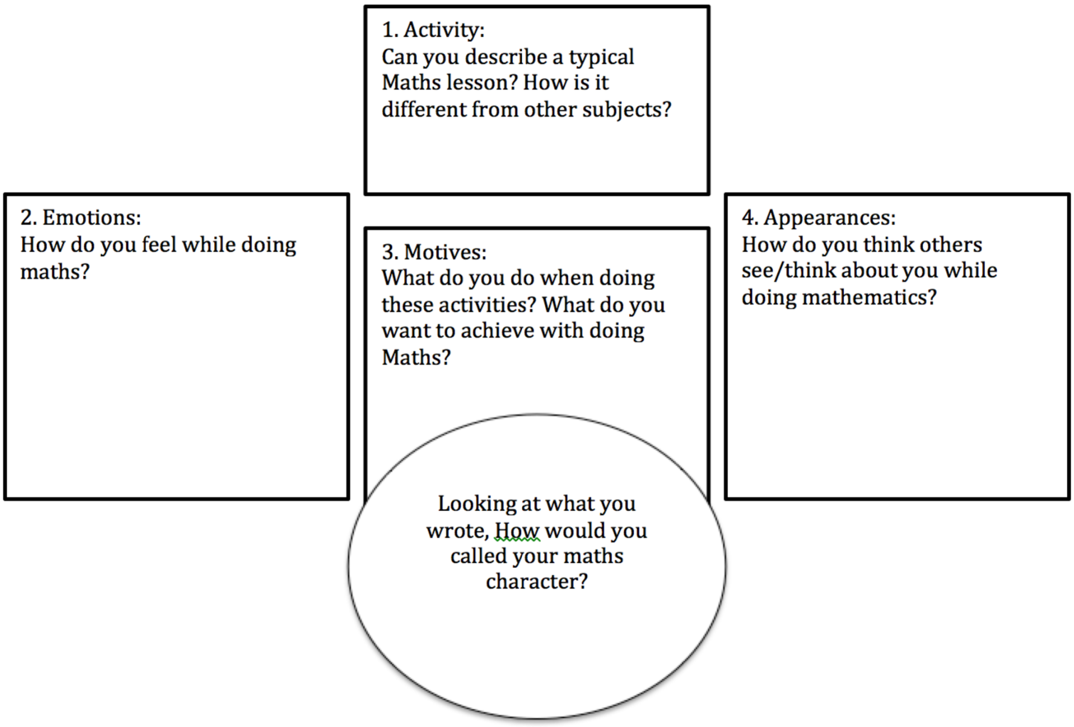


Figure 7.3: Mathematical Identity Mapping. Guiding questions inside the squares. Students were given instrument without promptings.

7.4.4. Data Analysis

7.4.4.1. Social perspective: mathematical practice

Using information from lesson observations and interviews, this analysis sought to answer how mathematics was described and practiced in this particular classroom. Lesson observations were coded in terms of their activities and tasks in order to describe lesson structures (Wells, 1993) and students' participation in classroom public discourse (any utterance that was somehow accepted or acknowledged by the teacher). For the latter, during lesson observations the lead researcher [DR] made notes on every public participation of every student in whole class discourse [e.g. what was the question/affirmation made and who made each of them]. A seating plan with students' names was used to mark each student's response to all of the teacher's questions. All spontaneous student interventions and signs of disruptive behavior were also marked on the seating plan. After observation this initial mapping was contrasted with audio-recordings, coding each interaction (usually a question from the teacher) regarding what response was expected (considering the response given and if it was accepted) (for similar approaches see Hiebert et al., 2003; Radovic & Preiss, 2010; Wells, 1999). These codes allowed us to identify open questions (often personal experiences, opinions and justifications) and closed questions (usually defining or naming a concept, analyzing an example or information given or applying a concept or procedure to an example), and how they were distributed between students in the classroom.

In relation to the analysis of the Students' Mathematical Interview, mathematical groups were coded thematically, identifying criteria to differentiate groups as well as definitions of competence in place. This coding procedure was highly driven by the data, following an iterative method and constant comparison analysis suggested by Strauss & Corbin (1998).

7.4.4.2. Intermediate level: map of the peer culture.

Different peer group clusters were identified using the Composite Social Maps software (SCM 4.0). By including information collected by the Natural Peer Groups interviews (each group reported by each student interviewed), the software computes a co-occurrence matrix and then identifies groups in terms of similarities between co-occurrences of individuals using a correlation matrix (Cairns, Gariepy, Kinderman & Leung, 1996). After groups were identified, a new matrix was created contrasting descriptions that the interviewees gave about their own and other groups. With this information, a general account for each peer cluster was constructed.

7.4.4.3. Subjective perspective: girls' identities analysis.

In order to analyze information regarding girls' mathematical identities, different data sources were organized into a single case document (one document per girl). This document included: (a) description of their individual participation in whole classroom discourse (lessons observations); (b) their own definitions of competence (Mathematical Groups Interviews); (c) their own positioning regarding these definitions (in which group they classified themselves); (d) their definitions of membership to their peer groups and how they distanced themselves from other groups (Natural peer clusters Interview); and e) their interviews regarding their mathematical stories and relationship with mathematics in the present (Mathematical Identities Interviews). In addition, information from other students and the teacher [quotes where each girl was mentioned] were included in the document, as a way of gaining access to how they were positioned by others in classroom practices. All of this data was transcribed in its original language (Spanish).

The analysis of each case document was firstly done individually and only later were comparisons between the three girls explored. In the analysis of each case, a comparison between different sources of data was central for identifying potential tensions and conflicts. A tension was regarded as an opposition between different aspects of mathematical identities (e.g. self-positioning v/s others positioning; narrative v/s practiced; past v/s present v/s future) and conflicts were identified when these tensions were associated with negative affect.

7.5. Results

As mentioned before, this paper aimed to contribute to the literature on mathematical identities by exploring **in detail the singularity of experiences of three girls** that were seen, and saw themselves, **as successful (high attainment) and engaged in their mathematical compulsory classroom**. We will present the main results of this study in the following way. Firstly, we offer a general account of the predominant mathematical practice in the classroom. Secondly, we describe in detail the process of MI construction for the three girls that we have selected to focus on.

7.5.1. Mathematical Practice

7.5.1.1. Spaces of participation

The analysis of lessons observations and students' interviews suggested a general traditional pedagogic practice, where curriculum, contents and interactions were highly controlled by the teacher. In relation to the pace of the lessons, it was observed that new contents were introduced gradually, with large amount of time being dedicated to rehearsal of previous contents. The teacher commented that she followed this gradual approach because she was concerned with making maths available for all of the students in the classroom, especially those that found it particularly difficult.

In terms of patterns of interaction, most whole class teaching was organized following the IRF pattern (Teacher 'Initiate' the interchange with a question, students 'Respond' and teacher provides some form of 'Feedback'). Most teacher elicitations (63%) were identified as closed questions (students were expected to find a correct answer), with more open questions (spaces for more participation) being saved for students' personal experiences and critical analysis of information (about

22%). In addition, a large variability was found between students' participation; while most students participated on only a few occasions (25 students participated only once or less per lesson), others were considerably more active (e.g. one of the focus girls, Carla, averaged close to 7 questions per lesson) (see figure 7.4).

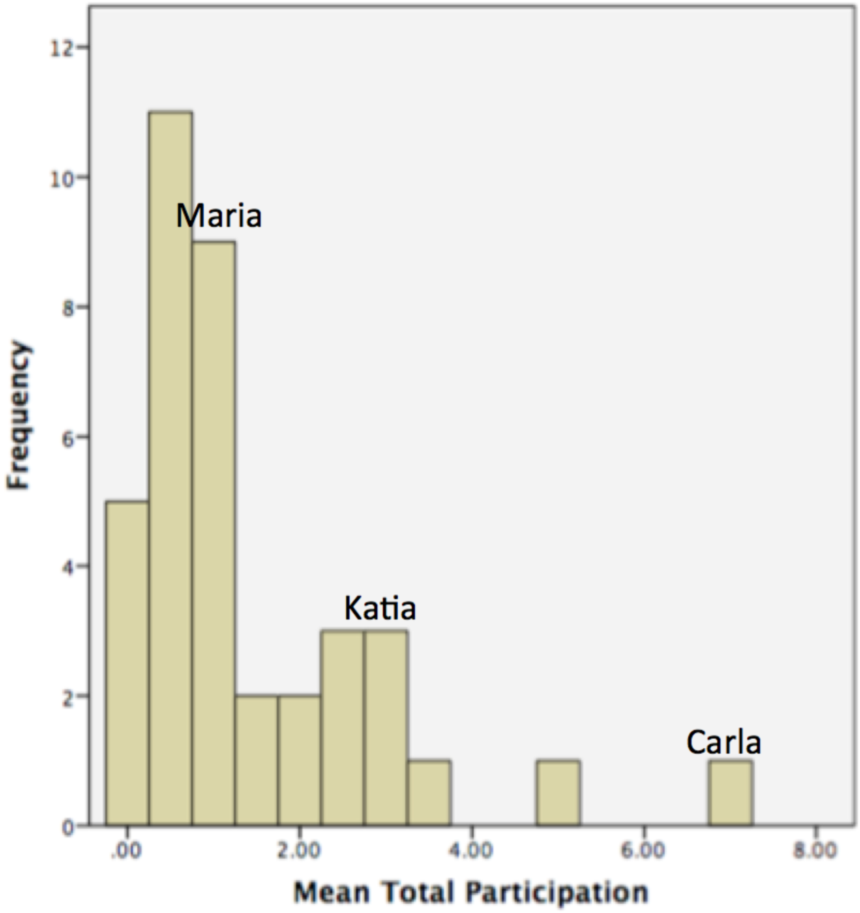


Figure 7.4: Histogram of mean participation per lesson. N= 38 students. Mean= 1.3; SD= 1.5 questions per lesson per student.

Data obtained from lesson observation regarding mathematics pedagogic practice in this classroom was consistent with information collected from students' interviews:

> First the teacher explains the purpose and what we are doing [the 'menu'] and she asks us to read it out loud. Then she explains if it is a new topic, she says how to do it. After she asks us to copy the problem and then, when we finish, always Carla, Christian, Andres and sometimes Daniela, raise their hands (...) Sometimes K does, but it is always because she gets asked and not because she wants to. After doing all the exercise, we finish and the teacher gives us stamps that we can later exchange for points for the test and she

> makes us revise what we did in the whiteboard. And all the lessons are the same...

7.5.1.2. Different kind of students

In the context of the mathematical group interviews, three different 'kinds of students' were identified: The 'Effortless' group, the 'Effortful' group and the 'Lazy' group. One student described these groups as following:

> The first group is the smart one. This group gets high grades even if they don't study, even if they don't come to class ['Effortless']. This one comes second, comes after the smart ones ['Effortful']. While the first one always gets good grades, sometimes this one does not do as good… but a little bad. In this group they have to study more and if they don't come, they get worse grades. And this last group is the one that always do bad, even in sports… they are the lazy ones ['Lazy'].

The description of the 'type of student' that belonged to the 'Effortless group' resembled the shared cultural model that being good at mathematics requires 'natural ability' (e.g Walkerdine, 1998). This group was described by the students as just 'good at it' in that they did not have to put in effort in order to achieve. In fact, the student that was perceived to be the best student in the class [Christian], was mentioned by all of his classmates as a member of the effortless group and was usually sleeping in his desk in the last row (as observed and described by his classmates). In addition to Christian, three girls and one boy were consistently mentioned as part of this group.

In contrast with the 'Effortless group', the 'Effortful group' was characterized as having a positive relationship with mathematics, but needed to 'put some effort in' in order to be successful. Students described the individuals that belonged to this group as frequently participating in class and showing interest. The Effortful group was predominantly described as composed by girls, with only a couple of boys mentioned as part of this group. It is possible to conclude from these observations that the effortless position was regarded as an imperative for high attainment boys in this classroom, but not for girls, who could position themselves as both effortless and also as effortful when being high attainers.

Finally, the Lazy group was consistently described as a group of low attaining boys who exhibited a 'bad relationship' with mathematics. This group was only comprised of boys, with 5 being mentioned in a relatively consistent way. It was perceived by almost all of the interviewees as a low achieving effortless group (13/16). Only three girls (that were friends with most of these boys - part of 'adolescent group' see below) said that their attainment was not related with lack of effort but to an actual difficulty in understanding and staying focused during lessons. In contrast, most of the interviewees (including some of the students from this group) regarded this group's lack of effort as laziness (9/13) and lack of interest (3/13).

In addition to this gendered dimension which seemed to mediate the definition of different kinds of 'mathematical people' in the classroom, there was a relationship between these mathematical groups and how students grouped in different peer clusters. Particularly, the peer groups to which the three focal girls belonged to ('Adolescent group', 'Normal group', and 'Korean group', see below) were comprised of contrasting 'mathematical people'. First, the 'Adolescent group' contained most of the boys that were defined as 'lazy' by their peers (4 out of 5) and all of the girls regarded as effortless (3 out of 3). The 'Korean' and the 'Normal' groups were mixed in terms of attainment and participation. High attaining girls from these groups (i.e. Carla, Daniela, one of Carla's friends, and Katia) were considered Effortful by their peers (see figure 7.5).

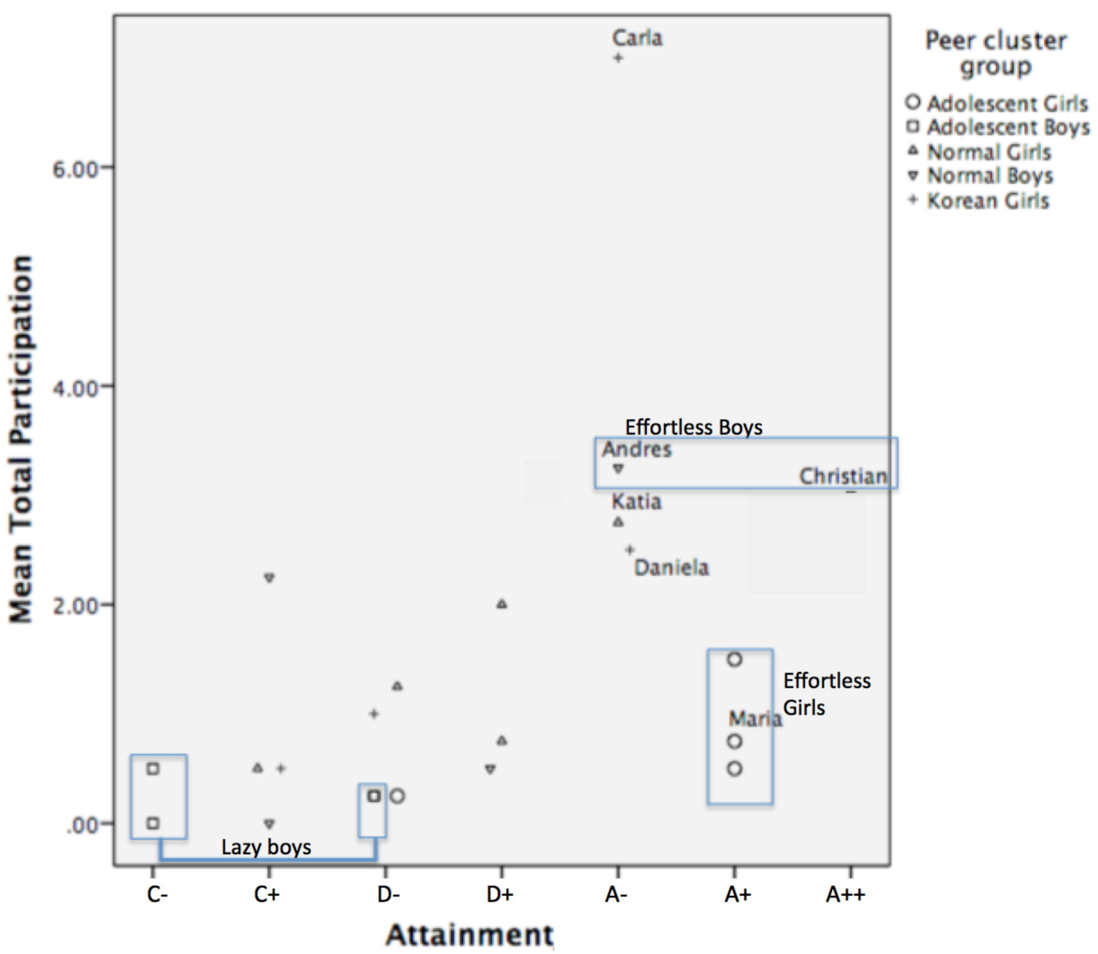


Figure 7.5: Students participation in whole classroom discourse (as measured by observations) and attainment (as measured by grades). Only focal students (Maria, Carla and Katia) and their peer clusters are included (n=23). Students that were consistently named as 'Effortless high attainment' and 'Effortless/Lazy low attainment' were also marked. Figure shows how Effortless girls paired up with Lazy boys (in the "Adolescent Group"), and how the Normal group and the Korean Group were diverse in terms of attainment and participation.

7.5.2. Girls Mathematical Identities in the Classroom

7.5.2.1. Maria: balancing mathematics and 'the normal life'.

According to data from the Natural Peer Interview, Maria belonged to a peer group that was comprised of 4 girls and 4 boys. Her classmates described this group as the 'Adolescent' group, for '*they acted as if they were older, thought they were the coolest, showed off about what they owned* (clothes/technology, i.e. making the link with higher economical capital), *and appeared not to be especially concerned about school*'. Some classmates mentioned that girls in this group were too preoccupied with their appearance; wearing skirts that were too short, using makeup, and always hugging and kissing each other. Some of them, they said, even had boyfriends. The boys in the group were described by all of their classmates as exhibiting 'low attainment', with peers from other peer clusters attributing this low attainment with laziness, but with the girls of their group attributing it to difficulties or low ability. This low attainment was persistent in these boys' mathematical stories, with three of them held-back a year (one of them held back two years) (see figure 7.5). During interviews, students from this adolescent peer group (both boys and girls) described themselves as 'the most mature group in the class'. They commented that other groups (especially the *'Normal'* group, see below) played like children, ran about during break times and were loud and euphoric. In contrast, girls in the Adolescent group said they preferred to hang out with boys and talk about different interests: e.g. Justin Bieber.

Data from classroom observations showed that Maria (and her female-friends, see figure 7.5) rarely participated during whole class public discourse. She was never observed offering an answer or raising hands, so her participation was limited to direct questions from the teacher. In addition, she was never asked to offer a personal opinion or to answer personal questions, and she never engaged in a prolonged exchange with the teacher. When observed doing peer-individual work with her group of friends, Maria presented a pattern of 'doing' mathematics characterized as working together as fast as they could in order to finish the mathematical activity and then engage in off-task conversations.

Maria's observed behavior during mathematical activity was highly consistent with the narratives generated by her classmates during the Mathematical Group Interviews. She was mentioned by 13/16 students as part of the Effortless group, where 'mathematics comes easy', a group considered to have the best relationship with mathematics. In this sense she was frequently mentioned as an example, a position that she assumed with no conflict:

> I am in the first group and the three of us are in this group [she mentions herself and two of her friends], because I think we are the ones that understand the most… Ah, and Christian is also in this group. They are the group that knows. I can ask them because they know. I think we…, I mean myself…, I don't study, I just pay attention during lessons and it gets recorded in my head, but I don't study.

The absence of conflict in Maria's account is also clear in how she described her emotions when doing mathematics. She did not describe strong positive feelings, despite naming mathematics as her favorite subject. On the contrary she mentioned emotional states related to the absence of discomfort, using words like 'comfortable', 'tranquil' and 'placid':

> I feel comfortable because I understand, because if I didn't understand I wouldn't probably feel comfortable.

When constructing her Mathematics' Life Story Maria states that this sense of tranquility had accompanied her all through her school life. However she comments that during the last few months these feelings have slightly changed and she has felt "less positive about it… or less worried about it". She links these feelings with the fact that, to her, mathematics (and school) has lost significance in her life, for new activities have acquired more relevance:

> What is happening is that I don't like to keep only with school, because when I was younger I cared only about school. But now I would like to do sports, or participate on a workshop… I mean, now I'm also going out with people [dating], so things are changing. School is still important, but I don't know if it is adolescence or what, but it is like you start to feel lazy.

It is this tension between different activities (socialising versus mathematics) that appears to shape Maria's definition of her mathematical character in the Identity Mapping. When discussing which name she would use to describe her mathematical personality, she chose to be called 'efficient':

> Maria: that's why I try to be quick doing the exercises, because if I do that I can think in maths for a moment and then I can return to the "normal life"... (Both laugh) (...) that's what I like to do... to do my work and then relax...
>
> DR: now the idea is to look at all the things you wrote and think about the things we talked and try to put a name to this character, what would you say defines you when doing maths...
>
> Maria: Efficient?... I don't know if I'm always efficient, but efficient is what... I'm gonna write efficient...

Consistent with their way of participating, Maria and her friends seemed to be working on managing the tension between mathematics and 'the normal life' (socialising) by balancing their different obligations: working on mathematics 'efficiently' and then returning to do their social activities. It is interesting to note that these obligations appeared to be somehow gendered, since she felt she had to choose activities that allowed her to be feminine/beautiful:

> DR: And you're also doing more things, don't you?
>
> Maria: yes, I'm the president of my class and I did swimming, but I left it because so much swimming was making me look bigger… so it is like choosing things to focus and leave things you don't want to deal with.

The links between Maria's mathematical positioning as an effortless high-attainment girl and her belonging to a particular social group in the classroom can be seen in two different ways. First, she defines her positioning as effortless in alignment with how she positions her girlfriends. Being an 'effortless high achiever' is possible because she shares this identity with her girlfriends, as noted by the use of 'we' in her description of the 'effortless group' above, and her description of how they share activities during mathematics lessons (to work and to chat). In addition, it can be hypothesized that Maria's identity is also resourced by her relationship with the boys that belong to her social group ['the adolescents'] in two different ways. Firstly, she presents her negotiation of effort as part of growing up and being 'mature'. Given the boys in this group were older than the rest of the class (three of them had been held-back for a year or two), their presence in the group supported the girls' notions of being mature which involved being able to balance their school life with other social obligations. Secondly, Maria positioned the boys in her group as ''having difficulties' (as opposed to lazy) which also played a part in allowing her to position herself as effortless i.e. having natural ability without needing to try. Therefore, it can be argued here that the 'natural ability as male' cultural model does not apply in Maria's case: she can identify with an effortless identity without perceiving it in conflict with her female identity.

Finally, it is interesting to note that the metaphor of 'balancing' is also present in Maria's account of the future, and how she foresees her relationship with mathematics. Here, for the first time, the tension that she is currently living appears to create conflicts, as she thinks she will need to manage her expectations in order to accomplish this 'balancing'. She says she would like to study something in relation to mathematics (as it is not a possibility to study something related with language), but that recently she has begun to lower her expectation in order to balance mathematics and life. From the aspiration of studying engineering, she has moved to administration:

> Maria: Engineering is like... everything is mathematics, so I don't want to start something for two semesters and then not being able to pass anything… I don't want to get stuck
>
> DR: Why do you think it is going to get this difficult? Have you ever felt this kind of difficulty in mathematics?
>
> Maria: No, never, because I have always had good grades, so for now I am relaxed, but I don't know if it's gonna be like this forever and I don't want to end up in a bad university and I also don't want to end up only doing mathematics all the time, because I like it but not that much, so administration is a balance between life and mathematics
>
> DR: And how do you see life in the future?
>
> Maria: I wouldn't like to leave home too old… it is not like I'm ultra hurried, but I wouldn't like to leave home too old, so my idea is to finish university, have a stable job and then leave… or maybe leave before, but go outside… into the real world, start my life… in general it is like living alone. I mean, I don't expect to marry too young, no grrrr, maybe marrying, yes, but not young, and not having kids young… it is like having everything that normal people have…
>
> DR: What you mean normal people? Does this happen with the people you know?
>
> M: yes and no… I know people that do this, but for example my mum has always been dependent and I don't want to be like this, I don't want to depend on nobody.

As the previous quote suggests, there is a gender dimension in how Maria envisions her future: in comparison to her mother, she would like to challenge the traditional role of a dependent-woman-wife by being a young-independent-person. This independency would require a career, but this career would have to allow her to do this 'independency role' before doing the 'wife and mother role', it would have to be fast (and therefore, not in a demanding engineering career).

In sum, Maria seems to be developing a mathematical identity with a high consistency between the way she acts, how her classmates position her in the classroom and how she perceives herself as well. All of these different sources of identification resource an emerging mathematical identity based on the cultural model of 'mathematics as a natural ability'. The resources provided by her classroom mathematical practice (which does not challenge her) and her relationship with her friends (aligning with her female friends and positioning her effortless engagement as consistent with her group 'maturity') seems to allow Maria to experience this as a positive mathematical identity that allows her to feel comfortable when doing maths. The main tension to manage in maintaining this identity seems to be related with balancing mathematics and what she calls 'real life'. While in the present, this real life involves being social (e.g. partying, dating, hanging out with friends), in the future it seems to be related to becoming independent (e.g. living alone before marrying and having kids). While her balancing of maths and real life is not experienced as conflicted in the present, the future is anticipated by her as rather challenging (as she will have to manage her expectations in order to be able to cope). The potential conflict that she anticipates appears to suggest some form of contradiction between pursuing a demanding career in mathematics and at the same time conforming to the gender role of being a wife and a mother.

7.5.2.2. Carla: The math that I am taught and the math that I like.

Data from the Natural Peer groups interview revealed that Carla belonged to a group comprised of 4 girls, with no boys from the classroom. All of their classmates identified this group as the '*Korean group*', since they liked Korean and 'Kitch-Pop' music. They were the only group in the classroom which liked this type of music, were described as 'weird' (one of the 'normal girls' even called them 'the weirdoes') and some of their classmates said that sometimes they got bullied because of this interest. However, one of them said they did not care if they were bullied: "they don't mind – they feel strongly about liking the Koreans". The girls themselves did not talk much about this interest; for them the important thing about their group was that they were 'inseparable' and that they did lots of different things together. They said they cared about each other. One of their classmates said that if someone said something to one of these girls, the others would defend them, especially Carla who was pictured as the leader of the group.

The analysis of classroom observations showed that Carla was the student that participated more actively during whole class discussions (see figure 7.5). Every time there was an episode of multiple questions/answers between teacher and students, she contributed with some answers. This high level of participation resulted in that she answered a wide range of different type of questions, from simple remembering questions (e.g. 'what did we do the previous class?') to open questions that required her to think on her personal experiences or opinions regarding the topic (e.g. 'considering the sample size, do you think it is possible to reach a conclusion regarding the population?'). In addition, Carla was also one of the few students that made spontaneous contributions to the public discourse. On a couple of occasions she made comments on what the teacher was saying, extended what the teacher was saying or answered a question with an unexpected answer. Following this, Carla's participation was rather different from that of her classmates and from what her teacher was sometimes expecting. This led to two particular features of some of Carla's engagement in classroom interaction. First, sometimes it was difficult for the teacher to actually respond to these forms of participation in her usual ways. For example, in the following extract the teacher was introducing the topic of surveys. She had asked a couple of students where they had seen surveys and they had responded with examples such as the census, the newspapers, on the internet, and so on. Carla's contribution to this discussion was to give an unexpected answer, and it appears the teacher does not know how to respond.

> Ms.P: who knows where else can we find surveys, Carla?
>
> Carla: I know some other place where several different surveys are done during the day…
>
> Ms.P: ah?
>
> Carla: In school… all day…
>
> Ms.P: Why?

> Carla: Because now you are asking us questions and in other subjects we are always answering questions and it is constantly questions from the teachers to the students…
>
> Ms.P: Ok, so you say in some way this could be regarded as a questionnaire or survey… now the difference is maybe that there are surveys where I can give you an option, possible answers, or open questions, where you can answer whatever you like…

A second feature was that in some of her interventions Carla actually risked making 'incorrect' answers (which was uncommon in these discussions).

This way of behaving was more difficult to categorize by her peers as a particular 'kind of student' as observed in the 'Mathematical groups interviews'. Although Carla was mentioned by some (4 classmates) and her teacher as having a positive relationship with the subject, her 'belonging' to this group was not clearly associated with ability, but more with interest and participation. As participation was linked by the students in the classroom with effort (see above about in 'different kind of students'), it was difficult for her peers to recognize Carla as a 'good mathematician'.

> For example, Carla supposedly knows a lot, but then in tests she does not do as good or sometimes she makes silly mistakes. Maybe she gets nervous or she does not care making these mistakes ('effortless-high-achiever' girl, individual interview).

Carla's self- positioning in relation to this public identity was also rather positive and somehow unconflicted: Carla perceived that her relationship with mathematics went beyond 'being right' or, as seen in the quote below, to have 'one main technique and just apply it'. Carla says:

> I feel good when doing mathematics, not like the majority who feel neutral about it. We see mathematics as wider, not like the majority who have one main procedure and just apply it. I have more technics and I reflect more when doing mathematics. Mathematics has confusion that I like… it is more like understanding it than applying a procedure.

It seems that Carla did not relate her participation and behavior to effort, but with engagement in an activity that goes beyond getting the correct answer. This perception shaped how she told her story in terms of her mathematical relationship in the 'Mathematical life story'. She described an increasing positive relationship with mathematics, saying that at the beginning it was too easy but then it became increasingly complex. Her more negative moment in life was described as a problem in how mathematics was taught to her by the teacher during year three and four:

> The teacher was too square; she did not give us the opportunity to open up to mathematics. She used to give as a technique and we had to apply it. But then in year five I felt that my mind opened up… I had to remember contents from previous years and to think on contents that were going to come in the future, so I felt like my mind was being opened by mathematics.

In the present she is not experiencing this problem. Although she says she resists traditional teaching practices, which we observed in Ms. P's general pedagogy, she insists Ms P's lessons are spaces for developing ideas and engaging with mathematical reflection and understanding:

> And this year I am with this teacher, and if I give her an idea, she says yes, but you have this error, and she helps me to develop my own strategies with mathematics.

In this sense it could be argued that Carla's positioning in relation to the mathematical practice in her classroom is that she sees spaces that her teacher offers to develop the kind of mathematics she likes and she takes them. This positioning is somehow perceived by Carla as being different: most students feel *'neutral'* about mathematics and *'have one main technique and just apply it'*. In contrast, she *'reflects more'*. In this sense, she considers that this kind of engagement is in part the student's responsibility: you can either comply with a repetitive practice (which she links with some sort of disengagement) or use the spaces provided (as consequently observed in her forms of participation).

How is this agency negotiated in her relationship with her peers and her belonging to a particular social cluster? Carla does not talk about her friends as similar to her in the mathematics classroom, but rather her constructed peer identity of being different or being weird seemed to support her unusual classroom participation and further mathematical identity. As mentioned before, Carla's participation was rather different from that of her classmates and from what her teacher was sometimes expecting. Although she did not comment on how her classmates took on her behavior, one of her friends (Daniela, another high attainment girl of the Korean group) did. This took place in a conversation about how students tended to bully others when talking or making mistakes in public which was associated with which group were you in:

> Sometimes it makes you feel anxious when the teacher asks you something because some of the guys bully you and that makes you feel stressed. For example, if this happens to one of them, they don't say anything. For example, if MV [low achieving boy from the adolescent group] says something silly, you have to stay quiet because if you didn't all the class shouts 'forever alone', even if you shouted with your four friends, they still shout you 'forever alone' (…) If we don't understand something, we just ask as many times as we need. Sometimes we get bullied, but we don't care.

