

The implementation, interpretation, and justification of likelihoods in cosmology

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Abstract

I discuss the formal implementation, interpretation, and justification of likelihood attributions in cosmology. I show that likelihood arguments in cosmology suffer from significant conceptual and formal problems that undermine their applicability in this context.

1 Introduction

In recent decades cosmologists have increasingly made use of arguments that involve the assignment of probabilities to cosmological models, usually as a way of guiding further theorizing about the universe. This despite cosmology being, on the face of it, an unlikely subject in which to employ probabilistic reasoning. In usual applications the utility of probabilities depends on their connection to empirical frequencies. In cosmology there is, so far as we know, only one universe. It would therefore seem to be an almost pointless exercise to attribute probabilities to the universe, its particular creation, or its particular history, as the assignment of probabilities would apparently be completely arbitrary. Nevertheless, perhaps owing to the significant observational limitations that exist in cosmology, cosmologists have sought to bolster the available empirical evidence with probabilistic reasoning, maintaining that it is both important and sensible to do so.¹

Not only is strictly probabilistic reasoning salient in cosmology, but so are various other arguments which are similar in style to probabilistic reasoning. I will refer to such reasoning in general as *likelihood* reasoning. For example, typicality and some topology-based arguments do not rely on probabilities per se, but, like many probabilistic arguments, they aim to show that some conclusion or kind of outcome is, for example, typical or atypical, probable or improbable, or favored or disfavored, i.e. *likely* or *unlikely*.²

While the logical structure of such arguments is similar, the formal implementation, interpretation, and justification of the likelihoods themselves can differ significantly. The aim of this paper is to investigate these three features of likelihoods in order to determine the applicability of likelihood reasoning in cosmology. Although it is not possible to show that such reasoning definitively fails in all cases, I will

¹It is not hard to find cosmologists expressing the importance of such arguments in cosmology: “The problem of constructing sensible measures on the space of solutions is of undeniable importance to the evaluation of various cosmological scenarios” (Gibbons and Turok, 2008, 1); “...the measure could play an important role in deciding what are the real cosmological problems which can then be concentrated on. In other words, we assume that our Universe is typical, and only if this was contradicted by the experimental data would we look for further explanations” (Coule, 1995, 455-6); “Some of the most fundamental issues in cosmology concern the state of the universe at its earliest moments, for which we have very little direct observational evidence. In this situation, it is natural to attempt to make probabilistic arguments to assess the plausibility of various possible scenarios (Schiffrin and Wald, 2012, 1).

²A referee notes the common technical usage of the term “likelihood” in statistics or more broadly in Bayesianism. I do not mean it in any technical sense but rather as a general term that covers kinds of reasoning similar to probabilistic reasoning.

argue that the various challenges I discuss do significantly undermine its viability in this context. These challenges include both conceptual and formal issues.

Before turning to these issues, however, it is appropriate to say a little more about the kind of arguments with which I am concerned. In an influential paper, Gibbons, Hawking, and Stewart (GHS) give a concise formula for how likelihood reasoning is applied in cosmology:

Cosmologists often want to make such statements as “almost all cosmological models of a certain type have sufficient inflation,” or “amongst all models with sufficient baryon excess only a small proportion have sufficient fluctuations to make galaxies.” Indeed one popular way of explaining cosmological observations is to exhibit a wide class of models in which that sort of observation is “generic.” Conversely, observations which are not generic are felt to require some special explanation, a reason why the required initial conditions were favoured over some other set of initial conditions.” (Gibbons et al., 1987, 736)

GHS here suggest how such arguments can be used to guide further theorizing in cosmology. As they explicitly say, if some observed feature of the universe can be shown to be likely among the physically reasonable cosmologies, then it requires no further explanation; if it is unlikely, then it requires further explanation. Another variant goes as follows: if some unobservable feature of the universe is shown to be likely among physically reasonable cosmologies, then one infers that it exists; if it is unlikely, then one infers that it does not. In the following section I will provide an important example, fine-tuning of the standard model of cosmology, that follows these formulas.

I emphasize that there exist various formal implementations of likelihood that can be used to support this kind of argument, e.g. using topology, measure theory, probability theory, etc. Cosmologists, however, have generally favored those that are similar to the application of likelihoods in statistical mechanics, a context where likelihood reasoning is acknowledged as successful. Simply inferring from the success of arguments in statistical mechanics to similar ones in cosmology presupposes, however, that the justification and interpretation of likelihoods in statistical mechanics appropriately carries over to the cosmological context. I will argue that this presupposition is incorrect. Indeed, a central claim defended in this paper is that the justification and interpretation of cosmological likelihoods cannot be secured by similar strategies used to justify and interpret the use of likelihoods in statistical mechanics. I draw attention to this particular strategy at the outset because many cosmologists appear to take the problematic inferences for granted, and it is important to see that it is not viable. This is not the only strategy, of course, so its failure does not completely undermine likelihood reasoning in cosmology. Hence, although there is an emphasis on this particular strategy in the paper, in the main it concerns general challenges to implementing, interpreting, and justifying likelihoods in cosmology.

Although investigating the full complement of formal implementations of likelihood notions would be of interest, for reasons of simplicity, familiarity, and relevance to arguments made in the literature, I will concentrate mostly on probabilistic measures of likelihood. Although I will usually not generalize the considerations raised in the following to other formal implementations of likelihood, many of them do so generalize; the reader is therefore invited to keep these other implementations in mind. Nonetheless, at times I do consider topology- and typicality-based arguments explicitly.

Concerning probabilistic likelihoods specifically, recall that an application of probability theory standardly requires three things: a set X of possible outcomes (the “sample space”), a σ -algebra \mathcal{F} of these possible outcomes (a collection of subsets that is closed under countable set-theoretic operations), and a probability measure P that assigns probabilities to elements of \mathcal{F} .³ The probability spaces relevant for

³Similarly, a topological space is specified by a set X (of possible spacetimes in this context) and a topology on X , i.e. a collection of subsets of X (the “open” sets). With a topology on X one can define a suitable notion of “negligible set” in the topology on X , for example a set whose closure has empty interior. The complements of negligible sets, “generic sets,” are then sets with properties that are “almost always” possessed by the set X . In this way topology can be used to define a rough notion of likelihood: “almost always” and “almost never.”

likelihood reasoning are those whose possible outcomes are possible cosmologies (models of the universe). Since the success of probabilistic arguments depends on an adequate justification of the relevant probability space and an adequate interpretation of probability in this context, I take as necessary conditions on a cosmological probability space that it be well-defined and that the choice of X and P must be justifiable and physically interpretable. (I take it that \mathcal{F} can be chosen on essentially pragmatic grounds.) These are the implementation, interpretation, and justification conditions required for a probabilistic likelihood attribution. The challenges I raise in the following concern meeting these conditions.

The plan of the paper is as follows. I first provide (§2) a concrete example, fine-tuning problems with the standard model of cosmology, to further motivate and focus the subsequent investigation. In §3 I consider general conceptual issues of probability measures in cosmology, including the specification of the appropriate reference class X , and the interpretation and the justification of the probability measure P . The main conclusions of this section are that implementing cosmological probabilities can only be understood as an assignment of probabilities to initial conditions of the universe and, more importantly, that there is indeed no acceptable justification for any particular probability measure in the context of (single universe) cosmology. I then investigate the potential for formally implementing a measure associated with the space of possible cosmologies permitted by the general theory of relativity in §4. I point out a variety of significant obstacles to providing any such measure. One can avoid (or at least ignore) most of these general issues by truncating the spacetime degrees of freedom so that the relevant probability space is finite-dimensional. This is the approach taken to define the most discussed measure, the Gibbons-Hawking-Stewart (GHS) measure (Gibbons et al., 1987). In §5 I argue that even setting aside the problems raised in §§3-4 there are serious interpretive and technical problems with taking this narrower approach, in particular for supporting the fine-tuning arguments presented in §2. I offer concluding remarks in §6.

2 Fine-Tuning Problems in Cosmology

To make the discussion more concrete, I will make use of a specific example involving likelihood arguments. Perhaps the most salient cases of likelihood reasoning in cosmology concern so-called “fine-tuning” problems.⁴ Two of the most important fine-tuning problems in recent history are the hot big bang (HBB) model’s flatness problem and horizon problem. They are important for my purposes because there is some reason to think that they are part of a successful chain of likelihood arguments, which I will briefly explain now.

The horizon and flatness problems begin with observations which suggest that the universe is, respectively, remarkably uniform at large scales and has a spatial geometry very close to flat. In the context of HBB model, the old standard model of cosmology, these presently observed conditions require very special initial conditions: an extraordinary degree of uniformity and flatness. If the conditions at the beginning of the universe were ever so slightly different than these initial conditions, the universe would be nothing at all like what it is now. The fine-tuning of the HBB model, then, is taken to be precisely this specialness of initial conditions.

How should one understand this specialness though? The most intuitive characterization makes use of likelihoods: spatial flatness and uniformity are (in some sense) unlikely given the relevant physical possibilities. What makes fine-tuning problematic, given this rendering, is unfortunately never made especially clear by cosmologists, but one might think that unlikely circumstances are either simply unlikely to be true or perhaps that such circumstances lack explanatory power (making such fine-tuning problems explanatory problems). In any case, it is in this way that the two fine-tuning problems can be understood as instances of likelihood arguments. Following the formula of GHS, in the context of the HBB model

⁴Fine-tuning problems also appear elsewhere in physics. For example, in high energy physics the failure of naturalness in the standard model of particle physics, known as the hierarchy problem, is often described as a fine-tuning problem.

flatness and uniformity are not generic; therefore they require an explanation.

Thus this is not the end of the story, since the fine-tuning problems create an explanatory demand. Indeed, the flatness and horizon problems are standardly used to motivate the introduction of the theory of cosmological inflation as a solution. According to many cosmologists, inflation makes uniformity and flatness generic outcomes of the inflationary epoch and is itself a generic mechanism found in a wide variety of spacetimes. Thus, following the formula of GHS, cosmologists claim to have exhibited a wide class of models in which flatness and uniformity are generic, thereby explaining the flatness and uniformity of our universe.

One might dismiss these arguments as spurious, but there is perhaps at least some reason to think that they might be trustworthy. Subsequent to the widespread adoption of the inflationary solution, it was realized that inflationary theory could be used to make empirical predictions of anisotropies in the cosmic microwave background. It is generally thought that these predictions have now been observationally confirmed. Unless inflationary theory was just an extraordinarily lucky guess, it might seem that the arguments that led to it exhibited good reasoning (Cf. (McCoy, 2017a)).

Although these arguments may be mistaken in various respects, my interest in them here concerns their apparent reliance on likelihoods. Certainly physicists can be easily read as adopting this probabilistic characterization of the fine-tuning problems and their resolution as likelihood arguments; some philosophers have discussed them along these lines as well (Earman, 1995; Earman and Mosterín, 1999; Smeenk, 2013; McCoy, 2015). The philosophical analyses made so far have not focused, however, on assessing the details of the likelihoods involved, i.e. on assessing the implementation, interpretation, and justification thereof. Thus the example described in this section is an important application of this study of likelihood reasoning.

Note that there are three steps in the fine-tuning arguments for inflation that depend on likelihoods (Ellis, 1988; Coule, 1995). First, it must be demonstrated (rather than merely supposed on the basis of intuition) that the uniform and flat spacetimes underlying the HBB model are unlikely. If they are unlikely, then they require some special explanation (as GHS say). This is why they should be considered fine-tuning problems. If inflationary theory is to solve these problems, then it must be shown that inflating cosmologies generically lead to spatial uniformity and flatness. This is the second place where likelihoods must be invoked. It must also be shown, however, that inflating cosmologies themselves are not unlikely, or that they are at least more likely than the special HBB spacetimes. This is the third place where likelihoods must be invoked.

Cosmologists have often relied on intuitive judgments of likelihood in making the arguments mentioned above, but the soundness of the arguments plausibly depends on there being an objective way of assessing the likelihoods of cosmological models (Gibbons et al., 1987; Hawking and Page, 1988) and some justified way of interpreting these likelihoods. Let us first look, then, at what problems stand in the way of justifying and interpreting likelihoods in cosmology.

3 General Conceptual Problems

In this section I discuss the significant conceptual issues that stand in the way of establishing a likelihood measure on the space of possible cosmologies. These general issues concern the choice of an appropriate reference class of cosmologies to serve as the sample space, and the justification and interpretation of a specific measure associated with the sample space.

The basic issue which makes applying probability theory to cosmology difficult has already been mentioned: there is, so far as we know, only one universe. The uniqueness of the universe has long been recognized as a problem for cosmology as a science, however its significance has often been overstated.⁵

⁵There are several papers discussing whether cosmology is even a science written in the middle of the 20th century: (Dingle, 1955; Munitz, 1962; Harré, 1962; Davidson, 1962). Many of the views expressed, however, have been justly criticized more recently

Thus it is necessary to draw the argument out in some detail in order to avoid an overly hasty conclusion. In the end, though, I do conclude that the point is decisive. There is simply no physical content to be found in the addition of cosmological probabilities to classical, single-universe cosmology.⁶

3.1 The Reference Class Problem

The first issue to face in defining a probability space in cosmology is deciding on the appropriate reference class, i.e. the appropriate sample space. Again, a probability space is standardly specified by a set X of possible outcomes, a σ -algebra \mathcal{F} of these possible outcomes, and a probability measure P that assigns probabilities to elements of \mathcal{F} . The problem of deciding the appropriate reference class is the problem of determining precisely what X should include.⁷

I begin with the reference class problem because likelihood reasoning depends very much on what X is. If the set of possibilities is in fact much, much larger than one assumes, an outcome that is likely according to one's assumptions may actually be unlikely. In contrast, the reference problem is less of an issue when modeling specific physical systems. So long as the target system can be modeled, one usually does not care so much which other systems are physically possible.

