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# Quantum Entanglement and Uncertainty Principle

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## ABSTRACT

We argue about quantum entanglement and the uncertainty principle through the tomographic approach. In the end of paper, we infer some epistemological implications.

## 1 Entanglement and Uncertainty principle

It is known that quantum mechanics is problematic in the sense that it is incomplete and needs the notion of a classical device measuring quantum observables as an important ingredient of the theory. Due to this, one accepts that there exist two worlds: the classical one and the quantum one. In the classical world, the measurements of classical observables are produced by classical devices. In the framework of standard theory, in the quantum world the measurements of quantum observables are produced by classical devices, too. Due to this, the theory of quantum measurements is considered as something very specifically different from classical measurements.

It is psychologically accepted that to understand the physical meaning of a measurement in the classical world is much easier than to understand the physical meaning

of analogous measurement in the quantum world. Using the relations of the quantum states in the standard representation and in the classical one (described by classical distributions), one can conclude that complete information on a quantum state is obtained from purely classical measurements of the position of a particle made by classical devices in each reference frame of an ensemble of classical reference frames, which are scaled and rotated in the classical phase space.

These measurements do not need any quantum language if we know how to produce, in the classical world (using the notion of classical position and momentum), reference frames in the classical phase space differing from each other by rotation and scaling of the axis of the reference frame and how to measure only the position of the particle from the viewpoint of these different reference frames.

Thus, we avoid the paradox of the quantum world which requires for its explanation measurements by a classical apparatus accepted in the framework of standard treatment of QM. The problem of wave function collapse reduces to the problem of a reduction of the probability distribution which occurs as soon as we “pick” a classical value of the classical random observable in the classical framework. This means that we “solved” the paradox of the wave function collapse reducing it to the problem of standard measurement of a classical random variable used in the probability theory. The measurement on a reference frame affects the distributions on the others (due to the underlying uncertainty principle). Can the nonlocal character of QM to be intrinsically present in a single system to emerge as subtle correlations among distributions of different reference frames?<sup>2</sup> We are going to analyze from another point of view this delicate question in the next section.

## 2 Entanglement in single system? A tomographic approach.

By using a tomographic approach (Mancini et al. 2003) to quantum states, we rise the problem of nonlocality within a single particle (single degree of freedom).

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<sup>2</sup>Wehner-Oppenheim (Wehner,Oppenheim,2010) have uncovered a fundamental link between the two defining properties of quantum physics: non-locality and uncertainty principle. According the authors, previously, researchers have treated non-locality and uncertainty as two separate phenomena. Now they have shown that two phenomenon are intricately linked. Moreover they show that this link is quantitative and have found an equation which shows that the “amount” of non-locality is determined by the uncertainty principle. The surprising result by Wehner and Oppenheim is that the uncertainty principle provides an answer. Two parties can only coordinate their actions better if they break the uncertainty principle, which imposes a strict bound on how strong non-locality can be. Oppenheim argue that it a surprising and perhaps ironic twist: Einstein and his co-workers discovered non-locality while searching for a way to undermine the uncertainty principle. Now the uncertainty principle appears to be biting back.

We propose (Asaninmov, Caponigro, Mancini, Man'ko, 2007) a possible way to look for such effects on a qubit. Although a conclusive answer is far from being reached, we provide some reflections on the foundational ground. QE is associated with the specific *nonlocal* correlations among the parts of a QS that has no classical analog. This assumes that the entangled system should consist of two or more parts. Although recently much interest has been dedicated to single particle entanglement, it relies to different degrees of freedom, hence to different parts of the system (subsystems). Typical Bell-type experiments involve, beside entangled (singlet) states, non commuting observables (on each subsystem). Thus, the nonlocal character might not solely be ascribed to the property of states (entanglement), but also to uncertainty principle (e.g. correlations that arise due to the noncommuting character of observables). As such it could somehow emerge even in a single system (single degree of freedom). Here, we address this possibility by resorting to quantum tomography in order to fix the meaning of *nonlocality* in this context. Results along this direction might shed light on the basic principles of QM, like the uncertainty principle, perhaps pointing out some form of self entanglement. The tomographic description can be applied to the systems with both continuous and discrete variables. Here we are interested in case of discrete variables, because we are going to deal with the "smallest" system—a qubit. As we have seen previous section, the problem of wave function collapse reduces to the problem of a reduction of the probability distribution which occurs as soon as we "pick" a classical value of the classical random observable in the classical framework. Nevertheless, measurement on a reference frame instantaneously affects the distributions on the others (due to the underlying uncertainty principle). In this sense nonlocality seems intrinsically present in a single system and should emerge as correlations among distributions on different reference frames (i.e. correlations of noncommuting observables measurement results). It immediately follows the question of whether such correlations can be reproduced by any hidden variable theory.