The links that Daniela makes between particular forms of participation (high participation and arguably, spontaneous contributions of clarification, which are consistent with form of participation that were observed in Carla) and her peer group are clear: she is talking about her group being made fun of other students (as noticed by the affirmation 'even if you shouted with your *four* friends' and by the use of the preposition *we* when describing this particular form of participation - 'we ask as many times as we need'). It can be hypothesized that the same mechanism reported by Daniela is providing Carla with support to her 'Mathematical identity as different'. As described before, Carla perceives her social group as different or crazy, which is consistent with her positioning against most of the rest of the class when talking about her mathematical identity. Following this, it can also be argued that in this case belonging to a particular group acts as resource for sustaining a particular mathematical identity.

As with Maria, therefore, it can be argued that Carla is also developing mathematical identifications that show high levels of consistency between her acts and narratives. While she values a 'wider mathematics' (beyond rote techniques), she engages and risks forms of participation that challenge the common practice (as she is observed as different from most of her classmates). This allows her to perceive the mathematical practice as consistent with the mathematics that she likes, her classmates as dis-aligned with this practice, and therefore an emerging mathematical identity based on the cultural model of 'mathematics as special or different'. At least two tensions are observed in managing these identifications. Firstly, there is a tension between the opportunities provided by the teacher (most activities and interventions seem to support the 'mathematics as rote') and the kind of opportunities she wants to take. However, it seems that she does not perceive this tension because she is able to participate (and maybe monopolize) encounters that support 'the mathematics that she likes'. Secondly, the unclear positioning of Carla as a good maths person' by her classmates (as they don't identify her as effortless and therefore she does not fit in the predominant cultural model of 'maths as natural ability') does not support her secure identification that she shows and narrates. This dis-alignment can be seen in how she and her friends get bullied (as perceived by her friend) and how her peers discount her relatively good attainment. However again Carla seems not particularly conflicted with this tension at least in the present as she does not recognize difficulties or effort in her engagement. Following these two tensions it can be argued that maintaining and managing positive and good identifications with mathematics for Carla requires quite a lot of confidence. This confidence seems to be again resourced by her belonging to a particular social cluster, which allows developing an identity as different, both in mathematics and the peer group/social relations world.

7.5.2.3. Katia: The complicated math and the simple math.

The analysis of the Natural peer group interviews showed that Katia belonged to a big mixed-gender group that was comprised of 6 girls and 4 boys. Other students in the class described the girls in this group as 'the quiet ones', since they were never told off by teachers. Girls from the Adolescent group also commented that these girls were childish, contrasting them with their own social self-identity as 'mature'. One girl and one boy within this group named themselves as *'the Normal Group'* (hence the adopted name) and, in distancing themselves from the *Adolescent Group*, they said that they enjoyed childhood as appropriate to their age, not wanting to grow up too fast. As one of them said "we live what we are suppose to live". In terms of their relationship with mathematics, the students that comprised this group were highly diverse in terms of grades, but none of them was regarded as lower attainers or lazy. In contrast to the girls that were described as 'quiet', the boys in this group were seen as disruptive and noisy (specially by Katia, see below).

In terms of frequency of participation, as revealed by the analysis of lesson observations, Katia was located somewhere between the high participation of Carla and the relatively peripheral positioning of Maria (see figure 7.5). In contrast with Maria and her friends, she usually raised her hand, offering answers to the teacher's questions, but as distinct from Carla, she was never seen making spontaneous contributions during whole class discourse. She often responded to questions that required an application or analysis of previous given information (e.g. how many students voted for a girl?) and also to questions that required an opinion or justification (e.g. what information do you think is missing or could improve this report?). However, with the latter, her answers were much more related to what the teacher was expecting or what had previously been said, an approach that we interpreted as compliance with the teacher's pedagogy.

Another feature of Katia's -and her friends- participation was observed during peer-individual work, and contrasted more strongly with Maria's detachment. Katia worked with her desk partner, but was also very active in looking for more help. She was frequently observed raising her hand, calling the teacher to check on her answers or asking other friends. Differently from Maria and her group, Katia usually moved around the classroom in this search for support. One occasion was very characteristic of this behavior. The whole group was trying to find the volume of a figure. Katia and her partner apparently got stuck and started calling the teacher. The teacher did not notice them, so they both stood up and went to the whiteboard, where the problem was written, and started trying to solve it there. Eventually the teacher came and helped them solve the problem, confirming their approach and checking on their final answer.

In the Mathematical groups interviews, similarly to Carla, Katia's good relationship with mathematics was noted by others (4 classmates mentioned her as having a good relationship with mathematics and the teacher) as not related to her ability, but to her interest and participation. Like Carla, Katia was seen as having to put in effort in order to achieve.

Although it could be argued that Carla and Katia were offered the same position in terms of how their peers and teacher saw them in the classroom, their response to this positioning was rather different. While Carla identified herself as someone who does not have to put effort in, but as someone that tries to push the boundaries of the mathematics that is presented in the classroom, Katia presented a much more conflicted self-position. In the 'Mathematical group interview' she seemed unsure about where to position herself:

> I'm in this group, but it is not like we are fast, because for example, Christian, he is like the smartest in the class… in general, he is good at everything… it is like in the personality of each of us. For example, Andres [one of the boys from her peer group], I see he has been continuously talking and I ask him if he did his work and he has it all done and he is even doing the second one and we haven't really finished the first one…

As observed in the previous quote, Katia found it difficult to see herself as part of the 'smart' group, the group that performs without effort. She is not fast and she does not have an intrinsic ability, a trait that according to her only a few (interestingly only males) have. In contrast she saw herself as responsible:

> I think I am responsible, and I think it is related with maturity because you need to be conscious of what is going to happen, you need to be responsible and do what you have to do (...) We [herself and one of her girlfriends] work together and if we have to do a list of exercises, we will be consequent with having to do them and we will finish them before starting to talk.

In order to explain this difference between herself and her immature able male classmates (i.e. Christian and Andres), Katia uses gendered oppositions, which allow her to give value to her approach:

> Women are more professional, like more focused. Boys are not mature enough. It is like they are not aware of causes and consequences. It is like they didn't know that if they don't do it they would have a low grade. But women it is like 'no… if I don't do it I would have a bad grade so I am going to do it and I'm gonna be consequent'.

However, this view also appeared to conflict with the mathematical practice she experienced. Katia felt that mathematics was often presented as difficult and complicated but, in reality, was simple and straightforward. For Katia, learning mathematics involved applying a method taught by the teacher and then waiting for the teacher to offer a new method if the first one was inadequate or too challenging.

> I would like the teacher to teach us techniques that weren't difficult so we could understand them in the moment and then we could move quickly, like the magic formula, easy procedures to understand (…) But in mathematics the teacher is like too serious and she explains everything as it is very complicated…

This self-positioning as responsible and effortful is clearly framed in relationship with her peer group. On the one hand, she shares a dutiful and compliant approach (in that they '*do what they have to do*') to mathematics with her girlfriends (particularly her closest friend, who is also a relatively high attainment girl in the classroom), and on the other hand she contrasts herself with the approach adopted by boys. Although she does this as a generalization (boys v/s girls) she uses one of the boys in her group (Andres) as an example. To her, Andres represents the idea that you don't need to be responsible in order to achieve. In addition, she sees responsibility in terms of helping those friends that (as boys) were not responsible and did not have the ability required to be effortless:

> I think we are 4 girls and we will have to be with the boys and we will have to help the boys to focus, we will have to be responsible, because I want to get a good grade…

In sum, Katia's positive relationship with mathematics is populated with tensions that are leading to conflicted identifications with cultural models available in the classroom. Although Katia is trying to maintain this positive relationship that she declares, she somehow fails because she does not fit with the available cultural model of 'mathematics as a natural ability' (and therefore - effortless). In other words, as she state herself, 'she does not have it in her personality'. In contrast with Carla's secure and confident positioning as 'different' (and maybe independent), she draws on the cultural model of 'responsible': someone dutiful, who does what is expected from others and looks for support or confirmation (specially from the teacher). This positioning was supported by her relationships within her peer cluster - by her alignment with her female friends and with women in general as responsible for others and by her opposition to the 'immature - irresponsible' male-friends in her peer cluster. Furthermore, being responsible was not only about herself, but also about taking responsibility for the poorly behaved boys.

It is possible that by constructing a view of mathematics as something simple Katia is trying to gain some sense of control over the activity, thus also enhancing her sense of efficacy. She can be an 'effortless good mathematician', but only when maths is stripped away from its unnecessary complexity. However, because the mathematics offered by the teacher is not straightforward, her expectation is not met and she finds herself somewhere in between a 'good at maths student' (who has it in him/herself and therefore does not need support) and a 'good compliant student' (who needs support and confirmation), figures that apparently do not fit together in this classroom.

7.6. Conclusion and Discussion

The main purpose of this paper was to systematically study three girls in compulsory education who presented a 'positive' relationship with mathematical, thus, describing the process by which their MIs were constructed through engagement in a particular classroom practice and in relationship with others. More specifically, this study was interested in exploring girls' self-reflections as well as their ways of acting inside the classroom and how these two were influenced by their relationship with peers and peer groups. By doing this, this paper addresses three relevant gaps in the existing literature on mathematical identities: a focus on girls' engagement and positive identifications with maths in compulsory rather than post compulsory mathematics, an interest in their developing narratives about themselves in relation to mathematics (a methodology also neglected in compulsory education) and an exploration on how this identification process is negotiated in relationship with peers and peer groups which appear to be so significant in early adolescence.

In general, the findings of this study confirm that, among girls that share a positive relationship with mathematical activity (high level of attainment and engagement with the practice), there is an important variability in how their MIs are constructed. Even though all three girls recognized some level of tension in managing their positive identification with maths, the nature of these tensions, and the level of conflict attached to them, was very different in each case. Maria and Carla experienced positive emotions when doing maths, confidence and consistency between the way they acted and the way they talked about mathematics. In contrast, Katia seemed emotionally more conflicted, trying to understand what was expected from her and how to deal with the unfair situation of being 'effortful' but not successful. In addition the three girls valued different forms of doing mathematics [independent-collaborative mathematics; wider-complex mathematics; straightforward-procedure oriented mathematics] and showed different forms of engagement while doing mathematics [detachment or engagement on the margins of practice; protagonist and challenging of the teacher's practice; compliant and support seeking] and positioned themselves differently when defining their identities as mathematicians [effortless/efficient; different; responsible].

A first conclusion that can be extracted from these three cases is that peer relations had a central role in the construction of each girl's MIs. Their participation in classroom activities was congruent with how other girls from the same group and their peer group identity was used as a resource for supporting their mathematics identities (and vice versa). These results provide further support to the argument that MIs are intertwined with the students' peer culture (Gholson & Martin, 2014). This is also consistent with the theoretical models that consider peer relations as a leading activity in adolescents' development (El'konin, 1971; Karpov, 2003). According to El'konin (1971) in particular, peer group relations pave the adolescent's trajectories to adult life. While for some girls these relationships will be rather gendered (e.g. figured as sexual relations or as caring/mothering), for others they will be figured in other ways (e.g. figured as being different by, for example, liking Korean music).

A second conclusion of this study relates to the rather dominant presence of the cultural model of mathematics as requiring natural/innate ability in this classroom. Effortless attainment, something innate or natural, has been recently described as a common aspiration of many students in schools (Jackson, 2006). Some authors have suggested that mathematics as requiring a natural ability may be valued among students since it helps in negotiating and maintaining popularity inside the peer culture (Francis, Skelton & Read, 2010). As a consequence, students can either identify themselves as a 'natural mathematicians' (which requires recognition from others) or attempt to find alternative ways to negotiate success with popularity and belonging in the peer culture. In this study, while identifying with an effortless position provided access to status in the peer group (i.e. Maria's example of being popular with her popular friends), finding an alternative MI required negotiating peer relations on the margins of the class peer structure (i.e. Carla's example of being different) or in opposition to those that seem effortless (i.e. Katia's example of her relationship with Andres as an opposition between being effortless and being responsible). Finding these alternative MIs appeared to be more conflicted than the relative alignment associated with Maria's effortless position.

It is interesting to note that the observed relationship between the cultural model of 'mathematics as requiring natural ability' and gender differed from other studies that focus on post-compulsory mathematics education. While the post-compulsory literature has described this cultural model as one explanation of girls/women finding little space to identify with mathematics - as it is linked with being male (Mendick, 2005a) - data from this study suggests that, in this specific compulsory mathematics classroom, the effortless attainment model is also a possibility for girls' identifications. One possible explanation for this discrepancy may well be that mathematics can have different meanings in compulsory and post-compulsory education. This case study suggests that in a compulsory mathematics classroom it is possible for students to see mathematics not as something masculine, because it is not that outstanding and because it is not marked as different or special (Damarin, 2000). In some way in this classroom the teacher is trying to make mathematics available for all, and she achieves (or tries to achieve) this by reducing the pace of the lessons and by lowering the difficulty of the mathematics taught.

Specifically in relation to gender, the data from this study suggests that gender appears to mediate girls' relationships with maths in at least two ways. Firstly, through interaction with peers, these girls are preparing themselves for future (gendered) relations, which influence their relation with mathematics 'in the moment' in classrooms as well as choice of mathematics as a career path. Theoretically, peer relations are a key developmental activity at this stage (Elkonin, 1971), and are used as a source for identity development (e.g. Erikson, 1968). It is 'peer relation activities' which pave the way for future development (Elkonin, 1971; Karpov, 2003): peers are not only a space of belonging but also a space for becoming. Maria is an example of how these peer relations are gendered and how this may have an effect on her aspirations about the future: balancing her 'efforts' and work was important in showing that she could acquire feminity for future gendered practices like dating, partying, having several different interests and looking pretty. Her aspirations also suggest that this balancing involves adjusting her expectations in the future, and therefore adjusting her role and place in the social world. It can be argued that through gendered peer relations Maria is exercising social roles and working out where she belongs to and where is her place in the labour market. Mathematics, and demanding mathematics careers are becoming a place where she feels she won't belong or where she won't be able to achieve her (gendered) goals.

Another interesting finding from this study, which is also gender-related, is that the discourse of natural ability as male seems to be a resource from which girls can draw upon to negotiate their difficulties. This finding is consistent with evidence from post-compulsory suggesting that 'math as a male domain' discourse can be used not only for discounting girls out of mathematics, but also as a resource ['having a male brain'] to enter the practice (Solomon, 2012). This phenomenon was mainly observed in Katia's story, who showed a highly gendered discourse about natural ability as a male attribute. It is interesting to note that, in her discourse, the effortless ones, the ones that 'have it in their personality', were exclusively men [e.g. Christian or Andres] which is surprising considering that in this classroom there were girls that also fit this definition [e.g. Maria]. This omission opens the question of why Katia adopts a gendered narrative, where men are skilled at math and women need to be responsible. A possible explanation is that such a narrative has a regulatory function in Katia's identifications; that by linking the cultural model 'maths as natural ability' with being male she can sustain a positive relationship with maths despite not being talented ('not fast like Andres or Christian'). By attaching this natural ability to being male she can normalize herself and preserve a positive relationship with maths. In other words, it is not her problem if she finds maths difficult, it is simply a struggle that all women have to face. However this discourse is not disembodied, but deeply embedded in her peer relations, and therefore resourced by her belonging to her particular peer group: a position that is required for the high attainment boys of the classroom, performed in practice by one of the boys of her group [e.g. Andres], and confirmed by her own positioning as a 'responsible-and-not-necessarily-talented-girl'.

In relation to limitations of this study, we have to mention the idiosyncratic nature of each mathematics classroom and how these processes will be highly context dependent. However this idiosyncrasy, this study provides evidence to the conceptual generalization that peer relations are pivotal in students' developing mathematical identities during adolescence. Following this and previous research on peer culture and social comparison influences on self-efficacy and self-concept (Francis, Skelton, & Read, 2010; Marsh, 1987), it is fair to state that future research, and summary readings of research (meta-analysis or reviews for policy etc.), may need to pay careful attention to the structure of peer groupings within a/the classroom(s).

Finally, there are a number of potential implications that emerge from these findings, specifically in relation to teaching practices in the mathematical classroom. This paper suggests that the subjective experience of doing maths, even among those that have a positive relationship with it, can differ in important ways between individuals. In consequence, teachers must acknowledge this variability and actively explore students' notions about mathematics, as well as understand how these notions are influenced by peer interaction. In our opinion, an approach that considers the unique tensions that emerge from the individual process of MIs construction has better chances to engage students in meaningful (for them) and productive (in terms of learning) mathematical activities.

Chapter 8. Discussion: Contributions, Limitations and Implications of this Thesis

8.1. Introduction

The main aim of this thesis was to provide a broad picture of the relationship between girls and school mathematics in Chile. Over the last 10 years, this relationship has been persistently portrayed as challenging, with national policies and social media describing differences in attainment as largely problematic. More importantly, the social construction of this problem has influenced the design of public policies on gender and mathematics with little empirical support, since research in Chile on this topic is almost non-existent and methodologically limited.

In contrast with the limited amount of research in the Chilean context, the specific study of gender and mathematics has been a focus of attention in the international community for decades. Nevertheless, despite the increased amount of studies addressing this problem, there are important gaps of a theoretical and methodological nature that still need to be considered. This is not a surprise if we keep in mind the complexity of the relationship between girls and mathematics. As a consequence, this thesis attempted to address, not only research questions that were relevant for the Chilean context, but also problems that were still a matter of debate in the international community.

Four main limitations, or conceptual gaps, of the existing knowledge on girls and mathematics were considered by this thesis. Firstly, it was found that gender, and particularly gender differences in attainment, is still predominantly studied as an isolated social category, without considering its intersection with ethnicity or social class (Grant & Sleeter, 1986; Lubienski, 2008). This intersection is extremely important, since gender differences and gender relations are ethnic- and culture-specific (e.g. Archer, 1996; Boaler Altendorff & Kent, 2011). Secondly, a tendency in the literature to focus on differences in performance or attainment was found, relatively neglecting dispositions and affective variables (Zan, Brown, Evans & Hannula, 2006). An exception to this approach can be observed in studies which use the concept of 'Mathematical Identities' as a tool in order to understand students' relationships with the subject (Black, Mendick & Solomon, 2008; Solomon, 2008) and explore how such a relationship is influenced by the identification with categories such as gender (e.g. Nasir & Cobb, 2007). Thirdly, although the study of Mathematical Identities has recently gained support in the mathematics education research community (Chronaki, 2013), important conceptual and operational limitations in the uses of identity have been identified (Brubaker & Cooper, 2000; Sfard & Pusak, 2005). Finally, the influence of relational and contextual variables in the development of different Mathematical Identities is not well-understood, thus remaining a matter of academic debate. In particular, the impact of different pedagogical practices (Boaler & Greeno, 2000), student-teacher relationships (Black, 2004) and peer relations (Gholson & Martin, 2014) appear to be central in mediating students' mathematical identities.

This thesis comprised a series of papers that not only addressed these gaps in the literature, but also offered novel evidence which is relevant to the gender and mathematics problem in Chile, particularly in relation to the development of policies and practices. In this final chapter each paper's contribution to knowledge will be discussed, thus elaborating further on the ideas presented in each chapter. Then, the main claims of this thesis will be articulated in order to discuss how they contribute to the literature on girls and mathematics in general. Following the discussion of each paper and general claims of the thesis, a theoretical synthesis will be discussed on each of these different contributions. Finally, the limitations and implications of these results will be discussed.

8.2. Synthesis of each Paper and Contributions to Knowledge

8.2.1. Gender Gaps in Mathematics Attainment in Chile

This section targeted a generalised discourse in Chile of girls and mathematics being a problematic match in terms of attainment, as well as the relative 'responsibility' often attributed to schools in reproducing this 'problem'. Although some authors have warned that a focus on the description of gender differences can contribute to the reproduction of gender stereotypes (Chipman, 2005; Caplan & Caplan, 2005), these stereotypes have permeated Chilean policies based on limited empirical support. For example, a recent Chilean policy which attempts to improve the assessment and ranking of schools, considered gender equity in terms of attainment as a relevant variable to measure (Agencia de Calidad de la Educacion, 2014). It is interesting to view that, although differences in Language, which in Chile commonly favour girls, are also considered in this new policy, most of the preliminary studies published by the agency in charge of this process are related to girls' disadvantages in mathematics[15]. The emphasis of this policy on gender differences reflects a powerful discourse for girls' failure in Mathematics, which has no strong empirical support. In view of this situation, Chapter 4 explored in detail the distribution of the gender gap in the Chilean population, thus offering evidence for the first time in order to discuss the magnitude of these differences as well as the empirical support to make schools accountable for them.

In relation to the size of the differences, this study replicated a general tendency reported by SIMCE over the last 10 years, where girls were consistently out-performed by boys in mathematics. However, the analyses offered new evidence in order to better understand girls' under-attainment. Firstly, modelling individual progress identified that girls are making less progress than boys at primary school. Secondly, although this effect was consistent, it was found that its size was negligible in comparison with the effect of previous attainment and socioeconomic status. Finally, gender differences interacted with socioeconomic background, with the gender effect being greater for less privileged backgrounds. This last result is alarming in that it suggests that differences may be particularly related to SES, thus reproducing social inequalities.

In relation to accountability which can be attached to schools based on the scale of the differences between boys and girls, this study also offers novel data. It suggests that between schools there is not much variation in relation to their differential effectiveness. In other words, although there was a significant variation between schools in terms of their gender gap, this variation was fairly small and un-comparable with differences in general effectiveness. These results question the use of attainment as a main source for assessing gender equity at school level. This provides an indicator which may be differential for schools that are doing poorly in comparison to the general trend, but not differential for any school that could be challenging gender differences.

[15] In their webpage they publish working documents about the development of the ordering methodology they are suggesting. In relation to gender equity, they have only published documents that focus or give emphasis on the under-attainment of girls (particularly in mathematics, but also in science). Boys under-attainment in language is relatively overlooked. http://www.agenciaeducacion.cl/estudios-e-investigaciones/investigaciones/apuntes/

8.2.2. Towards a Conceptual Model of Mathematical Identities

In order to contribute to the research on girls' relationships with mathematics, this thesis also attempted to address the lack of theoretical clarity on the conceptualisation of mathematical identities. In particular, Chapter 5 reviewed how the concept has been used in the mathematical research community, thus allowing a better operationalisation and a clearer map of the field. In addition, Chapters 6 and 7 are examples of different applications of the concept, discussing how these different approaches make use of an integrative model of identity and how these applications can be successfully implemented when exploring the relationship between girls and mathematics.

The critical review reported on chapter 5 was focused on two main questions: 1) to identify defining features of the different notions of identities and how they were emphasised by different studies; and 2) how different emphases in research were distributed in studies focused on different stages of schooling. The first finding of the critical review in Chapter 5 was the confirmation of a conceptual problem in the use of mathematical identities, something that has been noted by several authors: definitions were multiple, they draw on a diverse theoretical background and relied on a wide range of methods (e.g. Brubaker & Cooper, 2000; Sfard & Pusak, 2005). Interestingly, a more detailed analysis of the existing studies also allowed the identification of only a few dimensions in which these definitions contrasted markedly. All study conceptualisations could be articulated in only three main defining features: 1) identity's ontological 'nature' and the tension between individual and social identities (e.g. Cobb, Stephan, McClain & Gravemeijer, 2001; Lerman, 2000); 2) forms in which identities are expressed in the world and the tensions between representational and acted/enacted aspects of identities (Holland, Lachicotte, Skinner & Cain, 1998; Sfard & Pusak, 2005); and 3) notions of how identities develop and a shared notion of identities as learnt in time, either in the enacted versus crystallised (Black, 2004) or past-present-future dimensions of identity (Sfard & Pusak, 2005). Although these tensions represent an important and different emphasis in research that may be seen as incompatible, there were also important communalities between studies. This led me to suggest that any conceptualisation of identity needs to consider how its own stand relates with broader conceptualisations, understanding how the tensions mentioned above need to be understood as a dialectic relationship (Stetsenko & Arievitch, 2004). This dialectical approach has been considered before in order to address the problem of identity, particularly in relation to the tension between social and individual aspects of the developing self (Stetsenko & Arievitch, 2004):

> Conceptualizing these simultaneous processes of a mutual co-creation of the self and society helps us to move beyond the dichotomy of individual versus social and to reveal the *dialectical unity* (although not the equivalence) of these two equally important aspects of human life and development (p. 497).

The same can be said in relation to the tension between narrative and enacted/acted aspects of identities. For example, when distinguishing 'Figurative Identities' (i.e. those that are related with figures and representations) and 'Positional Identities (i.e. identities related with day-to-day and on-the-ground relations of power), Holland and colleagues (1998) highlight how these two 'facets' of identity relate in several ways:

> In order to highlight these facets, we make an analytic distinction between aspects of identities that have to do with figured worlds—storylines, narrativity, generic characters, and desire— and aspects that have to do with one's position relative to socially identified others, one's sense of social place, and entitlement. These figurative and positional aspects of identity interrelate in myriad ways. Sometimes they are completely coincident; sometimes one dominates over the other (p. 125).

Following this, one suggestion of chapter 5 was that an integrated approach to identity, one that considers individual/social and enacted/representational aspects and their process of development, was ideal. However, conscious of the challenges of such an approach, it was suggested that studies need to be aware of their own limitations in their own approaches in relation to an integrated approach. In the particular operationalisations of identity that were used in the research studies presented in this thesis, discussions were made towards this more integrative model.

In Chapter 7 a model of identity was used which derived from the theoretical conceptualisation presented in Chapter 5. A social and individual perspective of identity was integrated (following Cobb and colleagues, 2001) and data was collected in which identities were seen as expressed in narrative/representational forms and enacted forms. Regarding this final aspect, the analysis of this chapter revealed a dialectically co-constructed relationship between these two forms of expressing identities in the classroom. Narratives were stories of a reflected relationship with the practice, but also stories about different ways of behaving when engaging with it, and forms of behaviour were consequently supporting these narratives. In this sense, stories and forms of engagement were seen as two related sites of identity construction.

In contrast with the 'integrative approach' with an emphasis on detailed accounts of individual identities, in Chapter 6 a much more restricted operationalisation of identity/identification was used. This paper was concerned with exploring average 'effects', not diversity in different individuals' identities, so a survey approach was much more suitable. However, in order to avoid essentialising identity by linking it to the individual attributes measured by each survey, these 'attributes' were conceptualised as forms of identification with mathematics which the students 'performed' when answering the different surveys. In this sense, the students' answers to the surveys (and consequential scores) were not interpreted as fixed internal attributes, but as flexible aspects of a continuous process of identification. In addition, the social dimension, the process of how these identifications were constructed and a discussion of how these reported notions related with actual behaviour in the classroom all formed a matter of discussion in the conclusion of this paper. In this way, it was important to be aware of the limitations of a 'more restricted' approach to identity, and to discuss the results with a more integrative perspective.

Finally, both empirical studies that were focused on identity considered how identities and identifications unfolded over time. In Chapter 7 the temporal tension was captured in students' narratives, operationalising mathematical identities as changing during (school) life. For collecting these types of narratives an interview protocol was used which emphasised these changes (see 'Mathematical life story'). Chapter 6 focused on both current (affects and self-concepts) and designated (dispositions towards the future) aspects of mathematics' identifications. By including affects and self-concepts as current aspects of mathematics' identifications the emphasis was on trying to capture both changeable/dynamic aspects (emotions) and more stable/crystallized ones (self-concepts). By including designated aspects it was acknowledged that "identities give direction to one's actions and influence one's deeds to a great extent, sometimes in ways that escape any rationalisation" (Sfard & Pusak, 2005, p. 18).

In relation to the second research question which guided the literature review presented in Chapter 5 [how different emphases in research were distributed in studies focused on different stages of schooling], a marked contrast between compulsory and post compulsory studies in the way mathematical identities were conceptualised was found. Studies in compulsory education tended to conceive identity as attached to a particular shared practice and used methodologies that emphasised enacted dimensions of these identities. Post-compulsory studies, in contrast, focused on individual trajectories, thus, predominantly using narrative methods. This finding is extremely important, for it suggests that developmental factors may strongly influence the theorisation of how MIs are constructed across the lifespan, as well as what notions of what it means to be a child and what it means to be an adolescent are in place in particular societies.

During adolescence, for example, the emergence of new cognitive abilities, such as self-consciousness and self-reflection (Rutter & Rutter, 1993) use narratives as main tools in the development of the self and its sense of identity. However, these narratives are also developed as a social requirement. In western societies, during adolescence individuals are expected to make choices and take commitments (Erikson, 1968), which need to be explainable to others and to ourselves. In this sense, individuals are required to express their self-hood by choosing 'autonomously' amongst options (Smith, 2010). By choosing narrative methods, researchers are somehow reproducing these social requirements by misrecognising in the interview setting how these options (particularly regarding the process of choosing non-compulsory paths in education and work) are heavily influenced by our set of dispositions learnt in social (classed and gendered) relations (Bourdieu, 1977).

During childhood and formal schooling, learning in educational/formal settings becomes the activity that leads development (Karpov, 2003). In western societies, at this age, children have been described as dealing with a crisis between 'Industry and Inferiority', where it is expected that they develop a sense of competence (Erikson, 1980; 1982). In contrast with adolescents, who are supposed to learn to make ‘autonomous choices’, children are expected to adapt, comply and gain competence in their relation with shared practice, thus reorganising their relations by duties and the fulfilment of social roles (i.e. that of a student, Karpov, 2003). By neglecting individual trajectories and children’s perspectives, and by focusing on how they fit with a certain ‘adult world’ (for example, the world of certain pedagogies), again research can be reproducing children’s place in society as passive dependent beings (e.g. Renold, 2002). By highlighting this emphasis in childhood and adolescence/post-adolescence literature, in Chapter 5 it was suggested that research needs to extend its attempts to cover notions of identity that are not usually covered in each stage of development. This would provide a more complete picture of how children and adolescents develop their own relationships with this subject, thus making research on identity more complete and allowing research in the area to make stronger claims.

Although this work in relation to mathematical identities confirms the complexities of the concept, and the difficulties in providing robust theoretical and methodological approaches, it is believed that evidence from this thesis provides support to the usefulness of the concept in order to explore students’ relationships with mathematics. While some authors have suggested looking for more specific concepts in order to surpass ‘identity’ problems (Brubaker & Cooper, 2000), I think the benefits of an identity approach by far surpasses its weakness. Identity as a concept has provided useful tools for exploring issues of equity and discrimination, as well as subjective experiences of students. This has given a strong support to the relevance of situated practices in different developing kinds of mathematical students and the dynamic movement of identities in developing and changing practices. Following this, the analyses of chapters 5, 6 and 7 offer evidence suggesting the value of using this concept, but also the need for researchers to be more aware of the theoretical backgrounds and methodological implications of the identity concepts they use. This is so they can also acknowledge what is missing by specifically focusing on some defining features at the expense of others.

8.2.3. Mathematical Teaching and Mathematical Identifications

One of the goals of this thesis was to explore the influence of contextual factors, such as the mathematical practice itself (and how it is represented in different types of teaching), in the development of mathematical identities and gender differences. A main contribution of this thesis was the confirmation that students' perceptions of their teaching as being more student centred was associated with more positive student identification with mathematics, both for boys and girls. This result is quite novel, since most of the literature that has tried to link teaching with outcomes has focused on attainment (e.g. Desimone & Long, 2010; Hamilton et al., 2003).

