**Explanation, confirmation, and Hempel's paradox**

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ABSTRACT: Hempel's Converse Consequence Condition (CCC), Entailment Condition (EC), and Special Consequence Condition (SCC) have some prima facie plausibility when taken individually. Hempel, though, shows that they have no plausibility when taken together, for together they entail that *E* confirms *H* for any propositions *E* and *H*. This is "Hempel's paradox". It turns out that Hempel's argument would fail if one or more of CCC, EC, and SCC were modified in terms of explanation. This opens up the possibility that Hempel's paradox can be solved by modifying one or more of CCC, EC, and SCC in terms of explanation. I explore this possibility by modifying CCC and SCC in terms of explanation and considering whether CCC and SCC so modified are correct. I also relate that possibility to Inference to the Best Explanation.

KEYWORDS: Bayesian causal networks; confirmation; Converse Consequence Condition; explanation; Hempel; Hempel's paradox; Inference to the Best Explanation; screening-off; Special Consequence Condition

## 1 Introduction

Each of the following conditions has some prima facie plausibility:

> Converse Consequence Condition (CCC): For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H** and (ii) *H** is entailed by *H*, then *E* confirms *H*.

> Entailment Condition (EC): For any propositions *E* and *H*, if *E* entails *H*, then *E* confirms *H*.

> Special Consequence Condition (SCC): For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H** and (ii) *H** entails *H*, then *E* confirms *H*.

The same is true of the condition:

Non-Triviality Condition (NTC): For some propositions *E* and *H*, *E* does not confirm *H*.

Hempel (1965a), though, shows (in effect) that CCC, EC, and SCC together entail that NTC is false:

Hempel's Argument

(1) *E* entails *E* for any proposition *E*.

Thus

(2) *E* confirms *E* for any proposition *E*. [by (1) and EC]

(3) *E* is entailed by *E* & *H* for any propositions *E* and *H*.

Thus

(4) *E* confirms *E* & *H* for any propositions *E* and *H*. [by (2), (3), and CCC]

(5) *E* & *H* entails *H* for any propositions *E* and *H*.

Thus

(6) *E* confirms *H* for any propositions *E* and *H*. [by (4), (5), and SCC]

So, given that there is no questioning NTC (on any legitimate sense of confirmation), one or more of CCC, EC, and SCC should be rejected.[1] This is "Hempel's paradox".[2]

An intriguing possibility is that one or more of CCC, EC, and SCC should be modified in terms of explanation. Suppose, for example, CCC is modified so that "entailed" is replaced by "entailed and explained". Then (3) in Hempel's Argument would need to be modified accordingly. (3), though, would be false if it were so modified. For, as is uncontroversial, it is false that *E* is explained by *E* & *H* for any propositions *E* and *H*. Perhaps, then, Hempel's paradox can be solved by modifying one or more of CCC, EC, and SCC in terms of explanation.

It would not be enough—to solve Hempel's paradox—to simply modify one or more of CCC, EC, and SCC in terms of explanation so that Hempel's Argument fails. The new condition or conditions would need to be correct.

Does Hempel's paradox admit of a satisfactory solution? Are conditions such as CCC, EC, and SCC correct when modified in terms of explanation? These are the main questions of the paper.

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[1] The names "Converse Consequence Condition", "Entailment Condition", and "Special Consequence Condition" are Hempel's (1965a, pp. 31-32). Hempel formulates CCC, EC, and SCC in terms of sentences (as opposed to propositions) where *E* is an observation report. This difference in formulation is inconsequential for my purposes. The name "Non-Triviality Condition" is mine. Hempel (1965a, p. 32) discusses NTC but does not give it a name.

[2] This paradox is of course distinct from the ravens paradox (which sometimes goes by the name "Hempel's paradox"). See Hempel (1965a).

My interest in the second question is due in part to my interest in the first question. If the answer to the second question is affirmative, then perhaps so too is the answer to the first question.

There is more. Inference to the Best Explanation (IBE) is standardly construed in terms of categorical beliefs. The core idea can be put as follows:

> Idea 1 ($I_1$): Inferences to categorical beliefs (at least some of them) should be guided by explanatory considerations.

This idea has a counterpart concerning degrees of belief:

> Idea 2 ($I_2$): Changes in degrees of belief (at least some of them) should be guided by explanatory considerations.

$I_2$ has received much attention in recent years.[3] One issue discussed is whether $I_2$ runs counter to Bayesian conditionalization. Some researchers, for example, flesh out $I_2$ by developing an explanation-based alternative to Bayesian conditionalization. But there are alternative, and less radical, ways in which $I_2$ might be true. Suppose the answer to the second of the two main questions of the paper is affirmative and so conditions such as CCC, EC, and SCC are correct when modified in terms of explanation. Suppose, further, conditions such as CCC, EC, and SCC should be understood in terms of changes in degrees of belief. Then it follows that $I_2$ is true.

The remainder of the paper is organized as follows. In Section 2, I take a step towards solving Hempel's paradox by, in part, disambiguating CCC, EC, and SCC and showing that EC is true whereas CCC and SCC are false. In Section 3, I modify CCC and SCC in terms of explanation. The new conditions are $CCC_E$ and $SCC_E$. I also note that $CCC_E$ and $SCC_E$ are correct only if explanatory relations place constraints on probability distributions. In Section 4, I consider three obvious candidate constraints (placed by explanatory relations on probability distributions). I argue that $CCC_E$ and $SCC_E$ are incorrect when understood in terms of those candidate constraints. In Section 5, I turn to Schupbach's (this volume) articulation and defense of IBE in terms of power (explanatory power). I use it to construct a fourth candidate constraint. I argue that it too is no help for $CCC_E$ and $SCC_E$. In Section 6, I set out the basics of Bayesian causal networks and construct a fifth candidate constraint. This constraint improves on the four

[3] I have in mind here the growing literature on Bayesianism and IBE. See, e.g., Douven (1999, 2011a), Douven and Wenmackers (forthcoming), Huemer (2009), Iranzo (2008), Lipton (2001, 2004), McCain and Poston (2014), Okasha (2000), Poston (2014, Ch. 7), Psillos (2004), Roche and Sober (2013, 2014), Salmon (2001a, 2001b), Tregear (2004), van Fraassen (1989), and Weisberg (2009).

constraints considered in Section 4 and Section 5 in a certain key respect. I argue, though, that in the end the fifth candidate constraint too does not help with $CCC_E$ and $SCC_E$. In Section 7, I return to Hempel's paradox and set out my preferred solution. In Section 8, I conclude.

## 2 A step towards solving Hempel's paradox

It is standard in Bayesian confirmation theory to distinguish between *absolute* confirmation and *incremental* confirmation.[4] The distinction is this:

> Absolute Confirmation: For any propositions $E$ and $H$, $E$ absolutely confirms $H$ if and only if $\Pr(H \mid E) > \mathbf{t}$ where $\mathbf{t}$ is the threshold for high probability and $1 > \mathbf{t} \geq 0.5$.
>
> Incremental Confirmation: For any propositions $E$ and $H$, $E$ incrementally confirms $H$ if and only if $\Pr(H \mid E) > \Pr(H)$.

Note that I have in mind probabilities understood as "degrees of belief" (or "rational degrees of belief").