As said, in cosmology the appropriate reference class X will be the set of (physically) possible cosmologies. A cosmology is standardly taken to be a relativistic spacetime in contemporary cosmology, i.e. a model of the general theory of relativity (GTR).⁸ This is essentially because at large scales gravity appears to be the most important physical force in the universe, and general relativity is the best, most highly-confirmed theory of gravity that we have. Assuming that every element of X is a relativistic spacetime does not, however, obviously answer the question of what the possible cosmologies are. Which set of models of GTR are possible models of the universe?

Trust in our theories is usually thought to underwrite the belief that the models of that theory are physically possible. On this point of view our justified belief in the laws or modal structure of GTR would therefore determine X as the complete collection of models of GTR. This collection, the nomologically possible models, is the result of “the most straightforward reading of physical possibility” (Earman, 1995, 163)⁹.

The practice of cosmologists (and relativists) does not necessarily accord with this line of thinking however. By any measure general relativity is a permissive theory; any number of undesirable or pathological spacetimes is possible according to it.¹⁰ Many authors are for this reason inclined to exclude certain models from physical consideration, such as models with closed timelike curves (CTCs).¹¹ Should

for being overly skeptical towards addressing the scientific problems arising from the uniqueness of the universe (Kanitscheider, 1990; Ellis, 2007; Smeenk, 2008, 2013).

⁶Quantum cosmology and multiverse cosmology could make for a different conclusion. Although I focus on classical, single-universe cosmology here, I do believe further investigation into these larger contexts is warranted. Although some of my conclusions would carry over, there are some novelties which may make the case for cosmological probabilities better there. The interested reader should refer to the critiques in (Smeenk, 2014) and (Ellis et al., 2004) as a starting point.

⁷To some extent it does not matter too much precisely what X is so long as it is large enough, since one can always use the probability measure to assign zero probability to any subset of X , in effect counting them as impossible. Indeed, it may be mathematically convenient to include some “extra” objects in X for mathematical convenience, simplicity, etc. Nevertheless, the reference problem will remain, whether in the guise of choosing X or choosing elements of X to which probability zero is assigned.

⁸In full detail it should also include a physical model of relevant cosmological phenomena in that spacetime (Ellis and van Elst, 1999; Cotsakis and Leach, 2002; Ellis et al., 2012), but for my purposes it is only necessary to consider the spacetime component of a cosmology, setting aside the specific physics of the spacetime’s material contents.

⁹This straightforward reading of physical possibility is actually spelled out in two ways by Earman and his collaborators (Earman et al., 2009, 95), but here I will simply take this reading as the identification of nomological and physical possibility.

¹⁰Among the more exotic models are the “causally bizarre” Gödel spacetime and Taub-NUT spacetime which have CTCs. It must be acknowledged, however, that even the most familiar examples of spacetimes permitted by GTR have fairly unintuitive features: expanding space (Friedman-Robertson-Walker, de Sitter), spacetime singularities (Schwarzschild, Friedman-Robertson-Walker), etc. Therefore some distinction between the “undesirable” and the merely “unintuitive” need be made.

¹¹The inclination to disbar spacetimes with CTCs is sometimes characterized (grandiosely) as the “cosmic censorship conjecture.” Wald (1984, 304) states it simply (albeit imprecisely) as “all physically reasonable spacetimes are globally hyperbolic.” Since

one exclude pathological examples from the set of possible cosmologies because they do not strike one as “physically possible?”

I do not mean to take a stand on the question here. The point I rather wish to make is that in practice cosmologists do exclude certain relativistic spacetimes from consideration, thereby presuming some alternative physical modality to the “straightforward” nomological one given by the general theory of relativity. For the most part models are excluded merely because they are thought to be physically impossible or physically unreasonable, although (as a referee suggests) exclusions may be made on the basis of additional laws added to GTR as well. Nevertheless, the available justifications for these exclusions tend to be rather dubious (if given at all), as they do not rely on well-motivated physical principles or observational grounds (as has been noted by some commentators (Earman, 1987; Manchak, 2011)).

Some weight of consideration should be accorded to practice however, so the possibility of justifying the exclusion of pathological spacetimes should not be quickly dismissed merely because adequate justifications have not been so far given. If so, then the reference class problem cannot be regarded as solved simply because one can identify nomological and physical possibility by fiat (or by philosophical artifice).

In any case, even permitting the kinds of assumptions that exclude pathological spacetimes (such as global hyperbolicity which rules out CTCs) or mathematically inconvenient spacetimes (such as those that lack compact spatial sections), one is still left with a vast collection of cosmological models which will then be considered physically possible. If one furthermore arbitrarily restricts attention to spacetimes with some specific manifold M , as is common in the cosmology and relativity literature, i.e. to the subcollection of physically possible cosmologies with underlying manifold M and a physically possible metric g on this manifold (Lerner, 1973), one generically has an infinite-dimensional space (GTR is a field theory, after all). This leads to a second difficulty related to the nature of the reference class: although this space of models will possess some mathematical structure, there are difficulties with defining a probability measure (in particular) on such a space (see §4 below).

Whether because of these difficulties or in ignorance of them, physicists’ attention has so far been mostly directed at simple sets of cosmologies which can be presented as finite-dimensional state spaces, e.g. the state spaces on which one can define the GHS measure (the main topic of §5). While this brings the cosmological framework closer to the statistical mechanical one, where state spaces are generally taken to be finite-dimensional, this maneuver raises a (third) problem, which is not found in the statistical mechanical context. How can a measure contrived on a special (sub)set of physically possible spacetimes represent cosmological probabilities correctly?

On the one hand, if the collection S of simple models, e.g. spatially homogeneous and isotropic spacetimes, is taken to be the full set of physically possible models, then it is difficult to see how this can be justified on any well-motivated physical principle or on observational grounds. Surely, that is, GTR (or even pure physical intuition) suggests that there are spacetimes which may be physically possible cosmologies besides any particular simple collection of spacetimes S .

If, on the other hand, the collection S (equipped, say, with probability measure μ_S) is a subset of a larger possibility space X (equipped with probability measure μ_X), then the likelihood of a set of models $Z \subset S$ in the subspace must be a conditional likelihood $\mu_S(Z) = \mu_X(Z|S)$. In other words, the probability space S must be a conditional probability space of X .¹² Formally, the probability assigned to a collection

globally hyperbolic spacetimes do not have CTCs, it follows that all physically reasonable spacetimes do not have them either, at least if this version of the cosmic censorship conjecture is true. But then one wonders what it takes to be a physically reasonable spacetime. Whereas some find CTCs objectionable on philosophical grounds—for example, “those who think that time essentially involves an asymmetric ordering of events...are free to reject the physical possibility of a spacetime with CTCs” (Maudlin, 2012, 161)—others encourage a certain degree of epistemic modesty with respect to physical reasonableness. Manchak (2011), for example, demonstrates that given any physically reasonable spacetime there exists observationally indistinguishable spacetimes which exhibit undesirable or pathological features.

¹²Let X be the larger sample space with σ -algebra \mathcal{F} and probability measure μ_X . Suppose S is a measurable subset of X with non-zero measure according to μ_X . Then the conditional probability space S has σ -algebra $\{S \cap Z | Z \in \mathcal{F}\}$ and probability measure

of models $Z \subset S$ must be

$$\mu_S(Z) = \mu_X(Z|S) = \frac{\mu_X(Z \cap S)}{\mu_X(S)} = \frac{\mu_X(Z)}{\mu_X(S)}. \quad (1)$$

The probabilities of the large space X put a constraint on the probabilities of the smaller space S , a constraint that the probability measures which can be associated with the smaller space are not guaranteed to meet.

To illustrate these considerations in the most important case in cosmology, consider the subset of models of GTR that satisfy the cosmological principle (CP). The CP constrains the set of models from general relativity to those that are spatially homogeneous and isotropic. These models are known as the Friedman-Robertson-Walker (FRW) spacetimes. First, we may ask, is this assumption admissible as a way of specifying X ? Although the FRW models have been observationally successful, there is certainly no good argument that justifies the CP as definitively delimiting the space of physically possible spacetimes (Beisbart and Jung, 2006; Beisbart, 2009)—especially since the universe is not strictly speaking homogeneous and isotropic. More plausibly, then, the probabilities that we would obtain by making this assumption and building a probability measure on the set of FRW spacetimes are not unconditional probabilities: the space of physically possible cosmologies is surely larger. How large? If the space X of physically possible cosmologies is the space of nomically possible spacetimes (according to GTR), then the set S of FRW models is almost certainly negligible, a problematic result given the uses to which cosmologists want to put likelihood measures in cosmology. Even if S were not negligible, it is hard to see what the point of constructing measures associated with S , the set of FRW spacetimes, is without knowing what the measure is associated with the full possibility space X (at least if the motivation is to derive an objective probability measure). One simply needs to know the full measure (well enough, anyway) in order to know the correct conditional measure.

Summing up, three issues were raised in this section. I first drew attention to the justification of a particular set of possible spacetimes as the physically possible ones. In most applications of physical theory this issue is perhaps not pressing; for likelihood arguments to be successful in cosmology, however, it is crucial to choose the right X , since whether a spacetime or set of spacetimes is likely depends on precisely what the reference class is. Second I noted that the set of physically possible spacetimes is almost certainly infinite dimensional, unlike the possibility spaces used in statistical mechanics. This is a significant disanalogy and, moreover, creates serious technical difficulties in defining the possibility space as a probability space (as will be discussed below in §4). Finally I pointed out that restricting attention to a subset of physically possible spacetimes and assigning probabilities to the elements of this subset is a dubious strategy, since this subset must form a conditional probability space of the full probability space associated with the set of physically possible spacetimes.

3.2 Interpretation of Cosmological Likelihoods

The second issue to address is how to interpret cosmological likelihoods. I will say something in a moment on what I mean by interpretation, but I will begin with a preliminary issue: whether the likelihoods invoked in cosmology should be understood as epistemic or ontic. By ontic probabilities I mean physical probabilities; they describe chanciness (in one way or another) inherent in the physical system. Epistemic probabilities are probabilities which are attributed to agents; they are justifiable degrees of belief. If ontic probabilities exist and are known, then a plausible rule of rationality holds that epistemic probabilities should be set to the ontic probabilities. Nevertheless epistemic probabilities can be applicable to situations that do not involve ontic probabilities, e.g. when in situations of uncertainty.

On the one hand, one might expect that cosmological likelihoods should be ontic if they are to have physical significance and play a role in fine-tuning arguments. Perhaps this is why this approach is favored by most cosmologists who have written on the topic. The most well-known proposal in this vein

$\mu_S(S \cap Z) = \mu_X(Z|S)$.

is the already-mentioned canonical measure of (Gibbons et al., 1987), the GHS measure.¹³ Topological methods may be used to give an objective measure of likelihood as well.¹⁴ The basic strategy of these approaches is to begin with some physically motivated attribution of likelihoods to sets of cosmologies in some relevant space of possible cosmologies. The motivations may come, for example, from the structure of the space of models of GTR or from intuitions on how models in such spaces are physically related. With the likelihoods in hand, if one finds (for example) that spatially flat FRW spacetimes represent a negligible set of cosmologies and inflating FRW spacetimes are generic cosmologies, arguably one has a warranted basis for making an argument in favor of inflation.

On the other hand, purely epistemic notions of likelihood appear to be behind many cosmologists' intuitions about fine-tuning cases. There are relatively few places in the literature where more precise formal methods are used to substantiate these intuitions. Accordingly, it is difficult to analyze and assess the merits of purely epistemic measures of likelihood in cosmology in general. Examples do however exist, such as (Evrard and Coles, 1995) and (Kirchner and Ellis, 2003).¹⁵

A more thorough review of approaches to defining cosmological likelihoods would engage with the epistemic approaches, however I will restrict myself to addressing the more prominent physical approaches (apart from some comments on the principle of indifference below). This is due to the greater importance of the latter approaches in the physics literature and to maintain a reasonable scope in this paper.

I will also be more restrictive than is usual in the philosophical literature in how I employ the term "interpretation" in what follows. In philosophy an "interpretation of probability" is usually understood to refer to an account of how the concept of probability should be analyzed (Hájek, 2012). For the purposes of my argument it is not necessary to make use of the standard accounts, e.g. the logical interpretation, the frequentist, the propensity, etc. By "how probability is interpreted" I will mean "how randomness is understood". It is important to recognize some source of randomness in an application of probability theory in order for that application to be justified. As Hollands and Wald say in their discussion of applications of probability in cosmology, for example,

probabilistic arguments can be used reliably when one completely understands both the nature of the underlying dynamics of the system and the source of its 'randomness'. Thus, for example, probabilistic arguments are very successful in predicting the (likely) outcomes of a series of coin tosses. Conversely, probabilistic arguments are notoriously unreliable when one does not understand the underlying nature of the system and/or the source of its randomness. (Hollands and Wald, 2002b, 5)

Identifying possible sources of randomness is generally overlooked as a way of distinguishing accounts of probability. For purely epistemic probabilities this randomness is introduced by the agent, whether in terms of her independent choice, a standard of indifference, etc. For genuine chances this randomness comes from the physical situation (in some respect or another). This randomness need not be taken as a full-fledged feature of nature however. In Humean accounts of chance, for example, all that is understood to exist is a so-called "Humean mosaic" of events; laws and probability are understood as objective systematizations of this mosaic (Loewer, 2001, 2004).

