An Epistemicist Solution to the Alethic Paradoxes

Hasen Khudairi

Abstract

This paper targets a series of potential issues for the discussion of, and modal resolution to, the alethic paradoxes advanced by Scharp (2013). I aim, then, to provide a novel, epistemicist treatment of the alethic paradoxes. In response to Curry's paradox, the epistemicist solution that I advance enables the retention of both classical logic and the traditional rules for the alethic predicate: truth-elimination and truth-introduction. By availing of epistemic modal logic, the epistemicist approach permits, further, of a descriptively adequate explanation of the indeterminacy that is exhibited by epistemic states concerning liar-paradoxical sentences.

1 Introduction

This paper targets a series of potential issues for the discussion of, and resolution to, the alethic paradoxes advanced by Scharp, in his *Replacing Truth* (2013). I aim, then, to provide a novel, epistemicist treatment of the alethic paradoxes. In response to Curry's paradox, the novel, epistemicist solution that I advance enables the retention of both classical logic and the traditional rules for the alethic predicate: truth-elimination and truth-introduction. By availing of epistemic logic, the epistemicist approach permits, further, of a precise, descriptively adequate explanation of the indeterminacy exhibited by epistemic states concerning paradoxical sentences such as the liar.

In Section **2**, Scharp's replacement strategy is outlined, and his semantic model is described in detail. Sections **3-7** examine five, crucial issues for the approach and the semantic model that Scharp proffers. The five issues target

the following points of contention:

(i) Whether a positive theory of validity might be forthcoming on Scharp's approach, given that Scharp expresses sympathy with treatments on which – in virtue of Curry's paradox – validity is not identical with necessary truth-preservation;

(ii) The failure of compositionality in Scharp's Theory of Ascending and Descending Truth (ADT) and whether the theory is not, then, in tension with natural language semantics. The foregoing might be pernicious, given Scharp's use of consistency with natural language semantics as a condition for the success of approaches to the paradoxes. A related issue concerns whether it is sufficient to redress the failure of compositionality by availing of hybrid conditions which satisfy both Ascending and Descending Truth;

(iii) Whether ADT can generalize, in order to account for other philosophical issues that concern indeterminacy;

(iv) Whether Descending Truth and Ascending Truth can countenance the manner in which truth interacts with objectivity. It is unclear, e.g., how the theorems unique to each of Descending Truth and Ascending Truth – respectively, T-Elimination and T-Introduction – can capture distinctions between the reality of the propositions mapping to 1 in mathematical inquiry, by contrast to propositions – about humor, e.g. – whose mapping to 1 might be satisfied by more deflationary conditions; and

(v) Whether the replacement strategy in general and ADT in particular can be circumvented, in virtue of approaches to the alethic paradoxes which endeavor to resolve them by targeting constraints on the contents of propositions and the values that they signify.

Despite the critical issues that are raised for the approach, I believe that

the theory of ADT can be extended in order to solve several other types of paradox, beyond the paradoxes of self-reference. Scharp disavows of a unified approach to all paradoxical phenomena. However, because the theory of Ascending and Descending Truth appears to be extendible to several paradoxes, it might repay the attempt to examine the theory's possible strengths. In Section **5**, I outline, therefore, extensions of Scharp's ADT theory to the preface paradox; to the property version of Russell's paradox in the setting of unrestricted quantification; to probabilistic self-reference; and to the sorites paradox.

Finally, Section **8** examines the alethic paradoxes in the setting of epistemic logic. My epistemicist theory avoids the series of issues adducing against the ADT approach. I outline a novel, epistemicist solution to Curry's paradox, and provide a precise explanation of the epistemic indeterminacy expressed by agent's attitudes toward the liar sentence.

2   Scharp's Replacement Theory

Scharp avers that two main alethic principles target the use of the predicate as a device of endorsement and as a device of rejection. When the truth predicate is governed by (T-Out), then it can be deployed in the guise of a device of endorsement. When the truth predicate is governed by (T-In), then it can be deployed in the guise of a device of rejection.

Scharp's theory aims to replace truth with two distinct concepts. His explicit maneuver is to delineate the two, smallest inconsistent subsets of alethic principles; and then to pair one of the subsets with one of the replacement concepts, and the other subset with the second replacement concept.

Thus, one replacement concept will be governed by (T-In) and not by (T-Out); and the second replacement concept will be governed by (T-Out) and not

by (T-In).

Scharp refers to one of his two, preferred replacement concepts as *Descending Truth* (henceforth DT). DT is governed by (T-Out).

Scharp refers to the second of his two, preferred replacement concepts as *Ascending Truth* (henceforth AT). AT is governed by (T-In).

In his 'Syntactical Treatments of Modality, with Corollaries on Reflexion Principles and Finite Axiomatizability' (1963), Montague proved that, for any predicate $H(x)$, the following conditions on the predicate are inconsistent.

Montague's (1963) Lemma 3:

(i) All instances of $H(\phi) \to \phi$ are theorems.

(ii) All instances of $H[H(\phi) \to \phi]$ are theorems.

(iii) All instances of $H(\phi)$, where ϕ is a logical axiom, are theorems.

(iv) All instances of $H(\phi \to \psi) \to [H(\phi) \to H(\psi)]$ are theorems.

(v) Q – i.e., Robinson Arithmetic – is a subtheory.

Scharp notes that Montague's conditions target only Predicate-Elimination, and are thus apt for governing DT.

Scharp argues that (v) is necessary, in order for languages that express the theory to refer to their own sentences. Condition (i) is necessary, because it captures (T-Out). Condition (ii) is necessary, because denying iterations of DT entrains a version of the revenge paradox.

Thus, either Condition (iii) or Condition (iv) must be rejected. Condition (iii) states that all tautologies are Descending True. Condition (iv) is an instance of closure. In virtue of considerations pertaining to validity (see Section **3**), Scharp is impelled to reject (iv), s.t. DT cannot satisfy closure (151).

2.1 Properties of DT and AT

Scharp argues that the alethic principles, DT and AT, ought to include the following.

DT ought to satisfy:

[¬-Exc: $D(\neg\phi) \rightarrow \neg D(\phi)$];

[∧-Exc: $D(\phi \wedge \psi) \rightarrow D(\phi) \wedge D(\psi)$; and

[∨-Imb: $D(\phi) \vee D(\psi) \rightarrow D(\phi \vee \psi)$.

