



The University of
Nottingham

UNITED KINGDOM · CHINA · MALAYSIA

Bai, Ruibin and Kendall, Graham (2008) A model for fresh produce shelf-space allocation and inventory management with freshness-condition-dependent demand. *INFORMS Journal on Computing*, 20 (1). pp. 78-85. ISSN 1091-9856

Access from the University of Nottingham repository:

<http://eprints.nottingham.ac.uk/47513/1/A%20model%20for%20fresh%20produce%20shelf-space%20allocation%20and%20inventory%20management%20with%20freshness-condition-dependent%20demand.pdf>

Copyright and reuse:

The Nottingham ePrints service makes this work by researchers of the University of Nottingham available open access under the following conditions.

This article is made available under the University of Nottingham End User licence and may be reused according to the conditions of the licence. For more details see: http://eprints.nottingham.ac.uk/end_user_agreement.pdf

A note on versions:

The version presented here may differ from the published version or from the version of record. If you wish to cite this item you are advised to consult the publisher's version. Please see the repository url above for details on accessing the published version and note that access may require a subscription.

For more information, please contact eprints@nottingham.ac.uk

A Model for Fresh Produce Shelf-Space Allocation and Inventory Management with Freshness-Condition-Dependent Demand

Ruibin Bai

Automated Scheduling, Optimisation and Planning (ASAP) Research Group, Division of Computer Science,
The University of Nottingham, Ningbo 315100, China, ruibin.bai@nottingham.edu.cn

Graham Kendall

School of Computer Science, University of Nottingham, Nottingham NG8 1BB, United Kingdom, gkx@cs.nott.ac.uk

A significant amount of work has investigated inventory control problems associated with fresh produce. Much of this work has considered deteriorating inventory control with many models having been proposed for various situations. However, no researchers have specifically studied fresh produce, which has its own special characteristics. Most research categorizes fresh produce into more general deteriorating categories with random lifetimes and nondecaying utilities. However, this classification is not reasonable or practical because the freshness of an item usually plays an important role in influencing the demand for the produce. In this paper, a single-period inventory and shelf-space allocation model is proposed for fresh produce. These items usually have a very short lifetime. The demand rate is assumed to be deterministic and dependent on both the displayed inventory (the number of facings of items on the shelves) and the items' freshness condition (which decreases over time). Several problem instances of different sizes are provided and solved by a modified generalized reduced gradient algorithm.

Key words: inventory; shelf-space allocation; fresh produce; optimization

1. Scope and Purpose

The profit on general foods, such as canned goods, frozen vegetables, fruit juice, etc., is gradually decreasing due to highly competitive retail conditions. The demand for these products is also slowing. On the other hand, the demand for some other merchandise, such as fresh produce, organic food, and children's clothes, has increased dramatically due to improving living standards. This requires retailers to concentrate more in these areas (Johnson 2002). We formulate a model to assist in ordering and shelf-allocation decisions for the retail of fresh produce, such as vegetables, fruits, fresh meats, etc. We draw together two distinct areas: a packing problem (Bennell and Dowsland 2001; Burke et al. 2004, 2006) and inventory control (Nahmias 1982, Raafat 1991, Goyal and Giri 2001).

The main characteristics of the items we consider are their very short shelf life and decaying utilities over time. Preliminary experiments are conducted (using a modified generalized reduced gradient algorithm), on our proposed model, to demonstrate that good solutions can be found. Most of the literature has treated fresh produce as deteriorating items

with a random lifetime and nondecaying utilities (Nahmias 1982, Goyal and Giri 2001). We assume that the produce has a continuous utility and physically deteriorates over time. Freshness is one of the main criteria to evaluate a product's quality and could dramatically affect its demand if its condition is inferior. To obtain good financial performance from fresh goods requires adoption of strict temperature control and intelligent inventory and shelf management systems. Furthermore, although many deteriorating inventory models have been proposed, most of them are based on the analysis of a single item excluding the constraints of shelf space that arise when considering a range of goods. No researchers, to our knowledge, have yet integrated a deteriorating inventory model with a shelf-space allocation model (which plays a very important role in retail decision making due to the scarce shelf-space resources). We formulate a fresh produce management model that can simultaneously decide the ordering policy as well as allocate shelf space among different items, together with consideration of utility (i.e., freshness) deterioration.

2. Background

Perishable inventory has been intensively studied, and many models have been proposed. See Nahmias (1982), Raafat (1991), and Goyal and Giri (2001) for comprehensive reviews. However, most models assume that a fixed fraction of the inventory deteriorates over time but the utilities of the items do not decay before their expiration dates. Few models specifically consider fresh produce with the characteristics we mentioned in §1. Some of the shortcomings of previous models include the following:

(1) Most models (Liu 1990, Jain and Silver 1994) assume that fresh produce has a random lifetime (usually assuming an exponentially distributed lifetime) but the item utilities do not decay over time. Hence different ages of items capture the same demand however fresh they are as long as they are not completely spoiled. This is contradictory to the common sense view that freshness is one of the most important quality criteria for fresh produce.