In physics the structure of theories can helpfully suggest where objective randomness may be "realized". Conventional physical theories are described in terms of a set of physically possible states, a dynamics that determines the evolution of a system from physical state to physical state, and a set of

¹³The notable papers discussing their approach include (Henneaux, 1983; Gibbons et al., 1987; Hawking and Page, 1988; Coule, 1995; Gibbons and Turok, 2008; Carroll and Tam, 2010; Schiffrin and Wald, 2012).

¹⁴Hawking (1971), for example, proposes the application of such methods in cosmology. (Isenberg and Marsden, 1982) is another well-known example.

¹⁵Evrard (1996) and Evrard and Coles (1995) argue for a dissolution of the flatness problem using an epistemic approach to cosmological parameters. Their approach is criticized by Coule (1996), who favors the canonical measure.

functions that determine observable quantities defined on the set of states. Therefore there are naturally three ways randomness can enter into a physical description: the initial state of the system, the particular dynamical evolution of the system, and the realized observable properties of the system. From a metaphysical point of view one might question whether the association of randomness with these formal, descriptive features of theories is ontologically significant. Nevertheless, for present purposes it is sufficient to address the interpretation of probability at this theoretical level of description.

With this bit of terminology fixed, we can now ask how to interpret probability in cosmology, i.e. where to attribute randomness in the universe. Insofar as one takes the standard approach, according to which cosmological models are relativistic spacetimes, the space of states will be the set of possible spacetimes (or perhaps initial data on a spacelike hypersurface) permitted by GTR (or some subset thereof, as discussed in the previous section, depending on how one solves the reference class problem). The dynamics is given by the Einstein equation and the observables are going to be certain geometric properties of spacetime (which must in practice of course be supplemented by other physical models to derive proper observables like galaxy counts, galactic redshifts, light element abundances, etc.). Since general relativistic dynamics is essentially deterministic (setting to the side the issue of gauge and pathological spacetimes such as those with closed time-like curves, as these do not introduce randomness into the theory), one cannot locate the randomness there except by making the randomness so insignificant as to give rise to an essentially deterministic dynamics. Empirical considerations strongly militate against the idea that cosmological observables are substantially stochastic as well. In short, there is very little reason to think that the universe is “fluctuating” around the space of possible cosmologies dynamically and very little reason to think that its observable properties are either (insofar as one can even distinguish these). That leaves the initial state, the initial “choice” of spacetime, as the only way physical randomness can enter into cosmology.¹⁶

On this interpretation the (initial condition of the) universe is to be understood as the outcome of a random trial, whether literally or merely characterized as such. A cosmological probability measure, in other words, can only represent the objective chance of our universe being in a particular state (initially) or of a possible universe being realized. Naturally, the possibility of this viewpoint has suggested itself to some cosmologists, who compare the situation (usually pejoratively) to a blind-folded creator selecting a universe by throwing a dart at the dartboard of possible universes.¹⁷

Should one adopt such a point of view in cosmology? It is a coherent possibility at least. It is arguably tenable in statistical mechanics (where it is (tacitly) employed in typical Boltzmannian approaches) as one can at least verify the consistency of frequencies of initial microstates with empirical frequencies of observables (Hemmo and Shenker, 2012). In the absence of an analogous micro-theory, however, it is unclear why one would want to accept this interpretation in cosmology. As Loewer flatly observes, “one problem is that it does not make sense to talk of the actual frequency with which various initial conditions of the universe are realised” (Loewer, 2001, 615). A single-sample probabilistic scenario in cosmology is obviously observationally indistinguishable from a deterministic scenario that involves no probability at all, only an initial state. Moreover, there is relatively little theoretical reason to suppose that there was a random trial selecting among the space of relativistic spacetimes. Without any input from physics about the source and nature of this randomness of initial conditions (recalling the Hollands and Wald quotation above) and no way to verify it empirically, we should find the “dart throwing” interpretation highly unsatisfying as an explication of cosmological probabilities. If we were, however, to possess a trustworthy theory that did suggest such a random start to the universe (a multiverse theory or a theory of quantum gravity could do so, if sufficiently warranted), then we might have sufficient reason to introduce a probability measure and interpret it in this way. It would likely not be, however, associated with the full

¹⁶At least this is so at this level of description. The context of quantum cosmology would open up alternative possibilities. However the present discussion is, again, focused only on classical cosmology, in keeping with literature discussed.

¹⁷Cf. (Penrose, 1989b, 444) and (Hollands and Wald, 2002a, 2044). This view is, however, defended philosophically in (Demarest, 2016).

space of classical relativistic spacetimes.

I think it worth noting in passing that this issue of interpretation also infects foundational discussions of statistical mechanics when they move in response to the pressure to “globalize” the theory (Callender, 2011b), i.e. to treat the universe as a whole as a statistical mechanical system. As said, in the Boltzmannian approach the only possibility for interpreting statistical mechanical randomness is in understanding the initial conditions of the system as random (either in actuality or as the best systematization of the “Humean mosaic”). As before, we have relatively little reason to believe that the universe itself began as the outcome of a random trial in this scenario. It is only by generalizing from familiar statistical mechanical systems, i.e. subsystems of the universe, to the universe as whole (supposing that it is a statistical mechanical system too) that the idea has any degree of plausibility. Nevertheless this inference is disturbingly close to a composition fallacy.¹⁸

Therefore, in both theoretical contexts mentioned, general relativity and statistical mechanics, the only admissible interpretation of cosmological probabilities locates the associated randomness with initial conditions. Although the interpretation is coherent, I have suggested already that in both cases this interpretation has very little to recommend it. Whether one should adopt it, however, is really a matter of justification, so to this topic I turn.

3.3 Justification of Cosmological Likelihoods

The final issue to address in this section is the justification of cosmological likelihoods. The particular case on which I have been concentrating is that of probability distributions on some given sample space of cosmologies, assuming that initial conditions are subject to (real or imagined) randomness (since this is the only available ontic interpretation). To be sure there are significant technical problems with supplying such a probability space structure to these possible cosmologies, as will be discussed in §4. Even if these technical problems could be overcome, however, a more crucial issue is whether it is possible to adequately justify any particular cosmological probability measure. I will argue that it is not possible to do so.¹⁹

The first (admittedly obvious) point to make is that cosmological likelihoods cannot be empirically justified, at least insofar as cosmology concerns a single universe. If we suppose that probability theory applies to cosmology, then it must be the case that our universe is the outcome of a single random trial over possible initial conditions, as argued above. Cosmological probability measures are therefore vastly underdetermined. If one tries to make GTR into a probabilistic theory by defining a probability distribution over possible cosmologies, it is clearly the case that *any* choice of probability measure that assigns some probability to the cosmological model best representing our universe is empirically adequate. Note that the adequate probability measures include the probability distribution that makes our universe “quasi-determined”—assigns the cosmology representing our universe probability one.

The uniqueness of the universe therefore forces consideration of a non-empirical justification for cosmological likelihood measures. Several prominent cosmologists have accordingly relied on a priori principles, like the principle of indifference (PI) or some kind of objective “naturalness”, to justify uniform probability distributions (Kofman et al., 2002; Linde, 2007). The PI holds that if there is no salient reason to prefer any other probability distribution, given some sample space, one should assign a uniform probability density to that space (hence the probability measure is purely epistemic). A similar principle

¹⁸That is, an unsupported inference that a property of the parts is a property of the whole.

¹⁹This seems to be one of the main conclusions of (Schiffrin and Wald, 2012, 9). The authors claim that “the only way to justify the use of the Liouville measure in cosmology would be to postulate that the initial conditions of the Universe were chosen at random from a probability distribution given by the Liouville measure.” What they seem to mean is that the only possible *interpretation* of the Liouville measure (when it is a probability measure and not just a measure) is that it specifies the probabilities of specific initial conditions of the universe obtaining, and that the only “justification” of understanding it as such is as a theoretical posit. A posit, of course, is hardly a justification—they describe it (charitably) as an “unsupported hypothesis”—so I take it that they essentially would conclude that cosmological probabilities are unjustified.

is invoked in assigning a uniform probability distribution with respect to the natural Liouville measure associated with statistical mechanical phase spaces. Gibbons, Hawking, and Stewart’s approach follows the statistical mechanical strategy. Although they note the problematic nature of the PI—“indeed...it is not at all clear that every model should be given equal weight if one wishes the measure to provide an inductive probability” (Gibbons et al., 1987)—they assume that the probabilities should be uniform with respect to the Liouville measure. Carroll and Tam (2010) recognize that the Liouville measure does not in general entail any particular probability measure. As they say, however, “since the Liouville measure is the only naturally-defined measure on phase space, we often assume that it is proportional to the probability in the absence of further information; this is essentially Laplace’s ‘Principle of Indifference’.” They too go for the uniform mapping, however, and provide as precedent the practice of assuming a uniform probability distribution on the Liouville measure of phase space from statistical mechanics.²⁰

Unfortunately for one who wishes to apply such principles, in a reference class composed of an infinity of cosmologies there is no mathematically natural choice of probability measure and no probability distribution uniform with respect to the Liouville measure. In special cases (for example if the Liouville measure of a space equipped with it is finite) there may be a canonical choice of probability measure that is uniform with respect to the possibilities, but one then faces a dilemma raised earlier: either this space delimits the full space of possible cosmologies (which is highly implausible, if not clearly false) or its probabilities must be conditional probabilities in a larger space of possible cosmologies (which, insofar as this larger space has infinite total measure and therefore no uniform probability measure, cannot then be justified by the PI, naturalness, etc.). Therefore even if a justification of uniformity, by way of mathematical naturalness, the PI, etc., were possible in statistical mechanics, it would not easily carry over to the case of cosmology.

However, it has been made abundantly clear in the philosophical literature that a prioristic principles like the PI are not generally justifiable in statistical mechanics, mainly because empirical frequencies depend importantly on the nature of a physical system’s randomness and there is no reason to expect that the source of randomness acts uniformly on some space of possibilities (Shackel, 2007; Norton, 2008; North, 2010). If this is correct, then the simplest way of justifying the inference from statistical mechanics to cosmology fails, namely that the same principle may be used in both contexts. There is also no independent, compelling support for the PI or its cousins in cosmology (McMullin, 1993; Ellis, 1999; Earman, 2006; Norton, 2010; Callender, 2010). In cosmology very little at all is known about the mechanism that brings about the initial conditions of our models of the universe, and so assigning equal weights to distinct cosmological possibilities (especially if based merely on a lack of knowledge) is highly dubious, since it may well be the case that certain initial conditions are in fact more likely according to the true (presumably quantum) mechanism responsible for them.²¹

I have so far claimed that there is no direct empirical or a priori justification of cosmological probabilities, but there are indirect ways through which one might try to justify them. One prominent approach is to argue that the empirical justification of a uniform probability distribution in statistical mechanics indirectly justifies a uniform probability distribution in cosmology (for the moment setting aside the usual assumption that a cosmological model is a relativistic spacetime). In foundational discussions of statistical mechanics (especially in the context of the “past hypothesis” scenario described in (Albert, 2000)) philosophers often suppose that the justification of a uniform probability distribution for usual statistical

²⁰One sometimes sees the Liouville measure associated with a mechanical phase space called the “Lebesgue measure”. The Lebesgue measure standardly refers to the natural measure associated with \mathbf{R}^n . The measurable subsets U of phase space can be called “Lebesgue measurable sets” in the following sense: for all charts $\varphi : O \rightarrow \mathbf{R}^n$ on n -dimensional phase space Γ , $\varphi[U \cap O]$ is a Lebesgue measurable set in the usual sense. Since in general there is no canonical pull-back of the Lebesgue measure to Γ , however, it is somewhat misleading to call the Liouville measure the Lebesgue measure. Of course when the phase space is \mathbf{R}^n , the Liouville measure just is the usual Lebesgue measure and the terminology is justifiable, although consistency should incline one towards the former term.

²¹The only viable response I can see to these points is to modify the space of possibilities to make the PI hold, but then it is clear that the correct space of possibilities is not in fact justifiable a priori.

mechanical systems justifies a uniform probability distribution for the universe when it is modeled as a statistical mechanical system. I suggested at the end of the previous subsection that in the absence of an argument this inference commits the fallacy of composition. Let us see now whether there is any argument which could support the validity of the inference.

The most plausible way would be to argue that the universe is a statistical mechanical system because it is sufficiently similar to usual statistical mechanical systems. Our best understanding of the universe, however, suggests that it is not. Perhaps most importantly, gravitation plays a central role in cosmology unlike in conventional statistical mechanical systems, as has been pointed out and discussed in (Earman, 2006; Wallace, 2010; Callender, 2010, 2011a). This is, after all, why cosmological models are modeled using theories of gravitation.

Although this issue by itself seriously threatens the inference, there is a more serious problem. It is crucial to recognize that the cosmological probabilities discussed in this paper are (in statistical mechanical jargon) macroprobabilities, not microprobabilities. Cosmological probabilities are attributed to entire macroscopic histories (or initial states) of the universe. Statistical mechanical probabilities, by contrast, are microprobabilities; they are attributed to entire microscopic histories (or initial states) of statistical mechanical systems. There are, however, no unconditional non-trivial macroscopic probabilities in statistical mechanics. This is because the empirical content of statistical mechanics is related to microscopic frequencies. Macroscopic frequencies of course can be determined from these, and *conditional* macroscopic probabilities can certainly be defined as transition probabilities. But it makes no sense in statistical mechanics to ask what the probability is of a system beginning in a particular macrostate; the initial macrostate is taken as given in statistical mechanics. Thus one cannot hope to justify cosmological probabilities by way of the justification of statistical mechanical probabilities, since they are different kinds of probabilities (macro vs. micro).²² It would be a category mistake.

