The immanent contingency of physical laws in Leibniz's dynamics

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1. Introduction

The contemporary philosophical legacy of Leibniz's work is most visible in the domain of modality. The concept of possible worlds was already deployed systematically during Leibniz's "middle years" in the *Discours de métaphysique* (1686) and the ensuring correspondence with Arnauld, and in the later period, playing a central role in the *Essais de Théodicée* (1710). It was not only influential during its time but continues to have significance in the recent revival of the idea by David Lewis, his followers and opponents (Armstrong, Dretske, Tooley, Maudlin). It is, however, too obvious to bear repeating that in a time of Kripke semantics and possible worlds realism, contemporary possible worlds discourse bears only a distant family resemblance to Leibniz's use of the concept. Be this as it may, the question of the modality of natural laws is shared by Leibniz and contemporary analytic philosophers.

Lewis famously argues for a grounding of natural laws through the inductive and experimental conclusions we can draw from the arrangement of physical facts in the actual world. Natural laws, in this view, supervene (qua Humean supervenience) on the "mosaic" of facts that constitutes a given possible world.[1] Physical laws are different in different possible worlds because facts, different in these worlds, inductively constitute different laws. The contingency of natural laws is thus derived from the more basic ontological constitution of the possible world that these laws are "about". This is of course disputed. A number of theorists, represented today by the position of Tim Maudlin, have argued that possible worlds may be individuated by a set of natural laws.[2] The debate concerns whether facts or laws should be ontologically prioritized in understanding the fundamental constitution of a possible world and, in turn, how these different conceptions of the constitution of worlds determine the semantic content of natural laws. Of course this is not how Leibniz would have posed the problem. Certainly Leibniz does present possible worlds in terms of series of singular events and facts like "Caesar crossed the Rubicon" or "Cain killed Abel" that constitute the timeline (or "worldlines") of a possible world. However, these singular facts are, for Leibniz, mediated by the idea that what were fundamentally created by God were substances that logically contain these

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This paper was supported by research funds from the Università degli Studi di Milano. I thank the editors of this volume, Pietro Omodeo and Rodolfo Garau, for their invitation to present an early draft of this paper in their seminar at the Max Planck Institute for the History of Science. Their many remarks at the seminar and during the review of this article helped it take shape. I thank Stefano Di Bella and Daniel Garber for the many conversations that helped me solidify my views on the issues in this paper. I also thank Ian Lawson for his help with my often awkward use of the English language.

[1] Lewis, David. 1986. *Philosophical papers: Volume II*. Oxford and New York: Oxford University Press, ix.
[2] See Armstrong, David. 1983. *What is a Law of Nature?* Cambridge: Cambridge University Press; Dretske, Fred. 1977. Laws of Nature. *Philosophy of Science* 44: 333-344; Tooley, Michael. 1977. The Nature of Laws. *Canadian Journal of Philosophy* 7: 667-698. See in particular the second chapter in Maudlin, Tim. 2009. *The Metaphysics within Physics*. Oxford and New York: Oxford University Press.

properties and events. Facts, properties and events are predicates that belong, in the first place, to substances that then act in space and time. Although Leibniz does distinguish worlds through the presence or absence of a particular fact or event, the idea of a possible world as constituted by "worldlines" or a set of facts is foreign to Leibniz's work.

By saying that Leibniz had a different approach to the relation between possible worlds and natural laws is not to say that Leibniz could provide a better solution to contemporary problems. Our enigmas are our own. However, by looking at how Leibniz saw modality as significant in evaluating natural laws, we gain some needed distance from the closely nested problems discussed in the contemporary literature. In turn, what was the central issue concerning the contingency of natural laws was, for Leibniz, the capacity to distinguish between the necessary (geometrical laws) and the contingent (physical) laws of nature. This distinction implies the capacity to be able to separate the different kinds of causes equally at work in reality. In particular, for Leibniz, the fact that some laws of nature are contingent entails the fact that a form of causation beyond mechanical or efficient causes exists.

Now, within the context of Leibniz's scientific work, the distinction between contingent and necessary truths constitutes the fundamental role that the principle of sufficient reason plays in knowledge. In terms of physics, the extensional geometrical relations between physical things constitute a domain of necessary truths. Insofar as geometrical truths are necessary, they do not require the principle of sufficient reason. That is, since geometrical truths could not be otherwise, they need no sufficient reason to determine why they are one way rather than another. However, geometry does not determine the vast majority of physical laws such as gravitation, collisions or optics. Hence laws of nature are logically distinct from necessary geometrical laws.

The distinction between necessity and contingency in Leibniz's natural laws thus could be reduced to a choice between possible worlds. To put it simply, whereas geometrical truths cannot be otherwise, God creates the actual world according to a set of laws that could be otherwise but are chosen through divine wisdom. This relegation of natural laws, via sufficient reason, to a "choice" of possible worlds sustains a view of the natural world of bodies operating mechanically through a contingent set of laws and a contingent set of initial conditions. This was not quite Leibniz's view. The contingency of natural laws does not *merely* come down to the choice of laws. Rather, Leibniz saw the status of natural laws as arising from the action internal to physical substances. Hence, the actuality of physical laws is instantiated by the causal power inherent in substances rather than merely constituted by the divine arrangement of external relations between physical objects. It is with the aim of presenting this counterintuitive idea of the "immanence" of contingency that we proceed here.

In what follows, I will provide an interpretation of the distinction between necessary and contingent physical laws in terms of the causal structure that it implies. The aim is to make clear that, for Leibniz, the contingency of natural laws implies that final causes, beyond mechanical and efficient causes, operate in physical processes and relations. In turn, physical laws do not supervene but are instead the principles through which physical events and their aggregate effects are engendered. Here final causation is implied by the contingency of natural laws. This is of course incongruous with the range of causal theories we find acceptable today.

More than this, Leibniz was also, in his own time, arguing against the grain of his contemporaries, like Descartes, Hobbes and Spinoza, who sought to leave final causation behind with the Scholastics. Nonetheless, we grasp from this the role that Leibniz assigned to the concept of possible worlds for natural science. That is, our actual world can be evaluated against a number of possible worlds precisely because the actual physical laws realize the teleological optimality imbued into the world in divine creation. The meaning of contingency in Leibniz's physical theories is nothing other than final causation.

In order to argue for this interpretation, I shall proceed in three sections. In the first section I examine the distinction between final and efficient cause insofar as it maps onto physical and geometrical principles. This is to be found in the Leibniz's optical studies. Here I establish the difference Leibniz makes between geometrical and physical principles. In so doing, I provide the grounds for Leibniz's argument that efficient causation cannot suffice for a complete explanation of physical phenomena simply because physical relations realize the "optimization" of geometrical relations. From this, we see that the role played by contingency is to allow for a theory of optimality in the determination of scientific theories. In the second section I expand this theory of optimization to the case of collisions. Since the theory of collisions stood as the central terrain for the presentation of natural laws as such, I will examine the role that contingency plays in the nature of collision and rebounding. Here, continuity and elasticity are not taken to be necessary features of bodies. Rather, the geometrical realities that result from the phenomena of collision are the result of sufficient reason. Again, mere geometrical relations are inadequate at providing the needed account for the continuity of the laws of collision. Thirdly, I will turn to the role that contingency plays in the measure of Leibnizian force. Taking up only one of the many aspects of the problem of the measure of Leibnizian force, I will argue that beyond the idea of a supramundane selection of natural laws in divine creation, Leibniz places final causes within the physical systems themselves. Instead of blindly unfolding from a set of laws and initial conditions chosen by the divine in creation, bodies actively rather than passively realize the optimum. This final claim of course cannot be fully defended here but is instead explained in order to underline the centrality that contingency plays in Leibniz's natural philosophy. Nonetheless, what I demonstrate below is the claim that contingency is a central concept in Leibniz's physics and implicated within the operations of physical systems themselves. The contingency of physics is inseparable from the inherent actions of bodies.