(2) Some models (Mandal and Phaujdar 1989, Giri et al. 1996) formulate the demand as a deterministic function of instantaneous inventory with the assumption that all stock could be displayed on the shelves. However, this situation seldom occurs because the shelf space for fresh food is normally limited. It is also expensive due to the low temperature requirements. Therefore, only a part of the inventory can be displayed on the shelf. Shelf-space allocation among different items is especially important in this situation. The significance of shelf-space allocation for non-perishable merchandise has been addressed in Kotzan and Evanson (1969), Curhan (1972), Borin et al. (1994), Urban (1998), Yang and Chen (1999), and Bai and Kendall (2005).

(3) The approaches that were used to optimize the models (Ben-Daya and Raouf 1993, Kar et al. 2001) disregarded the integer nature of the solution and assumed that the objective function is a quasiconcave function and is differentiable. The last assumption is usually too strict for problems involving many constraints.

The literature has classified different deteriorating inventory models into two types: fixed-lifetime models and random-lifetime models. Examples of fixed-lifetime models include photographic films, medicine, computer chips, and canned food. A major characteristic of this type of model is that inventory allows for different ages of items with either a first-in-first-out (FIFO) or last-in-first-out (LIFO) issuing policy (Nandakumar and Morton 1990, Liu and Lian 1999). However, fresh produce is usually treated as a typical example of a random-lifetime product due to the uncertain spoilage (Liu 1990, Jain and Silver 1994). These models usually assumed a constant fraction of

inventory decay over time (called *exponential decay* in some publications).

Since fresh produce has only a very limited shelf life, most of the literature employed a single-period inventory model although different forms of demand function are used. Both stochastic and deterministic demand inventory models were proposed for the perishable products. Ben-Daya and Raouf (1993) proposed a multi-item, single-period perishable inventory model with a uniform distribution for demand. The objective was to maximize the total profit of all the items during one period. The “optimal” solution was calculated by a Lagrangian optimization with the assumption that the objective is differentiable. The integer nature of the variables was also disregarded. Furthermore, the method is not efficient when there are many constraints. Rajan et al. (1992) proposed a dynamic pricing and ordering decision-making model for decaying produce, in which the demand was assumed to be deterministic and dependent on the selling price. The products are assumed to have an exponential deterioration. Abad (1996) formulated the demand function as a function of instantaneous price. A closed-form mathematical procedure was carried out to solve the problem and parameter sensitivities were analyzed. However, the approach is heavily dependent on the mathematical description of the model so that adding even a single constraint could invalidate this approach. Some other models formulated the demand as a deterministic function of instantaneous inventory. Mandal and Phaujdar (1989) formulated a single-period inventory model for deteriorating items. The demand rate was linearly dependent on the instantaneous inventory level, and the inventory deteriorated according to a given function. Backordering was allowed, and holding and shortage costs were considered. The objective was to minimize the average cost. The variables included the time slots for different inventory stages and maximal stock level and maximal stock deficit. Giri et al. (1996) formulated the demand as a polynomial function of the instantaneous inventory in their perishable inventory model, which also assumed an exponential decay. The objective is to maximize the profit, with order quantity and reorder point (or cycle time) as decision variables. Some time-dependent demand functions were also proposed in deteriorating inventory models to capture changing demand over time. Xu and Wang (1990) assumed a linear time-dependent demand function within a limited time horizon. Exponentially time-dependent demand was also proposed to simulate a rapidly increasing/declining market (Hollier and Mak 1983, Zhou et al. 2003). Urban and Baker (1997) used a multiplicative demand function

of price, time, and inventory level in their single-period inventory model with the aim of finding optimal ordering and pricing policies for nonperishable products.

The first research to consider the effect of utility deterioration on demand was Fujiwara and Perera (1993) in the formulation of an economic order quantity (EOQ) perishable inventory model. An exponential penalty function $\alpha(e^{\beta t} - 1)$ ($\alpha > 0, \beta > 0$) was used to measure the cost of keeping an aging item in inventory. A closed form of economic order quantities was obtained by a quadratic approximation of exponential terms. The results show that this model is consistent with other EOQ models with exponential decay. Sarker et al. (1997) also attempted to incorporate the negative effect of aging inventory on demand. In their production-inventory model, the demand function in the inventory build-up phase and depletion phase considered a constant term and a negative term that is proportional to the instantaneous inventory (i.e., $f(t) = D - \beta I(t)$, where $f(t)$ is the demand function, $\beta > 0$, D is constant demand, and $I(t)$ is the instantaneous inventory level). However, illogically, the demand during the inventory-depletion phase is actually an increasing function due to the continuous decrease of the inventory $I(t)$ over time. This contradicts the authors' initial intention to represent a declining demand with the aging of the inventory.

