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A Novel Hybrid Algorithm for Mean-CVaR Portfolio Selection with Real-World Constraints

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Abstract. In this paper, we employ the Conditional Value at Risk (CVaR) to measure the portfolio risk, and propose a mean-CVaR portfolio selection model. In addition, some real-world constraints are considered. The constructed model is a non-linear discrete optimization problem and difficult to solve by the classic optimization techniques. A novel hybrid algorithm based particle swarm optimization (PSO) and artificial bee colony (ABC) is designed for this problem. The hybrid algorithm introduces the ABC operator into PSO. A numerical example is given to illustrate the modeling idea of the paper and the effectiveness of the proposed hybrid algorithm.

Keywords: Conditional Value at Risk; CVaR; Hybrid algorithm; Portfolio selection

1 Introduction

Portfolio selection is concerned with the allocation of a limited capital to a combination of securities in order to trade off the conflicting objectives of high profit and low risk [13, 17]. Since the introduction of mean-variance (MV) model developed by Markowitz, variance has become the most popular risk measure in portfolio selection. Variance considers high returns as equally undesirable as low returns because high returns will also contribute to the extreme of variance. Both theory and practice indicate the variance is not a good risk measure. Some alternative risk measures have been proposed [11, 18]. Value at Risk (VaR) is widely used by financial institution. However, it has its limitations, such as it is not a coherent risk measure [1]. Rockafellar and Uryasev [15] proposed the Conditional Value at Risk (CVaR), which is the conditional expectation of losses above the VaR.

In practice, problem of portfolio selection has some real-world constraints, which exacerbates the complexity. For example, it assumes that there exists a perfect market with no tax or transaction cost. In the present study, we will consider transaction cost, and floor and ceiling constraints. In addition, the least unit of trading is 100 shares in stock market of China, and shares must be subscribed a round lot. The modeling of such constraints involves the introduction of integer variables. We employ CVaR to measure the risk of portfolio, and a Mean-CVaR (MC) portfolio selection model with real-world constraints is proposed. In view of the difficulty to solve this model using classical optimization techniques, a hybrid meta-heuristics algorithm based Particle Swarm Optimization (PSO) and Artificial Bee Colony (ABC) is designed to handle this problem. The hybrid algorithm introduces the ABC operator into PSO. The added ABC operator is used to evolve personal experience of the particles. The hybrid approach elegantly combines the exploitation ability of PSO with the exploration ability of ABC.

The rest of the paper is organized as follows. Section 2 presents the backgrounds including PSO, ABC and CVaR. Section 3 the proposed MC portfolio selection model with real-world constraints. A hybrid algorithm based on PSO and ABC is provided in Section 4. In Section 5, a numerical example is given. The conclusions are drawn in Section 6.

2 Backgrounds

2.1 Particle Swarm Optimization

PSO was originally developed to emulate the flocking behavior of birds and fish schooling [5,9]. Each individual, called a particle, in the PSO population represents a potential solution of the optimization problem [2,19]. The population of PSO is referred to as a swarm, which consists of a number of particles. Particle i at iteration t is associated with a velocity vector $\boldsymbol{v}_i^t = [v_{i1}^t, v_{i2}^t, \cdots, v_{iD}^t]$ and a position vector $\boldsymbol{x}_i^t = [x_{i1}^t, x_{i2}^t, \cdots, x_{iD}^t]$ where $i \in \{1, 2, \cdots, NP\}$, NP is the population size. $x_{id} \in [l_d, u_d]$, $d \in \{1, 2, \cdots, D\}$, where D is the number of dimensions, and l_d and u_d are the lower and upper bounds of the dth dimension of search space, respectively. Each particle flies through space with a velocity. The new velocities and the positions of the particles for the next iterations are updated using the following two equations [3–5,9]:

$$v_{id}^{t+1} = wv_{id}^t + c_1 r_1 (pbest_{id}^t - x_{id}^t) + c_2 r_2 (gbest_d^t - x_{id}^t) \tag{1}$$

$$x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1} \tag{2}$$

where w is the inertia weight; $\boldsymbol{pbest}_i = [pbest_{i1}, pbest_{i2}, \cdots, pbest_{iD}]$ is the best position has been found by particle i, $\boldsymbol{gbest}_i = [gbest_{i1}, gbest_{i2}, \cdots, gbest_{iD}]$ is the historically best position has been found by the whole swarm so far; c_1 and c_1 are acceleration coefficients. The inertia weight w is used to trade off the exploration and exploitation; r_1 and r_2 represent two independently random numbers uniformly distributed on $[0, 1]$.

