How orientation and symmetry influence the way we perceive shapes

Journal Item

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Susan Forsythe looks at our perception of shapes

It is not at all uncommon for adults, let alone children, to be influenced by the material representation of a shape. The representation, say as a diagram on paper or as a square tile, has edges with thickness, maybe a coloured interior and it will be presented in a certain orientation. However, the abstract geometrical concept of a square is of a regular four sided polygon with only two dimensions. It is this difference between the abstract concept and the material representation which is responsible for the problems children may experience when working with 2 dimensional shapes.

The orientation of a shape is perceived in relation to the frame of reference within which the shape is seated. The frame of reference can be described as a container which is independent of the objects inside it. The most natural frame of reference is to use the vertical and horizontal axes which are derived from the physical world (Piaget and Inhelder, 1956). We can talk of a global or field axis of symmetry (in the environment) and a local or figural axis of symmetry (say, of a specific figure on a piece of paper).

shape **a** shape **b**

Figure 1

Although in figure 1, shape 'a' and shape 'b' are congruent squares they are commonly perceived to be different. Shape 'a' is usually thought of as being a square whilst shape 'b' is perceived as a diamond (or a rhombus, to give the correct mathematical name). This is because we tend to use the vertical axis, of the page, as our frame of reference so that the sides of the square would be described as being parallel and perpendicular to the reference orientation whereas the sides of the diamond would be described as being at a slant. Thus shapes are perceived within a perceptual reference frame. There is also a strong tendency to choose a reference frame orientated along an axis of reflection symmetry with a vertical axis being preferred.

Symmetry also has an important influence over how we perceive shapes. In a research project I undertook with pairs of students working in computer files in a Dynamic Geometry Software program, symmetry was observed to have a significant impact on the way the students perceived the shapes. The students were in year 8, but the findings of the study could have implications for the way we introduce shapes to primary pupils.

The specific Dynamic Geometry Software program which was used in the research project was the Geometers Sketchpad version 4 (Jackiw, 2001). A screen shot is shown in figure 2. Like other such programs it has tools for drawing basic objects such as points, lines and circles and the capability to construct objects based on those objects such as perpendicular lines or points of intersection. There is an arrow tool which is used to choose objects or to drag them across the screen thus manipulating any figure they are part of. Dragging objects allows them to be moved on the screen whilst defined relationships between them remain constant. Laborde (1993) explained that the introduction of Dynamic Geometry Software enables us to redefine the distinction between the theoretical object and its material representation. The figure on the computer screen is a dynamic representation of the theoretical shape which can be dragged on the screen and its behaviour when dragged is determined by the geometrical properties designed into its construction.

As an example (see figure 2 below), if a triangle is drawn using three line segments joined end to end then this figure can be dragged on the screen to display many different triangles. However, if we wish the triangle to be isosceles and to remain so when dragged then the properties of an isosceles triangle must be embedded into the figure when it is constructed.