2. The geometrical and the physical in the optics

This section provides a first perspective on the role of contingency in Leibniz's physical theories. To provide a first very general gloss, we rely on a very simple and abstract distinction between what is necessary and contingent in physical theories. The truths of geometry are taken to be necessary whereas physical laws are contingent. This simple distinction maps onto Leibniz's distinction between two kinds of principles: those that follow from the principle of non-contradiction (geometry, arithmetic and the like) and those that follow from the principle of sufficient reason (laws of nature). Leibniz's view here appears to satisfy current *ad hoc* views of such a distinction. But this was not an obvious view in Leibniz's time. Descartes, against whom

Leibniz often directed his criticisms, held that arithmetic truths were, in some sense, contingent insofar as God might have made an "eternal" reality where "it was not true that twice four make eight."[3] Of course, Descartes' view on this problem of the omnipotence of God was complicated and we shall not enter into it here. However, Leibniz was famously unequivocal in denying the "voluntaristic" view of God and held that we misunderstand the omnipotence of God when we assert that mathematical truths (broadly construed) could have been otherwise. The omnipotence of God is misunderstood when it implies the trespassing of logical limits. In other places, he argues against the distinction between abstract geometrical relations and concrete spatial relations. What is spatially concrete, Leibniz argues, cannot differ from what is geometrically abstract since geometrical truths cannot be otherwise. As he argues in the 1703 *Nouveaux essais sur l'entendement humain*:

> [T]here is no need to postulate two extensions, one abstract (for space) and the other concrete (for body). For the concrete one is as it is only by virtue of the abstract one: just as bodies pass from one position in space to another, i.e. change how they are ordered in relation to one another, so things pass also from one position to another within an ordering or enumeration. [4]

From a modal standpoint, we can argue that the principle of non-contradiction and the principle of sufficient reason can be used as a test to distinguish between those necessary principles that God will always uphold and those contingent principles where divine wisdom (sufficient reason) is required.

From this we have a rather simple epistemological picture. First there is a set of necessary logical and mathematical laws determined with respect to the principle of non-contradiction. At a second level, there are laws and principles whose (contingent) truth is owed to the wisdom and will of God. This remains an abstraction, of course, and we require a further step to concretize the distinction between these two levels of necessity and contingency. In this section, I will attempt this concretization, at least in Leibnizian terms, through the problem of optics.

An examination of Leibniz's optics is not the most direct way to view his approach to natural laws but it is the easiest path to grasp his use of the distinction between necessity and contingency in natural science.[5] The aim here is to provide a simple presentation of the necessity-contingency distinction in order then to understand why contingency is so crucial to the Leibnzian theory of physical causation.

Leibniz began his research on optics as early as 1671. Critical notes collected in the Akademie edition of his works reveals a study on a wide range of optical writings from Francesco Lana, Cavalieri, Dechales, Fabri, Descartes, Rohault and others.[6] In draft optical treatises of 1673, Leibniz had already attempted to formulate a theory of the uniformity of the laws of both

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[3] Descartes, CSM II 294. AT VII, 436.
[4] A VI, 6, 127; GP V, 115; Leibniz, Gottfried Wilhelm. 1981. *New Essays on Human Understanding*. Ed. and trans. by Peter Remnant and Jonathan Bennett. Cambridge: Cambridge University Press, 127.
[5] See McDonough, Jeffrey K. 2010. Leibniz's optics and contingency in Nature. *Perspectives on Science* 18(4): 432-455.
[6] See A VIII 1, 139-242.

refraction and reflection on the basis of the medium against which light strikes.[7] According to what we call the Snell-Descartes law, a mirrored surface reflects light at an angle (to the normal) equal to its angle of incidence (to the normal).[8] Refraction is a little bit more complicated. Light is refracted such that the proportion of the sine of the angle of incidence (to the normal) and the sine of the angle of refraction is inversely related to the "refractive indices" (to use an anachronism here) of the two media n_1 and n_2:

$$\frac{\sin \theta 1}{\sin \theta 2} = \frac{n_2}{n_1}$$

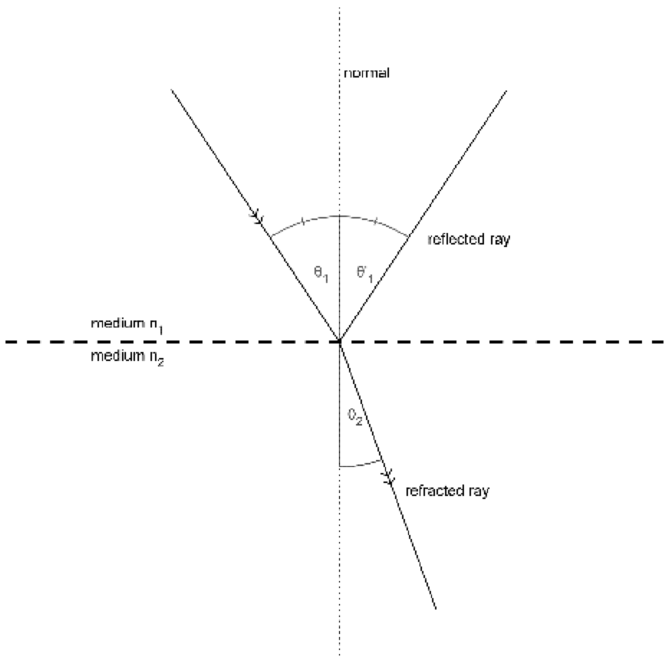

FIGURE 1

Now, moving past the history of the development of this law, we note that Descartes and Fermat famously disagreed as to whether the density of the medium was directly or inversely proportional to the speed of the motion of light traveling through it.[9] Although they did not disagree as to the fundamental proportion, the question of the mechanical cause of this result was under dispute. As we noted, the refractive index is determined by an inverse proportion of the sines of the angles of incidence and refraction. What this says about their speeds however, is a separate but obviously related issue. Although Descartes famously held the position of the infinite speed of light (the immediate propagation of light), he also modeled his optics on the motion of tennis balls. Within this model, Descartes embraced the counterintuitive view that light moves faster through a denser medium.[10] In this (erroneous) view, the speeds (v) of the law would be:

[7] A VIII 1, 168-179.
[8] See figure 1 below.
[9] Descartes, CSM II 162-163; AT VI 103-104.
[10] There may be a point of confusion about the idea of the "motion" of light here and the fact that Descartes also held the immediate propagation of light. Nonetheless, the demonstration of the dioptric

$$\frac{\sin \theta 1}{\sin \theta 2} = \frac{n_2}{n_1} = \frac{v_2}{v_1}$$

This view is one that Leibniz seems to have definitively rejected by 1673.[11]

At least until 1681, Leibniz addressed this fundamental law through an account of mechanical causation. Here the sine of the gradient of refraction is inversely proportional to the density of the medium through which light moves. Although Leibniz is careful to criticize Descartes and others in these writings concerning the proportions of the speeds of light, nothing suggests that Leibniz aimed to provide any other causal account than the intuitive and contemporaneous mechanical view.

By 1681, however, we find a simple derivation of the Snell-Descartes law from a different perspective. In an untitled text, we find an argument that makes use of what we would call an optimization argument in analytic geometry today. Taking the case of refraction, Leibniz argues, quite conventionally, that the proportion of the sine angle of incidence and the sine angle of refraction is the inverse proportion of the densities through which the light moves. However, he argues that the angles can be derived by taking the "minimum" of an equation. I shall simplify Leibniz's argument somewhat. We start with two densities that are proportional d/e. Taking the horizontal and vertical components of each line going into and out of point C, we have the incidence ray AC in two components, the horizontal x and the vertical l, and the refracted ray CB in two components, horizontal f-x and vertical m.