It is worth emphasizing the importance of this point, since failing to locate the empirical content of statistical mechanics in microprobabilities and to recognize the lack of an a prioristic justification of statistical mechanical probabilities has led to some popular, specious arguments. Particularly related to the present discussion is the claim that the “low entropy” state of the universe is improbable. The argument assumes that statistical mechanical probability assignments are determined by the so-called proportionality postulate, according to which a uniform probability distribution is applied to the entire phase space of the system (rather than to just the initial macrostate). Hence non-trivial unconditional macroprobabilities are assigned to macrostates. Macrostates with a small phase space volume will of course be assigned a small probability according to this principle. Since low entropy states are assumed to have a small phase space volume, the past “low entropy” state of the universe is accordingly improbable. Demands to explain this improbability of the initial macrostate (in accord with the GHS formula from the introduction), as in (Penrose, 1989a; Price, 2002, 2004), are therefore based on an ill-motivated application and interpretation of probability to cosmology (and statistical mechanics).²³

Cosmological probabilities therefore fail to be justifiable on all the plausible strategies obviously available. Perhaps, though, probability theory is simply the wrong way to formulate likelihoods in cosmology. As said, I cannot treat all possible formulations here, but there is one further case that is important to mention. Even if no unique probability distribution can be justified, it may be the case that there is a natural non-probabilistic measure on the space of possible cosmologies. This is indeed the case with the GHS measure (§5). While I grant the mathematical naturalness of such measures, I stress that mathematical naturalness is no guarantee of physical significance. It is a distinct step which requires its own justification, interpreting a mathematically natural measure as a physical likelihood measure. After all,

²²Of course one might suppose that the universe has a microstate, in analogy with statistical mechanics, but that is irrelevant to the argument here. The point is that cosmological probabilities are unconditional macroprobabilities, and such probabilities do not exist in statistical mechanics. Hence statistical mechanics cannot be used to justify them in cosmology.

²³Responses to these demands have accepted their presupposition—that the initial macrostate of the universe is improbable—and weighed whether and how some explanation could be provided (Callender, 2004). One ought to simply reject the presupposition as ill-motivated and inadequately justified.

measures can play a variety of roles in a physical theory (for example as a standard for integration along trajectories). There is simply no reason to assume that a mathematical measure must play the role of a likelihood measure in any theory that comes equipped with one.

Nevertheless, in statistical mechanics it has been argued that the Liouville measure can be used in such a way, i.e. as a typicality measure (Lebowitz, 1993a,b; Dürr, 2001; Goldstein, 2001, 2012). In this approach to interpreting statistical mechanics one makes do with the Liouville measure alone, and uses it as a standard of typical and atypical behavior. The basic schema of the typicality arguments used in this context is to show that some behavior or property is highly likely and its contrary is highly unlikely, in which case one can infer that that behavior or property holds; “In other words, typical phase space points yield the behavior that it was our (or Boltzmann’s) purpose to explain. Thus we should expect such behavior to be prevail in our universe” (Goldstein, 2001, 58).²⁴

One of course may try to infer from the putative success of typicality arguments in statistical mechanics to their applicability in cosmology. Yet, if typicality arguments are indeed successful in statistical mechanics, then they are because they depend importantly on the full complement of structures in statistical mechanics, e.g. the correct space of possibilities, the collection of macro-states, etc. One generally has empirical evidence that suggests the right structures for a system in statistical mechanics. One does not have this in the case of cosmology, in particular because of the issues mentioned above in connection to the reference class problem. Echoing the first point there, what is typical in cosmology depends very much on what set of cosmologies one is considering—and there is no guarantee that what is typical in one context is typical in another. Thus the transfer of justification from statistical mechanics to cosmology is again blocked.

Although the reference class problems and the issue of interpretation are important and significant, the most decisive issue is therefore with justification. I have argued that the justification of measures in statistical mechanics does not carry over straightforwardly to cosmology, so the most appealing indirect justification fails. I also argued that there is no independent justification for cosmological likelihoods. A priori justifications fail, just as they do in statistical mechanics. Empirical justification, the only sensible justification of likelihoods in statistical mechanics, is not possible in cosmology, due to the uniqueness of the universe. Likelihood reasoning, in short, should be considered inapplicable to cosmology because it cannot be adequately justified.

4 Likelihood in the Solution Space of General Relativity

In the remaining two sections of this paper I mostly set aside the conceptual problems which I have raised in the previous section and consider the prospects of rigorously defining some notion of likelihood on the space of cosmologies, i.e. without worrying too much about whether it makes much sense to do so. In this section I deal with the case where the space of possible universes is taken to be the solution space of GTR or some subspace thereof; in §5 I deal with the case where the solution space is restricted to be finite dimensional by specific modeling assumptions, focusing in particular on the GHS measure on minisuperspace.

I will follow the lead of GHS, etc. in looking for mathematically natural, objective likelihood measures in the structure of the solution space of GTR or subspaces thereof. Simply defining a likelihood measure is trivial. Finding one that is “picked out” by the mathematical structure of a theory, however,

²⁴Although some enthusiastic disciples of Boltzmann claim that typicality is the heart of all foundational matters in statistical mechanics—as Dürr (2001, 122) remarks, “we have the impression that we could get rid of randomness altogether if we wished to do so”—full reliance on typicality arguments clearly represents a significant retreat from the quantitative successes of statistical mechanics (Wallace, 2015), which depend on probability distributions to derive the empirical content of the theory (for example to predict fluctuation phenomena). As Pitowsky further notes, “the explanation...is a weak one, and in itself allows for no specific predictions about the behavior of a system within a reasonably bounded time interval” (Pitowsky, 2012, 41). Additional criticisms of the typicality account in statistical mechanics can be found in (Frigg, 2009, 2011; Frigg and Werndl, 2012).

suggests (to many) that it has special physical significance and is justified. If one has to make a special choice, then a special justification must be given. I will follow this line of thinking hence, despite the reservations I raised previously.

The full solution space of GTR is the space of relativistic spacetimes that satisfy the Einstein equation. The principal questions before us are, “what structure does this space have and can that structure be used to define a privileged notion of likelihood?” A relativistic spacetime \mathcal{M} is standardly defined as a differentiable four-dimensional manifold M that is Hausdorff, connected, and paracompact equipped with a Lorentzian metric g , a rank two covariant metric tensor field associated with M which has Lorentz signature. Given this definition of a relativistic spacetime, the space of possible cosmologies obviously will range over the set of four-dimensional topological manifolds; moreover, for each four-dimensional topological manifold, there is also a range of smoothness structures on these manifolds that make them into differentiable manifolds; finally, for each smooth, four-dimensional differentiable manifold, there is a (“kinematic”) range of Lorentzian metrics, which range is restricted by the Einstein equation to yield the possible (“dynamical”) set of cosmologies. I consider first the range of possibilities permitted by the manifold structure of a relativistic spacetime, then the range of possibilities permitted by the metric structure.

4.1 Manifold Possibility

There is presently relatively little that we can say about the structure of this large and complicated set of solutions in all its fullness. Since the applications of GTR that are most of interest to physicists concern particular spacetimes and perturbations thereof, far less attention has been paid to the set of solutions as a whole. Yet, as mentioned in the previous section, likelihood arguments depend on particular structural features of this space, so there is no getting around the need to understand it.

Of course, with specific modeling assumptions the set of relevant solutions can be reduced to something much more tractable. In the case of FRW spacetimes, for example, one restricts attention to manifolds that can be expressed as twisted products $I \times_a \Sigma$ (a is the scale factor), where I is an open timelike interval in the Lorentzian manifold $\mathbf{R}^{1,1}$ and Σ is a homogeneous and isotropic three-dimensional Riemannian manifold (McCabe, 2004, 530). Since one can fully classify these 3-manifolds (Wolf, 2010), one can enumerate the different possible FRW spacetime manifolds (McCabe, 2004, 561). With such an enumeration one could (at least conceivably) specify the likelihoods of each kind of product of 3-manifolds and 1-manifolds.

However, if one were to want a likelihood measure on the full space of possible cosmologies (according to arguments in §3 one certainly should), in particular one which is naturally motivated by the mathematical structure of that space, then it would require a means of classifying all four-dimensional manifolds.²⁵ This is not yet possible. Indeed, the classification of such manifolds is a notoriously difficult mathematical problem (and distinctly so in comparison to other dimensions, where classification has been established by geometrization or surgery techniques) (Freedman and Quinn, 1990; Donaldson and Kronheimer, 1997). Of course, one might try to avoid the mathematical difficulties by assuming that the relevant manifolds are of a particular simple kind or that many manifolds are unphysical. This opportunistic move is not so obviously well-motivated however.

Consider, for example, the discovery of so-called exotic smoothness structures on the topological manifold \mathbf{R}^4 , i.e. smoothness structures that are homeomorphic to \mathbf{R}^4 but not diffeomorphic to the standard Euclidean smoothness structure on \mathbf{R}^4 . This discovery reveals a large class of possible spacetime models, one that is almost entirely overlooked in cosmological work, where the Euclidean smoothness

²⁵A classification would not solve the justification problem though. There may of course be alternate ways of partitioning the set of possible cosmologies, and the mere existence of a classification scheme in any case does not entail that it is physically significant. Again, I am setting such concerns aside and concentrating on the problem of whether one simply has enough structure to define a likelihood measure of some kind or another on the assumption that natural mathematical structure has physical significance.

structure is automatically presumed on the topological manifold \mathbf{R}^4 . Is this presumption justified? Should spacetimes with exotic smoothness structures be excluded? The standard of physical equivalence in GTR suggests that they are not so easily dismissed:

The discovery of exotic smoothness structures shows that there are many, often an infinity, of nondiffeomorphic and thus physically inequivalent smoothness structures on many topological spaces of interest to physics. Because of these discoveries, we must face the fact that there is no a priori basis for preferring one such structure to another, or to the ‘standard’ one just as we have no a priori reason to prefer flat to curved spacetime models. (Asselmeyer-Maluga and Brans, 2007, 13)

One should therefore not neglect consideration of these spacetimes, at least in the absence of reasons to discount such spacetimes as physically possible.

Conceivably, one might retreat from physical likelihood claims concerning the full space of mathematically possible solutions and argue that some kind of epistemic likelihood measure is measure enough. One might hope, that is, that observation and induction thereon could be used to exclude enough “exotic” spacetimes to make classification of the remainder tractable. There is reason to doubt that this strategy would make any important difference, since alternative spacetime topologies or smoothness structures do present an underdetermination threat. Observationally indistinguishable spacetimes (Glymour, 1977; Malament, 1977) may have different global topological features, and so-called “small” exotic manifolds may be smoothly embedded outside our observational horizon in what we would otherwise have thought was a spacetime based on \mathbf{R}^4 . Insofar as these alternate spacetimes can have observable consequences, it would seem that they must be considered as epistemically possible given what we know about our observable universe.

Manchak (2009, 2011) in particular has forcefully argued that we do face a substantial epistemic predicament in cosmology because of the existence of observationally indistinguishable spacetimes. In his cases global properties of spacetime, such as inextendibility and hole-freeness, are underdetermined by the theoretical possibility of observationally indistinguishable spacetimes which do or do not possess these properties—even assuming robust inductive principles for local conditions on spacetime. His arguments have influenced several commentators to claim that knowledge of any global property of spacetime is indeed beyond our epistemic horizon (Beisbart, 2009; Norton, 2011; Butterfield, 2012, 2014). If Manchak is right, then the kind of underdetermination he discusses cannot be broken or bracketed. If the other kinds of underdetermination, e.g. by exotic manifolds, represent an epistemic threat as well, then they too cannot be broken or bracketed. Thus one must either show that these underdetermination arguments fail or accept that the success of likelihood arguments depends on the resolution of the manifold classification problem.

It may indeed be the case that these underdetermination arguments are unsuccessful, and perhaps just for the usual kinds of reasons Laudan and Leplin (1991). We may, for example, have grounds to favor a specific choice that breaks the underdetermination, or it may be the case that the underdetermination in question is of a superfluous feature that only arises because of our choice of theoretical framework.²⁶ So far, however, there have been only a few attempts (Magnus, 2005; McCoy, 2017b) at criticizing underdetermination arguments in the specific context of cosmology.

Yet there is reason to think that critiques of the underdetermination arguments mentioned above would not resolve the issue in favor of those who would want to exclude unusual spacetimes in favor of the simple ones. Indeed, some physicists claim, in effect, that underdetermination may be broken in favor of the unusual spacetimes. For example, some have argued that we may justifiably infer non-standard

²⁶One should also note that some assumptions have been made already to limit the theoretical possibilities in cosmology from the beginning. For example, we only consider locally Euclidean Hausdorff manifolds that are connected, and paracompact. There are relatively straightforward arguments to favor these particular choices (Ellis, 1971; Hawking and Ellis, 1973), but they could perhaps be questioned.

topologies for the universe if it best explains observational phenomena. As mentioned, exotic smoothness structures may have detectable astrophysical effects (Sładkowski, 2009), and if we inhabited a “small universe” (Ellis and Schreiber, 1986) in which light has had time to travel around the universe multiple times, we might be able to observe multiple images of galaxies, etc. which might favor a compact spatial topology.²⁷ Thus it is not enough to show that the underdetermination arguments fail in order to avoid the classification problem; one must also show that they fail in such a way to make one’s preferred set of solutions salient.