However, DT is not governed by:

[¬-Imb: $\neg T(\phi) \rightarrow T(\neg\phi)$; nor by

[∨-Exc: $T(\phi \vee \psi) \rightarrow T(\phi) \vee T(\psi)$.

AT ought to satisfy:

[¬-Imb: $\neg A(\phi) \rightarrow A(\neg\phi)$;

[∧-Exc: $A(\phi \wedge \psi) \rightarrow A(\phi) \wedge A(\psi)$; and

[∨-Imb: $A(\phi) \vee A(\psi) \rightarrow A(\phi \vee \psi)$.

However, AT is not governed by:

[¬-Exc: $T(\neg\phi) \rightarrow \neg T(\phi)$]; nor by

[∧-Imb: $T(\phi) \wedge T(\psi) \rightarrow T(\phi \wedge \psi)$.

Scharp argues, further:

–that classical tautologies are Descending True;

–that the axioms governing the syntax of the theory are Descending True;

–that the axioms of PA are Descending True, in order to induce self-reference via Gödel-numbering; and

–that the axioms of the theories for both AT and DT are themselves Descending True (152).

DT takes classical values, and, in Scharp's theory, there are no restrictions on the language's expressive resources. This is problematic, because 'a' := '¬A(x)'

and 'd' := '¬D(x)' can be countenanced in the language, and thereby yield contradictions:

Because A(x) is governed by (T-In), 'a' entails that A(a), although a states of itself that ¬A(x). Contradiction.

Because D(x) is governed by (T-Out), 'd' entails that replacing 'd' for x in 'd' is not a descending truth, i.e., ¬D(d)]. So – by condition (ii) – 'D[D(x)] → x' entails that it is not a descending truth that replacing 'd' for x in 'd' is not a descending truth [i.e., ¬D(¬d)].

Thus, Scharp concedes that there must be problematic sentences in the language for his theory, s.t. both the sentences and their negations are Ascending True, and s.t. the sentences and their negations are not Descending True (op. cit.).

Scharp endeavors to block the foregoing, by suggesting that DT can be governed by both unrestricted (T-Out), as well as a restricted version of (T-In). Similarly, AT can be governed by both unrestricted (T-In), as well as a restricted version of (T-Out).

To induce the foregoing, Scharp introduces a 'Safety' predicate, S(x). A sentence ϕ is *safe* if and only if ϕ is either (DT and AT) or not AT.

Thus,

S(ϕ) $\wedge$ ϕ → D(ϕ); and

S(ϕ) $\wedge$ A(ϕ) → ϕ.

A sentence ϕ is *unsafe* if and only if ϕ is AT and not DT:

S(ϕ) $\iff$ D(ϕ) $\vee$ ¬A(ϕ).

From which it follows that:

¬S(ϕ) $\iff$ ¬D(ϕ) $\wedge$ A(ϕ), s.t.

D(ϕ) → A(ϕ);

$\neg \exists \phi [D(\phi) \wedge \neg A(\phi)]$

$A(\phi) \rightarrow \neg D(\phi)$; and

$\neg A(\phi) \rightarrow \neg D(\phi)$ (153).

Scharp avers too that AT and DT are duals. Thus,

$D(\phi) \iff \neg A \neg(\phi)$; and

$A(\phi) \iff \neg D \neg(\phi)$ (152).

2.2 Scharp's Theory: ADT/Xeno Semantics

Scharp's Theory is referred to as ADT. The necessary principles comprising ADT are as follows (cf. 154):

- Descending Truth

(D1): $D(\phi) \rightarrow \phi$

(D2): $D(\neg \phi) \rightarrow \neg D(\phi)$

(D3): $D(\phi \wedge \psi) \rightarrow [D(\phi) \wedge D(\psi)]$

(D4): $[D(\phi) \vee D(\psi)] \rightarrow D(\phi \vee \psi)$

(D5): If ϕ is a classical tautology, then $D(\phi)$

(D6): If ϕ is a theorem of PA, then $D(\phi)$

(D7): If ϕ is an axiom of ADT, then $D(\phi)$.

- Ascending Truth

(A1): $\phi \rightarrow A(\phi)$

(A2): $\neg A(\phi) \rightarrow A(\neg \phi)$

(A3): $[A(\phi) \vee A(\psi)] \rightarrow A(\phi \vee \psi)$

(A4): $A(\phi \wedge \psi) \rightarrow [A(\phi) \wedge A(\psi)]$

(A5): If ϕ maps to the falsum constant, then $\neg A(\phi)$

(A6): If ϕ negates an axiom of PA, then $\neg A(\phi)$

- Transformation Rules

(M1): $D(\phi) \iff \neg A \neg (\phi)$

(M2): $S(\phi) \iff D(\phi) \vee \neg A(\phi)$

(M3): $S(\phi) \wedge \phi \to D(\phi)$

(M4): $A(\phi) \wedge S(\phi) \to \phi$

(E1): If s and t are terms; s = t; and replacing s with t in a sentence p yields a sentence q; then $D(p) \iff D(q)$

(E2): If s and t are terms; s = t; and replacing s with t in a sentence p yields a sentence q; then $A(p) \iff A(q)$

(E3): If s and t are terms; s = t; and replacing s with t in a sentence p yields a sentence q; then $S(p) \iff S(q)$.

2.3 Semantics for ADT

Scharp advances a combination of relational semantics for a non-normal modal logic, as augmented by a neighborhood semantics. (A modal logic is normal if and only if it includes axiom K and the rule of Necessitation; respectively '$\Box[\phi \to \psi] \to [\Box\phi \to \Box\psi]$' and '$\vdash \phi \to \vdash \Box\phi$'.) He refers to this as *xeno semantics.*

A model, M, of ADT is a tuple, $\langle D, W, R, I \rangle$, where D is a non-empty domain of entities constant across worlds, W denotes the space of worlds, R denotes a relation of accessibility on W, and I is an interpretation-function mapping subsets of D to W. The clauses for defining truth in a world in the model are familiar:

$\langle M,w \rangle \Vdash \phi$ iff $w \in V(\phi)$

$\langle M,w \rangle \Vdash \neg\phi$ iff it is not the case that $\langle M,w \rangle \Vdash \phi$

$\langle M,w \rangle \Vdash \phi \wedge \psi$ iff $\langle M,w \rangle \Vdash \phi$ and $\langle M,w \rangle \Vdash \psi$

$\langle M,w \rangle \Vdash \phi \vee \psi$ iff $\langle M,w \rangle \Vdash \phi$ or $\langle M,w \rangle \Vdash \psi$