Suppose the sense of confirmation at issue is absolute confirmation. Then CCC should be rejected while EC and SCC should be accepted. First, suppose a card is randomly drawn from a standard (and well-shuffled) deck of cards. Let $E$ be the proposition that the card drawn is a Heart, $H^*$ be the proposition that the card drawn is a Red, and $H$ be the proposition that the card drawn is a Diamond. $E$ confirms $H^*$, since $\Pr(H^* \mid E) = 1 > \mathbf{t}$. $H^*$ is entailed by $H$. But $E$ does not confirm $H$, since $\Pr(H \mid E) = 0 < 0.5 \leq \mathbf{t}$. So CCC is false. Second, for any propositions $E$ and $H$, if $E$ entails $H$, then $\Pr(H \mid E) = 1 > \mathbf{t}$. So EC is true. Third, for any propositions $E$, $H^*$, and $H$, if $H^*$ entails $H$, then $\Pr(H \mid E) \geq \Pr(H^* \mid E)$. It follows that for any propositions $E$, $H^*$, and $H$, if $\Pr(H^* \mid E) > \mathbf{t}$ and $H^*$ entails $H$, then $\Pr(H \mid E) > \mathbf{t}$. So SCC is true.[5]

Suppose, instead, the sense of confirmation at issue is incremental confirmation. Then each of CCC, EC, and SCC should be rejected. Return to the card case above. First, note

[4] See Carnap (1962, Preface to 2nd ed.) on "concepts of firmness" and "concepts of increase in firmness".

[5] Hempel (1965a) holds that CCC should be rejected while EC and SCC should be accepted. It does not follow, however, that he has in mind absolute confirmation. There is reason to believe, in fact, that he does *not* have in mind absolute confirmation. His "satisfaction criterion of confirmation" (1965a, sec. 9), which is motivated in part by appeal to EC and SCC, is obviously inadequate when absolute confirmation is at issue. Let $E$ be the proposition that Tweety is a raven and Tweety is black. Let $H$ be the proposition that all ravens are black. Suppose $\Pr(H)$ is low. Hempel's satisfaction criterion of confirmation implies that $E$ confirms $H$. Clearly, though, $\Pr(H \mid E)$ is low (not high) and thus is less than $\mathbf{t}$. See Carnap (1962, sec. 87) and Huber (2008) for discussion of how to understand Hempel on confirmation.

that $E$ confirms $H^*$, since $\Pr(H^* \mid E) = 1 > 1/2 = \Pr(H^*)$, and $H^*$ is entailed by $H$, but $E$ does not confirm $H$, since $\Pr(H \mid E) = 0 < 1/4 = \Pr(H)$. So CCC is false. Second, let $E$ be some contingent proposition and $H$ be a logically true proposition. Then $E$ entails $H$ but $\Pr(H \mid E) = 1 = \Pr(H)$. So EC is false. Third, let $E$ be the proposition that the card drawn is not a Heart, $H^*$ be the proposition that the card drawn is a Diamond, and $H$ be the proposition that the card drawn is a Red. Then $E$ confirms $H^*$, since $\Pr(H^* \mid E) = 1/3 > 1/4 = \Pr(H^*)$, and $H^*$ entails $H$, but $E$ does not confirm $H$, since $\Pr(H \mid E) = 1/3 < 1/2 = \Pr(H)$. So SCC is false.

The situation is a bit different if the sense of confirmation at issue is incremental confirmation and CCC, EC, and SCC are understood as restricted to propositions with non-extreme unconditional probabilities (i.e., unconditional probabilities greater than 0 and less than 1). CCC and SCC are still false, but EC is true. This is because for any propositions $E$ and $H$ with non-extreme unconditional probabilities, if $E$ entails $H$, then $\Pr(H \mid E) = 1 > \Pr(H)$. Hence CCC and SCC should be rejected while EC should be accepted.

There are senses of confirmation (or evidential support) in addition to absolute confirmation and incremental confirmation.[6] But none of them is such that EC should be rejected while CCC and SCC should be accepted.[7] And none of them is such that SCC should be rejected while CCC and EC should be accepted.[8,9]

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[6] See Douven (2011b), Roche (2012b, 2015), and Roche and Shogenji (2014a) for discussion.

[7] Suppose CCC and SCC are true. Take some propositions $E$ and $H^*$ such that $E$ confirms $H^*$. Then:

(1) $E$ confirms $H^*$.
(2) $H^*$ is entailed by $H^*$ & $H$ for any propositions $H^*$ and $H$.
Thus
(3) $E$ confirms $H^*$ & $H$ for any propositions $E$, $H^*$, and $H$ such that $E$ confirms $H^*$. [by (1), (2), and CCC]
(4) $H^*$ & $H$ entails $H$ for any propositions $H^*$ and $H$.
Thus
(5) $E$ confirms $H$ for any propositions $E$, $H^*$, and $H$ such that $E$ confirms $H^*$. [by (3), (4), and SCC]

Thus one or both of CCC and SCC should be rejected. This argument is noted in Skyrms (1966, p. 238).

[8] Suppose CCC and EC are true. Then:

(1) $E$ entails $E \vee H$ for any propositions $E$ and $H$.
Thus
(2) $E$ confirms $E \vee H$ for any propositions $E$ and $H$. [by (1) and EC]
(3) $E \vee H$ is entailed by $H$ for any propositions $E$ and $H$.
Thus
(4) $E$ confirms $H$ for any propositions $E$ and $H$. [by (2), (3), and CCC]

Thus one or both of CCC and EC should be rejected. This argument would fail if EC were understood as restricted to propositions with non-extreme unconditional probabilities. Suppose $H = {\sim}E$. Then $E$ entails $E$

I leave it for future investigation whether there are (interesting) senses of confirmation on which CCC and EC should be rejected while SCC should be accepted, or on which EC and SCC should be rejected while CCC should be accepted. I want to focus on confirmation in the sense of incremental confirmation. This is in part because of my interest in whether $I_2$ is true. So hereafter, unless otherwise noted, all talk of confirmation should be understood in terms of incremental confirmation.

No condition concerning incremental confirmation has any plausibility unless it is understood as restricted to propositions with non-extreme unconditional probabilities. So hereafter, unless otherwise noted, all talk of confirmation should be understood as restricted to propositions with non-extreme unconditional probabilities.

Given the focus on confirmation in the sense of incremental confirmation, and given the restriction to propositions with non-extreme unconditional probabilities, it follows that EC is true whereas CCC and SCC are false. Thus CCC and SCC should be rejected while EC should be accepted.

This is a step towards solving Hempel's paradox. But it is not enough. Each of CCC and SCC, it seems, contains a kernel of truth in that many cases where (i) $E$ confirms $H^*$ and (ii) $H^*$ is entailed by or entails $H$ are cases where, it seems, $E$ confirms $H$. Some such cases are cases where $E$ is a piece of observational evidence and $H^*$ and $H$ are scientific hypotheses one of which is more general than the other. Other such cases are more ordinary. Suppose, for example, a card is randomly drawn from a standard and well-shuffled deck of cards. Let $E$ be the proposition that Smith testified that the card drawn is

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$\vee H$, but, since $E \vee H$ has an extreme unconditional probability (of 1), it does not follow by EC that $E$ confirms $E \vee H$. The argument, though, can be readily modified to show that CCC and EC together entail an absurdity along the lines of the negation of NTC. See Moretti (2003) and Skyrms (1966, p. 238). Le Morvan (1999) gives an alternative argument for the thesis that CCC and EC together entail that NTC is false. Le Morvan's argument, like the argument above, would fail if EC were understood as restricted to propositions with non-extreme unconditional probabilities. But, it seems, Le Morvan's argument, unlike the argument above, cannot be readily modified to show that CCC and EC together entail an absurdity along the lines of the negation of NTC. See Moretti (2003) for discussion.

[9] The same is true with respect to SCC and the condition:

> Converse Entailment Condition (CEC): For any propositions $E$ and $H$, if $H$ entails $E$, then $E$ confirms $H$.

Suppose CEC and SCC are true. Take some propositions $E$ and $H$. Then:

(1) $E$ & $H$ entails $E$ for any propositions $E$ and $H$.
Thus
(2) $E$ confirms $E$ & $H$ for any propositions $E$ and $H$. [by (1) and CEC]
(3) $E$ & $H$ entails $H$ for any propositions $E$ and $H$.
Thus
(4) $E$ confirms $H$ for any propositions $E$ and $H$. [by (2), (3), and SCC]

Thus one or both of CEC and SCC should be rejected. This argument is noted in Tuomela (1976).

a Heart, *H** be the proposition that the card drawn is a Heart, and *H* be the proposition that the card drawn is the Jack of Hearts. Then, given certain rather natural ways of filling in the details, *E* confirms *H**, *H** is entailed by *H*, and, as per CCC, *E* confirms *H*. Now let *E* be the proposition that Smith testified that the card drawn is the Jack of Hearts, *H** be the proposition that the card drawn is the Jack of Hearts, and *H* be the proposition that the card drawn is a Jack, and the case is such that *E* confirms *H**, *H** entails *H*, and, as per SCC, *E* confirms *H*. So I want to find some adequate replacement conditions for CCC and SCC (conditions similar to them in content but not open to counterexample).