2.2 Artificial Bee Colony

ABC algorithm was proposed by simulating waggle dance and intelligent foraging behaviors of honeybee colonies [7]. In the ABC algorithm, there are two components: the foraging artificial bees and the food source [8]. The position of the a food source, $\boldsymbol{x}_i = [x_{i1}, x_{i2}, \cdots, x_{iD}]$, represents a possible solution and the nectar amount of a food source corresponds to the fitness of the associated solution. The colony of artificial bees contains three groups of bees: employed bees, onlookers and scouts [14].

The ABC algorithm consists of four phases: initialization, employed bee, onlooker bee and scout bee. In the initialization phase of the ABC, SN food source positions are randomly produced with the search space. After producing food sources and assigning them to the employed bees. In the employed bee phase of ABC, each employed bee tries to find a better quality food source based on $\boldsymbol{x}_i$. The new food source, denoted as $\boldsymbol{u}_i = [u_{i1}, u_{i2}, \cdots, u_{iD}]$, is calculated from the equation below.

$$u_{ij} = x_{ij} + \phi(x_{ij} - x_{sj}) \tag{3}$$

where $i \in \{1, 2, \cdots, SN\}$, where SN denotes the number of food source; j is a randomly generated integer number in the range $[1, D]$, ϕ is a randomly number uniformly distributed in the range $[-1, 1]$, and s is the index of a randomly chosen solution. ABC changes each position in only one dimension at each iteration. The source position $\boldsymbol{x}_i$ in the employed bee's memory will be replaced by the new candidate food source position $\boldsymbol{u}_i$ if the new position has a better fitness value. Each onlooker bee chooses one of the proposed food sources depending on the probability value p_i associated with the fitness value, where

$$p_i = fit_i / \sum_{j=1}^{SN} fit_j \tag{4}$$

where fit_i is the fitness of the food source i. After the food source is selected, a new candidate food source can be expressed by Eq. (3). If a food source, $\boldsymbol{x}_i$, cannot be improved for a predetermined number of cycles, referred to as *limit*, this food source is abandoned. Then, the scout produces a new food source randomly to replace $\boldsymbol{x}_i$.

2.3 Conditional Value at Risk

Let $L(x, y)$ be the loss function with weight vector x and the return rate vector y. Let $p(r)$ be the density function of the return rate vector y. Then $L(x, y)$ is random variable dependent on x. The probability of $L(x, y)$ not exceeding a threshold α is given by

$$\psi(x, \alpha) = \int_{L(x,y) \leq \alpha} p(y) dy \tag{5}$$

The VaR of the loss associated with x and a specified probability level β in $(0, 1)$ is the value

$$VaR_\beta(x) = \min\{\alpha \in R^m : \psi(x, \alpha) \geq \beta\} \tag{6}$$

As an improved risk measure, CVaR, is the expected portfolio return, conditioned on the portfolio returns being lower than VaR. It is defined as the Eq. (7). Compared with VaR, CVaR has some superior mathematical properties.

$$\begin{aligned} CVaR_\beta(x) &= E[L(x,y)|L(x,y)) \geq VaR_\beta(x)] \\ &= (1-\beta)^{-1}\int_{L(x,y)\geq VaR_\beta(x)} L(x,y)p(y)dy \end{aligned} \tag{7}$$

CVaR can be obtained by the following equation based on reference [15]

$$F_\beta(x,\alpha) = \alpha + (1-\beta)^{-1}\int_{y\in R^M} [L(x,y)-\alpha]^+ p(y)dy \tag{8}$$

where $(a)^+$ is defined as $\max(a, 0)$.

3 The Proposed Portfolio Selection Model

In this section, we discuss the MC portfolio selection model. Assume there n risky asset and one risk-free asset in a financial market for trading. An investor hopes to allocate his/her initial wealth m_0. For notational convenience, we first introduce the following notations:

- r_i: the return of risky asset i.
- r_f: the return of risk-free asset.
- t_i: the transaction cost of risky asset i;
- $s(x)$: the total return of the portfolio.
- p_i: the price of risky asset i each round lot;
- k_i: the round lot of risky asset i invested;
- σ_i: the highest limits on risky asset i;
- ε_i: the lowest limits on risky asset i;
- λ the acceptable return of the portfolio.