Almost all the models described above consider only a single item without any constraints, with the optimal solution usually obtained analytically. Recently, researchers have begun to incorporate shelf-space allocation technologies into their inventory systems. Kar et al. (2001) proposed a single-period inventory model for multi-deteriorating items with constraints on shelf space and investment. The problem considers selling the deteriorating items from two stores. At the beginning of the period, the ordered items are separated into fresh items and items that have begun to deteriorate. The fresh items are shipped to the main store, selling at a high price, and the deteriorating items are delivered to the second store and sold at a lower price. During the period, all decayed items in the main store are retained and delivered to the second store. The demand rate in the first store was formulated as a function of the item's selling price and instantaneous inventory. However, the demand in the second store was dependent only on the selling price. A generalized reduced gradient (GRG) method was used to optimize the model. However, as stated in Lasdon et al. (1978), GRG may not be efficient or robust for larger problem sizes and can guarantee only a local optimum. Besides, the non-integer variables and continuous objective assumption are the major drawbacks of this approach in

solving many NP-hard problems with integer variables. Hence, heuristic and metaheuristic approaches (Glover and Kochenberger 2003, Burke and Kendall 2005) have been used in this area to optimize these models. Borin et al. (1994) used a simulated-annealing approach to solve a product-assortment and shelf-space allocation problem. Genetic algorithms were employed in Urban (1998) to solve an integrated product-assortment, inventory, and shelf-space allocation model.

3. Model Formulation

Instead of assuming that fresh food has a random lifetime with an exponential decay, we assume that fresh food has predictable expiration dates but their freshness condition also decreases continuously according to a known function over time. The demand for the fresh produce is deterministic and is dependent on both the displayed inventory level and their freshness condition. The main difference between these two assumptions is that the former assumes that all items that have not yet deteriorated capture the same demand however fresh they are. This may sound reasonable for long-life perishable items (like photographic films and medicine) but is unrealistic for fresh produce as freshness is one of the most important aspects in evaluating their quality. In this paper, all fresh items are assumed to have a fixed, but very short, lifetime and will not entirely lose their utilities before their expiration date. However, freshness keeps decreasing over time, which has an effect on demand. The assumption of a fixed lifetime of fresh produce, with decreasing utilities, is realistic considering the advances in food planting, packing, and conservation technologies, especially the introduction of temperature-control systems in most supermarkets.

We use the following notation:

- $D_i(t)$ The demand function of item i over time.
- $f_i(t)$ A decreasing function (within range $[0, 1]$) representing the freshness condition of item i over time.
- α_i Scale parameter of item i .
- β_i Space elasticity of item i .
- σ_i Decaying rate of item i .
- $I_i(t)$ Inventory level of item i at time t .
- q_i Procurement quantity of item i .
- s_i The number of the facings assigned to item i .
- r_i The surplus of item i at the end of the cycle.
- W Total shelf space available.
- a_i Shelf space required for one facing of item i .
- p_i Unit selling price of item i .
- p_{di} Unit discounted price of item i . This price should be low enough such that all of the remaining items at the end of period can be sold out in a very short time at this price.

- c_{ai} Unit acquisition cost of item i (or unit procurement price).
- c_{hi} Unit holding cost of item i (including the costs caused by inventory loses, damage, maintenance, interest, insurance, etc.).
- c_s Shelf cost per unit space.
- C_{oi} Constant order cost of item i (independent of the order quantity).
- T_{ei} Lifetime of item i after which the item is rotten (i.e., cannot be sold).
- L_i Lower bound of the number of facings of item i .
- U_i Upper bound of the number of facings of item i .
- T_i Length of the cycle period of item i .
- HC_{1i} Total holding cost during $[0, t_{1i}]$.
- HC_{2i} Total holding cost during $[t_{1i}, T_i]$.

Many researchers (e.g., Kar et al. 2001, Urban 2002) use the function in Figure 1 to describe the change of inventory level over time t . From time 0 to t_{1i} , s_i facings of item i are displayed on the shelf with some of the stock stored in the backroom. As sales are made, the items in the backroom are moved to the shelf until the stock in backroom reaches zero (corresponding to the point when time reaches t_{1i}). Therefore, during this period, the shelf is fully stocked and the demand is a function only of product freshness. From time t_{1i} to t_{2i} , the shelf is only partly stocked and the demand is dependent on both the freshness and the instantaneous inventory level. Once the time reaches point T_i , a new order of quantity q_i is placed for item i (assuming no lead time), and the r_i surplus of item i is removed from the shelf and is assumed to be sold at a discount price, p_{di} , immediately. We adopt this representation together with a polynomial demand function that is widely used in many shelf-

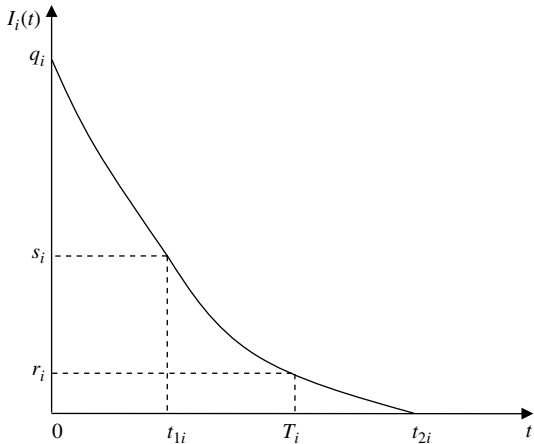


Figure 1 Graphical Representation of Inventory-Level Changes over Time

space allocation models (Corstjens and Doyle 1981, Giri et al. 1996, Urban and Baker 1997, Urban 1998)