I turn now to the possibility that CCC and SCC should be modified in terms of explanation.

## 3 $\mathbf{CCC_E}$ and $\mathbf{SCC_E}$

Consider:

> $CCC_E$: For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H**, (ii) *H** is entailed by *H*, and (iii) *H** is explained by *H*, then *E* confirms *H*.

> $SCC_E$: For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H**, (ii) *H** entails *H*, and (iii) *H** explains *H*, then *E* confirms *H*.

$CCC_E$ is CCC but with the added condition that *H** is explained by *H*. $SCC_E$, in turn, is SCC but with the added condition that *H** explains *H*.[10]

The expressions "explained by" and "explains" in $CCC_E$ and $SCC_E$ should be understood as short for "potentially explained by" and "potentially explains". The latter expressions, in turn, should be understood so that *H** can be potentially explained by or potentially explain *H* even if neither *H** nor *H* is true.[11]

If all explanation is deductive, then $CCC_E$ and $SCC_E$ are equivalent to the following:

> $CCC_{E*}$: For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H** and (ii) *H** is explained by *H*, then *E* confirms *H*.

[10] Brody (1968, 1974) considers a condition similar to $CCC_E$ but (a) without the constraint that *H** is entailed by *H* and (b) restricted to confirmation by positive instances. See Gower (1973), Koslow (1970), Martin (1972, 1975), and Tuomela (1976) for relevant discussion.

[11] See Lipton (2004, Ch. 4) for helpful discussion of the distinction between actual and potential explanations.

$SCC_{E*}$: For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H** and (ii) *H** explains *H*, then *E* confirms *H*.

It is a matter of controversy, though, whether all explanation is deductive.[12] So I shall assume just that some explanation is deductive and focus on $CCC_E$ and $SCC_E$.[13]

There is a clear respect in which $CCC_E$ and $SCC_E$ improve on CCC and SCC. If Hempel's Argument were modified in terms of $CCC_E$ and $SCC_E$, then it would have a false premise. It would read:

Hempel's Argument*
(1) *E* entails *E* for any proposition *E*.
Thus
(2) *E* confirms *E* for any proposition *E*. [by (1) and EC]
(3) *E* is entailed and explained by *E* & *H* for any propositions *E* and *H*.
Thus
(4) *E* confirms *E* & *H* for any propositions *E* and *H*. [by (2), (3), and $CCC_E$]
(5) *E* & *H* entails and explains *H* for any propositions *E* and *H*.
Thus
(6) *E* confirms *H* for any propositions *E* and *H*. [by (4), (5), and $SCC_E$]

Each of (3) and (5) is false.

Is it the case, though, that $CCC_E$ and $SCC_E$ are correct? The answer is negative if explanatory relations place no constraints on probability distributions. This can be seen as follows. Take some *H** and *H* such that *H** is explained by *H* or vice versa. Suppose explanatory relations place no constraints on probability distributions. Then the fact that *H** is explained by *H* or vice versa does not rule out any probability distributions and thus does not rule out any probability distributions on which CCC or SCC fails. It follows that neither $CCC_E$ and $SCC_E$ is correct.

The key question, then, is this: What constraints, if any, do explanatory relations place on probability distributions?

[12] For helpful discussion, and for additional references, see Salmon (1998, Ch. 9).
[13] The assumption is not that there are cases of explanation where the explanans explains the explanandum *by virtue of* entailing the explanandum. The assumption is simply that there are cases of explanation where *in fact* the explanans entails the explanandum.

## 4 Entailment, high probability, and increase in probability

There are some obvious candidate constraints (placed by explanatory relations on probability distributions). Consider (where, as above, $\mathbf{t}$ is the threshold for high probability and $1 > \mathbf{t} \geq 0.5$):

(c1) For any propositions $E$ and $H$, $H$ explains $E$ only if $H$ entails $E$ and thus $\Pr(E \mid H) = 1$.
(c2) For any propositions $E$ and $H$, $H$ explains $E$ only if $\Pr(E \mid H) > \mathbf{t}$.
(c3) For any propositions $E$ and $H$, $H$ explains $E$ only if $\Pr(E \mid H) > \Pr(E)$.

Each of (c1), (c2), and (c3) has some intuitive appeal. And each of them can be found in the literature.[14]

Are $\text{CCC}_\text{E}$ or $\text{SCC}_\text{E}$ correct when understood in terms of (c1), (c2) and (c3)? Suppose the only (non-trivial) constraints placed by explanatory relations on probability distributions are (c1), (c2), and (c3) (and their implications). Then whether $\text{CCC}_\text{E}$ and $\text{SCC}_\text{E}$ are correct hinges on whether the following are correct:

$\text{CCC}_{\text{Ec1-c3}}$: For any propositions $E$, $H^*$, and $H$, if (i) $E$ confirms $H^*$, (ii) $H^*$ is entailed by $H$, and (iii) $\Pr(H^* \mid H) = 1$, $\Pr(H^* \mid H) > \mathbf{t}$, and $\Pr(H^* \mid H) > \Pr(H^*)$, then $E$ confirms $H$.

$\text{SCC}_{\text{Ec1-c3}}$: For any propositions $E$, $H^*$, and $H$, if (i) $E$ confirms $H^*$, (ii) $H^*$ entails $H$, and (iii) $\Pr(H \mid H^*) = 1$, $\Pr(H \mid H^*) > \mathbf{t}$, and $\Pr(H \mid H^*) > \Pr(H)$, then $E$ confirms $H$.

But $\text{CCC}_{\text{Ec1-c3}}$ is equivalent to CCC, and $\text{SCC}_{\text{Ec1-c3}}$ is equivalent to SCC. This follows from the fact that in each case condition (iii) holds if condition (ii) holds. So, since CCC and SCC are incorrect, it follows that so too are $\text{CCC}_{\text{Ec1-c3}}$ and $\text{SCC}_{\text{Ec1-c3}}$.

The same is true, of course, if some but not all of (c1), (c2), and (c3) are set aside. And for the same reason: the entailment condition holds only if the posterior probability in question equals 1, is greater than $\mathbf{t}$, and is greater than the prior probability in question.

I turn now to a fourth candidate constraint.

[14] (c1) and (c2) can be found in Hempel (1965b). (c3) can be found in Salmon (1965). For discussion of the rather extensive extant literature on explanation, and for further references, see Salmon (2006) and Woodward (2014).

## 5 Explanatory power

Schupbach (this volume) sets out and defends a version of IBE on which explanatoriness is fleshed out in terms of power (explanatory power), where the degree to which $H$ has power over $E$ is measured by:

$$e(E,H)=\frac{\Pr(H\mid E)-\Pr(H\mid\sim E)}{\Pr(H\mid E)+\Pr(H\mid\sim E)}$$

$e(E, H)$'s range is from -1 to 1 (inclusive). $H$'s power over $E$ is positive if $e(E, H) > 0$. $H$ has no power over $E$ if $e(E, H) = 0$. $H$'s power over $E$ is negative if $e(E, H) < 0$.

It is not important for my purposes whether Schupbach's defense of his version of IBE, which includes his defense of $e$, succeeds. But a certain part of that defense is important for my purposes.