To address the above question, we consider the simultaneous measurement of spin projection along two directions specified by vectors  $\vec{a}, \vec{b} \in \mathbf{R}^3$ . We get the POVM elements for such a joint measurements as

$$\Pi_{r,s}(\vec{a}, \vec{b}) = \left( \frac{1}{4} + rs \vec{a} \cdot \vec{b} \right) \mathbf{1} + (r\vec{a} + s\vec{b}) \cdot \vec{\sigma}/2, \quad (3.5)$$

where  $r, s = \pm 1/2$  are the possible measurement results and  $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$  represents the vector of Pauli operators.

Due to the unsharpness of the measurements, the vectors  $\vec{a}, \vec{b}$  are constrained by

$$\|\vec{a} + \vec{b}\| + \|\vec{a} - \vec{b}\| \leq 2. \quad (3.6)$$

If we consider a qubit state  $\rho$ , the probability of outcomes  $r, s$  along  $\vec{a}, \vec{b}$  reads

$$P_{r,s}(\vec{a}, \vec{b}) = \text{Tr}[\rho \Pi_{r,s}(\vec{a}, \vec{b})]. \quad (3.7)$$

Then, we can write the correlation of measurement results. In doing so we suppose to have outcomes of the type  $\pm 1$  (rather than  $\pm 1/2$ ), thus obtaining

$$E(\vec{a}, \vec{b}) = \sum_{r,s=\pm 1/2} 4rs P_{r,s}(\vec{a}, \vec{b}). \quad (3.8)$$

Given the measurement correlations (3.8) one can test the nonlocal character of the quantum state through some Bell like inequality.

Let us consider the CHSH inequality (CHSH, 1969)

$$|E(\vec{a}, \vec{b}) + E(\vec{a}, \vec{b}') + E(\vec{a}', \vec{b}) - E(\vec{a}', \vec{b}')| \leq 2. \quad (3.9)$$

We restrict our attention to the  $x-z$  plane and consider

$$\vec{a} \propto (0, 0, 1), \quad (3.10)$$

$$\vec{a}' = \vec{b} \propto (\sin \phi, 0, \cos \phi), \quad (3.11)$$

$$\vec{b}' \propto (\sin(2\phi), 0, \cos(2\phi)), \quad (3.12)$$

with  $0 \leq \phi \leq \pi/2$ . Moreover, we take  $\rho \equiv |\psi\rangle\langle\psi|$  with

$$|\psi\rangle = \cos \frac{\theta}{2} |+\rangle + \sin \frac{\theta}{2} |-\rangle, \quad 0 \leq \theta < 2\pi. \quad (3.13)$$

We are now going to distinguish the four possible correlations (3.8). In each case we assume the condition (3.6) satisfied with equality and the two vectors having the same norm.

i)

$$\begin{aligned} \vec{a} &\equiv \frac{1}{\sqrt{1+\sin\phi}} (0, 0, 1) \\ \vec{b} &\equiv \frac{1}{\sqrt{1-\sin\phi}} (\sin\phi, 0, \cos\phi) \end{aligned} \Rightarrow E(\vec{a}, \vec{b}) = \frac{\cos\phi}{1+\sin\phi}. \quad (3.14)$$

ii)

$$\begin{aligned} \vec{a} &\equiv \frac{1}{\sqrt{1+\sin(2\phi)}} (0, 0, 1) \\ \vec{b}' &\equiv \frac{1}{\sqrt{1+\sin(2\phi)}} (\sin(2\phi), 0, \cos(2\phi)) \end{aligned} \Rightarrow E(\vec{a}, \vec{b}') = \frac{\cos(2\phi)}{1+\sin(2\phi)}. \quad (3.15)$$

iii)

$$\begin{aligned} \vec{a} &\equiv (\sin \phi, 0, \cos \phi) \\ \vec{b} &\equiv (\sin \phi, 0, \cos \phi) \end{aligned} \Rightarrow E(\vec{a}, \vec{b}) = 1. \quad (3.16)$$

iv)

$$\begin{aligned} \vec{a} &\equiv \frac{1}{\sqrt{1+\sin \phi}} (\sin \phi, 0, \cos \phi) \\ \vec{b} &\equiv \frac{1}{\sqrt{1-\sin \phi}} (\sin 2\phi, 0, \cos 2\phi) \end{aligned} \Rightarrow E(\vec{a}, \vec{b}) = \frac{\cos \phi}{1 + \sin \phi}. \quad (3.17)$$

Putting together Eqs.(3.14), (3.15), (3.16), (3.17) into Eq.(3.9), it is easy to see that the inequality is always verified (for any pure state of the qubit).