In relation to the differential effect between boys and girls, it is interesting to note that girls perceived student-centred activities as occurring less frequently than boys, even when they were in the same classrooms and arguably experiencing the same teaching. Unfortunately, in this study it was not explored why this difference appeared, but related possible explanations can be hypothesised. Firstly, it is possible that inside some classrooms girls will actually experience a less productive and active mathematics than boys (Black, 2004), with social relations positioning them as more compliant in their modes of participation (Walkerdine, 1998). A second related explanation is that some girls may actually perceive this 'different' mathematics as a way of explaining their relatively low self-concept, less positive affects towards mathematics and future dispositions. Regarding this last point, perceiving a less 'student-centred' pedagogy may also be a construction which could explain their aspirations, dispositions and their eventual decision to drop out. Although this study does not provide evidence to support these hypotheses, there is a tendency to think, in line with notions of identities as constructed in the dynamic interchange between subject and practice (Holland et al., 1998), that pedagogy is part of a process of identification with mathematics that mediates identities and vice-versa (Boaler & Greeno, 2000).

In addition, Chapter 6 showed that some of the difference between boys and girls in their different forms of identification could be accounted for by students' perceptions of their teaching as being less student-centred. In accordance with what was previously discussed about identities being constructed in dynamic transactions of the individual with its social context and practice, the less student-centred perception of the teaching practice was related with lower scores in the different forms of identification which girls reported. Even though this differential perception of teaching practice suggests a potential effect of teaching on gendered differences in identifications with mathematics, the data did not support the existence of a 'female friendly mathematics' (e.g. Becker, 1995). As described before, the effects of teaching were of a similar magnitude for boys and girls in all forms of identifications explored. This suggested that 'student-centred pedagogies' benefit the construction of both boys and girls' mathematical identities, whilst not offering any particular advantage for girls.

8.2.4. Diversity of Girls' Mathematical Identities

In the last research paper of this thesis (Chapter 7) the influence of peers, as a contextual variable in the construction of girls' mathematical identities, was explored through a series of case studies. More specifically, in relation to girls who presented a positive mathematical identity. The main question addressed by this paper was how these girls negotiated with their peers different positions which allowed them to develop and maintain their positive identifications with the mathematical activity. By focusing on girls who present positive identifications during compulsory schooling, this study addressed a relatively neglected area, mostly due to an overemphasis on girls' negative relationships with the subject. The girls studied here go 'against the odds' since they have managed to develop a positive relationship with mathematics, in a context where mathematics is predominantly constructed as masculine. This focus has been more common in post-compulsory education, where researchers have investigated how women manage to persevere in a mathematical career (e.g. Solomon, 2012; Solomon, Radovic & Black, 2015). It has also been used for understanding counter-stories of the success of young black students in mathematics (also in a context where black students are failing and leaving mathematics) (McClain, 2014; McGee & Martin, 2011; McGee & Pearman, 2014a). It is suggested that by extending this approach to compulsory schooling could help explain possibilities of success, but also dynamics which lead to dropping out: the drop out situation for girls who have both the skills and motivation to pursue a mathematical path.

A first result of this case series suggests that even amongst girls who have a positive relationship with mathematics, the process of identity construction is extremely idiosyncratic, and its development is closely related to the influence of particular social contexts such as peer group identity. These findings offer further support for the situated nature of developing mathematical identities, particularly in relation to how gendered relationships with activities depend on the nature of particular activities that occur inside the classroom (Boaler, 2002b) and particular groups that exist inside the classroom. The novelty of these findings relies on the fact that they extend previous ideas on the relevance of peer relationships in developing academic identities (e.g. Francis, Skelton & Read, 2010), to the problem of girls' mathematical identities.

This result is also novel in that it shows how belonging to a particular social cluster mediates not only mathematics participation but also the narratives and positionings which constitute students' mathematical identities. Girls developed meanings about who they were individually in relationship to mathematics, but at the same time, they also attributed meaning based on the peer groups they belonged to and how these groups perceived mathematical activity. These meanings and particular forms of participation were negotiated individually and collectively. Girls used peer group identities as resources for constructing and maintaining their individual maths identities (and vice-versa). These findings offer further support to the idea that identities are multiple and activity attached (e.g. Black & Williams, 2013; Varelas, Martin & Kane, 2012). However, results from this investigation are novel in that they show how individual and group mathematical identities influence and sustain each other. In some cases, as can be expected, the interaction between individual and group identities can also generate conflict and tensions.

In the exploration of girls who were seen as developing positive mathematical identities, results from this investigation contested evidence coming from post-compulsory studies, which have reported that a positive identification with mathematics is often lived as conflicted (e.g. Mendick, 2005a). In this study, some girls did not seem to present such conflicted positioning. This finding can perhaps be explained in relation to the classroom mathematical culture, which defined mathematics as a natural ability, thus influencing how competence was understood. As previously discussed, the natural ability cultural model played an important part in maintaining popularity, but remained a source of conflict when girls could not identify easily with it.

The gendered dimension of girls' peer relations and its relationship with their mathematical identities was observed in two different ways. Firstly, as previously described, it was found that some girls were limiting their mathematical expectations in view of a balance they predicted they would have to make in order to fit in a career and a role as a mother. In consequence, it was hypothesised that peer relations comprised an area where these girls prepared themselves for future (also gendered) relations. One of the case studies strongly supported this idea, by noting that the balancing efforts of maintaining popularity and social relations in the peer culture was also linked with balancing expectations for her future career and role as a mother. In this case, the expectation of mathematics becoming more difficult was not compatible with her need to balance a professional career with a family life.

Secondly, it was hypothesised that the gendered discourses of natural ability as male, and responsibility as female, were used as resources for maintaining a positive identification with maths. In one of the cases, a strong discourse of mathematics as a natural ability, and natural ability as male, was observed. Maintaining a positive relationship with mathematics (a positive mathematical identity) was seen as conflicted and difficult in this case. Here it was hypothesised that the cultural model of natural ability as being a male attribute, and females' success based on responsibility and hard work, could be used as a protective strategy, or an alignment with all the other women who can relate positively with maths, even when it is difficult for them. Jackson (2002, 2003) made a similar suggestion when exploring boys' under-attainment in the UK. She proposed that the construction of gendered attainment (particularly what was termed 'laddish behaviour') was used as a form of self-worth protection, protection from an implication of a lack of ability and of being viewed as feminine (2002).

8.3. General Claims of this Thesis

Although this thesis is comprised of a set of individual research papers, there is one common thread which brings these articles together: an interest in exploring girls' relationships with school mathematics. This interest comes out of the need to question the naturalisation of girls' relationships with mathematics and to challenge the idea that fixed attributes of 'being a girl' contribute to the historical problematic position that girls – and women – have had with mathematics (Boaler, 2002a). As previously stressed, this is particularly important in the Chilean context. Evidence from the papers presented in this thesis supports two main claims regarding girls' relationships with mathematics.

8.3.1. The Situated Nature of Gendered Relationships with Mathematics

As previously explained in the introduction (Chapter 1), this project adheres to a sociocultural perspective of learning and development. In other words, people's learning, development and identities are intimately related to the context in which they take place. This means that the particular social contexts where subjects engage in activities, and the meaning shared and negotiated in these contexts, are constitutional of individuals' particular relationships with these activities. As noted by Holland and colleagues (1998):

> Significant to our concept is the situatedness of identity in collectively formed activities. The "identities" that concern us are ones that trace our participation, especially our agency, in socially produced, culturally constructed activities—what we call figured worlds (p. 40 – 41) Figured worlds in their conceptual dimensions supply the contexts of meaning for actions, cultural productions, performances, disputes, for the understandings that people come to make of themselves, and for the capabilities that people develop to direct their own behaviour in these worlds (p. 60)

Adopting this paradigm in order to explore the relationship between girls and mathematics is is of great importance. A situated notion of learning, development and identities imply that the 'mathematical figured world' will depend on the 'context of meanings'. In other words, meanings of what mathematics (and doing school mathematics), and gender (and being a girl) mean in a particular cultural context (Atweh & Cooper, 1995). Data from this thesis has supported this view in different ways.

The first source of evidence is related to gender differences in attainment. Results from this thesis contrasts markedly with data obtained in other countries, offering supporting evidence to the context dependent nature of the gender and mathematics relationship. As previously presented in chapter 4 and chapter 6, girls in Chile were found to be *performing* and *identifying* less positively with mathematics than boys.

This evidence contrasts with data from international studies, which suggests an opposite trend. Several studies in the US (Hyde, Lindberg, Linn, Ellis & Williams, 2008; McGraw, Lubienski & Strutchens, 2006) and UK (Boaler, Altendorff & Kent, 2011; Gorard, Rees & Salisbury, 2001) have reported differences in attainment as small or inexistent, with girls' performance as good, and sometimes even better, than that of boys. Similarly, a recent study in Estonia also found that girls reported more positive views of mathematics than their male counterparts (Kaldo & Hannula, 2014). Discrepancies in data from different countries are important for two theoretical reasons. Firstly, it supports the idea that these gaps are heavily mediated by context and culture, and do not respond to natural differences between boys and girls. Secondly, it suggests that research needs to expand from developed countries to growing economies, putting special emphasis on cultural aspects which may be reproducing gender differences.

Another source of evidence comes from data which compares how the effects of gender varies within a country for students of different socioeconomic backgrounds. As was previously discussed in chapter 4, differences in attainment were found to change for different socioeconomic groups, becoming greater in less privileged socioeconomic backgrounds. This evidence suggests that the positive impact of higher socioeconomic status offsets the effects of gender (Lamb, 1996), thus, reproducing social differences not only between boys/men and girls/women, but also between girls from more deprived social classes and girls from more privileged backgrounds. In this sense, and in relation to the 'socioeconomic context', mathematics appears to work as 'an exclusive instrument of stratification' (Stinson, 2004, p. 9), regulating racial, ethnic, gender and class divisions and becoming what has been called a 'gatekeeper' (Stinson, 2004) also in the Chilean (and Latin-American) society (Ñopo, 2012).

Finally, evidence from this thesis also proposes that meanings, and forms of engagement in relation to mathematical activity, are also contextual inside the same classroom. As reported in Chapter 7, definitions of mathematics and the negotiation of successful positionings in the classroom were both heavily influenced by the peer clusters to which girls belonged. In other words, depending on which social group girls belonged, they experienced mathematics differently, and had dissimilar access to learning resources inside the same classroom (e.g. Black, 2004; Myhil, 2002). In consequence, the particular relationships that each girl established with the mathematical activity were situated within specific contexts of belonging, where different peer groups inside the classroom mediated the crystallisation of different forms of mathematical identities.

8.3.2. Girls and Mathematics: A socially Constructed Negative and Conflicted Relationship

As previously noted, this thesis offers novel data suggesting that the relationship between girls and mathematics in Chile is consistently more negative than that of boys. This was observed both as differences in attainment during year 4 and 8 (Chapter 4), and also as differences in how girls identified with mathematics (Chapter 6). Interestingly, this thesis also showed that even some girls who had a positive relationship with mathematics, in terms of attainment and emotional engagement, experienced their identification as mathematicians as conflictive, reflecting a struggle in maintaining a positive identification with the subject (Chapter 7).

These difficulties, however, cannot be solely accounted for by the significance of being a girl, as some girls in this same context (Chapter 7), and girls in other contexts, are performing successfully and identifying with fewer conflicts in mathematics. It is also a conclusion of this thesis that these difficulties are likely to be a product of existing social differences. As Bourdieu has noted (2001), differences between genders are a product of arbitrary distinction in a process that naturalises social constructions:

> The social world constructs the body as a sexually defined reality and as the depository of sexually defining principles of vision and division. This embodied social programme of perception is applied to all the things of the world and firstly to the *body* itself in its biological reality. It is this programme which constructs the difference between the biological sexes in conformity with the principles of a mythic vision of the world rooted in the arbitrary relationship of domination of men over women, itself inscribed, with the division of labour, in the reality of the social order (p. 11).

The 'social programme' referred by Bourdieu is related with the gendered division of labour in society. According to the same author (1980), every child learns or constructs their gendered identity at the same time as they construct the representation of their place in society, linking their expectations with their 'reality' as girls or boys:

> The child constructs its sexual identity, a central aspect of its social identity, at the same time as it constructs its representation of the division of labour between the sexes, on the basis of the same socially defined set of indissolubly biological and social indices. In other words, the growth of awareness of sexual identity and the incorporation of the dispositions associated with a particular social definition of the social functions assigned to men and women comes hand in hand with the adoption of a socially defined vision of the sexual division of labour (p. 78)

In line with these theoretical views, recent research has tried to link gendered social inequalities with differences in mathematical performance and attitudes (Else-Quest, Hyde & Linn, 2010; Guiso, Monte, Sapienza & Zingales, 2008). For example, Else-Quest et al. (2010) explored existing evidence for the gender stratification hypothesis. This hypothesis suggests that in patriarchal cultures, male students link their achievement to future opportunities and outcomes (Baker & Jones, 1993), while girls are offered fewer opportunities. In consequence, girls limit their expectations, influencing their perception of certain subject areas as less useful, thus impacting upon their performance and attitudes. As a result of their international meta-analysis, Else-Quest et al. (2010) found that cultural variations in opportunity structures for women explained, at least in part, a variation in differences in mathematics between countries (attainment and attitudes). In other words, differences between countries in relation to the size of gender differences in attainment and attitudes were explained in part by differences between countries in different indexes of gender equality. This result supported the idea that countries with higher levels of gendered inequalities will consequently present higher inequalities in mathematics attainment and dispositions.

Data from this thesis appears to support the application of the gender stratification hypothesis for the Chilean context. Despite a dramatic decrease in gender inequalities in Chile over the last 50 years, some remain markedly persistent. Chile still presents wage discrimination (Fuentes, 2005), which increases with higher levels of education and experience (Montenegro, 2001). Lower participation of women in the work-force (INE, 2012) as well as more involvement in less competitive careers in comparison with men (Cardenas, Correa & Prado, 2012; Voz de Mujer, 2010; Voz de Mujer, 2013) is still a problem. Additionally, Chilean culture still sees women as expected to conform with traditional 'feminine roles' (Avalos, 2003; Escobar, 2001) related to the care of the family and home (Encuesta Nacional Bicentenario UC-Adimark, 2012, in Stuven, 2013; Voz de Mujer, 2013). These social inequities may have an important role in explaining the persistent under attainment and less positive mathematical identities, exhibited by girls in Chile. It is possible that women may be educated throughout their years of schooling (as well as socialisation at home) in relation to their aspirations: girls may learn to keep their aspirations low, not because of lesser intelligence or capacity, but because of the demands posed by their roles as mothers and housewives, as well as the employment opportunities offered by society.

Consequently with the gender stratification hypothesis, this thesis showed that some girls seem to be downplaying their mathematics expectations in view of their social opportunities. For example, Chapter 7 showed how some girls' positive mathematical identities were negotiated simultaneously with gendered relationships with their peers and preparation for future life. The idea of balancing was central in one girls' narratives regarding her relationships with mathematics. Whilst in the present she was trying to balance her social and mathematical obligations in the classroom, in the future she predicted the necessity to balance her career aspirations in order to fit with her social (feminine) obligations (have a family and become a mother). This account provide support to the idea that if social opportunities (e.g. in the labour market) and social roles (e.g. in upbringing responsibilities) continue to be highly differentiated by gender, and if in social interactions (with peers but also other important socialisation figures) continue to reproduce these differences, chances are that differences in mathematics (and probably other areas) will persist.

8.4. Theoretical Synthesis of the Contributions of this Thesis

In summary, this thesis offers strong support to the usefulness of Mathematics Identities and identifications as a *mid-level* concept in order to explore how gender is implicated in the relationship between girls and mathematics (see figure 8.1). As noted by Holland and colleagues (1998), identities, or how people see themselves and are seen by other people, may guide further behaviour, motivating social life. In this sense, identities can help to explain how individuals or group of individuals, engage in the social world of school mathematics, and how this particular relationship can influence individuals' choices and performances.

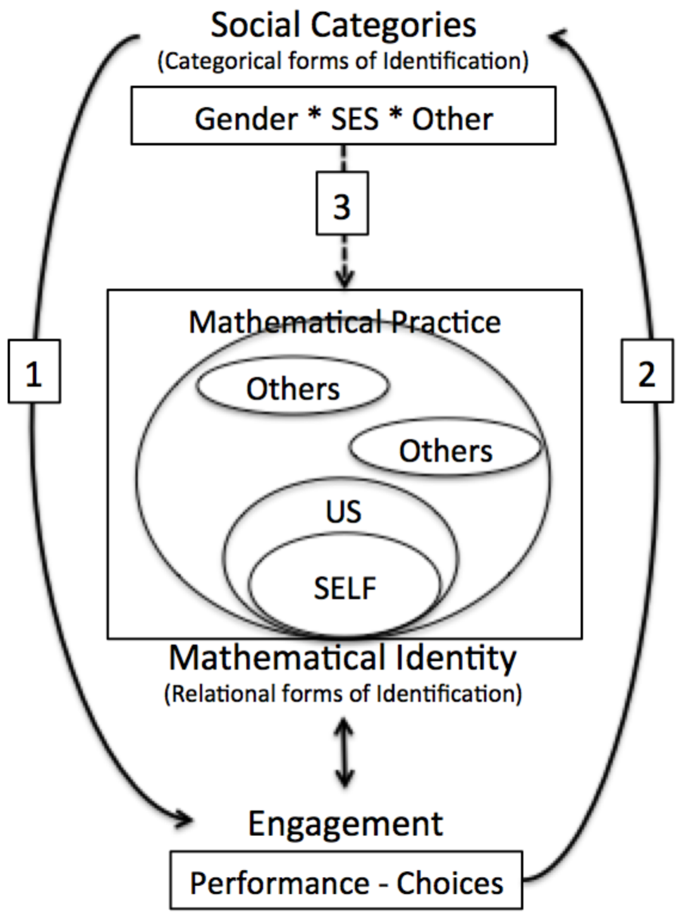


Figure 8.1: Theoretical synthesis of this thesis contributions. It shows how identities (categorical and relational forms of identification) have a motivational role influencing engagement, performance and choices. Numbers 1 and 2 show how the relationship between categorical forms of identification and engagement, performance and choices is bidirectional. Number 3 shows how the influence of categorical forms of identification in engagement, performance and choices is mediated by the local practice and how the relational forms of identification are lived in these local contexts.

In first place, gender can be conceptualised as a main social category to which individuals identify. This idea has also been proposed by authors in the field, who have talked about 'categorical modes of identification' (Brubaker & Cooper, 2000), or "identities that form in relation to major structural features of society: ethnicity, gender, race, nationality, and sexual orientation" (Holland et al., 1998, p.7.). According to Holland and colleagues (1998), these categorical modes of identification are associated with certain characteristics through the use of social artefacts. Thus, characteristics (e.g. not being competent at mathematics) become associated (stereotypically) with social categorical identities (e.g. being a girl), which can motivate different forms of engagement and influence choices and performances in social activities (e.g. students' attainment in Mathematics). This process is described as 1 in figure 8.1. In addition, this differential engagement (performances and choices) as expressed in different social artefacts (e.g. national comparisons between boys and girls in SIMCE attainment) will support the association of social characteristics with categorical identities (see figure 8.1, number 2). These two simultaneous processes generate a reproductive cycle, which can reinforce stereotypical associations and may make individuals' possibilities of breaking them difficult.

However, even though categorical modes of identification can help in understanding gender influences in girls' relationship with mathematics, these forms of identification are not enough to account for the complex processes which underpin the construction of mathematical identities in girls. In addition to gender as a social category to which people identify with, gender is also produced and reproduced in local practice, where individuals negotiate 'relational forms of identification' (Brubaker & Cooper, 2000). Bringing the local aspect into account allows one to understand why "conventions of privileged access associated with gender, caste, or some other major social division may or may not have been taken up, elaborated, and made hegemonic in a particular figured world or field of power" (Holland et al., 1998, p.131). This is extremely important since the 'effect' of categorical forms of identification in the subject has been described as not direct, but mediated by the particular local practice (what Holland and colleagues call "Figured worlds'. See figure 8.1, number 3). In different local contexts, individuals 'relate' with particular practices in place at the local level (e.g. pedagogic practice which defines the mathematical practices in place) and with others (e.g. teacher and other students) in a process which positions them as a different kind of mathematics student (i.e. different relational identities). These different mathematical identities are also negotiated as belonging to different social peer groups (a notion of 'us'), where students use peer group identities as resources for constructing and maintaining their individual maths identities (and vice-versa). All of these processes of identification (categorical and relational with practices and with others) become intertwined with each other, thus creating the complexity in which particular mathematical identities need to be understood.

Evidence provided by this thesis supports this theoretical model. Firstly, the complexity of the effects of gender in girls developing mathematical identities (at the categorical and relational level) might help to explain why the statistical relationship between gender and maths engagement/performance may be smaller and more nuanced than some other 'categorical forms of identification', such as class. Here, it is possible to hypothesise that the indirect effect of gender is taken up in local practices with less ubiquity than the effect of social class. While in Chile there are some powerful discourses in play regarding girls' possibilities of attaining at the same level of boys in mathematics (Mizala, Martinez & Martinez, 2015), these discourses are less powerful than those in relation to class (del Rio & Balladares, 2010). The comparison between the size of gender effects and SES, and the interaction between them, supports this hypothesis by showing that the effect of gender is smaller than the effect of SES and that both may have a summative effect on girls' attainment.

Secondly, the relationship between individual sense of one-self (termed here as identifications with mathematics), and the subjective experience of the mathematical practice, as shown in chapter 6 appear to confirm the idea that identities are built in relationship with a particular practice. In addition, in chapter 7, a two-way relationship was observed, where girls with different positionings in the classroom perceived and valued different ways of doing mathematics, which reinforced their particular forms of identifications. For example, Maria in chapter 7 positioned herself as an independent mathematical student who can perform a successful mathematician identity independently while performing as a popular girl in her peer group (adolescent group). These positionings in turn reinforced her perception of mathematics as a collaborative and independent enterprise, with space for engagement in other parallel activities.

Finally, the analysis at a local level also showed that girls' identification with mathematics, even those from the same classroom/school, are not 'determined' by their identification with gendered social categories. Evidence from chapter 7 suggests that some girls can develop positive mathematical identities in various ways in the classroom, while others perhaps cannot. In this sense, not all girls need to identify with dominant cultural models on girls' relationships with mathematics (e.g. girls as being less able and therefore, with less possibility of attaining at a high level), with some cultural groups resisting or not making use of these models. Following this, when considered in context as a cultural construct, mathematical identities can foster understanding on how girls can construct positive relationship with mathematics from even the most unlikely cultural resources. Furthermore, in local practice the relevance and salience of performance/attainment can become secondary to other cultural artefacts in play. For example, a girl can position herself as 'different' (using the cultural resource of Korean music in the example of Carla in chapter 7) in order to become a 'good mathematician' despite not displaying a high attainment level.

8.5. Limitations of this Study

The most general and obvious limitation of this study is its wide scope. Due to the fact that research on the topic in Chile is almost non-existent, this thesis adopted a broad approach in order to explore girls' relationships with mathematics in the country. For example, this thesis considered different aspects of the relationship as outcomes, thus not only including attainment variables [as many studies have done before] but also affective variables. This thesis also considered more than one level of analysis, in order to capture the influence of macro and local variables. Thus, it included (a) a national overview of gender differences in attainment; (b) an individual analysis of the influence of perceptions of teaching in mathematical identifications; and (c) an analysis of how individual identities interact with a particular classroom practice and social peer relationships. As previously discussed, these different levels also correspond with different aspects of identity which may help to explain the relationship between gender and mathematics.

The obvious limitation of such an approach is its lack of depth, in terms of the prevention of following a single research question via a sequence of successive experiments or observations. Its advantage, however, is that it allows one to obtain a very comprehensive picture of the problem in Chile, thus working effectively as an initial snapshot, or as a preliminary step in the process of comprehending the relationship between girls and mathematics. It is important to mention that the decision to follow a wide scope was also heavily influenced by concrete difficulties when implementing this research. For example, only a very short time period was available for collecting all of the data during the fieldwork in Chile. As a consequence, it was impossible to implement a second wave of data collection in order to test the hypotheses that had emerged from the initial data. Another quite concerning limitation was the lack of interest shown by Chilean schools for participating in this investigation, which appears to reflect important cultural resistances to research. This limited the possibility for acquiring a larger and more heterogeneous sample of classrooms for the survey study (Chapter 6), as well as restricted the possibility to extend the case studies (Chapter 7) to other classrooms.

Adopting a wide scope also entails methodological challenges, since different approaches –qualitative analysis, item response theory or multilevel modelling- were needed for each study. Nevertheless, a silver lining can be found within this challenge, since by using varied methodologies the individual limitations of each approach are somehow overcome (see Chapter 3 Methodology). For example, the emphasis on general trend differences in attainment (Chapter 4) and in different forms of identifications (Chapter 6) was complemented by a detailed analysis of the processes where these differences were produced (Chapter 7). While the first two chapters (4 and 6) evidenced an 'effect' of gender and teaching in students' attainment and identities, Chapter 7 suggested that these processes are dialectic and unified in a more complex notion of mathematics identity (one that is acted, experienced in relation to each other and in a specific context).

In addition, addressing different research questions with varied methodological perspectives allowed the tackling of different aspects of the overall research problem in the context of one research project. For example, a limitation of the study in Chapter 4 was an exclusive focus on attainment, thus neglecting affective and attitudinal variables. Some literature areas have suggested that these affective variables may be more relevant when exploring persistent differences in mathematically related careers (Stevens, Wang, Olivárez & Hamman, 2007), mainly because differences in attainment (as this study showed) are usually small. This limitation was somewhat addressed in chapter 6, where differences in affective variables (as forms of identifications) were considered an essential part of the design.

Another limitation of this study was related to the sample sizes and representation of the population in these samples. For example, in Chapter 6 due to difficulties in recruiting schools, the sample was limited to schools regarded as average in terms of attainment and socioeconomic status of their population. As can be expected, the obvious implication of this sampling limitation is that it did not allow the exploration of the distribution of differences between boys and girls from different contexts and socioeconomic statuses (as was carried out in relation to attainment in Chapter 4). Furthermore, the small number of classrooms included in this study limited the ability to explore if the effects of the perceptions of teaching could be regarded as a general effect of the teaching in the classroom, or mainly as an effect of the perception of each student inside the classrooms. Future studies with access to larger samples would allow the conduct of a multilevel analysis, exploring the relative contribution of individuals' perception and classrooms influences in students' perceptions of teaching. The classroom effect on students' identifications with mathematics would allow the generation of clearer guidelines for changes in teaching practice, rather than speculations about possible relationships between students' perceptions and classroom experiences.

Finally, in chapter 7 a methodological approach was followed which enabled the exploration of the dynamics between pedagogic practices and students' mathematical identity development. The analysis including narrative and observational data in relation to mathematical identities allowed the author to surpass the limitation of relying solely on self-reported data (as in chapter 6), providing support to the notion of the co-construction of classroom practices and students' identities. However, this study also had its own limitations: the idiosyncratic nature of the processes that were described does not allow generalisations to other contexts. In particular, even in the same school and in a different classroom, the different structuring and fragmentation of peers' relations and their relationships with the mathematics practice may house very different possibilities for girls' mathematical identities. It may be that some classrooms offer no peer groups in which a 'successful girl' would feel at home, and maybe a few where the opposite is the case. However, the model constructed can be used for studying mathematical identities arguably in any context in which mathematics is taught with a particular group of students. In addition, the theoretical grounding of this study allows a theoretical generalisation of the mediation of mathematical identities within particular peer relations and peer practices.

8.6. Implications for Policy and Practice

Results from this thesis suggest relevant implications for Chilean policy and general research practice. The first implication is the need to widen the notion of what gender equity means in the Chilean context. Today, equity in Chile is mainly defined in terms of outcomes, and in relation with attainment. This definition needs to be widened for two reasons. Firstly, different outcomes need to be considered when exploring gender equity, particularly the inclusion of affective variables. Affective variables have been recognised as an important source of further attainment differences (Fennema & Sherman, 1977) and, more importantly, have been described as influencing disposition in order to further engage in courses and related mathematical careers after compulsory education (Johnston, 1994).

Secondly, research on gender equity also requires a multidimensional definition of equity, a definition that considers not only outcomes but also differences in opportunities and treatment (Hart, 2003). Focusing only on outcomes does not allow the exploration of more systemic explanations, with the consequential risk of blaming individuals and naturalising gender differences (Boaler, 2002a).

In addition to the need for the inclusion of other outcomes and a multidimensional definition of equity, the notion of responsibility, of whom is to blame for gendered social differences in mathematics, also needs to be expanded. Although this study did not directly explore influences of social inequities and social roles, some support of this was found in chapter 7. In this chapter girls that were developing positive relationships with mathematics were viewed as having to negotiate these relationships contrary to their future self-placements in society (having children and having to negotiate their labour identities with their parental identities). This lends support to the need for questioning the responsible party for gender inequalities (currently this is the responsibility of schools or parents) and fostering a social responsibility of fighting against gendered social roles. Although advances have been observed in Chilean society, there are still marked differences in womens' opportunities, participation in the work force and deeply held beliefs of their suitability in traditional feminine roles. In addition with the need to expand responsibilities from families and schools to culture and society, this thesis also gave support for the need scale down these responsibilities. The detailed analysis of boys and girls' transactions and negotiations of identities in the mathematical classroom (Chapter 7) supported the need for becoming more aware of how traditional social roles are reproduced in micro interactions.

In terms of pedagogical implications, and coincidently with other studies, this thesis suggests that implementing teaching practices in alignment with student-centred principles (more collaborative, discussion-based, more active and independent) may be positive for both boys and girls. In general, by developing more positive identifications and dispositions in students this can assist with combatting a generalised disaffection with mathematics (Nardi & Steward, 2003), as well as improving possibilities for further engagement with the subject. The power of these pedagogical approaches in improving girls' relationships with mathematics is still unclear. Evidence from this study (chapter 6) suggests that girls tend to perceive their teaching as less student-centred, thus extending the debate for the potential impact of implementing this kind of pedagogy. However, this does not mean that teachers need to adapt different forms of teaching for boys and girls, but rather that they need to be aware and provide opportunities for girls inside their classrooms.

Chapter 9: Appendices

9.1 Appendix 1: Construction of Family SES Variable

Table 9.1:

SIMCE protocol for transforming level of education into years of education

Level of Education	Years of education
No studies	0
Primary Year 1	1
Primary Year 2	2
Primary Year 3	3
Primary Year 4	4
Primary Year 5	5
Primary Year 6	6
Primary Year 7	7
Primary Year 8	8
Secondary Year 1	9
Secondary Year 2	10
Secondary Year 3	11
Secondary Year 4	12
Non-complete Technical/Professional studies	14
Graduated Technical/Professional studies	16
Non-complete University studies	15
Graduated University studies	17
Masters degree	19
PhD degree	22

Note: Own translation.

Table 9.2:

SIMCE protocol for transforming range of Income to Median

Range of Income	Median
Bellow $100.000	50.000
Between $100.000 and $200.000	150.000
Between $201.000 and $300.000	250.000
Between $301.000 and $400.000	350.000
Between $401.000 and $500.000	450.000
Between $501.000 and $600.000	550.000
Between $601.000 and $800.000	700.000
Between $801.000 and $1.000.000	900.000
Between $1.001.000 and $1.200.000	1.100.000
Between $1.201.000 and $1.400.000	1.300.000
Between $1.401.000 and $1.600.000	1.500.000
Between $1.601.000 and $1.800.000	1.700.000
Between $1.801.000 and $2.000.000	1.900.000
Between $2.000.000 and $2.200.000	2.100.000
Above $2.200.000	2.300.000

Own translation.