$\langle M,w \rangle \Vdash \phi \rightarrow \psi$ iff, if $\langle M,w \rangle \Vdash \phi$, then $\langle M,w \rangle \Vdash \psi$

$\langle M,w \rangle \Vdash \phi \iff \psi$ iff $[\langle M,w \rangle \Vdash \phi$ iff $\langle M,w \rangle \Vdash \psi]$

$\langle M,w \rangle \Vdash \Box\phi$ iff $\forall w'[$If $R(w,w')$, then $\langle M,w' \rangle \Vdash \phi]$

$\langle M,w \rangle \Vdash \Diamond\phi$ iff $\exists w'[R(w,w')$ and $\langle M,w' \rangle \Vdash \phi]$

2.4 Further Montagovian Desiderata

Problem 1: Montague (op. cit.) proved that the predicate version of axiom K is inconsistent with (D1): $D(\phi) \rightarrow \phi$; (D5): If ϕ is a classical tautology, then $D(\phi)$; and (D6): If ϕ is a theorem of PA, then $D(\phi)$.

Scharp includes (D7) (159): If ϕ is an axiom of ADT, then $D(\phi)$. However, while Montague was one of the most precocious philosophers of the twentieth century, I suspect that Scharp does not mean to be attributing to Montague an anticipation of his theory.

Problem 2: Montague (op. cit.) proved that the predicate version of Necessitation is inconsistent with (D1): $D(\phi) \rightarrow \phi$, and (D6): If ϕ is a theorem of PA, then $D(\phi)$.

Problem 3: The third problem is supposed to concern whether the quantifier domain is constant or variable. Scharp opts for constant-domain models without argument, so it is unclear precisely what about the quantifier domain he is targeting as problematic. (Cf. 6.7.7: 172.)

Problem 4: The fourth problem concerns the status of Scharp's predicates, AT and DT, in the relational semantics, given that the modalities in the logic for the semantics are expressed by operators.

2.5 Satisfying the Desiderata

To address Problems 1 and 2, Scharp augments his relational semantics with a neighborhood semantics. $M = \langle D, W, R, I \rangle$ is thus enriched with a neighborhood function, N, which maps sets of subsets of W to each world in W.

Necessity takes then the revised clause:

$\langle M,w \rangle \Vdash \Box\phi$ iff $\exists X \in N(w) \forall w'[\langle M,w' \rangle \Vdash \phi \iff w' \in X]$

Possibility takes the revised clause:

$\langle M,w \rangle \Vdash \Diamond\phi$ iff $\neg[\exists X \in N(w) \forall w'[\langle M,w' \rangle \Vdash \neg\phi \iff w' \in X]]$

K and Necessitation need not hold on the neighborhood structure.

A counterexample to K in the neighborhood structure (160): Let M_k be a neighborhood model, s.t. $W_k = \{a,b,c,d\}$; $N_k(a) = \{\{a,b\}, \{a,c,d\}\}$; $N_k(b) = \{b\}$; $N_k(c) = \{c\}$; $N_k(d) = \{d\}$; $V_k(\phi) = \{a,b\}$; and $V_k(\psi) = \{a,c\}$. Thus, $\langle M_k,w \rangle \Vdash \Box\phi$; $\langle M_k,w \rangle \Vdash \Box[\phi \to \psi]$; but it is not the case that $\langle M_k,w \rangle \Vdash \Box\psi$.

So, K need not hold on the neighborhood structure. So, Montague's proof that the predicate version of K is inconsistent with Montague's conditions on predicates (interpreted, by Scharp, as expressing Descending Truth) is not pernicious.

A counterexample to Necessitation in the neighborhood structure (op. cit.): Let M_n be a neighborhood model, s.t. $W_n = \{a,b\}$; $N_n(a) = \{\{a\}\}$; $N_n(b) = \{\{b\}\}$; and $V_n(\phi) = \{a,b\}$. Thus, ϕ can be true in W, while $\Box\phi$ is false in $N_n(a)$.

So, Necessitation need not hold on the neighborhood structure. So, Montague's proof that the predicate version of Necessitation is inconsistent with the the conditions on Descending Truth is not pernicious.

So, Problems 1 and 2 can be answered.

However, the following rule is valid on neighborhood frames, and it entrains

a fifth problem:

(L) If $\vdash \phi \iff \psi$, then $\vdash \Box\phi \iff \Box\psi$.

Problem 5:

The problem with regard to replacing modal operators with Descending Truth predicates hinges on (L).

Scharp writes, e.g., that Gödel's (1931) Diagonalization Lemma 'guarantees that if our language can express Peano Arithmetic or its own theory of syntax [...] then it will have a sentence d, s.t. ¬D(d) is provably equivalent to d' (161).

Scharp concedes that Gödel's incompleteness theorem would yield that $\vdash_{ADT}$ ¬D(d). He argues for this as follows: Suppose D(d), so if D(d) then d; but – by the Lemma – if d, then ¬D(d); so if D(d), then ¬D(d).

Similarly, take another sentence in ADT, '0 = 0', then $\vdash_{ADT}$ ¬D(d) and $\vdash$ '0 = 0'. However, assuming (L) and replacing the operators therein with predicates entails that $\vdash_{ADT}$ D(d) $\iff$ D['0 = 0']. Given that it is true in ADT that D['0 = 0'], then $\vdash_{ADT}$ D(d).

Scharp argues that Problem 5 can be resolved by availing of the structure built into his xeno semantics.

Let L be a language with Boolean connectives, and the operators $\Box$, $\diamond$, and Σ. $\Box$ is the Descending Truth operator. $\diamond$ is the Ascending Truth operator. Σ is the Safety operator. A *xeno model* M = $\langle$F,R,N,V$\rangle$ where F denotes a xeno frame, R is an accessibility relation on wff in L, N is a function from W to 2^{2^w}, and V is an assignment-function from wff in L to the values [0,1].

Truth in a world is defined inductively as above.

The operators take the following clauses:

Descending Truth:

$\langle$M,w$\rangle \Vdash \Box\phi$ iff $\forall$w'$\in$W[R$_\phi$(w,w') $\rightarrow \exists X \in$N(w')$\forall$v$\in$W[$\langle$M,v$\rangle \Vdash \phi \iff$ v$\in X$]

Ascending Truth:

$P(\phi)$ denotes the neighborhood structure – i.e., the set of subsets of worlds – at which ϕ is true.