$$D_i^*(t) = \begin{cases} \alpha_i s_i^{\beta_i} & 0 \leq t \leq t_{1i} \\ \alpha_i [I_i(t)]^{\beta_i} & t_{1i} < t \leq t_{2i}, \end{cases}$$

where α_i and β_i are scale parameters and the space elasticity of item i , respectively, and $\alpha_i > 0$, $0 < \beta_i < 1$. We assume that the demand function conforms to a multiplicative form of the instantaneous inventory and the item's freshness condition, i.e., $D_i(t) = D_i^*(t)f_i(t)$, where $f_i(t)$ is a continuously decreasing function over time and $0 \leq f_i(t) \leq 1$. $f_i(t)$ could be a linear, quadratic, or exponential function of time. During the beginning of the period, the items are fresh and the value of the freshness function is almost 1. The demand rate is affected only by the displayed inventory level. However, as time passes, $f_i(t)$ gradually decreases, and the demand is scaled down according to how long an item has been kept in inventory. To be consistent with the exponential decay assumption in the literature, we assume that an item's freshness condition decreases exponentially over time, i.e., $f_i(t) = e^{-\sigma_i t}$, where $\sigma_i > 0$ is a constant decay rate. Hence we have

$$D_i(t) = D_i^*(t)f_i(t) = \begin{cases} \alpha_i s_i^{\beta_i} e^{-\sigma_i t} & 0 \leq t \leq t_{1i} \\ \alpha_i [I_i(t)]^{\beta_i} e^{-\sigma_i t} & t_{1i} < t \leq t_{2i}. \end{cases}$$

Based on the assumptions above, the inventory level of item i can be described by the differential equation $dI_i(t)/dt = -D_i(t)$. During time $[0, t_{1i}]$, we have

$$dI_i(t)/dt = -\alpha_i s_i^{\beta_i} e^{-\sigma_i t}, \quad (1)$$

with the boundary conditions $I_i(0) = q_i$ and $I_i(t_{1i}) = s_i$. The solution of (1) is $I_i(t) = q_i + (\alpha_i s_i^{\beta_i} / \sigma_i)(e^{-\sigma_i t} - 1)$ and $t_{1i} = (-1/\sigma_i) \ln(1 - (q_i - s_i)\sigma_i / (\alpha_i s_i^{\beta_i}))$. During time $[t_{1i}, t_{2i}]$, we have the differential equation

$$dI_i(t)/dt = -\alpha_i [I_i(t)]^{\beta_i} e^{-\sigma_i t}, \quad (2)$$

with the boundary conditions $I_i(t_{1i}) = s_i$ and $I_i(t_{2i}) = 0$. The solution of (2) is

$$I_i(t) = \left[\frac{\alpha_i (1 - \beta_i)}{\sigma_i} e^{-\sigma_i t} + K_i \right]^{1/(1-\beta_i)}$$

and

$$t_{2i} = -\frac{1}{\sigma_i} \ln \left(1 - \frac{(q_i - \beta_i(q_i - s_i))\sigma_i}{\alpha_i (1 - \beta_i) s_i^{\beta_i}} \right),$$

where

$$K_i = [q_i - \beta_i(q_i - s_i)] s_i^{-\beta_i} - \mu_i$$

and

$$\mu_i = \alpha_i (1 - \beta_i) / \sigma_i.$$

In general, we have the inventory function

$$I_i(t) = \begin{cases} q_i + \frac{\alpha_i s_i^{\beta_i}}{\sigma_i} (e^{-\sigma_i t} - 1) & 0 \leq t \leq t_{1i} \\ [\mu_i e^{-\sigma_i t} + K_i]^{1/(1-\beta_i)} & t_{1i} < t \leq t_{2i}. \end{cases} \quad (3)$$

The length of cycle period T_i ($I_i(T_i) = r_i$) is

$$T_i = -\frac{1}{\sigma_i} \ln \left[\frac{1}{\mu_i} (r_i^{(1-\beta_i)} - K_i) \right].$$

The holding cost during $[0, t_{1i}]$ is

$$\begin{aligned} HC_{1i} &= c_{hi} \int_0^{t_{1i}} \left(q_i + \frac{\alpha_i s_i^{\beta_i}}{\sigma_i} (e^{-\sigma_i t} - 1) \right) dt \\ &= c_{hi} \left[\left(q_i - \frac{\alpha_i s_i^{\beta_i}}{\sigma_i} \right) t_{1i} + (1 - e^{-\sigma_i t_{1i}}) \frac{\alpha_i s_i^{\beta_i}}{\sigma_i^2} \right]. \end{aligned}$$

The holding cost during $[t_{1i}, T_i]$ is

$$HC_{2i} = c_{hi} \int_{t_{1i}}^{T_i} ([\mu_i e^{-\sigma_i t} + K_i]^{1/(1-\beta_i)}) dt.$$