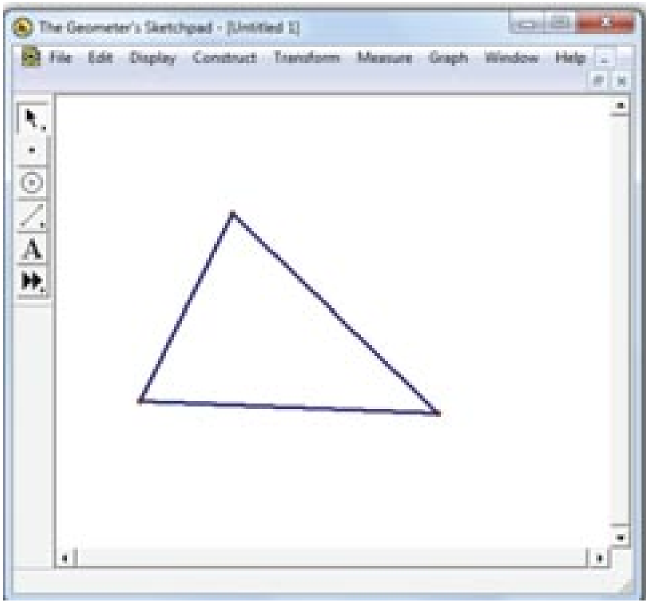

Figure 2

For the research project I designed some computer files in the Geometers Sketchpad which are constructed around two fixed length perpendicular line segments which I have named bars. Each bar was constructed using a vector to define the relationship between two points which were joined using a line segment. Like the bars of a toy kite they provide structure to the shape which is then constructed around them. If the bars of a toy kite were moved, the fabric which covers them would not be able to adapt but in Dynamic Geometry Software the outside edges and the interior of the shape transform as the bars move. This enables many different triangles and quadrilaterals to be generated.

I recorded the dialogue and on screen activity whilst pairs of students worked in the computer files. The students were asked to drag the bars and see what shapes they could make. The students typically generated shapes which have symmetry although they were also slightly interested in right angled triangles. There was little interest in the myriad irregular shapes it is possible to make. The bars (AC and BD in the figures) can be dragged to generate various shapes. Figure 3 shows some examples.

Clearly, the computer screen has its own frame of reference: vertical and horizontal axes which are parallel to the edges of the screen. Within this frame of reference the students appeared to prefer shapes with vertical symmetry rather than with horizontal symmetry. It is thought that perception of symmetry develops during infancy and an appreciation of vertical symmetry develops first (Bornstein and

kite

rhombus

Isosceles triangle

Arrowhead kite

Right angled triangle

Note that points A and D lie together.

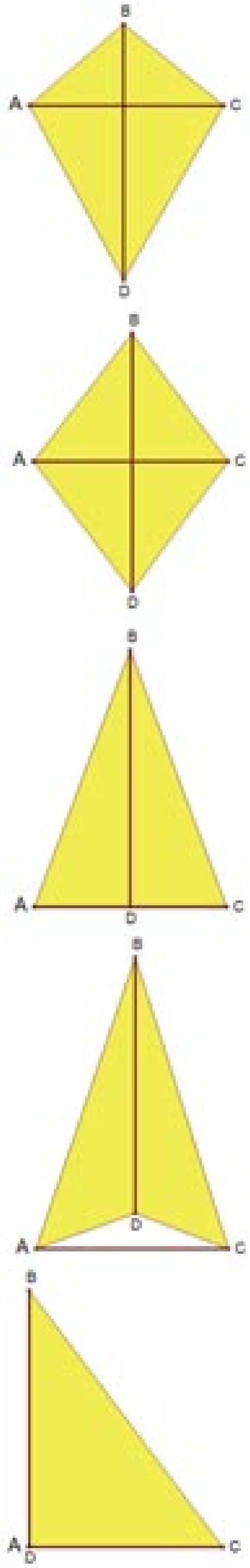

Figure 3

Krinsky, 1985, cited in Ortman and Shutte, 2010). Maybe this is connected to our own biology as reasonably symmetrical beings with a vertical axis.

The students often dragged one of the bars so that it maintained its status as a line of symmetry for the shape. I have called this strategy "dragging to maintain symmetry". In doing this they must have used a sense of symmetry and often talked about keeping the bar "in the middle of the shape". When trying to make a symmetrical shape the students talked about trying to make two halves look equal. This suggests that they possess a sense of symmetry which they used to guide them in moving the bars. When I asked the students to tell me what they understood about symmetry, their conception was often underpinned by their experience in mathematics lessons of folding shapes along an axis of symmetry. They were able to visualise the folding of the shape and use this visualisation to identify equal sides and angles using their understanding of line symmetry.

The vertical and horizontal bars sit easily within the frame of reference of the computer screen and so it seems natural that the students would generate shapes with vertical and horizontal symmetry. In the next stage of the project I tested my ideas about the students' use of symmetry when positioning the bars. I would make the tasks potentially more difficult by changing the orientation of the bars.