$\langle M,w \rangle \Vdash \Diamond\phi$ iff $\neg[\forall w' \in W[R_{\neg\phi}(w,w') \rightarrow P(\neg\phi) \in N(w')]$

Safety:

$\langle M,w \rangle \Vdash \Sigma\phi$ iff $\forall w' \in W[R_\phi(w,w') \rightarrow P(\phi) \in N(w')] \vee \exists w' \in W[R_{\neg\phi}(w,w') \wedge P(\neg\phi) \neg \in N(w')]$

A reflexive and co-reflexive xeno frame is equivalent to a neighborhood frame:

(Reflexivity) $\forall\phi\forall w \in W[R_\phi(w,w)] \wedge$

(Co-reflexivity) $\forall\phi\forall w \in W\forall w' \in W[R_\phi(w,w') \rightarrow w = w']$

In order to invalidate (L), Scharp constructs the following xeno model, M.

W = {a,b}

N(a) = {{a,b}}

N(b) = {{b}}

R_S = {(a,a),(b,b)}

R_T = {(a,a),(b,b),(a,b)}

P(S) = {(a,b)}

P(T) = {(a,b)}

$\forall w \in W \langle M,w \rangle \Vdash S$ iff $\langle M,w \rangle \Vdash T$. So, S $\iff$ T is valid in M.

However, $\forall w'[R_S(a,w')] \rightarrow P(S) \in N(w')$, so $\langle M,w \rangle \Vdash \neg\Box T$.

So, (L) is invalid in M, and Problem 5 can be answered.

A sentential xeno frame is *acceptable* iff

(i) $\forall w \in W \ N(w) \neq \emptyset$

(ii) $\forall w \in W \ \forall X \in N(w) \ X \neq \emptyset$

(iii) $\forall w \in W \ \forall X \in N(w) \ w \in X$

(iv) $\forall\phi \in L\forall w \in W[R_\phi(w,w)]$

(v) if ϕ and ψ are of the same syntactic type, then $R_\phi = R_\psi$

The interest of acceptable xeno frames is that they validate operator equivalents of (D1) and (D2) in ADT. [(D1): $D(\phi) \to \phi$; (D2): $D(\neg\phi) \to \neg D(\phi)$.]

Suppose that $\langle M,w \rangle \Vdash \Box\phi$, so $\forall w' \in W[R_\phi(w,w') \to P(\phi) \in N(w')]$. By (iv), $R_\phi(w,w)$, so $P(\neg\phi) \in N(w)$. By (iii), $\forall X \in N(w)[w \in X]$. So, $w \in P(\neg\phi)$. Thus, $\langle M,w \rangle \Vdash \Box\neg\phi$. So, it is not the case that w is in $P(\phi)$. By (iv), $P(\phi)$ is not in $N(w)$. By (iii), $\neg[\forall w' \in W[R_\phi(w,w') \to P(\phi) \in N(w')$. Thus, $\langle M,w \rangle \Vdash \neg\Box\phi$.

Problem 3 concerned whether the domain ought to be variable or constant. Scharp prescinds from targeting any issues with regard (i) to quantification and ontology, or (ii) to the interaction between DT, AT, and empirical sentences (172). He opts for a constant-domain semantics, and thus defines constant-domain xeno models (166):

A constant-domain xeno frame is a tuple, $F = \langle W,N,R_f,D \rangle$. A constant-domain xeno model adds an interpretation-function I to F, s.t. I maps pairs of F and worlds w to subsets of D, s.t. $M = \langle F,R_M,I \rangle$.

A substitution is a function from a set of variables to elements of D. A substitution v' is *x-variant* of v, if $v(y) = v'(y)$ for all variables y.

Thus,

$\langle M,w \rangle \Vdash_v F[(a_1), \ldots, F(a_m)]$, where

a_i is either an individual constant or variable iff $\langle f(a_1), \ldots, f(a_m) \rangle \in I(F,w)$, s.t. if a_i is a variable x_i, then $f(a_i) = v(x_i)$, and if a_i is an individual constant c_i, then $f(a_i) = I(c_i)$

$\langle M,w \rangle \Vdash_v \neg\phi$ iff it is not the case that $\langle M,w \rangle \Vdash_v \phi$

$\langle M,w \rangle \Vdash_v \phi \wedge \psi$ iff $\langle M,w \rangle \Vdash_v \phi$ and $\langle M,w \rangle \Vdash_v \psi$

$\langle M,w \rangle \Vdash_v \phi \vee \psi$ iff $\langle M,w \rangle \Vdash_v \phi$ or $\langle M,w \rangle \Vdash_v \psi$

$\langle M,w \rangle \Vdash_v \phi \to \psi$ iff, if $\langle M,w \rangle \Vdash_v \phi$ then $\langle M,w \rangle \Vdash_v \psi$

$\langle$M,w$\rangle \Vdash_v \phi \iff \psi$ iff $[\langle$M,w$\rangle \Vdash_v \phi$ iff $\langle$M,w$\rangle \Vdash_v \psi]$

$\langle$M,w$\rangle \Vdash_v \forall$x$[\phi$(x)$]$ iff for all x-variant v' $\langle$M,w$\rangle \Vdash_{v'} \phi$(x)

$\langle$M,w$\rangle \Vdash_v \exists$x$[\phi$(x)$]$ iff for some x-variant v' $\langle$M,w$\rangle \Vdash_{v'} \phi$(x)

$\langle$M,w$\rangle \Vdash \Box\phi$ iff $\forall$w'$\in$W$[R_\phi$(w,w') $\to \exists$X$\in$N(w')$\forall$v$\in$W$[\langle$M,w$\rangle \Vdash [\phi \iff$ v$\in$X$]$

$\langle$M,w$\rangle \Vdash \Diamond\phi$ iff $\exists$w'$\in$W$[R_{\neg\phi}$(w,w') $\land$ P$(\neg\phi)$ is not in N(w')

3 Issue 1: Validity

Scharp mentions Field's (2008) argument against identifying validity with necessary truth-preservation, although does not reconstruct the argument.

In order to argue against identifying validity with necessary truth-preservation, Field draws, inter alia, on Curry's Paradox.

