Studies in Logic 67

Many-Valued Logics

A Mathematical and Computational Introduction

Luis M. Augusto

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Preface

Although the title of this book indicates two main components, to wit, mathematical and computational, we wish to emphasize that this is first and foremost a book on many-valued logics. As such, the reader who might just be interested in the various many-valued logics will find abundant material in that which is the central part of this book, to wit, Part II. In this, the several main many-valued logics are presented from the viewpoints of their semantical properties, and their semantics are characterized mostly by means of truth tables and logical matrices. From the proof-theoretical viewpoint, for many of these logics partial or full axiomatizations are provided.

This Part II is, mainly for the objective of self-containment, preceded by an extensive introduction to "things" logical, i.e. logical languages, systems, and decisions. In this Part I, important notions such as logical consequence, adequateness of a logical system, and matrix semantics, among others, are rigorously defined. Further notions regarding decision procedures in classical logic are introduced, namely the satisfiability and validity problems (abbreviated as SAT and VAL, respectively), and they are put into relation with the several proof systems available. In Part III, we chose to approach the many-valued satisfiability problem (MV-SAT) via refutation, i.e. by proving unsatisfiability of a set of formulae (a theory), because of the many advantages, computational and other, that this proof procedure presents for the many-valued logics. This explains our extensive discussion of the analytic tableaux and resolution calculi for classical logic in this Part I. However, we wanted the reader to acquire a good grasp also of validity- and satisfiability-testing methods, and we thus provide sufficient material on this topic.

Part III deals with well-studied computational, or potentially computerizable, approaches to the many-valued logics, namely as far as automated deduction is concerned. Here, signed (clause) logic has a central place (Chapter 6). In effect, both signed tableaux and signed resolution are automatizable proof calculi that can very naturally be applied to many-valued logics, reason why we chose to elaborate on them at length. This elaboration constitutes Chapters 7 and 8, for signed tableaux and signed resolution, respectively.

We did not wish to emphasize the mathematical component, as this could scare away many a potential reader. Nevertheless, we believe that logic is a branch of mathematics, and as such it is both mathematically motivated and justified. (Incidentally, we do not think that logic is a foundation for mathematics–the reverse is true, in our opinion.) Given these two somehow conflicting views, we decided that the best thing to do would be to provide the mathematically literate, or just simply interested, readers with an Appendix in which the mathematical bases of our approach are expounded to the required extent. This is Part IV.

Logic is a subject that requires (many) years before one grasps satisfactorily what it is actually all about. This stage will arguably not be reachable without hands-on practice, much in the same way, perhaps, that many other fields require extensive, often arduous, practice. In this belief, we provide the readers with a large selection of exercises.[1]

In effect, this book is conceived for the reader who wishes to *do* something with many-valued logics, especially–but by no means only–in a computational context. With this we associate fields as diverse as cognitive modeling and switching theory. The literature on many-valued logics is abundant, but the "standard" monographs (Bolc & Borowik, 1992; Malinowski, 1993; Rescher, 1969; Rosser & Torquette, 1952), while not being obsolete, are now of mainly theoretical and historical interest. Some more recent work claiming to have practical applications in mind (e.g., Bolc & Borowik, 2003) follows to a great extent in the footsteps of these earlier monographs. On the other hand, recent work both in mathematical logic (e.g., Gottwald, 2001; Hähnle, 2001) and in philosophical logic / the philosophy of logic (e.g., Bergman, 2008) provides little or no material for the practical applications mentioned.

Surprisingly–or just plainly intriguingly–, the last comprehensive discussion of modern applications of many-valued logics dates from 1977 (Dunn & Epstein). The fact is that many-valued logics are today in more demand than ever before, due to the realization that inconsistency in information (and knowledge bases are frequently inconsistent) "is respectable and is most welcome" (Gabbay, 2014). We now know and accept that it is often the case that theories have truth-value gaps (some propositions appear to be neither true nor false), truth-value gluts (some propositions appear to be both true and false), vague concepts (e.g., cold, young, tall, sufficient), indefinite functions, lacunae, etc. Moreover, theories are in constant updating processes, and human and artificial reasoners require formal means to keep pace with these often fast and/or unpredictable processes, namely in order to review beliefs: what

[1] For reasons of timing, solutions to the exercises are not provided in this edition. In due time, there will be an Appendix with these solutions, which may actually be (also) posted online.

yesterday was a true/false proposition may today be neither true nor false, or both; what yesterday was clear-cut is now vague or indeterminate. The many-valued logics provide us with a powerful formalism to tackle these cases in terms of reasoning. In fact, this scenario calls for not only a practical-based approach to the many-valued logics per se, but also for hybridizations of these with other formal systems, as well as with more quantitative(-like) cognitive approaches (e.g., D'Avila Garcez, Lamb, & Gabbay, 2009). Importantly, the practical applications of many-valued logics have now gone beyond the mere industrial applications (e.g., fuzzy logics in washing machines), and promise to be of import for "higher," cognitive modeling. This *new logic* entails, in particular, a reappraisal of both "psychologism" and "formalism"–if not also of "logicism"–, especially because it proposes a reappraisal of what a cognitive, or reasoning, agent is (see, e.g. Gabbay & Woods, 2001). In order for this emerging work (Gabbay & Woods, 2003; 2005) to be further carried out and to incorporate the many-valued logics, we need a solid mathematical-computational grasp of these logics.

We hope the present book will provide the motivation for a new comprehensive treatment of the modern applications of many-valued logics that will include the *new-logic* factor as a central aspect of modern logic and its applications. Thus, a book like this on many-valued logics is now very timely, for *the times, they are a'changing*.

I wish to thank Dov M. Gabbay for including this book in the excellent Studies in Logic series of College Publications. My thanks go also to Jane Spurr for a smooth publication process.

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