The approximate expression of HC_{2i} is given in the appendix. However, this part is very small, and we use a simpler approximation (using $(s_i + r_i)/2$ as an approximation of average inventory during $[t_{1i}, T_i]$)

$$HC_{2i} = c_{hi}(s_i + r_i)(T_i - t_{1i})/2.$$

Therefore, the average profit of item i per unit time is the total income less any costs involved, divided by the time of the period

$$\begin{aligned} M_i &= \frac{1}{T_i} [p_i(q_i - r_i) + p_{di}r_i - c_{ai}q_i - C_{oi} \\ &\quad - HC_{1i} - HC_{2i}] - c_s s_i a_i. \end{aligned}$$

The objective is to maximize the overall profit of all items during the unit time.

$$\max \sum_{i=1}^n M_i(s_i, q_i, r_i) \quad (4)$$

$$\text{subject to } \sum_{i=1}^n s_i a_i \leq W \quad (5)$$

$$L_i \leq s_i \leq U_i \quad i = 1, 2, \dots, n \quad (6)$$

$$r_i \leq s_i \leq q_i \quad i = 1, 2, \dots, n \quad (7)$$

$$r_i < q_i \quad i = 1, 2, \dots, n \quad (8)$$

$$0 < T_i \leq T_{ei} \quad i = 1, 2, \dots, n \quad (9)$$

$$s_i, q_i \in \{1, 2, 3, \dots\} \quad i = 1, 2, \dots, n \quad (10)$$

$$r_i \in \{0, 1, 2, \dots\} \quad i = 1, 2, \dots, n. \quad (11)$$

The decision variables are shelf space, order quantity, and the amount of surplus at the end of the cycle.

Constraint (5) ensures that the total shelf space allocated to each item is no more than the total available shelf space. Constraint (6) makes sure that the space allocated to each item must be within an upper and a lower bound. Constraint (7) ensures that the order quantity of each item must be greater than the shelf displayed quantity, which itself should be greater than the number of surplus items. Constraint (9) ensures that the span of one cycle period must be less than the product-validity period. Constraints (10) and (11) ensure that the number of facings, order quantity, and the number of surplus items are integers. The model is a nonlinear combinatorial optimization problem and is difficult to optimize via conventional approaches.

If we have n products, the total number of variables is $3n$. From the model, we have the upper and lower bounds of variables r_i ($0 < r_i \leq s_i$) and s_i ($L_i < s_i \leq U_i$) and lower bound of q_i ($q_i \geq s_i$). The upper bound of q_i can be obtained from constraint (9). Since

$$T_i = -\frac{1}{\sigma_i} \ln \left[\frac{1}{\mu_i} (r_i^{(1-\beta_i)} - K_i) \right] \leq T_{ei},$$

we have

$$q_i \leq \frac{1}{(1-\beta_i)} r_i^{(1-\beta_i)} s_i^{\beta_i} + \frac{\alpha_i}{\sigma_i} s_i^{\beta_i} - \frac{\beta_i}{(1-\beta_i)} s_i - \frac{\alpha_i}{\sigma_i} e^{-\sigma_i T_{ei}} s_i^{\beta_i}.$$

If $[x]$ represent the largest integer no greater than x , the upper bound of order quantity q_i^{ub} is

$$q_i^{ub} = \left\lfloor \frac{1}{(1-\beta_i)} r_i^{(1-\beta_i)} s_i^{\beta_i} + \frac{\alpha_i}{\sigma_i} s_i^{\beta_i} - \frac{\beta_i}{(1-\beta_i)} s_i - \frac{\alpha_i}{\sigma_i} e^{-\sigma_i T_{ei}} s_i^{\beta_i} \right\rfloor.$$

An interesting implication of the model is that inventory depletes exponentially over time (see (3)), which is consistent with the exponential decay models in the literature. In addition, when $\sigma_i \rightarrow 0$, $e^{-\sigma_i t} \rightarrow 1 - \sigma_i t$, so the inventory function becomes the same polynomial function derived in Urban (2002).

4. Optimization of the Model

We use GRG algorithm to search for good solutions to the model (4–11). The underlying ideas behind the algorithm are described in Gabriele and Ragsdell (1977) and Lasdon et al. (1978). The algorithm has been shown to be efficient in solving nonlinear programming problems with smooth objective functions, and its applications in optimizing the inventory and shelf-space allocation model include Urban (1998) and Kar et al. (2001), with good results. The GRG algorithm is embedded in many spreadsheet software packages. The one we use is Solver, which is included in Microsoft Excel 2002. However, the GRG algorithm has two major drawbacks: (1) It can solve only continuous variable models. Although the package included