The capital invested in risk assets is $\sum_{i=1}^n k_i p_i$ and the remaining capital $m_0 - \sum_{i=1}^n k_i p_i$ invested in the risk-free asset. Obviously, it holds that$\sum_{i=1}^n k_i p_i \leq m_0$. The transaction cost are consider, and it denotes as $\sum_{i=1}^n t_i k_i$. Thus, the total return $s(x)$ of the portfolio can be described as follows:

$$\begin{aligned} s(x) &= \sum_{i=1}^n k_i p_i r_i + r_f(m_0 - \sum_{i=0}^n k_i p_i) - \sum_{i=1}^n t_i k_i \\ &= r_f m_0 + \sum_{i=1}^n [k_i p_i (r_i - r_f) - t_i k_i] \end{aligned} \tag{9}$$

The intention of the proposed model is to minimize the CVaR in the case of the return of the portfolio is equal or greater than λ.

$$\min \boldsymbol{z} = CVaR \tag{10}$$
$$s.t. \begin{cases} \varepsilon_i \leq x_i \leq \sigma_i & i = 1, 2, \cdots, n \\ s(x)/m_0 \geq \lambda & \\ \sum_{i=1}^{n} k_i p_i \leq m_0 & \\ k_i \geq 0, & \text{integer, } i = 1, 2, \cdots, n \end{cases}$$

where $x = (k_1 p_1/m_0, k_2 p_2/m_0, \cdots, k_n p_n/m_0)$ is the weight vector. In practice, asset i is chosen to be invested and the weight lies in $[\varepsilon_i, \sigma_i]$, where $0 \leq \varepsilon_i \leq \sigma_i \leq 1$. The first constraint is called floor and ceiling constraints. The second constraint is used to ensure the return of the portfolio.

4 A Hybrid Algorithm based on PSO and ABC

Due to the simple concept and efficiency of converging to reasonable solution fast, PSO has been successfully applied to a wide range of real-world problems. Despite the competitive performance of PSO, researchers have noted a major problem associated with the PSO is its premature convergence when solving complex problems [12]. ABC algorithm is good at exploration but poor at exploitation [20]. From the analysis of the merits and demerits of PSO and ABC, it is intuitive that hybridizing the PSO and ABC is a potential way to design an effective algorithm.

Generally, the locality of personal best position in PSO algorithm is distant from the global optimum. Once the swarm aggregates to such position, little opportunity is afforded for the swarm to explore for other solution and find the global optimum. This leads to the swarm suffer from premature convergence easily, especially when solving complicated multimodal problems. Thus, the evolution of the personal experience will promote the exploration of the personal experience space, which could potentially enhance PSO's performance. ABC has better ability to explore, which is beneficial to global search, but poor ability of exploitation. In this paper, we utilize the ABC operator to evolve the personal best position when the personal best position stagnated. It is expected that the proposed hybrid algorithm, PSOABC, combines the merits of PSO and ABC, and have capabilities of escaping from local optima and converge fast.

In PSOABC algorithm, we use PSO in the main loop. When the fitness of $\boldsymbol{pbest}_i$, denoted as $fit(\boldsymbol{pbest}_i)$, has not improved within a predefined number of successive iterations, denoted as k, it is considered to be stagnated and trapped into local optima. The setting of k is set to 3 in this paper. We only use the employed bee operator in ABC algorithm to evolve $\boldsymbol{pbest}_i$ in this work. The pseudo-code of the PSOABC algorithm is described in Algorithm 1. When $\boldsymbol{pbest}_i$ stagnated, we can use the employed bees operator to evolve $\boldsymbol{pbest}_i$. The mathematical expressions of this ABC operator described as follows:

$$z_{ij} = pbest_{ij} + \phi(pbest_{ij} - pbest_{sj}) \tag{11}$$

where s are randomly selected integers from the index of all solution with $s \neq i$. j is a randomly selected dimension number. ϕ is a randomly number uniformly distributed within the interval $[-1, 1]$.