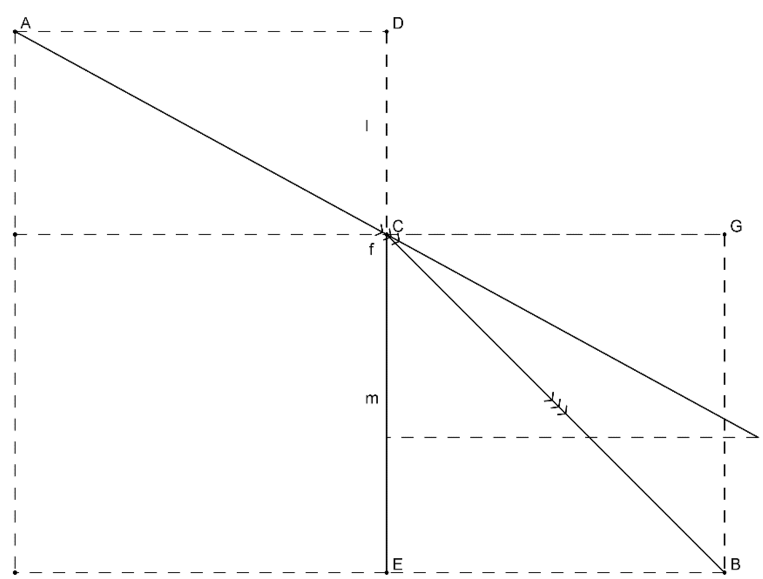

FIGURE 2[12]

law was modeled after the motion of a hypothesized tennis ball moving in and out of a medium at different angles. Although we cannot attempt to synthesize Descartes different models for light propagation, it is in this hypothetical sense that we speak of the motion of light.

[11] A VIII 1, n. 22, p. 183-184.

From this, Leibniz argues that “It will then be the case that $d\sqrt{l^2+x^2}+e\sqrt{(m^2}+f^2+x^2-2fx)$ is equal to a minimum.”[13] Now, before the eighteenth-century solidification of the concept of the function and the functional, what “minimum” means may be ambiguous, but Leibniz then asserts a key insight from his infinitesimal calculus. He argues then that:

$$2dx/d\sqrt{l^2+x^2}(AC)+(2ex-2ef)/\sqrt{(m^2}+f^2+x^2-2fx)(BC)=0$$

Taking the original equation as a “minimum” means precisely to make the derivative of that equation equal to zero. This is familiar to us through the concept of optimization in analysis and the principle of least action in physics.[14] Having shown this, Leibniz shows trivially that if the point C is the center of a circle and CA and CB its rays (of equal length), then the sine of the angle of incidence, line segment AD, and the sine of the angle of refraction, line segment EB, will “be reciprocal to the medium or densities”.[15]

With this text, it seems that Leibniz moves away from a mechanical argument. Of course this argument does not contradict any particular mechanical explanation. However, we see that a mechanical argument, like that of Descartes, relied on a mechanical hypothesis that a rare medium diffuses the light material and retards it while a dense medium is less “soluble” and allows light move faster through it.[16] However, rather than engaging with a mechanical argument, Leibniz engages in a theory that does not require such mechanical explanation. By a theory of the “minimum”, the optimization of the equation of triangles formed by the angles, Leibniz produces an explanation of the desired results.

The 1681 argument from the “minimum” comes very close to an argument from final causation. Sufficient reason “selects” the optimal (minimum) magnitude. From the optical texts of 1682 onward, Leibniz saw the Snell-Descartes law as determined through final causation. In

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[12] Figure reproduced from Leibniz, Gottfried Wilhelm. 1906. *Leibnizens Nachgelassene Schriften Physikalischen, Mechanischen und Technischen Inhalts*. Ed. by Ernst Gerland. Leipzig: B. G. Teubner; reprinted 2006. Ann Arbor: University of Michigan University Library, 73.

[13] Leibniz, *Leibnizens Nachgelassene Schriften Physikalischen, Mechanischen und Technischen Inhalts*, 73; translation by Jeffrey McDonough.

[14] For those unfamiliar to optimization in analysis, we can imagine that a voyage from Berlin to Milan can be optimized according to three (or more) variables: the cost of travel, the distance travelled and the speed travelled. Walking is cheap but lengthy and slow. Train is fast but lengthy. Flight is expensive but quick. Optimization analysis treats these three different variables together, given constraints, in order to find the “path of least action” between Berlin and Milan. In the rudimentary case here, we can liken the optical system as having to “choose” between “optimizing” different variables such as the shortest path, the shortest time or another relevant term. Fermat’s historical demonstration appealed to the optimization of the shortest time. Recall here that Leibniz’s main target was Descartes’ view that higher density of media resulted in greater speed of motion. Hence, Leibniz’s demonstration here is equivalent to Fermat’s but does not directly appeal to the same variable (shortest time). Instead, Leibniz assumes that given some passage of time t, which can be left out, the diagonals of line segments traced the motion of light and the area of the rectilinear figures constituted by those diagonal segments are such that they are optimized according to these paths.

[15] Leibniz, *Leibnizens Nachgelassene Schriften Physikalischen, Mechanischen und Technischen Inhalts*, 73; translated by Jeffrey McDonough.

[16] Descartes AT VI 103, CSM 1:163. See Mcdonough, Jeffrey K. 2016. Leibniz’s optics in *The Oxford Handbook of Leibniz*. Ed. by Maria Rosa Antognazza. Oxford: Oxford University Press, 6.

the June 1682 publication in the *Acta Erutitorum*, "Unicum opticae, catoptricae, et dioptricae principium", Leibniz argues, after his presentation of the sine law, "We have therefore reduced all the laws of rays confirmed by experience to pure geometry and calculation by applying a single principle, taken from final causes if you consider the matter correctly: for a ray setting out from C neither considers how it could most easily reach point E or D or G, nor is it directly through itself to these, but *the Creator of things created light so that from its nature that most beautiful result could arise*."[17]

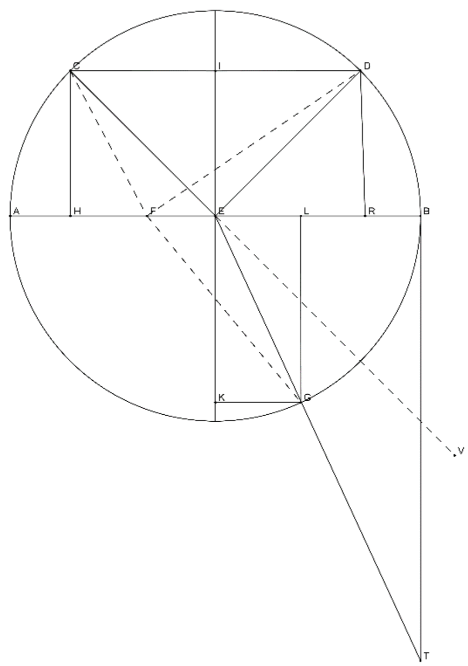

FIGURE 2[18]

Leibniz continues this passage with a criticism of the Cartesian rejection of final causes. But even examining this argument further, we see from this figure that, at least abstractly, the possible paths of motion from C to E, D or G are all coherent hypotheses seen from a geometrical perspective. Addressing the same kind of argument that he presented in 1681, Leibniz invokes final causality here in order to argue from sufficient reason that light passes through one of these paths (the most beautiful) instead of the others.

Using contingency and necessity as our guide, we can understand that alternative paths of motion in the phenomenon of refraction can be logically coherent even if they fail to be true in the actual world. As such, mere geometrical truths, mechanical implications, fail to supply the "sufficient reason" for determining why the refraction (catoptric) law is true.

What is most important for our discussion here is that Leibniz's appeal to final causation in physics is explanatory. That is, Leibniz never denies real mechanical realities that bring about

[17] Leibniz, Gottfried Wilhelm. 1682. Unicum opticae, catoptricae, et dioptricae principium. *Acta Eruditorum*, June 1682; 1768. *Opera Omnia* Vol. III. Ed. by L. Dutens. Geneva: Fratres De Tournes, 145-150; translated by Jeffrey McDonough.

[18] Figure reproduced from Table IV, figure 17 of *Opera Omina* III.

the desired result. From an epistemological perspective, the superior determination of the law of refraction does not, at the same stroke, determine the ontological constitution or mechanical realities that constitute such a phenomenon. It is hence important to notice that as Leibniz's optics matures, he continues to be generally agnostic on the ontological or mechanical constitution of these results. In the mature works like the *Tentamen Anagogicum* of 1696, we see Leibniz embracing the theory of final causes through the idea that light travels through the "most determined" path.[19] This does not say much about the mechanics of light propagation. Rather it allows Leibniz to hold the view that regardless of the mechanics, the behavior of light satisfies the optimization of its "function".