Schupbach notes that positive power places constraints (some of which are rather significant) on probability distributions. If $e(E, H) > 0$, then:

(a) $$\frac{\Pr(H\mid E)-\Pr(H\mid\sim E)}{\Pr(H\mid E)+\Pr(H\mid\sim E)}>0$$

(b) $$\Pr(H\mid E)-\Pr(H\mid\sim E)>0$$

(c) $$\frac{\Pr(E\mid H)}{\Pr(E)}>\frac{\Pr(\sim E\mid H)}{\Pr(\sim E)}$$

(d) $$\Pr(E\mid H)-\Pr(E\mid H)\Pr(E)>\Pr(E)-\Pr(E\mid H)\Pr(E)$$

(e) $$\Pr(E\mid H)>\Pr(E)$$

(f) $$\Pr(E\mid H)>\Pr(E\mid\sim H)$$

(g) $$\Pr(H\mid E)>\Pr(H)$$

It is worth noting that (a)-(g) follow from positive power as measured by all of the main measures of power in the literature.[15]

Now consider:

(c4) For any propositions $E$ and $H$, $H$ explains $E$ only if $e(E, H) > 0$.

[15] Here I have in mind the measures noted in Crupi and Tentori (2012) and Schupbach (2011).

Suppose the only (non-trivial) constraint placed by explanatory relations on probability distributions is (c4) (and its implications). Then whether $CCC_E$ and $SCC_E$ are correct hinges on whether the following are correct:

> $CCC_{Ec4}$: For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H**, (ii) *H** is entailed by *H*, and (iii) $e(H^*, H) > 0$, then *E* confirms *H*.
>
> $SCC_{Ec4}$: For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H**, (ii) *H** entails *H*, and (iii) $e(H, H^*) > 0$, then *E* confirms *H*.

Are $CCC_{Ec4}$ and $SCC_{Ec4}$ correct?

It is true that positive power places constraints on probability distributions. It is true in particular that if $e(E, H) > 0$, then (a)-(g) above all hold. It is also true, though, that any constraints placed on probability distributions by $e(H^*, H)$'s being positive are also placed on probability distributions by *H**'s being entailed by *H*, and that, similarly, any constraints placed on probability distributions by $e(H, H^*)$'s being positive are also placed on probability distributions by *H**'s entailing *H*. This is because any case where *H** is entailed by *H* is a case where $e(H^*, H)$ is positive (in fact equal to 1), and because any case where *H** entails *H* is a case where $e(H, H^*)$ is positive (in fact equal to 1). It follows that $CCC_{Ec4}$ is equivalent to CCC and that $SCC_{Ec4}$, in turn, is equivalent to SCC. So $CCC_{Ec4}$ and $SCC_{Ec4}$, as with CCC and SCC, are incorrect.[16]

The situation is this. It is not enough, for $CCC_E$ and $SCC_E$ to be correct, that explanatory relations place some constraints on probability distributions. It needs to be the case that they place constraints on probability distributions over and above the constraints already placed on probability distributions by entailment relations. This is why $CCC_{Ec1\text{-}c3}$, $SCC_{Ec1\text{-}c3}$, $CCC_{Ec4}$, and $SCC_{Ec4}$ all fail.

The candidate constraint constructed in the next section improves on (c1)-(c4) in that it provides a way of understanding $CCC_E$ and $SCC_E$ on which they are not equivalent to CCC and SCC. I turn now to that candidate constraint.

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[16] It does not help, of course, to modify CCC and SCC in terms of all four of the candidate constraints on the table at this point. The resulting conditions—$CCC_{Ec1\text{-}c4}$ and $SCC_{Ec1\text{-}c4}$—are equivalent to CCC and SCC and thus are incorrect.

## 6 Causation and screening-off

It is not implausible prima facie that some explanations are non-causal in that the explanans phenomenon is not a cause of the explanandum phenomenon.[17] Clearly, though, many explanations are causal. I want to focus on $CCC_E$ and $SCC_E$ understood so that the kind of explanation at issue is causal. I want to do this because, arguably, causes screen-off in a sense to be explained below and because screening-off places constraints on probability distributions over and above the constraints already placed on probability distributions by entailment. In Section 6.1, I explain the basics of Bayesian causal networks. In Section 6.2, I construct a fifth candidate constraint placed by explanatory relations on probability distributions. This constraint, (c5), involves screening-off. In Section 6.3, I return to CCC and SCC and examine why they fail. In Section 6.4, I evaluate $CCC_E$ and $SCC_E$ when understood in terms of (c5).

### 6.1 Bayesian causal networks

A Bayesian causal network consists of a set of variables $\mathbf{V} = \{V_1, V_2, \ldots, V_n\}$, a directed acyclic graph **G** over **V**, and a probability distribution Pr over **V**. **G** consists of nodes and directed edges (or arrows). Each node is a variable in **V** (and each variable in **V** is a node). Each directed edge connects exactly two nodes. A directed edge from one node to another indicates that the former is a direct cause of the latter. One node is a parent of another (and the latter is a child of the former) if and only if there is a directed edge from the former to the latter. One node is an ancestor of another (and the latter is a descendant of the former) if and only if there is a directed path (or series of directed edges), aligned tip-to-tail linking intermediate nodes, from the former to the latter. **G** is acyclic in that no node is an ancestor of itself.

An example of **V** (taken from Neapolitan 2004) is the set $\{B, C, F, H, L\}$, where:

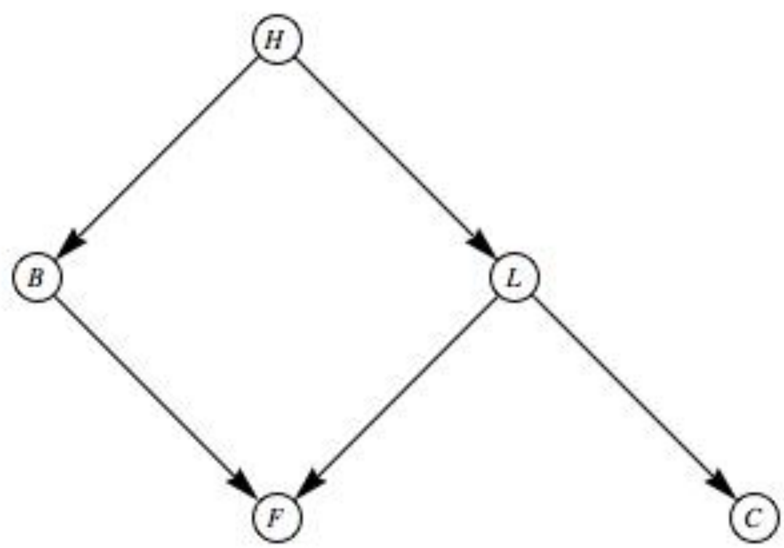
| Variable | Values |
|---|---|
| $B$ | $b_1$ = bronchitis is present<br>$b_2$ = bronchitis is not present |
| $C$ | $c_1$ = chest X-ray is positive<br>$c_2$ = chest X-ray is not positive |
| $F$ | $f_1$ = fatigue is present<br>$f_2$ = fatigue is not present |
| $H$ | $h_1$ = there is a history of smoking |

[17] See Lipton (2009) for discussion.

| | |
|---|---|
| | $h_2$ = there is not a history of smoking |
| $L$ | $l_1$ = lung cancer is present<br>$l_2$ = lung cancer is not present |

Here each variable is binary. But this is not required in general.

An example of **G** (also taken from Neapolitan 2004) is the directed acyclic graph:




Here $H$ is a parent of each of $B$ and $L$, $B$ is a parent of $F$, $L$ is a parent of each of $F$ and $C$, and though $H$ is not a parent of $F$ or of $C$, $H$ is an ancestor of $F$ and of $C$.