The argument from Curry's Paradox is such that – by (T-In) and (T-Out) – one can derive the following. If ϕ is a false sentence then,

1. $\phi \iff [T(\phi) \to \bot]$

2. $T(\phi) \iff [T(\phi) \to \bot]$

3. $T(\phi) \to [T(\phi) \to \bot]$

4. $[T(\phi) \land T(\phi)] \to \bot$ (by importation)

5. $T(\phi) \to \bot$

6. $[T(\phi) \to \bot] \to T(\phi)$

7. $T(\phi)$

8. $\bot$

So, necessary truth-preservation entails contradiction.

However, the argument need not be valid, if one preserves (T-In) and (T-Out) yet weakens the logic. One can avail of the strong Kleene valuation scheme, such that $|\phi|$ is ungrounded, i.e. maps to 1/2. One can then add a Determinacy

operator, such that it is not determinately true that ϕ and it is not determinately true that not ϕ; so, it is indeterminate whether ϕ.

Field argues, in virtue of the foregoing, that validity ought to be a primitive. In more recent work, Field (forthcoming) argues that validity is primitive if and only if it is 'genuine', such that the notion cannot be identical with either its model-theoretic or proof-theoretic analyses. As an elucidation of the genuine concept, he writes that 'to regard an inference or argument as valid is to accept a constraint on belief [. . . ; s.t.] (in the objective sense of 'shouldn't') we shouldn't fully believe the premises without fully believing the conclusion' (op. cit.). (The primitivist notion is intended to hold, as well, for partial belief.)

Scharp is persuaded by Field's argument, and endorses, in turn, a primitivist notion of validity, as a primitive canon of reasoning without necessary truth-preservation. Scharp takes this to be sufficient for the retention of Condition (iii), in Montague's Lemma (151). Scharp does not provide any further account of the nature of validity in the book. In later sections of the book, he reiterates his sympathy with Field's analysis, and also avails of Kreisel's 'squeezing' argument (section 8.8), to the effect that the primitive notion of validity extensionally coincides with a formal notion of validity (i.e., derivation in a first-order axiomatizable quantified logic with identity). However, one potential issue is that, in a subsequent passage, Scharp writes that: 'an argument whose premises are the members of the set G and whose conclusion is p is valid iff for every point of evaluation e [i.e., index], if all members of G are assigned tM-value [i.e., an AT- or DT-value of] 1 at e, then p is assigned tM-value 1 at e' (240); and this would appear to be a definition of validity as necessary truth-preservation.

The primitivist approach to validity is the primary consideration that Scharp

explicitly avails of, when arguing that closure ought to be rejected (Condition iv, in Montague's Lemma), rather than rejecting logical tautologies as candidates for the axioms of ADT (Condition iii, in Montague's Lemma) (151). So, further remarks about the nature of validity would have been welcome. An objection to prescinding from more substantial remarks about the nature of validity might also be that Scharp exploits claims with regard to its uses. So, e.g., he writes that 'a valid argument will never take one from descending truths to something not ascending true' (177). However, that claim is itself neither a consequence of either Kreisel's squeezing argument, nor the primitivist approach to validity.

4 Issue 2: Hybrid Principles and Compositionality

- $\wedge$-T-Imb.

  $D(\phi) \wedge D(\psi) \rightarrow A(\phi \wedge \psi)$

  v.

  $T(\phi) \wedge T(\psi) \rightarrow T(\phi \wedge \psi)$

- $\vee$-T-Exc.

  $D(\phi \vee \psi) \rightarrow A(\phi) \vee A(\psi)$

  v.

  $T(\phi \vee \psi) \rightarrow T(\phi) \vee T(\psi)$

  (cf. 147, 171)

Feferman's (1984) theory countenances a primitive truth predicate (Feferman-true, in what follows); a primitive falsity predicate; as well as a Determinacy

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operator (op. cit.). This is by salient contrast to Scharp's approach, on which truth is replaced with DT and AT. Scharp argues that Feferman overemphasizes the significance of the compositionality of his Determinacy operator, at the cost of not having either logical truths or the axioms of his own theory satisfy Feferman-truth. By contrast, Scharp believes that he can avail of hybrid principles, such that it is not a requirement of ADT that Descending Truth and Ascending Truth obey compositionality (157).

One objection to this maneuver is that AT and DT are separated, in the hybrid principles, between the antecendent and consequent of the conditional.[1] So, it is unclear whether Scharp's hybrid principles are sufficient to redress the failure of compositionality in ADT; i.e., there being truth-conditions for sentences whose component semantic values are, alternatively, DT and AT.

A further objection is that the foregoing might be in tension with Scharp's repeated mention of natural-language semantics, in order to argue against competing proposals. If natural-language semantics were to vindicate principles of compositionality, then this would provide a challenge to the empirical adequacy of Scharp's ADT theory, and thereby the viability of his replacement concepts for the traditional alethic predicate.

5   Issue 3: ADT and Indeterminacy

This issue concerns whether ADT might generalize, in order to account for other philosophical issues that concern indeterminacy. Whether ADT can be so extended to other issues, such as vagueness and types of indeterminacy, is not a necessary condition on the success of the theory. However, it might be a theoretical virtue of other accounts – e.g., classical, paracomplete, intuitionist,

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[1] Thanks here to xx.

and supervaluational approaches – that they do so generalize; and the extensions of logic and semantics to issues in metaphysics are both familiar and legion.[2]

E.g., McGee (1991) suggests replacing the truth predicate with (i) a vague truth predicate, and (ii) super-truth. The replacement predicates are not intended for deployment in inferences implicated in reasoning, such as conditional proof and arguments by reductio (155). McGee introduces a Definiteness operator, μ, in order to yield the notion of super-truth relative to a set of precisifications. There is thus a truth predicate and a super-truth predicate. Super-truth is governed by (T-In) and (T-Out). Vague truth is governed by neither.

Thus:

–If μ(p), then μ'T(p)'

(If p is definitely true, then 'p is true' is definitely true)

–If μ(¬p), then μ¬'T(p)'

(If p is definitely not true, then 'p is true' is definitely not true)

–If p is vague, then 'T(p)' is vague

(vagueness here is secured by availing of the strong Kleene valuation scheme, such that p is ungrounded, i.e. maps neither to true nor false, and rather to .5)

McGee endeavors to avoid Revenge, by arguing that

'u' := 'u is false or u is vague'

collapses to u is vague. So, u is not definitely true, and not definitely vague. Further, u is not derivable within McGee's supervaluationist theory, nor within a separate, fixed-point theory that he also advances.

Scharp raises several issues for the supervaluational approach. One issue is that vague sentences cannot be precisified via supervaluation – i.e., rendered determinately true – on pain of Revenge (156).