The status of final causation in this examination thus resides on the level of explanation and indicates how Leibniz sees the role of divine wisdom in the creation of the world. On this we can at least say that God chooses laws that "optimize" the motion of light through the medium. In this view, final causes are related only to the level of the choice of reason (*ratio*) for physical phenomenon and how far this extends to the particular behavior of individual bodies is limited. From this, we could argue that, for Leibniz, possible worlds are individuated according to natural laws optimized against a set of alternatives that are respectively geometrically coherent. We have thus established a minimal ground for the understanding of the role of contingency in Leibniz's natural philosophy. In what follows, we will expand this view to examine the fundamental role that contingency plays in Leibniz's determination of natural laws.

3. Modality and the elasticity of collision

In the discussion above, we have made concrete the idea that geometrical truths form a baseline of rigid necessary truths whereas another set of laws, chosen by God, constitute the contingent physical laws. Leibniz makes use of this distinction in order to argue for a general solution for optical laws - dioptric and catoptrics - through optimization arguments. In this section, we expand the idea of final causation by including arguments that are not so straightforwardly mathematical. We examine the sufficient reason that grounds why physical collision is continuous rather than discrete.

Now, the problem of corporeal collision was central to seventeenth-century investigations of the nature of corporeal motion for a number of different reasons. Although the causes of gravity, the shape of motion, and the nature of inertia were all important questions, it was within the arena of collision that the basic laws of motion and the key problem of conservation were dissected and demonstrated. Galileo famously penned an unfinished and unpublished dialogue (the sixth day) of the *Dialogo sopra i due massimi sistemi del mondo* precisely on the topic of collision.[20] Descartes spelled out his famous "laws of motion" on the basis of bodies in collision in the second book of *Principia philosophiae*. More pertinently, Leibniz read the 1669 entries on the laws of collision by Wallis, Wren and Huygens in the

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[19] GP VII 274-75; L 479. See Mcdonough, Jeffrey K. 2009. Leibniz on natural teleology and the laws of optics. *Philosophy and Phenomenological Research* 78(3): 505-544.

[20] Galilei, Galileo. 1898. *Le opere di Galileo Galilei* Vol. VIII. Ed. by Antonio Favaro, Isidoro Del Lungo. Florence: G. Barbera, 319-349.

Philosophical Transactions around the time when he understood physics as the gateway to his larger intellectual ambitions. Although the fundamental question of collision laws concerns the quantity conserved under collision, we will look at a related problem here. That is, we will examine the related question of the elasticity and continuity of bodies in collision.

To be clear, what we mean today by elasticity of collision means nothing more than the conservation of energy in collision. Leibniz's name is historically linked to this concept of the conservation of energy (or energy-work). However, not only is the Leibnizian quantity conserved under collision not ½ mv^2, his concept of elasticity is not rigorously tied to this quantity. First off, Leibnzian force, not to be confused with Newtonian force, is the conserved quantity mv^2, commonly referred to by near contemporaries and historical commentators as *vis viva*. This Leibnizian force is a conservation quantity and hence constant under physical transformations, rather than the cause of momentum change. From this, the Leibnizian force mv^2 is best analogized by the classical understanding of energy-work. We share with Leibniz the notion that it is this quantity that is conserved in elastic collisions. However, for Leibniz, elasticity does not primarily refer to collisions that conserve this quantity. Instead, elasticity refers to the continuous deformation of bodies in contact collision. More importantly, for Leibniz, as we shall examine, elasticity is absolute for all and any collisions. Since elasticity is absolute, it certainly follows that Leibnizian force is conserved. In the strictest sense, if elasticity were not absolute for collisions, Leibnzian force would, by definition, fail to be "force" because it is not universally conserved. We shall see what this means in what follows, but it is important to underline that, in this Leibnizian context, elasticity refers to the shape and hardness of bodies under collision rather than the conservation of energy in collision.

With this caveat, we examine the role that modality plays in the collision laws. Although collision involves many other issues, the aspect that we shall examine is the continuity of collision. That is, within this historical context, the nature of collision was not at all a settled matter. Could a pebble with near-infinite speed move a mountain? What is the threshold at which a massive body begins to carry off a smaller body? Are collisions continuous? The issue of continuity is important because of the context of mechanics within which Leibniz was writing. Of course, on the surface, continuity itself does not seem to have much to do with modality. We might think here of how nature might choose between different ways that bodies might behave in the case of collision. Hence, if we consider various theories of elastic and inelastic collisions as having, within themselves, geometric or mechanical coherence, then the different views on the elasticity collision will play a significant modal role. That is, only the principle of sufficient reason can distinguish between different models of collision as being "optimal" and hence chosen by the divine wisdom.

There is perhaps no easier way to grasp the significance of elasticity for Leibniz than to take his argument against the atomists. Physical extended atoms, at least in the way Leibniz understands them, are in principle unyielding since deformations imply the motion of smaller components under impact.[21] Composite bodies may deform in body-to-body impingements but

[21] A closer reading of contemporary atomists, especially Gassendi, would reveal that Leibniz's understanding, here and in many other occasions, constitutes a negligent or deliberate misunderstanding.

their behavior should be reduced to the more basic atom-to-atom relations which cannot exhibit deformation. In this atomistic view, physical transformations during collision in terms of speed and direction must be discontinuous. Here, perfect indivisible unities meet, come to rest and rebound at discrete times. Leibniz typically argues against this view in noting that atomism relies on a notion of mechanical or efficient causation which relies on discontinuity at its most basic level. In this sense, collisions in the atomistic world are fundamentally non-elastic. As he argues to De Volder in his correspondence of 27 December 1698:

> [H]owever small a body may be, it nonetheless stands in some ratio to a big one, and there is some collision force, although a small one I admit. And the flexing of each body through the resistance of its elastic element, through which the force of a collision is consumed little by little and transferred to the elastic element, must be returned by it in the same way, little by little [...] But as for inflexibly hard bodies, such as we imagine atoms to be, I, like you, fully believe that there can be no compression in them and no conservation of force either. But in turn, I think that there are no such things in nature…. I think that elasticity is essential to bodies on the basis of the order of things and metaphysical principles: although in nature it is accomplished by nothing other than a fluid moving around.[22]

Leibniz's theory of the absoluteness of elastic collision certainly provided Leibniz's main reason for his refutation of atomism but it was also aimed against the wider range of different theories of collisions. The problem of atomism concerned indivisible hard unities. However, Leibniz also held the possibility of inelastic collisions where, due to the softness of bodies, energy is lost. As he argues in the same letter to De Volder, "soft bodies absorb force not by destroying it but by receiving it into their small particles… as when a bullet is shot through many sheets of paper."[23] Here, Leibniz is generally cognizant of the "energy loss" in a system through inelasticity and friction but considers these events as micro-phenomena without detracting from the basic case of elastic collision. Moreover, if we consider energy loss from the perspective of the detachment of smaller parts of bodies in collision under impact, these smaller parts themselves remain elastic. In Leibniz's view, bodies are composed of an indefinite aggregation of smaller elastic bodies all the way down.

Given that Leibniz aimed his arguments about elasticity and the continuity of collision against more than just atomists, we should note that Leibniz's most concerted effort for demonstrating the modal significance of continuity in physics comes from his arguments against Descartes. Now we should note that Descartes himself famously opposed the atomism of Gassendi and hence we should not see these views as equivalent. However, what Descartes and Gassendi shared was a reduction of physical causation to efficient causation. Hence although Leibniz sought to defend the actuality of final causation against these positions, his argument

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Leibniz seems to have taken contemporary atomists as theorists of "void space" and paid less attention to their positive remarks on the structure of bodies.

[22] Leibniz, Gottfried Wilhem. 2013. *The Leibniz-De Volder Correspondences*. Ed. and trans. by Paul Lodge. New Haven: Yale University Press, 44-47.

[23] *The Leibniz-De Volder Correspondences*, 44-47.

against Descartes, as we will examine below, was significantly different from the argument starting from the composition of bodies in collision.