It is crucial to note that not just any probability distribution is admissible in a Bayesian causal network. Pr should be such that for any variable $V_i$ in **V** with parents (direct causes) and non-descendants (non-effects) in **G**, $V_i$'s parents screen-off $V_i$'s non-descendants from $V_i$, that is, render $V_i$'s non-descendants probabilistically irrelevant to $V_i$. This assumption is (a version of a condition) called "the causal Markov condition". Take the example above. Each of $L$ and $C$ is a non-descendant of $B$. So, by the causal Markov condition, Pr should be such that each of $L$ and $C$ is screened-off from $B$ by $H$ in that $\Pr(b_i \mid h_j \,\&\, l_k) = \Pr(b_i \mid h_j)$ for any $i, j, k$ and $\Pr(b_i \mid h_j \,\&\, c_k) = \Pr(b_i \mid h_j)$ for any $i, j, k$. If, say, it is given that $h_1$ (there is a history of smoking), then Pr should be such that none of $l_1$ (lung cancer is present), $l_2$ (lung cancer is not present), $c_1$ (chest X-ray is positive), and $c_2$ (chest X-ray is not positive) has any impact on the probability of $b_1$ (bronchitis is present) or the probability of $b_2$ (bronchitis is not present).

It might seem strange to talk of variables as causes. Bear in mind, though, that there are ways of understanding Bayesian causal networks on which variables are not causes. Consider, say, the directed edge in the example above from *H* to *B*. This edge need not be understood as indicating that *H* is a direct cause of *B*. It could instead be understood as indicating that a given subject's having (or not having) a history of smoking is a direct cause of her having (or not having) bronchitis. This in no way runs counter to the idea that causes (in singular causation) are events.[18]

Much more can be said about all this.[19] The important point for my purposes, though, is the idea that causal relations and screening-off walk hand in hand. This idea is important in that screening-off places rather significant constraints on probability distributions.

6.2 (c5)

Consider:

(c5) For any propositions *E* and *H*, *H* explains *E* only if there are partitions of propositions Γ, Γ*, and Γ** such that (a) *H* is a member of Γ, (b) *E* is a member of Γ*, (c) Γ** is a set of non-descendants of *E*, and (d) Γ screens-off Γ** from Γ*.

This condition is like the causal Markov condition except that it is framed in terms of explanation (as opposed to causation) and partitions (as opposed to variables).[20]

Return to the example above where **V** = {*B*, *C*, *F*, *H*, *L*}. It is straightforward to reinterpret that example in terms of (c5). Let *P* be the patient at issue. Take *H*, *B*, *L*, and the fact that *H* screens-off *L* from *B*. *H* is the partition {*P* has a history of smoking, *P* does not have a history of smoking}, *B* is the partition {*P* has bronchitis, *P* does not have bronchitis}, and *L* is the partition {*P* has lung cancer, *P* does not have lung cancer}. *H* screens-off *L* from *B* in that given either member of the partition {*P* has a history of smoking, *P* does not have a history of smoking}, neither member of the partition {*P* has lung cancer, *P* does not have lung cancer} has any impact on the probability of either member of the partition {*P* has bronchitis, *P* does not have bronchitis}.

[18] See Ehring (2009) for discussion of the relata of causation.
[19] For helpful introductory discussions of Bayesian causal networks, and for additional references, see Hitchcock (2009, 2012) and Williamson (2009).
[20] A similar condition can be motivated by appeal to Salmon's Statistical-Relevance (S-R) model of explanation (1971). See Woodward (2014) for discussion of that model.

Now suppose the only (non-trivial) constraints placed by explanatory relations on probability distributions are (c5) and its implications. Then whether $CCC_E$ and $SCC_E$ are correct hinges on whether the following are correct:

> $CCC_{Ec5}$: For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H**, (ii) *H** is entailed by *H*, and (iii) there are partitions Γ, Γ*, and Γ** such that (a) *H* is a member of Γ, (b) *H** is a member of Γ*, (c) Γ** is a set of non-descendants of *H**, and (d) Γ screens-off Γ** from Γ*, then *E* confirms *H*.
>
> $SCC_{Ec5}$: For any propositions *E*, *H**, and *H*, if (i) *E* confirms *H**, (ii) *H** entails *H*, and (iii) there are partitions Γ, Γ*, and Γ** such that (a) *H** is a member of Γ, (b) *H* is a member of Γ*, (c) Γ** is a set of non-descendants of *H*, and (d) Γ screens-off Γ** from Γ*, then *E* confirms *H*.

$CCC_{Ec5}$ improves on $CCC_{Ec1\text{-}c3}$ and $CCC_{Ec4}$ in that $CCC_{Ec5}$ is not equivalent to CCC. $SCC_{Ec5}$, in turn, improves on $SCC_{Ec1\text{-}c3}$ and $SCC_{Ec4}$ in that $SCC_{Ec5}$ is not equivalent to SCC. But are $CCC_{Ec5}$ and $SCC_{Ec5}$ correct?

I turn now to the issue of why CCC and SCC fail. Getting clear on that issue will help in evaluating $CCC_{Ec5}$ and $SCC_{Ec5}$.

### 6.3 Why CCC and SCC fail

Take any three propositions *E*, *H**, and *H*. Then (by a proof given in Shogenji forthcoming) it follows that:

$$(1)\qquad \Pr(H \mid E) - \Pr(H) = \begin{pmatrix} \left[\Pr(H \mid H^*) - \Pr(H)\right]\left[\Pr(H^* \mid E) - \Pr(H^*)\right] + \\ \left[\Pr(H \mid \sim H^*) - \Pr(H)\right]\left[\Pr(\sim H^* \mid E) - \Pr(\sim H^*)\right] + \\ \Pr(H^* \mid E)\left[\Pr(H \mid H^* \& E) - \Pr(H \mid H^*)\right] + \\ \Pr(\sim H^* \mid E)\left[\Pr(H \mid \sim H^* \& E) - \Pr(H \mid \sim H^*)\right] \end{pmatrix}$$

Let the first addend on the right side of (1) be "A", the second addend on the right side of (1) be "B", the third addend on the right side of (1) be "C", and the fourth addend on the right side of (1) be "D". Any case where the antecedent of CCC or SCC holds is a case where A and B are positive. It does not follow, though, that any case where the antecedent of CCC or SCC holds is a case where the left side of (1) is positive, for it might be that in some such cases the sum of C and D is negative and greater than or equal to in absolute value the sum of A and B.

Suppose, for example, a card is randomly drawn from a standard and well-shuffled deck of cards. Let $E$ be the proposition that the card drawn is a Club, the Ace of Spades, or the Ace of Hearts, $H^*$ be the proposition that the card drawn is not a Spade, and $H$ be the proposition that the card drawn is a Red. Then the antecedent of CCC holds and thus the sum of A and B in (1) is positive:

$$(2) \qquad \left[\frac{2}{3}-\frac{1}{2}\right]\left[\frac{14}{15}-\frac{3}{4}\right]+\left[0-\frac{1}{2}\right]\left[\frac{1}{15}-\frac{1}{4}\right]=\frac{11}{90}$$

The sum of C and D in (1), though, is negative:

$$(3) \qquad \left[\frac{14}{15}\right]\left[\frac{1}{14}-\frac{2}{3}\right]+\left[\frac{1}{15}\right][0-0]=-\frac{5}{9}$$

The right side of (3) is greater than in absolute value the right side of (2) and thus the left side of (1) is negative. So this case is a counterexample to CCC.

Now let $E$ be the proposition that the card drawn is not a Heart, $H^*$ be the proposition that the card drawn is a Diamond, and $H$ be the proposition that the card drawn is a Red. Then the antecedent of SCC holds and thus the sum of A and B in (1) is positive:

$$(4) \qquad \left[1-\frac{1}{2}\right]\left[\frac{1}{3}-\frac{1}{4}\right]+\left[\frac{1}{3}-\frac{1}{2}\right]\left[\frac{2}{3}-\frac{3}{4}\right]=\frac{1}{18}$$

The sum of C and D in (1), however, is negative:

$$(5) \qquad \left[\frac{1}{3}\right][1-1]+\left[\frac{2}{3}\right]\left[0-\frac{1}{3}\right]=-\frac{2}{9}$$

Since the right side of (5) is greater than in absolute value the right side of (4), it follows that the left side of (1) is negative. So this case is a counterexample to SCC.