in Microsoft Excel 2002 can deal with integer variables, it takes too long for the search to converge (1,800 seconds computation time is needed for a problem with only six items, running on a Pentium IV 1.8 GHz with 256 MB RAM; for a problem with 18 products, the algorithm does not converge even after one hour). (2) GRG usually gives only a local optimum that is closest to the starting point. Some preliminary experiments showed that, if the starting point is not carefully chosen, GRG performs very badly. Thus, we used a multithread GRG algorithm together with a solution-repair heuristic. Each thread of the algorithm can be divided into three subprocedures: initialization, GRG, and solution repair; shown in Figure 2.

```

Set MaxIter;
Set iter = 0;
Loop
//Initialization subprocedure
For each item i ( $1 \leq i \leq n$ ) set  $s_i = L_i$ ,  $q_i = s_i$ ,  $r_i = 0$ ;
Loop
  Select a random item j;
   $s_j = s_j + 1$ ;
Until no more facings can be added without violating the space
  constraint (5);
For each item i
  Increase  $q_i$  until no improvement can be obtained in the
  objective value;
  Increase  $r_i$  until no improvement can be obtained in the
  objective value;
Output solution  $S_0(q_i, s_i, r_i)$ 
//GRG calling subprocedure
 $S' = \text{Solver}(S_0)$ ;
//Solution repair subprocedure
Round every  $s_i, q_i, r_i$  ( $1 \leq i \leq n$ ) in  $S'$  to their nearest integers
While space constraint (5) is violated
  Rank the items by their unit space profit value  $M_i/(a_i s_i)$ ;
  Delete one facing of the item with the smallest unit space
  profit value (if this operation causes a constraint violation,
  the next item in the ranking list is considered);
If free shelf space > the size of the smallest item
  Loop
  Rank the items by their unit space profit value  $M_i/(a_i s_i)$ ;
  Add one facing of the item with the largest unit-space
  profit value (the next item in the ranking list is
  considered if the operation generates a constraint
  violation);
  Until no more facings can be added without violating the
  space constraint (5);
For each item i ( $1 \leq i \leq n$ )
  Increase/decrease  $q_i$  until no improvement can be obtained
  in the objective value;
  Increase/decrease  $r_i$  until no improvement can be obtained
  in the objective value;
Remember the best solution ( $S_{best}$ ) found so far;
iter++;
Until iter = MaxIter;
Output  $S_{best}$ ;

```

Figure 2 Pseudocode of the Multistart GRG Algorithm

To prevent GRG from getting stuck at a local optimum, *MaxIter* runs of GRG were executed using different initial states (solutions) and the best solution was output as the final solution. In this application, we set *MaxIter* = 5 after some preliminary experiments. The initialization subprocedure was used to generate a set of diverse solutions that can be used by GRG. Note that, because GRG is efficient only when handling continuous variables, a relaxed model (ignoring integrality constraints (10) and (11)) was input into the Excel Solver. Therefore, the solution output by GRG was not feasible. The solution-repair subprocedure is used to recover feasibility of the solution and further improve it by using a simple local-search method described in Figure 2 (several other rounding heuristics were tried, and this one generally performs best across the five problem instances we tested). All results were averaged over 10 runs on a Pentium IV 1.8 GHz CPU with 256 MB RAM, running Microsoft Windows 2000 Professional Version 5.

5. A Numerical Example

To provide a better understanding of the model and the solution procedure described above, a numerical example with six items was generated (denoted by BORIN94/6). The problem scale parameters (α_i) and space elasticities (β_i) are taken from Borin et al. (1994) and the other parameters are listed in Table 1. The GRG algorithm described in §4 was run 10 times with different initial random solutions. The algorithm consistently returned the same solution, which is shown in Table 2. For comparison, an exhaustive search was

Table 1 Parameters of the Numerical Example

Item	a_i	p_i	c_{ai}	c_{hi}	p_{di}	C_o	α_i	β_i	σ_i
1	0.028	5.03	2.46	0.19	1.23	34.3	28.53	0.1532	0.06
2	0.061	9.37	5.67	0.20	2.84	48.9	23.62	0.2273	0.07
3	0.025	5.10	2.70	0.26	1.35	35.6	25.59	0.2089	0.06
4	0.060	11.48	6.11	0.16	3.06	47.9	22.40	0.2143	0.04
5	0.036	6.74	3.53	0.30	1.77	33.9	15.62	0.2955	0.03
6	0.033	5.97	3.41	0.27	1.71	39.1	10.50	0.3104	0.03

Note. $W = 0.608$ (m²), $c_s = 5.0$ (pounds/m²/unit time), $L_i = 1$, $U_i = 12$, $T_{ei} = 7$ (days).

Table 2 Solution of the Numerical Example

Item	Solution by GRG				Optimal solution			
	q_i	s_i	r_i	T_i	q_i	s_i	r_i	T_i
1	83	3	0	2.68	81	2	0	2.78
2	78	2	0	3.17	78	2	0	3.17
3	77	3	0	2.61	77	3	0	2.61
4	88	3	0	3.35	88	3	0	3.35
5	64	3	0	3.17	64	3	0	3.17
6	50	1	0	5.19	56	2	0	4.68
Objective	347.45				347.58			