In an in-depth criticism of Descartes' laws of motion, the 1692 *Animadversiones in partem generalem Principiorum Cartesianorum*, Leibniz moves through the *Principia Philosophiae* systematically.[24] We will focus on his remarks on article 53 of the second part of the text.[25] Here Leibniz takes up the problem of corporeal collision, as Descartes does in these sections. According to Descartes' first rule, bodies of equal mass moving against each other at equal speeds will simply rebound from each other in perfectly elastic collisions. But what happens when we modulate the masses and speeds of the two colliding bodies? Famously, Descartes argues that if one body is more rapid, it will carry the other off in the same direction post collision. That is, if bodies B and C collide with B moving at -4v and C at 3v, according to the third and seventh rule, the two bodies will both move at -3.5v.[26] In the Cartesian view, the quantity of motion is divided equally between the two bodies ($\frac{4v+3v}{2} = 3.5v$). Now, to take a different case, if the same two bodies were moving in the same direction with 4v and 3v respectively, B will eventually meet C and the resulting collision will also result in both bodies traveling at 3.5v for the same reason. We can see more clearly from a chart what happens when we simply vary speed in the application of Cartesian laws.

Initial conditions			Cartesian laws	
v(B) initial	*v* (C) initial		*v(B)* final	*v* (C) final
-4	-4		-4	-4
-4	-3		-3,5	-3,5
-4	-2		-3	-3
-4	-1		-2,5	2,5
-4	0		3	-1
-4	1		-2,5	-2,5
-4	2		-3	-3
-4	3		-3,5	-3,5
-4	4		4	-4

[CHART 1]

Leibniz grasps that the Cartesian laws are contrary to phenomena but his reasons for rejecting them are due to the problem that they do not satisfy the conditions of continuity. Take our example above. Two bodies of equal mass in collision will result in a case where the body with the greater magnitude of speed carries the other off in its direction. However, what if the differences of speed become marginally different? Would a body moving at -4v also carry off an opposing body moving at 3.99v? Why would they not simply rebound as in the case of the first

[24] GP IV 350-392.
[25] GP IV 381-384.
[26] GP IV 382.

law of collision? Another way of saying the same thing is that the first law of collision is discontinuous with the other laws. What Leibniz argues in turn is to see the laws of collision as continuous with Descartes first law. The elastic rebounding of bodies of equal mass holds in cases when speeds are varied.

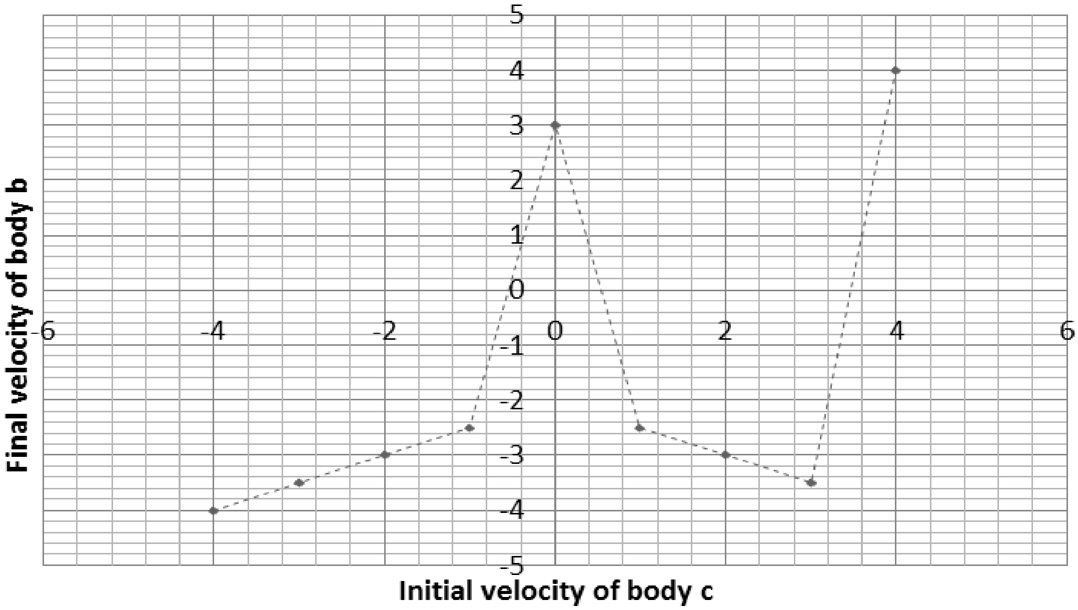
At the time of this text, Leibniz already understood that the Cartesian laws could be "fixed" by the addition of direction in the calculation of speeds. Indeed a more complex story concerning Leibniz's work on the laws of motion can be told here. Part of it will be examined in the next section. Nonetheless, in 1692 Leibniz sought to make another larger point through his examination of Descartes' laws of collision. By comparing his results of the effects of collisions under the same range of initial conditions, Leibniz argued that what occurs under collisions is simply the exchange of velocity (speed and direction). If a body B at -4v collides with a body C at 3v, body B rebounds with velocity 3v and C rebounds with velocity -4v. This holds under elasticity or the symmetry of collision in a non-accelerating inertial frame. As such, the chart of his experiment would produce the following chart.

Initial conditions			Leibnizian laws	
V(B) initial	*V* (C) initial		*V(B)* final	*V* (C) final
-4	-4		-4	-4
-4	-3		-3	-4
-4	-2		-2	-4
-4	-1		-1	-4
-4	0		0	-4
-4	1		1	-4
-4	2		2	-4
-4	3		3	-4
-4	4		4	-4

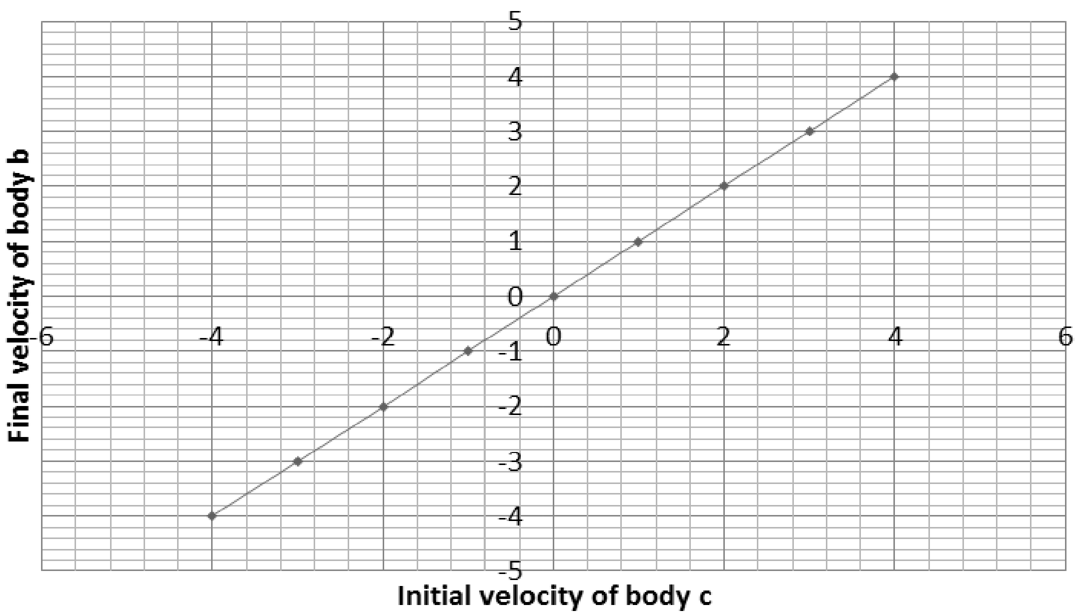
[Chart 2]

The result here is more apt for a description of the phenomena of collision. However, this is not what he emphasizes here. Leibniz argues instead that the Cartesian laws result in a "*delineatio monstrosa*" while his own graph results in a "*delineatio concinna*".[27]

[27] GP IV 382-383.