Summing up the discussion thus far, I have argued that a significant obstacle to defining likelihoods on the space of solutions is the problem of classifying manifolds. Without some means of classification, there seems to be no natural way of assigning likelihoods at this level of description of a relativistic spacetime. Presently all accounts of cosmological likelihoods, whether ontic or epistemic, ignore the issue completely, making specific choices of topology and smoothness structures without justification. This is not at all surprising, since the potential relevance of non-standard topologies and smoothness structures is little discussed in the philosophical or physical literature. It also does not appear easy to justify the choice cosmologists conventionally make given what the physical possibilities apparently are. Some of the alternative manifolds have physical consequences; if they are physically possible, it is difficult to see why they should be automatically considered unlikely (as is done tacitly in assuming particular manifolds).

4.2 Metric Possibility

The next level of description to consider is that in terms of the spacetime metric. So let us assume, along with workers investigating the solution space of general relativity and, derivatively, the space of cosmologies, a fixed spacetime manifold M (Isenberg and Marsden, 1982, 188). Then one can understand general relativity as a particular field theory on M using the framework of covariant classical field theory (Fischer and Marsden, 1979). This field bundle is a map $\pi : L(M) \rightarrow M$ with typical fiber L , where L is the vector space of Lorentzian metric tensors, e.g. for p in M , $L_p = \{g_p | T_p M \times T_p M \rightarrow \mathbf{R}\}$ with g_p normally non-degenerate, symmetric, and possessing a Lorentzian signature. A configuration of the field is represented by a section of this bundle, viz. a tensor field g on M . The canonical configuration space of the theory is thus the space of sections, which I denote hence as \mathcal{L} .

Does \mathcal{L} naturally have some structure which could be used to define likelihoods? This question has not been studied in nearly as much detail as the geometry of spacetime itself.²⁸ Some things are known. For example, it is desirable for many applications to treat subsets of \mathcal{L} as a manifold, but in general it is not possible to treat the entirety of these spaces as a manifold, because of, for example, the existence of conical singularities in the neighborhood of symmetric spacetimes (Fischer et al., 1980; Arms et al., 1982).²⁹

The entire canonical configuration space can be *given* some structure by, for example, topologizing it, as a way of introducing likelihoods topologically. Unfortunately, since there is an infinity of sections of the field bundle, there is an infinity of topologies which one can define on the set. How can one decide which topology is appropriate? Fletcher (2016) observes that some physicists have advocated a particular

²⁷Various other multiply-connected topologies with observable consequences are physically possible as well (Lachièze-Rey and Luminet, 1995; Luminet et al., 2003). At least in the case of multiply-connected topologies, the relevant techniques to test these possibilities have been developed and observation has largely ruled out that we occupy one of the distinguishable ones (Cornish et al., 1998, 2004).

²⁸“What is not nearly as well developed is the study of the space of Lorentzian geometries, which from the mathematical point of view includes questions about its topology, metric structure, and the possibility of defining a measure on it, and from the physics point of view is crucial for addressing questions such as when a sequence of spacetimes converges to another spacetime, when two geometries are close, or how to calculate an integral over all geometries” (Bombelli and Noldus, 2004).

²⁹That said, for vacuum spacetimes Isenberg and Marsden (1982) are able to show that near generic points the space of solutions is a symplectic manifold and as a whole is a stratified symplectic manifold, at least with their choice of topology, and restricting to globally hyperbolic spacetimes and spatially compact manifolds.

topology as appropriate for discussing similarity relations in general relativity. For example, Lerner (1973) favors the Whitney fine topology, a topology that is widely used to prove stability results in GTR (Beem et al., 1996). Geroch has furnished some examples, however, which suggest that this topology has “too many open sets,” i.e. the topology is intuitively too fine (Geroch, 1971)—at least for some purposes. Other topologies have unintuitive results as well. The compact-open topology for example, renders the verdict that chronology violating space-times are generic in \mathcal{L} in any of the compact-open topologies (Fletcher, 2016; Curiel, 2015, 12). Such considerations, and some further results of his own, lead Fletcher to conclude that “it thus seems best to accept a kind of methodological contextualism, where the best choice of topology is the one that captures, as best as one can manage, at least the properties relevant to the type of question at hand, ones that relevantly similar space-times should share” (Fletcher, 2016, 15).³⁰ Of course, whether any intuitions one has about which properties spacetimes should share can be adequately justified (in a particular context) is then an issue which must be addressed in each individual case. In any case, this “contextuality” at least makes clear what could be gleaned from the beginning: there is no natural choice of topology for the entire space \mathcal{L} .

There are also obstacles for defining \mathcal{L} to be a measure space. In general \mathcal{L} will be infinite-dimensional, essentially because spacetimes have an infinity of degrees of freedom (Isenberg and Marsden, 1982, 181).³¹ This presents a problem for a measure-theoretic approaches, a problem to which Curiel (2015) particularly draws attention. As he observes, “it is a theorem...that infinite-dimensional spaces of that kind do not admit non-trivial measures that harmonize in the right way with any underlying topology” (Curiel, 2015, 4). Thus it is not possible to substantiate claims of the form “most spacetimes (of some kind) are similar with respect to property X ”—where one interprets “most” is a measure-theoretic notion and “similar with respect to” is a topological notion (Curiel, 2015, 4). He motivates the issue in the following way:

Say we are interested in the likelihood of the appearance of a particular feature (having a singularity, e.g.) in a given family of spacetimes satisfying some fixed condition (say, being spatially open). If one can convincingly argue that spacetimes with that feature form a “large” open set in some appropriate, physically motivated topology on the family, then one concludes that such spacetimes are generic in the family, and so have high prior probability of occurring. If one can similarly show that such spacetimes form a meagre or nowhere-dense set in the family, one concludes they have essentially zero probability. The intuition underlying the conclusions always seems to be that, though we may not be able to define it in the current state of knowledge, there should be a physically significant measure consonant with the topology in the sense that it will assign large measure to “large” open sets and essentially zero measure to meagre or nowhere-dense sets. (Curiel, 2015, 3)

In finite-dimensional spaces it is possible to harmonize these notions in a way to make such claims have content. Although he points out that the natural infinite-dimensional extension of finite-dimensional manifolds depends on the differentiability class of the manifold with which one starts, Fréchet manifolds do cover the two relevant cases, and it is a theorem then that “the only locally finite, translation-invariant Borel measure on an infinite-dimensional, separable Fréchet space is the trivial measure (viz. the one that assigns measure zero to every measurable set)” (Curiel, 2015, 13). It follows that there is no sensible application of measure theory for the kinds of topological manifolds one would expect to use for rigorously discussing cosmological likelihoods.

Thus, even restricting attention to a specific manifold and its space of sections, there are significant challenges to defining a likelihood measure. Although there is no problem with simply supplying this

³⁰Hawking (1971, 396) advocates a similar contextualism: “A given property may be stable or generic in some topologies and not in others. Which of these topologies is of physical interest will depend on the nature of the property under consideration.”

³¹“In cosmology, however, the systems one most often focuses on are entire spacetimes, and families of spacetimes usually form infinite-dimensional spaces of a particular kind” (Curiel, 2015, 4).

space with a topology, there is no natural topology. One must make a choice and somehow justify it on some basis other than that given by the mathematical structure of the space. If one wants a measure-theoretic explication of likelihood, then there is a natural measure; unfortunately, this measure is just the trivial measure. Perhaps there are additional considerations which can be raised to aid in solving these problems, but at present one must conclude that likelihood reasoning concerning GTR's solution space has no adequate natural mathematical foundation.

5 Likelihood in FRW Spacetimes

Cosmologists have mostly bypassed the technical and conceptual difficulties that come with trying to define a notion of likelihood on the solution space of GTR. Instead they have focused on simpler finite-dimensional cases, presumably hoping that the results derived there are consistent with the larger, containing cosmological possibility spaces (recall the consistency requirement discussed in §3.1). Simplifying the problem in this way makes it technically feasible to define a variety of likelihood measures (although most of the conceptual problems of §3 of course remain).

In this section I discuss one such measure, the most well known account of cosmological likelihoods (Gibbons et al., 1987), to illustrate how some of the problems already mentioned arise in the simpler case and to present some interpretational issues that arise which are specific to the GHS measure. The measure defined by GHS is associated with the set of FRW spacetimes which have a scalar field as the only matter component. GHS choose the set of FRW spacetimes because it is the relevant set for describing the HBB universe. A scalar field is chosen as the matter content in order to represent the field driving inflation, as their primary aim is to investigate fine-tuning questions related to inflation (§2).

It is clear, of course, that one cannot count on any serious empirical confirmation of likelihood assignments to cosmologies due to the assumed uniqueness of the universe (§3). The exercise of contriving a likelihood measure might therefore appear futile, as GHS themselves observe:

The question of an appropriate measure, especially in cosmology, might seem to be more philosophical or theological rather than mathematical or physical, but one can ask whether there exists a 'natural' or privileged measure on the set of solutions of the field equations. (Gibbons et al., 1987, 736)

This is, of course, the question asked in the previous section, and it is the question they ask for a restricted set of relativistic spacetimes, the FRW spacetimes, thereby presuming that mathematical naturalness is a mark of physical significance. In particular, GHS argue that by adapting the canonical Liouville measure associated with phase space in statistical mechanics to the case of general relativity, i.e. formulating GTR as a phase space theory, one obtains a measure that can be used to make likelihood arguments. This construction is briefly presented below, followed by its application to the flatness problem, to the likelihood of inflation, and to the uniformity problem.

5.1 The Gibbons-Hawking-Stewart Measure

To assess these applications, it will be worth detailing some of the principal features of the GHS measure. I follow in outline the detailed derivations in (Schiffrin and Wald, 2012) and (McCoy, forthcoming), since the derivations in (Gibbons et al., 1987) and other papers making use of it are misleading or mistaken on some important points.

Since the GHS measure is intended to be the Liouville measure associated with the phase space of FRW spacetimes, one must first identify the appropriate phase space Γ for these spacetimes (with a scalar field as the matter source). This requires making use of the initial value formulation of GTR, where one takes a state of the system to be a spatial hypersurface Σ , the degrees of freedom of which are represented

by the spatial metric h and its extrinsic curvature π . Since FRW spacetimes are spatially homogeneous and isotropic, the spatial metric h is homogeneous and isotropic and the extrinsic curvature can be shown to be Hh , where H is the expansion coefficient known as the Hubble parameter. Thus the initial data for an FRW spacetime are adequately represented by h and H .

The Einstein equation for FRW spacetimes relates the hypersurface geometry (represented by h and H) with the parameters of the specified matter contents, i.e. the scalar field. The Einstein equation can be expressed in two equations with these assumptions, usually called collectively the Friedman equations (here incorporating the scalar field as the matter constituent of spacetime):

$$H^2 = \frac{8\pi}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) - \frac{k}{a^2}; \quad (2)$$

$$\dot{H} = -4\pi \dot{\phi}^2 + \frac{k}{a^2}, \quad (3)$$

where the scale factor a is related to the Hubble parameter H according to the equation $H = \dot{a}/a$ (dots represent differentiation with respect to cosmic time t), k represents whether space is negatively curved ($k = -1$), flat ($k = 0$), or positively curved ($k = 1$), ϕ is the field value of the scalar field sourcing the Einstein equation, and V is the scalar field's potential.

Making use of, among other things, the Friedman equations, one can show that the initial data h and H can be re-expressed in the four-dimensional space parameterized by $\{a, p_a, \phi, p_\phi\}$, where p_a (the conjugate momentum of a) is $-3a\dot{a}/4$ and p_ϕ (the conjugate momentum of ϕ) is $a^3\dot{\phi}$. As with the phase spaces of classical particle mechanics, this space comes equipped with the canonical symplectic form

$$\omega_{p_a, a, p_\phi, \phi} = dp_a \wedge da + dp_\phi \wedge d\phi, \quad (4)$$

In classical mechanics the configuration space is composed of all the possible positions of the particles and the phase space is composed of all the possible positions and momenta of the particles. In the case under consideration, the configuration space is composed of all the possible values of the scale factor and (homogeneous and isotropic) scalar field and the phase space is composed of these plus their associated conjugate momenta.

This space, however, is not yet the correct space of initial data and this form cannot be used to construct a natural measure. This is because the first of the two Friedman equations above is a constraint on this space that must be satisfied by the initial data. One can pull the symplectic form onto the surface in phase space that satisfies the constraint equation, but the result is only a (pre-symplectic) differential form since it is degenerate. There are, in other words, redundancies among the states in the three-dimensional constraint surface. These redundancies are due to states on the surface being dynamically-related (so they are part of the same phase space trajectory). Thus the natural next step would be to “solve the dynamics” so that one can take equivalence classes of phase points that are part of the same trajectory. There are difficulties with implementing this strategy in the context of GTR, so the simplest thing to do (what GHS do) is take a two-dimensional surface that intersects all the histories by setting p_a (or, with the usual substitution, H) to a particular value. This finally yields the GHS measure μ_{GHS} by defining a map from Lebesgue measurable sets U (Gibbons et al., 1987; Carroll and Tam, 2010; Schiffrin and Wald, 2012):

$$U \mapsto -6 \int_U a^2 \frac{(3H_*^2 + 2k/a^2)/8\pi - V}{((3/4\pi)(H_*^2 + k/a^2) - 2V)^{1/2}} da d\phi, \quad (5)$$

where $H = H_*$ is used here to pick out a specific two-dimensional surface. The topology of the a - ϕ space depends on k . If we swap a for $\alpha = 1/a$, then it is a half-cylinder if $k = 0$, a hemisphere if $k = +1$, or a hyperboloid if $k = -1$ (Schiffrin and Wald, 2012, 7). Although the measure is evidently fairly complex, what matters most for its application is the leading quadratic term a^2 .