Table 9.3:

Bivariate correlations amongst SES variables for Year 4 attainment students

	Z score of Fathers highest level of education Pearson Cor (n)	Z score of Mothers highest level of education Pearson Cor (n)	Z score of Family Income Pearson Cor (n)
Z score of Fathers highest level of education	1 (235965)	.693** (235965)	.567** (231016)
Z score of Mothers highest level of education	.693** (235965)	1 (235965)	.599** (231016)
Z score of Family Income	.567** (231016)	.599** (231016)	1 (234942)

** Correlation is significant at the 0.01 level (2-tailed).

Table 9.4:

Bivariate correlations amongst SES variables for Year 8 attainment students

	Z score of Fathers highest level of education Pearson Cor (n)	Z score of Mothers highest level of education Pearson Cor (n)	Z score of Family Income Pearson Cor (n)
Z score of Fathers highest level of education	1 (191647)	.611** (191647)	.466** (189835)
Z score of Mothers highest level of education	.611** (191647)	1 (191647)	.458** (189835)
Z score of Family Income	.466** (189835)	.458** (189835)	1 (190850)

** Correlation is significant at the 0.01 level (2-tailed).

Table 9.5:

Results from the Exploratory Factor Analysis of SES

		Year 4_2009	**Year 8_2009**
% Variance explained by Factor 1		75%	68%
Eigenvalues	Factor 1	2.237	2.026
	Factor 2	0.454	0.584
	Factor 3	0.309	0.39
Communalities after extraction	Income	0.688	0.584
	Mothers' Ed	0.763	0.718
	Fathers' Ed	0.787	0.724
Components Matrix	Income	0.829	0.764
	Mothers' Ed	0.873	0.847
	Fathers' Ed	0.887	0.851
Consistency analysis (cronbach's alpha)		.83	.76
Total sample		231016	189835

As observed in the tables, different items of the Family SES showed a medium correlation between them (all of them significant) and the extraction of a single factor is supported by the size of the Eigenvalues (factor 1 eigenvalues are higher than 1 and nearly four times the size of the second factor's eigenvalue). The first factor also accounted for a high proportion of the explained variance (year 4= 75%; year 8= 68%) and the final scale showed a high level of internal consistency (year 4= .83; year 8= .76).

9.2 Appendix 2: Missing Data Tables

Table 9.6:

Missing data on Ethnicity variable.

	Frequency	Percent
Ethnicity Total	178822	78.8
Missing	48157	21.2
Population of Year 8 Attainment data	226979	100

Table 9.7:

Sample, Missing data on family SES and Population.

	Sample	Missing Data (No-Data on Family SES)	Population
Year 4 Maths Scores	219682 (88%)	30810 (12%)	250492
Year 8 Maths Scores	189835 (84%)	37144 (16%)	226979
Progress	158853 (84%)	29147 (16%)	188000

9.3 Appendix 3: Analysis of Lessons' Activities

The main conclusions of this analysis were that: 1) most time during the lessons were organized as whole class teaching; 2) participation of the students during whole class teaching was guided closely by the teacher by using the IRF pattern of interaction [Initiation, Response, Feedback] (Mehan, 1979) (see bellow in appendix 9.4 on analysis of IRF Interactions); and 3) important part of the time was devoted to reinforce and revise previous contents. The distribution of time devoted to different formats of activities and different purposes are presented in figures 9.1 and 9.2

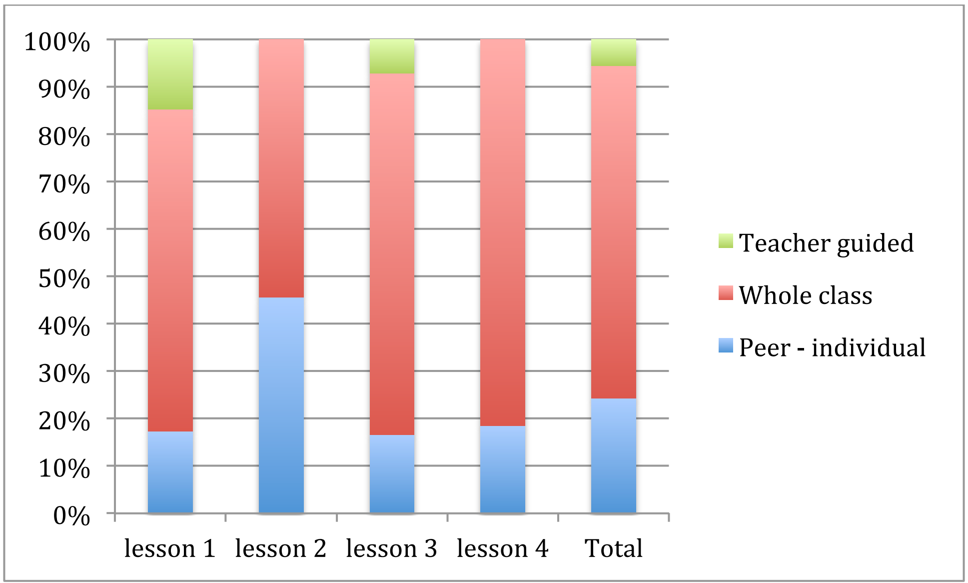


Figure 9.1: Distribution of time devoted to different formats of activities.

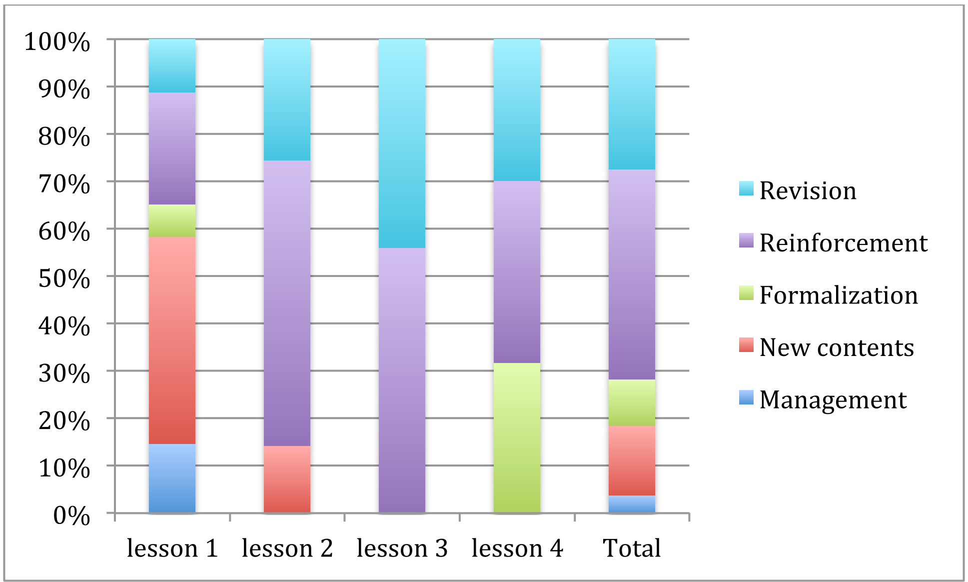


Figure 9.2: Distribution of time devoted activities with different purposes.

9.4 Appendix 4: Analysis of IRF Interactions

The main conclusion of this analysis was that most teacher elicitations (54%) were identified as closed questions (students were expected to find a correct answer), with more open questions (spaces for more participation) being saved for students' personal experiences and critical analysis of information (about 22%) (see figure 9.3).

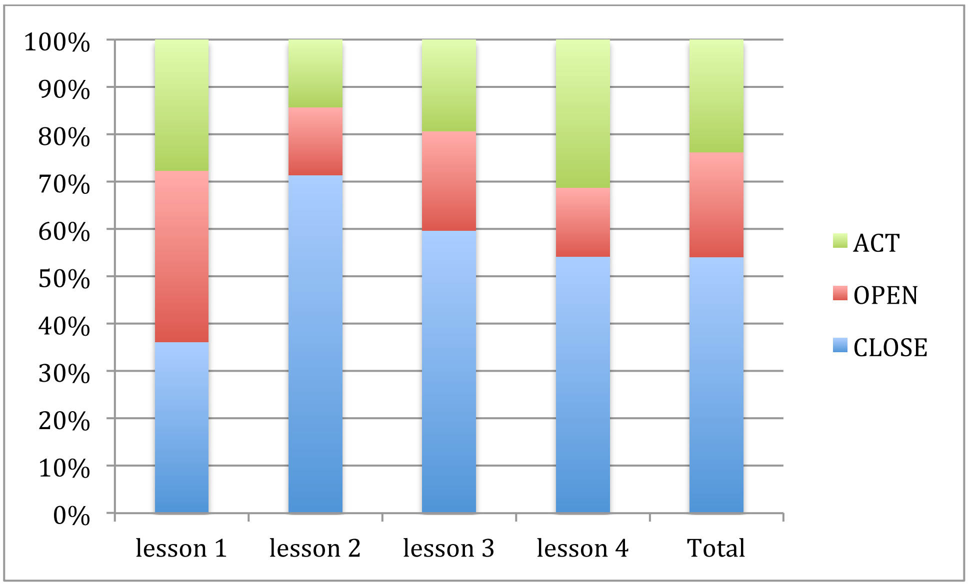


Figure 9.3: Distribution of questions in different lessons and total.

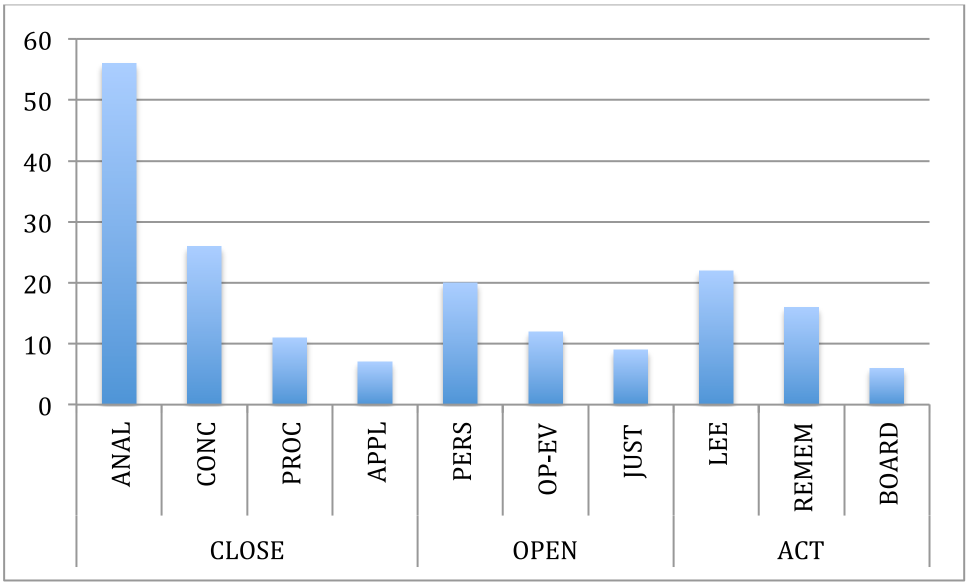


Figure 9.4: Distribution of type of questions total.

9.5 Appendix 5: Conformation of Peer Clusters in the Classroom

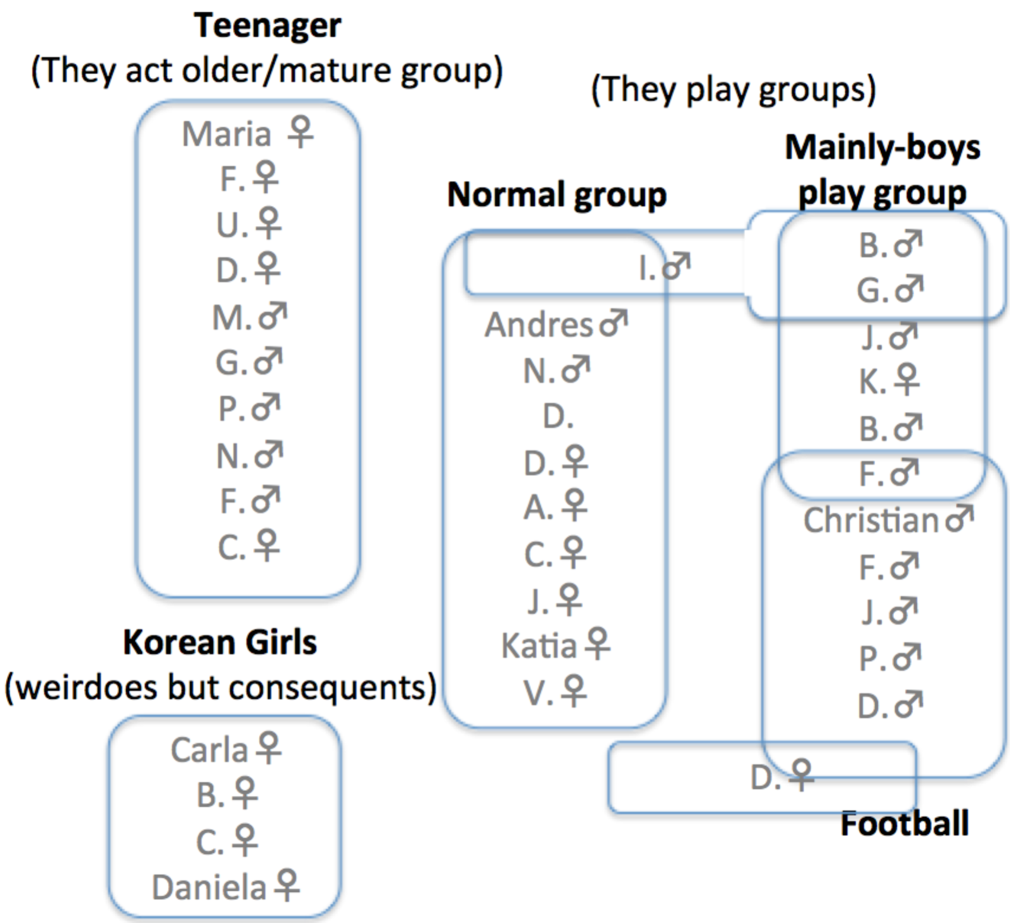


Figure 9.5: Map of students peer clusters groups. Names of groups represent names that were given by students in the interviews. To retain confidentiality, students that were the focus of the study or that were mentioned in the results section (see Chapter 7) were given pseudonymous, while other students are named by letter from their first or last names

9.6 Appendix 6: Characteristics of each Social Peer Cluster

Table 9.8:

Summary of students' accounts of characteristics of each social peer groups in the classroom.

	What the interviewee from these groups said about…				
	Teenager	**Mixed gender Play group**	**Korean Girls**	**Mainly boys Play group**	**Football**
Interviewees	(Maria♀, F♀, G♂)	(Katia♀, D♀, A♂)	(B♀, Carla♀)	(K♀, G♂)	(D♀, C♂)
Teenager	We are more mature, we have other mentality, we don't play foolish games. When we are only the boys we talk about boys things (?), when we are with the girls sometimes we talk about their things (G).	- (The girls) are friend and they hang out with boys and I don't hang out with them because I don't think like they. They try to behave as they were older and I don't think that's appropriate (D). - Los que se juran bakanes (they think they are the coolest). The girls act as they were older (A)	- (The girls) are always showing off, they use very short skirts and are continuously hugging and kissing and some of them have boyfriends (B). - They are the girls that hangout with the most disruptive boys. The girls like pop, J.Bieber and the boys do skate. They are like internet, like adolescent, like facebook (C).	- (The girls) talk about boys from other classes and music (K) - They have lunch together and they hang with each other. I don't know what they do. Sometimes the bother other people and they don't play much (G).	- This is the show off group, they are worried about brands, they discriminate and look at you from top to bottom - The boys have bullied me; they have called me "maraca" (prostitute – lesbian) (D). - They are usually together. They talk and have lunch together (C)
Mixed gender Play group	They are always together and they are always moving around. They laugh about everything and we call them the flies (F) - (The girls) are euphoric and loud. They are very different than me, they are more childish. Is not that I'm mature, but I think they are more inmature, more infantile (M)	- We live what we are. Childhood, and what we have to do (differently from "teenage" girls) (D). - We are the quieter ones. We have a good fun, but we care more (than the teenage group) about school. We don't think that we are the coolest. (A) - We hang out a lot. For J. birthday we are all coming, and the girls are staying for a sleepover. The boys aren't because they are too dirty (K)	The girls here listen to Justin Bieber and other listen to Calle trece. This is the quietest group, they are rarely naughty and it would be weird for one of them to get a bad annotation (C)	- They (the girls) are best friends, I don't know why (K). - I also participate in this group and it is one of the few mixed gender groups. We play together tag (G)	- I don't have a relationship with them, but they are my classmates. Allan is annoying but nothing terrible (D).

Table 9.8 (continuation):

Summary of students' accounts of characteristics of each social peer groups in the classroom.

	What the interviewee from these groups said about…				
	Teenager	**Mixed gender Play group**	**Korean Girls**	**Mainly boys Play group**	**Football**
Interviewees	(Maria♀, F♀, G♂)	(Katia♀, D♀, A♂)	(B♀, Carla♀)	(K♀, G♂)	(D♀, C♂)
Korean Girls	- They are friend because they like anime and kitsch pop. They are very childish (F).	- They are the weirdoes because they like weird things (D). - They are super childish (A)	- We do everything together. We've made sleepovers and we are inseparable (B)		- They like Koreans and "others" bully them because of this – but they don't mind – they feel strongly about liking the Koreans (D)
Mainly boys Play group			- They are always running and playing (B)	They are funny, they tell jokes (K)	- They throw airplanes from the second floor (D)
Football	- They are weird; they do silly (and boring) jokes. Like e.g ones they were acting like a leaf from outside was a marijuana leaf (G).		- Daniela does not hang out with girls because she says they bully her because she likes to play football (B).	- They (the boys) are friends because they play football (K)	- I started hanging out more with them because they have valued me (differently from other friends that have bullied me). They are different from other kids, they don't think about sex only (D) - We hang out, during the recess we usually play football and we talk about videogames (C)

Note. Rows are each groups' descriptions and columns represent who was interviewed (by pseudonym and group where they belonged)

9.7 Appendix 7: Information sheet for parents survey study

INFORMACIÓN PARA APODERADOS (Estudio 1)

Su pupilo(a) ha sido invitado a participar en un estudio de investigación parte del proyecto de una estudiante de Doctorado respecto de equidad de género en Matemática. Antes de decidir si está de acuerdo con la participación de su hijo es importante que entienda porqué esta investigación se está realizando y qué tipo de participación implicará. Por favor, lea la siguiente información detenidamente y discútala con otros si lo desea. Pregunte si hay algo que no le queda claro o si desea tener más información y tómese un tiempo para decidir si permite la participación de su pupilo. Muchas gracias por leer esto.

¿Quién realizará la investigación?

Darinka Radovic Sendra,
Psicóloga y Magister en Psicología Educacional,
Pontificia Universidad Católica de Chile.
Investigadora de Postgrado y Estudiante de Doctorado,
Universidad de Manchester, Reino Unido.

¿Cuál es el objetivo de la investigación?

Este estudio pretende desarrollar conocimiento respecto de cómo diferentes actividades de aprendizaje en clases de matemáticas producen diferente participación y compromiso de los estudiantes. En particular estará enfocada en explorar la relación entre prácticas y género.

¿Por qué su pupilo(a) ha sido elegido(a)?

Su pupilo(a) ha sido elegido(a) porque su director(a) y profesor(a) de matemáticas están de acuerdo con que este estudio sea llevado a cabo en el establecimiento de su pupilo(a). Idealmente este estudio involucrará la participación de todos los alumnos de la sala de clases.

¿Qué se le pedirá a mi pupilo(a)?

Si usted accede a que su pupilo(a) participe en esta investigación, se le solicitará que conteste un cuestionario sobre actividades de aprendizaje y actitudes hacia las Matemáticas. Este cuestionario no tardará más de 45 minutos.

¿Qué pasará con la información recolectada y cómo se mantendrá la confidencialidad?

La información recolectada será guardada sin identificación de los estudiantes o sus profesores, en un computador al que sólo la investigadora tiene acceso. Luego de la graduación, esta información será destruida. En caso de que los resultados de esta investigación sean publicados, la información será completamente anónima.

¿Qué sucede si no deseo que mi pupilo(a) participe o si cambio de idea?

Usted y su pupilo(a) deben decidir si desean participar en esta investigación. Si usted decide **NO** autorizar la participación de su pupilo, se le solicitará que firme la forma que se adjunta. En este caso no se le aplicará el cuestionario a su hijo(a) o se borrarán sus datos en caso de que el cuestionario ya haya sido aplicado.

Si usted da su autorización, no es necesario que firme el consentimiento: Si no devuelve la forma se asumirá que usted está autorizando la participación de su pupilo(a). Luego su pupilo(a) podrá decidir si desea contestar el cuestionario (al momento de la aplicación). Si usted decide autorizar la participación y cambia de idea, siéntase libre de retirar esta autorización en cualquier momento, sin necesidad de dar explicaciones.

¿Cuanto durará la investigación?

Esta investigación incluirá la aplicación de un cuestionario de 45 minutos. Luego de esto, es posible que el profesor(a) de su pupilo(a) desee continuar participando en un estudio que incluirá observaciones de clases y entrevistas. Si esto sucede usted será informado y se le solicitará que llene una carta de consentimiento nueva.

¿Dónde se llevará a cabo la investigación?

En el establecimiento de su pupilo(a) durante el horario de clases.

Contacto

En caso de requerir cualquier tipo de información, por favor contacte a Darinka Radovic:

Email: darinka.radovic@postgrad.manchester.ac.uk
Tel: 88625556

Agradeciendo de antemano su contribución a la realización de este estudio se despide,
Darinka Radovic S.
MsC. Psicología Educacional, P. Universidad Católica de Chile.

Parents Information Sheet (Study 1 – English Translation)

Your child is being invited to take part in a research study as part of a student PhD project on gender equity in mathematics. Before you decide if you agree with the participation of your child, it is important for you to understand why the research is being done and what it will involve. Please take time to read the following information carefully and discuss it with others if you wish. Please ask if there is anything that is not clear or if you would like more information. Take time to decide whether or not you agree with your child to take part. Thank you for reading this.

Who will conduct the research?

Darinka Radovic Sendra
MsC in Educational Psychology,
Pontificia Universidad Católica de Chile.
Postgraduate researcher,
The University of Manchester, UK.

What is the aim of the research?

This study is to develop understanding about how different teaching practices and activities in mathematics classrooms produce different participation of students involve in those practices. In particular it will be focused on exploring the relationship between practices and gender (mainly females) engagement/participation in mathematics.

Why has you child has been chosen?

Your child has been chosen to take part because his/her school and mathematics teacher agreed to take part in this study. The study, ideally, will involve the participation of all children in the class.

What would my child be asked to do if I agreed with his/her participation?

If you agree that your child can take part in this study, I will ask your child to fill in a short questionnaire about teaching practices and attitudes towards mathematics. This questionnaire should not take more than 45 minutes.

What happens to the data collected and how is confidentiality maintained?

All the data will be kept without identification of the students or teachers, in a secure computer, with passwords that will not allow access to anyone other than the main researcher and supervisors. After graduation, all the data will be destroyed. It is expected that results of this study will be published. If this happens, all information will be anonymous.

What happens if I do not want to take part or if I change my mind?

It is up to you to decide whether or not your child can take part. If you do decide that your child **CAN NOT** take part, you will be asked to sign a consent form. If you do not agree with the participation of your child no data will be collected about him/her. If you decide that your child can take part, you are still free to withdraw this agreement at any time without giving a reason.

If you give our permission you signature is not needed: If you don't return the form agreement of participation will be assumed. Your child will then be able to choose to participate. If you decide to give permission and change your mind, feel free to withdraw this authorization at any time without giving any reason.

What is the duration of the research?

This research will include an application of a students' survey, which will take less than 45 minutes. After this teachers may want to continue participating in the research which will include observations of lessons and interviews. If this happens you will be informed and asked to fill a new Consent form.

Where will the research be conducted?

All the activities of the research will be conducted in the school.

Contact for further information

For further information, please contact Darinka Radovic:

Darinka Radovic Sendra
Postgraduate researcher

Email: darinka.radovic@postgrad.manchester.ac.uk
Tel: 0161 275 8512

9.8 Appendix 8: Opt out consent form for parents survey study

CONSENTIMIENTO DE APODERADO (Estudio 1)

Si usted **NO ESTÁ DE ACUERDO** con que su pupilo(a) participe en este estudio, por favor, complete y firme la forma de consentimiento abajo. Si usted si da su autorización, por favor guarde este forma.

1. Yo **NO AUTORIZO** la participación de mi pupilo(a) en este estudio ☐

Establecimiento_____________________________________

Curso_____________

Nombre Apoderado(a)

____

Nombre Alumno(a)

____

____
Firma Apoderado

____
Fecha

PARENTS 'OPT OUT' CONSENT FORM (Study 1 – English Translation)

If you **DO NOT** agree with the participation of your child in this study, please complete and sign the consent form below. If you do agree, keep this form.

	Please Initial Box
1. I DO NOT allow the participation of my child in the surveys that are part of the study.	

School______________________________________ Classroom_____________

Parent's name ____

Student's name____

____
Signature

____
Date

9.9 Appendix 9: Information sheet for parents - case study classroom

INFORMACIÓN PARA APODERADOS (Estudio 2)

Su pupilo(a) ha sido invitado a participar en la **segunda etapa** de un estudio de investigación respecto de equidad de género en Matemática. Antes de decidir si está de acuerdo con la participación de su hijo es importante que entienda porqué esta investigación se está realizando y qué tipo de participación implicará. Por favor, lea la siguiente información y discútala con otros si lo desea. Pregunte si hay algo que no le queda claro o si desea tener más información, y tómese un tiempo para decidir si permite la participación de su pupilo.

¿Quién realizará la investigación?

Darinka Radovic Sendra,
Psicóloga y Magister en Psicología Educacional,
Pontificia Universidad Católica de Chile.
Investigadora de Postgrado y Estudiante de Doctorado,
Universidad de Manchester, Reino Unido.

¿Cuál es el objetivo de la investigación?

Este estudio pretende investigar cómo diferentes actividades de aprendizaje en clases de matemáticas producen diferente participación y emociones de los estudiantes. En particular estará enfocada en explorar la relación entre prácticas y género (principalmente mujeres).

¿Por qué su pupilo(a) ha sido elegido(a)?

Su pupilo(a) ha sido elegido(a) porque la dirección y profesor(a) de matemáticas están de acuerdo con que este estudio sea realizado en el establecimiento de su pupilo(a). Idealmente este estudio involucrará la participación de todos los alumnos de la sala de su pupilo(a).

¿Qué se le pedirá a mi pupilo(a) si yo autorizo su participación?

Si usted está de acuerdo, le solicitaré a su pupilo(a) que participe en las siguientes actividades.
1. Contestar cuestionarios sobre su relación con las matemáticas y con su sala de clases. Esta encuesta no tardara mas de 30 minutos.
2. Observaciones de al menos 8 clases de matemáticas. Si el profesor de su pupilo accede, estas observaciones serán grabadas por audio o video.
3. Entrevista individual y grupal: algunos alumnos serán invitados a participar en entrevistas individuales y/o grupales dentro del horario de clases.

¿Qué pasará con la información recolectada y cómo se mantendrá la confidencialidad?

Toda la información recolectada por el estudio (cuestionarios, audio y video grabaciones, transcripciones de entrevistas, etc.) será guardada sin identificación de los estudiantes o sus profesores, en un computador al que sólo la investigadora tiene acceso. Luego de la graduación, esta información será destruida. Se espera que los resultados de esta investigación sean publicados. De ser este el caso, la información será anónima.

¿Qué sucede si no deseo que mi pupilo(a) participe o si cambio de idea?

Usted debe decidir si autoriza la participación de su pupilo(a) y luego firmar la forma que se adjunta. Por favor, devuelva la forma al establecimiento tanto si desea como si no desea autorizar la participación de su pupilo(a). En el caso de que no autorice la participación de su pupilo(a), no se recogerá información sobre el(la). Esto incluye cuestionarios, entrevistas y bloqueo de la participación de su pupilo(a) en las observaciones.

Si usted entrega su autorización su pupilo(a) podrá decidir si desea participar en las actividades a las que se le invite (al momento de la aplicación). Si usted da su autorización y cambia de idea, siéntase libre de retirarla en cualquier momento, sin necesidad de dar explicaciones.

¿Cuanto durará y donde se realizará la investigación?

Esta investigación está planificada para durar parte del segundo semestre de 2013 y se realizará en el establecimiento de su pupilo(a) durante el horario de clases.

En caso de requerir cualquier tipo de información, por favor contácteme al email o teléfono a continuación. Agradeciendo de antemano su contribución se despide,

Darinka Radovic S.
MsC. Psicología Educacional, P. Universidad Católica de Chile.
Email: darinkaradovic@gmail.com o darinka.radovic@postgrad.manchester.ac.uk
Tel: 88625556

PARENTS INFORMATION SHEET (Study 2 – English Translation)

Your child has been invited to take part in the second stage of a research study as part of a student PhD project on gender equity in mathematics classrooms in Chile. Before you decide it is important for you to understand why the research is being done and what it will involve. Please take time to read the following information carefully and discuss it with others if you wish. Please ask if there is anything that is not clear or if you would like more information. Take time to decide whether or not you wish to take part. Thank you for reading this.

Who will conduct the research?

Darinka Radovic Sendra
MsC in Educational Psychology,
Pontificia Universidad Católica de Chile.
Postgraduate researcher,
The University of Manchester, UK.
What is the aim of the research?

This study is to develop understanding about how different teaching practices and activities in mathematics classrooms produce different participation of students involve in those practices. In particular it will be focused on exploring the relationship between practices and gender (mainly females) engagement/participation in mathematics.

Why has you child has been chosen?

Your child has been chosen to take part because his/her school and mathematics teacher agreed to take part in this study. The study, ideally, will involve the participation of all children in the class.

What would my child be asked to do if I agreed with his/her participation?

If you agree that your child can take part in this pilot study, this will involve the following:

1. I will ask your child to fill in a short questionnaire about teaching practices. This questionnaire should not take more than 30 minutes.

2. I will observe at least 8 mathematics lessons. Ideally these classes will be video and audio-recorded.

3. Some students will be invited to take part in a individual or group interview about their perceptions of mathematics (once parental consent has been gained). These interviews will happen during school hours.

What happens to the data collected and how confidentiality will be mantained?

All data will be kept without identification of the students or teachers, in a secure computer, with passwords that will not allow access to anyone other than the main researcher and supervisors. After graduation, all the data will be destroyed. It is expected that results of this study will be published. If this happens, all information will be anonymous.

What happens if I do not want to take part or if I change my mind?

It is up to you to decide whether or not your child can take part. If you do decide that your child can take part, you will be asked to sign a consent form. If you do not agree with the participation of your child no data will be collected about him/her, which includes surveys and interviews, and techniques to block out their participation in lessons from the data collection will also be used. If you decide that your child can take part, you are still free to withdraw this agreement at any time without giving a reason.

What is the duration of the research?

This research is planed to last part of the second semester of the school year of 2013 and it will be implemented in your child school.