It is clear, then, why CCC and SCC fail. Their antecedents leave it open that the sum of C and D in (1) is negative and greater than or equal to in absolute value the sum of A and B.

The crucial question vis-à-vis $\mathrm{CCC_{Ec5}}$ and $\mathrm{SCC_{Ec5}}$ is whether the same is true of their antecedents. If yes, then $\mathrm{CCC_{Ec5}}$ and $\mathrm{SCC_{Ec5}}$, as with CCC and SCC, are open to counterexample. If no, then $\mathrm{CCC_{Ec5}}$ and $\mathrm{SCC_{Ec5}}$, unlike CCC and SCC, hold without exception.

6.4 Why $CCC_{Ec5}$ and $SCC_{Ec5}$ fail

First, consider the following variant of (1):

$$(1^*) \qquad \Pr(H^*|E)-\Pr(H^*)=\begin{pmatrix}[\Pr(H^*|H)-\Pr(H^*)][\Pr(H|E)-\Pr(H)]+\\ [\Pr(H^*|{\sim}H)-\Pr(H^*)][\Pr({\sim}H|E)-\Pr({\sim}H)]+\\ \Pr(H|E)[\Pr(H^*|H\,\&\,E)-\Pr(H^*|H)]+\\ \Pr({\sim}H|E)[\Pr(H^*|{\sim}H\,\&\,E)-\Pr(H^*|{\sim}H)]\end{pmatrix}$$

It follows from (1*) that:

$$(6) \qquad \Pr(H^*|E)-\Pr(H^*)=\begin{pmatrix}[\Pr(H^*|H)-\Pr(H^*|{\sim}H)][\Pr(H|E)-\Pr(H)]+\\ \Pr(H|E)[\Pr(H^*|H\,\&\,E)-\Pr(H^*|H)]+\\ \Pr({\sim}H|E)[\Pr(H^*|{\sim}H\,\&\,E)-\Pr(H^*|{\sim}H)]\end{pmatrix}$$

Let the first addend on the right side of (6) be "A*", the second addend on the right side of (6) be "B*", and the third addend on the right side of (6) be "C*". Suppose the antecedent of $CCC_{Ec5}$ holds and thus the left side of (6) is positive and the first multiplicand in A* is positive. Suppose $\Gamma = \{H, {\sim}H\}$ and $E$ is included in $\Gamma^{**}$ (a set of non-descendants of $H^*$). It follows that:

$$(7) \qquad \Pr(H^*|H\,\&\,E)=\Pr(H^*|H)$$

$$(8) \qquad \Pr(H^*|{\sim}H\,\&\,E)=\Pr(H^*|{\sim}H)$$

By (7) and (8) it follows that the sum of B* and C* in (6) equals zero. Hence, as the left side of (6) is positive and the first multiplicand in A* in (6) is positive, it follows that the second multiplicand in A* in (6) is positive and so *E* confirms *H*. Hence it is not the case that the sum of C and D in (1) is negative and greater than or equal to in absolute value the sum of A and B in (1).

Next, suppose the antecedent of $SCC_{Ec5}$ holds. Suppose $\Gamma = \{H^*, {\sim}H^*\}$ and $E$ is included in $\Gamma^{**}$ (a set of non-descendants of $H$). Then:

$$(9) \qquad \Pr(H|H^*\&E)=\Pr(H|H^*)$$

(10) $\Pr(H \mid \sim H^* \& E) = \Pr(H \mid \sim H^*)$

By (9) and (10) it follows that each of C and D in (1) equals zero and thus the left side of (1) equals the sum of A and B in (1). Given that the first two conditions in $SCC_{Ec5}$ hold and thus the sum of A and B in (1) is positive, it follows that the left side of (1) is positive and thus *E* confirms *H*.

There are conditions, then, under which $CCC_{Ec5}$ and $SCC_{Ec5}$ hold without exception. $CCC_{Ec5}$ holds without exception under the condition that Γ = {*H*, ~*H*} and *E* is included in Γ** (a set of non-descendants of *H**). $SCC_{Ec5}$, in turn, holds without exception under the condition that Γ = {*H**, ~*H**} and *E* is included in Γ** (a set of non-descendants of *H*).

There is a potential problem however. The antecedents of $CCC_{Ec5}$ and $SCC_{Ec5}$ leave it open that *E* is not included in Γ**. Do $CCC_{Ec5}$ and $SCC_{Ec5}$ hold without exception even when *E* is not included in Γ**?[21]

It turns out that the answer is negative and that because of this the antecedents of $CCC_{Ec5}$ and $SCC_{Ec5}$ leave it open that the sum of C and D in (1) is negative and greater than or equal to in absolute value the sum of A and B in (1) (see Appendix for proof). Hence $CCC_{Ec5}$ and $SCC_{Ec5}$, as with CCC and SCC, are incorrect.

$CCC_{Ec5}$ and $SCC_{Ec5}$ could be modified by adding a condition to their antecedents to the effect that *E* is included in Γ**. This would shield them from counterexample. But then they would be open to a different objection. Their ranges of application would be too restrictive. Neither $CCC_{Ec5}$ nor $SCC_{Ec5}$ would have application in cases where *E* is not included in Γ**.

It does not follow that there is no plausible way of understanding explanation on which an explanatory relation between *H** and *H* suffices to close off the possibility that the sum of C and D in (1) is negative and greater than in absolute value the sum of A and B in (1). I suspect, though, that this is the case.

I want to set aside $CCC_E$ and $SCC_E$ and turn to a different approach.

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[21] The antecedents of $CCC_{Ec4}$ and $SCC_{Ec4}$ leave it open not just that *E* is not included in Γ** but also that the screening-off partition is not binary. So another question is whether $CCC_{Ec4}$ and $SCC_{Ec4}$ hold without exception even when the screening-off partition is not binary. I want to set aside this question and focus on the question of whether $CCC_{Ec4}$ and $SCC_{Ec4}$ hold without exception even when *E* is not included in Γ**. If, as I argue below, the answer to this question is negative, then it does not matter for my purposes whether $CCC_{Ec4}$ and $SCC_{Ec4}$ hold without exception even when the screening-off partition is not binary.

## 7 A solution to Hempel's paradox

It follows from (1) in Section 6.3 that:

> $CCC_{SO}$: For any propositions $E$, $H^*$, and $H$, if (i) $E$ confirms $H^*$, (ii) $H^*$ is entailed by $H$, and (iii) $\{H^*, \sim H^*\}$ screens-off $E$ from $H$ in that $\Pr(H \mid H^* \,\&\, E) \geq \Pr(H \mid H^*)$ and $\Pr(H \mid \sim H^* \,\&\, E) \geq \Pr(H \mid \sim H^*)$, then $E$ confirms $H$.
>
> $SCC_{SO}$: For any propositions $E$, $H^*$, and $H$, if (i) $E$ confirms $H^*$, (ii) $H^*$ entails $H$, and (iii) $\{H^*, \sim H^*\}$ screens-off $E$ from $H$ in that $\Pr(H \mid H^* \,\&\, E) \geq \Pr(H \mid H^*)$ and $\Pr(H \mid \sim H^* \,\&\, E) \geq \Pr(H \mid \sim H^*)$, then $E$ confirms $H$.

Any case where (i) and (ii) in $CCC_{SO}$ or $SCC_{SO}$ hold is a case where A and B in (1) are positive. So, given that any case where (iii) in $CCC_{SO}$ or $SCC_{SO}$ holds is a case where C and D in (1) are non-negative, any case where the antecedent of $CCC_{SO}$ or $SCC_{SO}$ holds is a case where the left side of (1) is positive. Hence $CCC_{SO}$ and $SCC_{SO}$ hold without exception.