5.2 The Flatness Problem

Recall that the HBB model’s fine-tuning problems, when interpreted as likelihood problems, depend on a demonstration that the uniform and flat spacetimes underlying the HBB model are unlikely. Let us begin with the flatness problem. If the flatness problem is indeed a problem, then what must be shown is that spatially flat ($k = 0$) spacetimes are unlikely according to the GHS measure. To show this the measure of flat spacetimes must be much smaller than the measure of non-flat spacetimes. Following GHS, we take the relevant reference class for this claim to be FRW spacetimes. Various physicists, including Hawking, Page, Coule, and Carroll, have argued that applying the GHS measure to this problem shows that spatially flat FRW spacetimes are in fact *likely*; thus they make the surprising claim that there is actually no flatness problem at all. Their arguments have recently been analyzed in detail in (McCoy, forthcoming). I will summarize the main points from there briefly, for they are relevant to the considerations raised in this paper and also to the remaining applications of the GHS measure.

First, note that the leading a^2 factor in the GHS measure causes the integral to diverge for large scale factors (the scale factor ranges from 0 to infinity) (Gibbons et al., 1987, 745)—crucially, this is so for each k —and (less obviously) to converge to 0 for small scale factors. Thus the total measure of the a - ϕ space is infinite for each k . Hence the GHS measure is not naturally a probability measure, since it is not normalizable without assuming some particular probability distribution on the a - ϕ space that normalizes it.

The divergence of the measure due to the integral over scale factors might be taken to suggest that almost all spacetimes have a large scale factor, since given any choice of scale factor a_* the measure of spacetimes with larger scale factor is infinite and the measure of spacetimes with smaller scale factor is finite. In this sense FRW spacetimes typically have large scale factors. But the claim is misleading to some extent. Consider an analogy with real numbers. Pick any number between zero and infinity; most numbers are going to be larger than the chosen number (according to the usual Lebesgue measure). Does that mean that most numbers are “large”? If one is precise about what one takes the claim to be, then it is correct. But nowhere have we introduced a standard of “largeness” and the real numbers certainly do not give us one. Likewise, the analogous claim in the context of FRW spacetimes is misleading, since there is no given or natural standard of “large” for scale factors.

In any case, Hawking and Page argue that this fact about scale factors should be taken to imply that almost all spacetimes are spatially flat, since the FRW dynamics insures that curved spacetimes become flat as the scale factor increases (the curvature $\kappa = k/a^2$). Similarly, Carroll and Tam (2010, 15) take the measure to show that there is a divergence at zero curvature, from which they conclude that curved spacetimes have negligible measure and that flat spacetimes have infinite measure. If these claims are correct, then flat spacetimes are typical and there is in fact no flatness problem as cosmologists have usually thought.

McCoy (forthcoming) argues, however, that the interpretations that Hawking and Page, and Carroll and Tam construe are highly misleading or mistaken in various respects. The first problem has already been mentioned: there is no natural standard of “large” scale factor and, hence, no standard of “flatness” for curvature, except $k = 0$. If one does take $k = 0$ as the natural standard of flatness, this leads to a second problem, namely that curved spacetimes do not have negligible measure according to the GHS measure. *Each* of the sets of negatively curved, flat, and positively curved spacetimes has infinite measure. It makes no difference whether one puts $k = -1, 0, \text{ or } +1$; the total measure of each of these three independent phase spaces is infinite. If a set and its complement both have infinite measure, then no conclusion can be drawn about whether it is likely or unlikely. Since the set of spatially flat FRW spacetimes has infinite measure and the set of spatially curved FRW spacetimes both have infinite measure, no conclusion can be drawn about the typicality of spacetime curvature on the basis of the GHS measure alone.

One way to make their claims about the flatness problem coherent is to suppose that they are tacitly introducing a “curvature cutoff” by including the “nearly flat” spacetimes with the flat spacetimes. Then

one could truly state that nearly flat spacetimes are typical. But what standard of “nearly flat” should one use? As suggested already, it seems that any choice would be arbitrary, since there is no natural standard of flatness in the context of FRW spacetimes (other than exact flatness). Moreover, since whether a spacetime is “nearly flat” can depend on its dynamical evolution (for non-flat spacetimes), one should not consider time slices of the cosmological histories; one should consider full histories (or else stipulate that “nearly flat” spacetimes must be “nearly flat” for their entire histories).

Despite these issues, Gibbons and Turok (2008) introduce just such a curvature cutoff. They argue that nearly flat spacetimes are empirically indistinguishable from flat spacetimes and therefore identify all the spacetimes flatter than the chosen cutoff. They argue that their results do not depend on the exact choice of cutoff. In a sense this claim is true, but it is one thing for their results to not depend on the *choice* of a cutoff and another for their results to not depend on *introducing* a cutoff. If their results did not depend on introducing a cutoff at all, then it seems that there would be no point in introducing a cutoff to begin with; they should just use the GHS measure. That their results differ, then, from what would be obtained by the GHS measure is suggestive. Indeed, by identifying the sufficiently flat spacetimes in the way that they do, they effectively assign them zero measure (as Carroll and Tam (2010) point out). They are no longer using the GHS measure but instead a probability measure obtained by defining a probability distribution on the a - ϕ space that assigns zero probability to the large scale factor sets. Obviously it does not matter where one puts in the cutoff if one wants to throw away infinite measure sets, but it does seem rather unjustified to discard them when making likelihood assessments.

Although (McCoy, forthcoming) goes into more details of the various arguments concerning the likelihood of flatness, it is enough for my purposes to mention these points. The main one is that the GHS measure by itself tells us nothing about the likelihood of flatness. The only way that it could be used to draw definitive conclusions about flatness in FRW spacetimes is to introduce more structure. Either one picks a standard of flatness, which appears to be rather arbitrary, or one adds a probability distribution, in which case one is no longer using the GHS measure so much as a probability measure that is defined by way of the GHS measure and the chosen probability distribution. Of course, in this latter case one expects some special justification for the choice of probability distribution, since it cannot be natural in the way the GHS measure is.

5.3 The Likelihood of Inflation in FRW spacetimes

Although the GHS measure cannot show that the flatness problem is real or illusory, let us suppose that it is indeed real and motivates the introduction of inflation. Does the GHS measure have any application to demonstrating that inflation solves the flatness problem? It would seem not, since it cannot even show that there is a flatness problem. For the purpose of illustration, let us nevertheless consider whether it can.

There is at least a precise condition for when inflation occurs—when the universe is expanding at an accelerated rate—so one can (potentially) pose precise questions about inflating spacetimes using the GHS measure. This can be given as a condition on the scalar field representing the physical field driving inflation: $\dot{\phi}^2 < V(\phi)$. In terms of the phase space variables used before this is

$$\frac{1}{4\pi} \left(H^2 + \frac{k}{a^2} \right) < V(\phi). \quad (6)$$

Two points are important to note at the outset. First, since the GHS measure is evaluated at a particular Hubble parameter H_* and whether inflation is occurring depends on H , the GHS measure cannot give a definitive assessment of the likelihood of inflation without considering full histories. It is not enough to consider some particular slice through the histories. Second, one also requires a specific model of inflation (a specific choice of $V(\phi)$) in order to make the assessment, since the given condition depends on the precise shape of the scalar field potential.

Let us turn now to the analysis. Recall from §2 that there are two conditions that must be met if inflation is to solve the flatness problem: flatness must be generic outcomes of inflation (Ellis, 1988) and inflating spacetimes themselves must be generic. As Carroll and Tam point out, however, the second question is not exactly the question that must be answered affirmatively to solve the fine tuning problems. It matters not, that is, that some spacetimes inflate only a little bit without solving the flatness problem. What one actually requires is that it is likely that an FRW universe undergoes sufficient inflation to solve the fine-tuning problems.

Consider the first question. Obviously for the $k = 0$ case the question is moot since space is flat regardless of whether there inflation occurs. Flatness cannot be a generic *outcome* of inflation if space is *assumed* to be flat. So it seems that we must consider non-flat spacetimes if the question is to have any significance. However, since spatially curved hypersurfaces cannot become (truly) flat hypersurfaces through the FRW dynamics, we require some specification of “nearly flat” in order to claim that some sort of flatness is a generic outcome of inflation. Not only is such a choice arbitrary, as noted before, it also makes the question moot. Since the GHS measure diverges as the curvature $\kappa \rightarrow 0$ (due to arbitrarily large scale factors in phase space), almost all spatially curved spacetimes are nearly flat to begin with, regardless of whether inflation occurs or does not. Near flatness cannot be a generic outcome of inflation if nearly all spacetimes are already nearly flat. Thus the GHS measure is no help in answering the first question (unless one unjustifiably throws out these initially nearly flat spacetimes).

Does the GHS measure fare any better in answering the second question? Are sufficiently inflating spacetimes generic? Obviously we must specify what it means for a spacetime to inflate sufficiently to solve the, e.g., flatness problem. If the universe is spatially flat, then obviously there can be no flatness problem for inflation to solve. If the universe is spatially curved, then it is clear that “solving the flatness problem” depends, once again, on an assessment of the problem in terms of “near flatness”. One could stipulate a standard, e.g. spacetimes that undergo a certain number of doublings in size (or alternatively e -folds as cosmologists usually describe expansion). Then one could at least attempt some calculations. This is the approach taken by, for example, Schiffrin and Wald (2012, §IV).

Schiffrin and Wald elect to treat the $k = 0$ case for a scalar field in a simple self-interaction potential $V = m^2\phi^2/2$, i.e. the slow roll inflation scenario. In this case the GHS volume element simplifies to

$$d\Omega \propto a^2 \sqrt{\frac{3}{4\pi} H_*^2 - m^2\phi^2} da d\phi. \quad (7)$$

They consider the histories of inflating spacetimes in the slow roll regime and show that spacetimes which undergo at least N e -folds of inflation are the ones for which $|\phi| \gtrsim 2\sqrt{N}$. Thus the GHS-measure of this set is proportional to

$$\int_0^\infty a^2 da \int_{2\sqrt{N}}^{\sqrt{3/4\pi}H_*/m} \sqrt{\frac{3}{4\pi} H_*^2 - m^2\phi^2} d\phi. \quad (8)$$

The ϕ integral is finite; the a integral obviously is not. If the measure of the complement of this set of spacetimes is finite or negligible, then sufficiently inflating spacetimes would be generic. However, it should be clear that the set of spacetimes which do not undergo at least N e -folds of inflation also has infinite GHS-measure, since the a integral in the measure of this set would still diverge. Therefore the likelihood of inflation is (unsurprisingly) indeterminate due to the large scale factor spacetimes.

Both Carroll and Tam (2010) and Gibbons and Turok (2008) attempt to overcome this problem with the GHS measure by regularizing the integrals to make them finite. They also choose to evaluate the likelihood of inflation at different values of H_* . Naturally they come up with different answers. This is because one must consider a full history to determine whether a spacetime has inflated and because regularizing the divergence in the GHS measure requires making some (potentially consequential) choice to make the measure finite. There is no canonical choice: different choices will lead to different results (this latter issue is thoroughly discussed in (Schiffrin and Wald, 2012)).

To illustrate the point in a simple way, consider the following. Since the GHS measure factorizes into two integrals, one over a and one over ϕ , one might think to define a probability measure μ_ρ with probability distribution ρ defined by

$$\rho = \frac{1}{\int d\Omega}. \quad (9)$$

Then μ_ρ is simply

$$\frac{\int_0^\infty a^2 da \int_{2\sqrt{N}}^{\sqrt{3/4\pi}H_*/m} \sqrt{\frac{3}{4\pi}H_*^2 - m^2\phi^2} d\phi}{\int d\Omega} = \frac{\int_{2\sqrt{N}}^{\sqrt{3/4\pi}H_*/m} \sqrt{\frac{3}{4\pi}H_*^2 - m^2\phi^2} d\phi}{\int \sqrt{\frac{3}{4\pi}H_*^2 - m^2\phi^2} d\phi}, \quad (10)$$

since the integral over the scale factor drops out. This measure is not invariant under time evolution, however, because a crucial part of the time-invariant measure has essentially been thrown away. Depending on how one chooses H_* one will therefore compute (potentially wildly) different probabilities of inflation (Schiffrin and Wald, 2012).³²

Schiffrin and Wald conclude their analysis with the following thoughts:

Should one impose a cutoff in a at, say, the Planck time and conclude that inflation is highly probable? Or, should one impose a cutoff in a at a late time and conclude that inflation is highly improbable? Or, should one impose an entirely different regularization scheme and perhaps draw an entirely different conclusion? Our purpose here is not to answer these questions but to emphasize that, even in this simple minisuperspace model, one needs more information than the GHS measure to obtain the probability of inflation. (Schiffrin and Wald, 2012, 12)

I am in agreement with their final point: one requires more than the GHS measure to obtain the probability of inflation. However this is the case because the GHS measure is not naturally a probability measure. More importantly, one also requires more than the GHS measure to draw conclusions about whether flatness is typical, whether inflation generically makes space flat, and whether inflation itself is typical. Thus, although the GHS measure is mathematically natural, it cannot tell one much about the generic properties of FRW spacetimes.