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[2]Cf. Williamson (forthcoming), for an argument for the retention of classical logic despite the semantic paradoxes, based on the abductive strength of its generalization.

Scharp argues that Descending Truth and Ascending Truth obey (T-Out) and (T-In), respectively, whereas – according to McGee – vague sentences do not. So, McGee's replacement restricts expressivity, whereas Scharp argues that there are no expressive restrictions on his proposal. Scharp notes, as well, that some of the axioms for McGee's theory are not definitely true – and are thus vague and not governed by T-Out or T-In – which would appear to be a considerable objection.

However, Field's (2008) approach – K3 plus a Determinacy operator, with a multi-valued semantics for the conditional – would appear to remain a viable proposal. Extensions of Field's proposal can be to an explanation of vagueness (Field, op. cit: ch. 5); to the logic of doxastic states (cf. Caie, 2012); and to the model-theory of metaphysics.

With regard, e.g., to the extension of Field's treatment of the paradoxes to the logic of doxastic states, Caie demonstrates that – rather than rejecting the Liar sentence – it would no longer be the case, by K3 and indeterminacy, that one could believe the Liar, and it would no longer be the case that one ought not to believe the Liar. In section **8**, I provide an epistemicist approach to Curry's paradox and the Liar which is able to retain both classical logic and the normal truth rules.

With regard to the extension of Field's treatment of the paradoxes to the logic and model-theory of metaphysics, consider the following. Given Curry's paradox, the validity of an epistemic norm might depend, for its explanation, on one's choice of logic. However, one's choice of logic might depend for its explanation on considerations from metaphysics. Suppose, e.g., that one distinguishes between fundamental and derivative metaphysical states of affairs. The fundamental states of affairs might concern the entities located in $3n$-dimensional

spacetime, such as whatever is represented by the wavefunction. The derivative states of affairs might concern emergent entities located in lower, 3-dimensional spacetime, such as the clouds. In order to capture the priority of the fundamental to the derivative, fundamental states of affairs could take the classical values, [0,1]; by contrast, derivative states of affairs could take the value .5 (in K3+indeterminacy), such that – while fundamental states are always either true or false – it is not determinate that a derivative state of affairs obtains, and it is not determinate that a derivative state of affairs does not obtain.

On the supervaluational treatment of the paradoxes, the approach can more generally be extended in order to account, e.g., for the metaphysical issues surrounding fission cases and indeterminate survival. Approaches which avail of a supervaluational response to fission scenarios, and similar issues at the intersection of nonclassical logic, metaphysical indeterminacy, and decision theory, can be found, e.g., in Williams (2014).

Thus, while it is not a necessary condition on the success of treatments to the alethic paradoxes that their proposals can generalize – in order, e.g, to aid in the resolution of other philosophical issues such as epistemic and metaphysical indeterminacy – there are viable proposals which can be so extended. The competing approaches thus satisfy a theoretical virtue that might ultimately elude ADT.

In his discussion of Priest's (2006) inclosure schema, Scharp disavows of a unified solution to the gamut of paradoxical phenomena (Scharp, 2013: 288). Despite the foregoing, I believe that there are at least four positive extensions of Scharp's theory of Ascending and Descending Truth that he does not discuss, and yet that might merit examination.

5.1 First Extension: The Preface Paradox

The first extension of the theory of ADT might be to the preface paradox. A set of credence functions is *Easwaran-Fitelson-coherent* if and only if (i) the credences are governed by the Kolmogorov axioms; and it is not the case both (ii) that one's credence is dominated by a distinct credence, s.t. the distinct credence is closer to the ideal, vindicated world, while (iii) one's credence is assigned the same value as the remaining credences, s.t. they are tied for closeness (cf. Easwaran and Fitelson, forthcoming).[3] Rather than eschew of consistency in favor of a weaker epistemic norm such as Easwaran-Fitelson coherence, the ADT theorist might argue that consistency can be preserved, because the preface sentence, 'All of the beliefs in my belief set are true, and one of them is false' might be Ascending True rather than Descending True. Because the models in Scharp's replacement theory can preserve consistency in response to the Preface, ADT might, then, provide a compelling alternative to the Easwaran-Fitelson proposal.

5.2 Second Extension: Absolute Generality

A second extension of Scharp's ADT theory might be to a central issue in the philosophy of mathematics; namely unrestricted quantification. A response to the latter might further enable the development of the property versions of AT and DT: i.e., being Ascending-True-of and being Descending-True-of. For example, Fine (2005) and Linnebo (2006) advance a distinction between sets and interpretations, where the latter are properties; and suggest that inconsistency might

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[3]A credence function is here assumed to be a real variable, interpreted as a subjective probability density. The real variable is a function to the [0,1] interval, and is further governed by the Kolmogorov axioms: normality, 'Cr(T) = 1'; non-negativity, 'Cr(ϕ) $\geq$ 0'; finite additivity, 'for disjoint ϕ and ψ, Cr($\phi \cup \psi$) = Cr(ϕ) + Cr(ψ)'; and conditionalization, 'Cr($\phi|\psi$) = Cr($\phi \cap \psi$) / Cr(ψ)'.

be avoided via a suitable restriction of the property comprehension scheme.[4] A proponent of Scharp's ADT theory might be able: (i) to adopt the distinction between extensional and intensional groups (sets and properties, respectively); yet (ii) circumvent restriction of the property comprehension scheme, if they argue that R is Ascending True-of yet not Descending True-of. The foregoing maneuver would parallel Scharp's treatment of the derivation, within ADT, of the Ascending and Descending Liars and their revenge analogues (see Section **2.1** above).

5.3 Third Extension: Probabilistic Self-reference

A third extension of ADT might be to a self-referential paradox in the probabilistic setting. Caie (2013) outlines a puzzle, according to which:

(1) '*' := $\neg CrT(*) \geq .5$

that is, (*) says of itself that it is not the case that an agent has credence in the truth of (*) greater than or equal to .5. As an instance of the T-scheme, (1) yields: 'T(*) $\iff \neg CrT(*) \geq .5$'. However, $CrT(*)$ ought to map to the interval between .5 and 1. Then, '$Cr(\phi) + Cr(\neg\phi) \neq 1$', violating the normality condition which states that one's credences ought to sum to 1.