Note that with both $CCC_{SO}$ and $SCC_{SO}$ the screening-off partition involves the *middle* proposition $H^*$ in the chain: $E$, $H^*$, $H$. Note also that with both $CCC_{SO}$ and $SCC_{SO}$ the screening-off at issue is *negative*-impact screening-off opposed to *no*-impact screening-off. Thus the use of "≥" instead of "=".[22]

I gave a case in Section 2 where a card is randomly drawn from a standard and well-shuffled deck of cards, $E$ is the proposition that Smith testified that the card drawn is a Heart, $H^*$ is the proposition that the card drawn is a Heart, and $H$ is the proposition that

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[22] $CCC_{SO}$ and $SCC_{SO}$ are special cases of a more general condition:

> For any propositions $E$, $H^*$, and $H$, if (i) $E$ confirms $H^*$, (ii) $H^*$ confirms $H$, and (iii) $\{H^*, \sim H^*\}$ screens-off $E$ from $H$ in that $\Pr(H \mid H^* \,\&\, E) \geq \Pr(H \mid H^*)$ and $\Pr(H \mid \sim H^* \,\&\, E) \geq \Pr(H \mid \sim H^*)$, then $E$ confirms $H$.

This condition is stronger than the condition:

> For any propositions $E$, $H^*$, and $H$, if (i) $E$ confirms $H^*$, (ii) $H^*$ confirms $H$, and (iii) $\{H^*, \sim H^*\}$ screens-off $E$ from $H$ in that $\Pr(H \mid H^* \,\&\, E) = \Pr(H \mid H^*)$ and $\Pr(H \mid \sim H^* \,\&\, E) = \Pr(H \mid \sim H^*)$, then $E$ confirms $H$.

The first of these conditions is established in Roche (2012a). The second is established in Shogenji (2003). See Roche (2014) for discussion of the first condition in the context of peer disagreement. See Roche and Shogenji (2014a) for discussion of the first condition in the context of Moore's proof of the existence of a material world. See Sober (2015, Ch. 5) for discussion of the first condition in the context of the problem of evil. See Roche and Shogenji (2014b) for discussion of the second condition in the context of degree of confirmation.

the card drawn is the Jack of Hearts. I also gave a case where instead $E$ is the proposition that Smith testified that the card drawn is the Jack of Hearts, $H^*$ is the proposition that the card drawn is the Jack of Hearts, and $H$ is the proposition that the card drawn is a Jack. In each case, given certain rather natural ways of filling in the details, $E$ confirms $H^*$, $H^*$ is entailed by or entails $H$, and $\{H^*, {\sim}H^*\}$ screens-off $E$ from $H$ in that $\Pr(H \mid H^* \,\&\, E) \geq \Pr(H \mid H^*)$ and $\Pr(H \mid {\sim}H^* \,\&\, E) \geq \Pr(H \mid {\sim}H^*)$. So, given $\mathrm{CCC}_{\mathrm{SO}}$ and $\mathrm{SCC}_{\mathrm{SO}}$, in each case $E$ confirms $H$.

Is it the case that $\{H^*, {\sim}H^*\}$ screens-off $E$ from $H$ if and only if $\{H, {\sim}H\}$ screens-off $E$ from $H^*$? And, regardless, is it the case that $\mathrm{CCC}_{\mathrm{SO}}$ and $\mathrm{SCC}_{\mathrm{SO}}$ would hold without exception if they were modified so that the screening-off partition involved the last, as opposed to the middle, proposition $H$?

The answer to each question is no. But the following conditions hold without exception:

> $\mathrm{CCC}_{\mathrm{SO}^*}$: For any propositions $E$, $H^*$, and $H$, if (i) $E$ confirms $H^*$, (ii) $H^*$ is entailed by $H$, and (iii) $\{H, {\sim}H\}$ screens-off $E$ from $H^*$ in that $\Pr(H^* \mid H \,\&\, E) \leq \Pr(H^* \mid H)$ and $\Pr(H^* \mid {\sim}H \,\&\, E) \leq \Pr(H^* \mid {\sim}H)$, then $E$ confirms $H$.
>
> $\mathrm{SCC}_{\mathrm{SO}^*}$: For any propositions $E$, $H^*$, and $H$, if (i) $E$ confirms $H^*$, (ii) $H^*$ entails $H$, and (iii) $\{H, {\sim}H\}$ screens-off $E$ from $H^*$ in that $\Pr(H^* \mid H \,\&\, E) \leq \Pr(H^* \mid H)$ and $\Pr(H^* \mid {\sim}H \,\&\, E) \leq \Pr(H^* \mid {\sim}H)$, then $E$ confirms $H$.

Recall (6) from Section 6.4. Any case where the antecedent of $\mathrm{CCC}_{\mathrm{SO}^*}$ or the antecedent of $\mathrm{SCC}_{\mathrm{SO}^*}$ holds is a case where the left side of (6) is positive, the first multiplicand in A* in is positive, and the sum of B* and C* is non-positive. It follows that any case where the antecedent of $\mathrm{CCC}_{\mathrm{SO}^*}$ or the antecedent of $\mathrm{SCC}_{\mathrm{SO}^*}$ holds is a case where the second multiplicand in A* is positive and thus $E$ confirms $H$.[23]

Note that the screening-off at issue in $\mathrm{CCC}_{\mathrm{SO}^*}$ and $\mathrm{SCC}_{\mathrm{SO}^*}$ is *positive*-impact screening-off as opposed to *negative*-impact screening-off. If, instead, the screening-off at issue in $\mathrm{CCC}_{\mathrm{SO}^*}$ and $\mathrm{SCC}_{\mathrm{SO}^*}$ were negative-impact screening-off, then $\mathrm{CCC}_{\mathrm{SO}^*}$ and $\mathrm{SCC}_{\mathrm{SO}^*}$ would be open to counterexample.[24]

$\mathrm{CCC}_{\mathrm{SO}}$, $\mathrm{SCC}_{\mathrm{SO}}$, $\mathrm{CCC}_{\mathrm{SO}^*}$, and $\mathrm{SCC}_{\mathrm{SO}^*}$ together have a rather large range of application. There are cases where $H^*$ is entailed by $H$, but there are also cases where $H^*$ entails $H$. There are cases where the screening-off partition involves the middle proposition $H^*$ and the screening-off is negative-impact screening-off, but there are also

[23] $\mathrm{CCC}_{\mathrm{SO}^*}$ and $\mathrm{SCC}_{\mathrm{SO}^*}$ are due essentially to Tomoji Shogenji (personal correspondence).

[24] Similarly, if the screening-off at issue in $\mathrm{CCC}_{\mathrm{SO}}$ and $\mathrm{SCC}_{\mathrm{SO}}$ were positive-impact screening-off, then $\mathrm{CCC}_{\mathrm{SO}}$ and $\mathrm{SCC}_{\mathrm{SO}}$ would be open to counterexample.

cases where the screening-off partition involves the last proposition *H* and the screening-off is positive-impact screening-off.

The situation is this. CCC and SCC are false. But each, it seems, contains a kernel of truth. This is borne out by $\text{CCC}_{\text{SO}}$, $\text{SCC}_{\text{SO}}$, $\text{CCC}_{\text{SO}*}$, and $\text{SCC}_{\text{SO}*}$.

My solution to Hempel's paradox is now complete. EC should be accepted while CCC and SCC should be rejected in favor of $\text{CCC}_{\text{SO}}$, $\text{SCC}_{\text{SO}}$, $\text{CCC}_{\text{SO}*}$, and $\text{SCC}_{\text{SO}*}$.

My solution is similar to the proposed explanation-based solution considered in Section 6 in that it involves the notion of screening-off. But with my solution on hand there is simply no need to appeal to explanation.

## 8 Conclusion

I noted in Section 1 that the core idea behind IBE can be put as follows:

> $\text{I}_1$: Inferences to categorical beliefs (at least some of them) should be guided by explanatory considerations.

I also noted that this idea, which concerns categorical beliefs, has a counterpart concerning degrees of belief:

> $\text{I}_2$: Changes in degrees of belief (at least some of them) should be guided by explanatory considerations.