Algorithm 1: The pseudo-code of PSOABC algorithm

Initialization: set up all parameters;
Set the maximum iteration number FEs; $t = 1$, $Stop = 0$;
Evaluate the fitness of the swarm and determine $\boldsymbol{pbest}_i$ and $\boldsymbol{gbest}$;
while *the stopping criteria is not satisfied* **do**
  for $i = 1 : NP$ **do**
    for $d = 1 : D$ **do**
      $v_{id}^{t+1} = wv_{id}^{t} + c_1 r_1 (pbest_{id}^{t} - x_{id}^{t}) + c_2 r_2 (gbest_{d}^{t} - x_{id}^{t})$;
      $x_{id}^{t+1} = x_{id}^{t} + v_{id}^{t+1}$;
    $i = i + 1$;
    Evaluate the fitness of the particle i; Update $\boldsymbol{pbest}_i$ and $\boldsymbol{gbest}$;
    if $fit(\boldsymbol{pbest}_i^t) - fit(\boldsymbol{pbest}_i^{t-1}) = 0$ **then**
      $Stop(i) = Stop(i) + 1$;
    else
      $Stop(i) = 0$;
  for $i = 1 : NP$ **do**
    if $Stop(i) \geq k$ **then**
      $z_{ij} = pbest_{ij} + \phi(pbest_{ij} - pbest_{sj})$;
    if $fit(\boldsymbol{z}_i < fit(\boldsymbol{pbest}_i)$ **then**
      $\boldsymbol{pbest}_i = \boldsymbol{z}_i$;
  $t = t + 1$

5 Numerical Example

The portfolio selection model constructed is a non-linear discrete optimization problem. The proposed hybrid algorithm based on PSO and ABC is suitable for real-valued problems. Kitayama *et al.* utilized penalty function approach handle the discrete decision variables [10]. In this approach, the discrete decision variables are handled as the continuous ones by penalizing at the intervals. The penalty function is given as the following the Eq. (12).

$$\phi(x) = \sum_{i=1}^{n} \frac{1}{2} \left[\sin \frac{2\pi\{x_{m+i}^{c} - 0.25(d_{i,j+1} + 3d_{i,j})\}}{d_{i,j+1} - d_{i,j}} + 1 \right] \tag{12}$$

where $d_{i,j}$ and $d_{i,j+1}$ represents the discrete decision variables. x_{m+i}^{c} is the continuous decision variables between $d_{i,j}$ and $d_{i,j+1}$.

We select 20 stocks from Chinese security market, as shown in Table 1. The symbol of $m(\%)$ in Table 1 denotes the expected return. The requirement of selecting the average yield is greater than 0. This paper selected raw data for the weekend's closing price.

Table 1. Stocks selected and expected return rate.

Ticker	m(%)	Ticker	m(%)
000002	0.45	600631	0.25
000039	0.46	600642	0.5
600058	0.77	600649	0.18
600098	0.63	600663	0.1
600100	0.12	600688	0.26
600115	0.35	600690	0.09
600183	0.4	600776	0.22
000541	0.26	600811	0.3
000581	0.53	600812	0.29
600600	0.37	600887	0.18

Assuming the investor has 500 million investment funds. According to the tax and commission in Chinese securities market, the transaction cost rate is set to 0.4%. The minimum invest weigh of each stock is 0, and the maximum weight is 10%. The risk-free return rate is equal to 4.14% based on one-year deposit rate in China, and λ is 4.5%.

Table 2. Experimental results comparison.

Algorithm	$\beta = 90\%$		$\beta = 95\%$		$\beta = 99\%$	
	Mean	SD	Mean	SD	Mean	SD
GA	0.0454	0.0019	0.0527	0.0064	0.0737	0.0042
PSO-w	0.0428	0.0014	0.0519	0.0044	0.0743	0.0057
ABC	0.0412	0.0015	0.0479	0.0027	0.0632	0.0024
PSOABC	**0.0336**	0.0009	**0.0343**	0.0016	**0.0443**	0.0013

Experimental results among genetic algorithm (GA), PSO-w [16], basic ABC [6] and PSOABC are compared. For a fair comparison, the population size is set to 40 for all algorithms, the maximum iteration is 3500. The selection rate, crossover rate and mutation rate is set to 0.9, 0.7 and 0.03, respectively. Other parameter settings in each algorithm are used according to their original references. All algorithms run 30 times independently. The experimental results are shown in the Table 2. In Table 2, "Mean" indicate the mean values of CVaR, and "SD" stands for the standard deviation. From Table 2, it can be seen that

PSOABC has a good performance and is a good alternative for the proposed portfolio selection model.