Contact for further information

For further information, please contact Darinka Radovic:

Darinka Radovic S.
MsC. Psicología Educacional, P. Universidad Católica de Chile.
Email: darinkaradovic@gmail.com or darinka.radovic@postgrad.manchester.ac.uk
Tel: 88625556

9.10 Appendix 10: Opt in consent form for parents case study classroom

CONSENTIMIENTO DE APODERADO (Estudio 2)

Por favor, lea y firme la forma de consentimiento a continuación y hágala llegar al establecimiento.

1. He leído la información entregada respecto del estudio y he tenido la oportunidad de considerar esta información y hacer preguntas que han sido contestadas satisfactoriamente.
2. Entiendo que la participación de mi pupilo(a) en el estudio es voluntaria y que soy libre de interrumpir su participación en cualquier minuto sin entregar razones.
3. Entiendo que las entrevistas serán grabadas y que si no doy mi autorización mi pupilo(a) no será entrevistado(a) ni encuestado(a).
4. Entiendo que las clases observadas (si el profesor consiente) serán grabadas por audio o video. Si no autorizo la participación de mi pupilo entiendo que su participación en las grabaciones será bloqueada.
5. Entiendo que los resultados de esta investigación podrán ser publicados de forma anónima en libros y/o revistas académicas.

Indique con un círculo si autoriza o no la participación de su pupilo(a)

Yo SI / NO autorizo la participación de mi pupilo(a) en la investigación.

Establecimiento______________________________Curso_____________

Nombre Apoderado(a) ____

Nombre Alumno(a) ____

____
Firma Apoderado

____
Fecha

Agradeciendo de antemano su contribución a la realización de este estudio se despide,
Darinka Radovic S.
MsC. Psicología Educacional, P. Universidad Católica de Chile.

PARENTS' CONSENT FORM (Study 2 – English Translation)

If you are happy to participate please complete and sign the consent form below

1. I confirm that I have read the attached information sheet on the above study and have had the opportunity to consider the information and ask questions and had these answered satisfactorily.

2. I understand that the participation of my child in the study is voluntary and that I am free to withdraw it at any time without giving a reason.

3. I understand that the interviews will be audio-recorded

4. I agree with the use of audio and video recordings during the classrooms observations

5. I agree that any data collected may be published in anonymous form in academic books or journals.

Mark with a circle if you agree or not your pupil's participation

I YES / NO allow my pupil to participate in the study.

School____ Classroom______________

Parent's name ____

Student's name____

____
Signature

____
Date

9.11 Appendix 11: Information sheet for teachers

INFORMACIÓN PARA PROFESORES

Usted ha sido invitado a participar en un estudio de investigación parte del proyecto de una estudiante de Doctorado respecto de equidad de género en Matemática. Antes de decidir si está de acuerdo con participar es importante que entienda porqué esta investigación se está realizando y qué tipo de participación implicará. Por favor, lea la siguiente información detenidamente y discútala con otros si lo desea. Pregunte si hay algo que no le queda claro o si desea tener más información y tómese un tiempo para decidir su participación. Muchas gracias por leer esto.

¿Quién realizará la investigación?

Darinka Radovic Sendra,
Psicóloga y Magister en Psicología Educacional,
Pontificia Universidad Católica de Chile.
Investigadora de Postgrado y Estudiante de Doctorado,
Universidad de Manchester, Reino Unido.

¿Cuál es el objetivo de la investigación?
Este estudio pretende desarrollar conocimiento respecto de cómo diferentes actividades de aprendizaje en clases de matemáticas producen diferente participación y compromiso de los estudiantes. En particular estará enfocada en explorar la relación entre prácticas y género (principalmente mujeres).

¿Por qué usted ha sido elegido(a)?

Usted ha sido elegido(a) para participar en este estudio porque el(la) director(a) del establecimiento está de acuerdo con que este estudio sea llevado a cabo y porque usted mostró interés en cooperar. Idealmente este estudio involucrará la participación de todas las personas que participan en las clases de matemáticas en su sala.

¿Qué se me solicitará si decido participar?

Si usted está de acuerdo en participar, le solicitaré su ayuda en algunas de las siguientes actividades:

1. La aplicación de cuestionarios a sus alumnos. Los cuestionarios serán de corta duración y los aplicaré en una fecha y horario que convengamos juntos.

2. Observación de clases. También en un horario en que convengamos juntos, observaré al menos 8 clases de matemáticas con su curso. Idealmente me gustaría contar con grabación de audio y/o video de la participación de sus alumnos.

3. Entrevistas. A lo largo del estudio me gustaría contar con su opinión respecto de las observaciones que realice. Para esto, durante el estudio le comentaré informalmente los avances que vaya realizando y lo invitaré a entrevistas más formales respecto de su propio punto de vista. Estas conversaciones también serán realizadas en un horario de mutuo acuerdo y tratando de entorpecer lo menos posible sus actividades.

4. Entrevistas con estudiantes: Finalmente invitaré a algunos de sus alumnos a conversaciones individuales y grupales. Le solicitaré ayuda en la gestión de estas conversaciones.

¿Qué pasará con la información recolectada y cómo se mantendrá la confidencialidad?

La información recolectada será guardada sin identificación de los estudiantes o sus profesores, en un computador al que sólo la investigadora tiene acceso. Luego de la graduación, esta información será destruida. Se espera que los resultados de esta investigación sean publicados. De ser este el caso, la información será completamente anónima.

¿Qué sucede si no deseo participar?

Usted debe decidir si desea participar en esta investigación. Si usted decide participar, se le solicitará que firme la forma que se adjunta.

Si usted decide participar y cambia de idea, siéntase libre de retirar esta autorización en cualquier momento, sin necesidad de dar explicaciones.

¿Cuanto durará la investigación?

Esta investigación está planificada para durar parte del segundo semestre de 2013.

¿Dónde se llevará a cabo la investigación?

Todas las actividades de la investigación serán realizadas en el establecimiento educacional donde usted trabaja y durante el horario de clases.

Contacto

En caso de requerir cualquier tipo de información, por favor contacte a Darinka Radovic:

Darinka Radovic Sendra

Email: darinka.radovic@postgrad.manchester.ac.uk
Tel: 88625556

TEACHER INFORMATION SHEET (English Translation)

You are being invited to take part in a research study as part of a student PhD project on gender equity in mathematics classrooms in Chile. Before you decide it is important for you to understand why the research is being done and what it will involve. Please take time to read the following information carefully and discuss it with others if you wish. Please ask if there is anything that is not clear or if you would like more information. Take time to decide whether or not you wish to take part. Thank you for reading this.

Who will conduct the research?

Darinka Radovic Sendra
MsC in Educational Psychology,
Pontificia Universidad Católica de Chile.
Postgraduate researcher,
The University of Manchester, UK.

What is the aim of the research?

This study is to develop understanding about how different teaching practices and activities in mathematics classrooms produce different participation of students involve in those practices. In particular it will be focused on exploring the relationship between practices and gender (mainly females) engagement/participation in mathematics.

Why have I been chosen?

You have been chosen to take part because your school's principal agreed to take part in it and because you showed interest in helping. The study, ideally, will involve the participation of all children in the class.

What would I be asked to do if I took part?

If you agree to take part in this pilot study, the following activities are likely to take place:

1. I will ask you and your students to fill in a short questionnaire about teaching practices and general identification information. Students' surveys will be collected in a time that we both agree.

2. In an agreed and convenient time for you, I will observe at least 8 lessons. Ideally these classes will be video and audio-recorded.

3. Along the study I would like to count with your opinion about observations. For this, I will informally talk to you about advances in my research and will invite you to more formal interviews about your opinions. These interviews will be made in a convenient time for you.

4. If parental consent is obtained, short interviews with some of your students about their perceptions of mathematics will also be conducted. I will ask you to help me with managing these interviews.

What happens to the data collected and how confidentiality is mantained?

All data will be kept without identification of the students or teachers, in a secure computer, with passwords that will not allow access to anyone other than the main researcher and supervisors. After graduation, all the data will be destroyed. It is expected that results of this study will be published. If this happens, all information will be anonymous.

What happens if I do not want to take part or if I change my mind?

It is up to you to decide whether or not to take part. If you do decide to take part you will be given this information sheet to keep and be asked to sign a consent form. If you decide to take part, you are still free to withdraw at any time without giving a reason.

What is the duration of the research?

This research is planed to last part of the second semester of the school year of 2013 and it will be implemented in your school.

For further information, please contact Darinka Radovic:

Darinka Radovic S.
MsC. Psicología Educacional, P. Universidad Católica de Chile.
Email: darinkaradovic@gmail.com or darinka.radovic@postgrad.manchester.ac.uk
Tel: 88625556

9.12 Appendix 12: Opt in consent form for teachers

CONSENTIMIENTO DE PROFESOR

Si usted **ESTÁ DE ACUERDO** con participar en este estudio, por favor, complete y firme la forma de consentimiento a continuación.

	Marque una cruz
1. Entiendo que mi participación en el estudio es voluntaria y que soy libre de interrumpir mi participación en cualquier minuto sin entregar razones.	
2. Entiendo que las entrevistas serán grabadas.	
3. Estoy de acuerdo con la grabación de audio durante las observaciones de clases.	
4. Estoy de acuerdo con la grabación de video durante las observaciones de clases.	
5. Estoy de acuerdo con el uso de citas anónimas en publicaciones respecto de este estudio.	
6. Estoy de acuerdo con la publicación anónima de resultados de este estudio en libros y revistas académicas.	

Establecimiento_____________________________________ Curso_____________

Nombre Profesor(a) ____

____

Firma Profesor

____

Fecha

Agradeciendo de antemano su contribución a la realización de este estudio se despide,
Darinka Radovic S.
MsC. Psicología Educacional, P. Universidad Católica de Chile.

TEACHERS' CONSENT FORM

If you are happy to participate please complete and sign the consent form below

	Please Initial Box
1. I understand that my participation in the study is voluntary and that I am free to withdraw at any time without giving a reason.	
2. I understand that the interviews will be audio-recorded	
3. I agree with the use of audio and video recordings during the classrooms observations	
4. I agree to the use of anonymous quotes	
6. I agree that any data collected may be published in anonymous form in academic books or journals.	

School_______________________________________ Classroom_____________

Teacher's name ____

____
Signature

____
Date

9.13 Appendix 13: Studies Included in Literature review reported in Chapter 5

Ahlqvist, S., London, B. & Rosenthal, L. (2013). Unstable identity compatibility: How gender rejection sensitivity undermines the success of women in science, technology, engineering, and mathematics fields. *Psychological Science,* 24(9), 1644-1652.

Anderson, D. D., & Gold, E. (2006). Home to school: Numeracy practices and mathematical identities. *Mathematical Thinking and Learning*, *8*(3), 261-286.

Andersen, L., & Ward, T. J. (2014). Expectancy-value models for the STEM persistence plans of ninth-grade, high-ability Students: A comparison between black, hispanic, and white students. *Science Education*, 98(2), 216-242.

Archer, L., Dewitt, J., Osborne, J., Dillon, J., Willis, B. & Wong, B. (2012). Balancing acts: Elementary school girls' negotiations of femininity, achievement, and science. *Science Education*, 96(6), 967-989

Axelsson, G. B. (2009). Mathematical identity in women: The concept, its components and relationship to educative ability, achievement and family support. *International Journal of Lifelong Education*, *28*(3), 383-406.

Bartholomew, H., Darragh, L., Ell, F., & Saunders, J. (2011). 'I'm a natural and I do it for love!': exploring students' accounts of studying mathematics. *International Journal of Mathematical Education in Science and Technology*, *42*(7), 915-924.

Berry, R.Q. (2008). Access to Upper-Level Mathematics: The Stories of Successful African American Middle School Boys. *Journal For Research In Mathematics Education*, 39(5), 464-488

Bishop, J.P. (2012). She's Always Been the Smart One. I've Always Been the Dumb One: Identities in the Mathematics Classroom. *Journal For Research In Mathematics Education*, 43(1), 34-74

Black, L. (2004). Differential participation in whole-class discussions and the construction of marginalised identities. Journal of Educational Enquiry, 5(1), 34–54.

Black, L. & Williams, J. (2013). Contradiction and conflict between 'leading identities': becoming an engineer versus becoming a 'good muslim' woman. *Educational Studies in Mathematics*, 84(1), 1-14

Black, L., Williams, J., Hernandez-Martinez, P., Davis, P., Pampaka, M. & Wake, G. (2010). Developing a 'leading identity': the relationship between students' mathematical identities and their career and higher education aspirations. *Educational Studies in Mathematics*, 73(1), 55-72

Boaler, J. & Staples, M. (2008). Participation, Knowledge and Beliefs: A Community Perspective on Mathematics Learning. *Educational Studies in Mathematics*, 40(3), 259-281

Boaler, J. (1999b). Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School. *Teachers College Record*, 110(3), 608-645

Boe, M.V. (2012). Science choices in Norwegian upper secondary school: What matters?. *Science Education*, 96(1), 1-20

Braathe, H.J. & Solomon, Y. (2015). Choosing mathematics: the narrative of the self as a site of agency. Educational Studies in Mathematics, 89(2), 151-166

Buschor, C.B., Kappler, C., Keck Frei, A., & Berweger, S. (2014). I want to be a scientist/a teacher: students' perceptions of career decision-making in gender-typed, non-traditional areas of work. *Gender and Education*, *26*(7), 743-758.

Buschor, C.B., Berweger, S., Frei, A.K. & Kappler, C. (2014). Majoring in STEM-What Accounts for Women's Career Decision Making? A Mixed Methods Study. *Journal Of Educational Research*, 107(3), 167-176

Buxton, C. A. (2005). Creating a culture of academic success in an urban science and math magnet high school. *Science Education*, *89*(3), 392-417.

Campbell, T., Lee, H., Kwon, H., & Kyungsuk, P. (2012). Student motivation and interests as proxies for forming STEM identities. Journal of the Korean Association for Science Education, 32(3), 532-540.

Chronaki, A. (2005). Learning about 'learning identities' in the school arithmetic practice: The experience of two young minority Gypsy girls in the Greek context of education. *European Journal Of Psychology Of Education*, 20(1), 61-74

Cobb, P., Gresalfi, M. & Hodge, L.L. (2009). An Interpretive Scheme for Analyzing the Identities That Students Develop in Mathematics Classrooms. *Journal For Research In Mathematics Education*, 40(1), 40-68

Craig, T. S. (2013). Conceptions of mathematics and student identity: implications for engineering education. *International Journal of Mathematical Education in Science and Technology*, *44*(7), 1020-1029.

Darragh, L. (2013). Constructing confidence and identities of belonging in mathematics at the transition to secondary school. *Research in Mathematics Education*, *15*(3), 215-229.

Darragh, L. (2015). Recognising 'good at mathematics': using a performative lens for identity. *Mathematics Education Research Journal*, *27*(1), 83-102.

de Abreu, G. (1995). Understanding how children experience the relationship between home and school mathematics. *Mind, Culture, and Activity,* 2(2), 119-142.

Empson, S.B (2003). Low-performing students and teaching fractions for understanding: An interactional analysis. *Journal For Research In Mathematics Education*, 34(4), 305-343

Epstein, D., Mendick, H. & Moreau, M-P. (2010). Imagining the mathematician: young people talking about popular representations of maths. *Discourse-Studies in the Cultural Politics of Education*, 31(1), 45-60

Forster, P.A. (2000). Katie Thought She Couldn't Do It but Now She Knows She Can. *Educational Studies in Mathematics*, 43(3), 225-242

Gholson, M., & Martin, D. B. (2014). Smart Girls, Black Girls, Mean Girls, and Bullies: At the Intersection of Identities and the Mediating Role of Young Girls' Social Network in Mathematical Communities of Practice. *Journal of Education*, 194(1), 19–34.

Hand, V.M. (2010). The Co-Construction of Opposition in a Low-Track Mathematics Classroom. *American Educational Research Journal*, 47(1), 97-132

Hernandez, P. R., Schultz, P., Estrada, M., Woodcock, A., & Chance, R. C. (2013). Sustaining optimal motivation: A longitudinal analysis of interventions to broaden participation of underrepresented students in STEM. *Journal of educational psychology*, *105*(1), 89.

Hernandez-Martinez, P., Williams, J., Black, L., Davis, P., Pampaka, M., & Wake, G. (2011). Students' views on their transition from school to college mathematics: rethinking 'transition'as an issue of identity. *Research in Mathematics Education*, *13*(2), 119-130.

Heyd-Metzuyanim, E. (2013). The co-construction of learning difficulties in mathematics-teacher-student interactions and their role in the development of a disabled mathematical identity. *Educational Studies in Mathematics*, 83(3), 341-368

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: On learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, *51*, 128-145.

Hodge, L. (2008). Student roles and mathematical competence in two contrasting elementary classes. *Mathematics Education Research Journal*, *20*(1), 32-51.

Holmegaard, H.T., Madsen, L.M. & Ulriksen, L. (2014). To Choose or Not to Choose Science: Constructions of desirable identities among young people considering a STEM higher education programme. *International Journal of Science Education*, 36(2), 186-215

Horn, I.S. (2008). Turnaround Students in High School Mathematics: Constructing Identities of Competence Through Mathematical Worlds. Mathematics Thinking and Learning, 10(3), 201-239

Hughes, R.M., Nzekwe, B. & Molyneaux, K. J. (2013). The Single Sex Debate for Girls in Science: a Comparison Between Two Informal Science Programs on Middle School Students' STEM Identity Formation. *Research in Science Education*, 43(5), 1979-2007

Kizzie Rouland, K., Johnson Rowley, S., & Kurtz-Costes, B. (2013). Self-views of African-American youth are related to the gender stereotypes and ability attributions of their parents. *Self and Identity*. 12(4), 382-399.

Krogh, L.B. & Andersen, H.M. (2013). Actually, I May be Clever Enough to do it. Using Identity as a Lens to Investigate Students' Trajectories Towards Science and University. *Research in Science Education*, 43(2), 711-731

Landers, M.G. (2013). Towards a theory of mathematics homework as a social practice. *Educational Studies in Mathematics*, 84(3), 371-391

Langer-Osuna, J.M. (2015). From Getting "Fired" to Becoming a Collaborator: A Case of the Coconstruction of Identity and Engagement in a Project-Based Mathematics Classroom. *Journal of the Learning Sciences*, 24(1), 53-92

Lee, JD. (1998). Which kids can "become" scientists? Effects of gender, self-concepts, and perceptions of scientists. Social Psychology Quarterly, 61(3), 199-219

Lee, JD. (2002). More than ability: Gender and personal relationships influence science and technology involvement. *Sociology of Education*, 75(4), 349-373

Lesko, A.C. & Corpus, J.H. (2006). Discounting the Difficult: How High Math-Identified Women Respond to Stereotype Threat. *Sex Roles,* 54(1-2), 113-125.

Lim, J. H. (2008a). Adolescent girls' construction of moral discourses and appropriation of primary identity in a mathematics classroom. *ZDM*, 40(4), 617-631.

Lim, J.H. (2008b). The road not taken: two African-American girls' experiences with school mathematics. *Race, Ethnicity and Education*, 11(3), 303-317

McClain, O. L. (2014). Negotiating Identity: A Look at the Educational Experiences of Black Undergraduates in STEM Disciplines. *Peabody Journal of Education*, *89*(3), 380-392.

McGee, E. O., & Pearman F. A. (2014a) Understanding Black Male Mathematics High Achievers from the Inside Out: Internal Risk and Protective Factors in High School. *The Urban Review*, 1(28).

McGee, E. & Pearman, F. A. (2014b). Risk and Protective Factors in Mathematically Talented Black Male Students Snapshots From Kindergarten Through Eighth Grade. *Urban Education*, 49(4), 363-393

McGee, E.O., Martin, D.B. (2011). You Would Not Believe What I Have to Go Through to Prove My Intellectual Value! Stereotype Management Among Academically Successful Black Mathematics and Engineering Students. *American Educational Research Journal*, 48(6), 1347-1389

Mendick, H. (2005). A beautiful myth? The gendering of being/doing 'good at maths'. *Gender and Education*, 17(2), 203-219

Nasir, N.S. (2002). Identity, Goals, and Learning: Mathematics in Cultural Practice. *Mathematics Thinking and Learning*, 4(40603), 213-247

Nasir, N.S. & Hand, V. (2008). From the court to the classroom: Opportunities for engagement, learning, and identity in basketball and classroom mathematics. *Journal of the Learning Science*, 17(2), 143-179

Oppland-Cordell, S. (2013). Mathematics and Racial Identity Co-Construction in Multiple Sociopolitical Contexts: A Case Study of a Latina Undergraduate Student from an Urban Community. *Journal of Urban Learning, Teaching and Research*, 9, 89-99.

Oppland-Cordell, S., & Martin, D. B. (2014). Identity, power, and shifting participation in a mathematics workshop: Latin@ students' negotiation of self and success. *Mathematics Education Research Journal*, *27*(1), 21-49.

Pronin, E., Steele, C.M. & Ross, L. (2004). Identity bifurcation in response to stereotype threat: Women and mathematics. *Journal of Experimental Social Psychology*, 40(2), 152-168.

Smith, C. (2010). Choosing more mathematics: happiness through work?. *Research in Mathematics Education*, *12*(2), 99-115.

Solomon, Y. (2007). Not belonging? What makes a functional learner identity in undergraduate mathematics?. *Studies in Higher Education*, 32(1), 79-96

Solomon, Y. (2012). Finding a voice? Narrating the female self in mathematics. *Educational Studies in Mathematics*, 80(40574), 171-183

Solomon, Y., Croft, T., & Lawson, D. (2010). Safety in numbers: mathematics support centres and their derivatives as social learning spaces. *Studies in Higher Education*, *35*(4), 421-431.

Solomon, Y., Lawson, D. & Croft, T. (2011). Dealing with 'fragile identities': resistance and refiguring in women mathematics students. *Gender and Education*, 23(5), 565-583

Stinson, D.W. (2008). Negotiating Sociocultural Discourses: The Counter-Storytelling of Academically (and Mathematically) Successful African American Male Students. *American Educational Research Journal*, 45(4), 975-1010

Tate, E. D., & Linn, M. C. (2005). How does identity shape the experiences of women of color engineering students?. *Journal of Science Education and Technology*, *14*(5-6), 483-493.

Turner, E., Dominguez, H., Maldonado, L. & Empson, S (2013). English Learners' Participation in Mathematical Discussion: Shifting Positionings and Dynamic Identities. *Journal For Research In Mathematics Education*, 44(1), 199-234

Ward, P.R., Johnston, W.S., & Lee, C. (2011). Exit interviews: undergraduates who leave mathematics behind. *For the Learning of Mathematics*, 21-26.

Walshaw, M. (2005). Getting political and unraveling layers of gendered mathematical identifications. *Cambridge Journal of Education*, *35*(1), 19-34.

Wood, M.B. (2013). Mathematical Micro-Identities: Moment-to-Moment Positioning and Learning in a Fourth-Grade Classroom. *Journal For Research In Mathematics Education*, 44(5), 775-808

9.14 Appendix 14: Complete students' survey (Spanish)

Estimad@ Alumn@,

El cuestionario que vas a completar se refiere a las clases de **MATEMATICAS** en tu establecimiento. Nadie más verá tus respuestas, ya que es estrictamente confidencial.

Por favor, responde todas las preguntas lo más honestamente posible.
Al responder y devolver este cuestionario entenderemos que estás de acuerdo con participar en este estudio. ¡Muchas gracias de antemano!

Parte A - Acerca de tu experiencia en tu establecimiento:

1. Completa las siguientes preguntas, escribiendo tus respuestas en los recuadros.

1a	¿Cuál es la asignatura que más prefieres?	
1b	¿Cual es la asignatura que menos prefieres?	

2. Tu promedio de notas está: (Marca la alternativa más apropiada para cada asignatura)

	Matemáticas	Lenguaje	Ciencias
Entre los 5 mejores promedios			
Sobre la mitad de los promedios del curso			
Bajo la mitad de los promedios del curso			
Entre los 5 promedios más bajos			

Durante el primer semestre mi promedio en matemáticas fue:

Part B – Sobre lo que planeas hacer en el futuro

1. ¿Cuales son tus planes respecto de la Enseñanza Media? Elige UNA de las siguientes:

1a	Planeo terminar la Enseñanza Media en este establecimiento	
1b	Planeo terminar la Enseñanza Media en otro establecimiento Técnico-Profesional	
1c	Planeo terminar la Enseñanza Media en otro establecimiento Científico-Humanista	
1d	Planeo dejar de asistir al colegio antes de terminar la Enseñanza Media	

2. ¿Cuales son tus planes para después de la Enseñanza Media? Elige UNA de las siguientes:

2a	Estudiar en una Universidad	
2b	Estudiar en un Instituto Profesional o un Centro de Formación Técnica	
2c	Trabajar	
2d	Otro (Si eliges otro, por favor, cuéntanos qué):	

3. Si planeas asistir a la Universidad o Instituto Profesional, ¿qué carrera es más posible que estudies? Si tienes más de una preferencia, escríbelas en orden, de la más a la menos posible.

Carrera(s):

Parte C – Acerca de como te sientes respecto a las matemáticas

1. ¿Que tan de acuerdo o desacuerdo estás con las siguientes afirmaciones? Usa la siguiente escala.

Muy en Des-Acuerdo	En Des-Acuerdo	Ni de acuerdo ni en Des-Acuerdo	De Acuerdo	Muy de Acuerdo
1	2	3	4	5

	(Marca un círculo o cruz en cada línea)	Muy en Des-Acuerdo	En Des-Acuerdo	Ni de acuerdo ni en Des-Acuerdo	De Acuerdo	Muy de Acuerdo
1	Las matemáticas son importantes para mi.	1	2	3	4	5
2	La mayoría de la gente puede aprender a ser bueno en matemáticas.	1	2	3	4	5
3	A mis apoderados les gustan las matemáticas.	1	2	3	4	5
4	Matemáticas es una de las más interesantes asignaturas del colegio.	1	2	3	4	5
5	Disfruto aprendiendo matemáticas.	1	2	3	4	5
6	Tengo una mente matemática.	1	2	3	4	5
7	Puedo obtener buenos resultados en matemáticas.	1	2	3	4	5
8	Me interesa aprender nuevas cosas en matemáticas.	1	2	3	4	5
9	En matemáticas eres recompensado por tu esfuerzo.	1	2	3	4	5
10	Ser bueno en matemáticas es algo con lo que se nace.	1	2	3	4	5
11	Puedo aprender matemáticas incluso si es difícil.	1	2	3	4	5
12	Prefiero usar procedimientos matemáticos que ya conozco antes que procedimientos nuevos.	1	2	3	4	5
13	La asignatura que más me preocupa es matemáticas.	1	2	3	4	5
14	Usualmente requiero ayuda en matemáticas.	1	2	3	4	5
15	Comparado con mis compañeros, soy bueno en	1	2	3	4	5

	matemáticas.					
16	Mis apoderados disfrutan resolviendo problemas matemáticos.	1	2	3	4	5
17	Me gustaría no tener que tomar cursos de matemáticas nunca más.	1	2	3	4	5
18	Preferiría que mis estudios futuros incluyeran muchas matemáticas.	1	2	3	4	5
19	Quiero seguir estudiando matemáticas después del colegio.	1	2	3	4	5
20	Me gustaría ser matemático.	1	2	3	4	5
21	Las matemáticas son importantes para mi futuro (después del colegio).	1	2	3	4	5

2. ¿Cómo te **sientes** usualmente en tus clases de **Matemáticas**? Usa la siguiente escala:

Muy levemente o nada	Un poco	Moderadamente	Bastante	Extremadamente
1	2	3	4	5

(Marca un círculo o cruz para cada emoción)	Muy Leve-mente o Nada	Un Poco	Modera-damente	Bas-tante	Extrema-damente	(Marca un círculo o cruz para cada emoción)	Muy Leve-mente o Nada	Un poco	Modera-damente	Bas-tante	Extrema-damente
Interesado	1	2	3	4	5	Irritable	1	2	3	4	5
Molesto	1	2	3	4	5	Alerta	1	2	3	4	5
Entusiasmado	1	2	3	4	5	Avergonzado	1	2	3	4	5
Enojado	1	2	3	4	5	Inspirado	1	2	3	4	5
Fuerte	1	2	3	4	5	Nervioso	1	2	3	4	5
Culpable	1	2	3	4	5	Decidido	1	2	3	4	5
Asustado	1	2	3	4	5	Atento	1	2	3	4	5
Hostil	1	2	3	4	5	Intranquilo	1	2	3	4	5
Optimista	1	2	3	4	5	Activo	1	2	3	4	5
Orgulloso	1	2	3	4	5	Temeroso	1	2	3	4	5
Aburrido	1	2	3	4	5	Ansioso	1	2	3	4	5

Parte D – Como las matemáticas son enseñadas y aprendidas.
1.¿Con qué **frecuencia** ocurre cada una de las siguientes situaciones en tus clases de matemáticas? Usa la siguiente escala:

Nunca	Rara Vez	Frecuentemente	Siempre o casi siempre
1	2	3	4

	(Marca un círculo o cruz en cada línea)	Nunca	Rara vez	Frecuen-temente	Siempre o casi siempre
1	Trabajamos en proyectos grupales.	1	2	3	4
2	Escuchamos al profesor hablar sobre contenidos.	1	2	3	4
3	Copiamos lo que el profesor escribió en el pizarrón.	1	2	3	4
4	Hablamos entre compañeros sobre cómo resolver problemas.	1	2	3	4

5	Le preguntamos a otros alumnos que expliquen sus ideas.	1	2	3	4
6	Hacemos proyectos o trabajos que incluyen otras asignaturas.	1	2	3	4
7	Hacemos ejercicios en el libro de clases.	1	2	3	4
8	Aprendemos cómo las matemáticas han cambiado en el tiempo.	1	2	3	4
9	Lo que aprendemos está relacionado con la vida fuera del colegio.	1	2	3	4
10	Aprendemos que matemáticas significa inventar reglas.	1	2	3	4
11	Investigamos contenidos por nosotros mismos.	1	2	3	4
12	Discutimos ideas entre todo el curso.	1	2	3	4
13	Explicamos nuestro trabajo a todo el curso.	1	2	3	4
14	Trabajamos en ejercicios de manera individual.	1	2	3	4
15	Respondemos preguntas del profesor.	1	2	3	4

2. ¿Qué **emociones** es más común que sientas cuando estás realizando estas actividades?

Emociones negativas (infeliz, molesto, insatisfecho, triste, aburrido)	Neutro	Emociones positivas (feliz, alegre, satisfecho, contento, optimista)

	(Marca un círculo o cruz en cada línea)	-				+
1	Trabajamos en proyectos grupales.					
2	Escuchamos al profesor hablar sobre contenidos.					
3	Copiamos lo que el profesor escribió en el pizarrón.					
4	Hablamos entre compañeros sobre cómo resolver problemas.					
5	Le preguntamos a otros alumnos que expliquen sus ideas.					
6	Hacemos proyectos o trabajos que incluyen otras asignaturas.					
7	Hacemos ejercicios en el libro de clases.					
8	Aprendemos cómo las matemáticas han cambiado en el tiempo.					
9	Lo que aprendemos está relacionado con la vida fuera del colegio.					
10	Aprendemos que matemáticas significa inventar reglas.					
11	Investigamos contenidos por nosotros mismos.					

12	Discutimos ideas entre todo el curso.					
13	Explicamos nuestro trabajo a todo el curso.					
14	Trabajamos en ejercicios de manera individual.					
15	Respondemos preguntas del profesor.					