[FIGURE 3: Descartes' *delineatio monstrosa*]



[FIGURE 4: Leibniz's *delineatio concinna*]

This *delineatio concinna* compared to Descartes *delineato monstrosa* can be made even clearer in seeing that if we take all the infinite continuous values between the initial conditions, the same continuous graph would result. Leibniz's argument thus relies on an appeal to sufficient reason. Descartes' laws, although erroneous and confused, are not logically incoherent. It demonstrates geometrical discontinuity, as the graph demonstrates. There is, in principle, nothing logically incoherent about the *discontinuity* of the Cartesian graph. Leibniz argues for the greater intelligibility of his graph on the basis of its continuity. In this sense, the presence of continuity is taken to have greater intelligibility insofar as it provides the case for the determination of sufficient reason. In this sense, the Leibniz's laws of collision crucially invoke the importance of contingency in his natural philosophy.

Now, from a larger perspective, Leibniz's problem with Descartes laws of motion and collision has to do with the conservation of energy-work or, more contextually, with Leibnizian force. However, in this argument, what Leibniz forgoes is the larger point concerning conservation and he concentrates instead on the error involved in the discontinuity in the different cases of collision.

As a rejoinder, this discontinuity of the Cartesian laws is also relevant to historical shifts in the concept of inertia. That is, against the earlier Keplerian idea of the *inclinatio ad quietem*, Leibniz held the Galilean and Huygensian view that motion is inertial-frame dependent. This is what he called the principle of the "equivalence of hypotheses", a principle that he used throughout his physical writings as early as 1676.[28] Hence, from the perspective of a non-accelerating frame there is no geometrical difference between a body A with velocity v striking a body C at rest, and a body A at rest struck by a body C with velocity –v. From this perspective, the Cartesian laws of rebounding, insofar as they rely on the absolute speeds of bodies, are fundamentally erroneous insofar as it could not take into account this (Galilean-Huygensian) relativity of inertial frames. If this "Galilean relativity" were to be in play within the Cartesian laws, the phenomenon of a more rapid body carrying off a slower body would be incoherent by definition. This indicates that, at least for Leibniz, the modern concept of inertia (the equivalence of hypotheses) falls under the domain of the principle of sufficient reason and should thus also be filed under the category of contingent rather than necessary laws of nature. Nonetheless, what Leibniz sees as resulting from this Cartesian error is the discontinuity of the Cartesian collision laws rather than its incoherence.

Final causation thus plays a fundamental role in Leibniz's collision laws. Among a set of different hypotheses about the behavior of masses in collision, Leibniz attempts to show that those theories that exhibit continuity are in fact those that are justified by sufficient reason. But rather than merely arguing from an appeal to the greater empirical adequacy of his theory, Leibniz argues for the higher optimality implied by his view; the more harmonious organization of nature, the *delineatio concinna*, through the continuity of the phenomena of collision.

The immanent contingency of motion is highlighted here since nothing in this argument implies the necessity of the continuity of motion or of the elasticity of collision. Rather, the elasticity and continuity of bodies and collision are grounded by the higher appeal to sufficient reason. In turn, if we speak in terms of efficient causes, whether the properties of bodies or some other mechanism cause this elasticity of collision is not directly addressed or strictly implied. The order of Leibniz's reasoning provides that it is sufficient reason that determines the absoluteness of elasticity. This determination of elasticity is thus a contingent one. The question of whether the continuity of collision is prior to or follows from the elasticity (malleability) of bodies is left unaddressed here. Nonetheless, Leibniz affirms only that the elasticity of collision

[28] See A VI, 3, 101-111; translated by Richard T.W. Arthur in 2013. *The Leibniz Review* 23, 101-116.

implies the elasticity of bodies (with no atomic elements) and that this follows from the principle of continuity.

4. The measure of force and the hierarchy of laws

In the last section we examined how structural physical properties like continuity are related to the laws of collision through sufficient reason. The role of contingency here implies, for Leibniz, the scientific importance of final causation. However, this view only implies the extrinsic determination of laws. Bodies are created to be continuous and collisions are created to be elastic under the notion of the creation of the world through the wisdom of God. Final causation is affirmed to be the choice of a possible world where these optimal mechanical laws hold.

In this section, we will examine how final causes, the domain of sufficient reason, are not only extrinsic determinations but also inherent in substances and how they intrinsically realize this teleology. As such, we will point to how nature, for Leibniz, is not only extrinsically determined by the modality of the judgment between possible worlds but is also part of the activity of the world itself in the realization of its cause.

At the start of this article, we examined the distinction between “mere” geometrical relations and “sufficient” physical laws. From the perspective of measurement and experiment, geometrical relations are always the effects of physical events and relations. Until now we have not examined the cause and effect relationship that this distinction implies. In this section I will argue that physical laws constitute the cause of geometrical effects in physical phenomena. Hence, although “mere” geometry is more epistemologically fundamental in an absolute sense, physical laws, insofar as causes, precede their effects ontologically. The aim of examining this ontological “priority” will allow us to distinguish an important but idiosyncratic aspect of Leibniz’s thinking about physical causality: the distinction between “merely” mechanical and “higher” mechanical causes.

The language of “mere” and “higher” mechanical causes may be rather clumsy. The point here is that “mere” mechanical principles are effects in physical phenomena that strictly follow geometrical relations while “higher” mechanical causes are due to the specific laws of god’s choosing. Leibniz explains in *Specimen dynamicum* that, “we acknowledge that all corporeal phenomena can be derived from efficient and mechanical causes, but we understand that these very mechanical laws as a whole are derived from higher reasons. And so we use this higher efficient cause only in establishing general and distant principles.”[29]

Although Leibniz was always careful to present his various theories of physical causality as consistent with a mechanistic account of physics, he stretches the terms in order to suggest a second-order notion of a “higher efficient cause” or a “higher reason”. As Leibniz explains

[29] AG 126.

further, this “higher efficient cause” turns out to converge with final cause. In the very next paragraph he explains:

“In general, we must hold that everything in the world can be explained in two ways: through the kingdom of power, that is, through efficient causes, and through the kingdom of wisdom, that is, through final causes, through God, governing bodies for his glory.... These two kingdoms everywhere penetrate each other without confusing or disturbing their laws, so that the greatest obtains in the kingdom of power at the same time as the best in the kingdom of wisdom.”[30]

Here, the “greatest” and the “best” are mirror properties between the two kingdoms of power and wisdom. This provides an indication of the metaphysical scaffolding that Leibniz supplied for a theory of optimization, the details of which we will not examine here, that allows us to see the “higher efficient cause” as a placeholder for final causality immanent in the physical world. In other moods, Leibniz would directly place dynamics in the role of final causes, “My dynamics requires a work of its own [...] You are right, Monsieur, to judge that it is in good part the foundation of my system, since there one learns the difference between truths whose necessity is brute and geometric and truths which have their source in suitability and in final causes.”[31] Hence the distinction between “mere” and “higher” efficient cause reflects the same distinction we have been examining throughout this article. On the one hand, we have an account of the physical world based on the geometrical relations between effects corresponding to size, shape and magnitude. On the other hand, we have an account of the physical world based on the sufficient reason for the laws by which geometrical arrangements are produced. It is this latter, irreducible to “mere” geometry, that corresponds to the true understanding of nature. Hence, despite the generous accommodations that Leibniz made for the mechanistic view of nature, it remains clear that “higher efficient causes” were aimed at carving out a space for the role of an innovated notion of final causation.

If we take this notion of “higher efficient causation” *qua* final cause seriously we should examine the aspect of this view that takes final causes to be an inherent aspect of physical motion. That is, we should take the physical action of a body as the agent that actualizes or realizes the effects of the final causality “impregnated” within it, as Leibniz famously remarks in the *Monadologie*.[32] It is this intrinsic activity of bodies that I will examine here.