5.4 The Uniformity Problem

So far I have only discussed the fine-tuning problems with respect to a small reference class, the set of FRW spacetimes with a single scalar field (minisuperspace). This is in several respects an inadequate reference class. Carroll and Tam (2010, 21) note that “examining a single scalar field in minisuperspace is an extremely unrealistic scenario;” Schiffrin and Wald (2012, 12-3) observe that “minisuperspace is a set of measure zero in the full phase space. Even if we are only interested in nearly [FRW] solutions, it is far from clear that the GHS measure will give a valid estimate of the phase space measure of the spacetimes that are ‘close’ to a given [FRW] solution.” One might therefore, as a first step beyond FRW models, move to examining the analog of the GHS measure on perturbed FRW spacetimes. Obviously this does not solve the “measure zero” problem, since one can run the same argument on perturbed FRW spacetimes as one did with FRW spacetimes—likelihoods assigned to the perturbed FRW spacetimes are only significant if they are consistent with likelihoods assigned to the full space of possible cosmologies (§3.1). Presumably, however, what one aims for is some “inductive” support for conclusions which are

³²Carroll has since acknowledged Schiffrin and Wald’s critique: “The procedure [Tam and I] advocated in (Carroll and Tam, 2010) for obtaining such a measure was faulty, as our suggested regularization gave a result that was not invariant under a choice of surface on which to evaluate the measure” (Carroll, forthcoming, 19).

consistent in both the containing and contained reference classes, since it is not entirely clear what the full set of possible cosmologies is.

Nevertheless, setting the reasonableness of proceeding to the side, for the horizon problem to be a problem of likelihoods one must obviously consider a larger set of spacetimes than the FRW spacetimes, since these are by definition spatially uniform. The technical details involved in constructing the relevant Liouville measure on perturbed FRW spacetimes are somewhat more complex than the technicalities so far discussed and not particularly illuminating, so I will only mention the relevant results and crucial assumptions.³³ The canonical volume element Ω on “almost” FRW models (according to Schiffrin and Wald (2012)) is

$$\Omega_{GHS} \wedge \left(\frac{a(H_*^2 + ka^{-2})}{H_*(3H_*^2 - V + 3ka^{-2})} \right)^{N_1} \times \prod_{n=1}^{N_1} (k_n^2 - 3k) d\Phi^{(n)} \wedge d\delta^{(n)} \wedge \left(\frac{1}{4} a^3 \right)^{N_2} \prod_{n=0}^{N_2} d\dot{h}^{(n)} \wedge dh^{(n)}. \quad (11)$$

Here there are additional terms (beyond those in the FRW volume element Ω_{GHS}) involving inhomogeneous scalar perturbations (Φ and δ) and tensor perturbations (\dot{h} and h).³⁴ N_1 and N_2 correspond to short-wavelength cutoffs for the scalar and tensor modes, respectively. These are necessary to make the phase space finite. One must also impose a long-wavelength cutoff, which Schiffrin and Wald implement by restricting attention to spatially compact spacetimes. Finally, some explication of “almost” FRW must be made: Schiffrin and Wald take it to mean that the magnitude of the metric perturbation Φ and the magnitude of the density perturbation δ are small in comparison to the background FRW metric, as do Carroll and Tam (2010).

Although some of the assumptions made to obtain this volume element may be challenged, the main problems revealed by adapting the GHS measure to perturbed FRW spacetimes are the same as before, and it would only belabor the points made already to go into detail. As before, the total measure of phase space is infinite, so probabilistic arguments cannot be made on the basis of the canonical measure alone. Indeed, Schiffrin and Wald note that “including more perturbation modes makes the large- a divergence more severe” (Schiffrin and Wald, 2012, 17). It follows that the results on the probability of inflation given by Carroll and Tam (2010) cannot to be trusted because an arbitrary choice has to be made to derive them. Once again, one can get any probability one wants by a particular choice of H_* (the value of the Hubble parameter where the measure of sets of spacetimes is evaluated).

Can one nevertheless make a likelihood argument with respect to uniformity? Is there a horizon problem according to the canonical measure? Carroll and Tam (2010, 25) claim that there is: “There is nothing in the measure that would explain the small observed values of perturbations at early times. Hence, the observed homogeneity of our universe does imply considerable fine-tuning; unlike the flatness problem, the horizon problem is real.” In some sense this conclusion is (intuitively) correct, since one expects that (nearly) uniform spacetimes are highly unlikely given all the spacetimes that seem physically possible. This goes for all spacetimes with symmetry, however, insofar as one takes GTR to delimit the space of possible cosmologies. Of course if one makes this claim, then one should say the same thing about spatial flatness (which is plausibly even less likely in the space of possible cosmologies). But these conclusions have nothing to do with the GHS measure; rather they are judgments based on expectations related to the space of possible cosmologies. This, though, is just the intuitive basis that objective measures were intended to avoid and improve upon.

6 Conclusion

I have discussed the formal implementation, interpretation, and justification of likelihood attributions in cosmology. A variety of arguments and issues were raised which, taken together, strongly suggest that

³³The interested reader is directed to (Schiffrin and Wald, 2012, §V) and to (Carroll and Tam, 2010, §5) for further details.

³⁴Note that h here is not the spatial metric as it was previously.

the use of probabilistic and similar reasoning is misplaced in the context of single-universe cosmology. Some of these concerned conceptual problems and some concerned technical problems. Some of these concerned independent considerations in cosmology and some concerned the application of considerations from statistical mechanics in cosmology. In all cases the verdict is the same: likelihood reasoning is problematic in cosmology. Since the discussion was widely ranging, a brief summary of the main points is in order.

The first issue I raised was the reference class problem (§3.1): What is the appropriate reference class of cosmologies for attributing cosmological likelihoods? The problem is particularly important in the context of likelihood arguments, since such arguments depend sensitively on the choice of possibility space. Although the space of models of GTR is a natural choice for this space, it is not necessarily the correct one. Nevertheless, it seems plausible to suppose that the space of models is “large” like the space of relativistic spacetimes. In this case one requires a way to attribute likelihoods on the full space of possibilities, since the appropriate likelihood attributions to subsets of this space may depend on the likelihoods on the full space. The formal challenges of implementing some likelihood measure on the full space were related in §4. The main issues involved the unavailability of natural structures which could be used as likelihood measures. Although there is potential for further work here, it does seem doubtful whether these challenges can be fully met.

However, even if they can be met, the attribution of likelihoods in cosmology faces significant conceptual difficulties. First is the issue of interpreting these likelihoods (§3.2). Some may be interpreted in terms of typicality, as is popular among some researchers working on the foundations of statistical and quantum mechanics. The only available interpretation of cosmological probabilities (in the particular sense of locating the origin of the probabilistic “randomness”), however, is that they pertain to an initial random trial to select an otherwise deterministically-evolving universe—the “god throwing darts” interpretation. While this strikes many as practically a *reductio ad absurdum* of the project of attributing probabilities to entire universes, it is at the very least coherent. If this is the only interpretation, though, then it becomes quite hopeless to justify any particular choice of probability measure (§3.3): any choice of measure which attributes a non-zero probability to the model(s) describing our universe is admissible, in the sense of being empirically adequate.

One might avoid this latter problem by proposing that the choice of measure is *a priori*. It has been claimed, for example, that probability and typicality measures are mathematically natural in statistical mechanics. There is indeed a relevant sense in which some mathematical objects “come for free” given certain mathematical structures (they don’t require any special choices other than the choice to make a definition), but that does not imply that their physical interpretation comes for free. In any case, for the spaces of possibilities considered by cosmologists there is no natural probability measure, since the total measures of these spaces are infinite. A choice has to be made, and none of the choices made by cosmologists is well-motivated, let alone well-justified.

Finally I considered a specific case considered in the physics literature. For certain spaces of possibilities, e.g. minisuperspace, there is a natural measure, namely the Liouville measure associated with the phase space of minisuperspace, called the GHS measure (§5). I showed (relying partly on arguments made in (McCoy, forthcoming) and (Schiffrin and Wald, 2012)) that the GHS measure cannot be used for the purposes to which it has been put by cosmologists: it cannot be applied to the flatness problem (§5.2); it cannot be used to calculate the likelihood of inflation in FRW spacetimes; it cannot be applied to the uniformity problem (§5.4). In each case the essential issue is that there is no typical spacetime in the spaces of possibilities considered. Thus one must introduce a choice (of cutoff, of probability measure, etc.), none of which is well-motivated, let alone justified.

It follows that the fine-tuning arguments presented in §2 are unsupported when fine-tuning is interpreted in likelihood terms. Therefore the fine-tuning arguments, as they stand, either fail, or else an alternative interpretation of fine-tuning must be sought which validates them. Alternative interpretations have been suggested (instability, lack of robustness, excess idealization, etc.) (McCoy, 2015), although

these have so far been little investigated. Given the problems facing a likelihood interpretation of fine-tuning, there does seem to be some reason to think that these alternative approaches to fine-tuning may be more promising,

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References

- Albert, David. *Time and Chance*. Cambridge, MA: Harvard University Press, 2000.
- Arms, Judith, Jerrold Marsden, and Vincent Moncrief. “The Structure of the Space of Solutions of Einstein’s Equations II: Several Killing Fields and the Einstein-Yang-Mills Equations.” *Annals of Physics* 144: (1982) 81–106.
- Asselmeyer-Maluga, Torsten, and Carl Brans. *Exotic Smoothness and Physics*. Singapore: World Scientific, 2007.
- Beem, John, Paul Ehrlich, and Kevin Easley. *Global Lorentzian Geometry*. New York: Marcel Dekker, 1996.
- Beisbart, Claus. “Can We Justifiably Assume the Cosmological Principle in Order to Break Model Underdetermination in Cosmology?” *Journal for General Philosophy of Science* 40: (2009) 175–205.
- Beisbart, Claus, and Tobias Jung. “Privileged, Typical, or Not Even That?—Our Place in the World According to the Copernican and the Cosmological Principles.” *Journal for General Philosophy of Science* 37: (2006) 225–256.
- Bombelli, Luca, and Johan Noldus. “The moduli space of isometry classes of globally hyperbolic spacetimes.” *Classical and Quantum Gravity* 21: (2004) 4429–4453.
- Butterfield, Jeremy. “Underdetermination in Cosmology: An Invitation.” *Aristotelian Society Supplementary Volume* 86, 1: (2012) 1–18.
- . “On Under-determination in Cosmology.” *Studies in History and Philosophy of Modern Physics* 46: (2014) 57–69.
- Callender, Craig. “There is no Puzzle about the Low-Entropy Past.” In *Contemporary Debates in the Philosophy of Science*, edited by Christopher Hitchcock, Hoboken: Wiley-Blackwell, 2004, chapter 12, 240–255.
- . “The Past Hypothesis Meets Gravity.” In *Time, Chance, and Reduction: Philosophical Aspects of Statistical Mechanics*, edited by Gerhard Ernst, and Andreas Hütteman, Cambridge: Cambridge University Press, 2010, 34–58.

- . “Hot and Heavy Matters in the Foundations of Statistical Mechanics.” *Foundations of Physics* 41: (2011a) 960–981.
- . “The Past Histories of Molecules.” In *Probabilities in Physics*, edited by Claus Beisbart, and Stephan Hartmann, Oxford: Oxford University Press, 2011b, 83–113.
- Carroll, Sean. “In What Sense Is the Early Universe Fine-Tuned?” In *Time’s Arrows and the Probability Structure of the World*, edited by Barry Loewer, Brad Weslake, and Eric Winsberg, Cambridge: Harvard University Press, forthcoming .
- Carroll, Sean, and Heywood Tam. “Unitary Evolution and Cosmological Fine-Tuning.” ArXiv Eprint, 2010. <http://arxiv.org/abs/1007.1417>.
- Cornish, Neil, David Spergel, and Glenn Starkman. “Circles in the sky: finding topology with the microwave background radiation.” *Classical and Quantum Gravity* 15: (1998) 2657–2670.
- Cornish, Neil, David Spergel, Glenn Starkman, and Eiichiro Komatsu. “Constraining the Topology of the Universe.” *Physical Review Letters* 92, 20: (2004) 1–4.
- Cotsakis, Spiros, and Peter Leach. “Is Nature Generic?” In *Cosmological Crossroads*, edited by Spiros Cotsakis, and Eleftherios Papantonopoulos, Berlin: Springer Verlag, 2002, 3–15.
- Coule, David. “Canonical measure and the flatness of a FRW universe.” *Classical and Quantum Gravity* 12: (1995) 455–469.
- . “Comparison of the minimal and canonical measures.” *Classical and Quantum Gravity* 13: (1996) 2029–2032.
- Curiel, Erik. “Measure, Topology and Probabilistic Reasoning in Cosmology.” PhilSci Archive Eprint, 2015. <http://philsci-archive.pitt.edu/11677/>.
- Davidson, William. “Philosophical Aspects of Cosmology.” *British Journal for the Philosophy of Science* 13: (1962) 120–129.
- Demarest, Heather. “The Universe Had One Chance.” *Philosophy of Science* 83, 2: (2016) 248–264.
- Dingle, Herbert. “Philosophical Aspects of Cosmology.” *Vistas in Astronomy* 1: (1955) 162–166.
- Donaldson, Simon, and Peter Kronheimer. *The Geometry of Four-Manifolds*. Oxford: Oxford University Press, 1997.
- Dürr, Detlef. “Bohmian Mechanics.” In *Chance in Physics*, edited by Jean Bricmont, Detlef Dürr, Maria Galavotti, Giancarlo Ghirardi, Francesco Petruccione, and Nino Zanghì, Berlin: Springer Verlag, 2001, number 574 in Lecture Notes in Physics, 115–131.
- Earman, John. “The Sap Also Rises: A Critical Examination of the Anthropic Principle.” *American Philosophical Quarterly* 24: (1987) 307–317.
- . *Bangs, Crunches, Whimpers, and Shrieks*. Oxford: Oxford University Press, 1995.
- . “The “Past Hypothesis”: Not even false.” *Studies in History and Philosophy of Modern Physics* 37: (2006) 399–430.
- Earman, John, and Jesus Mosterín. “A Critical Look at Inflationary Cosmology.” *Philosophy of Science* 66: (1999) 1–49.