In ADT, the probabilist self-referential paradox might be blocked as follows. Axiom (A2) states that $\neg A(\phi) \rightarrow A(\neg\phi)$; so if it is not an Ascending Truth that ϕ, then it is an Ascending Truth that not ϕ. However, (A2) does not hold for Descending Truth. Thus, in the instance of the T-scheme which states that 'T(*) $\iff \neg CrT(*) \geq .5$', the contraposition from '$\neg CrT(*) \geq$ .5' to '$Cr(\neg T(*) \geq .5$' is Ascending True, but not Descending True. So, if the

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[4]See Field (2004; 2008) for a derivation of the Russell property, R, given the 'naive comprehension scheme: $\forall u_1 \ldots u_n \exists y[\text{Property}(y) \wedge \forall x(x \text{ instantiates } y \iff \Theta(x, u_1 \ldots u_n)]$' (2008: 294). R denotes 'does not instantiate itself', i.e. $\forall x[x \in R \iff \neg(x \in x)]$, s.t. $R \in R \iff \neg(R \in R)$ (2004: 78).

contraposition from '¬CrT(*)' to 'Cr(¬T(*))' is not Descending True, then the transition from 'Cr(ϕ) + ¬Cr(ϕ) = 1' to 'Cr(ϕ) + Cr(¬ϕ) = 1' is not Descending True. Similarly, then, to the status of the Descending Liar in ADT, the derivation of contradiction from the probabilist self-referential sentence, (1), is Ascending True, but not Descending True.

5.4   Fourth Extension: The Sorites Paradox

A fourth extension of ADT might, finally, be to the sorites paradox. Scharp's xeno semanics is non-normal, such that the accessibility relation is governed by the axioms T (reflexivity) and 4 (transitivity), although not by axiom K. Suppose that there is a bounded, phenomenal continuum from orange to red, beginning with a color hue, c_i, and such that – by transitivity – if c_i is orange, then c_{i+1} is orange. The terminal color hue, in the continuum, would thereby be orange and not red. The transitivity of xeno semantics explains the generation of the sorites paradox. However, xeno semantics appears to be perfectly designed in order to block the paradox, as well: The neighborhood function in Scharp's xeno semantics for ADT is such that one can construct a model according to which transitivity does not hold. Let M_k be a neighborhood model, s.t. W_k = {a,b,c}; N_k(a) = {a,b}; N_k(b) ={a,b,c}; N_k(c) = {b,c}; V_k(ϕ) = {a,b}. Thus, $\langle M_k$,a$\rangle \Vdash \Box\phi$; but not $\langle M_k$,b$\rangle \Vdash \Box\phi$. So, it is not the case that $\langle M_k$,a$\rangle \Vdash \Box\Box\phi$; so transitivity does not hold in the model. Scharp's semantics for his ADT theory would thus appear to have the resources both to generate, and to solve, the sorites paradox.[5]

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[5]Scharp suggests that the truth predicate is contextually invariant, although assessment-sensitive (9.4). A second means by which the proposal could be extended in order to account for vagueness is via its convergence with the interest-relative approach advanced by Fara (2000; 2008).

6 Issue 4: Descending Truth, Ascending Truth, and Objectivity

Scharp claims that considerations of space do not permit him to elaborate on the interaction between Descending Truth, Ascending Truth, and objectivity (Section 8.3). Suppose that – depending on the target domain of inquiry – the truth-conditions of sentences might be sensitive to the reality of the objects and properties that the sentences concern. So, e.g., second-order implicit definitions for the cardinals might be true only if the terms therein refer to abstract entities. By contrast, what is said in sentences about humor might be true, if and only if the sentence satisfies deflationary conditions such as the T-schema.

Another objection to the replacement strategy, and of Scharp's candidate replacements in particular, is that it is unclear how – in principle – either Descending Truth or Ascending Truth can be deployed in order to capture the foregoing distinctions.

7 Issue 5: Paradox, Sense, and Signification

One final objection concerns the general methodology of the book. Scharp proceeds by endeavoring to summarize all of the extant approaches to the alethic paradoxes in the literature, and to marshall at least one issue adducing against their favor. However, there are two approaches to the paradoxes that Scharp overlooks. The first approach targets the notion of what is said by an utterance, i.e. the properties of sense and signification that a sentence might express. One such proposal is inspired by Bradwardine (c.1320/2010) and pursued by Read (2009). According to the proposal, if a sentence such as the Liar does not wholly signify that it is true, then one invalidates T-Introduction for the sentence. In

a similar vein, Rumfitt (2014) argues that paradoxical sentences are a type of Scheingedanken, i.e. mock thoughts that might have a sense, although take no value; so, T-Introduction is similarly restricted.

Scharp takes it to be a virtue of his account that he can retain the disquotational principles, even though they get subsequently divided among his replacement concepts. He might then reply to the foregoing proposal by suggesting that they similarly induce expressive restrictions in a manner that his approach can circumvent.

However, there are other approaches which avail of what I shall refer to as the sense and signification strategy, and which eschew of neither T-Elimination nor T-Introduction. Modulo a semantics for the conditional, K3 and indeterminacy at all orders ensures not only that hyper-determinacy – and therefore an assignment of classical values to the paradoxical sentences – is circumvented; but, furthermore, that revenge sentences cannot be derived either. Against this approach, Scharp reiterates his concern with regard to restrictions on expression. He writes, e.g., that 'Field avoids revenge only by an expressive limitation on his language' (107). However, a virtue of the approach is that, as in xeno semantics for ADT, T-Elimination and T-Introduction are preserved. Against ADT theory, K3+indeterminacy does not arbitrarily select the alethic principles that the semantic theory should satisfy. Crucially, moreover, the approach does not say more than one should like it to, as witnessed, e.g., by the derivability in ADT of both the DT and AT Liars and their revenge analogues. Rather, the language of paracompleteness and indeterminacy demonstrates that – without the loss of the foundational principles governing the alethic predicate – there are propositions which can satisfy the values in an abductively robust semantic theory, and thereby exhibit their content, although what they show cannot

ultimately be said.