3. La mayor parte del tiempo siento que las clases de matemáticas son:

a) Fáciles	b) Ni fáciles, ni difíciles	c) Difíciles

Parte F - Sobre ti

Mi nombre es:								
Asisto a este establecimiento desde el año:	2	0			Soy	Hombre	Mujer	

9.15 Appendix 15: Rasch analysis Student-centred teaching practice scale

Table 9.9:

Student-Centred Teaching Practice Summary of Measurement Statistics

	Person Mean (SD)	Item Mean (SD)
Measurement	0.01 (0.98)	0.00 (0.69)
Infit MNSQ	1.02 (0.66)	1.00 (0.10)
Outfit MNSQ	1.05 (0.75)	1.05 (0.09)
Separation Index	1.68	8.43
Reliability	0.74	0.99

Note. Person n= 286. Item n= 10. MNSQ= mean-square statistic, expectation of 1 (Values substantially below 1 indicate dependency data; values substantially above 1 indicate noise).

Table 9.10:

Student-Centred Teaching Practice Summary of Category Structure

Category	Observed Count (%)	Measurement Mean (Expected)	Infit MNSQ	Outfit MNSQ
1. Never	525 (18)	-1.09 (-1.09)	1.05	1.16
2. Rarely	899 (32)	-0.38 (-0.36)	0.87	0.89
3. Frequently	876 (31)	0.41 (0.37)	0.89	0.90
4. Always or almost always	539 (19)	1.17 (1.20)	1.07	1.15
Missing	21 (1)			

Note. Measurement statistics are expected to increase with category value. The expected values for Infit and Outfit MNSQ (mean-square statistic) for all categories are 1.0. Only values greater than 1.5 are problematic.

Table 9.11:

Student-Centred Teaching Practice Items Fit Statistics

Item	Measure	Model S.E	Infit MNSQ	Outfit MNSQ
We work together in group projects	0.60	0.08	0.89	0.91
We talk with other about how to solve a problem	-0.49	0.08	1.01	1.05
We ask others to explain their ideas	-0.42	0.08	1.01	1.07
We do projects that include other subjects	1.05	0.08	1.15	1.14
We learn how maths have changed	-0.40	0.08	0.96	0.97
We learn that maths is about inventing rules	0.57	0.08	1.08	1.08
We research topics on our own	0.85	0.08	0.98	1.07
We discuss ideas with he whole classroom	-0.50	0.08	0.90	1.04
We explain our work to the whole class	-0.10	0.08	0.85	0.92
We answer teacher's questions	-1.17	0.08	1.14	1.22
Mean	0.00	0.08	1.00	1.05
SD	0.69	0.00	0.10	0.09

Note. Measure higher values indicate item is more difficult to perceive higher frequency. The expected values for Infit and Outfit MNSQ (mean-square statistic) are 1.0. Values higher than 1.30 indicate misfitting items. Values lower 0.70 indicate overfitting items.

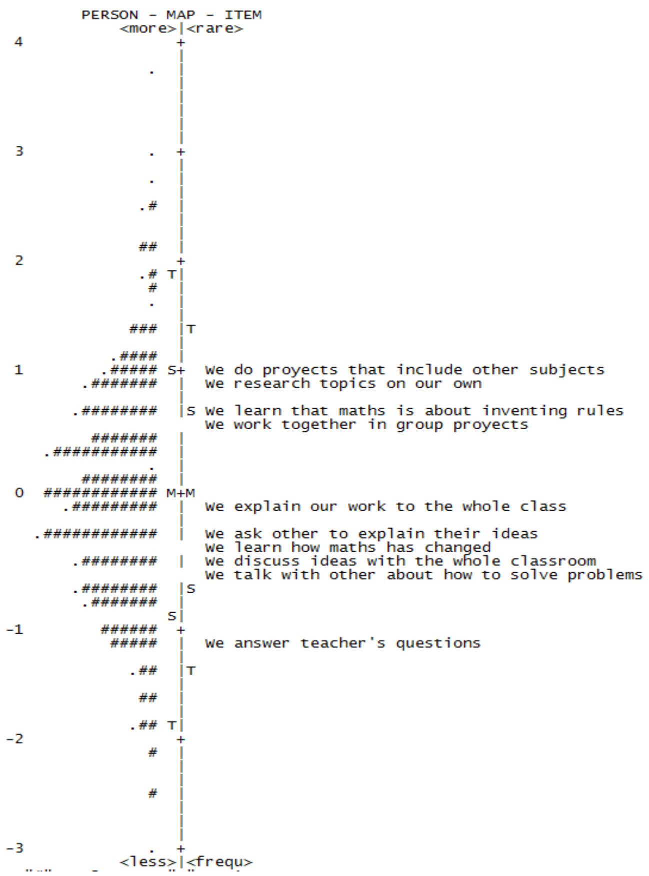


Figure 9.6: Student-Centred Teaching Practice Item – Person Map

9.16 Appendix 16: Rasch analysis Positive affect scale (PANAS)

Table 9.12:

Positive Affect Scale Summary of Measurement Statistics

	Person Mean (SD)	Item Mean (SD)
Measurement	0.04 (1.13)	0.00 (0.30)
Infit MNSQ	1.01 (0.71)	1.01 (0.23)
Outfit MNSQ	1.02 (0.73)	1.03 (0.21)
Separation Index	2.20	4.24
Reliability	0.83	0.95

Note. Person n= 285. Item n= 10. MNSQ= mean-square statistic, expectation of 1 (Values substantially below 1 indicate dependency data; values substantially above 1 indicate noise).

Table 9.13:

Positive Affect Scale Summary of Category Structure

Category	Observed Count (%)	Measurement Mean (Expected)	Infit MNSQ	Outfit MNSQ
1. Very slightly or not at all	440 (16)	-1.00 (-1.09)	1.19	1.24
2. A little	524 (19)	-0.65 (-0.56)	0.81	0.82
3. Moderately	773 (28)	-0.08 (-0.06)	0.92	0.95
4. Quite a bit	580 (21)	0.58 (0.52)	0.78	0.79
5. Extremely	439 (16)	1.32 (1.33)	1.13	1.19
Missing	84 (3)			

Note. Measurement statistics are expected to increase with category value. The expected values for Infit and Outfit MNSQ (mean-square statistic) for all categories are 1.0. Only values greater than 1.5 are problematic.

Table 9.14:

Positive Affect Scale Items Fit Statistics

Item	Measure	Model S.E	Infit MNSQ	Outfit MNSQ
Alert	0.72	0.07	1.45	1.43
Strong	0.30	0.07	1.22	1.23
Inspired	0.08	0.07	0.88	0.89
Excited	-0.02	0.07	0.72	0.73
Proud	-0.06	0.06	0.97	1.08
Enthusiast	-0.07	0.07	1.11	1.18
Determined	-0.12	0.07	0.94	0.90
Interested	-0.12	0.06	0.71	0.82
Attentive	-0.27	0.07	0.84	0.85
Active	-0.43	0.07	1.24	1.21
Mean	0.00	0.07	1.01	1.03
SD	0.30	0.00	0.21	0.21

Note. Measure higher values indicate item is more difficult to perceive higher frequency. The expected values for Infit and Outfit MNSQ (mean-square statistic) are 1.0. Values higher than 1.30 indicate misfitting items. Values lower 0.70 indicate overfitting items.

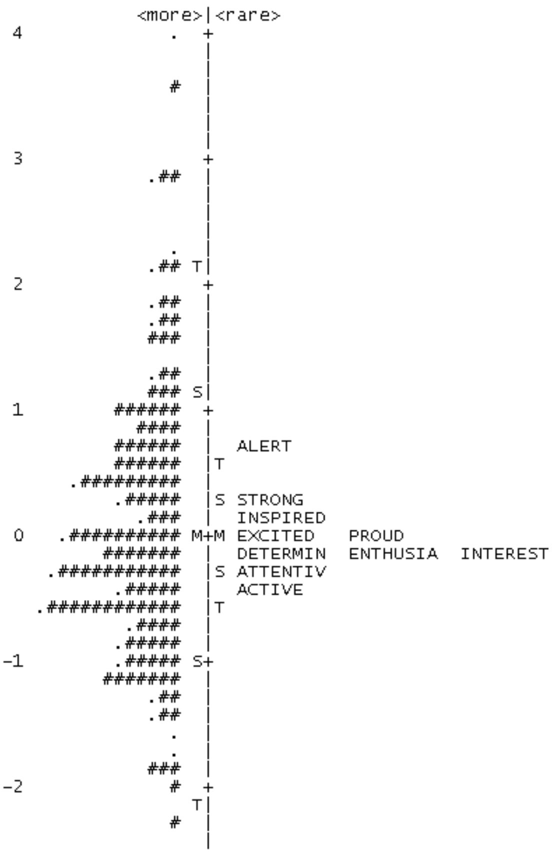


Figure 9.7: Positive Affect Scale Item – Person Map

9.17 Appendix 17: Rasch analysis Negative affect scale (PANAS)

Table 9.15:

Negative Affect Scale Summary of Measurement Statistics

	Person Mean (SD)	Item Mean (SD)
Measurement	-1.17 (1.10)	0.00 (0.41)
Infit MNSQ	1.02 (0.60)	1.04 (0.18)
Outfit MNSQ	0.98 (0.66)	0.99 (0.19)
Separation Index	1.33	5.52
Reliability	0.64	0.97

Note. Person n= 286. Item n= 10. MNSQ= mean-square statistic, expectation of 1 (Values substantially below 1 indicate dependency data; values substantially above 1 indicate noise).

Table 9.16:

Negative Affect Scale Summary of Category Structure

Category	Observed Count (%)	Measurement Mean (Expected)	Infit MNSQ	Outfit MNSQ
1. Very slightly or not at all	1432 (51)	-1.49 (-1.45)	1.04	1.03
2. A little	653 (23)	-0.83 (-0.93)	1.03	0.80
3. Moderately	368 (13)	-0.50 (-0.50)	0.97	0.90
4. Quite a bit	167 (6)	-0.07 (-0.08)	1.02	1.04
5. Extremely	166 (6)	0.27 (0.38)	1.17	1.20
Missing	63 (2)			

Note. Measurement statistics are expected to increase with category value. The expected values for Infit and Outfit MNSQ (mean-square statistic) for all categories are 1.0. Only values greater than 1.5 are problematic.

Table 9.17:

Negative Affect Scale Items Fit Statistics

Item	Measure	Model S.E	Infit MNSQ	Outfit MNSQ
Distressed	-0.04	0.07	0.66	0.76
Upset	0.29	0.08	0.84	0.69
Guilty	0.55	0.08	1.15	0.99
Scared	0.22	0.07	1.05	1.00
Hostile	0.23	0.07	0.98	1.02
Bored	-0.85	0.06	1.25	1.29
Irritable	0.16	0.07	0.91	0.80
Ashamed	0.28	0.08	1.10	0.96
Nervous	-0.35	0.06	1.26	1.28
Jittery	0.50	0.06	1.18	1.14
Mean	0.00	0.07	1.04	0.99
SD	0.41	0.01	0.18	0.19

Note. Measure higher values indicate item is more difficult to perceive higher frequency. The expected values for Infit and Outfit MNSQ (mean-square statistic) are 1.0. Values higher than 1.30 indicate misfitting items. Values lower 0.70 indicate overfitting items.

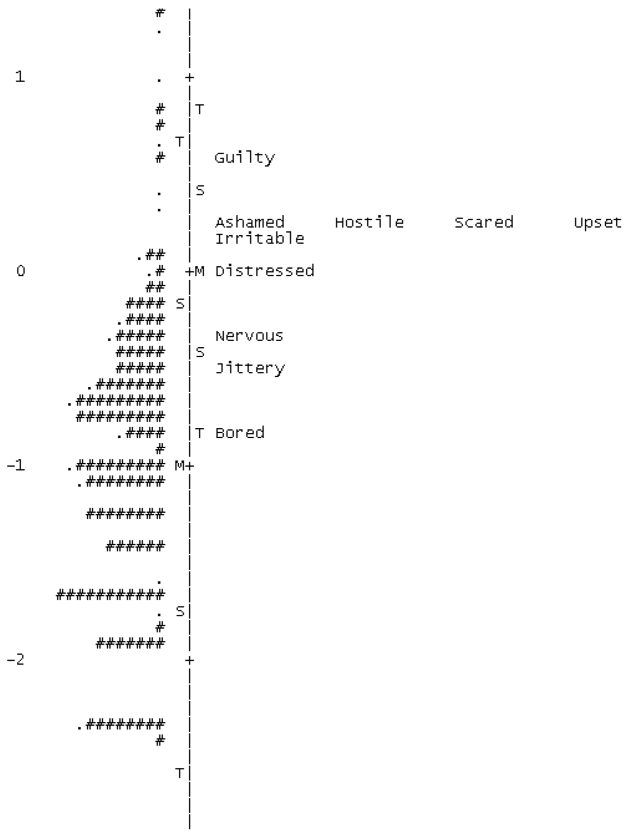


Figure 9.8: Negative Affect Scale Item – Person Map

9.18 Appendix 18: Rasch analysis Self-Concept scale

Table 9.18:

Self-Concept Summary of Measurement Statistics

	Person Mean (SD)	Item Mean (SD)
Measurement	1.07 (2.27)	0.00 (1.09)
Infit MNSQ	0.97 (0.85)	0.99 (0.18)
Outfit MNSQ	0.98 (0.88)	0.99 (0.24)
Separation Index	2.03	11.11
Reliability	0.80	0.99

Note. Person n= 291. Item n= 4. MNSQ= mean-square statistic, expectation of 1 (Values substantially below 1 indicate dependency data; values substantially above 1 indicate noise).

Table 9.19:

Self-Concept Summary of Category Structure

Category	Observed Count (%)	Measurement Mean (Expected)	Infit MNSQ	Outfit MNSQ
1. Strongly Disagree	81 (7)	-2.68 (-2.71)	1.06	1.07
2. Disagree	155 (13)	-1.33 (-1.24)	0.92	0.90
3. Unsure	327 (28)	0.13 (0.15)	0.86	0.83
4. Agree	340 (29)	1.85 (1.73)	0.86	0.97
5. Strongly Agree	253 (22)	3.47 (3.59)	1.29	1.22
Missing	21 (1)			

Note. Measurement statistics are expected to increase with category value. The expected values for Infit and Outfit MNSQ (mean-square statistic) for all categories are 1.0. Only values greater than 1.5 are problematic.

Table 9.20:

Self-Concept Items Fit Statistics

Item	Measure	Model S.E	Infit MNSQ	Outfit MNSQ
I have a mathematical mind	1.12	0.09	0.83	0.81
I can get good results in maths	-1.32	0.10	0.90	0.82
I can learn maths even if it’s hard	-0.84	0.10	1.30	1.39
Compare to my classmates, I’m good at maths	1.03	0.09	0.94	0.93
Mean	0.00	0.09	0.99	0.99
SD	1.09	0.00	0.18	0.24

Note. Measure higher values indicate item is more difficult to perceive higher frequency. The expected values for Infit and Outfit MNSQ (mean-square statistic) are 1.0. Values higher than 1.30 indicate misfitting items. Values lower 0.70 indicate overfitting items.

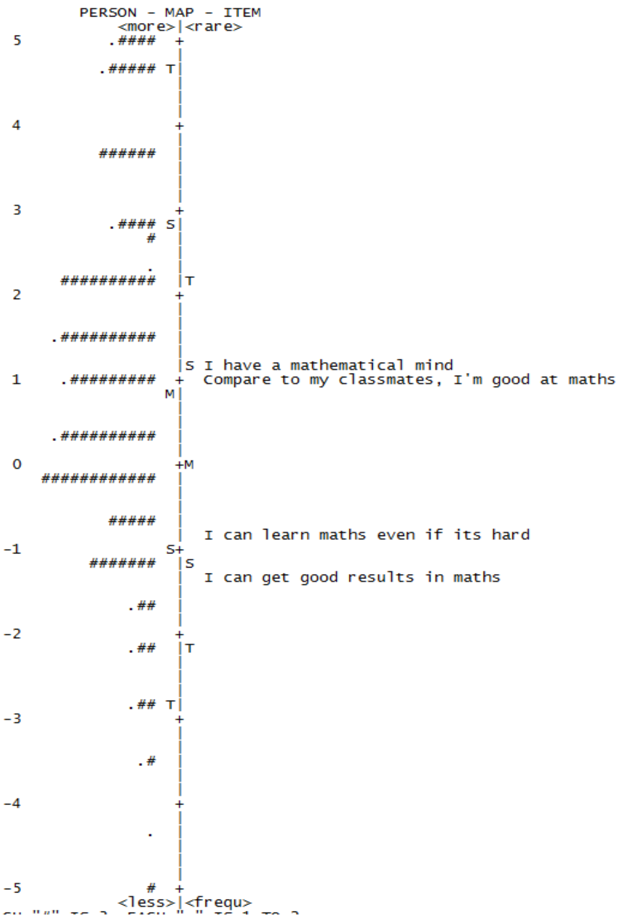


Figure 9.9: Self-Concept Item – Person Map

9.19 Appendix 19: Rasch Analysis Dispositions Scale

Table 9.21:

Dispositions Summary of Measurement Statistics

	Person Mean (SD)	Item Mean (SD)
Measurement	0.16 (1.82)	0.00 (1.26)
Infit MNSQ	0.98 (0.72)	0.98 (0.22)
Outfit MNSQ	0.99 (0.82)	0.99 (0.21)
Separation Index	2.30	14.79
Reliability	0.84	1.00

Note. Person n= 291. Item n= 6. MNSQ= mean-square statistic, expectation of 1 (Values substantially below 1 indicate dependency data; values substantially above 1 indicate noise).

Table 9.22:

Dispositions Summary of Category Structure

Category	Observed Count (%)	Measurement Mean (Expected)	Infit MNSQ	Outfit MNSQ
1. Strongly Disagree	319 (18)	-2.37 (-2.32)	0.88	0.90
2. Disagree	264 (15)	-1.20 (-1.17)	0.90	0.90
3. Unsure	456 (26)	0.01 (-0.06)	0.96	1.03
4. Agree	351 (20)	1.18 (1.12)	0.91	0.91
5. Strongly Agree	343 (20)	2.46 (2.55)	1.25	1.19
Missing	12 (1)			

Note. Measurement statistics are expected to increase with category value. The expected values for Infit and Outfit MNSQ (mean-square statistic) for all categories are 1.0. Only values greater than 1.5 are problematic.

Table 9.23:

Dispositions Items Fit Statistics

Item	Measure	Model S.E	Infit MNSQ	Outfit MNSQ
Maths is important for my future	-1.73	0.09	0.86	0.95
Maths is one of the most interesting subjects in school	-0.10	0.08	1.27	1.27
I would prefer my future studies to include a lot of maths	0.78	0.08	0.83	0.84
I would look forward to stuying more maths after school	0.89	0.08	0.75	0.73
I would like to be a mathematician	1.67	0.08	0.91	0.88
Maths is important for my future (after school)	-1.52	0.09	1.29	1.27
Mean	0.00	0.08	0.98	0.99
SD	1.26	0.00	0.22	0.21

Note. Measure higher values indicate item is more difficult to perceive higher frequency. The expected values for Infit and Outfit MNSQ (mean-square statistic) are 1.0. Values higher than 1.30 indicate misfitting items. Values lower 0.70 indicate overfitting items.

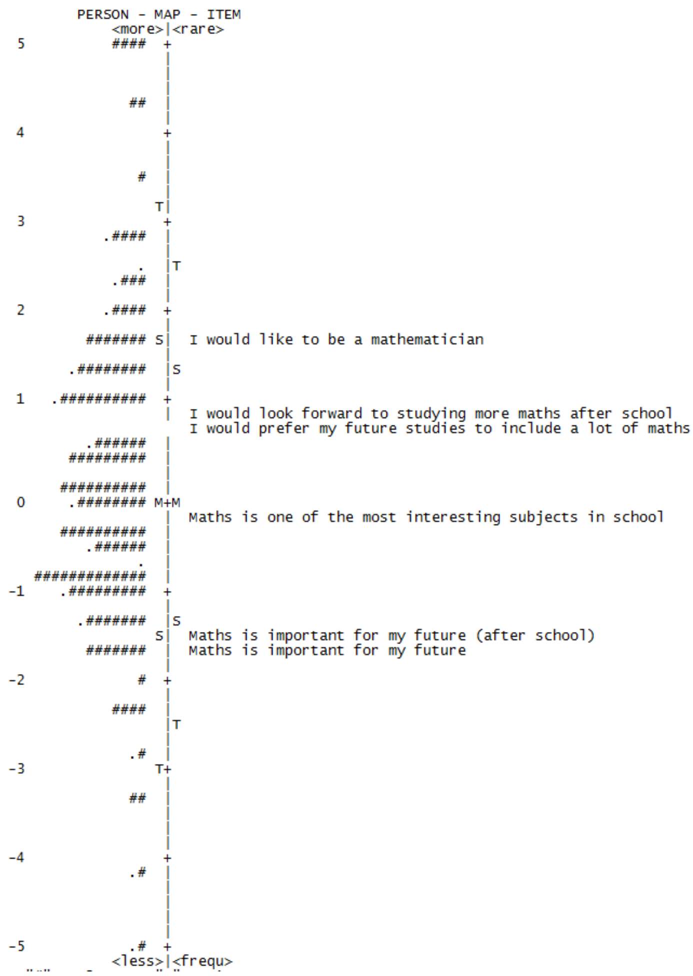


Figure 9.10: Dispositions Item – Person Map

Chapter 10. References

Abu-Hilal, M. M., Abdelfattah, F. A., Shumrani, S. A., Dodeen, H., Abduljabber, A. S., & Marsh, H. W. (2014). Mathematics and science achievements predicted by self-concept and subject value among 8th grade Saudi students: Invariance across gender. *International Perspectives in Psychology: Research, Practice, Consultation*, 3(4), 268.

Agencia de Calidad de la Educacion (2013a) Diferencias actitudinales entre hombres y mujeres en matemática: Análisis de los resultados de la Prueba PISA 2012. *Apuntes sobre la calidad de la Educacion, 1*(12) Retrieved from http://www.agenciaeducacion.cl/biblioteca-digital/apuntes-2/

Agencia de Calidad de la Educacion (2013b) Brecha de genero: Chile en la comparacion internacional. *Apuntes sobre la calidad de la Educacion, 1*(3) Retrieved from http://www.agenciaeducacion.cl/biblioteca-digital/apuntes-2/

Agencia de Calidad de la Educacion (2013c) Esa Brecha… Apuntes sobre la calidad de la Educacion, 1(2) Retrieved from http://www.agenciaeducacion.cl/biblioteca-digital/apuntes-2/

Agencia de Calidad de la Educacion (2014) Otros indicadores de calidad de la educacion. Retrieved from http://www.agenciaeducacion.cl/coordinacion-sac/otros-indicadores-de-calidad-educativa/

Aguayo, F., Correa, P., Cristi, P. (2011) *Encuesta IMAGES. Chile Resultados de la Encuesta Internacional de Masculinidades y Equidad de Género.* Santiago: CulturaSalud/EME

Aguinis, H. (1995). Statistical power with moderated multiple regression in management research. *Journal of Management*, *21*(6), 1141-1158.

Ahlqvist, S., London, B. & Rosenthal, L. (2013). Unstable identity compatibility: How gender rejection sensitivity undermines the success of women in science, technology, engineering, and mathematics fields. *Psychological Science,* 24(9), 1644-1652.

Ali-Khan, C., & Siry, C. (2014). Sharing seeing: Exploring photo-elicitation with children in two different cultural contexts. *Teaching and Teacher Education*, *37*, 194-207.

Alves Macedo, G. (2004). Fatores associados ao rendimento escolar de alunos da 5a série (2000)- uma abordagem do valor adicionado e da heterogeneidade, Centro de Desenvolvimento e Planejamento Regional (CEDEPLAR). Belo Horizonte: Faculdade de Ciencias Econômicas da Universidade Federal de Minas Gerais, Retrieved from http://www.abep.nepo.unicamp.br/site_eventos_abep/PDF/ABEP2004_609.pdf [Accessed January 10th 2015].

Andersen, L., & Ward, T. J. (2014). Expectancy-value models for the STEM persistence plans of ninth-grade, high-ability Students: A comparison between black, hispanic, and white students. *Science Education*, 98(2), 216-242.

Anderson, D. D., & Gold, E. (2006). Home to school: Numeracy practices and mathematical identities. *Mathematical Thinking and Learning*, *8*(3), 261-286.

Anglin, L. P., Pirson, M., & Langer, E. (2008). Mindful learning: A moderator of gender differences in mathematics performance. *Journal of Adult Development*, 15(3-4), 132-139.

Archer, J. (1996). Comparing women and men: What is being compared and why? *American Psychologist*, *51*(2), 153-154.

Archer, L., Dewitt, J., Osborne, J., Dillon, J., Willis, B. & Wong, B. (2012). Balancing acts: Elementary school girls' negotiations of femininity, achievement, and science. *Science Education*, 96(6), 967-989

Askew, M., Brown, M., Rhodes, V., Johnson, D. & Wiliam, D. (1997). *Effective teachers of numeracy*. London: Kings College.

Atweh, B. & Cooper, T. (1995). The construction of gender, social class and mathematics in the classroom. *Educational Studies in Mathematics*, 28, 293-310.

Avalos, B. (2003). *Gender parity and equality in Chile : a case study*. commissioned as a background paper for the UNESCO `Education for All' Global Monitoring Report 2003/4. Retrieved from http://portal.unesco.org/education/en/file_download.php/02c26938b259de80e7163ebd3c1865efGender+parity+and+equality+in+Chile.rtfhttp://portal.unesco.org/education/en/file_download.php/02c26938b259de80e7163ebd3c1865efGender+parity+and+equality+in+Chile.rtf

Axelsson, G. B. (2009). Mathematical identity in women: The concept, its components and relationship to educative ability, achievement and family support. *International Journal of Lifelong Education*, *28*(3), 383-406.

Baker, D. P., & Jones, D. P. (1993). Creating gender equality: Cross- national gender stratification and mathematical performance. *Sociology of Education, 66,* 91–103.

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory.* Engle- wood Cliffs, NJ: Prentice Hall.

Bartholomew, H., Darragh, L., Ell, F., & Saunders, J. (2011). 'I'm a natural and I do it for love!': exploring students' accounts of studying mathematics. *International Journal of Mathematical Education in Science and Technology*, *42*(7), 915-924.

Becker, J. R. (1995). Women's ways of knowing in mathematics. In Kaiser, G. & Rogers, P. (Eds.), *Equity in mathematics education: Influences of feminism and culture* (pp. 163–174). London: Falmer.

Belenky, M., Clinchy, B., Goldberger, N. & Tarule, J. (1986). *Women's ways of knowing: the development of self, voice and mind*. New York, Basic.

Berry, R.Q. (2008). Access to Upper-Level Mathematics: The Stories of Successful African American Middle School Boys. *Journal For Research In Mathematics Education*, 39(5), 464-488

Bishop, J.P. (2012). She's Always Been the Smart One. I've Always Been the Dumb One: Identities in the Mathematics Classroom. *Journal For Research In Mathematics Education*, 43(1), 34-74

Black, L. (2004). Differential participation in whole-class discussions and the construction of marginalised identities. *Journal of Educational Enquiry*, *5*(1), 34–54.

Black, L., Mendick, H. & Solomon, Y. (Eds.) (2008). *Mathematical relationships in education: Identities and participation.* New York: Routledge.

Black, L., & Williams, J. (2013). Contradiction and conflict between 'leading identities': becoming an engineer versus becoming a 'good muslim'woman. *Educational Studies in Mathematics*, *84*(1), 1-14.

Black, L., Williams, J., Hernandez-Martinez, P., Davis, P., Pampaka, M. & Wake, G. (2010). Developing a 'leading identity': the relationship between students' mathematical identities and their career and higher education aspirations. *Educational Studies in Mathematics*, 73(1), 55-72

Blanco, E.; De los Heros, M.; Florez, N.; Luna, M. & Zertuche, M. (2007). Factores asociados al logro educativo de matemáticas y español en la Prueba ENLACE 2007: un análisis multinivel, México: Facultad Latinoamericana de Ciencias Sociales (FLACSO), Retrieved from http://www.dgep.sep.gob.mx/enlace2008/flacso.pdf [Accessed January 10th 2015]

Boaler, J. (1999b). Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School. *Teachers College Record*, 110(3), 608-645

Boaler, J. (1999a). Participation, knowledge and beliefs: A community perspective on mathematics learning. *Educational studies in mathematics*, *40*(3), 259-281.

Boaler, J. (2000). Exploring Situated Insights into Research and Learning. *Journal for Research in Mathematics Education*, *31*(1), 113–119.

Boaler, J. (2002a). Paying the Price for " Sugar and Spice ": Shifting the Analytical Lens in Equity Research. *Mathematical Thinking and Learning*, *4*(2-3), 127–144.

Boaler, J. (2002b). *Experiencing School Mathematics: Traditional and Reform Approaches to Teaching and Their Impact on Student Learning*. New Jersey: Lawrence Erlbaum Associates.

Boaler, J. (2008). Promoting “relational equity” and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal*, *34*(2), 167–194.

Boaler, J., Altendorff, L., & Kent, G. (2011). Mathematics and science inequalities in the United Kingdom: when elitism, sexism and culture collide. *Oxford Review of Education*, *37*(4), 457-484.

Boaler, J., & Greeno, G. G. (2000). Identity, agency, and knowing in mathematics worlds. In J.Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171–200). Westport, CT: Ablex Publishing.

Boaler, J. & Staples, M. (2008). Participation, Knowledge and Beliefs: A Community Perspective on Mathematics Learning. *Educational Studies in Mathematics*, 40(3), 259-281

Boaler, J., William, D., & Zevenbergen, R. (2000). The construction of identity in secondary mathematics education. In J. Matos & M. Santos (Eds.), *Proceedings of the 2nd Mathematics Education and Society conference*. Lisbon: The University of Lisbon.

Boe, M.V. (2012). Science choices in Norwegian upper secondary school: What matters?. *Science Education*, 96(1), 1-20

Bond, T.G. & Fox, C.M. (2001). *Applying the Rasch Model: Fundamental Measurement in the Human Sciences.* New Jersey: Lawrence Erlbaum Associates.

Bourdieu, P. (1977). Outline of a theory of practice. Cambridge: Cambridge University Press.

Bourdieu, P. (1980). The Logic of Practice. Stanford, CA: Stanford University Press.

Bourdieu, P. (2001) *Masculine Domination.* Stanford: Stanford University Press.

Braathe, H.J. & Solomon, Y. (2015). Choosing mathematics: the narrative of the self as a site of agency. Educational Studies in Mathematics, 89(2), 151-166

Brew, C. (2001). Women, mathematics and epistemology: an integrated framework. *International Journal of Inclusive Education*, 5(1), 15-32.

Brown, B. (2004). Adolescent's relationships with peers. In R. Lerner & Steinberg, L. (Eds.) *Handbook of adolescent psychology*, 2nd edition. New Jersey: John Wiley & Sons, Inc.

Brown, M., Hodgen, J., Kuechemann, D., Coe, R. & Pepper, D.. (2010). Attitudes, Gender And Attainment: Evidence From A Large-Scale Survey In England. In Unknown. Praha 1 : Int Group Psychology Mathematics Educaction, (pp. 217 – 224) (Pme 34 Brazil: *Proceedings Of The 34th Conference Of The International Group For The Psychology Of Mathematics Education*, Vol 2).

Brubaker, R., & Cooper, F. (2000). Beyond "identity". *Theory and society*, *29*(1), 1-47.

Brunner, J.J. & Elacqua, G. (2003). Factores que inciden en una educación efectiva. Evidencia internacional. *Revista virtual La educación*, *139-140*, 1-11.