First I tested whether the students would still show a preference for vertical symmetry when the horizontal bar was longer than the vertical bar. I created the file with 6 cm vertical and 8 cm horizontal bars to test this as shown in the figure below.

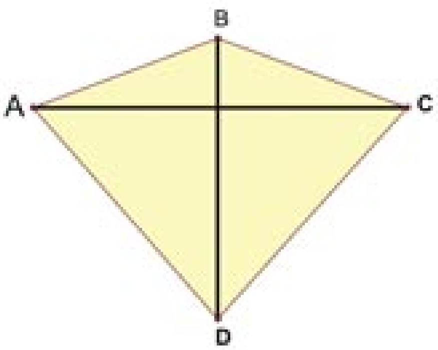

Figure 4

The students generated more shapes with vertical symmetry than horizontal symmetry in the file with the 8 cm horizontal and 6 cm vertical bars. This would indicate that the vertical axis is preferred even when it is shorter than the horizontal axis.

Next I created a file with perpendicular fixed length bars at an angle to the vertical as shown in figure 5. I was interested to see whether the students would still generate symmetrical shapes when the bars were orientated differently from the local frame of reference of the computer screen.

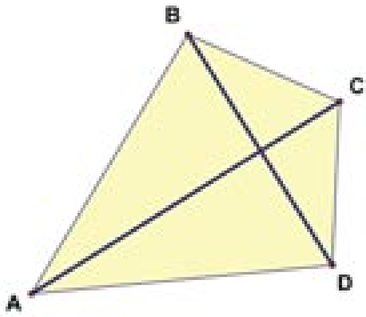

Figure 5

When dragging the bars to generate shapes in this task file the students still typically dragged one bar so that it kept its status as line of symmetry even when this entailed dragging diagonally across the screen, which must be more difficult than dragging vertically. They often talked about dragging 'up' or 'down' when this actually referred to dragging towards the top right hand corner of the screen, for example. It may be that the students were mentally rotating the figure on the screen (Pinker, 1997) so that they were dragging up in their own minds Some students found it harder to use dragging to maintain symmetry when they were dragging at an angle to the vertical which is indicated by the longer episodes of dragging due to the students going more slowly. If they are having to mentally rotate as well as keep symmetry constant then there must be a larger cognitive load, hence the longer times taken.

Another observation that emerged from the recordings was that there is a preferred position for a kite! The students, almost always positioned the 'cross bar' approximately three quarters of the way along (usually 'up') the bar which was the axis of symmetry as shown below. It could be argued that the reason for this is they have seen toy kites with the bars in this position. However another interpretation could be that humans find this kite position more aesthetically pleasing. One outcome of this preference was that some students thought kites should only be

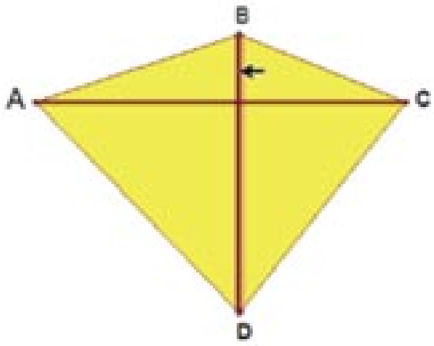

Figure 6

made in the 'three quarters' position and therefore they found it difficult to reason that moving the bars could generate an infinite number of kites. Other students reasoned that if you kept moving the cross bar bit by bit you could make many different kites.

In drawing conclusions from this work we may take into account the importance of orientation in the way humans perceive shapes. Teaching situations need to work with this rather than against it and so it is important to discuss orientation with students. We can be frank about our preference for shapes drawn the 'right way up' and talk about the shapes having been rotated when they are in a different orientation to the vertical.

Symmetry is another important aspect of the way we perceive shapes and there appears to be evidence that we focus on the line symmetry of a shape when we first visualise it in a holistic way. If that is the case then it may be more effective to teach symmetry of shapes first and to derive other properties from that.

References

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