- Earman, John, Chris Smeenk, and Christian Wüthrich. “Do the laws of physics forbid the operation of time machines?” *Synthese* 169: (2009) 91–124.
- Ellis, George. “Topology and Cosmology.” *General Relativity and Gravitation* 2: (1971) 7–21.
- . “Does inflation necessarily imply $\Omega = 1$?” *Classical and Quantum Gravity* 5: (1988) 891–901.
- . “83 years of general relativity and cosmology: progress and problems.” *Classical and Quantum Gravity* 16: (1999) A37–A75.
- . “Issues in the Philosophy of Cosmology.” In *Philosophy of Physics*, edited by John Earman, and Jeremy Butterfield, Amsterdam: Elsevier, 2007, volume B of *Handbook of the Philosophy of Science*, 1183–1285.
- Ellis, George, and Henk van Elst. “Cosmological Models.” In *NATO Advanced Study Institute on Theoretical and Observational Cosmology*, edited by Marc Lachièze-Rey. 1999, volume 541, 1–116.
- Ellis, George, Ulrich Kirchner, and William Stoeger. “Multiverses and Physical Cosmology.” *Monthly Notices of the Royal Astronomical Society* 347: (2004) 921–936.
- Ellis, George, Roy Maartens, and Malcolm MacCallum. *Relativistic Cosmology*. Cambridge: Cambridge University Press, 2012.
- Ellis, George, and G. Schreiber. “Observational and Dynamical Properties of Small Universes.” *Physics Letters A* 115: (1986) 97–107.
- Evrard, Guillaume. “Objective Prior for Cosmological Parameters.” In *Maximum Entropy and Bayesian Methods*, edited by Kenneth Hanson, and Richard Silver, Alphen aan den Rijn, Netherlands: Kluwer Academic Publishers, 1996, 381–386.
- Evrard, Guillaume, and Peter Coles. “Getting the measure of the flatness problem.” *Classical and Quantum Gravity* 12: (1995) L93–L97.
- Fischer, Arthur, and Jerrold Marsden. “The initial value problem and the dynamical formulation of general relativity.” In *General Relativity: An Einstein Centenary Survey*, edited by Stephen Hawking, and Werner Israel, Cambridge: Cambridge University Press, 1979, chapter 4, 138–211.
- Fischer, Arthur, Jerrold Marsden, and Vincent Moncrief. “The structure of the space of solutions of Einstein’s equations. I. One Killing field.” *Annales de l’Institut Henri Poincaré* 33: (1980) 147–194.
- Fletcher, Samuel. “Similarity, Topology, and Physical Significance in Relativity Theory.” *British Journal for the Philosophy of Science* 67: (2016) 365–389.
- Freedman, Michael, and Frank Quinn. *Topology of 4-Manifolds*. Princeton: Princeton University Press, 1990.
- Frigg, Roman. “Typicality and the Approach to Equilibrium in Boltzmannian Statistical Mechanics.” *Philosophy of Science* 76: (2009) 997–1008.
- . “Why Typicality Does Not Explain the Approach to Equilibrium.” In *Probabilities, Causes and Propensities in Physics*, edited by Mauricio Suárez, Berlin: Springer Verlag, 2011, volume 347 of *Synthese Library*, 77–93.
- Frigg, Roman, and Charlotte Werndl. “Demystifying Typicality.” *Philosophy of Science* 79: (2012) 917–929.

- Geroch, Robert. “General Relativity in the Large.” *General Relativity and Gravitation* 2: (1971) 61–74.
- Gibbons, Gary, Stephen Hawking, and John Stewart. “A natural measure on the Set of all Universes.” *Nuclear Physics B* 281: (1987) 736–751.
- Gibbons, Gary, and Neil Turok. “Measure problem in cosmology.” *Physical Review D* 77: (2008) 1–12.
- Glymour, Clark. “Indistinguishable Space-Times and the Fundamental Group.” In *Foundations of Space-Time Theories*, Minneapolis, MN: University of Minnesota Press, 1977, volume 8 of *Minnesota Studies in the Philosophy of Science*, 50–60.
- Goldstein, Sheldon. “Boltzmann’s Approach to Statistical Mechanics.” In *Chance in Physics*, edited by Jean Bricmont, Detlef Dürr, Maria Galavotti, Giancarlo Ghirardi, Francesco Petruccione, and Nino Zanghì, Berlin: Springer Verlag, 2001, 574, 39–54.
- . “Typicality and Notions of Probability in Physics.” In *Probability in Physics*, edited by Yemima Ben-Menahem, and Meir Hemmo, Berlin: Springer Verlag, 2012, chapter 4, 59–71.
- Hájek, Alan. “Interpretations of Probability.” In *The Stanford Encyclopedia of Philosophy*, edited by Edward Zalta, 2012. Winter 2012 edition. <https://plato.stanford.edu/archives/win2012/entries/probability-interpret/>.
- Harré, Rom. “Philosophical Aspects of Cosmology.” *British Journal for the Philosophy of Science* 50: (1962) 104–119.
- Hawking, Stephen. “Stable and Generic Properties in General Relativity.” *General Relativity and Gravitation* 1: (1971) 393–400.
- Hawking, Stephen, and George Ellis. *The large scale structure of space-time*. Cambridge: Cambridge University Press, 1973.
- Hawking, Stephen, and Don Page. “How Probable is Inflation?” *Nuclear Physics B* 298: (1988) 789–809.
- Hemmo, Meir, and Orly Shenker. “Measures over Initial Conditions.” In *Probability in Physics*, edited by Yemima Ben-Menahem, and Meir Hemmo, Berlin: Springer Verlag, 2012, The Frontiers Collection, chapter 6, 87–98.
- Henneaux, Marc. “The Gibbs Entropy Production in General Relativity.” *Lettere al Nuovo Cimento* 38: (1983) 609–614.
- Hollands, Stefan, and Robert Wald. “An Alternative to Inflation.” *General Relativity and Gravitation* 34: (2002a) 2043–2055.
- . “Comment on Inflation and Alternative Cosmology.” ArXiv Eprint, 2002b. <http://arxiv.org/abs/hep-th/0210001>.
- Isenberg, James, and Jerrold Marsden. “A slice theorem for the space of solutions of Einstein’s equations.” *Physics Reports* 89: (1982) 179–222.
- Kanitscheider, Bernulf. “Does Physical Cosmology Transcend the Limits of Naturalistic Reasoning?” In *Studies on Mario Bunge’s Treatise*, edited by Paul Weingartner, and Georg Dorn, Amsterdam: Radopi Bv Editions, 1990, volume 18 of *Poznan Studies in the Philosophy of the Sciences and Humanities*, 337–350.
- Kirchner, Ulrich, and George Ellis. “A probability measure for FLRW models.” *Classical and Quantum Gravity* 20: (2003) 1199–1213.

- Kofman, Lev, Andrei Linde, and Viatcheslav Mukhanov. "Inflationary Theory and Alternative Cosmology." *Journal of High Energy Physics* 2002: (2002) 1–14.
- Lachièze-Rey, Marc, and Jean-Pierre Luminet. "Cosmic Topology." *Physics Reports* 254: (1995) 135–214.
- Laudan, Larry, and Jarrett Leplin. "Empirical Equivalence and Underdetermination." *The Journal of Philosophy* 88: (1991) 449–472.
- Lebowitz, Joel. "Boltzmann's Entropy and Time's Arrow." *Physics Today* September: (1993a) 32–38.
- . "Macroscopic Laws, Microscopic Dynamics, Time's Arrow and Boltzmann's Entropy." *Physica A* 194: (1993b) 1–27.
- Lerner, David. "The Space of Lorentz Metrics." *Communications in Mathematical Physics* 32: (1973) 19–38.
- Linde, Andrei. "Inflationary Cosmology." In *Inflationary Cosmology*, edited by Martin Lemoine, Jerome Martin, and Patrick Peter, Berlin: Springer Verlag, 2007, volume 738 of *Lecture Notes in Physics*, 1–54.
- Loewer, Barry. "Determinism and Chance." *Studies in History and Philosophy of Modern Physics* 32: (2001) 609–620.
- . "David Lewis's Humean Theory of Objective Chance." *Philosophy of Science* 71, 5: (2004) 1115–1125.
- Luminet, Jean-Pierre, Jeffrey Weeks, Alain Riazuelo, Roland Lehoucq, and Jean-Philippe Uzan. "Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background." *Nature* 425: (2003) 593–595.
- Magnus, P. D. "Reckoning the Shape of Everything: Underdetermination and Cosmotopology." *British Journal for the Philosophy of Science* 56: (2005) 541–557.
- Malament, David. "Observationally Indistinguishable Space-Times." In *Foundations of Space-Time Theories*, edited by John Earman, Clark Glymour, and John Stachel, Minneapolis, MN: University of Minnesota Press, 1977, volume 8 of *Minnesota Studies in the Philosophy of Science*, 61–80.
- Manchak, John. "Can we know the global structure of spacetime?" *Studies in History and Philosophy of Modern Physics* 40: (2009) 53–56.
- . "What Is a Physically Reasonable Space-Time?" *Philosophy of Science* 78: (2011) 410–420.
- Maudlin, Tim. *Philosophy of Physics: Space and Time*. Princeton, NJ: Princeton University Press, 2012.
- McCabe, Gordon. "The structure and interpretation of cosmology: Part I—general relativistic cosmology." *Studies in History and Philosophy of Modern Physics* 35: (2004) 549–595.
- McCoy, Casey. "Does inflation solve the hot big bang model's fine-tuning problems?" *Studies in History and Philosophy of Modern Physics* 51: (2015) 23–36.
- . "Epistemic Justification and Methodological Luck in Inflationary Cosmology." *Phil-Sci Archive Eprint*, 2017a. <http://philsci-archive.pitt.edu/12771/>.
- . "Prediction in General Relativity." *Synthese* 194: (2017b) 491–509.

- . “Can Typicality Arguments Dissolve Cosmology’s Flatness Problem?” *Philosophy of Science* 84, 5: (forthcoming) 1–10.
- McMullin, Ernan. “Indifference Principle and Anthropic Principle in Cosmology.” *Studies in History and Philosophy of Science Part A* 24: (1993) 359–389.
- Munitz, Milton. “The Logic of Cosmology.” *British Journal for the Philosophy of Science* 13: (1962) 34–50.
- North, Jill. “An empirical approach to symmetry and probability.” *Studies in History and Philosophy of Modern Physics* 41: (2010) 27–40.
- Norton, John. “Ignorance and Indifference.” *Philosophy of Science* 75: (2008) 45–68.
- . “Cosmic Confusions: Not Supporting Versus Supporting Not.” *Philosophy of Science* 77: (2010) 501–523.
- . “Observationally Indistinguishable Spacetimes: A Challenge for Any Inductivist.” In *Philosophy of Science Matters: The Philosophy of Peter Achinstein*, edited by Gregory Morton, Oxford: Oxford University Press, 2011, chapter 13, 164–176.
- Penrose, Roger. “Difficulties with Inflationary Cosmology.” *Annals of the New York Academy of Science* 571: (1989a) 249–264.
- . *The Emperor’s New Mind*. Oxford: Oxford University Press, 1989b.
- Pitowsky, Itamar. “Typicality and the Role of the Lebesgue Measure in Statistical Mechanics.” In *Probability in Physics*, edited by Yemima Ben-Menahem, and Meir Hemmo, Berlin: Springer Verlag, 2012, The Frontiers Collection, chapter 3, 41–58.
- Price, Huw. “Boltzmann’s Time Bomb.” *British Journal for the Philosophy of Science* 53: (2002) 83–119.
- . “On the Origins of the Arrow of Time: Why There is Still a Puzzle about the Low-Entropy Past.” In *Contemporary Debates in the Philosophy of Science*, edited by Christopher Hitchcock, Hoboken, NJ: Wiley-Blackwell, 2004, 219–239.
- Schiffrin, Joshua, and Robert Wald. “Measure and probability in cosmology.” *Physical Review D* 86: (2012) 1–20.
- Shackel, Nicholas. “Bertrand’s paradox and the principle of indifference.” *Philosophy of Science* 74: (2007) 150–175.
- Sładowski, Jan. “Exotic Smoothness and Astrophysics.” *Acta Physica Polonica B* 40: (2009) 3157–3163.
- Smeenk, Chris. “The Logic of Cosmology Revisited.”, 2008. Unpublished Essay.
- . “Philosophy of Cosmology.” In *The Oxford Handbook of Philosophy of Physics*, edited by Robert Batterman, Oxford: Oxford University Press, 2013, chapter 17, 607–652.
- . “Predictability crisis in early universe cosmology.” *Studies in History and Philosophy of Modern Physics* 46: (2014) 122–133.
- Wald, Robert. *General Relativity*. Chicago: University of Chicago Press, 1984.

Wallace, David. "Gravity, Entropy, and Cosmology: in Search of Clarity." *British Journal for the Philosophy of Science* 61: (2010) 513–540.

———. "The quantitative content of statistical mechanics." *Studies in History and Philosophy of Modern Physics* 52: (2015) 285–293.

Wolf, Joseph. *Spaces of Constant Curvature*. Providence, RI: AMS Chelsea Publishing, 2010, 6th edition.