One way to flesh out $\text{I}_2$ is to modify conditions such as CCC, EC, and SCC in terms of explanation. There is no need, though, to modify EC in terms of explanation, for EC holds without exception (given the assumption that *E* and *H* have non-extreme unconditional probabilities). And, it seems, neither CCC nor SCC is correct when understood in terms of explanation.

It does not follow, of course, that $\text{I}_2$ is incorrect. There are other ways to flesh out $\text{I}_2$. McCain and Poston (2014, this volume), for instance, flesh out $\text{I}_2$ in terms of the notion of resiliency.[25]

The overall picture, however, is clearer. If changes in degrees of belief should be guided by explanatory considerations, then, it seems, this is not because conditions such as CCC, EC, and SCC should be modified in terms of explanation.

[25] See Roche and Sober (2014) for discussion.

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## Appendix

A Counterexample to $CCC_{Ec4}$

Consider the following probability distribution:

| $E$ | $H^*$ | $H$ | $P$ | Pr | | $E$ | $H^*$ | $H$ | $P$ | Pr |
|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | $\frac{1}{29}$ | | F | T | T | T | $\frac{2}{39}$ |
| T | T | T | F | $\frac{407473}{2841072}$ | | F | T | T | F | $\frac{8247278569811}{16455562695936}$ |
| T | T | F | T | $\frac{1}{156}$ | | F | T | F | T | $\frac{1}{531}$ |
| T | T | F | F | $\frac{1}{314}$ | | F | T | F | F | $\frac{1}{128}$ |
| T | F | T | T | 0 | | F | F | T | T | 0 |
| T | F | T | F | 0 | | F | F | T | F | 0 |
| T | F | F | T | $\frac{2}{37}$ | | F | F | F | T | $\frac{3}{56}$ |
| T | F | F | F | $\frac{5}{592}$ | | F | F | F | F | $\frac{320066869}{2383827712}$ |

It can be readily verified that:

(a) $\Pr(H^* \mid E) = 0.75 > 0.749662 \approx \Pr(H^*)$
(b) $\Pr(H^* \mid H) = 1$
(c) $\Pr(H^* \mid H \,\&\, P) = \Pr(H^* \mid H) = 1$
(d) $\Pr(H^* \mid H \,\&\, {\sim}P) = \Pr(H^* \mid H) = 1$
(e) $\Pr(H^* \mid {\sim}H \,\&\, P) = \Pr(H^* \mid {\sim}H) \approx 0.072$
(f) $\Pr(H^* \mid {\sim}H \,\&\, {\sim}P) = \Pr(H^* \mid {\sim}H) \approx 0.072$
(g) $[\Pr(H \mid H^*) - \Pr(H)][\Pr(H^* \mid E) - \Pr(H^*)] \approx 0.0000823$
(h) $[\Pr(H \mid {\sim}H^*) - \Pr(H)][\Pr({\sim}H^* \mid E) - \Pr({\sim}H^*)] \approx 0.000246$
(i) $\Pr(H^* \mid E)\ [\Pr(H \mid H^* \,\&\, E) - \Pr(H \mid H^*)] \approx -0.0191$
(j) $\Pr({\sim}H^* \mid E)\ [\Pr(H \mid {\sim}H^* \,\&\, E) - \Pr(H \mid {\sim}H^*)] = 0$

Given (a), $E$ confirms $H^*$. Suppose, consistent with (b), $H$ entails $H^*$. Let $\Gamma = \{H, \sim H\}$, $\Gamma^* = \{H^*, \sim H^*\}$, and $\Gamma^{**} = \{P, \sim P\}$, where $P$ and $\sim P$ are non-descendants of $H^*$, and where $E$ is a descendant of $H^*$ and thus is not a member of $\Gamma^{**}$. Then, given (c)-(f), there are partitions $\Gamma$, $\Gamma^*$, and $\Gamma^{**}$ such that $H$ is a member of $\Gamma$, $H^*$ is a member of $\Gamma^*$, $\Gamma^{**}$ is a set of non-descendants of $H^*$, and $\Gamma$ screens-off $\Gamma^{**}$ from $\Gamma^*$. Thus the antecedent of $\mathrm{CCC}_{\mathrm{Ec4}}$ holds. But, given (g)-(j), it is not the case that $E$ confirms $H$. Thus the consequent of $\mathrm{CCC}_{\mathrm{Ec4}}$ does not hold. Thus $\mathrm{CCC}_{\mathrm{Ec4}}$ is false. QED

B Counterexample to $\mathrm{SCC}_{\mathrm{Ec4}}$

Consider the following probability distribution:

| $E$ | $H^*$ | $H$ | $P$ | Pr | | $E$ | $H^*$ | $H$ | $P$ | Pr |
|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | $\frac{1}{16}$ | | F | T | T | T | $\frac{5}{43}$ |
| T | T | T | F | $\frac{1}{8}$ | | F | T | T | F | $\frac{1888104014513}{6010675728640}$ |
| T | T | F | T | 0 | | F | T | F | T | 0 |
| T | T | F | F | 0 | | F | T | F | F | 0 |
| T | F | T | T | $\frac{1}{310}$ | | F | F | T | T | $\frac{1}{11}$ |
| T | F | T | F | $\frac{2}{41}$ | | F | F | T | F | $\frac{1}{5}$ |
| T | F | F | T | $\frac{1}{146}$ | | F | F | F | T | $\frac{1}{256}$ |
| T | F | F | F | $\frac{27051}{7422640}$ | | F | F | F | F | $\frac{157449879}{6353779840}$ |

It can be readily verified that:

(k) $\Pr(H^* \mid E) = 0.75 > 0.618 \approx \Pr(H^*)$
(l) $\Pr(H \mid H^*) = 1$
(m) $\Pr(H \mid H^* \,\&\, P) = \Pr(P \mid H^*) = 1$
(n) $\Pr(H \mid H^* \,\&\, \sim P) = \Pr(P \mid H^*) = 1$
(o) $\Pr(H \mid \sim H^* \,\&\, P) = \Pr(H \mid \sim H^*) \approx 0.897$
(p) $\Pr(H \mid \sim H^* \,\&\, \sim P) = \Pr(H \mid \sim H^*) \approx 0.897$
(q) $[\Pr(H \mid H^*) - \Pr(H)][\Pr(H^* \mid E) - \Pr(H^*)] \approx 0.00518$
(r) $[\Pr(H \mid \sim H^*) - \Pr(H)][\Pr(\sim H^* \mid E) - \Pr(\sim H^*)] \approx 0.00837$
(s) $\Pr(H^* \mid E)\, [\Pr(H \mid H^* \,\&\, E) - \Pr(H \mid H^*)] \approx 0$
(t) $\Pr(\sim H^* \mid E)\, [\Pr(H \mid \sim H^* \,\&\, E) - \Pr(H \mid \sim H^*)] = -0.0163$

Given (k), $E$ confirms $H^*$. Suppose, consistent with (l), $H^*$ entails $H$. Let $\Gamma = \{H^*, \sim H^*\}$, $\Gamma^* = \{H, \sim H\}$, and $\Gamma^{**} = \{P, \sim P\}$, where $P$ and $\sim P$ are non-descendants of $H$, and where $E$ is a descendant of $H$ and thus is not a member of $\Gamma^{**}$. Then, given (m)-(p), there are partitions $\Gamma$, $\Gamma^*$, and $\Gamma^{**}$ such that $H^*$ is a member of $\Gamma$, $H$ is a member of $\Gamma^*$, $\Gamma^{**}$ is a set of non-descendants of $H$, and $\Gamma$ screens-off $\Gamma^{**}$ from $\Gamma^*$. Thus the antecedent of $SCC_{Ec4}$ holds. But, given (q)-(t), it is not the case that $E$ confirms $H$. Thus the consequent of $SCC_{Ec4}$ does not hold. Thus $SCC_{Ec4}$ is false. QED

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