6 Conclusions

In this work, we proposed a MC portfolio selection model. In this model, the portfolio risk is measured by CVaR and some real-world constraints are added. Note that the round lot, which involves the introduction of integer variables, is considered. We have proposed a novel hybrid algorithm to solve the portfolio selection problem. The proposed algorithm introduces the ABC operator to PSO in order to balance exploration and exploitation. A penalty function is adopted to transform the discrete portfolio selection model into a continuous one. A numerical example is given to illustrate the modeling idea of the paper, and the experimental results show that the proposed hybrid algorithm outperforms is highly competitive for this portfolio problem.

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References

1. Artzner, P., Delbaen, F., Eber, J.M., Heath, D.: Coherent measures of risk. Mathematical Finance 9(3), 203–228 (July 1999)
2. Chen, X., Li, Y.: A modified pso structure resulting in high exploration ability with convergence guaranteed. IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics 37(5), 1271–1289 (October 2007)
3. Cheng, S.: Population Diversity in Particle Swarm Optimization: Definition, Observation, Control, and Application. Ph.D. thesis, Department of Electrical Engineering and Electronics, University of Liverpool (2013)
4. Cheng, S., Shi, Y., Qin, Q.: Population diversity of particle swarm optimizer solving single and multi-objective problems. International Journal of Swarm Intelligence Research (IJSIR) 3(4), 23–60 (2012)
5. Eberhart, R., Kennedy, J.: A new optimizer using particle swarm theory. In: Proceedings of the Sixth International Symposium on Micro Machine and Human Science. pp. 39–43 (1995)

6. Karaboga, D.: An idea based on honey bee swarm for numerical optimization. Tech. rep., Erciyes University, Engineering Faculty, Computer Engineering Department (October 2005)
7. Karaboga, D., Basturk, B.: A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm. Journal of Global Optimization 39(3), 459–471 (November 2007)
8. Karaboga, D., Basturk, B.: On the performance of artificial bee colony (ABC) algorithm. Applied Soft Computing 8(1), 687–697 (2008)
9. Kennedy, J., Eberhart, R.: Particle swarm optimization. In: Proceedings of IEEE International Conference on Neural Networks (ICNN). pp. 1942–1948 (1995)
10. Kitayama, S., Arakawa, M., Yamazaki, K.: Penalty function approach for the mixed discrete nonlinear problems by particle swarm optimization. Structural and Multidisciplinary Optimization 32(3), 191–202 (2006)
11. Konno, H., Yamazaki, H.: Mean-absolute deviation portfolio optimization model and its applications to tokyo stock market. Management Science 37(5), 519–531 (May 1991)
12. Liang, J.J., Qin, A.K., Suganthan, P.N., Baskar, S.: Comprehensive learning particle swarm optimizer for global optimization of multimodal functions. IEEE Transactions on Evolutionary Computation 10(3), 281–295 (June 2006)
13. Markowitz, H.: Portfolio Selection. The Journal of Finance 7(1), 77–91 (March 1952)
14. Qin, Q., Cheng, S., Li, L., Shi, Y.: Artificial bee colony algorithm: A survey. CAAI Transactions on Intelligent Systems 9(2), 127–135 (2014)
15. Rockafellar, R.T., Uryasev, S.: Optimization of conditional value-at-risk. Journal of Risk 2(3), 21–41 (Spring 2000)
16. Shi, Y., Eberhart, R.: A modified particle swarm optimizer. In: Proceedings of the 1998 Congress on Evolutionary Computation (CEC1998). pp. 69–73 (1998)
17. Yoshimoto, A.: The mean-variance approach to portfolio optimization subject to transaction costs. Journal of the Operations Research Society of Japan 39(1), 99–117 (March 1996)
18. Young, M.R.: A minimax portfolio selection rule with linear programming solution. Management Science 44(5), 673–683 (May 1998)
19. Zhang, G., Li, Y.: Orthogonal experimental design method used in particle swarm optimization for multimodal problems. In: The Sixth International Conference on Advanced Computational Intelligence (ICACI 2013). pp. 183–188 (October 2013)
20. Zhu, G., Kwong, S.: Gbest-guided artificial bee colony algorithm for numerical function optimization. Applied Mathematics and Computation 217(7), 3166–3173 (2010)