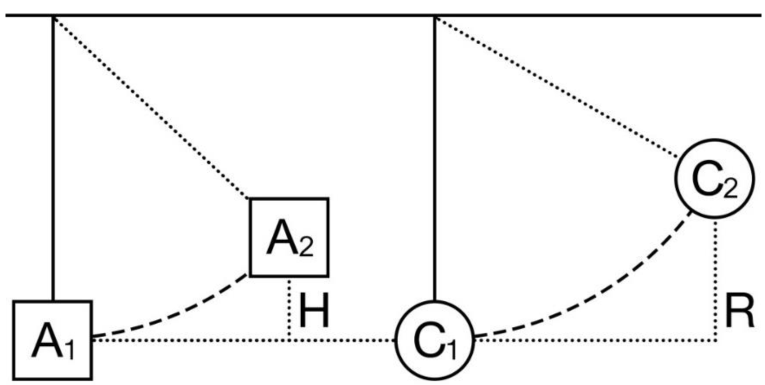
Leibniz’s mature physics, the dynamics, is known for its highlighting of the principle of energy-work and the conservation of mv^2.[33] Against the Cartesians, Leibniz argued that quantity of motion is not universally conserved but only conserved in horizontal elastic rectilinear collisions. Instead he argues that mv^2, a near cousin to the $\frac{1}{2}mv^2$ conservation of energy-work, is universally conserved in nature. The well-known details of this argument, the ensuing

[30] AG 127.
[31] Nicolas Remond on 22 June 1715, GP III 645.
[32] Monadologie §22; GP VI 610; AG 216.
[33] The term “dynamics” was coined by Leibniz in 1689, understood as a science based on the *causes* of motion. The project of the dynamics cannot stand in for the entirety of Leibniz’s mature physical theories but certainly forms a central aspect of it from around 1676 to 1701. See Duchesneau, François. 1994. *La Dynamique de Leibniz*. Paris: J. Vrin.

eighteenth-century vis viva controversy, and Leibniz's imprecisions shall be left out of the current discussion. What is important for us here is to see how work and gravity are related.

Now, in all those often-cited passages where Leibniz argues for the universal conservation of mv^2, he uses the Galilean principle that bodies fall at a rate in quadratic proportion to the height from which they fall. This is assumed throughout Leibniz's published articles on the issue. Concerning the error that the speed of a body in freefall is quadratic to the duration of fall, instead of the distance of fall, Leibniz seems to have insisted against this view even in mature writings.[34] Regardless of this error, Leibniz almost always assumes the quadratic distance theory to be the more universal one without defending it. Part of the reason for this, important for our discussion here, is that gravity was not considered to be the central aspect of physics until after Newton. It was one among a number of other fundamental problems of the time. Indeed, even Newton himself remained unwilling to provide an ontological account of the causes of gravity. Leibniz himself saw gravity as a feature of the larger problem of the vortex motion of the plenum-aether. In this, Leibniz used the Galilean law of falling bodies as an assumed background for the measure of freefalling bodies. It is against this background that we consider one of his standard arguments for the measure of force:

"Therefore, in order to obtain a measure of force, I considered whether those two bodies A and C, equal in size but different in speed, could produce any effects equal in power to their causes, and homogeneous with each other. [...] [L]et us assume that bodies A and C are heavy, and that their force is converted into ascent, which would come about if, at the very moment when they had the speeds they were said to have, a single unit of speed in A, and double that in C, they were understood to be at the ends of the vertical pendula[...] Now, it is well known from the demonstrations of Galileo and others that if body A, with a speed of one unit, ascents at its highest point... of one foot, then body C, with speed of two units, could ascend (at its highest) to a height of... four feet."[35]



[FIGURE 5][36]

What is important in this argument for the measure of force mv^2 is the quadratic relationship between maximum velocity and height $v^2 \propto h$. Comparing maximum speeds and maximum

[34] See GM VI 119-123; L 298-302.
[35] GM VI 245 ; AG 128.
[36] Figure recreated from figure 25 from GM VI; AG 128.

heights across different pendulums, Leibniz asserts that the linear proportions between speeds corresponds to quadratic proportions between heights. This form of measurement depends on what Leibniz called the equipollence of cause and effect. When Leibniz first formulated this principle definitively in 1676, he placed cause and effect in proportional relations such that, for c (cause) and e (effect): $\frac{c}{c'} = (\frac{e}{e'})^r$.[37] This was formulated in a context before Leibniz had arrived at the quadratic proportion but it shows that Leibniz considered the measure of force through a proto-functional view, something we might represent as $f(x)=x^r$ where x is the speed (the independent variable here) and the dependent variable as height. With respect to the "experiment" then, the pendulums represent cases where x is of different values. It turns out, insofar as Leibniz relies on an appeal to Galileo's law of falling bodies, that r=2.

Considering the demonstration itself then, apart from its underlying methodology, how is the quadratic relation between height and speed demonstrated by this form of experiment? The work done by the physical system in raising a body upwards to its maximal height corresponds to the maximum speed reached by the body as it comes to the base of the swing. Leibniz appeals to Galileo without much of a defense or even much detail. However, we can grasp the role of the Galilean law by asking: if gravitational acceleration were different, how would the pendulum bob behave differently?

This was precisely the question that Leibniz entertained when he first came upon the adoption of mv^2 as the conserved measure of force. In his 1678 *De corporum concursu*, Leibniz began his writing of the physical treatise with a different conservation principle, the Cartesian quantity of motion mv. Halfway through the work, he adopted the measure mv^2, the quantity that would then form the cornerstone of his dynamics. With diligence, he returned to the first page of his draft and crossed out the Cartesian principle and noted "This does not follow from our system".[38] In the scholium of his demonstration of the quadratic relation between maximal speed and maximal height in this text, he argued that perhaps this relationship would be different in a world with a different system.

"In our system, it is necessary that the moments are the square of the speeds because the effect is the height that the bodies could attain in their ascent; where the heights of ascent are the square of the speeds.

Perhaps in another system of the world, where the speeds have another relation to heights, another measure of forces would have to be made."[39]

Leibniz's remark here is not very extensive but we can grasp its immediate implications. If gravitational acceleration were different, either due to a different motion of the vortex or to the different attraction of masses, the relation between the two variables, maximal speed and maximal height, would be different according to that value. The conservation quantity mv^2, the measure of the conservation of work-energy, would thus be contingent, according to this account. Now obviously, in cases like the pendulum the physical system in the upward swing is

[37] I have heavily revised this expression from Leibniz's *De arcanis motus*. A VIII 2, 61.
[38] Leibniz, Gottfried Wilhelm. 1678. De corporum concorsu. In *G.W. Leibniz: La réforme de la dynamique: De corporum concursu (1678) et d'autres textes inédits*, ed Michel Fichant, 71. Paris: J. Vrin.
[39] Leibniz, "De corporum concorsu", 134.

working against the downward force of gravity. We can thus distinguish the concept of work and how it operates in different gravitational situations. Following the equipollence of cause and effect, the proportion $\frac{c}{c\prime} = (\frac{e}{e\prime})^r$ would hold even if the variable r were different. In principle the measure of the magnitude of work is a systematic or structural property, its measure is gained by the analysis of the transformation of the variable c or cause (the magnitude of speed in this case) and the corresponding transformation of the variable e or effect (the magnitude of height).

Leibniz's argument here is coherent as far as it goes. The reasoning here is, however, unfortunately circular. As interpreters like Iltis have argued, we should keep in mind that the conservation of Leibnizian force mv^2 had to already be assumed in order to have been "proved" in the first place.[40] This is no *experimentum crucis*. Hence Leibniz's arguments like that pendulums above are at best demonstrations of a theory. However as Duchesneau has also argued, this sort of *a posteori* demonstration of the measure of force was not meant to "prove" the conservation of mv^2, but only to supply an interpretation of given data.[41]

The particularly puzzling circularity is saliently pointed out above by Leibniz's appeal to the contingency implied by "another system of the world". If gravity behaved differently, then a different value for r would hold for the proportion $\frac{c}{c\prime} = (\frac{e}{e\prime})^r$. Now, this perspective does not directly adjudicate between different hypotheses behind the behavior of gravity. Leibniz famously rejected Newtonian "action at a distance" and appealed instead to a vortex-plenum theory of subtle matter circulating in the universe. Without getting into the details of the vortex theory to which Leibniz dedicated a number of published writings, we can point out that in order to spell out just such a vortex-plenum theory, one must first appeal to a fundamental theory of how bodies interact. Ontologically speaking, fundamental physical laws must come before any account of a plenum that is made up of circulating (subtle) bodies. Of course, for the purposes of an *a posteriori* measure of force, pendulums, insofar as they isolate the relation between speed and height (i.e. kinetic energy and work), are a legitimate method of demonstration. In its proper context, there is no real problem. *Ceteris paribus*, force is measured in the pendulum experiment against the backdrop of gravitational acceleration regardless of its ultimate causal basis. However, for the question of contingency the problem arises: on what exactly is the measure of force contingent?