Table 3 Parameters of Problem Instances

Parameters	Values	Parameters	Values
n	18/32/49/64	L_i	1
α_j	$U(10, 30)$	U_i	12
β_i	$U(0.15, 0.3)$	p_{di}	$0.5c_{ai}$
σ_i	$U(0.03, 0.1)$	c_s	5.0 pounds/m ² /day
a_i	$U(0.01, 0.09)$ m ²	C_o	$U(30, 50)$ pounds
c_{ai}	$N(100a_i, 0.4)$ pounds	T_{ei}	7 days
p_i	$N(1.8c_{ai}, 0.4)$ pounds	W	$2.5 * \text{minSpace}$
c_{hi}	$U(0.1, 0.3)$ pounds		

Notes. $U(a, b)$: uniform distribution. $N(c, d)$: normal distribution. minSpace : the minimal space requirement to satisfy products' number-of-facings lower bounds.

also carried out to get an optimal solution, also shown in Table 2. For this numerical example, the solution obtained by GRG is very close to the optimal solution. The relative deviation from optimality is only 0.04% ($(347.58 - 347.45)/347.58$ 100%).

6. Larger Problem Instances

Although numerical examples are helpful in understanding the model and testing the performance of the solution procedure, it is necessary to test the algorithm over larger problem instances. We created four benchmark problem instances using the parameters in Table 3. The problem sizes range from 18 to 64 products. These data sets can be downloaded from the Online Supplement to this paper (available at <http://joc.pubs.informs.org/ecompanion.html>). Here we provide the computational results of the modified GRG algorithm, shown in Table 4. The modified GRG algorithm is quite robust on the five tested problem instances. With BORIN94/6 and FRESH2, all 10 runs consistently returned the same solution although each run started from different, random initial solutions. For the other three instances, the difference between the best solution and worst solution, among the 10 runs, is very small and the standard deviations are less than one, a small value compared with the objective values. All solutions obtained by the algorithm

Table 4 The Computational Results of the GRG Algorithm on Five Problem Instances

	BORIN94/6	FRESH2	FRESH3	FRESH4	FRESH5
n	6	18	32	49	64
Av. obj.	347.45	1,129.60	2,056.46	3,163.98	4,387.16
Best obj.	347.45	1,129.60	2,057.15	3,164.59	4,387.73
Worst obj.	347.45	1,129.60	2,055.17	3,163.33	4,386.66
Std. dev.	0.00	0.00	0.97	0.51	0.43
Av. cpu	3.2	73.6	74.3	179.2	209.7

Notes. Av. obj.: the average objective value over 10 runs. Best obj.: The best objective value over 10 runs. Worst obj.: The worst objective value over 10 runs. Std. dev.: Absolute standard deviation of 10 results obtained by GRG. Av. cpu: Average cpu time consumed by GRG (in seconds).

satisfy the integer constraints and are therefore feasible solutions.

7. Conclusions

A single-period inventory and shelf-space allocation model has been proposed for fresh produce. The demand is assumed to be deterministic and conforms to a multiplicative form of the displayed stock-level and items' freshness conditions. The items' freshness condition is assumed to drop exponentially over time but could still capture some demand. The model is consistent with deteriorating inventory models reported in literature, in which an exponential decay in the inventory is assumed. Unlike other work, the proposed model considers the integer nature of the solution. Five benchmark problem instances were generated for the fresh produce inventory control and shelf-space allocation problem. A modified GRG algorithm was used to search for good solutions and their computational results were reported. The algorithm used in this paper ensures integrality of the decision variables.

Acknowledgments

The authors are very grateful to the United Kingdom Engineering and Physical Sciences Research Council (EPSRC) (Ref.: GR/R60577) for their funding and support. The authors also thank the anonymous reviewers for their comments, which led to a much improved paper.

Appendix

Denote $y(t) = [\mu_i e^{-\sigma_i t} + K_i]^{1/(1-\beta_i)}$. Divide the range $[t_{1i}, T_i]$ into k identical ranges by point $x_0 = t_{1i}$, $x_1, x_2, \dots, x_k = T_i$. We have

$$\begin{aligned}
 HC_{2i} &= c_{hi} \int_{t_{1i}}^{T_i} ([\mu_i e^{-\sigma_i t} + K_i]^{1/(1-\beta_i)}) dt \\
 &\approx \frac{c_{hi}(T_i - t_{1i})}{k} \left[\frac{1}{2}(y(x_0) + y(x_k)) + y(x_1) + \dots + y(x_{k-1}) \right].
 \end{aligned}$$

References

- Abad, P. L. 1996. Optimal pricing and lot-sizing under conditions of perishability and partial backordering. *Management Sci.* **42** 1093–1104.
- Bai, R., G. Kendall. 2005. An investigation of automated planograms using a simulated annealing based hyper-heuristic. T. Ibaraki, K. Nonobe, M. Yagiura, eds. *Metaheuristics: Progress as Real Problem Solvers—Oper. Res./Comput. Sci. Interfaces*, Vol. 32. Springer, Berlin, 87–108.
- Ben-Daya, M., A. Raouf. 1993. On the constrained multi-item single-period inventory problem. *Internat. J. Oper. Production Management* **13** 104–112.
- Bennell, J. A., K. A. Dowsland. 2001. Hybridising tabu search with optimisation techniques for irregular stock-cutting. *Management Sci.* **47** 1160–1172.
- Borin, N., P. W. Farris, J. R. Freeland. 1994. A model for determining retail product category assortment and shelf space allocation. *Decision Sci.* **25** 359–384.