### 3 Conclusions.

Although we have not found violations of Bell inequality, we cannot draw firm conclusions about the rised problem. In fact many other Bell-type inequalities could be considered, and moreover the effect could be sought in systems living in larger Hilbert spaces, even in continuous variable systems (which is an ongoing work). However, we can provide some reflections on the foundational ground. We can conceptually analyze the two possible scenarios:

- (Case A) impossibility to violate any Bell inequality;
- (Case B) possibility to violate some Bell inequality.

These scenarios bring us to the following reflections:

*Case A: Entanglement as basic level.* The Case A would be favorable to the assumption that the basic level of physical world could be the entanglement. This simple position may have important epistemological implications, like the rejection of individual object, and the rejection of individual intrinsic properties. As consequence, it is not possible to give a definition of the individual object in a spatio-temporal location and it is not possible to characterize the properties of the objects, in order to distinguish it from other ones. In other words, if we adopt the entanglement as basic level, we accept the philosophy of the relations and we renounce at the possible existence of intrinsic properties while we accept relational properties. We remember, for instance, that a mathematical model based on the relationist principle accept that the position of an object can only be defined respect

to other matter. We do not venture in the philosophical implications of the relationalism, as the monism which affirm that there are not distinction a priori between physical entities. An important advantage of these approach is the possibility to eliminate the privileged role of the observer. This is Rovelli's approach to QM where the founding postulate is the impossibility to talk about properties of systems in the abstract, but only of properties of systems relative to one system (we can never juxtapose properties relative to different systems). RQM is not the claim that reality is described by the collection of all properties relatives to all systems, rather, reality admits one description per each (observing) system, and any such description is internally consistent. As Einstein's original motivation with EPR was not to question locality, but rather to question the completeness of QM, so the relation interpretation can be interpreted as the discovery of the incompleteness of the description of reality that any single observer can give. In this particular sense, RQM can be said to show the "incompleteness" of single-observer Copenhagen interpretation.

*Case B: Uncertainty principle as basic level.* The Case B would show a sort of self-entanglement and would be favorable to the assumption that the basic level of physical world could be the uncertainty principle. As we know, Heisenberg's relation express ontological restrictions on the experiments that we can perform on quantum systems. The relation introduce a subject-object separation metaphorically called "the Heisenberg cut". For these reasons, there are many interpretations of the uncertainty principle. First, we note that the usual formalism of quantum theory does not incorporate notion such a "simultaneous observations", and thus no statement about them can be deduced from the same formalism. The question if the theoretical structure or the quantitative laws of quantum theory can be indeed derived on the basis of the uncertainty principle, as the same Heisenberg wished, is open. Recently, a proposal to construct QM as a theory of "principle" was provided by Bub; but this proposal does not use the uncertainty principle as one of its fundamental principles. Heisenberg's relation cut acts as a boundary between potentiality and actuality, a definite boundary between a QS and a classical apparatus. According to this position, in the world of potentiality should be possible to have precise value of measurable quantities: we see an evident contradiction with the assumption that physical quantities do not exist before a measurement process. In the perspective of the above relational approach to QM, Dickson (Dickson, 1996) proposes an original interpretation of uncertainty principle based on a refreshing reminder on the foundations of dynamics. According Dickson, the formulation of dynamical laws requires the notion of inertial frames. The tomographic approach seems in line with this idea.

We retain that the basic problem is how uncertainty principle consider the fundamental concept of "individuality" of a quantum event. First, we need to understand the definition of a quantum process, and not only to focus our attention on the unavoidable "disturbance" or "physical influence" of the observer on the observed. However, the new concept of nonlocality would change our vision of physical reality; probably we cannot anymore speak about simple individuality. The concept of individuality should be revisited. For instance, a forced equivalence between information and individuality (underlying a physical reality) is claimed by Zeilinger, putting forward an idea which connects the concept of information with the notion of elementary systems.

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