If this measure is contingent on the system of the world, then it is the extrinsic physical relations in the empirical world (considered as a world system) that determine force. On the other hand, if the system of the world is constituted by fundamental relations between bodies, it appears that the system of the world is contingent on these (other) fundamental relations. In other words, Leibniz's constant appeal to the law of falling bodies calls into question the logical integrity of his account of forces. In fact, this circularity was so immediate that it struck J. Bernoulli who challenged Leibniz precisely on this point during their correspondence.[42]

It is important to underline that there would be no problem here if the measure of force were not taken to be contingent. If Leibniz simply argued for the conservation quantity mv^2 and

[40] Iltis, Carolyn. 1971 Leibniz and the Vis Viva Controversy. *Isis* 62(1): 21-35.
[41] Duchesneau, *La dynamique de Leibniz*, 137.
[42] A III, 6, 398-411.

the quadratic relationship between speed and work, we would have been faced with a straightfoward case of hypothesis, deduction and verification. Instead, what we find in Leibniz is an argument for the measure of force that involves a hierarchy of relations, where it appears that the measure of force is contingent upon a cosmology with a certain notion of gravitational acceleration. From this perspective, we can finally stage the fundamental issue of Leibniz's engagement with the problem of contingency in natural science. Is the problem of contingency in Leibniz's physical theory simply God's choice of possible worlds? If so then different possible systems of worlds can certainly be governed by different gravitational laws. Possible worlds could even be individuated by these different gravitational laws. This was, however, not Leibniz's view.

Without changing the fundamental scientific claims that constituted Leibniz's mature physical theory, he saw, around 1689, the need to provide a foundational account of the intrinsic nature of forces. That is, Leibniz placed forces within the activity of bodies such that the theory of forces reached beyond the notion of extrinsic physical relations between bodies in a physical system. As such, the causal root of the measure of force could no longer be assigned to external laws governing different systems of the world but would instead extend from the intrinsic activities of bodies themselves. Here, beyond the assertion of an *a posteriori* measure of conservation, Leibniz pursued another kind of approach, moving to an *a priori* account for the measure of force.

Leibniz, starting from the 1689 dialogue *Phoranomus seu de potentia et legibus naturae* and repeated in the key treatise of the dynamics, *Dynamica de potentia et legibus naturae corporeae tentamen scientiae novae*, provided a new form of argument. Here the measure of force is the action (a) of the physical system in time. In brief, the action (a) of a physical system, assuming a one body system, is composed of two components: the displacement of a mass *in* a certain time (ms_t/t) and the speed of that mass *at* a certain time (v_t). For a one body system, unhindered, the action a of the system at time t, a_t is the product of ms_t and v_t. Since this one body system is unhindered, the evolution of s is constant and v is constant across times. Hence the action in time is $a/t=ms\cdot v/t$. As such, $a/t=ms/t\cdot v=mvv$. For a one bodied inertial system, the action of the system is mv^2. Now, how does this translate into work? In cases where work is involved, say a pendulum working against gravity, we can see that the first factor, ms/t, decelerates as it reaches the maximum height. Similarly, the evolution of the second factor, v, decelerates also. Insofar as $a/t=msv/t$, the rising pendulum exhibits the same quadratic relationship between action and the magnitude of v in this formula: $a=mv^2$.

Now, this presentation is flawed insofar as it fails to account for just how such a function with two factors (ms and v) evolves in time. From an algebraic perspective, however, the conception of action as determined by the two factors of ms/t and v preserves the quadratic relationship between action and speed. As such, bracketing larger problems, Leibniz does provide an *a priori* demonstration for the quadratic relationship between speed and work.

In this sense, Leibniz has produced an *a priori* argument for an invariant relation between work and speed as it temporally evolves in a physical system. The question now is to ask whether this is contingent. Despite the *a priori* status of the theory of action, we can say that it is contingent in an absolute sense since this principle of action could have been otherwise.

The evolution of action in time a/t remains on the left side of the equation a/t=msv/t. In another possible world, $a/t=(msv/t)^r$ could have been possible. However, this further development allows us to say that the conservation quantity $a/t=mv^2$ is contingent. But it is not contingent, or dependent, on the acceleration of gravity. It is, rather, contingent in a more fundamental or absolute sense. In other words, the measure of force is contingent in the sense that it could be otherwise (in a different possible world). However, it is not dependent on the cosmological system of the world or a possible version of the law of falling bodies. To put it rigorously, it is the measure of force that determines the law of falling bodies rather than vice-versa. In the *Dynamica* Leibniz boldly proposes a derivation of Galileo's law from an *a priori* account based on this very notion.[43] From this perspective it becomes more coherent to see the acceleration of gravity, in Leibniz's account, as contingent on the measure of force. Hence the circularity indicated above is, at least on the question of its modality, resolved.

With this modal investigation of the role of the relation between the measure of force and gravity, we have provided the grounds for understanding the hierarchy between mechanical and final causes. Even if we are agnostic about the ontological or cosmological causes of gravity, we can find that the relation between work and gravity is a contingent one. More importantly, we see how Leibniz could provide a contingent theory of forces, the foundational level of his physical theory, intrinsically rooted in the activity of bodies (or systems of bodies) themselves.

We have ventured here only to present Leibniz's view on the intrinsic activity of forces rather than to defend it. It does indeed suffer from some defects that we have only indicated. Nonetheless, the aim here is only to argue that the contingency of physical reality, for Leibniz, is not merely the divine choice between different physical systems under which mechanical relations operate blindly according to laws. Rather, the arrangement of physical reality radiates outward from the inherent action of bodies that realize the conservation of force *qua* mv^2. As such, the contingency of Leibniz's physical laws is an inherent or intrinsic property of the world as such, rather than the extrinsic choice between possible worlds.

4. Concluding remarks

In our examination of Leibniz's theory of contingency in natural laws, we have emphasized that he sought to reinvent final causes in a historical context in which they were regarded as an old bit of Aristotelian-scholastic folly. The basis of this defense of final causes lay in the use of the principle of sufficient reason, which allowed Leibniz to situate natural laws where geometrical truths were inadequate. The ground of Leibniz's argument was to show that a deep and formidable gap stood between geometrical-mechanical reality and actual physical reality. This gap itself demonstrated the need for a theory of causation beyond that of efficient causes. As such, we examined fundamental aspects of Leibniz's mature physical theory that demonstrate the concrete methodological implementation of this distinction between necessity and contingency in physical laws.

[43] GM VI 292.

Of course, viewed solely from the principle of sufficient reason, natural laws, even if they supersede the scope of necessary geometrical principles, would simply constitute the laws governing the external relations between objects in the actual physical system of the world. Hence, even if possible worlds could be individuated by sets of natural laws, contingency would fundamentally be a property of worlds rather than immanent in things. In view of this, we aimed at demonstrating that Leibniz wanted to take a step further, to make the contingency of natural laws a feature of the immanent constitution of bodies in action. In this sense, contingency can be seen as an immanent feature of the world, extending outward from the things of the world rather than simply a property of the order of things in a world system.

All of this stands in stark contrast not only with the contemporary Laplacian and Bayesian notion of contingency in the sciences but also with Leibniz's own legacy in the Kripkean and Lewisian modality of possible worlds. This stands as an obvious warning against reducing Leibniz's scientific heritage to the domain of possible worlds semantics. The lesser regarded aspect of Leibniz's theory of contingency is, however, the use of the principle of sufficient reason. In this investigation we have attempted to cast a historical spotlight on this principle and the origins of the concept of least action. In the attempt to address the gap between mere geometrical-mechanical principles and physical laws, the principle of sufficient reason and the theory of (least) action stubbornly defends an isolated citadel of final causation within the early modern period. Insofar as the concept of least action remains relevant even for the most speculative of contemporary physics, a reexamination of Leibniz on these grounds is pertinent and not a simply a visit to the antiquary shop of past curiosities.