- Burke, E. K., G. Kendall. 2005. *Search Methodologies: Introductory Tutorials in Optimization and Decision Support Methodologies*. Springer, Berlin.
- Burke, E. K., G. Kendall, G. Whitwell. 2004. A new placement heuristic for the orthogonal stock cutting problem. *Oper. Res.* **52** 655–671.
- Burke, E. K., R. S. R. Hellier, G. Kendall, G. Whitwell. 2006. A new bottom-left-fill heuristic algorithm for the two-dimensional irregular packing problem. *Oper. Res.* **54** 587–601.
- Corstjens, M., P. Doyle. 1981. A model for optimising retail space allocations. *Management Sci.* **27** 822–833.
- Curhan, R. 1972. The relationship between space and unit sales in supermarkets. *J. Marketing Res.* **9** 406–412.
- Fujiwara, O., U. L. J. S. R. Perera. 1993. EOQ models for continuously deteriorating products using linear and exponential penalty costs. *J. Oper. Res.* **70** 104–114.
- Gabriele, G. A., K. M. Ragsdell. 1977. The generalized reduced gradient method: A reliable tool for optimal design. *AMSE J. Engrg. Indust.* **99** 384–400.
- Giri, B. C., S. Pal, A. Goswami K. S. Chaudhuri. 1996. An inventory model for deteriorating items with stock-dependent demand rate. *Eur. J. Oper. Res.* **95** 604–610.
- Glover, F., G. A. Kochenberger. 2003. *Handbook of Metaheuristics*. Kluwer, Boston.
- Goyal, S. K., B. C. Giri. 2001. Recent trends in modeling of deteriorating inventory. *Eur. J. Oper. Res.* **134** 1–16.
- Hollier, R. H., K. L. Mak. 1983. Inventory replenishing policies for deteriorating items in a declining market. *Internat. J. Production Res.* **21** 813–826.
- Jain, K., E. A. Silver. 1994. Lot sizing for a product subject to obsolescence or perishability. *Eur. J. Oper. Res.* **75** 287–295.
- Johnson, M. 2002. Personal correspondence. Retail Vision Ltd., Huddersfield, UK.
- Kar, S., A. K. Bhunia, M. Maiti. 2001. Inventory of multi-deteriorating items sold from two shops under single management with constraints on space and investment. *Comput. Oper. Res.* **28** 1203–1221.
- Kotzan, J., R. Evanson. 1969. Responsiveness of drug store sales to shelf space allocations. *J. Marketing Res.* **6** 465–469.
- Lasdon, L. S., A. D. Waren, A. Jain, M. Ratner. 1978. Design and testing of a generalized reduced gradient code for nonlinear programming. *ACM Trans. Math. Software* **4** 34–50.
- Liu, L. 1990. (s, S) Continuous review models for inventory with random lifetimes. *Oper. Res. Lett.* **9** 161–167.
- Liu, L., Z. Lian. 1999. Continuous review models for inventory with fixed lifetimes. *Oper. Res.* **47** 150–158.
- Mandal, B. N., S. Phaujdar. 1989. An inventory model for deteriorating items and stock-dependent consumption rate. *J. Oper. Res. Soc.* **40** 483–488.
- Nahmias, S. 1982. Perishable inventory theory: A review. *Oper. Res.* **30** 680–708.
- Nandakumar, P., T. E. Morton. 1990. Near myopic heuristic for the fixed life perishability problem. *Management Sci.* **39** 241–246.
- Raafat, F. 1991. Survey of literature on continuously deteriorating inventory model. *J. Oper. Res. Soc.* **42** 27–37.
- Rajan, A., Rakesh, R. Steinberg. 1992. Dynamic pricing and ordering decisions by a monopolist. *Management Sci.* **38** 240–262.
- Sarker, B. R., S. Mukherjee, C. V. Balan. 1997. An order-level lot size inventory model with inventory-level dependent demand and deterioration. *Internat. J. Production Econom.* **48** 227–236.
- Urban, T. L. 1998. An inventory-theoretic approach to product assortment and shelf-space allocation. *J. Retailing* **74** 15–35.
- Urban, T. L. 2002. The interdependence of inventory management and retail shelf management. *Internat. J. Phys. Distribution Logist. Management* **32** 41–58.
- Urban, T. L., R. C. Baker. 1997. Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns. *Eur. J. Oper. Res.* **103** 573–583.
- Xu, H., H.-P. Wang. 1990. An economic ordering policy model for deteriorating items with time proportional demand. *Eur. J. Oper. Res.* **46** 21–27.
- Yang, M.-H., W.-C. Chen. 1999. A study on shelf space allocation and management. *Internat. J. Production Econom.* **60** 309–317.
- Zhou, Y.-W., H.-S. Lau, S.-L. Yang. 2003. A new variable production scheduling strategy for deteriorating items with time-varying demand and partial lost sale. *Comput. Oper. Res.* **30** 1753–1776.