8 Epistemicism and Alethic Paradox

Finally, the second approach that Scharp does not consider is one that has recently been developed by the present author. This approach provides an *epistemicist* solution to the alethic paradoxes, and is able to retain both classical logic and a univocal, non-replacement, alethic predicate which obeys both truth-elimination and truth-introduction. In the epistemic modal system at issue, the box-operator, $\Box$, is interepreted as 'the agent knows that' and the diamond-operator, $\diamond$, is interepreted as 'the agent believes that'. Belief is the dual of knowledge: 'B$\phi \iff \neg K \neg \phi$'. The epistemic system validates axiom T, which records the factivity of knowledge: 'K$\phi \to \phi$'. Curry's paradox is, again, the following:[6]

For any false sentence, ϕ,

1. $\phi \iff [T(\phi) \to \bot]$

2. $T(\phi) \iff [T(\phi) \to \bot]$

3. $T(\phi) \to [T(\phi) \to \bot]$

4. $[T(\phi) \wedge T(\phi)] \to \bot$ (by importation)

5. $T(\phi) \to \bot$ (by contraction)

6. $[T(\phi) \to \bot] \to T(\phi)$

7. $T(\phi)$

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[6]Read (2010) notes that there are at least three forms of Curry's paradox. One form, attributed to Albert of Saxony, is conjunctive: for any sentence A, 'ϕ' := A $\wedge$ F(ϕ). A second form, attributed both to Albert of Saxony and Bradwardine, is disjunctive: for any sentence A, 'ϕ' := A $\vee$ F(ϕ). A third form, owing to Löb (1955) is such that, for any sentence A: 'ϕ' := T(ϕ) $\to$ A. Read (op. cit.) argues that T-introduction is the culprit in the first three forms of the paradox. A fourth variation on Curry's paradox can be found in Beall and Murzi (2012), who replace the truth predicate with a validity predicate. A fifth form is targeted by Field (2008), and is the version at issue in this talk. This form of the paradox is such that a contradiction has a unique derivation, via the truth rules, given any *false* sentence.

8. $\perp$

Rather than weaken the logic in a type-free setting (Field, 2008); eschew of contraction (Beall and Murzi, 2012); eschew of T-introduction in virtue of defective properties of signification (Read, 2009); or retain classical logic for abductive reasons and disband of one of the truth rules (Williamson, forthcoming), the current approach argues that Curry's paradox is classicaly sound; that the normal truth rules can yet be retained; and that the paradox is problematic only becuase it exhibits an instance of epistemic indeterminacy.

The epistemic indeterminacy entrained by Curry's paradox occurs because step 5 invalidates axiom K on its epistemic interpretation: 'K($\phi \to \psi$) → (Kϕ → Kψ)'. This provides a counter-instance to epistemic closure. To see this, let ϕ denote 'T(ϕ)' and ψ denote '$\perp$'. Then:

(*) K(T$\phi \to \perp$) → [K(Tϕ) → K($\perp$)].

Our epistemic modal system validates reflexivity, or axiom T: 'K$\phi \to \phi$'.[7] Thus, in (*), K($\perp$) is false, because – by reflexivity – only truths can be known. However, K(ϕ) can be known, because it is an instance of T-introduction. Thus, the conditional in the consequent of (*) has itself a true antecedent and false consequent, and is thus false. Finally, the conditional in the antecedent of (*), 'K(T$\phi \to \perp$)' is true, in virtue of the proof of Curry's paradox. So the instance of K expressed in (*) has a true antecedent and false consequent, providing a counter-instance to K; and so K is not a valid axiom in our system of epistemic modal logic. Thus, what is problematic about Curry's paradox is that – despite

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[7]Whether the system validates further axioms is a question which requires separate treatment. Axioms 4 [K$\phi \to$ KKϕ]'; G [BK$\phi \to$ KBϕ]; 4.4 [K($\phi \wedge$ BKψ) → K($\phi \vee \psi$)]; GL [K[K(ϕ) → ϕ] → K(ϕ)]; and Grz [K[K($\phi \to$ Kϕ) → ϕ] → ϕ], have all been proposed as plausible axioms of epistemic logic. If one follows Priest (op. cit.) in arguing that the sorites paradox and the paradoxes of self-reference have the same form, and one believes that transitivity, i.e. axiom 4 on its epistemic interpretation, is the culprit in the sorites paradox, then perhaps eschewing of axiom 4 with regard to the paradoxes of self-reference might be a viable, similarly epistemicist approach. Cf. Williamson (1994; 2002), for the foundations of the latter proposal.

being classicaly sound and entailing contradiction via the normal truth rules – at least one of the steps in its derivation is epistemically indeterminate, by invalidating axiom K.

To complete the epistemicist approach to alethic paradox, we target the epistemic attitude that one ought to bear toward the liar sentence. As mentioned, Caie (2012) generalizes the strong Kleene valuation scheme, as augmented by the determinacy operator, to the doxastic setting. In order to induce consistency in an agent's belief set, the belief operator is taken to be its own dual, 'B(ϕ) $\iff$ ¬B¬ϕ'. The operator is further governed by K [B($\phi \to \psi$) $\to$ [B(ϕ) $\to$ B(ψ)]], D '[K(ϕ) $\to$ B(ϕ)], 4 [B($\phi \to$ BBϕ), and E [¬B(ϕ) $\to$ B¬B(ϕ)]. By contrast to Caie's approach, we target epistemic rather than doxastic logic. In our epistemic logic, we avoid validating the conjunction of axioms 4 and E, because their conjunction would entail that an epistemic agent's consistent theorizing is decidable, in conflict with Gödel's second incompleteness theorem (cf. Williamson, 2009). Because we should like epistemic agent's theorizing to be consistent, we avoid validating both 4 and E. The epistemic logic validates axiom T. However, Curry's paradox provides a case of epistemic indeterminacy, by invalidating axiom K. To complete the epistemicist solution to the liar paradox, I believe that the epistemic attitude toward liar sentences is one of belief, where belief is, again, the dual of knowledge: Bϕ if and only if ¬K¬ϕ. Thus, where λ is a sentence which asserts of itself that it is false, the descriptively adequate and normatively permissible attitude to bear toward the sentence is one of belief, such that one doesn't know whether it is not true. To avoid contradiction, we eschew finally of the D axiom, which records the seriality and safety conditions, according to which one ought to believe ϕ only if ϕ is known: K$\phi \to$ Bϕ. Conjoining seriality to 'K$\phi \iff$ ¬B¬ϕ', such that 'K$\phi \to$ Bϕ' and

'K$\phi \to \neg$B$\neg\phi$', would entail a consistency condition on belief that only ideal agents could satisfy. Thus, the derivation of contradiction in Curry's paradox is epistemically indeterminate, and the attitude of believing the liar sentence comprises a case of epistemic indeterminacy, as well: for all one knows it is not the case that the liar sentence is true.[8]

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