Brunner, J. J., Elacqua, G., Tillett, A., Bonnefoy, J., González, S., Pacheco, P., & Salazar, F. (2005). *Guiar el mercado. Informe sobre la educación superior en Chile*.

Buerk, D. (1985). The voices of women making meaning in mathematics. *Journal of Education*, 59-70.

Buschor, C.B., Berweger, S., Frei, A.K. & Kappler, C. (2014). Majoring in STEM-What Accounts for Women's Career Decision Making? A Mixed Methods Study. *Journal Of Educational Research*, 107(3), 167-176

Buschor, C.B., Kappler, C., Keck Frei, A., & Berweger, S. (2014). I want to be a scientist/a teacher: students' perceptions of career decision-making in gender-typed, non-traditional areas of work. *Gender and Education*, *26*(7), 743-758.

Buxton, C. A. (2005). Creating a culture of academic success in an urban science and math magnet high school. *Science Education*, *89*(3), 392-417.

Byrnes, J. P., Hong, L., & Xing, S. (1997). Gender differences on the math subtest of the Scholastic Aptitude Test may be culture-specific. *Educational Studies in Mathematics*, *34*(1), 49-66.

Cairns, R., Gariepy, J. L., Kinderman, T. & Leung, M. C. (1996). *Identifying social clusters in natural settings*. Unpublished manuscript, University of North Carolina at Chapel Hill.

Cairns, R., Xie, H., & Leung, M. C. (1998). The popularity of friendship and the neglect of social networks: toward a new balance. *New Directions for Child Development*, *81*, 25–53.

Campbell, T., Lee, H., Kwon, H., & Kyungsuk, P. (2012). Student motivation and interests as proxies for forming STEM identities. Journal of the Korean Association for Science Education, 32(3), 532-540.

Caplan, J. & Caplan, P. (2005). The Perseverative Search for Sex Differences in Mathematics Ability. In A.M.Gallagher & J.C. Kaufman (Eds.), *Gender differences in mathematics: An integrative psychological approach* (pp.1-24). Cambridge University Press.

Cárdenas, A., Correa, N. & Prado, X. (2012). *Mujeres y poder: participación en espacios de decisión*. ICSO, UDP. Retrieved from http://www.udp.cl/investigacion/repo_listado.asp

CASEN (2009) *Tabla Población De 0 A 24 Años Que Asiste Por Dependencia Según Región,* Retrieved from http://observatorio.ministeriodesarrollosocial.gob.cl/casen/casen-documentos.php?c=87&a=2009

CASEN (2013a) Una Medición de la Pobreza Moderna y Transparente para Chile. Retrieved from http://observatorio.ministeriodesarrollosocial.gob.cl/documentos/Presentacion_Resultados_Encuesta_Casen_2013.pdf

CASEN (2013b). Educacion, Sintesis de Resultados. Retrieved from http://observatorio.ministeriodesarrollosocial.gob.cl/documentos/Casen2013_Educacion.pdf

Cervini, R., & Dari, N. (2009). Género, escuela y logro escolar en matemática y lengua de la educación media: estudio exploratorio basado en un modelo multinivel bivariado. *Revista mexicana de investigación educativa*, *14*(43), 1051-1078.

Chipman, S.F. (2005) Research on the Women and Mathematics Issue: A Personal Case History. In A.M.Gallagher & J.C. Kaufman (Eds.), *Gender differences in mathematics: An integrative psychological approach* (pp.1-24). Cambridge University Press.

Chouinard, R., & Roy, N. (2008). Changes in high-school students' competence beliefs, utility value and achievement goals in mathematics. *British Journal of Educational Psychology*, *78*(1), 31-50.

Chronaki, A. (2005). Learning about 'learning identities' in the school arithmetic practice: The experience of two young minority Gypsy girls in the Greek context of education. *European Journal Of Psychology Of Education*, 20(1), 61-74

Chronaki, A. (2013). Identity work as a political space for change: The case of mathematics teaching through technology use. In M. Berger, K. Brodie, V. Frith & K. le Roux (Eds.), *Proceedings of the Seventh International Mathematics Education and Society Conference* (Vol. 1, pp. 1-18). Cape Town, South Africa: MES 7.

Clandinin, D. J., & Connelly, F. M. (2000). Narrative inquiry: Experience and story in qualitative research.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Sciences*, 10(1-2), 113-163.

Comunidad Mujer. (2014) Mujer y trabajo: ¿Qué estudian mujeres y hombres? *Serie Comunidad Mujer*, 26. Santiago, Chile. Retrieved from http://issuu.com/comunidadmujer/docs/boletin_enero_2014_vf

Connolly, P. (2006). The Effects of Social Class and Ethnicity on Gender Differences in GCSE Attainment: A Secondary Analysis of the Youth Cohort Study of England and Wales 1997-2001. *British Educational Research Journal*, *32*(1), 3-21.

Contreras, D., & Puentes, E. (2001). Is Gender Wage Discrimination Decreasing In Chile? Thirty Years Of 'Robust'Evidence. *Santiago, Chile: Universdiad de Chile: Departamento de Economía*.

Contu, A., & Willmott, H. (2003). Re-embedding situatedness: The importance of power relations in learning theory. *Organization science*, *14*(3), 283-296.

Cooper, K. S. (2013). Eliciting Engagement in the High School Classroom A Mixed-Methods Examination of Teaching Practices. *American Educational Research Journal*, 51(2), 363-402.

Cornejo, C.R., & Redondo,R, J. M. (2007). Variables y factores asociados al aprendizaje escolar: una discusión desde la investigación actual. *Estudios pedagógicos (Valdivia)*, *33*(2), 155-175.

Corporación Humanas (2009) *VI Encuesta Nacional: Percepción de las mujeres sobre su situación y condiciones de vida en Chile, año 2009*. Santiago: Corporación Humanas.

Craig, T. S. (2013). Conceptions of mathematics and student identity: implications for engineering education. *International Journal of Mathematical Education in Science and Technology*, *44*(7), 1020-1029.

Crawford, J. R., & Henry, J. D. (2004). The Positive and Negative Affect Schedule (PANAS): Construct validity, measurement properties and normative data in a large non-clinical sample. *British Journal of Clinical Psychology*, *43*(3), 245.

Creswell, J. (2003) *Research Design: Qualitative, Quantitative and Mixed Methods Approaches*. Thousand Oaks, CA: Sage.

Cuban, L. (1983). How did teachers teach, 1890–1980. *Theory into practice*, *22*(3), 159–165. Retrieved from http://www.tandfonline.com/doi/abs/10.1080/00405848309543056

Cudeck, R. (2000). Exploratory factor analysis. in H. Tinsley & S. Brown (Eds). *Handbook of applied multivariate statistics and mathematical modelling* (pp. 265-296). San Diego, California: Academic Press.

Damarin, S. K. (2000). The mathematically able as a marked category. *Gender and Education*, *12*(1), 69-85.

Daniels, L. M., Stupnisky, R. H., Pekrun, R., Haynes, T. L., Perry, R. P., & Newall, N. E. (2009). A longitudinal analysis of achievement goals: From affective antecedents to emotional effects and achievement outcomes. *Journal of Educational Psychology*, *101*(4), 948.

Darragh, L. (2013). Constructing confidence and identities of belonging in mathematics at the transition to secondary school. *Research in Mathematics Education*, *15*(3), 215-229.

Darragh, L. (2015). Recognising 'good at mathematics': using a performative lens for identity. *Mathematics Education Research Journal*, *27*(1), 83-102

de Abreu, G. (1995). Understanding how children experience the relationship between home and school mathematics. *Mind, Culture, and Activity,* 2(2), 119-142.

del Río, M. F., & Balladares, J. (2010). Género y Nivel Socioeconómico de los Niños: Expectativas del Docente en Formación. *Psykhe, 19*(2), 81-90.

del Río, M. F., & Strasser, K. (2013). Preschool children's beliefs about gender differences in academic skills. *Sex roles*, *68*(3-4), 231-238.

Denzin, N.K., & Lincoln, Y.S. (2000) Introduction: Entering the field of qualitative research. In Denzin, N.K. & Lincoln, Y.S. (Eds), *Handbook of qualitative research* (pp. 1 - 28). Thousand Oaks, CA: Sage.

Desimone, L., & Long, D. A. (2010). Teacher effects and the achievement gap: Do teacher and teaching quality influence the achievement gap between Black and White and high-and low-SES students in the early grades. *Teachers College Record*, *112*(12), 3024-3073.

Desimone, L., Smith, T., & Frisvold, D. (2009). Survey measures of classroom instruction: Comparing student and teacher reports. *Educational Policy*, 24(2), 267-329

DOEE (1998). *Informe de resultados. Tercer curso.* Asunción: Dirección de Orientación y Evaluación Educativa.

Dufey, M. & Fernández, A. M. (2012). Validez y confiabilidad del Positive Affect and Negative Affect Schedule (PANAS) en estudiantes universitarios chilenos. *Revista Iberoamericana de Diagnóstico y Evaluación Psicológica*, *34*(2), 157-173.

Eccles, J. S. (1994). Understanding women's educational and occupational choices. *Psychology of women quarterly*, 18(4), 585-609.

Eccles, J. S., Jacobs, J. E., & Harold, R. D. (1990). Gender role stereotypes, expectancy effects, and parents' socialization of gender differences. *Journal of Social Issues*, *46*(2), 183-201.

Eccles, J., Wigfield, A., Harold, R. D., & Blumenfeld, P. (1993). Age and Gender Differences in Children's Self- and Task Perceptions during Elementary School. *Child Development, 64*, 830-847.

El'konin, D. B. (1971). Toward The Problem of Stages in the Mental Development of Children. *Voprosy psikhologii*, (4), 6-20. Retrieved from Psychology and Marxism Internet Archive: http://www.marxists.org/archive/elkonin/works/1971/stages.htm

Ellis, M. W., Malloy, C. E., Meece, J. L., & Sylvester, P. R. (2007). Convergence of observer ratings and student perceptions of reform practices in sixth-grade mathematics classrooms. *Learning Environments Research*, *10*(1), 1-15.

Else-Quest, N.M., Hyde, J.S. & Linn, M.C. (2010). Cross-national patterns of gender differences in mathematics: a meta-analysis. *Psychological Bulletin*, *136*(1), 103-127.

Elwood, J. (1999). Equity issues in performance assessment: The contribution of teacher-assessed coursework to gender-related differences in examination performance. *Educational Research and Evaluation*, *5*(4), 321-344.

Empson, S.B (2003). Low-performing students and teaching fractions for understanding: An interactional analysis. *Journal For Research In Mathematics Education*, 34(4), 305-343

England, P. & Browne, I. (1992) Trends in women's economic status, *Sociological Perspective, 35*, 17–51.

Epstein, D., Mendick, H. & Moreau, M-P. (2010). Imagining the mathematician: young people talking about popular representations of maths. *Discourse-Studies in the Cultural Politics of Education*, 31(1), 45-60

Erikson, E. (1968) *Identity Youth and Crisis*. New York: Norton & Company

Erikson, E. (1980) *Identity and the Life Cycle*. New York: Norton & Company

Erikson, E. (1982) *The Life Cycle Completed*. New York: Norton & Company

Escobar (2001). Perspectivas históricas en torno al desarrollo de la educación de las mujeres en Chile, 1938-1952. In Escobar, Flores & Veneros (Eds.), *Investigaciones Red Nacional Universitaria Interdisciplinaria de Estudios de Género.* Santiago: SERNAM.

Esmonde, I., & Langer-Osuna, J. M. (2013). Power in numbers: Student participation in mathematical discussions in heterogeneous spaces. *Journal for Research in Mathematics Education*, 44(1), 288-315.

Esteve, A., & López-Ruiz, L. (2010). Union Formation Implications of Race and Gender Gaps in Educational Attainment: The Case of Latin America. *Population research and policy review*, *29*(5), 609-637.

Evans, J. (2000). *Adults' mathematical thinking and emotions: A study of numerate practice*. London: Routledge.

Feingold, A. (1992). Sex differences in variability in intellectual abilities: A new look at an old controversy. *Review of Educational Research*, *62*(1), 61-84.

Fennema, E., Carpenter, T. P., Jacobs, V. R., Franke, M. L., & Levi, L. W. (1998). New Perspectives on Gender Differences in Mathematics: A Reprise. *Educational Researcher*, *27*(5), 19–21.

Fennema, E. & Peterson, P. (1985) Authonomous Learning Behavious: A Possible Explanation of Gender-Related Differences in Mathematics, in L.C. Wilkinson & C. Marrett, C. (Eds.). *Gender influences in classroom interaction*. Orlando (pp.17-35), FL: Academic Press.

Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization and affective factors. *American educational research journal*, *14*(1), 51-71.

Field, A. (2009). *Discovering statistics using SPSS*. Sage publications.

Forster, P.A. (2000). Katie Thought She Couldn't Do It but Now She Knows She Can. *Educational Studies in Mathematics*, 43(3), 225-242

Francis, B., Skelton, C., & Read, B. (2010). The simultaneous production of educational achievement and popularity: How do some pupils accomplish it? *British Educational Research Journal*, *36*(2), 317-340.

Fredricks, J. A., & Eccles, J. S. (2002). Children's competence and value beliefs from childhood through adolescence: growth trajectories in two male-sex-typed domains. *Developmental psychology*, *38*(4), 519.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*, *111*(23), 8410-8415.

French, J. and French, P. (1984) 'Gender Imbalances in the Primary Classroom: An Interactional Account' *Educational Research 26 (2) p. 127-36*

Frenzel, A. C., Goetz, T., Pekrun, R., & Watt, H. M. (2010). Development of mathematics interest in adolescence: Influences of gender, family, and school context. *Journal of Research on Adolescence*, *20*(2), 507-537.

Frenzel, A. C., Pekrun, R., & Goetz, T. (2007). Girls and mathematics—A "hopeless" issue? A control-value approach to gender differences in emotions towards mathematics. *European Journal of Psychology of Education*, *22*(4), 497-514.

Friedman, L. (1989). Mathematics and the gender gap: A meta-analysis of recent studies on sex differences in mathematical tasks. *Review of Educationa1 Research***,** *59,* 185-213.

Fryer Jr, R. G., & Levitt, S. D. (2009). An empirical analysis of the gender gap in mathematics. *American Economic Journal: Applied Economics, 2*, 210-240.

Fuentes, J., Palma, A. & Montero, R. (2005) Discriminación salarial por género en Chile: Una mirada global, *Estudios de Economía*, *32*(2), 133-157, Gallagher, A. M., & De Lisi, R. (1994). Gender differences in scholastic aptitude test-Mathematics problem solving among high-ability students. Journal of Educational Psychology, 86(2), 204–211.

Gallagher, A. M., De Lisi, R., Holst, P. C., McGillicuddy-De Lisi, A. V., Morely, M., & Cahalan, C. (2000). Gender differences in advanced mathematical problem solving. Journal of Experimental Child Psychology, 75, 165–190.

Gaviria, J.; Martínez-Arias, R. & Castro, M. (2004). Un estudio multinivel sobre los factores de eficacia escolar en países en desarrollo: El caso de los recursos en Brasil, *Education Policy Analysis Archives, 12*(20), 1-31.

Gee, J. (2005). *An introduction to discourse analysis: Theory and method* (2nd ed.). London: Routledge.

Gee, J. P. (2000). Identity as an analytic lens for research in education. *Review of research in education*, *25*, 99–125

Gholson, M., & Martin, D. B. (2014). Smart Girls, Black Girls, Mean Girls, and Bullies: At the Intersection of Identities and the Mediating Role of Young Girls' Social Network in Mathematical Communities of Practice. *Journal of Education*, *194*(1), 19–34.

Gilbert, M. C., Musu-Gillette, L. E., Woolley, M. E., Karabenick, S. A., Strutchens, M. E., & Martin, W. G. (2014). Student perceptions of the classroom environment: Relations to motivation and achievement in mathematics. *Learning Environments Research*, *17*(2), 287-304.

Goetz, T., Frenzel, A. C., Hall, N. C., & Pekrun, R. (2008). Antecedents of academic emotions: Testing the internal/external frame of reference model for academic enjoyment. *Contemporary Educational Psychology*, *33*(1), 9-33.

Goldstein, H. (2003) *Multilevel Statistical Models*, 3rd edition. London: Edward Arnold

Gonzalez-Jiménez, R. M. (2003). Diferencias de género en el desempeño matemático de estudiantes de secundaria. *Educación matemática*, *15*(2), 129-162.

Gorard, S., & Taylor, C. (2004). *Combining methods in educational and social research*. McGraw-Hill Education (UK).

Gorard, S., Rees, G., & Salisbury, J. (2001). Investigating the patterns of differential attainment of boys and girls at school. *British Educational Research Journal*, 27(2), 125-139.

Graddol D & Swann J (1989) *Gender voices.* Oxford: Blackwell

Graham, J. W. (2009). Missing data analysis: Making it work in the real world. *Annual review of psychology*, *60*, 549-576.

Grant, C. a. & Sleeter, C.E. (1986). Race, Class, and Gender in Education Research: An Argument for Integrative Analysis. *Review of Educational Research*, *56*(2), 195-211.

Gray, J., Peng, W. J., Steward, S., & Thomas, S. (2004). Towards a typology of gender-related school effects: some new perspectives on a familiar problem. *Oxford Review of Education*, *30*(4), 529-550.

Guba, E.G. (1987) What have we learned about naturalistic evaluation? *Evaluation Practice*, 8, 23-43.

Guiso, L., Monte, F., Sapienza, P., & Zingales, L. (2008). Culture, gender, and math. *Science*, *320*(5880), 1164-1165.

Gunderson, E. A., Ramirez, G., Levine, S. C., & Beilock, S. L. (2012). The role of parents and teachers in the development of gender-related math attitudes. *Sex Roles*, *66*(3-4), 153-166.

Hamilton, L. S., McCaffrey, D. F., Stecher, B. M., Klein, S. P., Robyn, A., & Bugliari, D. (2003). Studying large-scale reforms of instructional practice: An example from mathematics and science. *Educational Evaluation and Policy Analysis*, *25*, 1–29.

Hammack, P. L. (2008). Narrative and the cultural psychology of identity. *Personality and social psychology review: an official journal of the Society for Personality and Social Psychology, Inc*, *12*(3), 222–47.

Hand, V.M. (2010). The Co-Construction of Opposition in a Low-Track Mathematics Classroom. *American Educational Research Journal*, 47(1), 97-132

Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: embodied and social theories. *Research in Mathematics Education*, *14*(2), 137-161.

Harre, R. & Crystal, D. (2004) Discursive analysis and the interpretation of statistics. In Todd, Z., Nerlich, B., McKeown, S & Clarke, D.D. (Eds). *Mixing Methods in Psychology: The integration of qualitative and quantitative methods in theory and practice* (pp. 59-78). Psychology Press.

Hart, L.E. (2003) Some directions for Research on Equity and Justice in Mathematics Education. In L. Burton. (Ed). *Which way social justice in mathematics education?* (pp. 27 – 49). Westport: Praeger Publisher.

Hedges, L. V., & Nowell, A. (1995). Sex differences in mental test scores, variability, and numbers of high-scoring individuals. *Science*, *269*(5220), 41-45.

Hernandez-Martinez, P., Williams, J., Black, L., Davis, P., Pampaka, M., & Wake, G. (2011). Students' views on their transition from school to college mathematics: rethinking 'transition' as an issue of identity. *Research in Mathematics Education*, *13*(2), 119-130.

Hernandez, P. R., Schultz, P., Estrada, M., Woodcock, A., & Chance, R. C. (2013). Sustaining optimal motivation: A longitudinal analysis of interventions to broaden participation of underrepresented students in STEM. *Journal of educational psychology*, *105*(1), 89.

Herzig, A. H. (2004). "Slaughtering this beautiful math": Graduate women choosing and leaving mathematics. *Gender and Education*, 16(3), 379–395.

Heyd-Metzuyanim, E. (2013). The co-construction of learning difficulties in mathematics-teacher-student interactions and their role in the development of a disabled mathematical identity. *Educational Studies in Mathematics*, 83(3), 341-368

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: On learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, *51*, 128-145.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J.; & Stigler, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study*. Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Hodge, L. (2008). Student roles and mathematical competence in two contrasting elementary classes. *Mathematics Education Research Journal*, *20*(1), 32-51.

Hodges, D. C. (1998). Participation as dis-identification with/in a community of practice. *Mind, culture, and activity*, *5*(4), 272-290.

Hodgetts, K. (2008) Underperformance or 'getting it right'? Constructions of gender and achievement in the Australian inquiry into boys' education, *British Journal of Sociology of Education*, *29*(5), 465-477

Holland, D. & Skinner, D. (1987) Prestige and intimacy: the cultural models behind Americans' talk about gender types. in Quinn, D. (Ed.), *Cultural models in language and thought* (pp. 78-111) New York: Cambridge University Press

Holland, D., & Lachicotte, W. (2007). Vygotsky, Mead and the new sociocultural studies of identity. In H. 582 Daniels, M. Cole, & J. Wertsch (Eds.), *The Cambridge companion to Vygotsky* (pp. 101–135). Cambridge: Cambridge University Press.

Holland, D., Lachicotte, W. J., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Cambridge, Massachusetts: Harvard University Press.

Holmegaard, H.T., Madsen, L.M. & Ulriksen, L. (2014). To Choose or Not to Choose Science: Constructions of desirable identities among young people considering a STEM higher education programme. *International Journal of Science Education*, 36(2), 186-215

Horn, I.S. (2008). Turnaround Students in High School Mathematics: Constructing Identities of Competence Through Mathematical Worlds. Mathematics Thinking and Learning, 10(3), 201-239

Howe, K.R. (1988). Against the quantitative-qualitative incompatibility thesis or dogmas die hard. *Educational Researcher,* 17(8), 10-16.

Hox, J. (2010). *Multilevel analysis: Techniques and applications*. Routledge.

Hughes, R.M., Nzekwe, B. & Molyneaux, K. J. (2013). The Single Sex Debate for Girls in Science: a Comparison Between Two Informal Science Programs on Middle School Students' STEM Identity Formation. *Research in Science Education*, 43(5), 1979-2007

Hyde, J. S., Fennema, E., & Lamon, S. J. (1990). Gender differences in mathematics performance: a meta-analysis. *Psychological Bulletin*, *107*(2), 139–55.

Hyde, J. S., Lindberg, S. M., Linn, M. C., Ellis, A. B., & Williams, C. C. (2008). Gender similarities characterize math performance. *Science*, *321*(5888), 494-495.

IIMEC (1997). *Informe nacional sobre el desarrollo, validación y aplicación de las pruebas de diagnóstico de conocimientos*, San José de Costa Rica: Ministerio de Educación Pública.

INE (Instituto Nacional de Estadisticas). (1992) Resultados Generales Censo de Poblacion y Vivienda. Retrieved from http://www.ine.cl/canales/usuarios/cedoc_online/censos/pdf/censo_1992.pdf

INE (Instituto Nacional de Estadisticas). (2012) Sintesis de Resultados Censo 2012. Retrieved from http://www.latercera.com/noticia/negocios/2014/02/655-567075-9-documento-de-la-sintesis-de-los-resultados-del-censo-2012.shtml

Inhelder, B. & Piaget, J. (1958). *Growth of logical thinking: from childhood to adolescence*. New York: Basic Books.

Ireson, J., & Hallam, S. (2005). Pupils' liking for school: Ability grouping, self-concept and perceptions of teaching. *British Journal of Educational Psychology*, *75*(2), 297-311.

Jackson, C. (2002). 'Laddishness' as a self-worth protection strategy. *Gender and Education*, *14*(1), 37-50.

Jackson, C. (2003). Motives for 'laddishness' at school: Fear of failure and fear of the 'feminine'. *British Educational Research Journal, 29*(4), 583-598.

Jackson, C. (2006). *Lads and ladettes in school: Gender and a fear of failure.* Buckingham: Open University Press.

Jacobs, J. E., Davis-Kean, P. E., Bleeker, M., Eccles, J. S., & Malanchuk, O. (2005). “I can, but I don't want to”: The impact of parents, interests, and activities on gender differences in math. In A. M. Gallagher & J. C. Kaufman (Eds.), *Gender differences in mathematics: An integrative psychological approach* (pp. 246– 263). New York: Cambridge University Press.

Jacobs, J. E., Lanza, S., Osgood, D. W., Eccles, J. S., & Wigfield, A. (2002). Changes in children’s self-competence and values: Gender and domain differences across grades one through twelve. Child Development, 73, 509 – 527.

Jerolmack, C., & Khan, S. (2014). Talk is cheap ethnography and the attitudinal fallacy. *Sociological Methods & Research*, 1-32.

Johnston, S. (1994). Choosing Mathematics ‘You Need it Even if You Don't Want to Do it’. *Australian journal of education*, 38(3), 233-249.

Johnson, R., & Onwuegbuzie, A. (2004). Mixed methods research: A research paradigm whose time has come. *Educational researcher*, *33*(7), 14–26.

Kahle, J. B., Meece, J., & Scantlebury, K. (2000). Urban African-American middle school science students: Does standards-based teaching make a difference?. *Journal of Research in Science Teaching*, *37*(9), 1019-1041.

Kaiser, G. & Rogers, P. (1995). Introduction: Equity in Mathematics Education. In Rogers, P. & Kaiser, G. (Eds), *Equity in Mathematics Education: Influences of Feminism and Culture* (pp. 1 – 10) London – Washington DC: The Falmer Press.

Kaldo, I. & Hannula, M. (2014). Gender differences favouring females in Estonian university students’ views of mathematics. Nordic Studies in Mathematics Education, 19(1), 3-22.

Karpov, Y. V. (2003). Development through the lifespan: A neo-Vygotskian approach. in Kozulin, A. (Ed). *Vygotsky's educational theory in cultural context*. (pp. 138-155). Cambridge University Press.

Kember, D., & Gow, L. (1994). Orientations to teaching and their effect on the quality of student learning. *The Journal of Higher Education*, 58-74.

Kimball, M. M. (1989). A new perspective on women's math achievement. *Psychological Bulletin*, *105*(2), 198.

Kizzie Rouland, K., Johnson Rowley, S., & Kurtz-Costes, B. (2013). Self-views of African-American youth are related to the gender stereotypes and ability attributions of their parents. *Self and Identity*. 12(4), 382-399.

Krogh, L.B. & Andersen, H.M. (2013). Actually, I May be Clever Enough to do it. Using Identity as a Lens to Investigate Students' Trajectories Towards Science and University. *Research in Science Education*, 43(2), 711-731

Kyriakides, L. (2004). Differential School Effectiveness in Relation to Sex and Social Class: Some Implications for Policy Evaluation. *Educational Research and Evaluation*, *10*(2), 141–161.

Lamb, S. (1996). Gender differences in mathematics participation in Australian schools: Some relationships with social class and school policy. *British Educational Research Journal*, *22*(2), 223–240.

Lamprianou, I., & Athanasou, J.A. (2009). *A teacher’s guide to educational assessment*. Rotterdam: Sense Publishers.

Landers, M.G. (2013). Towards a theory of mathematics homework as a social practice. *Educational Studies in Mathematics*, 84(3), 371-391

Langer-Osuna, J.M. (2015). From Getting "Fired" to Becoming a Collaborator: A Case of the Coconstruction of Identity and Engagement in a Project-Based Mathematics Classroom. *Journal of the Learning Sciences*, 24(1), 53-92

Lavasani, M. G., & Khandan, F. (2011). Mathematic anxiety, help seeking behavior and cooperative learning. *Cypriot Journal of Educational Sciences*, 6(2), 61-74.

Le, V. N., Lockwood, J. R., Stecher, B. M., Hamilton, L. S., & Martinez, J. F. (2009). A longitudinal investigation of the relationship between teachers' self-reports of reform-oriented instruction and mathematics and science achievement. *Educational Evaluation and Policy Analysis*, *31*(3), 200-220.

Leder, G.C. (1992). Mathematicas and gender: Changing perspectives. In. D.A.Grouws (Ed.) *Handbook of research on mathematics teaching and learning* (pp. 597-622). New York: Macmillan.

Lee, JD. (1998). Which kids can "become" scientists? Effects of gender, self-concepts, and perceptions of scientists. Social Psychology Quarterly, 61(3), 199-219

Lee, JD. (2002). More than ability: Gender and personal relationships influence science and technology involvement. *Sociology of Education*, 75(4), 349-373

Leontiev, A. N. (2009). *The Development of Mind*. Retrieved from Psychology and Marxism Internet Archive: www.marxists.org/archive/leontev/works/development-mind.pdf

Lerman, S. (2000). The social turn in mathematics education research. In J.Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19-44), Westport, CT: Ablex Publishing.

Lerman, S. (2001) Cultural, Discursive Psychology: A Sociocultural Approach to Studying the Teaching and Learning of Mathematics, *Educational Studies in Mathematics*, 46, 87-113.

Lesko, A.C. & Corpus, J.H. (2006). Discounting the Difficult: How High Math-Identified Women Respond to Stereotype Threat. *Sex Roles,* 54(1-2), 113-125.

Lewis, G. (2013). Emotion and disaffection with school mathematics. *Research in Mathematics Education*, 15(1), 70-86.

Lim, J.H. (2008a). Adolescent girls' construction of moral discourses and appropriation of primary identity in a mathematics classroom. *ZDM*, 40(4), 617-631.

Lim, J.H. (2008b). The road not taken: two African-American girls' experiences with school mathematics. *Race, Ethnicity and Education*, 11(3), 303-317

Linacre, J. M. (2002). Optimizing rating scale category effectiveness. *Journal of Applied Measurement*, *3*(1), 85-106.

Linacre, J.M. (2011) *Winsteps* (version 3.72.3) [Computer software]. Chicago: winsteps.com

LLECE (2000). *Segundo informe del Primer Estudio Internacional Comparativo sobre Lenguaje, Matemática y Factores Asociados para alumnos del Tercer y Cuarto Grado de Educación Básica,* Santiago de Chile: UNESCO.

LLECE (2008) *Segundo Estudio Regional Comparativo y Explicativo (SERCE).* Santiago: Oficina Regional de Educación de la UNESCO para América Latina y el Caribe (OREALC)- UNESCO.

Lubienski, S. T. (2000). A clash of social class cultures? Students' experiences in a discussion-intensive seventh-grade mathematics classroom. *The Elementary School Journal*, 377-403.

Lubienski, S. T. (2008). On" gap gazing" in mathematics education: The need for gaps analyses. *Journal for Research in Mathematics Education*, *39*(4), 350-356.

Ma, X. (2008). Within-School Gender Gaps in Reading, Mathematics, and Science Literacy. *Comparative Education Review*, *52*(3), 437-460.

Ma, X., & Cartwright, F. (2003). A longitudinal analysis of gender differences in affective outcomes in mathematics during middle and high school. *School Effectiveness and School Improvement*, *14*(4), 413-439.

Mac an Ghaill, M. (1994). *The making of men: Masculinities, sexualities and schooling*. Buckingham: Open University Press.

Maccoby, E. & Jacklin, C. (1974). *The psychology of sex differences*, Stanford, CA: Stanford University Press.

Machin, S., & McNally, S. (2005). Gender and student achievement in English schools. *Oxford review of economic policy*, *21*(3), 357-372.

Manzi, J., Strasser, K., San Martin, E. & Contreras, D. (2008) *Quality of education in Chile*. Working Paper, Mide, UC, Santiago, Chile.

Marcel, M., & Tokman, C. (2005). ¿ Cómo se financia la educación en Chile. Santiago: Ministerio de Hacienda.

Marks, G. N. (2008). Accounting for the gender gaps in student performance in reading and mathematics: Evidence from 31 countries. *Oxford Review of Education*, *34*(1), 89-109.

Marsh, H. W. (1987). The big-fish-little-pond effect on academic self-concept. *Journal of Educational Psychology*, 79(3), 280e295.

Marsh, H. W. (1990). Causal ordering of academic self-concept and academic achievement: A multiwave, longitudinal panel analysis. *Journal of educational psychology*, *82*(4), 646.

Marsh, H. W. (2007). Self-concept theory, measurement and research into practice: The role of self-concept in educational psychology. Leicester: British Psychological Society.

Marsh, H. W., Trautwein, U., Lüdtke, O., & Köller, O. (2008). Social comparison and big- fish-little-pond effects on self-concept and other self-belief constructs: Role of general- ized and specific others. Journal of Educational Psychology, 100, 510–524.

Martin, D. (2012). Learning mathematics while Black. *The Journal of Educational Foundations*, 26, 47–66.

Martinic, S. & Vergara, C. (2007) Gestión Del Tiempo e Interacción del Profesor-Alumno en la Sala de Clases de Establecimientos con Jornada Escolar Completa en Chile. *REICE. Revista Iberoamericana sobre Calidad, Eficacia y Cambio en Educación,* 5, 3-20

Maxcy, S.J. (2003) Pragmatic threads in mixed methods research in the social science: The search for multiple modes of inquiry and the end of the philosophy of formalism. In Tashakkori, A., & Teddlie, C. (Eds). *Handbook of mixed methods in social and behavioural research*. Thousand Oaks, CA: Sage Publications.

McClain, O. L. (2014). Negotiating Identity: A Look at the Educational Experiences of Black Undergraduates in STEM Disciplines. *Peabody Journal of Education*, *89*(3), 380-392.

McClelland, G. H., & Judd, C. M. (1993). Statistical difficulties of detecting interactions and moderator effects. *Psychological bulletin*, *114*(2), 376.

McCombs, B. L., & Quiat, M. (2002). What makes a comprehensive school reform model learner centered?. *Urban Education*, *37*(4), 476-496.

McEwan, P. J. (2001). The effectiveness of public, catholic, and non-religious private schools in Chile's voucher system. *Education Economics*, *9*(2), 103-128.

McEwan, P. J. (2004). The indigenous test score gap in Bolivia and Chile. *Economic Development and Cultural Change*, *53*(1), 157-190.

McGee, E.O., Martin, D.B. (2011). You Would Not Believe What I Have to Go Through to Prove My Intellectual Value! Stereotype Management Among Academically Successful Black Mathematics and Engineering Students. *American Educational Research Journal*, 48(6), 1347-1389

McGee, E. O., & Pearman F. A. (2014a) Understanding Black Male Mathematics High Achievers from the Inside Out: Internal Risk and Protective Factors in High School. *The Urban Review*, 1(28).

McGee, E. & Pearman, F. A. (2014b). Risk and Protective Factors in Mathematically Talented Black Male Students Snapshots From Kindergarten Through Eighth Grade. *Urban Education*, 49(4), 363-393

McGraw, R., Lubienski, S. T., & Strutchens, M. E. (2006). A closer look at gender in NAEP mathematics achievement and affect data: Intersections with achievement, race/ethnicity, and socioeconomic status. *Journal for Research in Mathematics Education*, 129-150.

Meckes, L., & Carrasco, R. (2010). Two decades of SIMCE: an overview of the National Assessment System in Chile. *Assessment in Education: Principles, Policy & Practice*, *17*(2), 233-248.

Mella, O. (2002). Factores que afectan los resultados de la escuela pública Chilena. *REICE: Revista Electrónica Iberoamericana sobre Calidad, Eficacia y Cambio en Educación*, *4*(1), 29-37.

Mendick, H. (2005a). A beautiful myth? The gendering of being/doing 'good at maths'. *Gender and education*, *17*(2), 203-219.

Mendick, H. (2005b). Only connect: Troubling oppositions in gender and mathematics. *International Journal of Inclusive Education*, *9*(2), 161-180.

Mendick, H. (2005c). Mathematical stories: why do more boys than girls choose to study mathematics at AS level in England? *British Journal of Sociology of Education*, 26(2), 235–251.

MINEDUC (2005). Análisis de las diferencias de logro en el aprendizaje escolar entre hombres y mujeres. Santiago de Chile: SIMCE. Retrieved from http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Análisis+de+las+diferencias+de+logro+en+el+aprendizaje+escolar+entre+hombres+y+mujeres#0

MINEDUC (2010) *Estadisticas Educación 2009*, Santiago, MINEDUC.

MINEDUC (2011) *Análisis del Sistema Escolar desde la Perspectiva de Género 2011*. Santiago, Chile. Retrieved from http://sgdce.mineduc.cl/sphider/search.php?query=&t_busca=1&results=&search=1&dis=0&category=8

Ministerio de Educacion Peru (2001). Resultados de las pruebas de matemática y lenguaje: ¿Qué aprendimos a partir de la evaluación CRECER 1998, Lima: Unidad de Medición de Calidad Educativa Ministerio de Educación, Retrieved from http://www2.minedu.gob.pe/umc/admin/images/publicaciones/boletines/Boletin-0506.pdf

Mittelberg, D., Rozner, O., & Forgasz, H. (2011). Mathematics and gender stereotypes in one Jewish and one Druze grade 5 classroom in Israel. *Education Research International*, 1 – 10.

Mizala, A., Martínez, F., & Martínez, S. (2015). Pre-service elementary school teachers' expectations about student performance: How their beliefs are affected by their mathematics anxiety and student's gender. *Teaching and Teacher Education*, *50*, 70-78.

Mizala, A. & Romaguera, P. (2000). Determinación de los factores explicativos de los resultados escolares de la Enseñanza Media en Chile. Serie Economía, *85*, 1-32.

Mizala, A., Romaguera, P., & Urquiola, M. (2007). Socioeconomic status or noise? Tradeoffs in the generation of school quality information. *Journal of Development Economics*, *84*(1), 61-75.

Mizala, A., & Torche, F. (2012). Bringing the schools back in: the stratification of educational achievement in the Chilean voucher system. *International Journal of Educational Development*, *32*(1), 132–144.

Montenegro, C. (2001). *Wage Distribution in Chile: Does Gender Matter? A Quantile Regression Approach*. The World Bank, Poverty Reduction and Economic Management Network.

Moriondo, M., Palma, P., Medrano, L., & Murillo, P. (2012). Adaptación de la Escala de Afectividad Positiva y Negativa (PANAS) a la población de adultos de la ciudad de Córdoba: Análisis psicométricos preliminares. *Universitas Psychologica*, *11*(1), 187-196.

Morrow, C., & Morrow, J. (1995). Connecting women with mathematics. . In Kaiser, G. & Rogers, P. (Eds.), *Equity in mathematics education: Influences of feminism and culture* (pp. 13-26). London: Falmer.

Mullis, I.V.S., Martin, M.O., Foy, P., & Arora, A. (2012). *TIMSS 2011 International Results in Mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College

Myhill, D. (2002) Bad Boys and Good Girls? Patterns of Interaction and Response in Whole Class Teaching, *British Educational Research Journal*, 28: 3, 339-352

Nagy, G., Trautwein, U., Baumert, J., Köller, O. & Garrett, J. (2006) Gender and course selection in upper secondary education: Effects of academic self-concept and intrinsic value, *Educational Research and Evaluation*, 12:4, 323-345,

Nardi, E., & Steward, S. (2003). Is mathematics TIRED? A profile of quiet disaffection in the secondary mathematics classroom. *British Educational Research Journal*, *29*(3), 345-367.

Nasir, N.S. (2002). Identity, Goals, and Learning: Mathematics in Cultural Practice. *Mathematics Thinking and Learning*, 4(40603), 213-247

Nasir, N. I. S., & Cobb, P. (2007). Improving access to mathematics: Diversity and equity in the classroom. *Multicultural Education Series. New York: Teachers College Press*.

Nasir, N.S. & Hand, V. (2008). From the court to the classroom: Opportunities for engagement, learning, and identity in basketball and classroom mathematics. *Journal of the Learning Science*, 17(2), 143-179

Nasir, N. S., & Saxe, G. B. (2003). Ethnic and Academic Identities: A Cultural Practice Perspective on Emerging Tensions and Their Management in the Lives of Minority Students. *Educational Researcher*, *32*(5), 14–18.

Navarrete, C.; López, R. & Laguna, J. (2008). *Los factores asociados en la Evaluación Nacio- nal del Rendimiento Académico 2006: Un análisis multinivel*, Managua: Ministerio de Educación de Nicaragua, Retrieved from http://www.mined.gob.ni/Bolet_MINED/Articulos %20y%20Opiniones/diciembre/pdf.

Nguyen, H. H. D., & Ryan, A. M. (2008). Does stereotype threat affect test performance of minorities and women? A meta-analysis of experimental evidence. *Journal of Applied Psychology*, *93*(6), 1314.

Noland, C. M. (2006). Auto-photography as research practice: Identity and self-esteem research. *Journal of research Practice*, *2*(1), 1.

Ñopo, H. (2012). New Century, Old Disparities: Gender and Ethnic Earnings Gaps in Latin America and the Caribbean. Retrieved from http://elibrary.worldbank.org/doi/book/10.1596/978-0-8213-8686-6http://elibrary.worldbank.org/doi/book/10.1596/978-0-8213-8686-6

Nowell, A. & Hedges, L.V. (1998). Trends in Gender Differences in Academic Achievement from 1960 to 1994 : An Analysis of Differences in Mean, Variance, and Extreme Scores. *Sex Roles*, *39*(1), 21-43.

Noyes, A. (2012). It matters which class you are in: student-centred teaching and the enjoyment of learning mathematics. *Research in Mathematics Education*, *14*(3), 273-290.

Nurmi, J. E. (2004). Socialization and Self-developmentchanneling, Selection, Adjustment, and Reaction. In R. Lerner & Steinberg, L. (Eds.) Handbook of adolescent psychology, 2nd edition. New Jersey: John Wiley & Sons, Inc.

Nuttall, D. L., Goldstein, H., Prosser, R., & Rasbash, J. (1989). Differential school effectiveness. *International Journal of Educational Research*, *13*(7), 769-776.

OECD (2013), *PISA 2012 Results: What Students Know and Can Do – Student Performance in Mathematics, Reading and Science (Volume I),* PISA, OECD Publishing.

OECD (2015), In this together: Why Less Inequality Benefits all, OECD Publishing, Paris. Retrieved from http://www.oecd.org/social/inequality.htm

Op't Eynde, P., De Corte, E., & Verschaffel, L. (2006). "Accepting emotional complexity": A socio-constructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics*, *63*(2), 193-207.

Oppland-Cordell, S. (2013). Mathematics and Racial Identity Co-Construction in Multiple Sociopolitical Contexts: A Case Study of a Latina Undergraduate Student from an Urban Community. *Journal of Urban Learning, Teaching and Research*, 9, 89-99.

Oppland-Cordell, S., & Martin, D. B. (2014). Identity, power, and shifting participation in a mathematics workshop: Latin@ students' negotiation of self and success. *Mathematics Education Research Journal*, *27*(1), 21-49.

Paikof, R., & Brooks-Gunn, J. (1991). Do parent-child relationships change during puberty? Psychological Bulletin, 110, 1: 47-66.

Pajares, F. (1996). Self-efficacy beliefs in academic settings. *Review of Educational Re- search,* **66,** 543–578.

Pajares, F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. *Contemporary educational psychology*, *24*(2), 124-139.

Palardy, G. J. (2008). Differential school effects among low, middle, and high social class composition schools: A multiple group, multilevel latent growth curve analysis. *School Effectiveness and School Improvement*, *19*(1), 21-49.

Palardy, G. J., & Rumberger, R. W. (2008). Teacher effectiveness in first grade: The importance of background qualifications, attitudes, and instructional practices for student learning. *Educational Evaluation and Policy Analysis*, *30*, 111–140.

Pampaka, M., Kleanthous, I., Hutcheson, G. D., & Wake, G. (2011b). Measuring mathematics self-efficacy as a learning outcome. Research in Mathematics Education, 13(2), 169-190.

Pampaka, M., Williams, J. S., Hutchenson, G., Black, L., Davis, P., Hernandez-Martines, P., & Wake, G. (2013). Measuring alternative learning outcomes: Dispositions to study in higher education. Journal of Applied Measurement, 14(2), 197-218.

Pampaka, M., Williams, J., Hutcheson, G., Wake, G., Black, L., Davis, P., & Hernandez-Martinez, P. (2011a). The association between mathematics pedagogy and learners' dispositions for university study. *British Educational Research Journal*, 1–24.

Pampaka, M. & Wo, L. (2014). Revisiting Mathematical Attitudes of Students in Secondary Education. In Liljedahl, P., Oesterle, S., Nicol, C., & Allan, D. (Eds.) *Proceedings of the Joint Meeting 4 - 385 of PME 38 and PME-NA 36*,Vol. 4, 385-392. Vancouver, Canada: PME.

Parker, P. D., Marsh, H. W., Ciarrochi, J., Marshall, S., & Abduljabbar, A. S. (2014). Juxtaposing math self-efficacy and self-concept as predictors of long-term achievement outcomes. *Educational Psychology*, 34(1), 29-48.

Patton, M. (2002). *Qualitative Research & Evaluation Methods*. California: Sage Publications.

Penuel, W.R., & Wertsch, J.J.V. (1995). Vygotsky and identity formation : A sociocultural approach. *Educational psychologist*, *30*(2), 83–92.

Peterson, P. L., & Fennema, E. (1985). Effective teaching, student engagement in classroom activities, and sex-related differences in learning mathematics. *American Educational Research Journal*, 22(3), 309-335.

Philbin, M., Meier, E., Huffman, S., & Boverie, P. (1995). A survey of gender and learning styles. *Sex Roles*, 32(7-8), 485-494.

Piñeros, L. & Rodríguez, A. (1998). *Los insumos escolares en la educación secundaria y su efecto sobre el rendimiento académico de los estudiantes: un estudio en Colombia*, Human Development Department, LCSHD paper series, núm. 36, Washington, DC: The World Bank.

Preiss, D., (2009). The Chilean instructional pattern for the teaching of language: A video-survey study based on a national program for the assessment of teaching. *Learning and Individual Differences*, 19(1), 1-11.

Pronin, E., Steele, C.M. & Ross, L. (2004). Identity bifurcation in response to stereotype threat: Women and mathematics. *Journal of Experimental Social Psychology*, 40(2), 152-168.

Radovic, D. (2015). *Towards conceptual coherence in research on mathematics learner identity: a systematic review of the literature.* Manuscript in preparation.

Radovic, D., Black, L., Salas, C. & Williams, J. (2015) *Being a Girl Mathematician: Analysis of the diversity of positive mathematical identities in a secondary classroom.* Manuscript submitted for publication.

Radovic, D., Black, L., Salas, C. & Williams, J. (in press) *The Intersection of girls' Mathematics and Peer Group positionings in a mathematics' classroom.* Manuscript accepted for publication in Proceddings of the CERME 9: Congress of European Research in Mathematics Education, Charles University, Prague Czech Republic

Radovic, D., & Preiss, D. (2010). Patrones de Discurso Observados en el Aula de Matemática de Segundo Ciclo Básico en Chile. *Psykhe (Santiago)*, *19*, 65–79

Raîche G. (2005) Critical Eigenvalue Sizes in Standardized Residual Principal Components Analysis, *Rasch Measurement Transactions*, 19:1 p. 1012.

Ramirez, M. . (2006). Understanding the low achievement of Chilean students: A cross-national analysis using TIMSS data. *International Journal of Educational Research*, 1–29.

Ramirez, M. (2007). Diferencias dentro de las salas de clases. Distribución de rendimiento en Matemática. *Estudios Públicos*, 106(otoño), 5-22.

Raudenbush, S.W. & Bryk, A.S. (2002). *Hierarchical Linear Models: Applications and Data Analysis Methods*. California: Sage.

Renold, E. (2002). Presumed innocence (Hetero) sexual, heterosexist and homophobic harassment among primary school girls and boys. *Childhood*, *9*(4), 415-434.

Renold, E., & Allan, A. (2006). Bright and Beautiful: High achieving girls, ambivalent femininities, and the feminization of success in the primary school. *Discourse: Studies in the Cultural Politics of Education*, 27(4), 457–473.

Robles, R., & Páez, F. (2003). Estudio sobre la traducción al español y las propiedades psicométricas de las escalas de afecto positivo y negativo (PANAS). *Salud mental*, *26*(1), 69-75.

Rodd, M., & Bartholomew, H. (2006). Invisible and special: young women's experiences as undergraduate mathematics students. *Gender and Education*, 18(1), 35–50.

Ross, J. A., & Bruce, C. D. (2012). The Gender Confidence Gap in Fractions Knowledge: Gender Differences in Student Belief – Achievement Relationships. *School Science and Mathematics*, *112*(5), 278–288.

Roth, W. M., & Radford, L. (2011). *A cultural-historical perspective on mathematics teaching and learning*. Springer Science & Business Media.

Rutter, M., & Rutter, M. (1993). *Developing minds*. London: Penguin

Sammons, P., Nuttall, D., & Cuttance, P. (1993). Differential school effectiveness: Results from a reanalysis of the Inner London Education Authority's Junior School Project data. *British Educational Research Journal*, *19*(4), 381-405.

Scheerens, J. (1990). School effectiveness research and the development of process indicators of school functioning. *School effectiveness and school improvement*, *1*(1), 61-80.

Schmader, T. (2002). Gender identification moderates stereotype threat effects on women's math performance. *Journal of Experimental Social Psychology*, *38*(2), 194-201.

Schuh, K. L. (2004). Learner-centered principles in teacher-centered practices? *Teaching and Teacher Education*, *20*(8), 833–846.

Sells, L. W. (1978). The forum: Mathematics — a critical filter. *The Science Teacher*, 45(2), 28-29.

Servicio Nacional de la Mujer (SERNAM). (2008). *Análisis de género en el aula* (Documento de trabajo N° 117). Santiago, Chile. Retrieved from http://estudios.sernam.cl/documentos/?eMTE0NDczNw==An%C3%A1lisis_de_Genero_en_el_Aula._http://estudios.sernam.cl/documentos/?eMTE0NDczNw==An%C3%A1lisis_de_Genero_en_el_Aula._

Sfard, A., & Prusak, A. (2005). Telling Identities: In Search of an Analytic Tool for Investigating Learning as a Culturally Shaped Activity. *Educational Researcher*, 34(14), 14–22.

SIMCE (2006). *Informe de Resultados SIMCE 2005*. Santiago, Ministerio de Educacion.

SIMCE (2010a). *Resultados Nacionales SIMCE 2009*. Santiago, Ministerio de Educacion.

SIMCE (2010b). Metodologia para la construccion de Grupos Socioeconomicos. Santiago: MINEDUC. Retrieved from http://www.agenciaeducacion.cl/wp-content/uploads/2013/02/Metodologia_de_Construccion_de_Grupos_Socioeconomicos_en_SIMCE_4to_Basico_20091.pdf

SIMECAL (1998). *Rendimientos de 3° y 6° de educación primaria en lenguaje y matemática y factores asociados*, La Paz: Ministerio de Desarrollo Humano.

Smith, C. (2010). Choosing more mathematics: happiness through work?. *Research in Mathematics Education*, *12*(2), 99-115.

Smith, R.M., Schumacker, R.E. & Busch, M.J (1995). *Using Item Mean Squares to Evaluate Fit to the Rasch Model*. Paper presented at the 1995 Annual Meting of the American Educational Research Asociation: San Francisco.

Snijder, T. & Bosker, R. J. (1999). *Multilevel Analysis: An Introduction to Basic and Advanced Multilevel Modeling*. Sage.

Solomon, Y. (2007a). Not belonging? What makes a functional learner identity in undergraduate mathematics? *Studies in Higher Education*, 32(1), 79-96

Solomon, Y. (2007b). Experiencing mathematics classes: Gender, ability and the selective development of participative identities. *International Journal of Educational Research.* 46(1–2), 8–19.

Solomon, Y. (2008). *Mathematical Literacy: Developing Identities of Inclusion*. London and New York: Routledge.

Solomon, Y. (2012). Finding a voice? Narrating the female self in mathematics. *Educational studies in mathematics*, *80*(1-2), 171-183.

Solomon, Y., Croft, T., & Lawson, D. (2010). Safety in numbers: mathematics support centres and their derivatives as social learning spaces. *Studies in Higher Education*, *35*(4), 421-431.

Solomon, Y., Lawson, D., & Croft, T. (2011). Dealing with 'fragile identities': resistance and refiguring in women mathematics students. *Gender and Education*, 23(5), 565-583.

Solomon, Y., Radovic, D. & Black, L. (2015) I can actually be very feminine here": contradiction and hybridity in becoming a female mathematician. *Educational Studies in Mathematics*, DOI: 10.1007/s10649-015-9649-4

Spencer, S. J., Steele, C. M., & Quinn, D. M. (1999). Stereotype threat and women's math performance. *Journal of experimental social psychology*, *35*(1), 4-28.

Steinberg, L. (1987). Impact of puberty on family relations: Effects of pubertal status and pubertal timing. *Developmental Psychology, 23(3)*: 451-460

Steinberg, L. (1989). *Adolescence* (2nd Ed). New York: McGraw- Hill Publishing Company.

Stentoft, D., & Valero, P. (2009). Identities-in-action. *Nordic Studies in Mathematics Education*, *14*(3), 55-77.

Stetsenko, A., & Arievitch, I. M. (2004). The self in cultural-historical activity theory. Reclaiming the unity of social and individual dimensions of human development. *Theory & Psychology*, *14*(4), 475-503.

Stevens, T., Wang, K., Olivárez Jr, A., & Hamman, D. (2007). Use of self-perspectives and their sources to predict the mathematics enrollment intentions of girls and boys. *Sex roles*, *56*(5-6), 351-363.

Stinson, D. W. (2004). Mathematics as “gate-keeper” (?): Three theoretical perspectives that aim toward empowering all children with a key to the gate. *The Mathematics Educator*, 14(1), 8–18.

Stinson, D.W. (2008). Negotiating Sociocultural Discourses: The Counter-Storytelling of Academically (and Mathematically) Successful African American Male Students. *American Educational Research Journal*, 45(4), 975-1010

Stipek, D.J & Gralinski, J.H. (1991) Gender differences in children's achievement-related beliefs and emotional responses to success and failure in mathematics. *Journal of Educational Psychology*, 83(3), 361-371

Stodolsky, S. (1985). Telling math: Origins of math aversion and anxiety. *Educational Psychologist*, 20, 125–133.

Strand, S. (2010). Do some schools narrow the gap? Differential school effectiveness by ethnicity, gender, poverty, and prior achievement. *School Effectiveness and School Improvement*, *21*(3), 289–314.

Strauss, A., & Corbin, J. (1998). Basics of qualitative research. Thousand Oaks, CA: Sage.

Stuven, A. M. (2013). La mujer ayer y hoy: un recorrido de incorporación social y política. *Centro de Políticas Públicas UC*, *8,* 61, 1-20.

Swan, M. (2006a). Learning GCSE mathematics through discussion: what are the effects on students?. *Journal of Further and Higher Education*, *30*(3), 229-241.

Swan, M. (2006b) Designing and using research instruments to describe the beliefs and practices of mathematics teachers, *Research in Education,* 75, 58–70.

Tabachnick, B. G., & Fidell, L. S. (2007). Chapter 15: Mu1tilevel Linear Modeling. In *Using multivariate statistics*. Pearson.

Tashakkori, A., & Teddlie, C. (2003). *Handbook of mixed methods in social and behavioural research* (pp. 51 - 89). Thousand Oaks, CA: Sage Publications.

Tate, E. D., & Linn, M. C. (2005). How does identity shape the experiences of women of color engineering students?. *Journal of Science Education and Technology*, *14*(5-6), 483-493.

Teddie, C., & Tashakkori, A. (2009). *Foundations of Mixed Methods Research: Integrating Quantitative and Qualitative Approaches in the Social and Behavioral Sciences*. Thousand Oaks, CA: Sage Publications.

Teese, R., Davies, M., Charlton, M., & Polesel, J. (1995). *Who wins at school?* Melbourne: Department of Education Policy and Management, The University of Melbourne.

The World Bank (2015). Chile: Country at Glance. Retrieved from http://www.worldbank.org/en/country/chile.

Thomas, J., & Harden, A. (2008). Methods for the thematic synthesis of qualitative research in systematic reviews. *BMC medical research methodology*, *8*(1), 45.

Thomas, S., Sammons, P., Mortimore, P., & Smees, R. (1997). Differential secondary school effectiveness: comparing the performance of different pupil groups. *British Educational Research Journal*, *23*(4), 451-469.

Timmermans, R. E., Van Lieshout, E. C., & Verhoeven, L. (2007). Gender-related effects of contemporary math instruction for low performers on problem-solving behavior. *Learning and Instruction*, *17*(1), 42-54.

Todd, Z., Nerlich, B. & McKeown, S. (2004) Introduction. In Todd, Z., Nerlich, B., McKeown, S & Clarke, D.D. (Eds). *Mixing Methods in Psychology: The integration of qualitative and quantitative methods in theory and practice* (pp. 2-15). Psychology Press.

Torche, F. (2005). Privatization reform and inequality of educational opportunity: The case of Chile. *Sociology of Education*, *78*(4), 316-343.

Treviño, E., Valdés, H., Castro, M., Costilla, R., Pardo, C., y Donoso, F. (2010). *Factores asociados al logro cognitivo de los estudiantes de América Latina y El Caribe*. Santiago: OREALC/UNESCO Santiago & LLECE.

Troncoso, P., Pampaka, M., & Olsen, W. (2015). Beyond traditional school value-added models: a multilevel analysis of complex school effects in Chile. *School Effectiveness and School Improvement*. DOI: 10.1080/09243453.2015.1084010

Turner, E., Dominguez, H., Maldonado, L. & Empson, S (2013). English Learners' Participation in Mathematical Discussion: Shifting Positionings and Dynamic Identities. *Journal For Research In Mathematics Education*, 44(1), 199-234

United Nations (2015) Facts and Figures: Leadership and Political Participation. Retrieved from See more at: http://www.unwomen.org/en/what-we-do/leadership-and-political-participation/facts-and-figures#sthash.P3hp0K25.dpuf, or http://www.ipu.org/wmn-e/classif.htm

Urdan, T., & Midgley, C. (2003). Changes in the perceived classroom goal structure and pattern of adaptive learning during early adolescence. *Contemporary educational psychology*, *28*(4), 524-551.

Vaisey, S. (2009). Motivation and Justification: A Dual-Process Model of Culture in Action1. *American Journal of Sociology*, *114*(6), 1675-1715.

Valentine, J. C., DeBois, D. L., & Cooper, H. (2004). The relation between self-beliefs and academic achievement: A meta-analytic review. *Educational Psychologist*, 39, 37–41.

Varelas, M., Martin, D. B., & Kane, J. M. (2012). Content learning and identity construction: A framework to strengthen African American students' mathematics and science learning in urban elementary schools. *Human Development*, 55(5), 319-339.

Voz de Mujer (2010). Encuesta Nacional Mujer y Trabajo. Santiago: Comunidad Mujer.

Voz de Mujer (2013). Mujer y Trabajo: factores culturales, roles de género y comportamiento laboral de las chilenas”. Santiago: Comunidad Mujer.

Vygotsky, L. S. (1998). *Child development, The Collected Works of L. S. Vygotsky*, Vol. 5. New York: Plenum.

Walkerdine, V. (1998). *Counting girls out: Girls and Mathematics*. Second edition. London and New York: RoutledgeFalmer.

Walls, F. (2009). *Mathematical subjects: Children talk about their mathematics lives*. Springer Science & Business Media.

Walshaw, M. (2001). A Foucauldian gaze on gender research: What do you do when confronted with the tunnel at the end of the light?. *Journal for Research in Mathematics Education*, 471-492.

Walshaw, M. (2005). Getting political and unraveling layers of gendered mathematical identifications. *Cambridge Journal of Education*, *35*(1), 19-34.

Ward, P.R., Johnston, W.S., & Lee, C. (2011). Exit interviews: undergraduates who leave mathematics behind. *For the Learning of Mathematics*, 21-26.

Watson, D., Clark, L. A., & Tellegen, A. (1988). Development and validation of brief measures of positive and negative affect: the PANAS scales. *Journal of personality and social psychology*, *54*(6), 1063.

Watt, H. M. G. (2004). Development of adolescents' self-perceptions, values, and task perceptions according to gender and domain in 7th- through 11th-grade Autralian students. *Child Development*, 75(5), 1556–1574.

Wells, G. (1993). Reevaluating the IRF sequence: A proposal for the articulation of theories of activity and discourse for the analysis of teaching and learning in the classroom. *Linguistics and education*, 5(1), 1-37.

Wells, G. (1999). *Dialogic inquiry: Towards a sociocultural practice and theory of education*. New York, NY: Cambridge University Press.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, England: Cambridge University Press

Wigfield, A., & Eccles, J. S. (1994). Children's competence beliefs, achievement values, and general self-esteem: Change across elementary and middle school. *Journal of Early Adolescence*, 14(20), 107–138.

Wigfield, A., Battle, A., Keller, L. B., & Eccles, J. S. (2002). Sex differences in motivation, self-concept, career aspiration, and career choice: implications for cognitive development. In R. De Lisi & A. McGillicuddy-De Lisi (Eds.), *The development of sex differences in cognition* (pp. 93–124). Westport, CT, US: Ablex Publishing.

Willingham, W. & Cole, N. (1997). Research on gender differences, in W. W. Willingham & N. S. Cole (Eds.), *Gender and Fair Assessment,* Hillsdale, NJ: Lawrence Erlbaum.

Willis, P. (1977) *Learning to Labour: How Working Class Kids get Working Class Jobs*. London: Saxon House

WISE (2012) *Women in Science, Technology, Engineering and Mathematics: from Classroom to Boardroom*, Retrieved from http://www.wisecampaign.org.uk/files/useruploads/files/wise_stats_document_final.pdf, [Accessed Jun 30th 2014]

Wood, M.B. (2013). Mathematical Micro-Identities: Moment-to-Moment Positioning and Learning in a Fourth-Grade Classroom. *Journal For Research In Mathematics Education*, 44(5), 775-808

Yin, R. K. (2003). *Case Study Research: Design and Methods* (Vol. 5). California: Sage.

Ylvisaker, M., Mcpherson, K., Kayes, N., & Pellett, E. (2008). Metaphoric identity mapping: Facilitating goal setting and engagement in rehabilitation after traumatic brain injury. *Neuropsychological Rehabilitation*, 18(5-6), 713-741.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational studies in mathematics*, *63*(2), 113-121.

Zeidner, M., & Schleyer, E. J. (1999). The big-fish-little-pond effect for academic self- concept, test anxiety, and school grades in gifted children. *Contemporary Educational Psychology*, 24, 305-329.

Zorrilla, M. y Muro, F. (2004). *La enseñanza secundaria en México 2002. Una exploración de modelos explicativos de resultados de aprendizaje y características del alumno, del entorno familiar y escolar*, México: Instituto Nacional para la Evaluación de la Educación, Retrieved from http://www.inee.edu.mx/images/stories/documentos_pdf/ Resultados_Evaluaciones/Estudios/Anteriores_2003/10_ensenanza_secund_mx02.pdf